Societal Consensus and Redistributive Taxation

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Abstract

This paper aims to update the normative analysis of income redistribution by conditioning it on a societal consensus. An inequality-averse social planner chooses a linear income tax schedule while accounting for the possibility that certain policy choices are consonant with the individual social preferences of a plurality of groups in the population, i.e. that they lead to a societal consensus. It is assumed that consensual choices induce ethical labour supply choices, and thus a greater potential for redistribution. The planner’s trade-offs lie between pursuing consensual policies that differ from those otherwise chosen optimally, and retaining its free rein. In a two skill-type economy, it proves worthwhile for the planner to choose a consensual tax schedule, even if it does not correspond to the unconstrained optimal choice, as long as it remains moderate. This occurs the greater is the proportion of high-skilled agents, and the lower is the pre-tax wage inequality.

JEL classification codes: H2, D6, D71.

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1 Introduction

Implicit to all economic analyses are potentially contentious moral values, and the judgements based upon them. In the case of normative economics, these moral values and judgments should be (and typically are) made explicit, due to its chief focus being to characterize a

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social optimum, through the choice of institutions and policies based on their strict desirability. Yet, by their very nature, normative analyses involve greater degrees of contention than their positive counterparts, none more so than those preoccupied with the redistribution of income. Among the many complaints addressed towards normative economics, let us note the following, of particular significance to the questions of income redistribution addressed herein: first, the lack of justification for the choice of a social welfare function (SWF), which must include views about the distribution of utilities (and income), and also requires interpersonal comparisons of utilities, for which there may not be a clear basis; and second, the lack of concern given to political institutions, context, and apparent constraints to policy choices in many normative and welfare economics settings. Such criticisms weaken the social acceptability of the analysis being conducted, and blunt the applicability of its policy prescriptions. They have thus been seen as contributing factors for the apparent falling from grace of normative economics (cf. Atkinson, 2011; Bhagwati, 2011; Shiller and Shiller, 2011), including normative public economics, and the greater use of analyses rooted in political economy and social choice instead to address related research questions.

Yet, some of those very authors, namely Atkinson (2011) and Shiller and Shiller (2011), called for a renewal of normative economics, rather than its continued abandonment. Furthermore, they see it as passing through a greater emphasis on moral concerns, rather than a move away from this bone of contention, and instead towards the greater inclusion of political concerns. Indeed, approaches rooted in political economy, as applied for instance to the questions of income redistribution, rest on the assumed preferences of the electorate, their expression through the political competition process, and the various vested interests that influence it. As such, little space remains for the role of policy advocacy from a normative standpoint, and the resulting social-political consensus around redistributive policies is then, by and large, deterministic.

Thus, while such approaches serve to explain how a certain redistributive outcome may arise given particular conditions, it does not address how such conditions may arise, and whether they can or, more pointedly, should be influenced. This was presciently and thoroughly discussed by Boadway (2002): the inclusion of binding political constraints in normative economics unduly restricts policy advocacy on the part of economists, thus potentially rendering it pointless, and inducing a status quo bias in the choice of policies. Even in dynamic models of political competition where past policy choices shape and constrain the policy space available to parties (e.g. Forand, 2014), convergence to a subset of the policy space is guaranteed given certain initial conditions, and no allowance is made for any drastic changes in what is viewed to be desirable policies, aside from strong exogenous – i.e. unexplained – shocks.
So, while Atkinson (2011) favours a renewal involving a greater emphasis on moral concerns through further work on the Rawlsian notion of egalitarian justice, Sen’s concept of capabilities, and fairness objectives, alongside the use of personal moral principles and ethics as a basis for individual and societal decision-making, Shiller and Shiller (2011) instead emphasize the need for normative and welfare economics to rest on moral values and judgements that are representative of a broad societal consensus, rather than one solely favoured by economists. This emphasis on conditioning normative economics on a societal consensus, and its elusiveness in matters pertaining to the redistribution of income, is also found in Boadway (2002).

While very wide-ranging, these recommendations remain useful and form a basis for the present work, which aims to condition the normative analysis of income redistribution on a societal consensus surrounding those questions. It does so by first supposing that individuals, who differ in their marginal product but have identical preferences for leisure, also have social preferences linked to an ideal level of income redistribution, and proxied by an ideal tax rate – one that is in turn linked to the purely-redistributive linear tax schedule chosen by an inequality-averse social planner (or decision-maker at large). Second, a choice of tax rate within a certain range of one’s ideal is deemed to relax the postulate of pure self-interested behaviour in the individual supply of labour at the intensive margin. Instead, individuals are then assumed to supply labour ethically, which is to say, following Boadway, Marceau, and Mongrain (2007), as if they were not taxed. Thus, for certain distributions of individual social preferences, it may be that at least a plurality of individuals or groups within society are inclined to simultaneously supply labour ethically for a range of tax rates: this is how a societal consensus around questions of redistribution is defined. Third, the social planner, given a typical Bergson-Samuelson SWF exhibiting an aversion to inequality in utilities, must then choose an optimal linear income tax schedule. This would be a well-studied problem (its origins lying in the work of Sheshinski, 1972), were it not for the fact that the societal consensus, or lack thereof, now acts as a constraint on the planner’s problem.\footnote{This may appear not unlike the approach of Winer, Tridimas, and Hettich (2014), where the planner’s optimal tax problem is constrained by the acceptable level of coercion exerted on individuals, in a model where taxation serves to provide a pure public good. Coercion is then defined as the difference between the individuals’ indirect utilities at their optimal self-interested taxation-and-spending choices, and the planner’s. This paper, in contrast, does not take the view that tax choices not aligned with individual preferences induce any coercion. It is rather the opposite stance that is adopted: tax choices sufficiently aligned with the social preferences of individuals induce ethical behaviour on their part. Also, the present emphasis is on purely redistributive policies, rather than the provision of pure public goods.} The social planner may then foster a societal consensus though its choice of linear income tax, or instead choose to forsake it and retain free rein in its policy choice, while also then inducing a societal dissensus.
Ethical behaviour in the individual supply of labour leads to jumps in the individual supply function for certain ranges of tax rates. This in turn leads to discontinuities in the aggregate supply of labour, and the aggregate taxable labour earnings, this for different tax rates, whether more or less consensual. As a result of such ethical behaviour and societal consensus considerations, it follows that the social planner’s problem, rather than being well-behaved with a unique local and global interior optimum like that of many such optimal tax problems, becomes highly discontinuous and potentially characterized by many local optima. The main trade-off that the social planner may now face is between favouring a societal consensus, and thus a higher potential for redistribution, through an otherwise non-optimal choice of linear tax function; and choosing its desired linear tax scheme, while potentially thus inducing purely self-interested behaviour in individual labour supply.

In a model limited to two types of agents, of high- or low-skill, results show that for most moderate individual social preferences for redistribution, thus leading to a moderate social consensus, it is in the interest of social welfare for the planner to pursue a consensual choice of tax policy, even if that choice does not correspond to the unconstrained optimum under ethical behaviour. Thus, the alternative, to retain free rein over the policy optimum, while inducing a dissensus characterized by purely self-interested behaviour by all concerned, only prevails when consensual societal preferences are sufficiently extreme (i.e. involving too low or too high a tax rate). The emphasis on choosing a consensual tax rate is directly related to how instrumental the higher-skilled individuals are to the consensus (i.e. how great a proportion of the population they represent), for it is their ethical behaviour which induces the greatest increase in the total taxable labour earnings, and the potential for redistribution. On the contrary, the greater is the pre-tax wage inequality, then the more likely it is that the social planner will opt for its preferred optimal tax scheme along with a dissensus, when the allowable consensual tax schedules differ from what would be optimally chosen.

1.1 Related Literature

Beyond its attempt at conditioning the normative analysis of income redistribution on a societal consensus, expanded upon earlier, this paper draws from and contributes to the literature on the welfare state, and the potential for income redistribution. Various authors have sought to explain what determines the possible extent of income distribution, and the perennity of the welfare state. The determinants of individual preferences for redistribution, both self-interested and not, have comprehensively been covered in surveys and original works by Alberto Alesina and multiple co-authors (Alesina and La Ferrara, 2005; Alesina and Giuliano, 2009; Alesina et al., 2012). The various explanations suggested, and empirically-
tested, include the following: the individuals’ self-interested expected future income and social mobility; their indirect concern for inequality (e.g. as it affects growth prospects and thus future earnings and consumption); their direct concern for inequality, or how a certain degree thereof induces a utility loss that depends on their aversion to it; their concern for fairness; and their beliefs in how inequality affects work incentives, in linking rewards and effort.

