Health insurance, income protection insurance, benefit limits, and economic growth

Abstract

In a small open economy where human capital can yield higher expected profit than physical capital, we build an overlapping generation framework. We compared the cases with health insurance (HI) and income protection insurance (IPI). HI provides individuals with a fixed amount of benefit, while IPI protects a certain proportion of individual income. First, we find that when individuals can choose insurance coverage freely, HI and IPI are mostly the same, and individuals will choose to cover a fixed proportion of their risks. Freely chosen insurance can either enhance or deter human capital accumulation. Second, with proper benefit limit, HI can benefit human capital accumulation and economic growth and that the insurance should be just enough to eliminate private saving. Also, limited IPI can be better in boosting human capital than limited HI. Finally, HI, IPI and limits will be more important for economic growth when the return to human capital is moderate.

Keywords: health insurance, income protection insurance, benefit limit, economic growth

I. Introduction

Human capital, which can offset diminishing returns of physical capital, is crucial for economic growth (Lucas, 1988). Individuals need innate ability (Zilcha, 2003), money (Arawatari and Ono, 2009) and time (Soares, 2003; Yakita, 2003) in the formation of human capital. However, the realization of human capital may be uncertain due to health risks. Individuals may fall ill and lose the ability to work, which is a considerable disincentive for investment in education and training.

To defend against those uncertainties and smooth future consumption, individuals have two options: saving and insurance. In this paper, we intend to compare those two ways and find how they will intertwine with each other. Specifically, we investigated two insurance policies: health insurance (HI) and income protection insurance (IPI). HI provides individuals a specific amount of money when they fall ill. IPI provides individuals with a proportion of their full income when they lose the ability to work.

First, we assume individuals to choose insurance coverage freely. We want to find how individuals will choose their hedging tools when they have access to both saving and insurances and how they will choose HI and IPI coverage. When individuals can choose freely, we find individuals prefer the more efficient insurance instead of saving and would choose to cover a fixed proportion of their future income. Free HI and free IPI will yield the same result and are mostly the same. Insurance can either increase or decrease education investment.

Next, we bring the benefit limits into our model. HI benefit limit means the compensation money cannot exceed a certain amount. IPI limit means the insurance coverage proportion cannot exceed a certain fraction. We find with proper limits, HI and IPI can boost human capital accumulation. Finally, the growth-optimal IPI is better than growth-optimal HI in enhancing education.

II. The model

Imagine there is a small economy which has long been isolated from the outside world due to war or some political reasons, like North Korea or some countries in Africa. It has managed to establish a stable government and begins to open to the outside world, like what Singapore did in the 1960s. Due to the long-time isolation, it falls far behind the world, both in physical and human capital. Now, capital can flow freely in or out of the small economy. The world’s interest rate \( r \) is exogenously fixed for the small economy.

We build a discrete-time overlapping-generation model. All individuals live for two periods: young and old, which are denoted by 1 and 2, respectively. Those who spend their young period in period \( t \) are called generation \( t \).

There is no public education system; children learn all their skills by watching parents do their work. Following Yakita (2003) Lu and Yanagihara (2013), we assume that the human capital of the parents proportionately determines the human capital of their children.

\[
h_{1t} = \chi h_{2t}, \chi > 0
\]

(1)

Because of the free capital flow, capital will peruse low wage, which will cause the wage rates inside the country to be equal
to the world’s wage rate. We normalize the wage per efficient labor unit to be 1. Base on this level of human capital, they get an income, \( i_{1,t} \).

\[
\begin{align*}
  i_{1,t} &= h_{1,t} 
\end{align*}
\]

They can allocate their income to three parts: consumption in the young period, investment in education, saving, and payments for insurance. Therefore, the budget constraint in the young becomes:

\[
\begin{align*}
  c_{1,t} + e_{1,t} + s_{1,t} &= i_{1,t} = h_{1,t} 
\end{align*}
\]

As the same with Krebs (2003) and Grossmann (2008), the human capital of generation \( t \) in their old period, \( h_{2,t+1} \), is proportionally determined by the amount of education investment as:

\[
\begin{align*}
  h_{2,t+1} &= \theta e_{1,t} 
\end{align*}
\]

Here \( \theta > 1 \) is a parameter measuring education efficiency. Except for simplicity, the linearity of human capital production reflects that when a country is far backward in knowledge and technology than the world’s level, copying the existing knowledge and learn some necessary skills do not show decreasing marginal return. We assume those skills will quickly get outdated, so all the skills in this period will be useless in the next period. The zero intercept means if an individual does not invest in learning new skills, all the skills he possesses now will be outdated next period.

