Optimal income taxation in the presence of networks of altruism *

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Abstract

This paper characterizes the optimal income tax in the presence of networks of altruism. Networks of altruism arise when individuals take into account the private utility of other individuals as well as their own, into their utility. In our paper, individuals form groups of altruism, that materialize through monetary transfers that they make to each other. Individuals thus have two dimensions of choices: their labour income and the transfers they make to other individuals. First, we establish the structure of incomes and transfers under the laissez-faire situation. Second, we characterize the optimal linear income tax and show that depending on the distribution of the homogeneity of groups, because of partial crowding out transfers, the optimal tax rate should adjust for the elasticity of transfers to the tax rate. Finally, the optimal non-linear income tax has a more ambiguous effect.

Keywords: Optimal taxation, networks, altruism, private transfers.
JEL classification: D31, H21, H3, D64, D85.

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1 Introduction

Individuals form networks of altruism when they take into account the private utilities of other individuals into their own social utility. As in the standard optimal income taxation literature, they also have different abilities to earn income. Taking into account their choice of labor supply, this then determines a network of private transfers between individuals. Supposing the network of altruism is composed of different components – for example, extended families, or communities – we characterize the labor supply and transfer choices of individuals when there is no tax. We then find the optimal linear tax rate for a utilitarian social planner. The equity-efficiency trade-off is now affected by the transfers taking place between individuals. Taxes here do not crowd out one-for-one transfers, and should thus be, in some cases, adjusted to the elasticity of transfers to the tax rate. This depends inter alia on the average of group diversity in the society in question.

Two motivations for looking at this question stand out.

In many countries and settings, small monetary transfers between individuals are pervasive. In developing countries in particular, total private transfers between individuals represent important fractions of total income. Cox and Jimenez (1990) provide examples of average transfer amounts as percentages of average (global) income in rural India where over the years 1975-1983 they represented 8% of average global income in those regions, in Kenya where this figure was 3% in 1974, or Malaysia where this was approximately 11%. There could at least however, be three other (or simultaneously important) motives for transfers in these contexts: insurance, social norms, or, to a lesser extent perhaps, warm-glow of giving. In the current paper, we focus on altruistic motives for transfers, and look at optimal tax design in this setting. Given that these are societies where the State is not very large, it could be interesting, from a policy perspective, to provide some insights into the way tax policy should be designed according to this feature of the economy. Let us note, however, that we leave informal economy considerations aside.

The second motivation is a theoretical one. Standard optimal income taxation literature (see Salanié (2011)) commonly considers individuals as separate entities, or as belonging to a (small) household – generally made up of two spouses and eventually, children. In this paper, we consider larger networks linking contemporaneous individuals and look at how this affects our standard results.

Abstracting away from informal sector considerations, and supposing that State capacity is such that a major part of incomes could be taxed, our aim is thus to characterize the optimal income tax if there are private transfers taking place between individuals. To the extent that a government has to finance public infrastructures (such as schools, hospitals,...), it is important
to determine how the equity-efficiency trade-off of the tax they have to levy on the population is affected by the fact that there are altruistic links and transfers between individuals.

We build on the setting of Bourlès, Bramoullé and Perez-Richet (2017). Individuals are all part of a network as described at the beginning of this section – the adjacency matrix of this network having for entries, the weights associated to the private utilities of the other individuals in the society. In their paper, those weights are random non-negative numbers that are less than one. In our setting, the structure of the network of altruism is such that individuals belong to different components of a network (so that the adjacency matrix is block diagonal). Each component is as in Arrow (1981), with different proportions of high skilled and low skilled individuals. Different components may represent extended families or villages for example.

Unlike Bourlès et al. (2017) and Arrow (1981), however, earnings are endogenous. Individuals thus have to choose both their labor supply and the transfers they make. We model this as a simultaneous game, where each individual chooses their income and transfers taking the choices of the other individuals as given. This induces a new network, one of transfers. Quite intuitively, transfers do not take place between individuals of different components.

We characterize the laissez-faire economy in this case. We find that transfers will only flow from the high skilled to the low skilled, and do so only if altruism is high enough or if skill levels are different enough. This might give a sense of why this setting would not be appropriate for low altruism or high homophily environments. Moreover, this is consistent with the fact that even though low-ability individuals value the private utilities of high-ability individuals in their component, they will still not make transfers to high-ability individuals. Indeed, since it is more costly for them to work, it would not make sense for them to work more in order to make a transfer. In the same way, it does not make sense for one individual to do all the transfers. All individuals of the same type thus behave identically.

Now when a linear tax on income is introduced, we find partial crowding out of transfers. This is due to two factors: on the one hand, the tax rate reduces the size of transfers, and on the other hand, the resulting lump-sum redistribution reduces the incentives for transfers. If lump-sum redistribution is high enough and if on average, groups are not very heterogeneous, then at the optimum, transfers are crowded out. The optimal income tax formula should thus take into account the elasticity of transfers to the tax rate. However, despite the fact that there are no other motives for transfers than (pure) altruism, distortive taxes are not neutral. This is because all individuals are not linked, because altruism is two-sided and because the government is not perfectly informed about the labour supply of individuals (their type is private information).

The remainder of this paper is organized as follows. The next section discusses related lit-
erature. In Section 3 we present the formal framework. In section 4, we characterize the laissez-faire economy, and in Section 5, we characterize the optimal income tax in this setting. We conclude in Section 6.

2 Related literature

There are three seminal papers looking at altruistic links between individuals. Barro’s 1974 article (Barro (1974)) features an overlapping generations model with finitely lived individuals and a "chain of operational [i.e., positive] intergenerational transfers" – that is, a connection from the young to the old or from the old to the young. In Becker’s 1974 article (Becker (1974)), a family head is altruistic towards the rest of their family. Finally, Bergstrom, Blume and Varian (1986) consider a society where all wealthy individuals have one-sided altruism for all poor individuals. The structure of altruism in such a society thus allows the income of poor individuals to be considered as a public good. These three articles hold neutrality results, whereby private transfers and exogenous, publicly decided redistribution of wealth (public debt and social security in the case of Barro (1974)) achieve the same equilibrium (see Mercier Ythier (2006) for a detailed exposure of these results). Bernheim and Bagwell (1988) push Barro’s results further by making the point that through dynastic considerations, operative linkages are widespread in the society. They find neutrality results which they deem untenable, such as the irrelevance of all public redistribution, distortionary taxes, and prices. In this paper, such neutrality results do not arise for at least three reasons. First, our network of altruism is made of components, so that there is no path of altruism connecting any two individuals in the society. Second, altruism is two-sided. And finally labor supply is chosen endogenously, and is not known to the government: there is asymmetry of information on the abilities of individuals.

As we are considering two-sided altruism, our paper is more closely related to another seminal paper: Arrow (1981), in which individuals of different wealth levels are all linked with the same altruistic weights. Moreover, the most recent paper we rely on is Bourlès et al. (2017), which characterizes the Nash equilibria of transfers in any network of altruism, but with a given wealth distribution for individuals, hence no endogenous labor supply. They do, however, characterize the risk-sharing implications of such networks by introducing stochastic incomes in a sequel paper, Bourlès, Bramoullé and Perez-Richet (2018). The incomes in our paper, on the other hand, are deterministic (they depend on the abilities of individuals and public policy in place) and do not face shocks.

