Wide-band nonlocal impedance boundary condition model for high-conductivity regions in integral equation framework

Sándor BILICZ and Zsolt BADICS and József PÁVÓ

Budapest University of Technology and Economics, Faculty of Electrical Engineering and Informatics, Department of Broadband Infocommunications and Electromagnetic Theory,
Budapest, Hungary

Abstract

A surface integral equation method for modeling inductors in a wide frequency range is presented. The unknowns are surface current and surface charge densities on the conductor surfaces. In the dielectric medium, a full-wave formulation is used, whereas in the conductor, a magneto-quasistatic model is considered. The full-wave formulation ensures that the model correctly takes into account high-frequency effects, such as resonance or radiation. The magneto-quasistatic model within the conductor yields accurate results for the real part of the impedance, i.e., the ohmic loss, starting from very small frequencies, where the 0th order surface impedance boundary condition is no longer valid due to the large skin-depth.

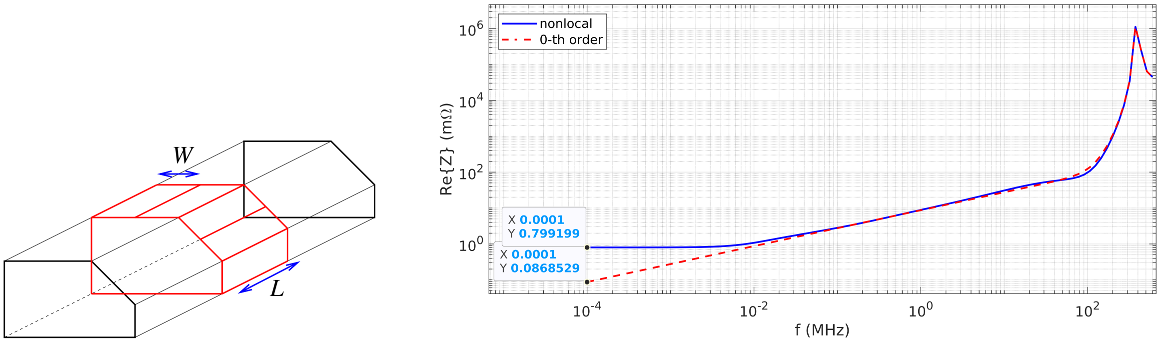
1 Introduction

The accurate modeling of ohmic losses of inductors is of key importance in many applications, e.g., various on-chip inductors, or wireless power transmission. This requires the proper modeling of the current distribution within the conductor by means of a magneto-quasistatic (MQS) model. However, in the dielectric medium, electro-magneto quasistatic (EMQS) or full-wave formulation is needed to model resonances or radiation phenomena, respectively. The models are usually coupled by a surface impedance boundary condition (SIBC) at the conductor-insulator boundary. In [1] a full-wave integral equation formulation is presented, where the current density within the conductor are approximated in a semi-analytical form that relies on the special rectangular shape of the cross section. In [2] also rectangular cross section is considered, a spiral-shaped inductor on a substrate is modeled, yet this model is valid for low frequencies only.

In [3], a nonlocal generalization of the SIBC by incorporating a two-dimensional (2D) MQS model of the eddy-currents within the cylindrical wire has been presented. In the present paper, this method is further extended to model wires with general cross section, which includes but not limited to the rectangular cross section, that is typical in spiral inductors.

2 Methodology

The proposed formulation is briefly summarized as follows. The magnetic vector potential $\underline{A}$ and electric scalar potential Φ are introduced in the dielectric medium. A full-wave integral equation is written for the potentials and the surface current and surface charge densities on the wire surfaces. The wires that can have an arbitrary cross section, and their length is considerably larger than their transversal extension. Their surface is discretized by rectangular elements along the length and the circumference. The SIBC, i.e., the ratio of the tangential electric and tangential magnetic field within one surface element is not a local relation, but there is interaction between the surface elements within the *same* length segment (in Fig. 1a they are



(a) Discretization of the wire surface for an arbitrary cross section

(b) Real part of the impedance of the loop in the example

Figure 1: Illustration of the method: discretization and results of the example

marked by red color). This interaction consists in a 2D MQS boundary value problem within the conductor's cross section, that is actually solved by the finite element method. Note that in the case of certain regular shapes, analytical or semi-analytical solutions also exist for this 2D MQS problem. The approach is presented in detail for circular cross section in [3], hence we omit the equations here. The original contribution of this paper over [3] consists in the extension of the method for an arbitrary cross section.

3 Example

Let us consider a circular loop made of copper. The radius of the wire is 1.5 mm and the radius of the loop is 50 mm. The dc resistance of the loop is 0.78 mΩ. The impedance of the loop has been calculated by means of the surface integral equation formulation, using 100×12 surface elements along the length and the circumference of the wire, respectively. Both 0th order SIBC and the proposed nonlocal SIBC are tested. The results are presented in Fig. 1b. One can observe that both SIBCs correctly capture the resonant behavior of the impedance around 400 MHz, however, for small frequencies, the nonlocal SIBC results in a solution tending to the analytically calculated dc resistance, whereas the 0th order SIBC yields non-realistic impedance values. In the full version of the paper, examples with more general cross sections and inductor shapes will be presented, and the resource demand of the method will also be analyzed.

Acknowledgements

This research is supported by the Hungarian Scientific Research Fund under Grant K-135307.

References

[1] Mohammed Al-Qedra et al. "A Novel Skin-Effect Based Surface Impedance Formulation for Broadband Modeling of 3-D Interconnects With Electric Field Integral Equation". In: *IEEE Transactions on Microwave Theory and Techniques* 58.12 (2010), pp. 3872–3881.

[2] Sathya Sree Jayaraman et al. "A Scalable, Broadband, and Physics-Based Model for On-Chip Rectangular Spiral Inductors". In: *IEEE Transactions on Magnetics* 55.9 (2019), pp. 1–6.

[3] Sándor Bilicz et al. "A Full-Wave Integral Equation Method Including Accurate Wide-Frequency-Band Wire Models for WPT Coils". In: *IEEE Transactions on Magnetics* 54.3 (2018), pp. 1–4.