Consider that individuals face uncertainty in their old period. If they are healthy, they can fully realize their human capital. On the contrast, if they are unhealthy, as we are discussing the situation in the developing world, where the health care industry is also under-developed, for simplicity, we assume the illness will cause the total loss of the ability to work. So, his consumption in the second period will be:

\[
\begin{align*}
  c_{2,t+1} &= \begin{cases} 
    (1 + r)s_{1,t} + h_{2,t+1}, & \text{if healthy} \\
    (1 + r)s_{1,t}, & \text{if unhealthy} 
  \end{cases}
\end{align*}
\]

Denoting the probability of being unhealthy as \( \pi \), we can rewrite the expected utility function into the following:

\[
\begin{align*}
  E[u(c_{1,t}, h_{2,t+1}, s_{1,t})] &= \frac{1}{\sigma} c_{1,t}^\sigma + \frac{1}{1 + \beta} \left[ (1 - \pi) \left( (1 + r)s_{1,t} + h_{2,t+1} \right)^\sigma \right] + \frac{1}{1 + \beta} \left[ \pi \left( (1 + r)s_{1,t} \right)^\sigma \right] 
\end{align*}
\]

Now we notice the zero-risk interest rate is \( r \), and the expected return from investing in education is \( \theta(1 - \pi) - 1 \), if \( \theta(1 - \pi) - 1 < r \), a risk-averse individual will prefer to deposit his money in the bank instead of invest in the risky human capital, which will lead to zero human capital in the second period. Things will be quite simple, and little discussion is needed. The more exciting is, what will happen if the return from education is high. In the following discussion, I would focus on the situation where \( \theta(1 - \pi) - 1 > r \).

**Assumption 1:** The expected return from investment in human capital is higher than the zero-risk interest rate, or \( \theta(1 - \pi) - 1 > r \).

From (3) and (4), we can get the simplified budget constraint without IPI in the young period:

\[
\begin{align*}
  c_{1,t} + \frac{h_{2,t+1}}{\theta} + s_{1,t} &= i_{1,t} = h_{1,t} 
\end{align*}
\]

As in Yaari (1965) and Blanchard (1985), we assume that there are actuarially fair insurance companies, and the insurance market is perfectly competitive. At the end of the first period, an individual buys the insurance. Then the insurance companies invest the money they get from individuals to the zero-risk capital market in the second period. Finally, at the end of the second period, they pay the original money and capital returns to individuals who fall ill.

If individuals have access to IPI, they can choose to protect a fraction \( \delta \) of their income when they fall ill in the second period but must pay the premium in the first period. Then the expected utility function and budget constraint can be modified as follows:

\[
\begin{align*}
  E[u(c_{1,t}, h_{2,t+1}, s_{1,t})] &= \frac{1}{\sigma} c_{1,t}^\sigma + \frac{1}{1 + \beta} \left[ (1 - \pi) \left( (1 + r)s_{1,t} + h_{2,t+1} \right)^\sigma \right] + \frac{1}{1 + \beta} \left[ \pi \left( (1 + r)s_{1,t} + \delta h_{2,t+1} \right)^\sigma \right] 
\end{align*}
\]

\[
\begin{align*}
  c_{1,t} + \frac{h_{2,t+1}}{\theta} + s_{1,t} + \frac{\delta h_{2,t+1}}{1 + r} &= i_{1,t} = h_{1,t} 
\end{align*}
\]
We can see in the first period, an individual need to pay \( \frac{\delta h_{2,t+1}}{1+r} \) to cover a fraction \( \delta \) of his potential loss when he falls ill. And when he falls ill, he will get \( \delta h_{2,t+1} \).

Now we want to find how an individual will allocate his first-period income among current consumption, saving, education investment, and insurance. Maximizing (8) subject to (9), we get:

**Lemma 1**: When individuals can choose IPI and HI freely, there will be no private saving: \( s_{1,t} = 0 \). Individuals would instead use the combination of investment in education and income protection to finance their second-period consumption.