It is thus on the basis of different equilibrium beliefs in the relative influence of luck and effort on the determination of economic outcomes that Bénabou and Tirole (2006) seek to positively explain differences in the degree of income redistribution between Europe and the United States. Likewise, Cervellati et al. (2010) explain how culturally-similar countries and populations can have diametrically-opposed redistributive outcomes – with some ending up in a low-hours, low-redistribution equilibrium, and others in a high-hours, high-redistribution equilibrium – on the basis of a median voter model that includes social norms related to work hours, and the possibility for multiple work-hours equilibria to arise for identical social preferences and taxation choices.

Meanwhile, the importance of social norms for the continued existence of a redistributive welfare state, and their fragility, has been investigated and emphasized by many, notably by Assar Lindbeck (alone and with co-authors Sten Nyberg and Jörgen Weibull, cf. Lindbeck (1995, 1996); Lindbeck et al. (1999, 2003); Lindbeck and Nyberg (2006); Lindbeck (2006)). In particular, Lindbeck (1996) highlights the potential for social norms regarding work participation to be weakened following severe macroeconomic shocks. These shocks, having thrown large numbers of people onto the social safety net, would normalize such behaviour, and thus cause the welfare state to ultimately be imperiled, through an increase in benefits uptake and dependency. Lindbeck and Nyberg (2006), for their part, claim that the very presence of a social safety net, and people receiving transfers from the state, may undermine the intergenerational transmission of social norms inducing a strong work ethic.

Such results are however dismissed as implausible by Corneo (2012) both on theoretical and empirical grounds. Svendsen and Svendsen (2016) agree, and refer to Lindbeck as the “Cassandra of the welfare state” (p. 3), quoting thus the consecrated expression of Paldam (1986). This does not however diminish the fact that to them, explaining the continued success of the welfare state only with the use of self-interested individual behaviour amounts to past attempts at explaining the flight of the bumblebee; the inevitable conclusion being that, being so heavy and lacking in aerodynamics, it should simply not fly. Svendsen and Svendsen’s preferred approach is to emphasize the importance of social capital, and thus trust, in keeping the Scandinavian welfare state flying.

This paper therefore adds to the seeming plethora of explanations for the extent of income
redistribution, and the welfare state’s perennity by assuming ethical behaviour in labour supply. Yet, it goes beyond that mere assumption by linking such behaviour to the existence of a societal consensus, and the policies conducive to it. In this regard, despite the emergence and characteristics of a political consensus in matters of income redistribution having been described extensively by Grünert (2009), it is closest to the work of Boadway and Martineau (2016), due to its emphasis on normative analysis. Therein, a non-linear optimal tax problem is modified to account for the presence of a social norm affecting work participation. When this norm leads to multiple equilibria in labour force participation, the choice of an optimal tax schedule may be compatible with many equilibria, but may thus also only constitute a local optimum. This can spur the social planner to seek a global, rather than strictly local optimum, by instigating a transition from a low- to a high-participation equilibrium. This is done through the choice of a less redistributive tax schedule than would be locally optimal, which then leads to a self-reinforcing increase in labour force participation until the new equilibrium is reached. A greater potential for redistribution is then possible at the high-participation equilibrium, such that the optimal tax schedule may at last be implemented.

On this note, we may now turn to the second main area of the literature that this paper both draws from and seeks to contributes to, pertaining to the influence of economic ideas in shaping a societal consensus around a certain set of institutions and policies. The seminal work of Hall (1993), investigated the process by which Keynesian ideas were adopted in the interwar and post-war eras. First, Hall made the hypothesis that the capacity for new ideas to serve vested interests in the state’s policymaking apparatus dictated the speed of their adoption; what Farrell and Quiggin (2012), following Heclo (1974), refer to as “powering”. Yet, Hall also emphasized the need for new ideas and theories to answer lingering questions that the endemic set of ideas cannot answer; this is referred to as “puzzling”, still in Heclo’s terminology, as used by Farrell and Quiggin. Thus, Hall developed a theory of institutional change based on Kuhn’s view of ideas paradigms in science. His theory of policy paradigms posits that institutional change may occur on three levels, following the diffusion of economic ideas through both powering and puzzling mechanisms. In descending order of importance, they are: the overarching objectives of policymaking; the techniques used to attain such objectives; and finally, the policy instruments themselves. For there to be a paradigm shift, all three levels must be upended; Hall argues that the 1976-1981 period in Great Britain corresponds to such a shift, away from Keynesian macroeconomic stabilization and state intervention, and towards Monetarism, economic deregulation, and laissez-faire. Periods of relative stability are therefore punctuated by crises, which are resolved by a change in the prevailing paradigm.

Blyth (2002) builds on Hall and others to develop a theory of institutional change based
on ideas and interests, which emphasizes the importance of Knightian uncertainty in bringing about such changes. In particular, Blyth adds to Hall’s theory by investigating the process by which crises unfold, and the role of ideas therein. The process is as follows: ideas serve to reduce uncertainty in times of crises; provide a cement to form coalitions; help question and attack existing institutions, and establish new ones; and finally, serve to stabilize them. This theory is then used to study the transition towards Keynesianism in the 1930s, and away from it, in the 1970s, in the United States and Sweden.

Finally, Farrell and Quiggin (2012) study the brief renewal of Keynesian ideas following the 2007-2009 global Great Recession, both in academic and policymaking circles. They innovate relatively to Hall and Blyth by emphasizing the importance of expert opinions in shaping a consensus around ideas. They make use of recent developments in network theory to argue that the spread of ideas is akin to a contagion process, stemming from a few influential and well-connected individuals. Yet, this contagion process is conditional on the network’s structure: for instance, clustered networks are not conducive to contagion processes outside of the cluster; networks with a greater variance in the number of links will potentially have greater contagion effects; and individuals with more links may act either as “firebreaks” or “disseminators”, either propagating or halting contagion. Of particular interest to the present paper, Farrell and Quiggin define the endemic set of ideas as an “apparent consensus”; in the absence of such a set of ideas, a “dissensus” then prevails. Furthermore, they characterize policies as being either “consonant” or “dissonant” with the consensus, the cost of implementing the former being lower than the latter, but free rein in policy choices being greater overall when there is a dissensus.

The rest of the paper is organized as follows. The model and analysis is found in Section 2. Within it, the model’s fundamental assumptions and underlying individual optimizing behaviours are first characterized. Then, a societal consensus is defined, and the planner’s problem is conditioned upon its existence, in an economy with two skill types. Results are then derived, in terms of optimal linear income tax schedules and welfare, for a full consensus, and partial consensus. They are then compared with similar results for a dissensus. Lastly, Section 3 concludes, and outlines directions for future work.

2 Model and Analysis

To study the effect that a societal consensus surrounding questions of income redistribution may have on the planner’s choice of policies, we start with a typical social planning problem. The timing is as follows. First, the inequality-averse social planner chooses an optimal linear income tax scheme: this involves a tax rate, $\tau$, and a demogrant (or lump-sum transfer), $T$,
the latter being determined through a balanced-budget requirement. Then, observing their
tax burdens, the agents make their consumption-labour (leisure) choices. As is typical, this
problem is then solved by backward induction, with the optimal consumption-labour choices
being made first, and anticipated by the planner, who incorporates them in his problem
before choosing the linear income tax schedule. The planner’s informational constraints are
also familiar: while labour choices may be inferred, they cannot be directly observed, and
thus taxation can only be based on realized incomes.

Where the present optimal linear tax problem differs from the canonical case is in the
inclusion of individual social preferences for redistribution, which may affect consumption-
leisure choices. These individual social preferences need not be consonant with the person’s
otherwise self-interested behaviour, and may for instance either be proxied by a degree of
aversion to inequality, or (as chosen here) a preferred tax rate, \( \tau_i \), where \( i = 1, 2, \ldots, n \)
denotes the agent’s type. The planner, despite knowing fully by assumption both the agents’
self-interested and social preferences, does not seek to aggregate such preferences into a new,
updated social welfare function, since doing so would obviously run into the serious problems
first raised by [Arrow (1951)] with his Impossibility Theorem. Rather, the agents’ behaviour is
deemed to differ according to whether or not their individual social preferences are satisfied
by the planner’s choices. If the chosen tax rate is sufficiently close to their preferred rate (i.e.
within Euclidian distance \( \theta > 0 \), assumed to be exogenous\(^2\)), agents are deemed to behave
ethically in choosing labour supply. Following [Boadway et al. (2007)], this ethical behaviour
could mean, at its paroxysm, that individuals supply labour as if they were not taxed, which
we also assume here.