Even if the informal economy is large in many developing countries, we abstract away from such considerations. We thus rely on two strands of the empirical literature in developing countries – one that has focused on motives for sharing and the other that estimates the elasticity of private transfers to taxes.
Cox, Hansen and Jimenez (2004) for example find that patterns of transfers are consistent with both risk sharing and altruism. Boltz, Marazyan and Villar (2019), however, find that private transfers are more consistent with social norms motives, as they highlight evidence of income hiding behavior. Tackling these other motives would require different frameworks, which may be of interest for future work.

Moreover, as mentioned in the introduction and as discussed in Bénabou and Tirole (2006), there exist of course many other motives for pro-social behavior, such as the well-known warm-glow of giving motive as first formalized in Andreoni (1990). The latter, however, might be more pertinent when considering motives for charity contributions (e.g., Diamond (2006), Saez (2004), Fack and Landais (2010)), in which the literature does not find complete crowding out of private transfers, because of warm-glow of giving.

There is also an extensive literature on the formation of risk sharing networks (see Fafchamps and Gubert (2007), Bloch, Genicot and Ray (2008), Bramoullé and Kranton (2007), De Weerdt and Dercon (2006)). We take the network of altruism as given.

The literature focusing on the elasticity of private transfers to taxes and subsidies in developing countries (e.g. Strupat and Klohn (2018), Heemskerk, Norton and De Dehn (2004)) finds evidence for partial crowding-out. For example, Cox and Jimenez (1992) find that in Peru, social security reduced the amount of private transfers from young to old by approximately 20 percent. Another perspective on these questions is to consider the coexistence of (formal) markets and informal networks. Theoretically, Kranton (1996) finds results that depend on the initial share of reciprocal exchange, and Gagnon and Goyal (2017) find that resulting aggregate welfare, inequality and strength of social ties depend on whether the market and the network are complements or substitutes. In India, Mobarak and Rosenzweig (2013) and Mobarak and Rosenzweig (2012) find a positive interaction between formal and informal insurance.

After having analyzed the Nash equilibrium of the laissez-faire situation, our framework is an optimal income taxation one, as developed in Mirrlees (1971), Piketty (1997) and Saez (2001) and exposed in Piketty and Saez (2013) and Salanié (2011).

As mentioned in the introduction, optimal income taxation of couples has been vastly studied both theoretically and empirically (see the most recent works by Gayle and Shephard (2019), Obermeier (2018)). Notably, the extensive work by Chiappori (1988), Chiappori (1992), Alderman, Chiappori, Haddad, Hoddinott and Kanbur (1995) on household members’ collective and individual decisions or Kleven, Kreiner and Saez (2009) which accounts for relative home and labour production of secondary earners but not of bargaining weights between spouses. In the former articles, sharing of resources is done in a first stage, before the labor supply decision has been made. In these models, the progressivity of the income tax schedule as well as the
Network considerations are starting to appear more and more in the taxation literature but through another perspective: tax evasion. Indeed, when taking into account State capacity considerations, it is increasingly noticeable that the extent to which individuals have access to tax avoidance schemes depends on their networks – see for example Nordblom and Ohlsson (2006), Di Porto and Ohlsson (2016) for tax avoidance in the context of Italian families and a tax on real estate, or Alstadsæter, Kopczuk and Telle (2018) in the context of Norway, where social ties may give access to tax-avoidance shelters.

Finally, even though we set ourselves within a country, this may help think about optimal taxation in the context of international transfers and remittances, even though, of course, this would then be at an international level; see for example Kopczuk, Slemrod and Yitzhaki (2005).

3 Theoretical framework

Private preferences. Individuals have a utility function $u$ that is increasing in private goods consumption, or after-tax and -transfer income, $c$, and decreasing in earnings $y$. Utility also depends on a measure of the individual’s productive ability $\omega$. There are two types of skill levels – low and high: $\omega_L$ and $\omega_H$, with $\omega_L < \omega_H$. We refer to $\omega$ as the individual’s type. It is not observable by the government. The private utility$^1$ that an individual with type $\omega$ derives from $c$ and $y$ is denoted by $u(c, y, \omega)$. It is increasing in consumption and decreasing in income, thus reflecting the cost of working.

Spence-Mirrlees single crossing property. We assume that $-\frac{u_y(c, y, \omega)}{u_c(c, y, \omega)}$, the marginal rate of substitution between labour and consumption is decreasing in the individual’s skills, i.e. for any pair $(c, y)$ and any $\omega, \omega'$ with $\omega' > \omega$,

$$\frac{u_y(c, y, \omega')}{u_c(c, y, \omega')} \leq \frac{u_y(c, y, \omega)}{u_c(c, y, \omega)}.$$

In other words, individuals with higher ability need less compensation in consumption to work more.

We also assume that an individual’s marginal utility of consumption $u_c(c, y, \omega)$ is both non-increasing in $c$ and non-increasing in $\omega$, i.e. $u_{cc}(c, y, \omega) \leq 0$ and $u_{c\omega}(c, y, \omega) \leq 0$.

Groups and altruistic ties. Individuals belong to groups of different proportions $\pi$ of high skill individuals (individuals of ability $\omega_H$).$^2$ These groups may be thought of as extended families or villages for example. The distribution of proportions in the population is represented

$^1$This may also be referred to, as in Pareto (1916) ophelimity.

$^2$\(\pi\) is thus between 0 and 1.
by a cumulative distribution function $F(\pi)$ with density $f(\pi)$. All groups have the same size $n$, and constitute the different components of the total network of altruism. Note that the distribution function of $\pi$’s will be an important feature of our model. It will constitute a proxy for the homogeneity of groups in the society. Moreover, since the structure of the network is given, this distribution is exogenous.

Within the groups, individuals have the same altruistic links to each other. As a result, as in Bourlès et al. (2017) the social utility of an individual $i$ belonging to group $C$ is equal to:

$$U_i(c, y, \omega_L, \omega_H) = u(c_i, y_i, \omega_i) + \sum_{j \neq i}^{j \in C} \alpha u(c_j, y_j, \omega_j)$$

with $\alpha < 1$.

The vectors $c$ and $y$ represent the vectors of consumption levels and incomes in the group.

Thus, at the society level, the adjacency matrix of altruistic ties may be represented by a block diagonal matrix, whose blocks are matrices with a diagonal equal to 1 and all other entries equal to $\alpha$. $\alpha$ represents the level of altruism in the society.

Note that $\alpha < 1$ implies that an individual always values their own marginal consumption more than they do for the other individuals in their group.

Individuals make transfers to each other, so that

$$c_i = y_i - T(y_i) - \sum_{j \neq i}^{j \in C} t_{ij} + \sum_{j \neq i} t_{ji}$$

Where $t_{ij}$ are transfers from individual $i$ to individual $j$ and $t_{ji}$ from individual $j$ to individual $i$ (so that $t_{ij}$ and $t_{ji}$ are both non negative), and $T(.)$ is the income tax decided upon by the government. Under the *laissez-faire* situation, $T(y) = 0$.

