Abstract

This paper studies the properties of interest limitation rules designed to limit the profit-shifting activity of multinational enterprises (MNEs). The focus is on how the two leading alternative earnings stripping rules (ESRs) affect profit-shifting and investment incentives, and which of the two is preferable from society’s point of view. By using a dynamic investment model, the paper reveals quite different investment incentives and neutrality properties between the EBIT and EBITDA rules. While the cost of capital remains independent of the useful lives of assets under the EBIT rule, this neutrality is lost under the EBITDA rule. The EBITDA rule is also more sensitive to the key interest limitation parameter, the fixed ratio, than the EBIT rule. However, the EBITDA rule still provides higher social welfare in most cases. The cases where the EBIT rule implies higher welfare are usually those with large differences between the useful lives of assets.

JEL codes: H25, H26, H32, G32

Keywords: Corporate income tax, Multinational enterprises (MNEs), Interest limitation rules, Capital structure, Profit-shifting, Anti-tax avoidance rules, Earnings stripping rules (ESRs), Interest barriers (IBs)

1. Introduction

Profit shifting activity of multinational enterprises (MNEs) and the anti-tax avoidance measures, targeted to reduce this activity, have been one of the most discussed topics in corporate taxation in recent years. Profit shifting is found to have severe adverse effects: it reduces the worldwide tax revenue, redistributes from high-tax countries to low-tax countries, distorts the competition between national enterprises and MNEs, and may affect tax morale. The literature is also well-aware of several profit shifting channels, with intra-group transfer pricing, lending\(^1\) and strategic location of patents being the most well-known. (Heckemeyer and Overesch 2013; Dharmapala 2014, 2016)

As a response to profit shifting many countries have implemented anti-tax avoidance measures (see Table 1). Even if their aim is to reduce profit shifting and protect tax bases, the consequences of these measures for the society remain unfortunately unknown in many cases. We focus in this paper on investment incentives and social welfare implications of measures targeted to reduce debt-related profit shifting (debt-shifting). While the focus of these measures is to tackle profit shifting, it should be done with little distortion on real economic activity.

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\(^1\) See Desai et al. (2004), Huizinga and Laeven (2008) and Dharmapala and Riedel (2013) for evidence of profit-shifting using internal debt as the means.

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The incentive for debt-shifting arises in the current international corporate tax system as most of the national tax systems allow the deduction of interest expenses in the country of the debtor and constitute taxable income in the country of the lender. This way, the MNE can effectively transfer profits from high-tax to low-tax countries by locating intra-group lending activities in a country with a particularly low tax rate. As a result, the overall tax burden of the MNE is reduced.

In order to address debt-shifting countries have introduced essentially two types of variants of partial limitations to interest deductions. While the so-called thin capitalization rules (TCRs) impose a cap on deductible expenses by disallowing deductions above a fixed debt-to-equity ratio, the so-called earnings stripping rules (ESRs) cap deductions if the net interest expenses exceed a fixed share of the firm’s gross earnings.

While earlier most countries limiting the interest deductions applied the first approach (TCR), the profit-based ESRs have become increasingly common in recent years. Several countries have adopted this system following the example of Germany, which introduced the system in 2008 (see Table 1). Parallel to these unilateral decisions by individual countries, Action 4 of the OECD Base Erosion and Profit Shifting (BEPS) project recommends an ESR type constraint limiting the deductibility of interest expenses to 10% – 30% (fixed ratio) of the firm’s EBITDA or EBIT.

In line with the OECD guidelines, the EU’s Anti-Tax Avoidance Directive (ATAD), enacted in 2016 and implemented in 2019, includes an ESR. According to article 4 of the ATAD, the minimum level of protection is achieved when borrowing costs are limited to 30% of EBITDA. An EBIT-based profit measure and a tighter restriction (lower fixed ratio) can also be applied. If the EBIT base is chosen, the incentive for debt-shifting arises in the current international corporate tax system as most of the national tax systems allow the deduction of interest expenses in the country of the debtor and constitute taxable income in the country of the lender. This way, the MNE can effectively transfer profits from high-tax to low-tax countries by locating intra-group lending activities in a country with a particularly low tax rate. As a result, the overall tax burden of the MNE is reduced.

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the way the directive suggests that the fixed ratio should be adjusted is to make the interest limitation effectively equal to the EBITDA-based limit.

Even if both the EBITDA-rule and the EBIT-rule are in line with the ATAD, it remains unknown which one of the two is preferable. Neither the directive nor other literature has studied their implications thoroughly. We compare these rules by studying their implications for profit-shifting, investment incentives and social welfare.

The paper proceeds as follows. The follow-up section illustrates the pre-ATAD practices of the interest limitation rules. Section 3 then reviews the related literature. Section 4 introduces our investment model, where a two-country MNE can invest and shift profits between countries. Section 5 analyses the incentives of interest limitation rules for MNE investment and tax-planning by using the investment model. Section 6 studies the optimal policies of the government in choosing the optimal interest limitation rule by comparing their implications for the social welfare. Section 7 concludes.

2. Pre-ATAD Practices of Interest Limitation Rules

The ATAD will align by and large the design of interest limitation rules across EU countries from 2019 onwards. However, the directive still leaves some space by allowing both the EBIT and EBITDA to serve as a basis for the ESR. Also the details within each ESR are left to some extent for countries to choose. Regarding the different implications and about the question of which one of the ESRs is better, the directive remains silent. Also the countries seem to be unaware about which of the rules is optimal from the society point of view, something which may be reflected in the variety of ESRs being implemented before the directive. Table 1 provides some key aspects of ESR systems in selected European countries.

Table 1: Pre-ATAD ESR systems in selected European countries

<table>
<thead>
<tr>
<th>Country</th>
<th>Applied from</th>
<th>All or related party debt</th>
<th>De minimis threshold</th>
<th>Profit-related measure</th>
<th>Fixed ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Denmark*</td>
<td>2007</td>
<td>All debt</td>
<td>DKK 21.3M</td>
<td>EBIT</td>
<td>80 %</td>
</tr>
<tr>
<td>Finland</td>
<td>2014</td>
<td>Related party debt</td>
<td>€ 0.5M</td>
<td>EBITDA</td>
<td>25 %</td>
</tr>
<tr>
<td>France**</td>
<td>2008</td>
<td>Related party debt</td>
<td>€ 3M</td>
<td>EBITDA</td>
<td>25 %</td>
</tr>
<tr>
<td>Germany</td>
<td>2008</td>
<td>All debt</td>
<td>€ 3 M</td>
<td>EBITDA</td>
<td>30 %</td>
</tr>
<tr>
<td>Italy</td>
<td>2008</td>
<td>All debt</td>
<td>0</td>
<td>EBITDA</td>
<td>30 %</td>
</tr>
<tr>
<td>Norway</td>
<td>2014</td>
<td>Related party debt</td>
<td>NOK 5M</td>
<td>EBITDA</td>
<td>25 %</td>
</tr>
<tr>
<td>Sweden***</td>
<td>2019</td>
<td>All debt</td>
<td>0</td>
<td>EBIT</td>
<td>35 %</td>
</tr>
<tr>
<td>UK</td>
<td>2017</td>
<td>All debt</td>
<td>£ 2M</td>
<td>EBITDA</td>
<td>30 %</td>
</tr>
</tbody>
</table>


*Note that Denmark has three interest limitation rules in place: TCR, ESR and an interest ceiling

**France has two rules in place: TCR and ESR

***See Government bill (2017). Sweden also has a specific interest limitation adopted in 2009 and applicable in the case of acquisitions.

7 In 2016, of 57 countries that applied some kind of formal interest limitation rule, 50 countries had a TCR and 7 countries an ESR rule (IMF 2016). Denmark, Germany, France and Italy were among the first ones and adopted the ESRs in 2007-2008, while other countries in the table followed a few years later. Swedish government launched in 2017 a proposal to introduce ESR, which has not been approved by the parliament yet (Government bill, 2017).
The table shows that the ESRs differ from each other along several dimensions. First, while some countries have targeted their rules only at intra-group interest payments (related party debt), others also apply them to interest expenses paid to third-party lenders (all debt). Second, the ‘de minimis’ thresholds, which commonly bite only above a fairly high threshold and therefore effectively rule small firms out of their scope, differ greatly between countries. While there is no de minimis threshold in Italy, it is set to the level of €3M in Germany and France. Third, even if most countries in the table have chosen their profit-related measure to be EBITDA, also EBIT and EBITD do occur. Finally, the values for the fixed ratio differ to some extent. Although for the most cases, the fixed ratio remains between 25% and 30%, exceptions do exist: In Denmark the fixed ratio is as high as 80% and for Sweden it has been suggested to be 35%.

Even if there are several differences across the interest limitation rules, we focus in this paper on the profit-related measures and the corresponding fixed ratios. The differences between the EBIT and EBITDA concepts arise as they are calculated based on different items each of which is reported for taxation purposes. Whereas both concepts employ on similar footing the earnings (E) that refer to taxable profit, the difference arises especially via depreciation (D), which refers to depreciation allowances deducted in tax accounts. The latter therefore includes accelerated deprecations exceeding deprecations reported for accounting purposes and temporary increased deprecations.

A priori, the differences between the two variants of ESR should have implications for various margins. First, with a given fixed ratio a narrow profit basis corresponds to a tighter restriction, and therefore provides a high incentive not to engage in debt-shifting. However, a lower limit might also affect activities other than those conducted artificially by weakening the conditions for financing investment. Second, because the difference between EBIT and EBITDA lies in the fact that they include a different set of cost items, they are likely to create differing incentives for activities generating these costs. Finally, the two variants of ESR might be associated with fixed ratios of different sizes (as was even suggested in the ATAD). The effects of the alternative systems might well be sensitive to the size of the fixed ratio.

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8 The ATAD applies the wider definition (all debt).
9 The ATAD allows a country to choose a high threshold but does not require one.
10 In Finland’s (pre-ATAD) practice the base is defined as earnings before interest, taxes and depreciation (EBITD). One further special aspect of the system is that the Finnish group contribution is taken into account when calculating the base (given contribution is added and received contribution is subtracted from the base).
11 Also Bulgaria applies an EBIT-based interest limitation rule with a high fixed ratio of 75%.
3. Related Literature

Our study is related to a growing literature on anti-tax avoidance measures designed to constrain profit-shifting activities by MNEs. The empirical literature on the effects of interest limitation rules has mostly focused on TCRs. It has shown that these restrictions have decreased the debt levels of companies (Buettner et al. 2012; Blouin et al., 2014; Ruf and Schindler 2015; Buettner et al., 2015), but at the same time it also acknowledges the flip side of these rules: they tend to decrease the foreign direct investments (Buettner et al., 2014). The scarce empirical research on the effects of ESR includes Buslei and Simmler (2012), Dressler and Scheuering (2012), Alberternst and Sureth-Sloane (2016), which study the effects of German interest limitation rule (ESR) introduced in 2008, and Harju et al. (2017), who consider the effects of Finnish ESR introduced in 2014. The evidence shows that the ESRs decrease the debt-to-equity ratio and the financial expenses of companies.

Regarding the theoretical studies, three of them have considered the choice between TCR- and ESR-type interest limitations. Kalamov (2015) finds that a switch from TCR to ESR occurs assuming that firms are able to manipulate the interest rate on internal debt. Gresik et al. (2017) also find that ESR largely dominates over TCR. Mardan (2017) studies the choice of the optimal rule in a framework where firms may face financial constraints. He argues that TCR is optimal when the share of financially constrained firms is high. If, instead, financial markets work well ESR may dominate.

Haufler and Runkel (2012) study the strategic choices of a country which has two tax instruments, a corporate tax rate and an interest limitation. They show that countries compete over real investment mainly through TCR rather than a low tax rate. This is because TCR can be targeted at MNEs, while tax rate cuts also affect the tax revenue collected from purely national firms. Haufler et al. (2016) consider the optimal policy of a country that has multiple means to prevent debt-shifting, a controlled foreign company (CFC) rule and a TCR. They show that both are included in the optimal policy mix, and that increasing economic integration tends to lead to stricter TCRs but laxer CFC rules.

Debt-shifting and interest limitation rules have attracted much attention in both empirical and theoretical research in the past few years. Nevertheless, we are unaware of economic studies dealing with positive and normative aspects of the design of ESR-based limitation rules. This paper aims to fill this gap by analyzing the relative benefits of EBIT- and EBITDA-based limitations. For that purpose we construct in the following section a dynamic investment model, which is then used in studying the optimal choices of an MNE. After the MNE choices we focus on the optimal government policies in choosing the desirable interest limitation rule together with the optimal level for fixed ratio in order to maximize the social welfare.
4. Dynamic Investment Model

Let us consider a value-maximizing MNE which has operations in two countries, home and foreign. The company is fully owned and financed by domestic investors. The parent company resides and produces in the home country and owns a subsidiary in the foreign country, and may transfer profits between the countries by using intra-group lending. It finances from gross profit, $\pi(K)$, new debt financing, $L$, and foreign intra-company dividends, $D^*$. The funds are used on investment, $I$, dividends, $D$, taxes, $T$, and interest expenses on external, $iB$, and internal, $S$, loans. $B$ denotes the stock of external debt and $i$ the (constant) rate of interest on debt. Intra-group loans are provided by the subsidiary located in the foreign country. Budget constraint of parent is:

$$\pi(K) + L + D^* = I + D + T + iB + S$$

Gross profit, $\pi(K)$, from domestic operations is generated using a single input, capital, $K$, and carries the standard properties. The evolution of the capital stock is determined by investments, $I$, and the rate of economic depreciation $\delta \in (0,1)$: $\dot{K} = I - \delta K$. New loans, $L$, add to the stock of external debt as follows: $\dot{B} = L$.

External debt is constrained to a share $b$ of total assets, $B \leq bK \equiv B^{max}$, where $b \in (0,1]$ is the maximum debt-to-assets ratio. Hence the agency problem between lenders and borrowing firms is modelled by simply assuming that the unit cost of debt, $i$, is constant up to a threshold, $bK$, at which the availability of new third-party debt stops entirely.

We do not model the stock of internal loans or the unit cost of these loans. This reflects the simplifying assumption that the prime purpose of intra-company interest payments is to shift profits. While $S$ can be manipulated with no costs, it is constrained by an exogenous ceiling: $S \leq S^{max}$.

The parent pays source-based corporate tax, $T$, at rate $\tau \in (0,1)$ on profit net of fiscal depreciation and tax-deductible interest costs, which include both external and intra-group interest payments. Repatriated foreign profit, $D^*$, is not included in the tax base due to the exemption method applied by the home government to intra-company dividends:

$$T = \tau[\pi(K) - \alpha k - C],$$

Here $\alpha \in [\delta, 1]$ is the rate of (possibly accelerated) fiscal depreciation and $k$ is the stock of book capital. $k$ develops as follows: $\dot{k} = I - \alpha k$. $C$ stands for tax-deductible interest expenses, which depend on tax rules in the way described below.

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12 Following the common practice in international trade literature, home country variables are unstarred and foreign country variables are starred.

13 In section 5.3, we will make the amount deducted as fiscal depreciation endogenous in order to address some interactions between the ESR rules and depreciations which cannot be addressed using this model version.
As we consider an ESR regime either based on an EBITDA or an EBIT limit and with no ‘de minimis’ rule or any other exemption,\textsuperscript{14} the tax-deductible interest expenses, $C$, are defined as follows:

$$C^{ESR} = \min(iB + S; zM^j), \quad j \in \{\text{ebit, ebitda}\}.$$  

If the actual expenses ($iB + S$) are lower than the ceiling ($zM^j$) all interest expenses are deductible for taxation purposes. And if they exceed the ceiling only the amount corresponding to the ceiling ($zM^j$) can be deducted.