Targeting a single individual’s (or household’s, or group’s) preferred tax rate will therefore
yield a lesser distortion (that is, none at all) on their labour supply. Such targeting, extended
to many such individuals, households or groups at once by choosing a single linear income
tax schedule, can further lessen distortions, and enhance the potential for redistribution due
to their ethical behaviour in terms of labour supply. Put differently, greater amounts (in
the form of a higher demogrant) can then be redistributed for a given tax rate, or a certain
level of redistribution can then be met with a lower rate. Therein lies why the planner might

\(^2\)This tolerance for different policies, \( \theta \), may in fact be thought of as endogenous, and dependent on past
policy choices. However, it is not immediately apparent how it might depend on it. For instance, in the
context of redistributive policies being historically long-enacted, it may be that even individuals with a low
preference for redistribution are willing to behave ethically in their supply of labour for policy choices vastly
different than their ideal. On the contrary, it may also be that the historical choice of tax rates falling within
a certain range of the policy space narrows the tolerance of like-minded individuals for deviations outside
that very range, assuming that \( \theta_i \) can differ with \( i \). Pursuing policies that promote a societal dissensus,
rather than those reinforcing a consensus, may then be key in expanding their tolerance, rather than the
opposite.
be concerned with the existence of a societal consensus, defined (for now) as a range of tax rates where ethical behaviour in terms of labour supply prevails for at least a plurality of individuals or groups in society. However, fostering such a societal consensus may come with trade-offs, in that it may conflict with the social planner’s degree of aversion to inequality. For instance, the societal consensus may be around lower tax rates than would otherwise choose the planner, or vice versa. The question then becomes a matter of when such trade-offs are worth making, and what shape the consensus might then take.

2.1 Fundamental Assumptions

Let us assume an economy populated by \( m \) individuals, households or groups (henceforth: agents), indexed by \( i \), whose respective weight in the population is \( n_i \), where \( \sum_{i=1}^{m} n_i = 1 \). Each of these agents is assumed to have identical, self-interested preferences over consumption and labour. To abstract from income effects on labour supply, quasi-linearity in consumption is also assumed, such that: \( u(c_i, l_i) = c_i - h(l_i) \), with \( h(l_i) \) being strictly increasing and convex; to obtain closed-form analytical and numerical solutions, \( h(l_i) = (1/2)l_i^2 \) is sometimes used. Agents however differ in their marginal product of labour, \( w_i \), such that \( 1 = w_1 < w_2 < w_3 < \ldots < w_m \), with the lowest marginal product \( w_1 \) being normalized to 1, along with the price of the composite consumption good and numéraire, \( p = 1 \).

As discussed earlier, agents also have heterogeneous and possibly non-self-interested social preferences concerning redistribution, proxied by an ideally-preferred tax rate \( \tau_i \). Given the planner’s chosen tax rate \( \tau \), agents will behave ethically in supplying labour, meaning that they will supply it as if it were not taxed, if \( |\tau - \tau_i| \leq \theta \), where \( \theta > 0 \) is an exogenous tolerance for deviations from the preferred tax rate. To the contrary, if \( |\tau - \tau_i| > \theta \), then agents behave self-interestedly in their choice of labour supply.

Let us now turn to the optimal consumption and labour choices of agents, followed by a formal definition of societal consensus, and the social planner’s general problem.

2.2 Optimal Consumption and Labour Choices

2.2.1 Self-interested behaviour

An agent behaves self-interestedly if \( |\tau - \tau_i| > \theta \), and solves the following problem:

\[
\max_{c_i, l_i} u(c_i, l_i) = c_i - h(l_i)
\]

subject to (s.t.):

\[
c_i = w_il_i(1-\tau) + T.
\]
This reduces to:

$$\max_{l_i} w_i l_i (1 - \tau) + T - h(l_i)$$

which yields, as a first-order condition for an interior solution:

$$w_i (1 - \tau) = h'(l_i).$$  \hspace{1cm} (1)

Intuitively, and as is standard, labour is supplied by the agent until the point where the marginal benefit of working more (expressed in terms of consumption, on the left-hand side), is equal to the marginal disutility of doing so. A corner solution where $l_i^* = 0$ and $c_i^* = T$ could also be possible, provided that:

$$w_i (1 - \tau) < h'(l_i)$$

for all $l_i \geq 0$. In other words, that the marginal disutility of labour always exceeds its marginal benefit in terms of consumption.

The equation (1) implicitly determines $l_i^*(w_i, \tau)$, and therefore the agent’s indirect utility function, $v(w_i(1 - \tau), T)$. For the purpose of explicitly deriving indirect utility functions, let us suppose that $h(l_i) = (1/2) l_i^2$. The above then reduces to:

$$l_i^* = w_i (1 - \tau)$$  \hspace{1cm} (2)

and hence an individual’s indirect utility function, when labour is supplied self-interestedly, can be written as:

$$v(w_i(1 - \tau), T) = (1/2) w_i^2 (1 - \tau)^2 + T.$$  \hspace{1cm} (3)

\subsection*{2.2.2 Ethical behaviour}

In contrast, when $|\tau - \tau_i| \leq \theta$, then labour is supplied ethically, i.e. as if the agent were not being taxed. Setting $\tau = 0$ in (1) then yields:

$$w_i = h'(l_i)$$  \hspace{1cm} (4)

while for the same functional form of $h(l_i) = (1/2) l_i^2$, this reduces to:

$$l_i^e = w_i$$  \hspace{1cm} (5)
where \( l_i \) denotes the agent’s ethical choice of labour supply. Lastly, the indirect utility function then becomes:

\[
v(w_i, \tau, T) = (1/2)w_i^2(1 - 2\tau) + T
\]

Putting these together, we can write:

\[
v(w_i, \tau, T) = \begin{cases} 
(1/2)w_i^2(1 - \tau)^2 + T & |\tau - \tau_i| > \theta \\
(1/2)w_i^2(1 - 2\tau) + T & |\tau - \tau_i| \leq \theta.
\end{cases}
\] (6)

### 2.3 The General Problem, and Defining a Societal Consensus

The social planner’s problem must account for the possibility of self-interested and ethical behaviours across agents of all types. Yet such changes in behaviour, which depend upon the planner’s choice of tax rate, lead to discrete jumps in the agents’ labour supply functions (from (2) to (5)), the indirect utility function found in (6), and thus in the social welfare function, this irrespectively of the social planner’s degree of aversion to inequality. The general social planning problem is therefore rather difficult to solve, as it translates into maximizing a piece-wise continuous function with many jumps: many local optima may therefore exist, and establishing the global optimum is either an algebraically-laborious or a computationally-intensive task.

Yet, reframed in terms of being conditioned on a societal consensus, this rather complex problem may be simplified. If approached in parts, by considering ranges of the policy space, \( \tau \in (0, 1) \) where a societal consensus prevails, and where it does not, it leads to straightforward optimization problems on the part of the planner for each range. The welfare levels associated with each optimal policy choices may then be compared with each other, so as to determine where a global optimum may lie. Still, this approach has evident limits, as it amounts to manually doing a grid-search for the global optimum, which becomes impractical or inefficient if the policy space is characterized by many small intervals where a form of consensus prevails. However, some important insights may be gathered in this particular fashion, and possibly extended to more general settings. The question then becomes how to formally define a societal consensus.

**Definition 1.** A societal consensus means that at least a plurality, if not all agents in society supply labour ethically. If all do, then the consensus is full; if only a plurality does, then the consensus is partial.

For there to be a consensus, a non-empty set (or interval) \( C \equiv [\tau, \overline{\tau}] = \bigcap_{i=1}^m C_i \) must be formed by the intersections of individual sets \( C_i \equiv [\tau_i - \theta, \tau_i + \theta] \). If \( \bigcap_j C_j = \emptyset \) for all \( j \), then there is a dissensus.
Note that a consensus does not imply unanimity about the choice of a tax rate, or even that a majority (or plurality) agrees on one such rate, but only that the choice of tax rate is sufficiently close to what is considered desirable by individuals so as to induce behaviour that departs from pure self-interest.

The existence of a societal consensus therefore depends on the individuals’ preferences for redistribution, proxied by the preferred tax rate, and the prescribed distance, θ. This will of course further depend on the number of skill groups within society, and the distribution of social preferences.

2.4 The Simplest Case: \( m = 2 \)

Let us now consider the case where there are only two types of agents within society, i.e. where \( m = 2 \). Given the number of agent types being considered, we need only concern ourselves with two possibilities: there there should be either a full consensus among all agents, or no consensus at all, i.e. a dissensus. We initially set aside the cases where only one of the two agent types behaves ethically, only to return to them later.

2.4.1 Full consensus

A full consensus here arises if and only if \([\tau_1 - \theta, \tau_1 + \theta] \cap [\tau_2 - \theta, \tau_2 + \theta] \neq \emptyset\). The consensual set is again denoted by \( C = [\underline{\tau}, \overline{\tau}] \).