Labour supply and transfers are then determined as a Nash Equilibrium of a simultaneous game, where individuals choose their labor supply at the same time as their transfers, taking the actions of the other individuals as given.

Finally, we shall use the following specification for the private utility functions. We take them to be concave but with no income effect; specifically:

$$u(c_i, y_i, \omega_i) = v(c_i - k\left(\frac{y_i}{\omega_i}\right))$$

where $v$ is increasing and concave and $k$, which reflects the cost of effort, is increasing and convex.
4 Transfers and incomes under the laissez-faire

Let us first note that given the structure of the network of altruism, it is never the case that an individual belonging to a component makes a transfer to an individual belonging to another component of the network.

Let us thus place ourselves in a group – or component – $C^\pi$ with a proportion $\pi$ of individuals of high type.

The program of individual $i$ is the following:

$$\max\{y_i, t_{ij} \mid j \neq i\} U_i(c, y_i, \omega_L, \omega_H)$$

Their first order conditions are then the following:

- with respect to earnings $y_i$: \( u_c(c_i, y_i, \omega_i) = u_y(c_i, y_i, \omega_i) \)
- with respect to transfer $t_{ij}$: \( -u_c(c_i, y_i, \omega_i) + \alpha u_c(c_j, y_j, \omega_j) \leq 0 \)

which can be rewritten as

\[ \omega_i = k'\left(\frac{y_i}{\omega_i}\right) \tag{1} \]

\[ -v'\left(y_i - \sum_{j \neq i} t_{ik} + \sum_{j \neq i} t_{ki} - k\left(\frac{y_i}{\omega_i}\right)\right) + \alpha v'\left(y_j - \sum_{k \neq j} t_{jk} + \sum_{k \neq j} t_{kj} - k'\left(\frac{y_j}{\omega_j}\right)\right) \leq 0 \tag{2} \]

As in Bouriès et al. (2017), if $t_{ij} > 0$, then (2) binds.

Moreover, if $t_{ij} > 0$, then $t_{ji} = 0$.

Indeed, suppose that both are positive. Then,

\[ u_c(c_i, y_i, \omega_i) = \alpha u_c(c_j, y_j, \omega_j) < u_c(c_j, y_j, \omega_j) \]
\[ u_c(c_j, y_j, \omega_j) = \alpha u_c(c_i, y_i, \omega_i) < u_c(c_i, y_i, \omega_i) \]

which is a contradiction.

Importantly, note that since there are no income effects, the labor supply of individuals does not take transfers into account. It is the same as it would have been without altruism.

**Lemma 1** The longest path of transfers is of length 1.

As in Arrow (1981), suppose that individual $i$ makes a transfer to individual $j$, who in turn makes a transfer to individual $k$. Then, from (2),

\[ u_c(c_i, y_i, \omega_i) = \alpha u_c(c_j, y_j, \omega_j) \quad \text{and} \quad u_c(c_j, y_j, \omega_j) = \alpha u_c(c_k, y_k, \omega_k) \]

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Then: \(u_c(c_i, y_i, \omega_i) = \alpha^2 u_c(c_k, y_k, \omega_k)\).

But from (2), we also have that \(u_c(c_i, y_i, \omega_i) \geq \alpha u_c(c_k, y_k, \omega_k)\). So that \(\alpha \geq 1\), which is a contradiction.

**Corollary 1** It is never the case that an individual of low skill makes a transfer to an individual of high skill, nor that individuals of same skill make transfers to each other.

Indeed, since working is more costly for lower skilled individuals than it is for higher skilled individuals, then even though they have altruistic links with high skilled individuals, low skilled individuals will never earn more income than them and hence make then a transfer. In the same way, since individuals prefer a extra marginal unit of consumption for themselves than for the others in their component (since \(\alpha < 1\)), they will never want to work the same as an individual of the same type to then consume less.

In particular, if the groups are completely homogeneous, there are no transfers taking place.

Hence, the only kinds of transfers that might take place at equilibrium are transfers from high types to low types.

The formal proof is in the Appendix.

**Proposition 1** The Nash equilibrium is such that within each component \(C^n\)

(i) There is a unique allocation for each type: the profile of incomes and consumption are identical within types.

(ii) The total sum of transfers is unique.

In the Appendix, we first show that if an individual of type H makes a transfer, then all individuals of type H make transfers. Then, the sum of their transfers and their consumption is the same. Finally, all individuals of type L receive the same sum of transfers and thus have the same consumption.

However, the profile of transfers is not unique. For example, suppose we are in a situation with two individuals of high type, 1 and 2, and two of low type, 3 and 4. Suppose there are transfers taking place at the equilibrium. Then, as established above, 1 and 2 each make the same sum of transfers \(t\). This could however take place in an infinity of ways. In particular, 1 could transfer \(t\) to 3, and 2 could do the same to
4. Or 1 could transfer \( t/2 \) to 3 and \( t/2 \) to 4, and 2 the same.

We have thus shown that there was only one equilibrium possible. In this equilibrium, within each component:
- either there is no transfer at all.
- or all high type individuals make positive transfers to low type individuals of their component.
In this case the total sum of transfers is unique (i.e. each high type individual makes the same total amount of transfers).

All individuals of high types thus make the same total transfer \( t \geq 0 \) to individuals of low type. Individuals of low type receive \( \frac{\pi}{1-\pi} t \) each.
Hence, if \( i \) is of high type,
\[
c_i = c_H = y_H - t
\]
And if \( j \) is of low type,
\[
c_j = c_L = y_L + \frac{\pi}{1-\pi} t
\]

Application 1. (The proofs are in the Appendix)

Let us use the following specifications for our utility functions:
\[
v(c - k\left( \frac{y}{\omega} \right)) \text{ such that } v(x) = \frac{x^{1-\eta}}{1-\eta} \text{ and } k\left( \frac{y}{\omega} \right) = \left( \frac{y}{\omega} \right)^{1+\frac{1}{\epsilon}}.
\]
Now condition (1) above implies that \( k'\left( \frac{y}{\omega} \right) = \omega \) and thus \( y = \omega^{1+\epsilon} \).

As mentioned earlier, labour supply does not adjust to transfers, and this stems from the fact that there are no income effects.

The equilibrium with transfers arises if and only if \( \alpha^{1/\eta} > \left( \frac{\omega_L}{\omega_H} \right)^{1+\epsilon} \).

So that transfers are optimal if and only if \textbf{there is enough altruism or if the differences in skills are large enough}. This is a condition that depends on exogenously fixed parameters, but that does not depend on \( \pi \). If transfers take place in one component, they will take place in all the other components and \textit{vice-versa}.

In that case, in component \( C^\pi \): \( t = \frac{1}{1 + \epsilon} \frac{\alpha^{1/\eta} \omega_H^{1+\epsilon} - \omega_L^{1+\epsilon}}{\alpha^{1/\eta} + \frac{\pi}{1-\pi}} \).