The ESR ceiling is in turn defined via the fixed-ratio parameter $z > 0$ times $M^j$, where

$$M^{\text{ebitda}} = \pi(K)$$
$$M^{\text{ebit}} = \pi(K) - \alpha k$$

Under the EBITDA rule the calculation base is gross profit before interest expenses, taxes and fiscal depreciation.\textsuperscript{15} This equals $\pi(K)$ in our model. Under the EBIT rule the base is gross profit before interest expenses and taxes, which we model as the difference between gross profit, $\pi(K)$, and fiscal depreciation allowances, $\alpha k$.\textsuperscript{16}

There are two benchmark regimes: standard corporate tax, CIT, where the firm may deduct all interest expenses, and comprehensive business income tax, CBIT, where no interest expenses are deductible. $C$ takes the following values in these regimes:

$$C^{\text{cit}} = iB + S$$
$$C^{\text{cbit}} = 0$$

The subsidiary is located in the foreign country, which has a low tax rate, $\tau^* \in (0, \tau)$. It finances its activities from exogenous profit, $\pi(K^*)$, and internal interest payments from the parent, $S$. The funds are used on profit repatriations, $D^*$, and foreign corporate taxes, $T^* = \tau^*[\pi(K^*) + S]$.

$$\pi(K^*) + S = D^* + T^*.$$  

The objective of the MNE is to maximize its market value $V(0)$:

$$\max_{\{D, L, S, D^*\}} V(0) = \int_0^\infty De^{-\rho t} dt,$$

where $\rho$ is the discount rate of the MNE. The details of the model are summarized in Appendix A.

\textsuperscript{14} Among the aspects of real-life applications which the model ignores are so-called carry forward rules which let companies carry forward interest expenses that exceed the limit.

\textsuperscript{15} We assume no other changes in the market value of capital than those represented by economic depreciation. Therefore, amortizations are zero in our model.

\textsuperscript{16} The ATAD interest limitation rule stipulates that profit concepts shall be calculated based on items reported for taxation purposes; therefore it is the fiscal depreciation allowances which are deducted in calculating $M^{\text{EBIT}}$. 


5. Optimal Policies of MNE

In this section we employ the investment model described above and derive the optimal policies for an MNE. We start by considering two questions related to debt and interest expenses in section 5.1. First we consider how the deductibility of interest expenses affects the incentives regarding the choice between the two financial forms, debt and equity, in the presence of an ESR. Next we focus on the choice between the internal and external debt when the ESR constraints the interest expenses (ESR is binding), and consider the question about which of the expenses firm prefers to cut first, the internal or external debt expenses. Following the financial debt-related choices we study the investment incentives of the EBIT and EBITDA rules in section 5.2.

5.1 The Effects on Debt Financing

Debt vs. Equity

The choice between equity and external debt as the financial form can be addressed by considering the first-order condition of the Lagrangian with respect to \( B \) in the long-run equilibrium (condition (A2g') in Appendix A):

\[
\rho - (i - \tau \frac{\partial C}{\partial B}) = \mu_2.
\]

The condition compares the after-tax costs of equity (\( \rho \)) and debt (\( i - \tau \frac{\partial C}{\partial B} \)). If all interest costs are tax-deductible in the margin (deductible interest expenses increase with \( B \) the external debt level at the rate of interest \( \frac{\partial C}{\partial B} = i \)), the left-hand-side (lhs) is positive, implying that the firm prefers debt to equity.\(^{17}\) Unless other considerations prevent it, the firm now chooses the maximum debt level (\( \mu_2 > 0 \Rightarrow B = bK \)). The tax-deductibility of interest costs in different cases are illustrated in Table 2 (column with \( \frac{\partial C}{\partial B} \)). It implies that the external debt is tax-favored over equity in the standard CIT model and under the ESRs when the maximum of the deductible interest expenses is has not been reached (cases with non-binding ESR limit, \( iB + S \leq zM \)).

<table>
<thead>
<tr>
<th>( \frac{\partial C}{\partial B} )</th>
<th>CIT ( iB + S \leq zM )</th>
<th>EBITDA ( iB + S &gt; zM )</th>
<th>EBIT ( iB + S \leq zM )</th>
<th>EBIT ( iB + S &gt; zM )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \frac{\partial C}{\partial K} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \frac{\partial C}{\partial S} )</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

\(^{17}\) Recall that we assume \( \rho = i \).

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Table 2. The partial derivatives of tax deductable interest expenses (C)
The other case arises when the interest costs are non-deductible in the margin \( \frac{\partial C}{\partial B} = 0 \). The lhs of equation (8) takes the value of zero implying that debt and equity are equally costly, and the firm becomes indifferent between the financing forms. This occurs also in the CBIT model, which forbids the deductibility of interest entirely, like the ESRs if the interest constraint is binding.

Let us next focus on comparison between the internal debt (profit shifting, \( S \)) and equity financing. The optimality condition for the profits transferred to the foreign country using debt-shifting (condition A2d) can be written as follows:

\[
\frac{\partial C}{\partial S} - \left(1 - \tau \frac{\partial C}{\partial S}\right) + (1 - \tau^*) = \mu_3
\]

The left hand side of the equation shows the net tax saving from a one unit increase in internal interest expenses \( S \). Like we see in the last row in table 2 the tax-deductible expenses increase one-to-one with additional costs \( \frac{\partial C}{\partial S} = 1 \) under the CIT and both of the ESRs when the ceiling is non-binding. In this case the tax benefit is \( \tau - \tau^* > 0 \) implying that the firm prefers increasing \( S \).

Another case shown in Table 2 is when the additional expenses on internal debt cannot be deducted \( \frac{\partial C}{\partial S} = 0 \). In these cases (CBIT and ESRs with a binding ceiling) the debt shifting incentive disappears as the gain becomes negative \( \mu_3 = -\tau^* \).

Regarding the choices between the debt and equity our model has different implications for the ESRs depending on whether the ceiling is binding or not. In cases where the ESR is binding, the interest limitation rules place the external debt to equal footing with the equity (like CBIT), but in the non-binding cases debt remains tax-favored over equity (like CIT). For the internal debt choice (profit shifting) our model implies that the ESRs remove the debt-shifting motive when the ceiling is binding.

**Internal vs. external debt**

Next we focus on one of the crucial points regarding the performance of ESRs, namely the choice between internal and external debt. While the ESRs limit the deductibility of total interest expenses when they exceed the ceiling, they provide incentives for an MNE to cut these expenses. An important question is then which type of interest payments would an MNE prefer to reduce first, internal or external. This choice shows whether the ESRs restrain the artificial profit shifting, like intended, or the external debt typically used for the real business.

We consider this question by using a simple experiment, where the firm adds one unit to its deductible interest expenses on external debts, \( iB \), and reduces its internal expenses, \( S \), by the same amount so that both total interest expenses, \( C \), and the stock of capital, \( K \), are unchanged. The question is whether this operation is profitable or not.

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18 Here we have assumed a steady state and used the condition (A2c). See Appendix A for the details.
Now $i\Delta B = 1$ and the impact of this on $K$ is eliminated by reducing equity capital by an equal amount. Reduction in the equity capital decreases the firm’s after-tax profits by more than what the increase in the external debt adds, and therefore, the net effect on the firm’s after-tax profit becomes positive ($1 - (1 - \tau) = \tau > 0$). Due to the deductibility of interest expenses the firm gains a tax saving of $\tau$. The reduction in the internal interest expenses, $S$, leads to a loss in saved taxes of $-(\tau - \tau^*) < 0$ (see equation 9). The experiment’s net value then is: $\tau + (\tau^* - \tau) = \tau^* > 0$, which implies that interest expenses on external debt are more valuable for the firm than internal interest.

Therefore, the firm adapts its debt stock and internal interests to the ESR limit such that it first reduces internal loans and the expenses on them. This result is summarized in Proposition 1:

**Proposition 1.** Assuming that $\tau^* > 0$, the firm prefers external interest to internal interest, under an ESR system.

Ultimately, the optimal size of debt, $\hat{B}$, is determined as part of the investment decision. Therefore, we cannot exclude the case where $i\hat{B} < zM^j$. The implications on $S$ are as follows:

**Corollary 1.** Under a binding ESR ceiling, the MNE chooses $S = 0$ if $i\hat{B} \geq zM^j$ and $S > 0$ if $i\hat{B} < zM^j$.

Hence the preference order between external and internal expenses leads to a situation where the MNE entirely gives up the opportunity to transfer profits using debt-shifting if external interest expenses are high in the optimum. However, if a low level of external debt is chosen ($i\hat{B} < zM$), then the MNE sets $S = zM - i\hat{B} > 0$.

### 5.2 The Effects on Investment

This section considers the effects of ESRs on investment incentives. It focuses on an internal equilibrium in the presence of a binding ceiling ($i\hat{B} + S \geq zM^j$). The ESRs are compared to two benchmark tax systems, CIT and CBIT. To study the investment incentives we start by considering the following optimal condition derived from the Lagrangian (see Appendix A for details):\(^{22}\)

\(^{19}\) This can also be seen using condition (8): external debt is increased by $\Delta B = 1/i$; now the benefit can be measured as follows: $\mu \Delta B = t\Delta B = \tau$.

\(^{20}\) We assume that the reduction in the subsidiary’s profit is shifted forward to the parent via a change in $D^*$. This has no tax implications since $D^*$ is paid out of after-tax (foreign) profits and the dividend is tax-free in the hands of the parent due to the exemption method.

\(^{21}\) The internal solution results if it is profitable to continue issuing new debt when the interest expenses on external loans surpass the ESR ceiling. Hence, at the level of the ceiling, the marginal return on investment is higher than the cost of capital of debt-financed investment. The other possible solutions are 1) a corner solution, which occurs when debt financed investment is profitable up to the ESR ceiling but unprofitable above the ceiling, and 2) an internal solution with a non-binding ESR limit ($i\hat{B} < zM^j$).

\(^{22}\) This is equation (A5) in Appendix A.
\[(1 - \tau)p' + \tau \frac{\partial c}{\partial K} = (\rho + \delta) \left(1 - \tau \frac{\alpha + \frac{\partial c}{\partial K}}{a + \rho}\right) - bp + b(i - \tau \frac{\partial c}{\partial B}).\]

The equation implicitly shows the cost of capital for each tax systems considered: CIT, CBIT, EBIT and EBITDA. By substituting the regime-specific partial derivatives \(\frac{\partial c}{\partial K}, \frac{\partial c}{\partial a}, \text{ and } \frac{\partial c}{\partial B}\) given in Table 2 we get the MNE’s cost of capital in each of the four systems.\(^{23}\) These are provided in Table 3.

**Table 3. Cost of capital \((p = p' - \delta)\) for debt-financed investments**

<table>
<thead>
<tr>
<th>Cost of capital, (p)</th>
<th>CIT</th>
<th>CBIT</th>
<th>EBITDA</th>
<th>EBIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - (\tau A) (\frac{1 - \tau}{1 - \tau}((1 - \tau)i + \delta) - \frac{\tau i}{1 - \tau}\tau A - \delta)</td>
<td>1 - (\tau A) (\frac{1 - \tau}{1 - \tau}(i + \delta) - \delta)</td>
<td>1 - (\tau A) (\frac{1 - \tau}{1 - \varepsilon}(i + \delta) - \delta)</td>
<td>1 - (\varepsilon A) (\frac{1 - \varepsilon}{1 - \varepsilon}(i + \delta) - \delta)</td>
<td></td>
</tr>
</tbody>
</table>

Note: \(A = \frac{a}{i + a}\) is present value of fiscal depreciation allowances, and \(\varepsilon = (1 - z)\tau\) is effective tax rate.

The table shows the cost of capital \((p = p' - \delta)\) for the MNE’s domestic debt-financed investment. Let us start by considering the cost of capital for the CIT. In the first of the two terms in the cost of capital equation the taxation affects via three channels. First, it reduces the marginal cost of debt financing \(((1 - \tau)i\) in the parenthesis), which is an after-tax cost due to the deductibility of interest expenses; second, it affects the tax saving from fiscal depreciation allowances \((\tau A\) in the numerator); and, finally, it affects the future net returns (after-tax returns) on a marginal investment project \((1 - \tau\) in the denominator).\(^{24}\)

Compared to CIT, under CBIT the first channel disappears due to the non-deductibility of interest expenses, but the two others survive. A further change is that the second term disappears.

The EBITDA and EBIT systems retain the property in CBIT that the marginal cost of debt is a pre-tax cost, \(i\), (not an after-tax cost, \((1 - \tau)i\), as under CIT). However, the marginal tax rates on future returns differ. In EBIT and EBITDA it is the effective tax rate, \(\varepsilon = (1 - z)\tau < \tau\), while in CBIT it is the statutory tax rate, \(\tau\). The intuition to this result is that the returns on the marginal project release the interest constraint and allow the firm to increase its interest deductions. This is a further mechanism compared to CBIT, and its effect translates into a decline in the effective tax rate and, further, in the cost of capital. We summarize the main common elements behind the cost of capital for EBIT- and EBITDA-based interest limitation rules in Proposition 2.\(^{25}\)

**Proposition 2.** The EBITDA- and EBIT-based caps on interest expenses increase the cost of capital through a higher marginal cost of debt financing, but reduce it through a lower effective tax rate on future returns on marginal investment.

---

\(^{23}\) To simplify the exposition we assume an investment project that is financed entirely with debt, \(b = 1\).

\(^{24}\) The second term on the right-hand-side (rhs) of \(p\) in CIT, in Table 3, is an interaction term of fiscal depreciation and interest deduction which is of lesser importance here.

\(^{25}\) The proposition concerns only the interior solution as discussed in the beginning of the section.
The cost of capital under EBIT involves one further novelty: the tax rate affecting the tax saving from depreciation allowances now is $\varepsilon A$. This arises because the ESR ceiling depends negatively on the amount deducted as depreciation allowances. An increase in $k$ tightens up the ESR limit and leads to increased taxes. Thus the EBIT-based cap also affects the cost of capital through the tax saving from depreciation allowances.

Let us next turn into considering the comparative statics of the cost of capital with respect to the tax parameters ($\tau, z, \delta, \alpha$). The results for the four tax systems are provided in Table 4.