Given a SWF exhibiting a constant aversion to inequality, measured by \( \rho \in [0, +\infty) \), the social planner’s problem is then:

\[
\max_{\tau \in C} \sum_{i=1}^{2} n_i \frac{1}{1 - \rho} \left[ \left( \frac{1}{2} \right) w_i^2 (1 - 2\tau) + T \right]^{1 - \rho}
\]

s.t.

\[
\sum_{i=1}^{2} n_i w_i^2 \tau = T
\]

where the constraint is simply a balanced-budget requirement that the total sums collected, i.e. the tax rate times the average earnings \( \tau \overline{y} \equiv \tau \sum_{i=1}^{2} n_i w_i^2 \), equal the total transfer \( T \). Furthermore, this can be reduced to:

\[
\max_{\tau \in C} \sum_{i=1}^{2} n_i \frac{1}{1 - \rho} \left[ \left( \frac{1}{2} \right) w_i^2 (1 - 2\tau) + \tau \overline{y} \right]^{1 - \rho}.
\]
The first-order condition for an interior solution then yields:

\[
\sum_{i=1}^{2} n_i \left[ \frac{1}{2} w_i^2 (1 - 2\tau) + \tau \bar{y} \right]^{-\rho} \left( -w_i^2 + \bar{y} \right) = 0 \tag{7}
\]

This yields a unique solution, for all \( \rho \in (0, +\infty) \), \( n_2 = 1 - n_1 \), and \( w_2 > 1 \): \( \tau^c = 1/2 \). Evaluated at \( \tau^c = 1/2 \), we then find that \( T^c = (1/2) (1 + n_2 (w_2^2 - 1)) \). In turn, this yields \( v(w_1, \tau^c) = v(w_2, \tau^c) = T^c \), such that the (indirect) utilities of all agent types are equalized. This results mirrors that of Boadway et al. (2007), and is a consequence of the ethical supply of labour, which results in the elasticity of total earnings to the tax rate being zero. The tax rate is then set to meet the planner’s redistributive objective, which no longer involves any equity-efficiency trade-off. It follows that any inequality-averse planner will then equalize utilities, regardless of \( \rho > 0 \) given some of the above assumptions (crucially, \( m = 2 \), and the separability of utility in consumption and labour, such that one abstracts from wealth effects on labour supply). Even a purely utilitarian (inequality-neutral) planner, for whom \( \rho = 0 \), would then be indifferent as to which tax rate to set.

Meanwhile, the effective tax rates \( t_i \), defined as the net tax burden \( \tau y_i - T \) divided by pre-tax earnings \( y_i \), are respectively for each type of agent:

\[
t_1 = \frac{\tau (y_1 - \bar{y})}{y_1}, \quad t_2 = \frac{\tau (y_2 - \bar{y})}{y_2}.
\]

Evaluated at \( \tau = \tau^c = 1/2 \), \( w_1 = 1 \) and \( n_1 = 1 - n_2 \), these further reduce to:

\[
t_1 = (1/2) (-n_2 (w_2^2 - 1)), \quad t_2 = \frac{(1/2) (w_2^2 - 1) (1 - n_2)}{w_2^2}
\]

wherefore:

\[
\frac{\partial t_i}{\partial n_2} < 0, \quad \frac{\partial t_i}{\partial w_2} > 0 \ \forall i = 1, 2.
\]

These can also be summarized, for certain values of \( w_2 \) and \( n_2 \), in the table below.
Due to the agents’ ethical behaviour freeing the planner from considering efficiency concerns, this solution cannot evidently be bested by any other choice of tax rate when no societal consensus (i.e. a dissensus) prevails. Yet, $\tau^c = 1/2$ may not lie in $C$. It follows that to achieve a societal consensus, the planner may have to compromise with its optimal choice of tax, and redistributive objective, and instead choose $\tau \succ 1/2$. It would then correspond to one of the bounds of set $C$, that closest to $\tau^c = 1/2$. However, whether set $C$ lies to the left or to the right of $\tau^c$ also has implications for social welfare, as the consensual welfare function may not be symmetric around the optimal rate. Given the above assumptions, this is in evidence in the figure below.

<table>
<thead>
<tr>
<th>$w_2 = 2$</th>
<th>$w_2 = 4$</th>
<th>$w_2 = 8$</th>
<th>$w_2 = 16$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_2 = 1/8$</td>
<td>-0.188 0.328</td>
<td>-0.938 0.410</td>
<td>-3.98 0.430</td>
</tr>
<tr>
<td>$n_2 = 1/4$</td>
<td>-0.375 0.281</td>
<td>-1.88 0.352</td>
<td>-7.88 0.369</td>
</tr>
<tr>
<td>$n_2 = 3/8$</td>
<td>-0.563 0.234</td>
<td>-2.81 0.293</td>
<td>-11.8 0.308</td>
</tr>
<tr>
<td>$n_2 = 1/2$</td>
<td>-0.750 0.188</td>
<td>-3.75 0.234</td>
<td>-15.8 0.246</td>
</tr>
<tr>
<td>$n_2 = 5/8$</td>
<td>-0.938 0.141</td>
<td>-4.69 0.176</td>
<td>-19.7 0.185</td>
</tr>
<tr>
<td>$n_2 = 3/4$</td>
<td>-1.13 0.0938</td>
<td>-5.63 0.117</td>
<td>-23.6 0.123</td>
</tr>
<tr>
<td>$n_2 = 7/8$</td>
<td>-1.31 0.0469</td>
<td>-6.56 0.0586</td>
<td>-27.6 0.0615</td>
</tr>
</tbody>
</table>

Table 1: Numerical point estimates of $t_1^c$ and $t_2^c$, for $\tau^c = 1/2$, $\rho \rightarrow 1$, and different values of $w_2$ and $n_2$.
2.4.2 Dissensus

A dissensus arises if and only if \([\tau_1 - d, \tau_1 + d] \cap [\tau_2 - d, \tau_2 + d] = \emptyset\). While this completely frees the planner in choosing any linear tax schedule, it does so at the cost of greater distortions in labour supply, which is now supplied purely self-interestedly.
The social planning problem may then now be written as:

$$\max_\tau \sum_{i=1}^{n} n_i \frac{1}{1 - \rho} \left[ \frac{1}{2} w_i^2 (1 - \tau)^2 + T \right]^{1 - \rho}$$

s.t.

$$\sum_{i=1}^{n} n_i w_i^2 (1 - \tau) \tau = T.$$ 

By using once more $\bar{y}$ to denote average earnings, this reduces to:

$$\max_\tau \sum_{i=1}^{n} n_i \frac{1}{1 - \rho} \left[ \frac{1}{2} w_i^2 (1 - \tau)^2 + (1 - \tau) \bar{y} \bar{y} \right]^{1 - \rho}.$$

The first-order condition for an interior solution is then:

$$\sum_{i=1}^{n} n_i \left[ \frac{1}{2} w_i^2 (1 - \tau)^2 + \tau (1 - \tau) \bar{y} \right]^{\rho} \left( -w_i^2 (1 - \tau) + (1 - 2\tau) \bar{y} \right) = 0$$

which further reduces to, by applying the previously-assumed $w_1 = 1$ and $n_1 = 1 - n_2$:

$$\begin{align*}
(1 - n_2) \left[ \frac{1}{2} (1 - \tau)^2 + \tau (1 - \tau) \bar{y} \right]^{-\rho} \\
\times \left( -(1 - \tau) + (1 - 2\tau) \bar{y} \right) + n_2 \left[ \frac{1}{2} w_2^2 (1 - \tau)^2 + \tau (1 - \tau) \bar{y} \right]^{-\rho} \\
\times \left( -w_2^2 (1 - \tau) + (1 - 2\tau) \bar{y} \right) = 0. \quad (8)
\end{align*}$$

Unlike in the case of a consensus, the general solution $\tau^d$ is conditional on $\rho$, $n_2$, and $w_2$. The possibility for multiple roots also complicates the search for a general solution. As is often done in such cases, certain values of $\rho$ may be chosen, chiefly among them $\rho \to 1$, which makes the social welfare function a population-weighted sum of logarithmic transformations of individual utilities. This is also equivalent to a logarithmic transformation of a multiplicative social welfare function.

