Research on ridge regression with initial values used for strain measurement in DVC

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Abstract

A ridge regression with initial values (RR) is proposed to improve the measurement accuracy of digital volume correlation (DVC) for full-gradient strain field. It is a combination of the strengths of both the power window weighted digital volume correlation (PW-DVC) and widely used pointwise least square (PLS) methods. The calculation examples of synthetic images indicate that, for image with large strain gradient, the minimum strain error of PW-DVC+RR is 20.3% less than that of PW-DVC+PLS, but only 2 microstrains more than that of PW-DVC. For image with small strain gradient, the minimum strain error of PW-DVC+RR is 56.7% less than that of PW-DVC, but only 44.7 microstrains more than that of PW-DVC+PLS.

1 Introduction

Power window weighted digital image correlation (PW-DIC) [1] is an effective method for fields with large strain gradients. This method can also be extended to three dimensions, referred to as power window weighted digital volume correlation (PW-DVC). Conversely, widely used pointwise least square (PLS) [2] is better suited for fields with small strain gradients. In this study, a ridge regression and initial values (RR), combining the strengths of both PW-DVC and PLS techniques, is proposed. This method aims to significantly enhance the measurement accuracy of DVC when applied to full-gradient strain fields.

2 Ridge regression with initial values

PW-DVC is used to calculate the initial displacement and strain results of grid points. A shrinkage penalty is introduced into PLS method and this method evolutes into the so-called ridge regression to fit the undetermined displacement polynomial in the strain calculation box [2], described as below:

$$J\left(\theta_{1,2,3}\right)=\left\|X\theta_{1,2,3}-Y_{1,2,3}\right\|_2^2+\lambda\left\|\theta_{1,2,3}\right\|_2^2 \tag{1}$$

where, $J(\theta_i)$ is the sum of squared residuals, $\theta_1=[a_0, a_1, a_2, a_3]^T$, $\theta_2=[b_0, b_1, b_2, b_3]^T$ and $\theta_3=[c_0, c_1, c_2, c_3]^T$ are the coefficients of the displacement fitting polynomial, $\|\bullet\|_2^2$ is the square of 2-Norm, λ is the shrinkage penalty, $X=[1\ x_1\ y_1\ z_1;\ \cdots\cdots\ ;1\ x_n\ y_n\ z_n]$ is a $n\times4$ matrix, n is the number of grid points, x_j, y_j, z_j are the coordinates of the j^{th} grid point, $Y_1=[u_1, u_2,\cdots, u_n]^T$, $Y_2= [v_1, v_2,\cdots, v_n]^T$ and $Y_3= [w_1, w_2,\cdots, w_n]^T$ are corresponding to θ_1, θ_2, and θ_3, u_j, v_j and w_j are the displacement components of the j^{th} grid point, respectively.

Then the initial strain results are introduced into the ridge regression, and Eq.(1) is transformed to the below form:

$$J\left(\theta_{1,2,3}\right)=\left\|X\theta_{1,2,3}-Y_{1,2,3}\right\|_2^2+\lambda\left\|\theta_{1,2,3}-\theta'_{1,2,3}\right\|_2^2 \tag{2}$$

where $\theta'_1=[0, u_x, u_y, u_z]^T$, $\theta'_2=[0, v_x, v_y, v_z]^T$ and $\theta'_3=[0, w_x, w_y, w_z]^T$.

The solution of Eq.(2) takes the form:

$$\theta_{1,2,3}=\left(X^TX+\lambda I\right)^{-1}\left(X^TY_{1,2,3}+\lambda\theta'_{1,2,3}\right) \tag{3}$$

The Cauchy strains or Green strains can be calculated based on the obtained coefficients θ_1, θ_2 and θ_3. The value of λ is determined by a given selection strategy.

3 Simulations

The accuracy of the proposed method is verified through two image sets with imposed inhomogeneous deformation in this section. The strain fields in the x direction are shown in Fig.1. There is no deformation in the y and z direction. All algorithms are implemented in Matlab.

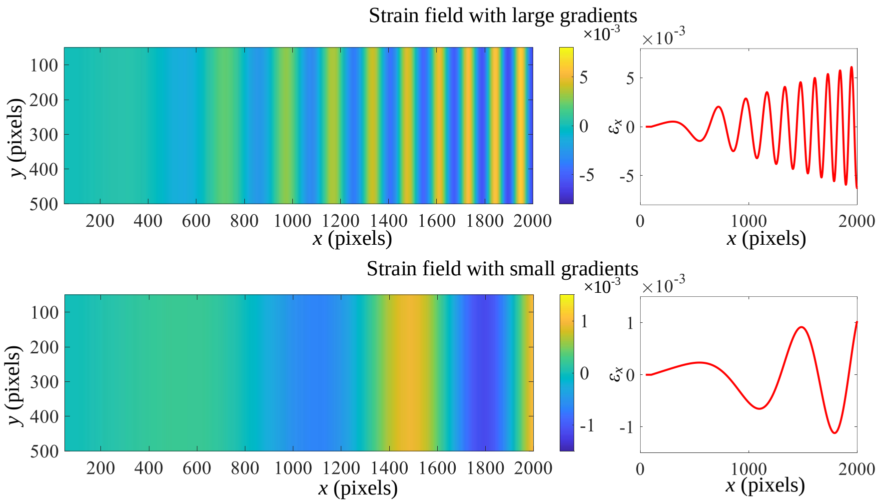

Fig.1: The strain fields in x direction of two image sets

The sizes of subvolume are 31×31×31, 37×37×37, 43×43×43, and 49×49×49 and the sizes of strain calculation box are 3×3×3, 7×7×7, 11×11×11, 15×15×15, and 19×19×19. The minimum strain errors of PW-DVC, PW-DVC+PLS and PW-DVC+RR are shown in Table 1.

Table 1: The minimum strain errors of three methods

	Methods		
	PW-DVC	PW-DVC+PLS	PW-DVC+RR
Image with large strain gradients	3.35×10^{-4}	4.23×10^{-4}	3.37×10^{-4}
Image with small strain gradients	2.21×10^{-4}	5.09×10^{-5}	9.56×10^{-5}

It is seen that, the error of PW-DVC is lower for image with large strain gradient, whereas the error of PW-DVC+PLS is lower for image with small strain gradient. Furthermore, the errors of PW-DVC+RR can nearly be maintained at its minimum for full-gradient strain field. For image with large strain gradient, the minimum strain error of PW-DVC+RR is 20.3% less than that of PW-DVC+PLS, but only 2 microstrains more than that of PW-DVC. For image with small strain gradient, the minimum strain error of PW-DVC+RR is 56.7% less than that of PW-DVC, but only 44.7 microstrains more than that of PW-DVC+PLS.

4 Conclusion

A ridge regression with initial values is established to combine the strengths of both PW-DVC and PLS. Simulations indicate that the accuracy of PW-DVC+RR is comparable to that of PW-DVC when dealing with fields with large strain gradients, and is comparable to that of PW-DVC+PLS when dealing with fields with small strain gradients. PW-DVC+RR can yield better accuracy for full-gradient strain field.

References

[1] X. Song, K. Xiong, Research on a new power window weighted digital image correlation for accurate measurement (Under review).

[2] B. Pan, D. Wu, Z. Wang, Internal displacement and strain measurement using digital volume correlation: A least-squares framework, Meas. Sci. Technol. 23 (2012).