Table 4. Comparative statics of the cost of capital

<table>
<thead>
<tr>
<th></th>
<th>CIT</th>
<th>CBIT</th>
<th>EBITDA</th>
<th>EBIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\partial p}{\partial \tau}$</td>
<td>$\frac{(1 - A)}{(1 - \tau)\varphi} &gt; 0$</td>
<td>$\frac{1 - A}{(1 - \tau)\varphi} &gt; 0$</td>
<td>$\frac{1 - z - A}{(1 - \varepsilon)^2\varphi} &gt; 0$</td>
<td>$\frac{(1 - A)(1 - z)}{(1 - \varepsilon)^2\varphi} &gt; 0$</td>
</tr>
<tr>
<td>$\frac{\partial p}{\partial z}$</td>
<td>na</td>
<td>na</td>
<td>$-\frac{\tau(1 - \varepsilon A)}{(1 - \varepsilon)^2\varphi} &lt; 0$</td>
<td>$-\frac{\tau(1 - A)}{(1 - \varepsilon)^2\varphi} &lt; 0$</td>
</tr>
<tr>
<td>$\frac{\partial p}{\partial \delta}$</td>
<td>0</td>
<td>0</td>
<td>$-\frac{\tau}{1 - \varepsilon} &lt; 0$</td>
<td>0</td>
</tr>
<tr>
<td>$\frac{\partial p}{\partial \alpha}$</td>
<td>$-\frac{\tau i \omega}{(1 - \varepsilon)(\alpha + i)}$</td>
<td>$-\frac{\tau i \omega}{(1 - \varepsilon)(\alpha + i)}$</td>
<td>$-\frac{\tau i \omega}{(1 - \varepsilon)(\alpha + i)}$</td>
<td>$-\frac{\tau i \omega}{(1 - \varepsilon)(\alpha + i)}$</td>
</tr>
</tbody>
</table>

Note: $\varphi = i + \delta, \omega = \frac{1}{(1 - \varepsilon)(\alpha + i)}$

The first row in the table illustrates the cost of capital responses to statutory tax rate increases. For CIT, CBIT and EBIT the cost of capital is increasing with the tax rate, while for EBITDA it depends on the magnitudes of the fixed ratio ($z$) and the present value of fiscal depreciation allowances ($A$). The cost of capital for EBITDA increases with the statutory tax rate if $z + A < 1$, decreases if $z + A > 1$, and does not depend on the tax rate if $z + A = 1$.

Regarding the fixed ratio ($z$) we observe that it reduces the cost of capital in both ESR regimes. This is not surprising since this change releases the ceiling on deductible interest expenses, bringing the firm a tax saving. However, since $\tau(1 - \varepsilon A) > \tau(1 - A)$, the reducing effect is stronger under the EBITDA rule than under the EBIT rule. Thus the cost of capital under the EBITDA rule depends more heavily on the fixed ratio. We summarize this result in Proposition 3.

**Proposition 3.** For EBIT and EBITDA interest limitation rules the cost of capital is lower the higher is the fixed ratio ($\frac{\partial p}{\partial z} < 0$). The cost of capital under EBITDA is more responsive to fixed ratio than under EBIT ($|\frac{\partial p}{\partial z}^{\text{EBITDA}}| > |\frac{\partial p}{\partial z}^{\text{EBIT}}|$).

An important difference between the ESRs arises when considering the effects of economic depreciation ($\delta$) on the cost of capital. While the cost of capital does not depend on it under EBIT (or CIT, or CBIT), the cost of capital decreases with economic depreciation under EBITDA. This means that the EBITDA rule distorts the choice of investment by favoring assets with a short economic life at the cost of assets with longer lives. Proposition 4 summarized this observation.
Proposition 4. The cost of capital under the EBIT-rule does not depend on economic depreciation \( (\partial p/\partial \delta = 0) \). For the EBITDA-rule the cost of capital lower the higher is the economic depreciation \( (\partial p/\partial \delta < 0) \).

We further observe that an increase in the rate of fiscal depreciation \((\alpha)\) reduces the cost of capital in all systems. This is less of a surprise. However, under the EBIT rule, if \( z = 1 \) then \( \varepsilon = 0 \) and the effect disappears.

In order to illustrate the cost of capital differences, Table 5 presents the numerical values for cost of capital for a fully debt-financed investment in the four tax regimes (CIT, CBIT, EBITDA and EBIT) using some typical parameter values. We consider two assumptions concerning the limitation. In the first, the fixed-ratio rate is equal under both rules, \( z^{\text{ebitda}} = z^{\text{ebit}} \); in the second, the interest limitation is equally stringent: \( z^{\text{ebitda}} M^{\text{ebitda}} = z^{\text{ebit}} M^{\text{ebit}} \).

We use the following parameter values: \( i = 0.05, \delta = 0.15, \alpha = 0.25, r = 0.25, z^{\text{ebitda}} = 0.25 \) and \( M^{\text{ebitda}} / M^{\text{ebit}} = 1.7. \)\(^{26}\) At these values, limiting interest deductibility raises the cost of capital of debt-financed investment. In the case of the EBITDA rule the effect is small, while under the EBIT rule the effect is larger. Using these parameter values the figure for EBITDA is close to CIT and the figure for EBIT is close to CBIT.\(^{27}\)

Table 5. Capital cost for investment financed from debt in the four tax regimes, %

<table>
<thead>
<tr>
<th>( z^{\text{ebitda}} = z^{\text{ebit}} )</th>
<th>CIT</th>
<th>CBIT</th>
<th>EBITDA</th>
<th>EBIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z^{\text{ebitda}} M^{\text{ebitda}} = z^{\text{ebit}} M^{\text{ebit}} )</td>
<td>TBA</td>
<td>TBA</td>
<td>TBA</td>
<td>TBA</td>
</tr>
</tbody>
</table>

Next we illustrate the results provided in Propositions 3 and 4. Figures 1a and 1b show how the cost of capital depends on the fixed ratio, \( z \), and economic depreciation, \( \delta \).\(^{28}\) The figures show that under the EBITDA rule the cost of capital is very sensitive to the level of both \( z \) and \( \delta \). When, for example, \( z \) increases from 0.2 to 0.3 the cost of capital quickly drops below the level in CIT. Hence, the EBITDA ceiling does not necessarily restrain debt-financed investment; if \( z \) is sufficiently high, it rather provides a special incentive to invest using debt.

Similarly, at a low rate of economic depreciation, the cost of capital of EBIT and EBITDA do not differ much and are well above that in CIT. However, for assets with a short useful life the cost of capital of EBITDA can be substantially lower than under EBIT or CIT. This implies that the EBITDA rule distorts investment based on useful economic life, the result given in Proposition 4.

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26 The value of \( M^{\text{ebitda}} / M^{\text{ebit}} \) is the average across all corporate firms in Finland.
27 Also Södersten (2014) calculates the effects of interest limitations on the cost of capital. His focus is on CBIT- and EBIT-type limits. Some of the observations are in line with ours.
28 The latter assumes \( z^{\text{ebitda}} = z^{\text{ebit}} \).
Figure 1a. Cost of capital at different values of fixed ratio, $z$

Figure 1b. Cost of capital at different values of economic depreciation, $\delta$
6. Optimal ESR Policies of Government

In this section we consider the home country government’s optimal choice between EBITDA and EBIT rules. The analysis starts from the case where capital is homogenous (Section 6.1) and then moves on to consider the case of two types of capital which differ in their useful lives (Section 6.2).

6.1. Optimal ESR when Capital is Homogenous

The government maximizes national welfare, which is measured as a weighted sum of domestic tax revenue, $T$, and domestic, $\pi^{AT}$, plus foreign, $D^*$, after-tax profits:\(^{29}\)

\[
W = T + \lambda(\pi^{AT} + D^*)
\]

$\lambda \in [0,1]$ is the relative welfare weight placed on the MNE’s profit. A weight lower than one can be interpreted as reflecting that raising corporate tax revenue is valuable for society for redistributive or efficiency reasons.\(^{30}\) $\lambda = 0$ represents the case of a Leviathan government which only values tax revenue.

Domestic tax revenue is calculated as tax at rate $\tau$ on gross profit, $\pi(K)$, minus depreciation allowances, $\delta K$, and deductible interest expenses, $z^jM^j (j \in \{EBIT, EBITDA\})$.\(^{31}\)

\[
T = \tau[\pi(K) - \delta K - z^jM^j]
\]

Domestic after-tax profit is gross profit minus economic depreciation, opportunity cost and domestic taxes:

\[
\pi^{AT} = \pi(K) - \delta K - \rho K - T = (1 - \tau)[\pi(K) - \delta K] - \rho K + \tau z^jM^j
\]

Foreign after-tax profit is:

\[
D^* = (1 - \tau^*)\bar{\pi}^*
\]

where $\bar{\pi}^*$ is the subsidiary’s gross profit from foreign operations. It is considered to be exogenous in the current decision problem.

In our model the government maximizes its welfare by choosing between the EBITDA and the EBIT interest limitation rules, including their optimal parameter values. In its optimal choice a government takes also the firm responses into account. The optimal solution for our model is thus derived in three steps. The firm first chooses its capital level ($K$) to maximize its net-of-tax profits with a given interest limitation rule ($EBITDA$ or $EBIT$) and the corresponding fixed ratio ($z^{EBITDA}$ or $z^{EBIT}$).

\(^{29}\) We broadly follow Mardan (2017) here. See Feldstein and Hartman (1979) for an early treatment.

\(^{30}\) In cases where $\lambda < 1$ one tax revenue unit in the hands of tax authority is considered more valuable than one net-of-tax profit unit in the hands of firm. The higher tax revenue valuation reflects the costs arising from tax collection.

\(^{31}\) In this section we assume that fiscal depreciation corresponds to economic depreciation ($\alpha = \delta$) and that there is no profit shifting ($S = 0$). We also focus on the case where the interest limitation rule is binding. This corresponds to the case, where $iB + S \geq zM$ (see equation 3 in Section 4).
\( z^{\text{EBIT}} \). In the second step the government chooses the optimal fixed ratio for each interest limitation rule (\( \text{EBITDA} \) or \( \text{EBIT} \)) to maximize the welfare of the society while taking into account the firm response. Finally, the government chooses the interest limitation rule according to the welfare maximizing policy while taking the optimal firm choices and the optimal fixed ratio choices as given.

**Step 1: Firm Choice**

The firm chooses its capital level \( (K) \) to maximize the net-of-tax profits \( (\pi^{AT} + D^*) \). While an increase in the capital level increases the net-of-tax profits, it increases capital depreciation and causes an opportunity cost. Balancing between the opposite effects yields the following cost of capital equations for the EBITDA and the EBIT (see Appendix C):

\[
\begin{align*}
\pi'(K^{\text{EBITDA}}) &= \frac{\rho + (1-\tau)\delta}{1-\tau + z^{\text{EBITDA}}} = \frac{\rho + (1-\tau)\delta}{1-\epsilon^{\text{EBITDA}}} \\
\pi'(K^{\text{EBIT}}) &= \delta + \frac{\rho}{1-\tau + z^{\text{EBIT}}} = \delta + \frac{\rho}{1-\epsilon^{\text{EBIT}}}
\end{align*}
\]

Each cost of capital equation implicitly gives us the optimal capital level as a function of given parameter values, \( K^{\text{EBITDA}} = K^{\text{EBITDA}}(\tau, \rho, \delta, z^{\text{EBITDA}}) \) and \( K^{\text{EBIT}} = K^{\text{EBIT}}(\tau, \rho, \delta, z^{\text{EBIT}}) \). For both the EBITDA and EBIT rules the cost of capital is increasing with respect to the (per-unit) opportunity cost, \( \rho \), and the depreciation rate, \( \delta \). The relationship between the cost of capital and the tax rate, \( \tau \), is not as straightforward, but depends on the fixed ratio. Under the EBITDA (EBIT) rule the cost of capital increases with respect to tax rate when \( z^{\text{EBITDA}} < \frac{\rho}{\rho + \delta} \) (\( z^{\text{EBIT}} < 1 \)) and decreases when \( z^{\text{EBITDA}} > \frac{\rho}{\rho + \delta} \) (\( z^{\text{EBIT}} > 1 \)). When \( z^{\text{EBITDA}} = \frac{\rho}{\rho + \delta} \) or \( z^{\text{EBIT}} = 1 \), the cost of capital becomes independent of the tax rate (\( \pi'(K) = \rho + \delta \) in both cases).\(^3\) A particularly important issue regarding the government welfare maximization problem is that each optimal capital level depends on a fixed ratio \( (z^{\text{EBITDA}} \) or \( z^{\text{EBIT}} \)). For both the EBITDA and EBIT rules the cost of capital decreases with respect to the fixed ratio and thus the optimal level for capital is higher the higher is the fixed ratio. However, the exact dependencies between the fixed ratio and the optimal capital level differ across the rules. Next let us consider the fixed ratio choices taking place in the second step.

**Step 2: Fixed Ratio Choice**

The government chooses the fixed ratio for both the EBITDA and the EBIT rule \( (z^{\text{EBITDA}} \) and \( z^{\text{EBIT}} \)) by maximizing the welfare, while taking into account the firm response to these choices. The government optimization problem is thus the following \( (j \in \{\text{EBIT}, \text{EBITDA}\}) \):

\[
\max_{z^j} W^j = T + \lambda(\pi^{AT} + D^*) = T(K^*(z^j), z^j) + \lambda(\pi^{AT}(K^*(z^j), z^j) + D^*)
\]

The first order conditions (FOCs) for the government optimization problems are:

\(^3\) These cases appear in the optimum when \( \lambda = 1 \). See “Case: \( \lambda = 1 \)” below.
The optimal fixed ratios are functions of both the parameter values and the fixed ratio. Thus we have solutions for both the optimal capital levels \( K^{*\text{EBITDA}} \) and \( K^{*\text{EBIT}} \) and the optimal fixed ratios \( z^{*\text{EBITDA}} \) and \( z^{*\text{EBIT}} \) as follows: \( z^{*\text{EBITDA}} = z^{*\text{EBITDA}}(\tau, \rho, \delta, K^{*\text{EBITDA}}(t, \tau, \rho, \delta, z^{*\text{EBITDA}})) \) and \( z^{*\text{EBIT}} = z^{*\text{EBIT}}(t, \tau, \rho, \delta, z^{*\text{EBIT}}) \). In the third step the government makes a discrete choice between the EBITDA and the EBIT interest limitation rules to maximize the welfare. Before turning to the third step let us consider a particular special case in order get a first grasp on how the different steps determine the optimal choices.

**Case: \( \lambda = 1 \)**

An interesting special case where the government becomes indifferent between an additional unit of tax revenue and an additional unit of net-of-tax profits occurs when \( \lambda = 1 \). In this case the optimal choices for the fixed ratio rules given in equations (22) and (23) reduce to the following:

\[
(22') \quad z^{*\text{EBITDA}} = \frac{\pi'(K^{*\text{EBITDA}}) - 1}{\pi'(K^{*\text{EBITDA}})} \quad (< 1)
\]
Thus the fixed ratio is chosen in the optimum to be higher for the EBIT rule than for the EBITDA rule, and it also becomes independent of the parameter values. Inserting the optimal fixed ratios into the cost of capital equations given in (17) and (18) shows us that the condition for the optimal level of capital is the same for both rules:

\begin{align}
(17') & \quad \pi'(K^*,\text{EBITDA}) = \rho + \delta \\
(18') & \quad \pi'(K^*,\text{EBIT}) = \rho + \delta
\end{align}

As the cost of capital equations are the same for both rules, also the optimal capital levels become the same ($K^*,\text{EBITDA} = K^*,\text{EBIT}$). Another thing worth observing is that the cost of capital equation is now independent of the tax parameters under both rules. Thus the taxation does not distort the capital choice in either of the systems, making both rules neutral in this respect. Moreover, we observe that the welfare functions become the same for EBITDA and EBIT when $\lambda = 1$:

\begin{align}
(13') & \quad W^{*}\text{EBITDA} = \pi(K^*,\text{EBITDA}) - (\rho + \delta)K^*,\text{EBITDA} + (1 - \tau^*)\pi^* \\
(13'') & \quad W^{*}\text{EBIT} = \pi(K^*,\text{EBIT}) - (\rho + \delta)K^*,\text{EBIT} + (1 - \tau^*)\pi^*
\end{align}

Given that the optimal capital levels are equal as well, we find that the welfares become equally large between EBITDA and EBIT rules. Thus the government becomes indifferent between them. This is the case even if the optimal choices imply different tax revenues and net-of-tax profits for different rules. With the optimal choices the tax revenues are the following:

\begin{align}
(14') & \quad T^{*}\text{EBITDA} = \frac{\tau\delta}{\rho + \delta}[\pi(K^*,\text{EBITDA}) - (\rho + \delta)K^*,\text{EBITDA}] \\
(14'') & \quad T^{*}\text{EBIT} = 0
\end{align}

The net-of-tax profits are in these cases the following:

\begin{align}
(15+16') & \quad \pi^{AT,*}\text{EBITDA} + D^* = (1 - \frac{\tau\delta}{\rho + \delta})[\pi(K^*,\text{EBITDA}) - (\rho + \delta)K^*,\text{EBITDA}] + D^* \\
(15+16'') & \quad \pi^{AT,*}\text{EBIT} + D^* = \pi(K^*,\text{EBIT}) - (\rho + \delta)K^*,\text{EBIT} + D^*
\end{align}

The equations show that the EBIT gives all the welfare to firms and receives tax revenue of zero, whereas the EBITDA apportions the welfare more equally between the tax revenues and the net-of-tax profits.