For $\rho \to 1$, the solution $\tau^d$ is then:

$$\tau^d(w_2, n_2) = \frac{1}{2 ((2n_2 - 1)(w_2^2 - 1) + 1)} \left[ 3n_2(n_2 - 1)(w_2^2 - 1)^2 - w_2^2 \\
+ \sqrt{(w_2^4 - n_2^2(w_2^2 - 1)^2 + 2n_2(3w_2^2 - 4)(w_2^2 - 1))(1 + n_2(w_2^2 - 1))} \right]. \quad (9)$$

This yields the following rates for different values of $n_2$ and $w_2$, found in Figure 2 on page...
(a) In terms of $w_2^2$, for certain values of $n_2^2$

(b) In terms of $n_2^2$, for certain values of $w_2^2$

Figure 2: The optimal tax rate in the event of a dissensus, $\tau^d$, for different values of $n_2^2$ and $w_2^2$, and $\rho \to 1$

As for effective tax rates, they are now given by:

$$
t_1 = -n_2^2 \tau^d (w_2^2 - 1)$$

$$
t_2 = \frac{\tau^d (1 - n_2^2) (w_2^2 - 1)}{w_2^2}
$$

which translates into the following point estimates for certain values of $w_2$ and $n_2$.

|       | $w_2 = 2$ |       | $w_2 = 4$ |       | $w_2 = 8$ |       | $w_2 = 16$ |
|-------|---|-------|---|-------|---|-------|---|-------|
|       | $\tau^d$ | $t_1^d$ | $t_2^d$ | $\tau^d$ | $t_1^d$ | $t_2^d$ | $\tau^d$ | $t_1^d$ | $t_2^d$ |
| $n_2 = 1/8$ | 0.149 | -0.0559 | 0.0978 | 0.335 | -0.628 | 0.275 | 0.416 | -3.28 | 0.358 | 0.440 | -14.0 | 0.383 |
| $n_2 = 1/4$ | 0.191 | -0.143 | 0.107 | 0.341 | -1.28 | 0.240 | 0.390 | -6.14 | 0.288 | 0.403 | -25.7 | 0.301 |
| $n_2 = 3/8$ | 0.201 | -0.226 | 0.0942 | 0.323 | -1.82 | 0.189 | 0.358 | -8.46 | 0.220 | 0.367 | -35.1 | 0.228 |
| $n_2 = 1/2$ | 0.195 | -0.293 | 0.0731 | 0.296 | -2.22 | 0.139 | 0.324 | -10.2 | 0.159 | 0.331 | -42.2 | 0.165 |
| $n_2 = 5/8$ | 0.178 | -0.334 | 0.0501 | 0.265 | -2.48 | 0.0932 | 0.287 | -11.3 | 0.106 | 0.293 | -46.7 | 0.109 |
| $n_2 = 3/4$ | 0.151 | -0.339 | 0.0283 | 0.225 | -2.53 | 0.0527 | 0.244 | -11.5 | 0.0600 | 0.248 | -47.4 | 0.0618 |
| $n_2 = 7/8$ | 0.108 | -0.284 | 0.0101 | 0.168 | -2.21 | 0.0197 | 0.184 | -10.1 | 0.0226 | 0.188 | -41.9 | 0.0234 |

Table 2: Numerical point estimates of $\tau^d$, $t_1^d$, and $t_2^d$, for different values of $w_2$ and $n_2$, and $\rho \to 1$
Lastly, as $\rho \to 1$, $W^d(\tau^d)$ becomes:

$$W^d(\tau^d, n_2, w_2) = (1 - n_2) \ln \left[ \left(\frac{1}{2}\right)(1 - \tau^d)^2 + \tau^d(1 - \tau^d)\eta \right]$$

$$+ n_2 \ln \left[ \left(\frac{1}{2}\right)w_2^2(1 - \tau^d)^2 + \tau^d(1 - \tau^d)\eta \right].$$

(10)

It is this function which, evaluated at $\tau^d(w_2, n_2)$, must then be compared with $W^c(\tau, n_2, w_2)$, in order to ascertain when pursuing a full societal consensus may or may not be worthwhile.

### 2.4.3 Welfare comparisons between full consensus and dissensus

If the social planner could implement $\tau^c$, the question of whether it should answers itself: a full consensus around a certain tax rate brings about ethical labour supply in all agent types, implying an elasticity of total earnings with respect to the tax rate of zero, and thus no efficiency-equity trade-off in setting the tax rate. In no way a social planner would (and should) then pursue a dissensus. However, as was previously discussed, it may be that individual social preferences do not include $\tau^c$ as part of the consensual policy space. The equity-efficiency trade-off then reasserts itself fully, given that the planner must then either conform to the consensual wishes of the population by choosing $\tau \in C$ or choose to retain free rein over the optimal choice of tax rate, albeit at the cost of a dissensus and therefore a negative elasticity of total earnings with respect to the tax rate. It then becomes an open question as to which choice should be made, one that depends greatly on the parameters of the model, and especially on the consensual policy space $C$. The figure and table below show the difference between dissensus and consensus welfare, $\Delta \equiv W^d - W^c$, as a function of $\tau$, the tax rate(s) dictated by the consensual policy space. They lead to the following proposition, and corollary.

**Proposition 1.** Given most moderate individual social preferences for redistribution, leading to equally-moderate consensual policy spaces $C$, it is in the interest of social welfare for the planner to pursue a societal consensus by choosing $\tau \in C$, even if $\tau^c \notin C$. Conversely, extreme consensual values, corresponding to very low or very high tax rates, should bring the inequality-averse social planner to retain free rein over its optimal choice of $\tau$. Thus, a societal dissensus coupled with an optimal choice of linear tax can be preferable to a societal consensus characterized by extreme choices.

**Corollary 1.** For a given level of pre-tax wage inequality, measured by $w_2/w_1$, the greater is the proportion of high-skilled agents $n_2 = 1 - n_1$, then the greater is the proportion of the policy space $\tau \in [0,1]$ where the planner should defer its choice of optimal linear tax to that dictated by the societal consensus, even if $\tau^c \notin C$. Conversely, for a given proportion
of high-skilled agents, the greater the level of pre-tax wage inequality, then the greater is the proportion of the policy space where the social planner should choose $\tau = \tau^d$, for all possible consensual policy intervals $C$.

Figure 3: The welfare differential between dissensus and consensus, $\Delta \equiv W^d - W^c$, as a function of $\tau$, for different values of $w_2$ and $n_2$, and $\rho \to 1$
Indirect utility is given by
\[ v(w_2(1 - \tau), T) = (1/2)w_2^2(1 - 2\tau) + T. \]
In contrast, the high-skilled agents supply labour self-interestedly, which means that their indirect utility is then given by
\[ v(w_1(1 - \tau), T) = (1/2)w_1^2(1 - 2\tau) + T. \]

In the absence of a full consensus spanning the entirety of the population – an elusive prospect to be sure – the social planner could potentially pursue a partial societal consensus, composed of a plurality of agent types. In a two-type world, this can be understood as encouraging the ethical labour supply of a single type, by targeting their social preferences through the planner’s choice of \( \tau \). Despite the obvious limitations of this approach (not least that it does not sit well with Definition 1 going as far as contradicting it), it offers some insight as to what characteristics this partial consensus should satisfy for the planner to pursue it, especially at the expense of its full discretion in policy choices.

### 2.4.4 Partial consensus: targeting the ethical behaviour of population sub-groups

A “partial consensus” where the low-skilled supply labour ethically In such a partial consensus, the low-skilled agents – of relative population weight \( n_1 = 1 - n_2 \), with social individual preferences given by \( \tau_1 \), and marginal product \( w_1 = 1 - \) supply labour ethically, which means that their indirect utility is then given by
\[ v(w_1(1 - \tau), T) = (1/2)w_1^2(1 - 2\tau) + T. \]

<table>
<thead>
<tr>
<th>( n_2 = 1/8 )</th>
<th>( w_2 = 2 )</th>
<th>( w_2 = 4 )</th>
<th>( w_2 = 8 )</th>
<th>( w_2 = 16 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.0761, 0.711)</td>
<td>(0.173, 0.606)</td>
<td>(0.214, 0.579)</td>
<td>(0.226, 0.573)</td>
<td></td>
</tr>
<tr>
<td>(0.0997, 0.769)</td>
<td>(0.179, 0.681)</td>
<td>(0.205, 0.658)</td>
<td>(0.212, 0.653)</td>
<td></td>
</tr>
<tr>
<td>(0.106, 0.829)</td>
<td>(0.172, 0.757)</td>
<td>(0.191, 0.738)</td>
<td>(0.196, 0.734)</td>
<td></td>
</tr>
<tr>
<td>(0.104, 0.896)</td>
<td>(0.161, 0.839)</td>
<td>(0.176, 0.824)</td>
<td>(0.180, 0.820)</td>
<td></td>
</tr>
<tr>
<td>(0.0960, 0.973)</td>
<td>(0.145, 0.934)</td>
<td>(0.158, 0.923)</td>
<td>(0.161, 0.921)</td>
<td></td>
</tr>
<tr>
<td>(0.0820, 1)</td>
<td>(0.125, 1)</td>
<td>(0.136, 1)</td>
<td>(0.139, 1)</td>
<td></td>
</tr>
<tr>
<td>(0.0584, 1)</td>
<td>(0.0951, 1)</td>
<td>(0.105, 1)</td>
<td>(0.108, 1)</td>
<td></td>
</tr>
</tbody>
</table>