Here, as stated above, \( y_H \) and \( y_L \) are the same as if there wasn’t any altruism. The inequality in consumption levels, however, is lower than if there wasn’t any altruism. As expected, the total transfers \( t \) from each high skilled individual decrease when \( \pi \) increases. Moreover, the gap
between \( y_H - t \) and \( y_L + \frac{\pi}{1-\pi} t \) decreases as \( \pi \) increases. Hence, in groups with higher shares of low skill individuals, not only do high skill individuals each operate more redistribution, but they also do so at a higher "sacrifice" from their individual incomes.

This gives a sense as to why, if the government decided to tax incomes without taking into account transfers, then if transfers still took place, this might be deemed unfair for groups where there is a higher share of low skill individuals, and where each high skilled individuals make higher transfers (in absolute and relative terms) – thus donating a higher share of their income to low skilled members of their network component – while at the same time reducing consumption gaps more.

5 Optimal income taxation

The first consideration to examine when looking at a normative objective of the government is the social welfare function it chooses to maximize. It could maximize a function either of private utilities, \( SWF_p \), or of social utilities, \( SWF_s \), where

\[
SWF_p = \int z_i G(u^i(c_i, y_i)) \, d\nu(i)
\]

and

\[
SWF_s = \int z_i G(U^i(c_i, y_i)) \, d\nu(i)
\]

where \( z_i \) are (arbitrary) Pareto weights, \( G \) is increasing, and \( d\nu(i) \) is the distribution of individuals.\(^3\)

Let us note that if the tax maintains a symmetric equilibrium – that is, if individuals of the same type still behave identically in the Nash equilibrium – then it is the same to maximize an unweighted utilitarian social welfare function\(^4\) of private utilities or of social utilities. The proof is in the appendix.

In the following, however, we will consider a paternalistic social planner. Our social welfare function of interest will thus be \( SWF_p \), which we will refer to as \( SWF \) from now on. This enables us to avoid reaching illiberal conclusions, as discussed in Sen (1970), Fleurbaey and Maniquet (2011) and very clearly summed up in Bierbrauer (2019).

5.1 Optimal linear income taxation

Suppose the government uses a linear tax rate \( \tau \in [0, 1] \), a demogrant \( R \), and potentially has additional exogenous non-transfer spending \( E \). The demogrant represents a lump-sum distribution of the tax proceeds to the individuals in the population.

\(^3\)Here, \( c_i \) and \( y_i \) are the vectors of consumption levels and incomes in the component \( i \) belongs to.

\(^4\)That is, for which \( z^i = 1 \) for all \( i \) and \( G \) is the identity function.
The results that there is a unique profile of incomes and consumption and that the total sum of transfers is unique stay the same. We are still in a situation where at equilibrium, in each component, either there are no transfers taking place, or all high ability individuals make (the same total) transfers to low ability individuals. The reasoning follows very closely that of the laissez-faire situation.

Suppose the social planner is unweighted utilitarian. Then their program is the following:

$$\max_{\tau,R} \int_0^1 \left( \pi u((1 - \tau)y_H + R - t, y_H, \omega_H) + (1 - \pi)u((1 - \tau)y_L + R + \frac{\pi}{1 - \pi}t, y_L, \omega_L) \right) f(\pi) \, d\pi$$

under the (government budget) constraint that

$$\tau \int_0^1 (\pi y_H(\pi, \tau) + (1 - \pi) y_L(\pi, \tau)) \, f(\pi) \, d\pi \geq R + E$$

Define $Y(\tau) := \int_0^1 (\pi y_H(\pi, \tau) + (1 - \pi) y_L(\pi, \tau)) \, f(\pi) \, d\pi$, the aggregate income.

This enables us to define, in turn, $e = \frac{(1 - \tau)}{Y} \frac{dY}{d(1 - \tau)}$ the elasticity of aggregate earnings with respect to the net-of-tax rate.

**Proposition 2** The optimal linear tax rate is of the form: $\tau = \frac{1 - \overline{g} - \phi - e}{1 - \overline{g} - \phi + e}$.

Where $\overline{g}$ is the usual average normalized social marginal welfare weight weighted by pre-tax incomes:

$$\overline{g} = \frac{\int_0^1 (\pi y_H u_H^c + (1 - \pi) y_L u_L^c) \, f(\pi) \, d\pi}{Y \int_0^1 (\pi u_H^c + (1 - \pi) u_L^c) \, f(\pi) \, d\pi}$$

And $\phi$ is a new factor to take into account:

$$\phi = \frac{\int_0^1 \pi \frac{\partial}{\partial \tau} (u_H^c - u_L^c) \, f(\pi) \, d\pi}{Y \int_0^1 (\pi u_H^c + (1 - \pi) u_L^c) \, f(\pi) \, d\pi}$$

which depends on how private transfers vary with the tax rate.

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5Note that $t$ might be equal to 0.
6With a slight abuse of notation, we write $y_L$ and $y_H$ instead of $y_L(\pi, \tau)$ and $y_H(\pi, \tau)$ and $u_L^c$ and $u_H^c$ for $u_c(c_L(\pi, \tau), y_L(\pi, \tau), \omega_L)$ and $u_c(c_H(\pi, \tau), y_H(\pi, \tau), \omega_H)$.
7In the same way, here, we also mean $t(\pi, \tau)$. 

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We may compare this with the standard formula \( \tau = \frac{1 - \bar{g}}{1 - \bar{g} + e} \). For identical \( \bar{g} \) and \( e \), this is higher or equal than our optimal tax rate if \( \phi > 0 \). In particular, if transfers decrease with the tax rate and if at the optimum there is no complete crowding out of transfers, then the tax rate will be lower than it would have been if the social planner had applied the usual formula without taking into account the \( \phi \) term. Indeed, by taking into consideration the partial crowding out of transfers, the government may take advantage of the fact that there is already some redistribution occurring at a private level – a redistribution that does not distort labor supply incentives – and thus decrease the marginal tax rate used to redistribute at the society level – thus also reducing the amount of distortion created by the tax.

**Application 1. (sequel)** (The proofs are in the Appendix)

Let us follow-up with our example from the previous section. This will give a sense of when the social planner will find it efficient to encourage partial redistribution through private transfers while at the same redistributing at the society level.

Remember we use the following specifications for our utility functions:

\[
v(c - k(\frac{y}{\omega})) \quad \text{such that} \quad v(x) = \frac{x^{1-\eta}}{1-\eta} \quad \text{and} \quad k(\frac{y}{\omega}) = \left(\frac{y}{\omega}\right)^{1+\frac{1}{\epsilon}}.
\]

Here, incomes of high and low ability individuals will be the following: \( y_H = (1 - \tau)^{\epsilon}\omega_H^{1+\epsilon} \) and \( y_L = (1 - \tau)^{\epsilon}\omega_L^{1+\epsilon} \).

Again, let us note that labor supply does not depend on transfers. It is the same as it would have been without any altruism in the society. This is because of the fact that there are no income effects. The linear income tax, however, now distorts the decisions of labor supply.