---

34 With the optimal capital levels the fixed ratios are $z^*,\text{EBITDA} = \frac{\rho}{\rho + \delta}$ and $z^*,\text{EBIT} = 1$. 

18
Next we relax the assumption that \( \lambda = 1 \) and consider the more general cases where \( \lambda \neq 1 \). The comparison between the rules becomes more difficult because we do not have closed form solutions, and therefore the comparison is done in step 3 by simulations for these cases.

**Step 3: Choice of the System (EBITDA vs EBIT)**

For the third step we employ the optimal solutions from the first two steps (equations (17) and (22) for the EBITDA rule; equations (18) and (23) for the EBIT rule) and compare the welfare under the EBITDA rule to that under the EBIT rule. The equations from the first two steps do not allow a straightforward way for comparing the welfare implications, because in general the optimal capital levels and the optimal fixed ratios differ across the two rules \( (K^{*\text{EBITDA}} \neq K^{*\text{EBIT}} \text{ and } z^{*\text{EBITDA}} \neq z^{*\text{EBIT}}) \). We therefore use simulations in the third step in what follows.

The procedure being used in the simulations to find the optimal choices goes as follows. We first fix the parameter values for \( \tau, \rho, \delta \) and \( \lambda \) and choose the functional form for the production function \( \pi \). After that we tentatively pick a value for the fixed ratio under a given interest limitation rule \( (z^{\text{EBITDA}} \text{ or } z^{\text{EBIT}}) \). For the EBITDA rule we plug the chosen value for the fixed ratio \( z^{\text{EBITDA}} \) into equation (17) in order to produce the optimal capital level \( (K^{*\text{EBITDA}}(z^{\text{EBITDA}})) \) corresponding to this particular fixed ratio. The optimal capital level is then plugged into equation (22) in order to provide the best response of the government in choosing the fixed ratio. The procedure is continued as long as this best response corresponds to the chosen tentative value and we have found the optimal fixed ratio \( z^{*\text{EBITDA}} \) (and thus also the corresponding optimal capital level, \( K^{*\text{EBITDA}} \)). The corresponding procedure (with equations (18) and (23)) is used for finding the optimal fixed ratio and the corresponding optimal capital level for the EBIT rule \( (z^{*\text{EBIT}} \text{ and } K^{*\text{EBIT}}) \). After finding the optimal fixed ratios and the corresponding capital levels for each rule we compare the implied welfares, which then determine the optimal choice in the third step:\(^{35}\)

\[
W^{*\text{EBITDA}} = T(K^{*\text{EBITDA}}, z^{*\text{EBITDA}}) + \lambda(\pi^{\text{AT}}(K^{*\text{EBITDA}}, z^{*\text{EBITDA}}) + D^*)
\]

\[
W^{*\text{EBIT}} = T(K^{*\text{EBIT}}, z^{*\text{EBIT}}) + \lambda(\pi^{\text{AT}}(K^{*\text{EBIT}}, z^{*\text{EBIT}}) + D^*)
\]

Let us first focus on the following case: \( \tau = 0.219, \delta = 0.25, \rho = 0.05 \) and \( \lambda = 1.\)\(^{36}\) The functional form of the production function is chosen to be \( \pi(K) = Scale \ast K^{1/2} \) (\( \pi'(K) = \frac{1}{2} \ast Scale \ast K^{-1/2}, \pi''(K) = -\frac{1}{4} \ast Scale \ast K^{-3/2} \)). The upper panel in Table 6 presents the results with these parameter choices. The table shows that the optimal fixed ratios differ between EBIT and EBITDA rule, but the optimal capital levels of the firm do not. EBITDA yields fixed ratio of 0.17, whereas EBIT yields much higher fixed ratio of 1. Due to difference in the fixed ratios and the base for the deduction EBITDA collects more taxes than EBIT, but leaves the company with less after-tax profits. However, despite of these differences both interest limitation rules provide equally high welfare

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\(^{35}\) We disallow a government to choose the fixed ratio in a way that would imply a subvention and focus on cases where the tax revenue remains non-negative. We also choose \( D^* = 0 \) in the simulations.

\(^{36}\) \( \tau = 0.219 \) is the average of top tax rate on corporate income within EU in 2017 (EC 2017). Scale = 20.
and thus the government becomes indifferent between the rules in our baseline case ($W^{*}.EBITDA = W^{*}.EBIT$). Note that this was already observed above when considering the case where $\lambda = 1$. 

Table 6. Simulation Results (variation in depreciation rate and relative weight)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\tau = 0.219$</th>
<th>$\delta = 0.25$</th>
<th>$\rho = 0.05$</th>
<th>$\lambda = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal values</td>
<td>$z_{EBITDA} = 0.17$</td>
<td>$z_{EBIT} = 1$</td>
<td>$K_{EBITDA} = 1111$</td>
<td>$K_{EBIT} = 1111$</td>
</tr>
<tr>
<td>Taxes and Profits</td>
<td>$T_{EBITDA} = 61$</td>
<td>$T_{EBIT} = 0$</td>
<td>$\pi^{AT,EBITDA} = 272$</td>
<td>$\pi^{AT,EBIT} = 333$</td>
</tr>
<tr>
<td>Solution:</td>
<td>EBITDA and EBIT imply equally high welfare</td>
<td>$(W_{EBITDA} = 333; W_{EBIT} = 333)$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\tau = 0.219$</th>
<th>$\delta = 0.25$</th>
<th>$\rho = 0.05$</th>
<th>$\lambda = 0.99$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal values</td>
<td>$z_{EBITDA} = 0.14$</td>
<td>$z_{EBIT} = 0.35$</td>
<td>$K_{EBITDA} = 1093$</td>
<td>$K_{EBIT} = 1052$</td>
</tr>
<tr>
<td>Taxes and Profits</td>
<td>$T_{EBITDA} = 65$</td>
<td>$T_{EBIT} = 55$</td>
<td>$\pi^{AT,EBITDA} = 268$</td>
<td>$\pi^{AT,EBIT} = 278$</td>
</tr>
<tr>
<td>Solution:</td>
<td>EBIT implies 0.1% higher welfare than EBIT</td>
<td>$(W_{EBITDA} = 331; W_{EBIT} = 330)$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\tau = 0.219$</th>
<th>$\delta = 0.25$</th>
<th>$\rho = 0.05$</th>
<th>$\lambda = 0.98$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal values</td>
<td>$z_{EBITDA} = 0.11$</td>
<td>$z_{EBIT} = 0$</td>
<td>$K_{EBITDA} = 1075$</td>
<td>$K_{EBIT} = 1014$</td>
</tr>
<tr>
<td>Taxes and Profits</td>
<td>$T_{EBITDA} = 69$</td>
<td>$T_{EBIT} = 84$</td>
<td>$\pi^{AT,EBITDA} = 264$</td>
<td>$\pi^{AT,EBIT} = 248$</td>
</tr>
<tr>
<td>Solution:</td>
<td>EBIT imply 0.1% higher welfare than EBIT</td>
<td>$(W_{EBITDA} = 328; W_{EBIT} = 328)$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\tau = 0.219$</th>
<th>$\delta = 0.15$</th>
<th>$\rho = 0.05$</th>
<th>$\lambda = 0.99$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal values</td>
<td>$z_{EBITDA} = 0.22$</td>
<td>$z_{EBIT} = 0.63$</td>
<td>$K_{EBITDA} = 2459$</td>
<td>$K_{EBIT} = 2392$</td>
</tr>
<tr>
<td>Taxes and Profits</td>
<td>$T_{EBITDA} = 89$</td>
<td>$T_{EBIT} = 51$</td>
<td>$\pi^{AT,EBITDA} = 441$</td>
<td>$\pi^{AT,EBIT} = 449$</td>
</tr>
<tr>
<td>Solution:</td>
<td>EBITDA implies 0.1% higher welfare than EBIT</td>
<td>$(W_{EBITDA} = 496; W_{EBIT} = 495)$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\tau = 0.219$</th>
<th>$\delta = 0.05$</th>
<th>$\rho = 0.05$</th>
<th>$\lambda = 0.99$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal values</td>
<td>$z_{EBITDA} = 0.46$</td>
<td>$z_{EBIT} = 0.87$</td>
<td>$K_{EBITDA} = 9824$</td>
<td>$K_{EBIT} = 9713$</td>
</tr>
<tr>
<td>Taxes and Profits</td>
<td>$T_{EBITDA} = 125$</td>
<td>$T_{EBIT} = 42$</td>
<td>$\pi^{AT,EBITDA} = 875$</td>
<td>$\pi^{AT,EBIT} = 958$</td>
</tr>
<tr>
<td>Solution:</td>
<td>EBITDA implies 0.1% higher welfare than EBIT</td>
<td>$(W_{EBITDA} = 991; W_{EBIT} = 990)$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In the other panels in Table 6 we vary the welfare weight that the government places on the firm after-tax profits ($\lambda$) and the depreciation rate ($\delta$). The second and third panels in the table show the results for cases where the welfare weights are $\lambda = 0.99$ and $\lambda = 0.98$. For these cases we observe that like in the baseline case the optimal fixed ratios differ between the EBITDA rule and the EBIT rule (0.14 vs 0.35 and 0.11 vs 0). We also observe that the fixed ratio under the EBIT rule changes very rapidly due to changes in the welfare weights, whereas the fixed ratio under the EBITDA rule is more stable. Regarding the optimal capital levels we observe that they are higher under the EBITDA rule. For the case where $\lambda = 0.99$ the EBITDA rule collects more taxes, but leaves companies with lower net-of-tax profits than the EBIT rule. For the case $\lambda = 0.98$ the opposite holds. However, in both cases the EBITDA rule implies higher welfare then the EBIT rule, even if the differences are not very large.

The two lower panels in the table study how the choices change when we vary the depreciation rate to receive values $\delta = 0.15$ and $\delta = 0.05$ and choose the welfare weight to be $\lambda = 0.99$. For both cases we observe that the EBIT rule implies clearly higher fixed ratio than the EBITDA rule. The EBITDA rule implies higher capital levels and higher tax revenues, but lower the net-of-tax profits for the companies. The welfare remains higher for EBITDA rule in all cases. However, again the difference is not very large.

Figure 2 studies in more detail how the welfare responses to the depreciation rate and the weight placed for the net-of-tax profits under the EBITDA and EBIT interest limitation rules. In addition to
overall welfare responses it also shows the responses of the components of the welfare as well as their central determinants, the optimal fixed ratios and the corresponding capital levels. The figure shows the optimal fixed ratios \((z^{*,EBITDA}, z^{*,EBIT})\), the capital levels \((K^{*,EBITDA}, K^{*,EBIT})\), taxes collected \((T^{*,EBITDA}, T^{*,EBIT})\), net-of-tax profits \((\pi^{AT,*,EBITDA}, \pi^{AT,*,EBIT})\) and the welfares \((W^{*,EBITDA}, W^{*,EBIT})\) for both EBITDA and EBIT rule when the depreciation rate varies from 0 to 0.25 and the relative weight from 0.95 to 1. First, we observe that the cases with no depreciation (left hand side columns) imply the exact same choices, because in the absence of depreciation the bases for EBITDA and EBIT become the same. Regarding the case with relative weight equal to 1 we have that the welfare implications are the same under both rules, like already discussed above. EBITDA rule collects more taxes in each case, but leaves less net-of-tax profits for the companies.

Regarding the other cases \((\delta \neq 0 \text{ and } \lambda \neq 1)\) we observe that both the depreciation rate and the welfare weight placed for the net-of-tax profits of a company affect the optimal choices of fixed ratios under both rules. With a given depreciation rate the optimal fixed ratio is larger the larger is the welfare weight for net-of-tax profits of a company. With a given welfare weight the optimal fixed ratio is smaller the larger is the depreciation rate. The table also shows that the optimal fixed ratio varies faster under the EBIT rule than in the EBITDA rule. Another thing we find is that the capital levels are higher under EBITDA rule in each case. However, sometimes the tax revenues are larger under the EBIT rule and sometimes under the EBITDA rule. The same holds for the net-of-tax profits of companies. Even if the tax revenues and net-of-tax profits of companies vary depending on the case, the welfare is higher (or equal to) under EBITDA rule than under the EBIT rule. Therefore, in each of the cases being studied the optimal choice of the government in the third step would be to choose the EBITDA interest limitation rule instead of the EBIT rule.

\[ M^{EBIT} = \pi(K) - \delta K \quad \text{and} \quad M^{EBITDA} = \pi(K) \] are the same when \(\delta = 0\).
Figure 2. EBITDA vs EBIT: Welfare and its components under different parameter values.

\[
\text{Welfare}(\tau=0.219; \rho=0.05)
\]
6.2 Optimal ESR when Capital is Heterogeneous

In this section we consider a two-asset model where the capital levels $K_1$ and $K_2$ depreciate at rates $\delta_1$ and $\delta_2$ respectively. Like above the government maximizes the welfare of a society ($W$), which is composed of a linear combination of the tax revenues ($T$) and the after-tax profits ($\pi^{AT} + D^*$). The model is again solved in three steps. Now the firm first chooses its (two) capital levels ($K_1$ and $K_2$) to maximize its net-of-tax profits with a given interest limitation rule ($EBITDA$ or $EBIT$) and the corresponding fixed ratio ($z^{EBITDA}$ or $z^{EBIT}$). Then with the given optimal firm choices the government chooses the optimal fixed ratio to maximize the welfare of a society for each rule ($EBITDA$ and $EBIT$). In the third step the government chooses the interest limitation rule in a welfare maximizing way and takes the optimal firm choices and the optimal fixed ratio choices as given. We consider a separable production function: $\pi(K_1, K_2) = \pi(K_1) + \pi(K_2)$. Thus we now have:

\begin{align}
W &= T + \lambda(\pi^{AT} + D^*) \\
T &= \tau[\pi(K_1) + \pi(K_2) - \delta_1 K_1 - \delta_2 K_2 - z^j M^j] \\
\pi^{AT} &= \pi(K_1) + \pi(K_2) - \delta_1 K_1 - \delta_2 K_2 - \rho(K_1 + K_2) - T = (1 - \tau)[\pi(K_1) + \pi(K_2) - \delta_1 K_1 - \delta_2 K_2] - \rho(K_1 + K_2) + \tau z^j M^j \\
D^* &= (1 - \tau^*)\bar{\pi}^*
\end{align}

**EBITDA: Firm Choice**

Let us first consider the EBITDA rule. The EBITDA-base is the gross profit of the firm:

\begin{equation}
M^{EBITDA} = \pi(K_1) + \pi(K_2)
\end{equation}

In the first step the firm chooses its capital levels to maximize the company profits ($\pi^{AT} + D^*$) given in equations (28) and (29). Using the EBITDA-base and maximizing the profits with respect to capital levels $K_1$ and $K_2$ gives the following expressions for the costs of capital for the two assets (see Appendix C):

\begin{align}
\pi'(K_1^{EBITDA}) &= \frac{\rho^+(1-\tau)\delta_1}{1-\tau + \tau z^{EBITDA}} = \frac{\rho^+(1-\tau)\delta_1}{1-\epsilon^{EBITDA}} \\
\pi'(K_2^{EBITDA}) &= \frac{\rho^+(1-\tau)\delta_2}{1-\tau + \tau z^{EBITDA}} = \frac{\rho^+(1-\tau)\delta_2}{1-\epsilon^{EBITDA}}
\end{align}

Each of the cost of capital equations implicitly gives us the optimal capital level as a function of given parameter values, $K_1^* = K_1^*(\tau, \rho, \delta_1, z^{EBITDA})$ and $K_2^* = K_2^*(\tau, \rho, \delta_2, z^{EBITDA})$. A particularly important issue to notice is that the optimal capital levels depend on the fixed ratio ($z^{EBITDA}$). For the asset that depreciates faster the cost of capital is higher and thus the optimal capital level lower ($\delta_1 > \delta_2 \Rightarrow K_1 < K_2$; the functional forms in the production function are the same for $K_1$ and $K_2$).
The cost of capital equations show us also another thing: the ratio of the cost of capital for the two types of assets is constant \( \frac{\pi'(K_{1}^{\text{EBITDA}})}{\pi'(K_{2}^{\text{EBITDA}})} = \frac{\rho+(1-\tau)\delta_1}{\rho+(1-\tau)\delta_2} \), and especially it does not depend on the fixed ratio, \( z^{\text{EBITDA}} \). Next let us consider the optimal fixed ratio determined in the second step in our procedure.