Proof: If we use \( X_{1,t} = \frac{\delta h_{2,t+1}}{1+r} \) to represent the insurance premium in the first period, then the budget constraint is:

\[ c_{1,t} + e_{1,t} + X_{1,t} + s_{1,t} = i_{1,t} = h_{1,t} \]  

(10)

The expected utility will be:

\[ E[u(c_{1,t}, e_{1,t}, s_{1,t}, X_{1,t})] = \frac{c_{1,t}^\sigma}{\sigma} + \frac{1}{1 + \beta} \left[ (1 - \pi)(\theta e_{1,t} + (1 + r)s_{1,t})^\sigma \right] + \frac{1}{1 + \beta} \left[ \pi \left( \frac{1 + r}{\pi} X_{1,t} + (1 + r)s_{1,t} \right)^\sigma \right] \]

(11)

\[ \frac{\partial E}{\partial s_{1,t}} = \frac{1}{1 + \beta} \left( 1 - \pi \right) \left( \theta \frac{h_{2,t+1}}{\theta} + (1 + r)s_{1,t} \right)^{\sigma-1} \left( 1 + r \right) \left[ 1 + \frac{1}{1 + \beta} \left( \frac{1 + r}{\pi} X_{1,t} + (1 + r)s_{1,t} \right)^{\sigma-1} \right] \]

\[ \frac{\partial E}{\partial e_{1,t}} = \frac{1}{1 + \beta} \left( 1 - \pi \right) \left( \theta \frac{h_{2,t+1}}{\theta} + (1 + r)s_{1,t} \right)^{\sigma-1} \]

\[ \frac{\partial E}{\partial X_{1,t}} = \frac{1}{1 + \beta} \left( 1 + r \right) \left( \frac{1 + r}{\pi} X_{1,t} + (1 + r)s_{1,t} \right)^{\sigma-1} \]

So, we have \( \frac{\partial E}{\partial s_{1,t}} = \frac{\partial E}{\partial e_{1,t}} + \frac{\partial E}{\partial X_{1,t}} \).

According to Assumption 1, \( \theta (1 - \pi) < 1 + r \), so \( \frac{1 + r}{\theta} < 1 - \pi \).

\[ \frac{\partial E}{\partial s_{1,t}} < \left( (1 - \pi) \frac{\partial E}{\partial e_{1,t}} + \pi \frac{\partial E}{\partial X_{1,t}} \right) \]

So, \( \frac{\partial E}{\partial s_{1,t}} < \max \left( \frac{\partial E}{\partial e_{1,t}}, \frac{\partial E}{\partial X_{1,t}} \right) \).

In other words, when the private saving \( s_{1,t} \) is not 0, an individual will always find it optimal to reduce saving and increase education investment or insurance payment. In the end, we get the corner solution of \( s_{1,t} = 0 \).

Now as we know private saving \( s_{1,t} = 0 \), saving is not an option; we can rewrite our budget constraint and expected utility function:

\[ E[u(c_{1,t}, h_{2,t+1})] = \frac{c_{1,t}^\sigma}{\sigma} + \frac{1}{1 + \beta} \left[ (1 - \pi)(h_{2,t+1})^\sigma \right] + \frac{1}{1 + \beta} \left[ \pi \left( \frac{\delta h_{2,t+1}}{\pi} \right)^\sigma \right] \]

(12)

\[ c_{1,t} + h_{2,t+1} + \frac{\delta n h_{2,t+1}}{1+r} = i_{1,t} = h_{1,t} \]

(13)

Next, we want to find how individuals will choose their coverage level.

**Proposition 1**: Without limit, an individual will choose the optimal IPI coverage \( \delta^* = \left( \frac{1+r}{(1-\pi)\theta} \right)^{\frac{1}{\sigma}} \in (0, 1) \). And IPI and HI will yield the same result.

