Application of Modified ICCGH to Linear System Appearing in Shielding Current Analysis of Uncracked HTS Film: Improvement of $\mathcal{H}$-matrix-based preconditioning

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Abstract

The $\mathcal{H}$-matrix-based preconditioning has been improved, and the performance of the ICCGH incorporated with the proposed preconditioning has been investigated numerically. The result of computations shows that the modified ICCGH is faster than the conventional one.

1 Introduction

For analyzing the shielding current of the high-temperature superconductor (HTS), the time evolution of the shielding current density must be solved numerically [1, 2]. After discretizing with respect to space and time, the above problem can be transformed into a linear system. Since the linear system has a symmetric dense coefficient matrix, a direct method has been so far adopted as a solver.

Recently, the $\mathcal{H}$-matrix-based acceleration technique has been proposed [3]. This technique consists of the $\mathcal{H}$-matrix arithmetic and the $\mathcal{H}$-matrix-based preconditioning. By applying the acceleration technique to an iterative method, the solver speed in the shielding current analysis can be increased. However, the $\mathcal{H}$-matrix-based preconditioning has some disadvantages. First, the appropriate number of skips cannot be determined in advance. Second, the skip interval is always constant. Therefore, it is difficult to determine the appropriate preconditioner previously.

The purpose of the present study is to improve the $\mathcal{H}$-matrix-based preconditioning for overcoming the above difficulties and to investigate the performance of the solver incorporated with the proposed preconditioning numerically.

2 $\mathcal{H}$-Matrix-Based Preconditioning

In this study, we apply the shielding current analysis in an uncracked HTS film to the scanning permanent magnet method [3]. Under the thin-plate approximation, the time evolution of a scalar function T is governed by the following equation:

$$\mu_0 \partial_t \, (\hat{W} T) + (\nabla \times \boldsymbol{E}) \cdot \boldsymbol{e}_z = -\partial_t \langle \boldsymbol{B} \cdot \boldsymbol{e}_z \rangle, \qquad (1)$$

where $\boldsymbol{E}$, $\boldsymbol{B}$ and $\boldsymbol{e}_z$ are the electric field, the applied magnetic flux density and a unit vector along z-axis, respectively. Also, μ_0 and b the permeability of vacuum and the film thickness, respectively. Furthermore, $\langle \ \rangle$ is an average operator over the thickness of the HTS film. In addition, $\hat{W}$ is the operator corresponding to the magnetic flux density that is generated by shielding current density.

By using not only the discretization with respect to time and space but also the linearization by means of the Newton method, the initial-boundary value problem of (1) can finally be transformed into a linear

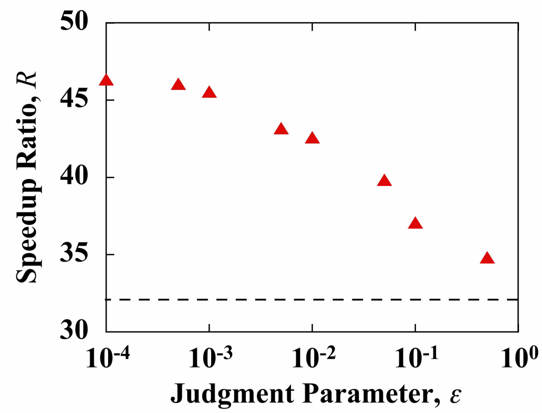

Figure 1: Dependence of speedup ratio R on the judgement parameter ε. In addition, the dashed line denotes the speedup ratio for the conventional ICCGH.

system. In order to solve the resulting linear system, the CG method with the $\mathcal{H}$-matrix-based acceleration technique (ICCGH) is adopted.

As mentioned in Sec. 1, in conventional the $\mathcal{H}$-matrix based preconditioning, the appropriate preconditioner cannot be determined at each time step. In the proposed preconditioning, the preconditioner is generated only when $\|J(\boldsymbol{T}_{\rm new}) - J(\boldsymbol{T}_{\rm old})\|_{\rm F}/\|J(\boldsymbol{T}_{\rm new})\|_{\rm F} \geq \varepsilon$ is satisfied. Here, $T_{\rm new}$ and $T_{\rm old}$ are nodal vectors for T obtained for the current and one previous cycles of Newton method, respectively. In addition, ε and $\| \; \|_{\rm F}$ indicate a judgment parameter and the Frobenius norm, respectively. Throughout the present study, the ICCGH incorporated with the proposed preconditioning is called the modified ICCGH.

3 Numerical Result

In this section, we evaluate the speed performance of the modified ICCGH. As parameters for the solver, the judgment of convergence $\epsilon_{\rm CG}$ and the initial guess $\delta\boldsymbol{T}_0$ are given by $\epsilon_{\rm CG} = 10^{-9}$ and $\delta\boldsymbol{T}_0 = \boldsymbol{0}$, respectively. Moreover, parameters for $\mathcal{H}$-matrix-based acceleration technique are fixed as follows: $k = 2$ and $\varepsilon_{\rm H} = 0.5$. In addition, the number N of unknowns is employed as $N = 11235$. The values of physical and geometric parameters The values of physical and geometric parameters refer the reader to [2].

As the measure of the speed of the linear-system solver, the speedup ratio $R = \tau/\tau_{\rm p}$ is employed. R is calculated as a function of the judgment parameter ε and is plotted in Fig. 1. Here, $\tau_{\rm p}$ and τ denote CPU times for the modified ICCGH and the CG method without any acceleration technique, respectively. We see from this figure that the speed-up rate of modified ICCGH increases with a decrease of ε. In addition, the modified ICCGH is up to about 1.44 times faster than the conventional one.

References

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