**EBITDA: Fixed Ratio Choice**

In the second step the government takes the firm optimal choices \((K_1^* = K_1^{(*)}(\tau, \rho, \delta_1, z^{\text{EBITDA}}) \) and \( K_2^* = K_2^{(*)}(\tau, \rho, \delta_2, z^{\text{EBITDA}}) \)) as given when choosing the optimal fixed ratio \((z^{\text{EBITDA}})\). Therefore, the government optimization problem is the following:

\[
\begin{align*}
\max_{z^{\text{EBITDA}}} W^{\text{EBITDA}} &= T + \lambda(\pi^A + D^*) = T(K_1^*(z^{\text{EBITDA}}), K_2^*(z^{\text{EBITDA}}), z^{\text{EBITDA}}) + \\
&+ \lambda(\pi^A(K_1^*(z^{\text{EBITDA}}), K_2^*(z^{\text{EBITDA}}), z^{\text{EBITDA}}) + D^*)
\end{align*}
\]

The FOC for the government optimization problem is:

\[
\begin{align*}
\frac{dW^{\text{EBITDA}}}{dz^{\text{EBITDA}}} &= \frac{\partial T}{\partial K_1^{\text{EBITDA}}} \frac{dK_1^{\text{EBITDA}}}{dz^{\text{EBITDA}}} + \frac{\partial T}{\partial K_2^{\text{EBITDA}}} \frac{dK_2^{\text{EBITDA}}}{dz^{\text{EBITDA}}} + \frac{\partial T}{\partial z^{\text{EBITDA}}} + \\
&+ \lambda \left( \frac{\partial (\pi^A + D^*)}{\partial K_1^{\text{EBITDA}}} \bigg|_{z^{\text{EBITDA}} = 0} \frac{dK_1^{\text{EBITDA}}}{dz^{\text{EBITDA}}} + \frac{\partial (\pi^A + D^*)}{\partial K_2^{\text{EBITDA}}} \bigg|_{z^{\text{EBITDA}} = 0} \frac{dK_2^{\text{EBITDA}}}{dz^{\text{EBITDA}}} + \frac{\partial (\pi^A + D^*)}{\partial z^{\text{EBITDA}}} \right) = 0
\end{align*}
\]

The two terms on the right hand side vanish due to the firm maximization in the first step (envelope theorem). Like in the single asset case, the FOCs compare the change in the net-of-tax profits to that in the tax revenue following from both the mechanical changes due to the change in the tightness of interest barrier and the firm capital responses. The FOC implies the following optimal fixed ratio (see Appendix C):

\[
\begin{align*}
z^{*,\text{EBITDA}} &= \\
\left(\pi'(K_1^{*,\text{EBITDA}}) - \delta_1\right) \frac{dK_1^{*,\text{EBITDA}}}{dz^{*,\text{EBITDA}}} + \left(\pi'(K_2^{*,\text{EBITDA}}) - \delta_2\right) \frac{dK_2^{*,\text{EBITDA}}}{dz^{*,\text{EBITDA}}} - (1 - \lambda)(\pi(K_1^{*,\text{EBITDA}}) + \pi(K_2^{*,\text{EBITDA}})) \\
\pi'(K_1^{*,\text{EBITDA}}) \frac{d\pi'\left(K_1^{*,\text{EBITDA}}\right)}{dz^{*,\text{EBITDA}}} + \pi'(K_2^{*,\text{EBITDA}}) \frac{d\pi'\left(K_2^{*,\text{EBITDA}}\right)}{dz^{*,\text{EBITDA}}}
\end{align*}
\]

where \( \frac{dK_1^{*,\text{EBITDA}}}{dz^{*,\text{EBITDA}}} = \frac{-\tau(\pi'\left(K_1^{*,\text{EBITDA}}\right))^2}{\pi'\left(K_1^{*,\text{EBITDA}}\right)(\rho+(1-\tau)\delta_1)} \) and \( \frac{dK_2^{*,\text{EBITDA}}}{dz^{*,\text{EBITDA}}} = \frac{-\tau(\pi'\left(K_2^{*,\text{EBITDA}}\right))^2}{\pi'\left(K_2^{*,\text{EBITDA}}\right)(\rho+(1-\tau)\delta_2)} \).

Equation (35) shows that the optimal fixed ratio is a function of both the parameter values and the optimal levels of capital, and the optimal capital levels are in turn functions of both the parameter values and the fixed ratio: \( z^{*,\text{EBITDA}} = z^{*,\text{EBITDA}}(\tau, \rho, \delta_1, \delta_2, K_1^{*}(\tau, \rho, \delta_1, z^{*,\text{EBITDA}}), K_2^{*}(\tau, \rho, \delta_2, z^{*,\text{EBITDA}})) \). Therefore, the optimal fixed
ratio is given implicitly in equations (31), (32) and (35). They also imply the corresponding optimal capital levels $K_1^{*,EBITDA}$ and $K_2^{*,EBITDA}$.

**EBIT: Firm Choice**

Let us next consider the EBIT rule. The EBIT-base is the gross profit minus the depreciation of capital (with the given capital levels the EBIT-base is narrower than the EBITDA-base):

$$M^{EBIT} = \pi(K_1) + \pi(K_2) - \delta_1 K_1 - \delta_2 K_2$$

(36)

The firm maximization yields the following cost of capital equations (see Appendix C):

$$\pi'(K_1^{*,EBIT}) = \delta_1 + \frac{\rho}{1 - \tau + \epsilon^{EBIT}} = \delta_1 + \frac{\rho}{1 - \epsilon^{EBIT}}$$

(37)

$$\pi'(K_2^{*,EBIT}) = \delta_2 + \frac{\rho}{1 - \tau + \epsilon^{EBIT}} = \delta_2 + \frac{\rho}{1 - \epsilon^{EBIT}}$$

(38)

The equations determine the optimal capital levels in the presence of EBIT interest limitation rule. Like for the EBITDA rule, also now the optimal capital levels depend on the fixed ratio (now on $z^{EBIT}$) and each cost of capital equation implicitly gives the optimal capital level as a function of given parameter values, $K_1^* = K_1^*(\tau, \rho, \delta_1, z^{EBIT})$ and $K_2^* = K_2^*(\tau, \rho, \delta_2, z^{EBIT})$. Unlike for the EBITDA rule, the ratio of cost of capitals depends now on the fixed ratio, whereas for the EBITDA rule it was a constant. The differences between the equations ((37) and (38) vs (31) and (32)) also imply different capital levels in the optimum for the EBITDA and EBIT rules.

**EBIT: Fixed Ratio Choice**

In the second step the government chooses the optimal fixed ratio to maximize the welfare and takes the firm choices as given. The FOC reads as follows:

$$\frac{dW^{EBIT}}{dz^{EBIT}} = \frac{\partial T}{\partial K_1^{*,EBIT}} \frac{dK_1^{*,EBIT}}{dz^{EBIT}} + \frac{\partial T}{\partial K_2^{*,EBIT}} \frac{dK_2^{*,EBIT}}{dz^{EBIT}} + \frac{\partial T}{\partial z^{EBIT}} +$$

$$+ \lambda \left( \frac{\partial (\pi^{AT} + D^*)}{\partial K_1^{*,EBIT}} \frac{dK_1^{*,EBIT}}{dz^{EBIT}} + \frac{\partial (\pi^{AT} + D^*)}{\partial K_2^{*,EBIT}} \frac{dK_2^{*,EBIT}}{dz^{EBIT}} + \frac{\partial (\pi^{AT} + D^*)}{\partial z^{EBIT}} \right) = 0$$

(39)

The FOC implies the following optimal fixed ratio (see Appendix C):

$$z^{*,EBIT} =$$

$$\left( \pi'(K_1^{*,EBIT}) - \delta_1 \right) \frac{dK_1^{*,EBIT}}{dz^{*,EBIT}} + \left( \pi'(K_2^{*,EBIT}) - \delta_2 \right) \frac{dK_2^{*,EBIT}}{dz^{*,EBIT}} - (1 - \lambda)\left( \pi(K_1^{*,EBIT}) + \pi(K_2^{*,EBIT}) - \delta_1 K_1^{*,EBIT} - \delta_2 K_2^{*,EBIT} \right)$$

$$\left( \pi'(K_1^{*,EBIT}) - \delta_1 \right) \frac{dK_1^{*,EBIT}}{dz^{*,EBIT}} + \left( \pi'(K_2^{*,EBIT}) - \delta_2 \right) \frac{dK_2^{*,EBIT}}{dz^{*,EBIT}}$$

(40)

where $\frac{dK_1^{*,EBIT}}{dz^{*,EBIT}} = -\frac{\tau}{\pi''(K_1^{*,EBIT})} (\delta_1)^2$ and $\frac{dK_2^{*,EBIT}}{dz^{*,EBIT}} = -\frac{\tau}{\pi''(K_2^{*,EBIT})} (\delta_2)^2$. 

25
Equation (40) shows the optimal fixed ratio as a function of both the parameter values and the optimal levels of capital, where the optimal capital levels are again functions of both the parameter values and the fixed ratio: 

\[ z^{*,\text{EBIT}} = z^{*,\text{EBIT}}(\tau, \rho, \delta_1, \delta_2, K_1^*(\tau, \rho, \delta_1, z^{*,\text{EBIT}}), K_2^*(\tau, \rho, \delta_2, z^{*,\text{EBIT}})). \]

Therefore, the optimal fixed ratio is given implicitly in equations (37), (38) and (40). It also implies the corresponding optimal capital levels \( K_1^{*,\text{EBIT}} \) and \( K_2^{*,\text{EBIT}} \).

**Case: \( \lambda = 1 \)**

Let us now consider the special case where \( \lambda = 1 \). Now the optimal fixed ratios for EBITDA and EBIT rules become the following:

\[
(35') \quad z^{*,\text{EBITDA}} = \frac{(\pi'(K_1^{*,\text{EBITDA}})-\delta_1)dk_1^{*,\text{EBITDA}}}{(\pi'(K_2^{*,\text{EBITDA}})-\delta_2)dk_2^{*,\text{EBITDA}}} (< 1)
\]

\[
(40') \quad z^{*,\text{EBIT}} = 1
\]

What we see here is that the optimal fixed ratio is lower under the EBITDA rule. The cost of capital equations become the following: \(^{38}\)

\[
(31') \quad \pi'(K_1^{*,\text{EBITDA}}) = \frac{[\rho+(1-\tau)\delta_1][\rho+\delta_1]}{[\rho+(1-\tau)\delta_1][\rho+\delta_1]}\frac{dk_1^{*,\text{EBITDA}}}{dz^{*,\text{EBITDA}}} + \frac{\rho+\delta_2}{\rho+\delta_2}\frac{dk_2^{*,\text{EBITDA}}}{dz^{*,\text{EBITDA}}}
\]

\[
(32') \quad \pi'(K_2^{*,\text{EBITDA}}) = \frac{[\rho+(1-\tau)\delta_2][\rho+\delta_2]}{[\rho+(1-\tau)\delta_2][\rho+\delta_2]}\frac{dk_1^{*,\text{EBITDA}}}{dz^{*,\text{EBITDA}}} + \frac{\rho+\delta_1}{\rho+\delta_1}\frac{dk_2^{*,\text{EBITDA}}}{dz^{*,\text{EBITDA}}}
\]

\[
(37') \quad \pi'(K_1^{*,\text{EBIT}}) = \delta_1 + \rho \text{ and}
\]

\[
(38') \quad \pi'(K_2^{*,\text{EBIT}}) = \delta_2 + \rho
\]

The equations for the fixed ratios and the cost of capitals show that under the EBIT rule the optimal capital choices become independent of the taxation, whereas under the EBITDA rule they depend on the taxation in a non-transparent way.

By substituting the optimal values for EBIT rule into equations (26) – (29) gives us the welfare, the taxes and the net-of-tax profits implied by these optimal choices straightforwardly:

\[
(26') \quad W^{*,\text{EBIT}} = \pi(K_1^{*,\text{EBIT}}) + \pi(K_2^{*,\text{EBIT}}) - (\rho + \delta_1)K_1^{*,\text{EBIT}} - (\rho + \delta_2)K_2^{*,\text{EBIT}} + D^*
\]

---

\(^{38}\) For the EBITDA rule the cost of capital is derived as follows: first insert (35') separately into (31) and (32) and solve for \( \pi'(K_1^{*,\text{EBITDA}}) \) and \( \pi'(K_2^{*,\text{EBITDA}}) \). Then multiply the first of the equations by \( \frac{dk_1^{*,\text{EBITDA}}}{dz^{*,\text{EBITDA}}} \) and the second one by \( \frac{dk_2^{*,\text{EBITDA}}}{dz^{*,\text{EBITDA}}} \) to get \( \pi'(K_1^{*,\text{EBITDA}}) \frac{dk_1^{*,\text{EBITDA}}}{dz^{*,\text{EBITDA}}} + \pi'(K_2^{*,\text{EBITDA}}) \frac{dk_2^{*,\text{EBITDA}}}{dz^{*,\text{EBITDA}}} \). Then use the fact that \( \pi'(K_1^{*,\text{EBITDA}}) = \frac{\rho+(1-\tau)\delta_1}{\rho+\delta_1} \) to get the cost of capital equations (31') and (32').
Again the EBIT rule collects no taxes, but leaves all the welfare for companies in terms of net-of-tax profits. For the EBITDA case the welfare, the taxes and the net-of-tax profits read as follows:

\[(26')\]
\[W^{*,\text{EBITDA}} = T^{*,\text{EBITDA}} + \lambda (\pi^{AT,*,\text{EBITDA}} + D^*)\]

where

\[T^{*,\text{EBITDA}} = T^{*,\text{EBIT}} + \lambda (\pi^{AT,*,\text{EBITDA}} + D^*)\]

Here \(\pi^{AT,*,\text{EBITDA}}\), \(\pi^{AT,*,\text{EBITDA}}\) and \(z^{*,\text{EBITDA}}\) are given in equations (31'), (32') and (35') respectively and \(K^{*,\text{EBITDA}}\) and \(K^{*,\text{EBITDA}}\) are implicitly given in equations (31') and (32').

