

# Physics-Informed Neural Networks with Transfer Learning for Beam Impedance Simulations

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## Abstract

Physics-informed neural networks (PINNs), which is a powerful approach for solving partial differential equations with deep learning, has been recently applied to modeling of electrodynamic interaction problems including a relativistic beam in charged particle accelerators. In the present study, the transfer learning (TL) is applied to the PINNs based on the total-field (TF) formulation. It is shown that TL can accelerate significantly the training process of the TF-PINNs in the simulation of the beam impedance of an infinitely long beam pipe of circular cross section.

## 1 Introduction

Physics-informed neural networks (PINNs) [1],[2] is a powerful approach for solving partial differential equations (PDEs) via deep learning. The key idea is to embed the PDEs into the loss function of a deep neural network (DNN) using automatic differentiation.

Recently, this approach has been successfully applied to the modeling of electrodynamic interaction problems including a relativistic beam in charged particle accelerators [3]-[6]. The PINNs based on the total-field (TF) formulation is proposed in [3]-[5]. We shall call this approach as TF-PINNs. The concept of transfer learning (TL) [7],[8] was introduced into the PINNs based on the scattered field formulation [6], where the basic idea on introducing TL for the simulation of the beam impedance [9] is briefly described. However, TL is not yet discussed in the framework of TF-PINNs [3]-[5].

The purpose of the present study is to apply TL into TF-PINNs [3].

## 2 Application of Transfer Learning to TF-PINNs

It is well-known in [9] that beam impedances are obtained from electromagnetic fields excited by a relativistic charged particle beam moving in an accelerator component in the frequency domain. Here the TL [7],[8] is used to accelerate the training process of TF-PINNs [3] in the frequency domain. In many cases, a frequency range of interest is specified. The DNNs are trained as approximate solutions at the frequency sampling points. In the training process, DNN parameters trained at a low frequency point can be used to initialize a DNN at the adjacent higher frequency point. If the frequency step is enough small, the TF solution obtained at one frequency point can be similar to that of the adjacent higher frequency point. Then, it is expected to reduce the number of iterations for the optimization of the corresponding DNN.

## 3 Numerical Result

To show the effect of TL on the TF-PINNs [3], we calculate the beam impedance of a round Gaussian charge density with the total charge  $q=1\text{pC}$ , the Gaussian parameter  $\sigma_r=1\text{mm}$  in the radial direction and the relativistic factor  $\gamma=100$  in infinitely long circular beam pipe with radius  $R=1\text{cm}$  and perfectly electric conductor wall.

Throughout this study, we adapt a fully connected neural network and the Swish activation function for the DNN architecture. We use three hidden layers and 30 neurons per layer. At each sampling point, the DNN parameters (weights and bias) are updated to minimize the loss function  $L$  by using the L-BFGS algorithm, until  $L$  get smaller than a threshold  $\epsilon = 10^{-6}$  or the number of iterations get larger than 30,000.

The left side of Fig.1 shows the number of iterations for training in TF-PINNs with and without the TL. From the second sampling frequency point, the number of iterations is decreased at each sampling point by using the TL. This result demonstrates that the training process of the TF-PINNs can be accelerated with the TL.

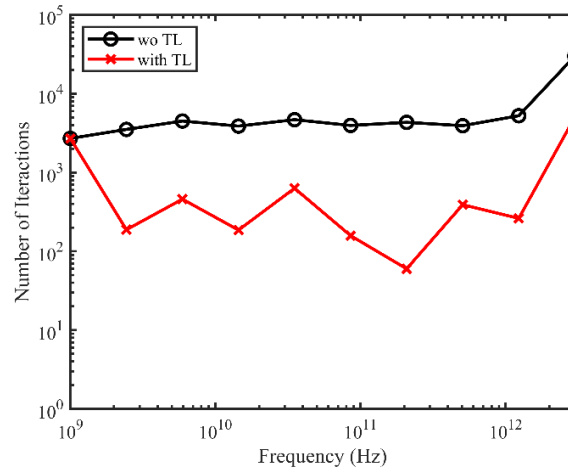


Figure 1: Effect of Transfer Learning on TF-PINNs in the beam impedance simulation

## 4 Conclusion

The TF-PINNs combined with the TL has been developed, and successfully applied to the beam impedance of a round Gaussian charge density in an infinitely long circular beam pipe with perfectly electric conductor wall. It has been found that the TL is useful for accelerating the training processes in TF-PINNs.

## References

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