We are now in an equilibrium of transfers if and only if: \( \tau < \overline{\tau} \),

where \( \overline{\tau} = \frac{\alpha^{1/\eta}\omega_H^{1+\epsilon} - \omega_L^{1+\epsilon}}{(\alpha^{1/\eta}\omega_H^{1+\epsilon} - \omega_L^{1+\epsilon}) + (1 - \alpha^{1/\eta})(1 + \epsilon)(\mu_\pi\omega_H^{1+\epsilon} + (1 - \mu_\pi)\omega_L^{1+\epsilon})} \).

\( \alpha^{1/\eta} > \left(\frac{\omega_L}{\omega_H}\right)^{1+\epsilon} \) thus remains a necessary condition.

In that case, \( t = \frac{(1-\tau)^{1+\epsilon}(\alpha^{1/\eta}\omega_H^{1+\epsilon} - \omega_L^{1+\epsilon}) - R(\tau)(1 - \alpha^{1/\eta})}{\alpha^{1/\eta} + \frac{\pi}{1-\tau}} \),

where \( R(\tau) = \tau(1 - \tau)^{\epsilon}(\mu_\pi\omega_H^{1+\epsilon} + (1 - \mu_\pi)\omega_L^{1+\epsilon}) \), with \( \mu_\pi := \mathbb{E}(\pi) \), the average of \( \pi \)'s over

---

8We know from the proof of Corollary 1 that whether there are transfers or not, \( u_H^t < u_L^t \), so that the sign of \( \phi \) depends on the sign of \( \frac{\mu_\pi}{\overline{\tau}} \), which since there are no income effects here, will generally be negative or 0.
the population.

This enables us to notice that $\tau$ is a function of exogenous parameters only, and does not depend on specific values of $\pi$. Hence, the equilibria are of the same type in all components of the network: at equilibrium, either there are transfers taking place in all groups (except the ones with $\pi = 0$ or 1) or no transfers are taking place. Moreover, $\tau$ is decreasing in $\mu_\pi$: the more skewed the distribution is towards groups of high shares of low skilled individuals, the more likely it is that the optimal tax rate will encourage some private transfers. Indeed, as we saw in the laissez-faire situation, the same comparative static can be made here: groups with lower shares of high skilled individuals operate better redistribution between high and low skilled individuals. If there are enough such groups, the social planner will thus reach more efficiency if they encourage partial private redistribution.

Let us note also that transfers will be lower than in the laissez-faire case for two reasons. On the one hand, the term $\alpha^{1/\eta} \omega_H^{1+\epsilon} - \omega_L^{1+\epsilon}$ is now weighted by $(1 - \tau)^{1+\epsilon} < 1$: the linear tax rate partially crowds out transfers, as high ability individuals now have lower disposable income. On the other hand, since $\alpha^{1/\eta} < 1$, lump-sum redistribution $R(\tau)$ decreases the incentives to make transfers, as the government is already partially taking care of redistribution.

Now, as in a non altruistic model, it cannot be that at the optimum, $\frac{dR}{d\tau} < 0$. Hence, $\tau^* < \frac{1}{1 + \epsilon}$ and at the optimum, $\frac{\partial t}{\partial \tau} \leq 0$.

Finally, this enables us to partly characterize the optimum. We will have two cases:

If $\tau > \frac{1}{1 + \epsilon}$, then the optimal tax rate $\tau^*$ is such that there are transfers taking place at the optimum, and the rate has to be adjusted downwards so as to encourage these to take place.

Note that this is more likely the lower $\mu_\pi$ is.

If on the other hand $\tau \leq \frac{1}{1 + \epsilon}$, then the optimal tax rate might be in $[0, \tau]$ and hence encourage transfers or in $[\tau, \frac{1}{1 + \epsilon}]$ and hence crowd out transfers. This will ultimately depend on the distribution of $\pi$’s in the society.

5.2 Optimal non-linear income taxation

Introducing a non-linear income tax now raises the question of whether the different proportions of high skill individuals in the different groups are known to the government or constitute private information for the group.
We shall suppose, as is certainly the case, that the government does not have this information.

Now since the tax is non-linear, this may change the *laissez-faire* characterization of transfers within components. This will depend on the shape of the tax schedule, and more specifically on the marginal tax rates at different incomes.

## 6 Concluding remarks

This paper develops a conceptual framework to study optimal taxation in the presence of altruism and thus private transfers between individuals. First, we establish the structure of incomes and transfers under the *laissez-faire* situation. Second, we characterize the optimal linear income tax and show that depending on the distribution of the homogeneity of groups, because of partial crowding out transfers, the optimal tax rate should take into account the elasticity of transfers to the tax rate. Indeed, the social planner might find it advantageous to encourage transfers at a private level before operating redistribution at the society level: by taking into consideration the partial crowding out of transfers, the government may take advantage of the fact that there is already some redistribution occurring at a private level – a redistribution that does not distort labor supply incentives – and thus decrease the marginal tax rate used to redistribute at the society level – thus also reducing the amount of distortion created by the tax. Further, the more skewed the distribution is towards groups of high shares of low skilled individuals, the more likely that the optimal tax rate will encourage some private transfers. Indeed, groups with lower shares of high skilled individuals operate better redistribution between high and low skilled individuals, but at a higher cost for high skilled individuals. Enough of such groups will allow the social planner to reach more efficiency if they encourage partial private redistribution.

Further questions are:

In this context, does the Atkinson and Stiglitz (1976) result still hold?

How do our results change when the motives for transfers are different?

### Appendix

### Proofs

**Proof of Corollary 1.** Let us place ourselves in a component.

- We show this for transfers from individuals of type L to individuals of type H.
  WLOG, suppose that individual 1 is of type L and individual 2 is of type H, and that individual 1 makes a transfer to individual 2.
Then, from Lemma 1, 1 is a donor and 2 is a receiver. Let $S_1$ be the set of individuals 1 makes transfers to, and $S_2$ the set of individuals who make transfers to 2.

Then from (2), $v' \left( c_1 - k \left( \frac{y_1}{\omega_1} \right) \right) = \alpha v' \left( c_2 - k \left( \frac{y_2}{\omega_2} \right) \right)$, so that $v' \left( c_1 - k \left( \frac{y_1}{\omega_1} \right) \right) < v' \left( c_2 - k \left( \frac{y_2}{\omega_2} \right) \right)$, and since $v$ is concave, $c_1 - k \left( \frac{y_1}{\omega_1} \right) < c_2 - k \left( \frac{y_2}{\omega_2} \right)$.

So:

\[ y_1 - t_{12} - \sum_{i \in U_1} t_{1i} - k \left( \frac{y_1}{\omega_1} \right) > y_2 + t_{12} + \sum_{j \in U_2} t_{j2} - k \left( \frac{y_2}{\omega_2} \right) \]

and

\[ 2t_{12} < \left( y_1 - k \left( \frac{y_1}{\omega_1} \right) \right) - \left( y_2 - k \left( \frac{y_2}{\omega_2} \right) \right) - \sum_{i \not\in U_1} t_{1i} - \sum_{j \not\in U_2} t_{j2} \quad (\ast) \]

Now from (1), we know that the incomes of all individuals of type H are the same, and all incomes of individuals of type L are the same. And that since $\omega_L < \omega_H$, $y_H > y_L$.