(a) Intervals of \( \tau \)

<table>
<thead>
<tr>
<th>( n_2 = 1/8 )</th>
<th>( w_2 = 2 )</th>
<th>( w_2 = 4 )</th>
<th>( w_2 = 8 )</th>
<th>( w_2 = 16 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>63.5</td>
<td>43.3</td>
<td>36.5</td>
<td>34.7</td>
<td></td>
</tr>
<tr>
<td>66.9</td>
<td>50.2</td>
<td>45.3</td>
<td>44.1</td>
<td></td>
</tr>
<tr>
<td>72.3</td>
<td>58.5</td>
<td>54.7</td>
<td>53.8</td>
<td></td>
</tr>
<tr>
<td>79.2</td>
<td>67.8</td>
<td>64.8</td>
<td>64.0</td>
<td></td>
</tr>
<tr>
<td>87.7</td>
<td>78.9</td>
<td>76.5</td>
<td>76.0</td>
<td></td>
</tr>
<tr>
<td>91.8</td>
<td>87.5</td>
<td>86.5</td>
<td>86.1</td>
<td></td>
</tr>
<tr>
<td>94.2</td>
<td>90.5</td>
<td>89.5</td>
<td>89.2</td>
<td></td>
</tr>
</tbody>
</table>

(b) As a percentage (%) of policy space

Table 3: Where the full consensus welfare \( W^c \) exceeds the dissensus welfare \( W^d \), for different values of \( w_2 \) and \( n_2 \), and \( \rho \to 1 \)
sensus to arise, the planner’s policy choice must lie in the interval $C_1 = [\tau_1 - \theta, \tau_1 + \theta]$. The planner’s problem is then generally:

$$\max_{\tau \in C_1} \frac{n_1}{1 - \rho}((1/2)w_1^2(1 - 2\tau) + T)^{1-\rho} + \frac{n_2}{1 - \rho}((1/2)w_2^2(1 - \tau)^2 + T)^{1-\rho}$$

s.t.

$$n_1 w_1^2 \tau + n_2 w_2^2 (1 - \tau) \tau = T$$

which thereafter reduces to:

$$\max_{\tau \in C_1} \frac{n_1}{1 - \rho}((1/2)w_1^2(1 - 2\tau) + n_1 w_1^2 \tau + n_2 w_2^2 (1 - \tau) \tau)^{1-\rho}$$

$$+ \frac{n_2}{1 - \rho}((1/2)w_2^2(1 - \tau)^2 + n_1 w_1^2 \tau + n_2 w_2^2 (1 - \tau) \tau)^{1-\rho}$$

and leads to the following first-order condition for an interior solution:

$$n_1((1/2)w_1^2(1 - 2\tau) + n_1 w_1^2 \tau + n_2 w_2^2 (1 - \tau) \tau)^{-\rho}$$

$$\times (-n_1) w_1^2 + n_2 w_2^2 (1 - 2\tau))$$

$$+ n_2((1/2)w_2^2(1 - \tau)^2 + n_1 w_1^2 \tau + n_2 w_2^2 (1 - \tau) \tau)^{-\rho}$$

$$\times (-w_2^2(1 - \tau) + n_1 w_1^2 + n_2 w_2^2 (1 - 2\tau)) = 0. \quad (11)$$

Making use of $w_1 = 1$ and $n_1 = 1 - n_2$, this further reduces to:

$$n_2(1 - n_2)((1/2)(1 - 2\tau) + (1 - n_2) \tau + n_2 w_2^2 (1 - \tau) \tau)^{-\rho}$$

$$\times (-1 + w_2^2(1 - 2\tau))$$

$$+ n_2((1/2)w_2^2(1 - \tau)^2 + n_1 \tau + n_2 w_2^2 (1 - \tau) \tau)^{-\rho}$$

$$\times ((1 - n_2)[1 - w_2^2(1 - \tau)] - \tau n_2 w_2^2) = 0. \quad (12)$$

Let us denote the solution as $\tau^l$, and to perform numerical simulations, found below, let us again focus on the case where $\rho \rightarrow 1$. These results are summarized by the following proposition and its corollary.

**Proposition 2.** Optimally-chosen linear tax rates when low-skilled agents supply labour ethically, $\tau^l$, exceed those when all agent types supply labour self-interestedly, $\tau^d$.

Intuitively, this is the result of less elastic total earnings with respect to the tax rate, which reduces the equity-efficiency trade-off. However, that trade-off remains more present than in the case of a full consensus, where labour is supplied ethically by all types.
Corollary 2. The difference $\tau^l - \tau^d$ is greatest when pre-tax wage inequality $w_2/w_1$ is low and the proportion of high-skilled agents, $n_2$, is low. Conversely, $\tau^l - \tau^d \to 0$ as $w_2/w_1 \to \infty$, or as $n_2 \to 1$.

When the wage inequality increases, the inequality-averse nature of the planner tends to emphasize the need for redistribution, and thus the equity motive over its efficiency counterpart. Therefore, even in the absence of ethical labour supply on the part of the low-skilled agents, the chosen linear tax rate $\tau^d$ is relatively high, such that the difference between it and $\tau^l$ is small. For a high proportion of high-skilled agents, $n_2$, both $\tau^l$ and $\tau^d$ are lower, and not much different from each other; what matters most then in both efficiency and equity terms is the elasticity of high-skilled earnings with respect to the tax rate, as the ethical behaviour of a small portion of the population with a low wage is of little consequence. Thus, not only is the high-skilled utility then proportionately more important in the social welfare function than for low values of $n_2$ – which should, ceteris paribus, call for lower values of $\tau$ – but a certain level of redistribution can be achieved at a much lower tax rate, ceteris paribus.

<p>| $w_2 = 2$ | $w_2 = 4$ | $w_2 = 8$ | $w_2 = 16$ |</p>
<table>
<thead>
<tr>
<th>$\tau^d$</th>
<th>$\tau^l$</th>
<th>$\tau^d$</th>
<th>$\tau^l$</th>
<th>$\tau^d$</th>
<th>$\tau^l$</th>
<th>$\tau^d$</th>
<th>$\tau^l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_2 = 1/8$</td>
<td>0.149</td>
<td>0.270</td>
<td>0.335</td>
<td>0.395</td>
<td>0.416</td>
<td>0.434</td>
<td>0.440</td>
</tr>
<tr>
<td>$n_2 = 1/4$</td>
<td>0.191</td>
<td>0.253</td>
<td>0.341</td>
<td>0.364</td>
<td>0.390</td>
<td>0.396</td>
<td>0.403</td>
</tr>
<tr>
<td>$n_2 = 3/8$</td>
<td>0.201</td>
<td>0.234</td>
<td>0.323</td>
<td>0.334</td>
<td>0.358</td>
<td>0.361</td>
<td>0.367</td>
</tr>
<tr>
<td>$n_2 = 1/2$</td>
<td>0.195</td>
<td>0.213</td>
<td>0.296</td>
<td>0.302</td>
<td>0.324</td>
<td>0.326</td>
<td>0.331</td>
</tr>
<tr>
<td>$n_2 = 5/8$</td>
<td>0.178</td>
<td>0.187</td>
<td>0.265</td>
<td>0.267</td>
<td>0.287</td>
<td>0.288</td>
<td>0.293</td>
</tr>
<tr>
<td>$n_2 = 3/4$</td>
<td>0.151</td>
<td>0.154</td>
<td>0.225</td>
<td>0.226</td>
<td>0.244</td>
<td>0.244</td>
<td>0.248</td>
</tr>
<tr>
<td>$n_2 = 7/8$</td>
<td>0.108</td>
<td>0.108</td>
<td>0.168</td>
<td>0.169</td>
<td>0.184</td>
<td>0.184</td>
<td>0.188</td>
</tr>
</tbody>
</table>

Table 4: Numerical point estimates of $\tau^l$ and $\tau^d$, for different values of $w_2$ and $n_2$, and $\rho \to 1$