Proof:

\[ c_{1,t} + e_{1,t} + X_{1,t} = i_{1,t} = h_{1,t} \]

(14)

\[ E[u(c_{1,t}, e_{1,t}, X_{1,t})] = \frac{c_{1,t}^\sigma}{\sigma} + \frac{1}{1 + \beta} \left[ (1 - \pi)(\theta e_{1,t})^\sigma \right] + \frac{1}{1 + \beta} \left[ \pi \left( \frac{1+r}{\pi} X_{1,t} \right)^\sigma \right] \]

(15)
\[ \frac{\partial E}{\partial e_{1,t}} = \frac{1}{1 + \beta} (1 - \pi) \theta^\sigma e_{1,t}^{\sigma - 1} = \frac{1}{1 + \beta} (1 - \pi) \theta^\sigma \left( \frac{h_{2,t+1}}{\theta} \right)^{\sigma - 1} \]

\[ \frac{\partial E}{\partial X_{1,t}} = \frac{1}{1 + \beta} \pi \left( \frac{1 + r}{\pi} \right) ^\sigma X_{1,t}^{\sigma - 1} = \frac{1}{1 + \beta} \pi \left( \frac{1 + r}{\pi} \right) \delta \left( \frac{h_{2,t+1}}{1 + r} \right)^{\sigma - 1} \]

To maximize expected utility, as the cost of \( e_{1,t} \) and \( X_{1,t} \) are the same, we need:

\[ \frac{\partial E}{\partial e_{1,t}} = \frac{\partial E}{\partial X_{1,t}} \]

\[ \frac{1}{1 + \beta} (1 - \pi) \theta^\sigma \left( \frac{h_{2,t+1}}{\theta} \right)^{\sigma - 1} = \frac{1}{1 + \beta} \pi \left( \frac{1 + r}{\pi} \right) \left( \frac{\delta h_{2,t+1}}{1 + r} \right)^{\sigma - 1} \]

We get

\[ \delta = \left( \frac{1 + r}{1 - \pi} \right) \frac{1}{\pi} \equiv \delta^* \]

Also, \( \lim_{c_{1,t} \to 0} \frac{\partial E}{\partial c_{1,t}} = \lim_{X_{1,t} \to 0} \frac{\partial E}{\partial X_{1,t}} = \lim_{e_{1,t} \to 0} \frac{\partial E}{\partial e_{1,t}} = +\infty, \lim_{c_{1,t} \to h_{1,t}} \frac{\partial E}{\partial c_{1,t}} = \lim_{X_{1,t} \to h_{1,t}} \frac{\partial E}{\partial X_{1,t}} = \lim_{e_{1,t} \to h_{1,t}} \frac{\partial E}{\partial e_{1,t}} = \) limited numbers,

\[ \frac{\partial^2 E}{\partial c_{1,t}^2} < 0, \frac{\partial^2 E}{\partial X_{1,t}^2} < 0, \frac{\partial^2 E}{\partial e_{1,t}^2} < 0. \]

As an individual decrease \( c_{1,t} \) from \( h_{1,t} \) and increase \( X_{1,t} \) and \( e_{1,t} \) from 0 based on the rule of

\[ \frac{\partial E}{\partial c_{1,t}} = \frac{\partial E}{\partial X_{1,t}} = \frac{\partial E}{\partial e_{1,t}} \]

So, there must exist a \( \delta^* \equiv \left( \frac{1 + r}{1 - \pi} \right) \frac{1}{\pi} \) to make \( \frac{\partial E}{\partial c_{1,t}} = \frac{\partial E}{\partial X_{1,t}} = \frac{\partial E}{\partial e_{1,t}} \).

Obviously, \( \delta^* \) is increasing in \( r \) and decreasing in \( (1 - \pi) \theta \), which is the expected return of human capital.

When individuals can invest in insurance freely, the insurance amount and insurance coverage proportion are both determined, so HI and IPI will yield the same result.

And we define the economic growth rate as

\[ \theta_t \equiv \frac{\theta_{t+1} - \theta_{t+1} - 1}{\theta_{t+1}} = \frac{\theta_{t+1} - \theta_{t+1}}{\theta_{t+1}} = \frac{1}{\theta_{t+1}} \]

Obviously, higher \( e_{1,t} \) relative to \( h_{1,t} \) means higher economic growth.

**Proposition 2:** When \( \Omega < 1 \), the freely chosen IPI or HI will increase the human capital accumulation and economic growth rate, where \( \Omega \equiv \frac{(1 - \pi)(\theta - (1 + r))}{\pi(1 + r)} \frac{1}{\theta^\sigma} - \left[ \frac{(1 - \pi)(1 + r)}{1 + \pi} \right] \frac{1}{\theta^\sigma} + (1 + r) \left[ \frac{1 + r}{1 + r} \right] \frac{1}{\theta^\sigma} \right. \]

vice versa. The lower the expected return of education relative to the interest rate is, the more likely freely chosen IPI or HI will increase human capital accumulation and economic growth, vice versa.