If the assets depreciate at the same rate (\(\delta_1 = \delta_2 = \delta\)) the equations simplify a lot for the EBITDA rule and the optimal choices resemble those in the one-asset model in the earlier subsection. The optimal fixed ratio becomes: \(z^{*,\text{EBITDA}} = \frac{\rho}{\rho + \delta}\) and the optimal capital levels are determined from:

\[
\pi'\left(K^{*,\text{EBITDA}}\right) = \frac{\rho}{\rho + \delta}; \quad \pi'\left(K^{*,\text{EBITDA}}\right) = \frac{\rho}{\rho + \delta}.
\]

Like in the single asset model the government becomes also now indifferent between the rules. Moreover, the choices become independent of the taxation in this case as well. The welfare, the taxes and the net-of-tax profits become for both the EBIT and EBITDA rules the same as in equations (13'), (13''), (14'), (14''), (15+16') and (15+16'').

Next we relax the \(\lambda = 1\) assumption.

Choice of the System (EBITDA vs EBIT)

For the third step we employ the optimal solutions from the first two steps (equations (31), (32) and (35) for the EBITDA rule; equations (37), (38) and (40) for the EBIT rule) and compare the welfare under the EBITDA rule to that under the EBIT rule. Because the optimal capital levels and the optimal fixed ratios differ across the two rules, the equations from the first two steps do not allow us to compare their welfare implications straightforwardly, but the simulations are needed.

The procedure for the comparison goes in a similar manner than in the single asset model. We first fix the parameter values for \(\tau, \rho, \delta_1, \delta_2\) and \(\lambda\) and choose the functional form for the production function \(\pi\). After that we tentatively pick a value for the fixed ratio under a given interest limitation rule (\(z^{\text{EBITDA}}\) or \(z^{\text{EBIT}}\)). For the EBITDA rule we plug the chosen value for the fixed ratio \(z^{\text{EBITDA}}\) into equations (31) and (32) in order them to produce the optimal capital levels (\(K_1^{*}\) and \(K_2^{*}\)) corresponding to this particular fixed ratio. The resulting optimal capital levels are then plugged into equation (35) in order to provide the best response for the government in choosing the fixed ratio. The procedure is continued as long as this best response corresponds to the chosen tentative value and we have found the optimal fixed ratio \(z^{*,\text{EBITDA}}\) (and thus also the corresponding optimal capital
levels). The corresponding procedure is used for finding the optimal fixed ratio for the EBIT rule, $z^\cdot_{EBIT}$. After finding the optimal fixed ratios for both EBITDA and EBIT cases we compare their implied welfares, which then determine the optimal choice in the third step.

Let us first consider the following set of parameter values: $\tau = 0.219$, $\rho = 0.05$, $\delta_1 = 0.25$, $\delta_2 = 0.25$ and $\lambda = 1$. The functional form of the production function is chosen to be $\pi(K) = Scale \times K^{1/2}$, $\pi'(K) = -\frac{1}{4} Scale \times K^{-3/2}$. This case in the two-asset model corresponds to that in the single asset model, but now there are two assets that depreciate at the same rate $\delta_1 = \delta_2 = 0.25$. The upper panel in Table 7 depicts the results for this case. Compared to the corresponding case for a single asset given in the upper panel in Table 6, we see that the optimal fixed ratios are the exact same ones as in the single asset model. Each capital level is also chosen like the asset in the single asset model. Because there are now two assets instead of one, the tax revenues and the net-of-tax profits are now doubled compared to the single asset model. The welfare implications are the exact same as in the single asset model: the EBITDA rule implies the exact same welfare as the EBIT rule.

Table 7. Simulation Results (variation in depreciation rate and relative weight)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\tau = 0.219$</th>
<th>$\delta_1 = 0.25$</th>
<th>$\delta_2 = 0.25$</th>
<th>$\rho = 0.05$</th>
<th>$\lambda = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal fixed ratio</td>
<td>$z^\cdot_{EBITDA} = 0.17$</td>
<td>$z^\cdot_{EBIT} = 1$</td>
<td>$z^\cdot_{EBIT} = 1$</td>
<td>$z^\cdot_{EBIT} = 1$</td>
<td>$z^\cdot_{EBIT} = 1$</td>
</tr>
<tr>
<td>Optimal capital level</td>
<td>$K^\cdot_{EBITDA} = 1111$</td>
<td>$K^\cdot_{EBIT} = 1111$</td>
<td>$K^\cdot_{EBIT} = 1111$</td>
<td>$K^\cdot_{EBIT} = 1111$</td>
<td>$K^\cdot_{EBIT} = 1111$</td>
</tr>
<tr>
<td>Taxes and Profits</td>
<td>$T^\cdot_{EBITDA} = 122$</td>
<td>$T^\cdot_{EBIT} = 0$</td>
<td>$\pi^\cdot_{EBITDA} = 545$</td>
<td>$\pi^\cdot_{EBIT} = 667$</td>
<td></td>
</tr>
<tr>
<td>Solution:</td>
<td>EBITDA and EBIT imply equally high welfare</td>
<td>($W^\cdot_{EBITDA} = 667; W^\cdot_{EBIT} = 667$)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\tau = 0.219$</th>
<th>$\delta_1 = 0.25$</th>
<th>$\delta_2 = 0.25$</th>
<th>$\rho = 0.05$</th>
<th>$\lambda = 0.99$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal fixed ratio</td>
<td>$z^\cdot_{EBITDA} = 0.14$</td>
<td>$z^\cdot_{EBIT} = 0.35$</td>
<td>$z^\cdot_{EBIT} = 0.35$</td>
<td>$z^\cdot_{EBIT} = 0.35$</td>
<td>$z^\cdot_{EBIT} = 0.35$</td>
</tr>
<tr>
<td>Optimal capital level</td>
<td>$K^\cdot_{EBITDA} = 1093$</td>
<td>$K^\cdot_{EBIT} = 1052$</td>
<td>$K^\cdot_{EBIT} = 1052$</td>
<td>$K^\cdot_{EBIT} = 1052$</td>
<td>$K^\cdot_{EBIT} = 1052$</td>
</tr>
<tr>
<td>Taxes and Profits</td>
<td>$T^\cdot_{EBITDA} = 130$</td>
<td>$T^\cdot_{EBIT} = 110$</td>
<td>$\pi^\cdot_{EBITDA} = 536$</td>
<td>$\pi^\cdot_{EBIT} = 556$</td>
<td></td>
</tr>
<tr>
<td>Solution:</td>
<td>EBITDA implies 0.1% higher welfare than EBIT</td>
<td>($W^\cdot_{EBITDA} = 661; W^\cdot_{EBIT} = 661$)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^{39}$ Scale = 20

$^{40}$ Note that the similarity in the optimal conditions follows from the production function being separable.

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In the two consecutive panels in Table 7, we study the variation in the relative weight ($\lambda$) and consider relative weights $\lambda = 0.99$, and $\lambda = 0.98$. They show that the optimal conditions seen already in Table 6 are preserved in Table 7. The fixed ratios and the capital levels are chosen in the exact same way, and the tax revenues and the net-of-tax profits are doubled compared to that in Table 6 (because now there are two assets instead of one). A higher weight for the net-of-tax profits (higher $\lambda$) implies higher optimal fixed ratio, higher capital levels, lower tax revenues, higher net-of-tax profits and higher welfare. It is worth noting that the fixed ratio changes w.r.t $\lambda$ faster under the EBITDA than under the EBIT rule. The EBITDA rule also implies higher welfare than the EBIT rule in each case (exactly like in Table 6).

The two remaining panels in Table 7 vary the depreciation rates of the assets, while retaining them the same with each other ($\delta_1 = \delta_2 = 0.15; \delta_1 = \delta_2 = 0.05$). The table shows that the variation in the depreciation rates does not change the optimal fixed ratio choices from the single asset model as long as the depreciation rates do not differ from each other. Each individual asset is chosen like in the single asset model, but as there are now two assets the taxes collected and the net-of-tax profits are both twice the magnitude of those in the single asset model. Also the implications regarding the comparison between the EBITDA rule and the EBIT rule are the same: with the given depreciation rates they find that the EBITDA implies higher welfare than the EBIT.

### Table 8. Baseline Case + Differences in Depreciation Rates

<table>
<thead>
<tr>
<th>Parameters (baseline)</th>
<th>Solution:</th>
<th>EBITDA implies 0.1% higher welfare than EBIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau = 0.219$</td>
<td>$\delta_1 = 0.25$</td>
<td>$\delta_2 = 0.25$</td>
</tr>
<tr>
<td>Optimal fixed ratio</td>
<td>$z^{EBITDA} = 0.14$</td>
<td>$z^{EBIT} = 0.35$</td>
</tr>
<tr>
<td>Optimal capital level</td>
<td>$K^1_{EBITDA} = 1093$</td>
<td>$K^1_{EBIT} = 1052$</td>
</tr>
<tr>
<td>Taxes and Profits</td>
<td>$T^{EBITDA} = 130$</td>
<td>$T^{EBIT} = 110$</td>
</tr>
<tr>
<td>Solution:</td>
<td>$EBITDA$ implies 0.1% higher welfare than $EBIT$</td>
<td>$(W^{EBITDA} = 661; W^{EBIT} = 661)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Solution:</th>
<th>EBITDA implies 0.1% higher welfare than EBIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau = 0.219$</td>
<td>$\delta_1 = 0.25$</td>
<td>$\delta_2 = 0.20$</td>
</tr>
<tr>
<td>Optimal fixed ratio</td>
<td>$z^{EBITDA} = 0.15$</td>
<td>$z^{EBIT} = 0.43$</td>
</tr>
<tr>
<td>Optimal capital level</td>
<td>$K^1_{EBITDA} = 1103$</td>
<td>$K^1_{EBIT} = 1060$</td>
</tr>
<tr>
<td>Taxes and Profits</td>
<td>$T^{EBITDA} = 141$</td>
<td>$T^{EBIT} = 107$</td>
</tr>
<tr>
<td>Solution:</td>
<td>$EBITDA$ implies 0.1% higher welfare than $EBIT$</td>
<td>$(W^{EBITDA} = 727; W^{EBIT} = 727)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Solution:</th>
<th>EBITDA implies 0.1% higher welfare than EBIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau = 0.219$</td>
<td>$\delta_1 = 0.25$</td>
<td>$\delta_2 = 0.15$</td>
</tr>
<tr>
<td>Optimal fixed ratio</td>
<td>$z^{EBITDA} = 0.18$</td>
<td>$z^{EBIT} = 0.55$</td>
</tr>
<tr>
<td>Optimal capital level</td>
<td>$K^1_{EBITDA} = 1122$</td>
<td>$K^1_{EBIT} = 1072$</td>
</tr>
<tr>
<td>Taxes and Profits</td>
<td>$T^{EBITDA} = 154$</td>
<td>$T^{EBIT} = 99$</td>
</tr>
<tr>
<td>Solution:</td>
<td>$EBITDA$ implies 0.1% higher welfare than $EBIT$</td>
<td>$(W^{EBITDA} = 826; W^{EBIT} = 826)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Solution:</th>
<th>EBITDA implies 0.1% higher welfare than EBIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau = 0.219$</td>
<td>$\delta_1 = 0.25$</td>
<td>$\delta_2 = 0.10$</td>
</tr>
<tr>
<td>Optimal fixed ratio</td>
<td>$z^{EBITDA} = 0.24$</td>
<td>$z^{EBIT} = 0.70$</td>
</tr>
<tr>
<td>Optimal capital level</td>
<td>$K^1_{EBITDA} = 1157$</td>
<td>$K^1_{EBIT} = 1085$</td>
</tr>
<tr>
<td>Taxes and Profits</td>
<td>$T^{EBITDA} = 173$</td>
<td>$T^{EBIT} = 84$</td>
</tr>
<tr>
<td>Solution:</td>
<td>$EBITDA$ implies 0.1% higher welfare than $EBIT$</td>
<td>$(W^{EBITDA} = 991; W^{EBIT} = 990)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Solution:</th>
<th>EBIT implies higher welfare than EBITDA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau = 0.219$</td>
<td>$\delta_1 = 0.25$</td>
<td>$\delta_2 = 0.05$</td>
</tr>
<tr>
<td>Optimal fixed ratio</td>
<td>$z^{EBITDA} = 0.37$</td>
<td>$z^{EBIT} = 0.84$</td>
</tr>
<tr>
<td>Optimal capital level</td>
<td>$K^1_{EBITDA} = 1236$</td>
<td>$K^1_{EBIT} = 1098$</td>
</tr>
<tr>
<td>Taxes and Profits</td>
<td>$T^{EBITDA} = 193$</td>
<td>$T^{EBIT} = 64$</td>
</tr>
<tr>
<td>Solution:</td>
<td>$EBIT$ implies higher welfare than $EBITDA$</td>
<td>$(W^{EBITDA} = 1320; W^{EBIT} = 1320)$</td>
</tr>
</tbody>
</table>

Note that these are the same ($\delta_1 = \delta_2 = \delta$) as in the corresponding panels in Table 6.
A particularly intriguing issue in the two-asset model is that it allows to study, beyond the single asset model, the optimal choice when the useful lives of assets differ (δ₁ ≠ δ₂). We study these cases in Table 8. The first panel in the table reports the results when the depreciation rates are the same (δ₁ = δ₂ = 0.25; let us call this the baseline case). The three consecutive panels show the cases where the useful life of asset 1 is kept at the baseline level, δ₁ = 0.25, but the useful life of the other asset varies. The table shows that the lower the depreciation rate for the second asset (δ₂) and the higher the difference between the depreciation rates, the higher is the optimal fixed ratio for both rules. The optimal fixed ratio is also observed to respond to depreciation rate change more rapidly under the EBIT rule than under the EBITDA rule. The lower depreciation rate implies higher capital levels under both rules. The lower rate implies higher tax revenue under the EBITDA rule, whereas for the EBIT rule the tax revenue remains lower the lower is the depreciation rate. EBITDA implies higher social welfare in all but one case: the case with largest difference between the depreciation rules (when δ₁ = 0.25 and δ₂ = 0.05).

Figure 3. Optimal Fixed Ratios for EBITDA and EBIT
Let us next consider the optimal fixed ratio choices and the welfare implications between the rules in a more detailed level. Let us first consider in Figure 3 the optimal fixed ratios when the depreciation rates ($\delta_1$ and $\delta_2$) vary between 0.01 and 0.25. The upper right hand corner of the figure shows the optimal fixed ratios for the EBITDA rule and the EBIT rule when the depreciation rates are both 0.25 ($\delta_1 = \delta_2 = 0.25$). The upper of the numbers (0.14) stands for the EBITDA rule and the lower one (0.35) stands for the EBIT rule. Note that these are the numbers observed also in the first panel in Table 8 and in the second panel of Table 7, which considers the cases where the depreciation rates of the two assets coincide with each other. Notice also that given that the optimal fixed ratio choices are the same also with a single asset, these fixed ratios are also seen in the second panel of Table 6 as well. Moreover, given that the figure covers all the cases where the depreciation rates vary between 0.01 and 0.25, it also covers all the fixed ratio results in tables 6 – 8 where the relative weight is 0.99 ($\lambda = 0.99$).