A “partial consensus” where the high-skilled supply labour ethically On the contrary, in such a partial consensus, the high-skilled agents – of relative population weight $n_2$, with social individual preferences given by $\tau_2$, and marginal product $w_2 > 1$ – supply labour ethically, which means that their indirect utility is then given by $\nu(w_2(1 - \tau), T) = \frac{1}{2}w_2^2(1 - 2\tau) + T$. In contrast, the low-skilled agents now supply labour self-interestedly, which means that their indirect utility is given by $\nu(w_1(1 - \tau), T) = \frac{1}{2}w_1^2(1 - \tau)^2 + T$. For such a partial consensus to arise, the planner’s policy choice must lie in the interval $C_2 = [\tau_2 - \theta, \tau_2 + \theta]$. The planner’s problem is then generally:

$$\max_{\tau \in C_2} \frac{n_1}{1 - \rho} \left(\frac{1}{2}w_1^2(1 - \tau)^2 + T\right)^{1 - \rho} + \frac{n_2}{1 - \rho} \left(\frac{1}{2}w_2^2(1 - 2\tau) + T\right)^{1 - \rho}$$
s.t. \[ n_1 w_1^2(1 - \tau) + n_2 w_2^2 \tau = T \]

which now reduces to:

\[
\max_{\tau \in \mathbb{C}_1} \frac{n_1}{1 - \rho} \left( \frac{1}{2} w_1^2(1 - \tau)^2 + n_1 w_1^2(1 - \tau) + n_2 w_2^2 \tau \right)^{1 - \rho} \\
+ \frac{n_2}{1 - \rho} \left( \frac{1}{2} w_2^2(1 - 2\tau) + n_1 w_1^2(1 - \tau) + n_2 w_2^2 \tau \right)^{1 - \rho}
\]

and leads to the following first-order condition for an interior solution:

\[
n_1((1/2)w_1^2(1 - \tau)^2 + n_1 w_1^2(1 - \tau) + n_2 w_2^2 \tau)^{-\rho} \\
\times \left( - \frac{n_1}{1 - \rho} \left( \frac{1}{2} w_1^2(1 - \tau) - n_1 w_1^2 \tau + n_2 w_2^2 \right) \\
+ n_2((1/2)w_2^2(1 - 2\tau) + n_1 w_1^2(1 - \tau) + n_2 w_2^2 \tau)^{-\rho} \\
\times \left( - \frac{n_2}{1 - \rho} \left( \frac{1}{2} w_2^2(1 - 2\tau) + n_2 w_2^2 \tau \right) + (1 - n_2)(1 - \tau) \right) \right) = 0.
\] (13)

Making once again use of \( w_1 = 1 \) and \( n_1 = 1 - n_2 \), this further reduces to:

\[
(1 - n_2)((1/2)(1 - \tau)^2 + (1 - n_2)(1 - \tau) + n_2 w_2^2 \tau)^{-\rho} \\
\times \left( - \frac{n_1}{1 - \rho} \left( (1 - n_2)(1 - \tau) - (1 - n_2)w_1^2 \tau + n_2 w_2^2 \right) \\
+ n_2(1 - n_2)((1/2)w_2^2(1 - 2\tau) + (1 - n_2)(1 - \tau) + n_2 w_2^2 \tau)^{-\rho} \\
\times \left( - w_2^2 + (1 - 2\tau) \right) \right) = 0.
\] (14)

For \( \rho \to 1 \), this yields the values found below, which are summarized by the following proposition and corollary.

**Proposition 3.** Optimally-chosen linear tax rates when high-skilled agents supply labour ethically, \( \tau^h \), exceed those when all agent types supply labour self-interestedly, \( \tau^d \).

The intuition behind this result is the same as in the low-skilled case. It is however now further amplified by the fact that the more productive individuals have an elasticity of labour earnings with respect to the tax rate of zero.

**Corollary 3.** The difference \( \tau^h - \tau^d \) is greatest when both pre-tax wage inequality, \( w_2/w_1 \), and the proportion of high-skilled agents, \( n_2 \), are high. Conversely, \( \tau^l - \tau^d \to 0 \) as \( w_2/w_1 \to \infty \) and as \( n_2 \to 0 \).

This result is strikingly different from the low-skilled case. As \( n_2 \) increases, *ceteris paribus*, \( \tau^h \) is monotonically increasing, while \( \tau^d \) is generally decreasing. The latter is due to the effects
that higher values of \( n_2 \) have in making total earnings more elastic with respect to \( \tau \), and in increasing the relative weight of high-skilled agents in the SWF. The former, however, differs from the behaviour of \( \tau^d \) and \( \tau^l \) with respect to \( n_2 \) (and \( w_2 \)), as the total earnings then become less elastic with respect to \( \tau \) as \( n_2 \) (and \( w_2 \)) increases: a greater proportion of agents supply labour ethically (and are relatively more productive), which allows for greater optimal linear tax rates so as to meet the planner’s redistributive objectives.

\[
\begin{array}{cccccccc}
\text{\( n_2 \)} & \text{\( \tau^d \)} & \text{\( \tau^h \)} & \text{\( \tau^d \)} & \text{\( \tau^h \)} & \text{\( \tau^d \)} & \text{\( \tau^h \)} & \text{\( \tau^d \)} & \text{\( \tau^h \)} \\
\hline
\text{1/8} & 0.149 & 0.246 & 0.335 & 0.466 & 0.416 & 0.494 & 0.440 & 0.499 \\
\text{1/4} & 0.191 & 0.339 & 0.341 & 0.473 & 0.390 & 0.494 & 0.403 & 0.499 \\
\text{3/8} & 0.201 & 0.370 & 0.323 & 0.474 & 0.358 & 0.494 & 0.367 & 0.499 \\
\text{1/2} & 0.195 & 0.384 & 0.296 & 0.475 & 0.324 & 0.494 & 0.331 & 0.499 \\
\text{5/8} & 0.178 & 0.392 & 0.265 & 0.476 & 0.287 & 0.494 & 0.293 & 0.499 \\
\text{3/4} & 0.151 & 0.397 & 0.225 & 0.476 & 0.244 & 0.494 & 0.248 & 0.499 \\
\text{7/8} & 0.108 & 0.400 & 0.168 & 0.476 & 0.184 & 0.494 & 0.188 & 0.499 \\
\end{array}
\]

Table 5: Numerical point estimates of \( \tau^h \) and \( \tau^d \), for different values of \( w_2 \) and \( n_2 \), and \( \rho \to 1 \)

### 2.4.5 Further welfare comparisons: “partial consensus” and dissensus

As shown above, the optimal linear tax rate chosen in a partial consensus will only approximate the one corresponding to a dissensus when: for a low-skilled partial consensus, the higher is the pre-tax wage inequality or the proportion of high-skilled agents; for a high-skilled partial consensus, the higher is pre-tax wage inequality and the lower is the proportion of high-skilled agents. Given that ethical labour supply leads to both an increase in total labour earnings (thus allowing for a higher degree of income redistribution for a given tax rate, \textit{ceteris paribus}), and an increase in the indirect utility of the ethical individuals, all things being equal elsewhere, it follows that a partial consensus with an optimally-chosen rate is better than a dissensus, and the more so the two optimal rates differ.

However, as with the comparison between a full consensus and a dissensus, it may be that the planner’s optimal partially-consensual rate is not part of the partially-consensual policy space. This triggers a further trade-off between favouring a partial consensus and giving the social planner free rein over the policy choice, while then triggering a dissensus. Accordingly, this trade-off is once more studied in terms of the differences between dissensus and partial consensus welfare. It is illustrated in the figures and tables below, for \( \rho \to 1 \), as a function of \( \tau \), the rate(s) dictated by the partial consensus. It also leads to the following propositions.

**Proposition 4.** Only small ranges of the low-skilled’s social preferences may lead the planner
to compromise with its optimal choice of linear tax rate, and instead favour a rate compatible with a partial consensus. These ranges monotonically decrease both in pre-tax wage inequality, $w_2/w_1$, and in the proportion of high-skilled agents, $n_2$.

The low-skilled agents’ ethical supply of labour does not affect much the elasticity of total earnings with respect to the tax rate: the less so the greater is the pre-tax wage inequality, and the smaller is their proportion of the population, $n_1$. It follows that it is then rarely preferable for the planner to compromise with its choice of optimal tax rate for the sake of pursuing a partial consensus, when $\tau' \notin C_1$. 
Figure 4: The welfare differential between dissensus and partial (low-skill) consensus, $\Delta^l \equiv W^d - W^l$, as a function of $\tau$, for different values of $w_2$ and $n_2$, and $\rho \rightarrow 1$
Table 6: Where the partial (low-skill) consensus welfare $W_l$ exceeds the dissensus welfare $W_d$, for different values of $w_2$ and $n_2$, and $\rho \to 1$

**Proposition 5.** Broader ranges of the high-skilled’s social preferences may lead the planner to compromise with its optimal choice of linear tax rate, and instead favour a rate compatible with a partial consensus. These ranges monotonically increase in the proportion of high-skilled agents, $n_2$.