Suppose that $y_L - k \left( \frac{y_L}{\omega_L} \right) > y_H - k \left( \frac{y_H}{\omega_H} \right) \quad (\ast \ast)$.

Now, since $\omega_H > \omega_L$, $k \left( \frac{y_L}{\omega_L} \right) > k \left( \frac{y_H}{\omega_H} \right)$.

So (\ast \ast) implies that $y_L - k \left( \frac{y_L}{\omega_L} \right) > y_H - k \left( \frac{y_H}{\omega_H} \right) \quad (\ast \ast \ast)$.

Now the function $y \mapsto y - k \left( \frac{y}{\omega_H} \right)$ is increasing over $[0, y_H]$, since its derivative cancels at $y_H$ and since $k$ is convex, so that $y_L - k \left( \frac{y_L}{\omega_L} \right) \leq y_H - k \left( \frac{y_H}{\omega_H} \right)$, and (\ast \ast \ast), thus (\ast \ast) do not hold.

So that (\ast) implies that $t_{12} \leq 0$, a contradiction.

- The argument is even more straightforward with two individuals of same type, since we know that they earn the same income.

**Proof of Proposition 1.** This proof can be made in four steps.

Let us place ourselves in a component.

- Suppose individuals $i$ and $j$ are both of type H.

Suppose that $i$ makes transfers and that $j$ makes no transfer.

Denote $T > 0$ the sum of $i$’s transfers. Let $l$ be one of $i$’s beneficiaries.

Then: $u_c(c_i, y_i, \omega_i) = \alpha u_c(c_i, y_i, \omega_l)$, and since $j$ does not make a transfer to $l$, $u_c(c_j, y_j, \omega_j) \geq \alpha u_c(c_j, y_j, \omega_l) = u_c(c_j, y_j, \omega_l)$.

So that $y_j - k \left( \frac{y_j}{\omega_H} \right) \leq y_i - T - k \left( \frac{y_i}{\omega_H} \right)$, and since $y_i = y_j = y_H$, this implies that $T \leq 0$. and
we have a contradiction.
So if an individual H makes a transfer, then all individuals of type H make a transfer, and if
an individual H does not make a transfer, then no other individual of type H makes a transfer.

- Now suppose individuals $i$ and $j$ both make transfers, the sum of which are respectively $T_i$ and $T_j$.
If they both make transfers to a same individual $l$, then $u(c_i, y_i, \omega_i) = \alpha u(c_l, y_l, \omega_l) = u(c_j, y_j, \omega_j)$. So that $y_j - T_j - k\left(\frac{y_l}{\omega_l}\right) = y_i - T_i - k\left(\frac{y_l}{\omega_l}\right)$. Since $y_i = y_j = y_H$, we have that $T_i = T_j$.
Suppose they make transfers to different individuals, $l$ and $m$.
Then $u(c_i, y_i, \omega_i) \geq \alpha u(c_m, y_m, \omega_m) = u(c_j, y_j, \omega_j)$.
And $u(c_j, y_j, \omega_j) \geq \alpha u(c_i, y_i, \omega_i) = u(c_i, y_i, \omega_i)$.
So that $u(c_i, y_i, \omega_i) = u(c_j, y_j, \omega_j)$ and by the same arguments as above $T_i = T_j$.

- Finally, suppose there are transfers. Suppose an individual $l$ of type L receives a transfer (say from individual $i$), and an individual $m$ receives no transfer. Denote $t_l > 0$ the sum of transfers received by $l$.
Then: $u(c_i, y_i, \omega_i) = \alpha u(c_l, y_l, \omega_l)$ and $u(c_i, y_i, \omega_i) \geq \alpha u(c_m, y_m, \omega_m)$.
So: $u(c_i, y_i, \omega) \geq u(c_m, y_m, \omega_m)$, and since $u$ is concave in $c$, $y_l + t_l - k\left(\frac{m}{\omega_L}\right) \leq y_m - k\left(\frac{m}{\omega_L}\right)$.
So since $y_l = y_m = y_L$, this implies $t_l \leq 0$, and we have a contradiction.
So if one individual of type L receives a transfer, then all individuals of type L receive a transfer.

- Suppose one individual $l$ receives a transfer from an individual $i$ and an individual $m$ receives a transfer from an individual $j$ ($i$ and $j$ might be the same). Denote $t_l$ and $t_m$ the sum of transfers received by each individual. Then, we know that $c_i = c_j$ and $y_i = y_j$, so $u(c_i, y_i, \omega_i) = \alpha u(c_l, y_l, \omega_l)$ and $u(c_j, y_j, \omega_j) = \alpha u(c_m, y_m, \omega_m)$ imply that $u(c_i, y_i, \omega_i) = u(c_m, y_m, \omega_m)$, and by the same argument as above ($y_l = y_m = y_L$, $t_l = t_m$).
So the sum of transfers received by individuals of low type is the same.

Proof of Application 1. We use the following specifications for our utility functions:

\[ v\left(c - k\left(\frac{y}{\omega}\right)\right) \text{ such that } v(x) = \frac{x^{1-\eta}}{1-\eta} \text{ and } k\left(\frac{y}{\omega}\right) = \left(\frac{y}{\omega}\right)^{1+\eta} \frac{1}{1 + \frac{\eta}{\epsilon}}. \]

- We know that from (1), it follows that $y_H = \omega_H^{1+\epsilon}$ and $y_L = \omega_L^{1+\epsilon}$.

- In each component, we also know that an equilibrium is either one where all high skilled individuals make transfers to low skilled individuals or where no transfers take place.

Suppose we are in an equilibrium where no transfer takes place. Then by definition, this
means that it is not worth deviating for any individual. The only deviation that could take
place would be for a high skilled individual to make a transfer to a low skilled one (or to many
low skilled individuals).
This would imply that the social utility is increasing at \( t = 0 \).
That is
\[
-v'(y_H - k \left( \frac{y_H}{\omega_H} \right)) + \alpha v'(y_H - k \left( \frac{y_H}{\omega_H} \right)) > 0
\]
i.e. \( \alpha^{1/\eta} > \left( \frac{\omega_L}{\omega_H} \right)^{1+\epsilon} \).
So if \( \alpha^{1/\eta} \leq \left( \frac{\omega_L}{\omega_H} \right)^{1+\epsilon} \) then the no-transfer situation is a Nash equilibrium.
On the contrary, if \( \alpha^{1/\eta} > \left( \frac{\omega_L}{\omega_H} \right)^{1+\epsilon} \) then we are in a situation in which all individuals of
high types make the same total transfer \( t \) to individuals of low type. Individuals of low type
receive \( \pi_1 - \pi t \) each. Transfers are then determined as the solution to
\[
v'(y_H - t - k \left( \frac{y_H}{\omega_H} \right)) = \alpha v' \left( y_L + \frac{\pi}{1 - \pi} t - k \left( \frac{y_L}{\omega_L} \right) \right)
\]
Which gives us: \( t = \frac{1}{1 + \epsilon} \frac{\alpha^{1/\eta} \omega_H^{1+\epsilon} - \omega_L^{1+\epsilon}}{\alpha^{1/\eta} + \frac{\pi}{1 - \pi}} \).
• The gap in consumption levels in component \( C^\pi \) is then: \( c_H - c_L = y_H - y_L - (t + \frac{\pi}{1 - \pi} t) = y_H - y_L - \frac{1}{1 - \pi} t \).
So that since \( y_L \) and \( y_H \) do not depend on \( \pi \), \( \frac{\partial}{\partial \pi} (c_H - c_L) = \frac{\partial}{\partial \pi} (- \frac{1}{1 - \pi} t) > 0 \).
The gap in consumption levels is higher in components with higher proportions of high ability
individuals.