**Proof:**

Now we want to compare the situation with free insurance and without insurance.

When there isn’t IPI or HI, people control the risk by saving. To distinguish the two situations, we denote the expected utility without insurance as \( E^\sigma \) and the expected utility with insurance as \( E^\sigma \).

By maximizing (15) subject to (14), maximizing (8) subject to (9), and \( h_{2,t+1} = \theta e_{1,t} \), we get the optimal human capital investment in the first period with and without insurance:

\[ e_{1,t}^{free} = \frac{\left[ (1 - \pi)(\theta - (1 + r)) \right]^\frac{1}{\theta^\sigma}}{\theta + \left[ (1 - \pi)(\theta - (1 + r)) \right]^\frac{1}{\theta^\sigma}} \theta_{t+1}^\sigma \]

\[ e_{1,t}^{O} = \frac{\left[ (1 - \pi)(\theta - (1 + r)) \right]^\frac{1}{\theta^\sigma}}{\theta + \left[ (1 - \pi)(\theta - (1 + r)) \right]^\frac{1}{\theta^\sigma}} \theta_{t+1}^\sigma \]

\[ h_{1,t}^O = \frac{\left[ (1 - \pi)(\theta - (1 + r)) \right]^\frac{1}{\theta^\sigma}}{\theta + \left[ (1 - \pi)(\theta - (1 + r)) \right]^\frac{1}{\theta^\sigma}} \theta_{t+1}^\sigma \]

Here \( T^I \equiv \left[ \frac{(1 + r)}{(1 - \pi)\theta} \right] \frac{1}{\theta^\sigma} \frac{1}{\theta + \left[ (1 - \pi)(\theta - (1 + r)) \right]^\frac{1}{\theta^\sigma}} \theta_{t+1}^\sigma \)

\( T^O \equiv \left[ \frac{(1 - \pi)(\theta - (1 + r))}{\pi(1 + r)} \right]^\frac{1}{\theta^\sigma} \frac{1}{\theta + \left[ (1 - \pi)(\theta - (1 + r)) \right]^\frac{1}{\theta^\sigma}} \theta_{t+1}^\sigma \)

\( \frac{1}{\theta + \left[ (1 - \pi)(\theta - (1 + r)) \right]^\frac{1}{\theta^\sigma}} \theta_{t+1}^\sigma \)

is the ratio of saving divided by human capital investment.
\[ e_{1,t}^{\text{free}} > e_{1,t}^{0} \Leftrightarrow \frac{1}{1+\beta} \begin{pmatrix} \frac{1}{1+\beta} & \frac{1}{1+\beta} \end{pmatrix} \begin{pmatrix} T^i + 1 \end{pmatrix} < (1+r)T^0 + \frac{1}{1+\beta} \begin{pmatrix} \frac{1}{1+\beta} & \frac{1}{1+\beta} \end{pmatrix} \begin{pmatrix} T^0 + 1 \end{pmatrix} \]

\[ \Leftrightarrow \Omega \equiv \left[ \frac{1}{(1-\pi)(\theta - (1+r))} \right]^{\frac{1}{\pi}} - \left[ \frac{1}{1+\beta} \right]^{\frac{1}{\pi}} + \frac{1}{1+r} \left( \frac{1+\beta}{1+r} \right)^{\frac{1}{\pi}} \frac{1}{\pi} < 1 \]

\[ \Omega \equiv \left[ \frac{1}{\pi} (t-1) \right]^{\frac{1}{\pi}} - \left[ \frac{1}{1+\beta} + (1+r) \right]^{\frac{1}{\pi}} \frac{1}{\pi}, \text{where} \ t \equiv \frac{(1-\pi)\theta}{1+r} \]

Obviously, when \( t \equiv \frac{(1-\pi)\theta}{1+r} \to 1, \left[ \frac{1}{\pi} \left( \frac{(1-\pi)\theta}{1+r} - 1 \right) \right]^{\frac{1}{\pi}} \to 0, \Omega < 1. \)

As we assumed, \( t \equiv \frac{(1-\pi)\theta}{1+r} > 1, \) so, \( t \equiv \frac{(1-\pi)\theta}{1+r} \to 1 \) implies a very small \( \theta \) and very big \( \pi. \) At that time, IPI or HI will enhance education investment and economic growth.