In contrast with the low-skilled partial consensus, it is in the planner’s interest to compromise with its optimal choice of tax rate for much larger intervals of the policy space compatible with a high-skilled partial consensus. This underlines the importance of the high-skilled agents’ ethical behaviour in a full consensus, and more generally for expanding the potential for redistribution, and thus lessening the equity-efficiency trade-off.
Figure 5: The welfare differential between dissensus and partial (high-skill) consensus, $\Delta^h \equiv W^d - W^h$, as a function of $\tau$, for different values of $w_2$ and $n_2$, and $\rho \to 1$
we should then expect the results of Proposition 1 to carry through for Corollary 1 to also hold, albeit with differences in the measurement of pre-tax consensus policy space $\theta$ across all groups, and for a higher tolerance parameter $\rho$ requires this overlap to involve them all. This is more likely for similar preferred tax rates $C$ groups, and which in turn defined the consensual policy space $\mathcal{C}$. Furthermore, one might expect the consensus policy space $\mathcal{C}$, to the extent that it is not empty, to be on average smaller. However, whereas all that was required for a full consensus with $m > 2$ groups, develop an intuitive, and indeed qualitative understanding of how they may translate to future research endeavours. However, one may, on the basis of the above results for prospect, is mired with difficulty due to the sheer myriad of possible cases, and the added computational complexity of the task. A thorough analysis is therefore perhaps better left to future research endeavours. However, one may, on the basis of the above results for $m = 2$ groups, develop an intuitive, and indeed qualitative understanding of how they may translate to $m > 2$ groups.

First turn to a full consensus involving $m = 3$ groups. For a single tolerance parameter $\theta$, and all possible preferred tax rates for all three groups, such a consensus is more elusive than previously was the case. Whereas all that was required for a full consensus with $m = 2$ groups was overlap in the ranges of tax rates giving rise to ethical behaviour across both groups, and which in turn defined the consensual policy space $\mathcal{C}$, the presence of three groups requires this overlap to involve them all. This is more likely for similar preferred tax rates across all groups, and for a higher tolerance parameter $\theta$. Furthermore, one might expect the consensus policy space $\mathcal{C}$, to the extent that it is not empty, to be on average smaller. However, we should then expect the results of Proposition 1 to carry through for $m > 2$. Similarly, we should expect Corollary 1 to also hold, albeit with differences in the measurement of pre-tax welfare $W$.

### Table 7: Partial (high-skill) consensus welfare $W^h$

<table>
<thead>
<tr>
<th>$n_2 = 1/8$</th>
<th>$n_2 = 1/4$</th>
<th>$n_2 = 3/8$</th>
<th>$n_2 = 1/2$</th>
<th>$n_2 = 5/8$</th>
<th>$n_2 = 3/4$</th>
<th>$n_2 = 7/8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_2 = 2$</td>
<td>(0.0884, 0.391)</td>
<td>(0.107, 0.531)</td>
<td>(0.110, 0.593)</td>
<td>(0.106, 0.645)</td>
<td>(0.0968, 0.699)</td>
<td>(0.0821, 0.762)</td>
</tr>
<tr>
<td>$w_2 = 4$</td>
<td>(0.181, 0.579)</td>
<td>(0.182, 0.645)</td>
<td>(0.174, 0.712)</td>
<td>(0.161, 0.784)</td>
<td>(0.145, 0.866)</td>
<td>(0.125, 0.972)</td>
</tr>
<tr>
<td>$w_2 = 8$</td>
<td>(0.217, 0.574)</td>
<td>(0.206, 0.650)</td>
<td>(0.192, 0.728)</td>
<td>(0.176, 0.811)</td>
<td>(0.158, 0.906)</td>
<td>(0.136, 1)</td>
</tr>
<tr>
<td>$w_2 = 16$</td>
<td>(0.227, 0.572)</td>
<td>(0.212, 0.651)</td>
<td>(0.196, 0.731)</td>
<td>(0.180, 0.817)</td>
<td>(0.161, 0.920)</td>
<td>(0.139, 1)</td>
</tr>
</tbody>
</table>

(a) Intervals of $\tau$

<table>
<thead>
<tr>
<th>$w_2 = 2$</th>
<th>$w_2 = 4$</th>
<th>$w_2 = 8$</th>
<th>$w_2 = 16$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_2 = 1/8$</td>
<td>30.3</td>
<td>39.8</td>
<td>35.7</td>
</tr>
<tr>
<td>$n_2 = 1/4$</td>
<td>42.4</td>
<td>47.2</td>
<td>44.4</td>
</tr>
<tr>
<td>$n_2 = 3/8$</td>
<td>48.3</td>
<td>53.8</td>
<td>53.6</td>
</tr>
<tr>
<td>$n_2 = 1/2$</td>
<td>53.9</td>
<td>62.3</td>
<td>63.5</td>
</tr>
<tr>
<td>$n_2 = 5/8$</td>
<td>60.2</td>
<td>72.1</td>
<td>74.8</td>
</tr>
<tr>
<td>$n_2 = 3/4$</td>
<td>68.0</td>
<td>84.7</td>
<td>86.4</td>
</tr>
<tr>
<td>$n_2 = 7/8$</td>
<td>79.2</td>
<td>90.5</td>
<td>89.5</td>
</tr>
</tbody>
</table>

(b) As a percentage (%) of the policy space

2.5 Generalizing certain results to $m > 2$ groups

Obtaining general estimates when more than two groups are present, while a very interesting prospect, is mired with difficulty due to the sheer myriad of possible cases, and the added computational complexity of the task. A thorough analysis is therefore perhaps better left to future research endeavours. However, one may, on the basis of the above results for $m = 2$ groups, develop an intuitive, and indeed qualitative understanding of how they may translate to $m > 2$ groups.
wage inequality, and the weighting of the higher-skilled population groups.

As concerns forms of partial consensus for $m > 2$ groups, they now make proper sense, but also take many more forms. For instance, for $m = 3$, one may envision three potential forms of partial consensus: high-low, high-mid, mid-low. Yet, while this complicates the derivation of formal results, one should again expect Propositions 4 and 5 to qualitatively hold. In other words: the greater is the average pre-tax wage in a partial consensus, and the greater is the population weight of the groups involved in this partial consensus, then the more likely is the social planner going to choose a partially-consensual tax rate rather than one optimally-chosen, but leading to a dissensus.

3 Concluding Remarks and Further Research

This paper sought to condition the normative analysis of redistribution on a societal consensus rooted in individual social preferences, when consensual policy choices were deemed to induce a departure from purely self-interested behaviour in the individuals’ choice of labour supply, and instead promote ethical behaviour. In a two skill-type model where an inequality-averse social planner chose a linear income tax schedule, it was shown that to pursue a societal consensus could be beneficial, even if it meant not choosing the otherwise-optimal tax rate, as long as the consensus was over moderate tax rates. For extreme consensual ranges, the social planner should instead forsake the consensus, and retain free rein in its choice of a tax schedule. The range of welfare-maximizing consensual tax rates was found to increase with the proportion of high-skilled agents, while also to decrease with the degree of pre-tax wage inequality. The former result illustrates the importance of high-skilled agents to a societal consensus, through the effect of their ethical behaviour on the total labour earnings, and thus on the potential for redistribution. The latter result illustrates how a dissensus may be freeing for the social planner to achieve its redistributive objective, when the typical trade-off between equity and efficiency is biased towards the former.

Through its analysis and results, this paper highlighted the possibility that the optimal choice of a linear income tax scheme may not be as straightforward as typically envisioned in normative public finance. Indeed, if consensual policy choices lead to changes in behaviour, especially of a discrete nature, then it may be that the social planning problem no longer is characterized by a unique global and local optimum. Instead, discontinuities in the SWF may lead to multiple local optima, among which lies one or many global optima. This paper’s model avoided many of the complications of such a problem by limiting the number of skill types to two. Further research should seek to produce formal, general results for more than two skill types, and for many forms of partial consensus.
Another avenue for further work is to move away from this paper’s rather basic modelling of the formation of a societal consensus, and its determinants, and towards a deeper understanding of the matter. In particular, endogenizing $\theta$, the tolerance of individuals for deviations of policy choices from their ideal, or even endogenizing the social preferences of individuals for them to depend on past policy choices, could deepen our understanding of the interaction between policy choices and societal consensus. It could also further relate this work to the importance of economic ideas and policy experiments for inducing shifts in policy paradigms.

References