Figure 3 shows that the optimal fixed ratio is smaller under the EBITDA rule than under the EBIT rule in each case. It also shows that with a fixed depreciation rate $\delta_2$ ($\delta_1$) the higher the other depreciation rate $\delta_1$ ($\delta_2$) the lower the optimal fixed ratio typically is. Consider for instance, the first row where $\delta_2 = 0.25$. Here the optimal fixed ratio for the EBITDA rule varies between 0.14 and 0.65, with 0.14 corresponding to the case ($\delta_1, \delta_2$) = (0.25,0.25) and 0.65 corresponding to the case ($\delta_1, \delta_2$) = (0.01,0.25). When $\delta_2 = 0.25$, the optimal fixed ratio for the EBIT rule varies between 0.35 and 0.93. Even if the variation is quite large for the cases, where $\delta_2 = 0.25$, it is much less, say when $\delta_2 = 0.01$. The first column in the figure shows these cases. Here the optimal fixed ratio for EBITDA rule varies between 0.65 and 0.79. For the EBIT rule the corresponding variation remains between 0.93 and 0.94. Without some exceptions the optimal fixed ratio is higher the higher the depreciation rate. One of the exceptions is the case for the EBITDA rule, when moving from the case ($\delta_1, \delta_2$) = (0.06,0.24) to the case ($\delta_1, \delta_2$) = (0.06,0.25). Here the fixed ratio changes from 0.33 to 0.34. For the EBIT rule we observe the same when moving from the case ($\delta_1, \delta_2$) = (0.06,0.22) to the case ($\delta_1, \delta_2$) = (0.06,0.23). Now the fixed ratio changes from 0.81 to 0.82.

Figures 4 and 5 shows the optimal choices (EBIT or EBITDA) with given optimal fixed ratios depicted in Figure 3 and the corresponding underlying optimal capital levels. The figure shows that for the most of the cases the EBITDA rule implies higher welfare and is therefore chosen by the government. However, there are some cases where the EBIT rule is chosen instead. These are the case where the depreciation rate of at least one of the assets is very low. For instance, for all the cases where $\delta_1=0.01$ ($\delta_2=0.01$) and $\delta_2 \geq 0.03$ ($\delta_1 \geq 0.03$), the EBIT rule becomes preferable. In cases where the depreciation rate of at least one of the assets is greater than or equal to 0.06 makes the EBITDA rule the optimal choice of the government. In each case where the depreciation rates do not differ, the EBITDA rule is preferable choice for the interest limitation rule. The cases where the EBITDA rule is preferable is depicted in dark gray in figure 5. The EBIT rule is preferable in the light gray areas of the figure.
Figure 4. Detailed optimal choices between the EBITDA and EBIT rules
Figure 5. Optimal choices between the EBITDA and EBIT rules
7. Conclusions

We have studied the properties of interest limitation rules designed to limit profit-shifting activity of multinational enterprises (MNEs). The focus is on two leading earnings stripping rules (ESRs), one of which places a restriction on the tax-deductibility of interest expenses based on EBIT, another based on EBITDA. We have studied the implications of these rules on profit-shifting, investment incentives and social welfare.

Given that the interest limitation rules work by limiting the deductibility of total interest expenses of a company, an essential aspect is whether companies reduce their internal or external debt as a response to these rules. While the target of these rules is to cut debt-related profit-shifting (artificial; debt-shifting) they should at the same time not to disrupt the financing of real activity. We find that when the foreign country tax rate is greater than zero, the MNE response to a binding ESR is that it first reduces internal interest expenses and only after that the external interest expenses. The rules hence target first at profit-shifting which is good news for policy makers.

We also find quite different investment incentives and neutrality properties between the EBIT and EBITDA rules. While the cost of capital remains independent of the useful lives of assets under the EBIT rule, this neutrality is lost under the EBITDA rule. A binding EBITDA rule has the property that it distorts the allocation of capital by favoring the assets with short economic life. The EBITDA rule is also more sensitive to the key interest limitation parameter, the fixed ratio, than the EBIT rule.

Regarding the social welfare we find that the optimal fixed ratios chosen by the government differ between the EBITDA rule and the EBIT rule. Also the optimal corresponding choices of the capital levels of the company differ between the rules. These differences in turn imply differences in the tax revenues and the net-of-tax profits of companies as well as in the social welfare. We find that the EBITDA rule provides higher social welfare in most cases, yet the results depend on the parameter values. The cases where the EBIT rule implies higher welfare are usually those with large differences between the useful lives of assets.
References


European Commission (2017): Taxation Trends in the European Union, Data for the EU Member States, Iceland and Norway, DG Taxation and Customs Union


Government bill (2017): ... [TBA]


Appendix A. Basic model and MNE’s optimal policies

Model

The model introduced in Sec. 2 is as follows:

\[(A1a)\] \[\max_{(D, L, S, D^*)} \int_0^\infty De^{-\rho t} dt\]

\[(A1b)\] \[\dot{K} = I - \delta K\]

\[(A1c)\] \[\dot{k} = I - ak\]

\[(A1d)\] \[\dot{B} = L\]

\[(A1e)\] \[\dot{K}^* = I^*\]

\[(A1f)\] \[(1 - \tau)\pi(K) + \tau(C + ak) + L + D^* = I + D + iB + S\]

\[(A1g)\] \[ (1 - \tau^*)[\pi(K^*) + S] = D^* + I^* \]

\[(A1h)\] \[D \geq 0, \ B \leq bK, \ S \leq S^*\]

Equation \((A1a)\) gives the objective of the firm and equations \((A1b), (A1c), (A1d)\) and \((A1e)\) the equations of motion for the domestic capital stock \(K\), accounting stock of capital (book capital) \(k\), the stock of debt \(B\), and foreign stock of capital \(K^*\), respectively. \((A1f)\) represents the parent’s and \((A1g)\) the subsidiary’s budget constraint. The conditions in \((A1h)\) place inequality constraints on the control variables \(D\) and \(S\), as well as the stock of debt \(B\).

The Lagrangean of the model is:

\[H = D + q_1\{(1 - \tau)\pi(K) + L + D^* + \tau(ak + C) - iB - S - D - \delta K\} + \]
\[+q_2\{(1 - \tau)[\pi(K) - ak] + L + D^* + \tau C - iB - S - D\} + q_3L + \]
\[+q_4\{(1 - \tau^*)[\pi(K^*) + S] - D^*\} + \mu_1D + \mu_2(bK - B) + \mu_3(S - S^*) \]

where \(q_j, j = 1, 2, 3, 4\) are the co-state variables for \(K, k, B\) and \(K^*\) respectively, and \(\mu_1, \mu_2\) and \(\mu_3\) are Lagrange multipliers of the lower boundary of \(D\) and the upper boundaries of \(B\) and \(S\), respectively.

The optimality conditions are:

\[(A2a)\] \[\frac{\partial H}{\partial D} = 1 - q_1 - q_2 + \mu_1 = 0\]

\[(A2b)\] \[\frac{\partial H}{\partial L} = q_1 + q_2 + q_3 = 0\]
\[ \frac{\partial H}{\partial D} = q_1 + q_2 - q_4 = 0 \]

\[ \frac{\partial H}{\partial S} = -(q_1 + q_2) + \tau \frac{\partial C}{\partial S} (q_1 + q_2) + (1 - \tau^*)q_4 - \mu_3 = 0 \]

\[ \dot{q}_1 = \rho q_1 - [(1 - \tau)\pi' + \tau \frac{\partial C}{\partial K}](q_1 + q_2) + \delta q_1 - b\mu_2 \]

\[ \dot{q}_2 = \rho q_2 - \tau(\alpha + \frac{\partial C}{\partial k})(q_1 + q_2) + \alpha q_2 \]

\[ \dot{q}_3 = \rho q_3 + (i - \tau \frac{\partial C}{\partial B})(q_1 + q_2) + \mu_2 \]

\[ \dot{q}_4 = \rho q_4 - q_4(1 - \tau^*)\pi'(K^*) \]

The partial derivatives of deductible interest expenses $C$ with respect to $K$, $B$, $k$ and $S$ are summarized in Table 2 in the main text.

We analyze the MNE’s optimal choices in a steady state. We can consider this state by assuming $D > 0 \rightarrow \mu_1 = 0$. From conditions (A2a), (A2b), (A2c) and (A2d) we now obtain:

\[ q_1 + q_2 = q_4 = 1, \quad q_3 = -1, \quad \Rightarrow \dot{q}_1 + \dot{q}_2 = \dot{q}_3 = \dot{q}_4 = 0. \]

In this state also $\dot{q}_1 = \dot{q}_2 = 0$ separately.

**Financing decision**

Inserting the steady state values of $\dot{q}_2$, $q_1 + q_2$ and $q_3$ into (A2g) we can rewrite the condition for debt, $B$, as follows:

\[ \rho - \left( i - \tau \frac{\partial C}{\partial B} \right) = \mu_2. \]

The condition for optimal profit shifting, $S$, is obtained by inserting the steady-state values of $q_1 + q_2$ and $q_4$ into (A2d).:

\[ -\left( 1 - \tau \frac{\partial C}{\partial S} \right) + (1 - \tau^*) = \mu_3. \]

**Investment decision**

We continue by considering the MNE’s choice of $K$. Using steady-state values of $q_1 + q_2$ and $\dot{q}_2$ and solving condition (A2f) for $q_2$, we obtain the following expression:

\[ q_2 = \frac{\tau}{\alpha + \frac{\partial C}{\partial k}} \frac{\alpha \frac{\partial C}{\partial k}}{\alpha + \rho} \]

A condition for the firm’s cost of capital can be derived by solving condition (A2g) for $\mu_2$ and substituting this and $q_2$ from (A4) in condition (A2e):


\[(A5)\quad [(1 - \tau)\pi' + \tau \frac{\partial C}{\partial K}] = (\rho + \delta) \left(1 - \tau \frac{\alpha + \frac{\partial C}{\partial K}}{\alpha + \rho}\right) - b\rho + b(i - \tau \frac{\partial C}{\partial B}).\]

In what follows we assume that investment is financed entirely with debt, \( b = 1. \)

In standard CIT all interest expenses are deductible without a ceiling, therefore \( \frac{\partial C}{\partial B} = i \) and \( \frac{\partial C}{\partial K} = 0. \) The tax saving from depreciation allowances, \( q_2, \) now is:

\[(A4')\quad q_2 = \tau \frac{\alpha}{a+i} \equiv \tau A.\]

The expression for the cost of capital becomes:

\[(A5')\quad \pi' = \frac{1 - \tau A}{1 - \tau} \left((1 - \tau)i + \delta\right) - \frac{\tau i}{1 - \tau} \tau A.\]  \quad \text{(CIT)}

In the case of CBIT interest costs are non-deductible without any threshold. Therefore \( \frac{\partial C}{\partial B} = \frac{\partial C}{\partial K} = 0. \)

Substituting these into \((A5)\) we obtain:

\[(A5^i)\quad \pi' = \frac{1 - \tau A}{1 - \tau} (i + \delta).\]  \quad \text{(CBIT)}

Under the EBITDA rule the interest cap is \( C = z\pi(K). \) This means that under a binding cap \( \frac{\partial C}{\partial B} = \frac{\partial C}{\partial K} = 0 \) and \( \frac{\partial C}{\partial k} = z\pi'(K). \) Inserting these values into the condition \((A5)\) we obtain the following expression for the cost of capital:

\[(A5^ii)\quad \pi' = \frac{1 - \tau A}{1 - \epsilon} (i + \delta),\]  \quad EBITDA-rule

where \( \epsilon = \tau(1 - z) \) is the effective tax rate.

The EBIT rule caps interest payments at \( z(\pi(K) - \alpha k). \) In the case of a binding constraint, \( \frac{\partial C}{\partial B} = -z\alpha, \frac{\partial C}{\partial k} = z\pi'(K) \) and \( \frac{\partial C}{\partial B} = 0. \)

Using equation \((A5)\) we obtain the following expression for the cost of capital

\[(A5^v)\quad \pi'(K) = \frac{1 - \epsilon A}{1 - \epsilon} (i + \delta).\]  \quad \text{(EBIT rule)}
Appendix B. Model with endogenous fiscal depreciation

We make the following changes to the basic model. The firm chooses the amount of fiscal depreciation $F$ from the range $F \in [\delta k, \alpha k]$, where $\alpha > \delta$ is the maximum rate of (accelerated) fiscal depreciation and $\delta$ is the minimum depreciation rate. Now, book capital $k$ develops as $\dot{k} = I - F$, corporate tax $T$ is calculated $T = \tau[\pi(K) - F - C]$, and the gross income concept of the ESR threshold is $M^{EBITDA} = \pi(K)$, $M^{EBIT} = \pi(K) - F$.

The Lagrangean of the model is:

\[ (B1) \quad H = D + q_1\{(1 - \tau)\pi(K) + L + D^* + \tau(F + C) - iB - S - D - \delta K\} + \\
+ q_2\{(1 - \tau)[\pi(K) - F] + L + D^* + \tau C - iB - S - D\} + q_3L + \\
+ q_4\{(1 - \tau^*)[\pi(K^*) + S] - D^*\} + \mu_1D + \mu_2(bK - B) + \mu_3(S - S) + \\
+ \mu_4(F - \delta k) + \mu_5(\alpha k - F) \]

where $\mu_4$ and $\mu_5$ are shadow prices of the lower and upper boundaries of fiscal depreciation $F$.

The optimality conditions are as in Appendix A with the following changes. First, there is a new condition that considers the optimal choice of $F$:

\[ (B2) \quad \frac{\partial H}{\partial F} = \tau(1 + \frac{\partial C}{\partial F})(q_1 + q_2) - q_2 + \mu_4 - \mu_5 = 0, \]

Second, instead of (A2f), the equation of motion for $q_2$ now is:

\[ (B3) \quad \dot{q}_2 = \rho q_2 + \mu_4 \delta - \mu_5 \alpha. \]

By considering the conditions in a steady state ($q_1 + q_2 = 1, \dot{q}_2 = 0$.) and substituting $q_2$ from (B3) in (B2), we can rewrite the optimality condition for $F$ as follows:

\[ (B2') \quad \tau \left(1 + \frac{\partial C}{\partial F}\right) = \frac{\alpha + \rho}{\rho} \mu_5 - \frac{\delta + \rho}{\rho} \mu_4. \]
Appendix C. Welfare calculations

We first show how the cost of capital equations (17) and (18) are derived.