**Proof that** \( SWF_p = SWF_s \) **in the case of an unweighted utilitarian social welfare function and symmetric Nash equilibrium.** Indeed, in that case all low (resp. high) skill
individuals earn the same income \( y_L \) (resp. \( y_H \)) and have the same consumption \( c_L \) (resp. \( c_H \)).
So that all low skill individuals will have the same social utility
\[
U_L(c_L, c_H, y_L, y_H, \omega_H, \omega_L) = (1 + ((1 - \pi)n - 1)\alpha)u(c_L, y_L, \omega_L) + \pi n \alpha u(c_H, y_H, \omega_H)
\]
and high skill individuals too:
\[
U_H(c_L, c_H, y_L, y_H, \omega_H, \omega_L) = (1 + (\pi n - 1)\alpha)u(c_H, y_H, \omega_H) + (1 - \pi) n \alpha u(c_L, y_L, \omega_L)
\]
So that
\[ \pi U_H(c, y, \omega_H, \omega_L) + (1 - \pi) U_L(c, y, \omega_H, \omega_L) = [1 + (n-1)\alpha][\pi u_H(c_H, y_H, \omega_H) + (1 - \pi) u_L(c_L, y_L, \omega_L)] \]
So since \( 1 + (n-1)\alpha \) does not depend on \( \pi \), it is the same to maximize
\[
\int_0^1 (\pi U_H(c, y, \omega_H, \omega_L) + (1 - \pi) U_L(c, y, \omega_H, \omega_L)) f(\pi) \, d\pi
\]
as it is to maximize
\[
\int_0^1 (\pi u_H(c_H, y_H, \omega_H) + (1 - \pi) u_L(c_L, y_L, \omega_L)) f(\pi) \, d\pi
\]

**Proof of Proposition 2.** Let us restate the problem of the unweighted utilitarian social planner.

- Their program is the following (noting that \( t(\pi, \tau) \) may be positive or equal to 0):

\[
\max_{\tau, R} \int_0^1 \left( \pi u(1 - \tau) y_H + R - t, y_H, \omega_H) + (1 - \pi) u((1 - \tau) y_L + R + \frac{\pi}{1 - \pi} t, y_L, \omega_L) \right) f(\pi) \, d\pi
\]

under the (government budget) constraint that
\[
\tau \int_0^1 (\pi y_H(\pi, \tau) + (1 - \pi) y_L(\pi, \tau)) f(\pi) \, d\pi \geq R + E
\]
(At the optimum, the budget constraint is binding.)

- Define \( Y(\tau) := \int_0^1 (\pi y_H(\pi, \tau) + (1 - \pi) y_L(\pi, \tau)) f(\pi) \, d\pi \), the aggregate income, which enables us:

  - to define \( e = \frac{(1 - \tau)}{Y} \frac{dY}{d(1 - \tau)} \) the elasticity of aggregate earnings with respect to the net-of-tax rate

  - and to rewrite \( R \) as a function of \( \tau \): \( R(\tau) = \tau Y(\tau) - E \).

- The problem of the social planner can then be rewritten as

\[
\max_{\tau} \int_0^1 \left( \pi u((1 - \tau) y_H + \tau Y - t, y_H, \omega_H) + (1 - \pi) u((1 - \tau) y_L + \tau Y + \frac{\pi}{1 - \pi} t, y_L, \omega_L) \right) f(\pi) \, d\pi
\]
so that the first order condition of the problem of the social planner is (using the envelope theorem thanks to the individuals’ first order conditions$^9$):

\[
dSWF \frac{d}{d\tau} = 0
\]

\[
\Leftrightarrow \int_0^1 \left( \pi(-y_H + \tau \frac{dY}{d\tau} + Y - \frac{\partial t}{\partial \tau} u^H_c + (1 - \pi)(-y_L + \tau \frac{dY}{d\tau} + Y + \frac{\pi}{1 - \pi} \frac{\partial t}{\partial \tau}) u^L_c \right) f(\pi) \, d\pi = 0.
\]

Let us replace \( \frac{dY}{d\tau} \) by \( \frac{dY}{d\tau} = -e \frac{Y}{(1 - \tau)} \).

So that our above first order condition becomes

\[
\int_0^1 Y \left( 1 - \frac{\tau}{1 - \tau} e \right) \left( \pi u^H_c + (1 - \pi) u^L_c \right) f(\pi) \, d\pi
\]

\[
\quad = \int_0^1 \left( \pi(y_H + \frac{\partial t}{\partial \tau}) u^H_c + (1 - \pi)(y_L - \frac{\pi}{1 - \pi} \frac{\partial t}{\partial \tau}) u^L_c \right) f(\pi) \, d\pi
\]

\[
\quad = \int_0^1 \left( \pi y_H u^H_c + (1 - \pi) y_L u^L_c \right) f(\pi) \, d\pi + \int_0^1 \left( \frac{\partial t}{\partial \tau} u^H_c - (1 - \pi) \frac{\partial t}{\partial \tau} u^L_c \right) f(\pi) \, d\pi
\]

So:

\[
Y \left( 1 - \frac{\tau}{1 - \tau} e \right) = \frac{\int_0^1 \left( \pi y_H u^H_c + (1 - \pi) y_L u^L_c \right) f(\pi) \, d\pi}{\int_0^1 \left( \pi u^H_c + (1 - \pi) u^L_c \right) f(\pi) \, d\pi} + \frac{\int_0^1 \frac{\partial t}{\partial \tau} u^H_c - (1 - \pi) \frac{\partial t}{\partial \tau} u^L_c \, f(\pi) \, d\pi}{\int_0^1 \left( \pi u^H_c + (1 - \pi) u^L_c \right) f(\pi) \, d\pi}
\]

- Denoting \( \bar{g} = \frac{\int_0^1 \left( \pi y_H u^H_c + (1 - \pi) y_L u^L_c \right) f(\pi) \, d\pi}{Y \int_0^1 \left( \pi u^H_c + (1 - \pi) u^L_c \right) f(\pi) \, d\pi} \)

and \( \phi = \frac{\int_0^1 \frac{\partial t}{\partial \tau} (u^H_c - u^L_c) \, f(\pi) \, d\pi}{Y \int_0^1 \left( \pi u^H_c + (1 - \pi) u^L_c \right) f(\pi) \, d\pi} \)

we have:

\[
1 - \frac{\tau}{1 - \tau} e = \bar{g} + \phi.
\]

- Note that, as in the standard case, we still have that \( R(0) = R(1) = 0 \), so that the derivative of \( R \) with respect to \( \tau \) changes sign at least once (it is generally U-shaped).