The firm chooses its capital level \( K \) to maximize the net-of-tax profits

\[
\pi^{AT} + D^* = \pi(K) - \delta K - \rho K - T + (1 - \tau^*)\tilde{\pi}^* = (1 - \tau)[\pi(K) - \delta K] - \rho K + \tau z^i M^i + (1 - \tau^*)\tilde{\pi}^*
\]

Here the foreign after-tax profits \((1 - \tau^*)\tilde{\pi}^*\) are considered as exogenous and therefore do not affect the optimization. The first order condition for the maximization is:

\[
\frac{\partial}{\partial K} (\pi^{AT} + D^*) = (1 - \tau)[\pi'(K) - \delta] - \rho + \tau z^i \frac{\partial M^i}{\partial K} = 0 \quad (j \in \{EBIT, EBITDA\})
\]

For EBITDA \( M^{EBITDA} = \pi(K), \frac{\partial M^{EBITDA}}{\partial K} = \pi'(K) \) and thus we have:

\[
\frac{\partial}{\partial K} (\pi^{AT} + D^*) = 0 \iff (1 - \tau + \tau z^{EBITDA})\pi'(K) - (1 - \tau)\delta - \rho = 0
\]

The term \((1 - \tau + \tau z^{EBITDA})\pi'(K)\) corresponds to an increase in the net-of-tax profits, \((1 - \tau)\delta\) stands for additional cost of increased depreciation and \(\rho\) is the opportunity cost. Solving the equation gives us the cost of capital equation for EBITDA \(\epsilon^{EBITDA} = \tau(1 - z^{EBITDA})\):

\[
\pi'(K^{*,EBITDA}) = \frac{\rho + (1 - \tau)\delta}{1 - \tau + \tau z^{EBITDA}} = \frac{\rho + (1 - \tau)\delta}{1 - \epsilon^{EBITDA}} \quad \text{ (equation 17)}
\]

For EBIT \( M^{EBIT} = \pi(K) - \delta K, \frac{\partial M^{EBIT}}{\partial K} = \pi'(K) - \delta \) and thus (inserting into equation C2):

\[
\frac{\partial}{\partial K} (\pi^{AT} + D^*) = 0 \iff (1 - \tau + \tau z^{EBIT})\pi'(K) - \delta - \rho = 0
\]

Now \((1 - \tau + \tau z^{EBIT})\pi'(K)\) stands for an increase in the net-of-tax profits, \((1 - \tau + \tau z^{EBIT})\delta\) is the cost of increased depreciation, and \(\rho\) is the opportunity cost. Compared to EBITDA there is an additional term \((\tau z^{EBIT} \delta)\) for the cost of depreciation for EBIT. For EBIT we have the following cost of capital equation \(\epsilon^{EBIT} = \tau(1 - z^{EBIT})\):

\[
\pi'(K^{*,EBIT}) = \delta + \frac{\rho}{1 - \tau + \tau z^{EBIT}} = \delta + \frac{\rho}{1 - \epsilon^{EBIT}} \quad \text{ (equation 18)}
\]

Next we show how the equations (22) and (23) for the optimal fixed ratios are derived.

The government maximizes its welfare with respect to the fixed ratio and takes into account the optimal firm choice. The government optimization problem is thus to maximize the following welfare function with respect to the fixed ratio \(j \in \{EBIT, EBITDA\}\):

\[
W^j = T + \lambda(\pi^{AT} + D^*) = T(K^*(z^j), z^j) + \lambda(\pi^{AT}(K^*(z^j), z^j) + D^*)
\]
For EBITDA the FOC is:

\[ \frac{dW^\text{EBITDA}}{dz^\text{EBITDA}} = \frac{\partial T}{\partial K^\text{EBITDA}} \frac{dK^\text{EBITDA}}{dz^\text{EBITDA}} + \frac{\partial T}{\partial z^\text{EBITDA}} + \lambda \left( \frac{\partial (\pi'^\text{AT} + D^*)}{\partial K^\text{EBITDA}} \frac{dK^\text{EBITDA}}{dz^\text{EBITDA}} + \frac{\partial (\pi'^\text{AT} + D^*)}{\partial z^\text{EBITDA}} \right) = 0 \]

\[ \lambda \left( \pi'^\text{EBITDA} - \delta - z^\text{EBITDA} \pi^\text{EBITDA} \right) \frac{dK^\text{EBITDA}}{dz^\text{EBITDA}} = (1 - \lambda)(\pi(K^\text{EBITDA}) - \delta K^\text{EBITDA}) \]

Here the first term in brackets vanish due to the first stage firm maximization. Solving for the equation for the EBITDA fixed ratio gives us:

\[ z^\text{EBITDA} = \frac{(\pi'^\text{EBITDA} - \delta) \frac{dK^\text{EBITDA}}{dz^\text{EBITDA}} - (1 - \lambda)(\pi(K^\text{EBITDA}) - \delta K^\text{EBITDA})}{\pi'^\text{EBITDA} \frac{dK^\text{EBITDA}}{dz^\text{EBITDA}}} \]  

(equation 22)

The terms within the optimal fixed ratio equation can be calculated by using the cost of capital equation (C4), which implicitly gives the optimal capital level to be used here in (C9), and the term \( \frac{dK^\text{EBITDA}}{dz^\text{EBITDA}} \) can be shown to be \( \frac{dK^\text{EBITDA}}{dz^\text{EBITDA}} = -\tau \pi'^\text{EBITDA} \). To see this let us take a derivative of the cost of capital equation (C4) with respect to the fixed ratio:

\[ \frac{d}{dz^\text{EBITDA}} \left( \frac{\partial}{\partial z^\text{EBITDA}} \left( \pi'^\text{AT} + D^* \right) \right) = \tau \pi'(K) + (1 - \tau + \tau z^\text{EBITDA}) \pi''(K) \]

\[ \frac{dK^\text{EBITDA}}{dz^\text{EBITDA}} = \frac{-\tau \pi'(K^\text{EBITDA})}{\pi''(K^\text{EBITDA})(1 - \tau + \tau z^\text{EBITDA})} \]

where the last equality follows from equation (C3).

For EBIT the FOC is:

\[ \frac{dW^\text{EBIT}}{dz^\text{EBIT}} = \frac{\partial T}{\partial K^\text{EBIT}} \frac{dK^\text{EBIT}}{dz^\text{EBIT}} + \frac{\partial T}{\partial z^\text{EBIT}} + \lambda \left( \frac{\partial (\pi'^\text{AT} + D^*)}{\partial K^\text{EBIT}} \frac{dK^\text{EBIT}}{dz^\text{EBIT}} + \frac{\partial (\pi'^\text{AT} + D^*)}{\partial z^\text{EBIT}} \right) = 0 \]

\[ \lambda \left( \pi'(K^\text{EBIT}) - \delta - z^\text{EBIT} (\pi'(K^\text{EBIT}) - \delta) \right) \frac{dK^\text{EBIT}}{dz^\text{EBIT}} = (1 - \lambda)(\pi(K^\text{EBIT}) - \delta K^\text{EBIT}) \]

Solving the equation for the fixed ratio gives us:

\[ z^\text{EBIT} = \frac{(\pi'(K^\text{EBIT}) - \delta) \frac{dK^\text{EBIT}}{dz^\text{EBIT}} - (1 - \lambda)(\pi(K^\text{EBIT}) - \delta K^\text{EBIT})}{(\pi'(K^\text{EBIT}) - \delta) \frac{dK^\text{EBIT}}{dz^\text{EBIT}}} \]  

(equation 23)

The cost of capital is given in equation (C6) and the term \( \frac{dK^\text{EBIT}}{dz^\text{EBIT}} = \frac{-\tau \pi'(K^\text{EBIT}) - \delta)^2}{\pi''(K^\text{EBIT})} \) is derived like the corresponding equation for the EBITDA by taking a derivative of the cost of capital equation with respect to the fixed ratio and then using the cost of capital condition given in (C5).
Let us next derive the results for the two-asset model. The optimal capital level for each asset ($n \in \{1,2\}$) is derived from a firm net-of-tax profit maximization, where net-of-tax profits are:

(C13) \[ π^{AT} + D^* = (1 - τ)[π(K_1) + π(K_2) - δ_1K_1 - δ_2K_2] - ρ(K_1 + K_2) + τz^{1M} + (1 - τ^*)\tilde{π}^* \]

For EBITDA $M^{EBITDA} = π(K_1) + π(K_2)$, for EBIT $M^{EBIT} = π(K_1) + π(K_2) - δ_1K_1 - δ_2K_2$ and the subsidiary net-of-tax profits are exogenous. The FOCs for each capital under both systems are:

(C14) \[ \frac{∂}{∂K_1}(π^{AT} + D^*) = (1 - τ + τz^{EBITDA})π'(K_1) + (1 - τ)\delta_1 - ρ = 0 \]

(C15) \[ \frac{∂}{∂K_2}(π^{AT} + D^*) = (1 - τ + τz^{EBITDA})π'(K_2) + (1 - τ)\delta_2 - ρ = 0 \]

(C16) \[ \frac{∂}{∂K_1}(π^{AT} + D^*) = (1 - τ + τz^{EBIT})[π'(K_1) - δ_1] - ρ = 0 \]

(C17) \[ \frac{∂}{∂K_2}(π^{AT} + D^*) = (1 - τ + τz^{EBIT})[π'(K_2) - δ_2] - ρ = 0 \]

Solving for $π'(K_1)$ and $π'(K_2)$ in each case gives straightforwardly the cost of capital equations (31), (32) for EBITDA and (37) and (38) for EBIT.

(C18) \[ π'(K_1^{EBITDA}) = \frac{ρ + (1-τ)\delta_1}{1 - τ + τz^{EBITDA}} = \frac{ρ + (1-τ)\delta_1}{1 - ε^{EBITDA}} \] (equation 31)

(C19) \[ π'(K_2^{EBITDA}) = \frac{ρ + (1-τ)\delta_2}{1 - τ + τz^{EBITDA}} = \frac{ρ + (1-τ)\delta_2}{1 - ε^{EBITDA}} \] (equation 32)

(C20) \[ π'(K_1^{EBIT}) = δ_1 + \frac{ρ}{1 - τ + τz^{EBIT}} = δ_1 + \frac{ρ}{1 - ε^{EBIT}} \] (equation 37)

(C21) \[ π'(K_2^{EBIT}) = δ_2 + \frac{ρ}{1 - τ + τz^{EBIT}} = δ_2 + \frac{ρ}{1 - ε^{EBIT}} \] (equation 38)

Each of the cost of capital equations implicitly gives the optimal capital level, which depends on the parameter values and especially on the fixed ratio ($z^{EBITDA}$ or $z^{EBIT}$).

Let us next show how the optimal fixed ratios are derived. The government maximizes its welfare under each of the systems (EBITDA and EBIT) and takes into account the firm choices. Thus the welfare for each system reads as follows:

(C22) \[ W^{EBITDA} = T + \lambda(π^{AT} + D^*) = T(K_1^*(z^{EBITDA}), K_2^*(z^{EBITDA}), z^{EBITDA}) + \lambda(π^{AT}(K_1^*(z^{EBITDA}), K_2^*(z^{EBITDA}), z^{EBITDA}) + D^*) \]

(C23) \[ W^{EBIT} = T + \lambda(π^{AT} + D^*) = T(K_1^*(z^{EBIT}), K_2^*(z^{EBIT}), z^{EBIT}) + \lambda(π^{AT}(K_1^*(z^{EBIT}), K_2^*(z^{EBIT}), z^{EBIT}) + D^*) \]
The FOCs for the government optimization problems are:

(C24) \[
\frac{dW^{EBITDA}}{dz^{EBITDA}} = \frac{\partial T}{\partial K_1^{EBITDA}} \frac{dK_1^{EBITDA}}{dz^{EBITDA}} + \frac{\partial T}{\partial K_2^{EBITDA}} \frac{dK_2^{EBITDA}}{dz^{EBITDA}} + \frac{\partial T}{\partial z^{EBITDA}} + \\
\lambda \left( \frac{\partial (\pi^{AT+D'})}{\partial K_1^{EBITDA}} \frac{dK_1^{EBITDA}}{dz^{EBITDA}} + \frac{\partial (\pi^{AT+D'})}{\partial K_2^{EBITDA}} \frac{dK_2^{EBITDA}}{dz^{EBITDA}} + \frac{\partial (\pi^{AT+D'})}{\partial z^{EBITDA}} \right) = \tau \left( \pi^{'}(K_1^{EBITDA}) - \delta_1 - \\
z^{EBITDA}\pi^{'}(K_1^{EBITDA}) \right) \frac{dK_1^{EBITDA}}{dz^{EBITDA}} + \left( \pi^{'}(K_2^{EBITDA}) - \delta_2 - z^{EBITDA}\pi^{'}(K_2^{EBITDA}) \right) \frac{dK_2^{EBITDA}}{dz^{EBITDA}} - (1 - \lambda) \left( \pi(K_1^{EBITDA}) + \pi(K_2^{EBITDA}) \right)
\]

(C25) \[
\frac{dW^{EBIT}}{dz^{EBIT}} = \frac{\partial T}{\partial K_1^{EBIT}} \frac{dK_1^{EBIT}}{dz^{EBIT}} + \frac{\partial T}{\partial K_2^{EBIT}} \frac{dK_2^{EBIT}}{dz^{EBIT}} + \frac{\partial T}{\partial z^{EBIT}} + \\
\lambda \left( \frac{\partial (\pi^{AT+D'})}{\partial K_1^{EBIT}} \frac{dK_1^{EBIT}}{dz^{EBIT}} + \frac{\partial (\pi^{AT+D'})}{\partial K_2^{EBIT}} \frac{dK_2^{EBIT}}{dz^{EBIT}} + \frac{\partial (\pi^{AT+D'})}{\partial z^{EBIT}} \right) = \tau \left( \pi^{'}(K_1^{EBIT}) - \delta_1 - z^{EBIT}\pi^{'}(K_1^{EBIT}) - \delta_1 \right) \frac{dK_1^{EBIT}}{dz^{EBIT}} \\
+ \left( \pi^{'}(K_2^{EBIT}) - \delta_2 - z^{EBIT}\pi^{'}(K_2^{EBIT}) - \delta_2 \right) \frac{dK_2^{EBIT}}{dz^{EBIT}} - (1 - \lambda) \left( \pi(K_1^{EBIT}) + \pi(K_2^{EBIT}) \right)
\]

Solving for the fixed ratio gives the equations for the optimal fixed ratios in (35) and (40):

(C26) \[
z^{*EBITDA} = \\
\frac{\left( \pi^{'}(K_1^{EBITDA}) - \delta_1 \right) \frac{dK_1^{EBITDA}}{dz^{EBITDA}} + \left( \pi^{'}(K_2^{EBITDA}) - \delta_2 \right) \frac{dK_2^{EBITDA}}{dz^{EBITDA}} - (1 - \lambda) \left( \pi(K_1^{EBITDA}) + \pi(K_2^{EBITDA}) \right)}{\pi^{'}(K_1^{EBITDA}) \frac{dK_1^{EBITDA}}{dz^{EBITDA}} + \pi^{'}(K_2^{EBITDA}) \frac{dK_2^{EBITDA}}{dz^{EBITDA}}} \] (equation 35)
\( z^{*,EBIT} = \)

\[
\frac{\left(\pi'(K_{1}^{*,EBIT}) - \delta_1\right)^2 dK_{1}^{*,EBIT}}{(\pi''(K_{1}^{*,EBIT}))^2 (\rho + (1-\tau)\delta_2)} + \frac{\left(\pi'(K_{2}^{*,EBIT}) - \delta_2\right)^2 dK_{2}^{*,EBIT}}{(\pi''(K_{2}^{*,EBIT}))^2 (\rho + (1-\tau)\delta_2)}
\]

\( \) (equation 40)

The optimal capital levels are implicitly given in the cost of capital equations (C18) – (C21) and for the capital responses to the fixed ratio we have

\[
d_{K_{1}^{*,EBITDA}} = \frac{-\tau(\pi'(K_{1}^{*,EBIT}) - \delta_1)^2}{\pi''(K_{1}^{*,EBIT})\rho} \quad \text{and} \quad d_{K_{2}^{*,EBITDA}} = \frac{-\tau(\pi'(K_{2}^{*,EBIT}) - \delta_2)^2}{\pi''(K_{2}^{*,EBIT})\rho}. \]

The first two of these equations is derived by differentiating the cost of capital equations (C18) and (C19) with respect to \( z^{*,EBITDA} \) and (C20) and (C21) with respect to \( z^{*,EBIT} \) and then solving for the corresponding derivative by using also one of the equations (C14) – (C18) in each case.