- Now, either the optimal tax is such that \( t = 0 \) for all \( \pi \), in which case the government generally wouldn’t be mistaken by setting \( \tau \) such that \( \frac{\tau}{1 - \tau} e = 1 - \bar{g} \) (unless we are at the frontier, and for some \( \pi \)'s, \( \frac{\partial t}{\partial \tau} \neq 0 \) even though \( t(\pi, \tau) = 0 \). Or, it is not and we know that

\[9\]For all \( i \), the first order condition defining individual \( i \)'s labour supply is \( (1 - \tau) u^i_c + u^i_y = 0. \)
since \( u^H_c < u^L_c \) whether there are transfers or not, the sign of \( \phi \) depends on the sign of \( \frac{\partial t}{\partial \tau} \).

**Proof of Application 1 sequel.** Using the functional specifications defined above,

- our first order conditions defining the labor supply individuals choose are now

\[
(1 - \tau) = \frac{y_H^{1/\epsilon}}{\omega_H^{1+1/\epsilon}}
\]

(3)

\[
(1 - \tau) = \frac{y_L^{1/\epsilon}}{\omega_L^{1+1/\epsilon}}
\]

(4)

So \( y_H = (1 - \tau)^{1/\epsilon} \omega_H^{1+\epsilon} \) and \( y_L = (1 - \tau)^{1/\epsilon} \omega_L^{1+\epsilon} \).

- For ease of exposure, suppose that \( E = 0 \). In this case the tax is a purely redistributive one.

Then, \( R(\tau) = \tau (1 - \tau)^{1/\epsilon} \int_0^1 \left( \pi \omega_H^{1+\epsilon} + (1 - \pi) \omega_L^{1+\epsilon} \right) f(\pi) \, d\pi \),

which may also be rewritten \( R(\tau) = \tau (1 - \tau)^{1/\epsilon} (\mu \omega_H^{1+\epsilon} + (1 - \mu) \omega_L^{1+\epsilon}) \), where \( \mu := \mathbb{E}(\pi) \).

- Then, following a similar reasoning to that in the *laissez-faire* case, a Nash equilibrium with transfers takes place if and only if:

\[
\tau < \frac{\alpha^{1/\eta} \omega_H^{1+\epsilon} - \omega_L^{1+\epsilon}}{(\alpha^{1/\eta} \omega_H^{1+\epsilon} - \omega_L^{1+\epsilon}) + (1 - \alpha^{1/\eta})(1 + \epsilon)(\mu \omega_H^{1+\epsilon} + (1 - \mu) \omega_L^{1+\epsilon})}
\]

Let us note also that since \( \tau \in [0, 1] \) this is only possible if we have the same condition as under the *laissez-faire*; that is, \( \alpha^{1/\eta} > \left( \frac{\omega_L}{\omega_H} \right)^{1+\epsilon} \). This remains a necessary condition.

Let us denote \( \overline{\tau} = \frac{\alpha^{1/\eta} \omega_H^{1+\epsilon} - \omega_L^{1+\epsilon}}{(\alpha^{1/\eta} \omega_H^{1+\epsilon} - \omega_L^{1+\epsilon}) + (1 - \alpha^{1/\eta})(1 + \epsilon)(\mu \omega_H^{1+\epsilon} + (1 - \mu) \omega_L^{1+\epsilon})} \).

Then we will have an equilibrium with transfers if \( \tau^* < \overline{\tau} \), and a no-transfer equilibrium if \( \tau^* \geq \overline{\tau} \).

- Let us now show that at the optimum, \( \frac{dR}{d\tau}(\tau^*) \geq 0 \).

Suppose that \( \tau^* \geq \overline{\tau} \).

Then, there are no transfers taking place, and

\[
v \left( (1 - \tau)y_H + R(\tau) - k \frac{y_H}{\omega_H} \right) = v \left( \frac{\omega_H^{1+\epsilon}}{1 + \epsilon} (1 - \tau)^{1+\epsilon} + R(\tau) \right)
\]
\[ v \left( (1 - \tau) y_L + R(\tau) - k \left( \frac{y_L}{\omega_L} \right) \right) = v \left( \frac{\omega_L^{1+\epsilon}}{1 + \epsilon} (1 - \tau)^{1+\epsilon} + R(\tau) \right) \]

Then, if at \( \tau^* \), \( \frac{dR}{d\tau}(\tau^*) < 0 \), since \( v \) is increasing and \( \tau \mapsto (1 - \tau)^{1+\epsilon} \) is decreasing in \( \tau \), the government could reduce the tax rate by a small increment, increase all individuals’ utility levels as well as revenue – a contradiction.

Suppose now that there are transfers taking place, i.e., \( \tau^* < \tau \).

Then, transfers are defined by the following equation:

\[ ((1 - \tau)y_H - t + R(\tau) - k \left( \frac{y_H}{\omega_H} \right))^{-\eta} \geq \alpha ((1 - \tau)y_L + \frac{\pi}{1 - \pi} t + R(\tau) - k \left( \frac{y_L}{\omega_L} \right))^{-\eta} \]

So that:

\[ t = \frac{(\frac{1 - \tau}{1 + \epsilon})^\eta (\alpha^{1/\eta} \omega_H^{1+\epsilon} - \omega_L^{1+\epsilon}) - R(\tau)(1 - \alpha^{1/\eta})}{\alpha^{1/\eta} + \frac{\pi}{1 - \pi}} \]

And

\[ v \left( (1 - \tau)y_H + R(\tau) - t - k \left( \frac{y_H}{\omega_H} \right) \right) = v \left( \frac{(1 - \tau)^{1+\epsilon} \left( \pi \omega_H^{1+\epsilon} + (1 - \pi) \omega_L^{1+\epsilon} \right) + R(\tau)}{\alpha^{1/\eta} (1 - \pi) + \pi} \right) \]

and

\[ v \left( (1 - \tau)y_L + R(\tau) + \frac{\pi}{1 - \pi} t - k \left( \frac{y_L}{\omega_L} \right) \right) = v \left( \alpha^{1/\eta} \frac{(1 - \tau)^{1+\epsilon} \left( \pi \omega_H^{1+\epsilon} + (1 - \pi) \omega_L^{1+\epsilon} \right) + R(\tau)}{\alpha^{1/\eta} (1 - \pi) + \pi} \right) \]

Then, with exactly the same arguments as above, it cannot be that at the optimum, \( \frac{dR}{d\tau}(\tau^*) < 0 \).

So at the optimum, \( \frac{dR}{d\tau}(\tau^*) \geq 0 \).

Hence, since \( R(\tau) = \tau(1 - \tau)^\epsilon (\mu \omega_H^{1+\epsilon} + (1 - \mu) \omega_L^{1+\epsilon}) \), this is equivalent to

\[ \tau^* \leq \frac{1}{1 + \epsilon} \]

- This enables us, also, to know the sign of \( \frac{\partial t}{\partial \tau}(\pi, \tau) \) at the optimum. Indeed, since at the optimum \( \frac{dR}{d\tau}(\tau^*) \geq 0 \), \( \frac{\partial t}{\partial \tau}(\pi, \tau^*) < 0 \).

References