On the contrary, when \( t \to +\infty, \lim_{t \to +\infty} \Omega \to +\infty > 1. \) So, when the human capital return is very large relative to physical capital return, IPI or HI will deter education investment and economic growth.

III. Benefit limit and human capital accumulation

In the above analysis, we find without limit, IPI and HI are the same regardless of the names. However, both IPI and HI have benefit limits. Of course, those limits will lower the risk of moral hazard. Now we shall see those limits can also boost economic growth.

**Proposition 3:** With proper limit, HI will boost human capital accumulation and economic growth. The growth optimal HI is

\[ X_{1,t}^{\text{HI}} = \frac{h_{1,t}}{A+1}, \text{where} \ A = \left( \frac{1+r}{(1-\pi)\theta} \right)^{\frac{1}{\pi}} \frac{1}{(\pi+1)^{\frac{1}{\pi}}} \frac{1}{\pi} \left( \frac{(1-\pi)(\theta - (1+r))}{\pi(1+r)} \right)^{\frac{1}{\pi}}. \]

When \( X_{1,t} < X_{1,t}^{\text{HI}}, s_{1,t} > 0, \) as \( X_{1,t} \) increases, \( s_{1,t} \) will decrease; when \( X_{1,t} > X_{1,t}^{\text{HI}}, s_{1,t} = 0. \) The growth-optimal HI is lower than the freely chosen HI, \( X_{1,t}^{\text{HI}} < X_{1,t}^{\text{free}}, \)

and the education investment at the freely-chosen point will also be lower, \( e_{1,t}^{\text{free}} < e_{1,t}^{\text{HI}}. \)

Proof:

When there exists private saving, we have:

\[ c_{1,t} + e_{1,t} + X_{1,t} + s_{1,t} = h_{1,t} \]

\[ E \left[ u(c_{1,t}, h_{2,t+1}, \delta) \right] = \frac{e_{1,t}^\sigma}{\sigma} + \frac{1}{1+\beta} \left[ \frac{1}{(1-\pi)(\theta e_{1,t} + (1+r)s_{1,t})} \right]^\sigma \]

\[ + \frac{1}{1+\beta} \left[ \left( \frac{1+r}{\pi} X_{1,t} + (1+r) s_{1,t} \right) \right]^\sigma \]

(20)

Now we regard \( X_{1,t} \) as a parameter instead of a variable, maximizing (21) subject to (20), we get:

\[ e_{1,t} = \frac{h_{1,t} + (BC\frac{1+r}{\pi} - 1) X_{1,t}}{(D + CF\theta)} \]

where \( C = \frac{(1+r)}{(1-\pi)(\theta - (1+r))^{\frac{1}{\pi}}}, B = \frac{(1-\pi)(\theta - (1+r))^{\frac{1}{\pi}}}{(\pi+1)^{\frac{1}{\pi}}}, \)

\[ D = \left( \frac{1+r}{(1-\pi)\theta} \right)^{\frac{1}{\pi}} + 1, F = \left( \frac{1+r}{(1-\pi)(\theta - (1+r))^{\frac{1}{\pi}} - (\pi+1)^{\frac{1}{\pi}}} \right)^{\frac{1}{\pi}} (1+r) \]
Obviously, $C > 1$, $B \frac{1+r}{\pi} = \frac{1}{\pi^{1+\rho}} \left(\frac{1}{(1-\pi)(\theta-(1+r))} \frac{1}{(\pi\star(1+r))} \frac{1}{1-\beta} \right)$ according to Assumption 1, 

$$(1-\pi)(\theta-(1+r)) - (\pi \star (1+r)) = \theta(1-\pi) - (1+r) > 0,$$ so $B \frac{1+r}{\pi} > \frac{1}{\pi} > 1$.

So, $CB \frac{1+r}{\pi} - 1 > 0$. As $X_{1,t}$ increases, education investment will also increase.

At the same time, we have

$$\frac{h_{1,t}}{1+\frac{1}{\pi^{1+\rho}} \left(\frac{1}{(1-\pi)(\theta-(1+r))} \frac{1}{(\pi\star(1+r))} \frac{1}{1-\beta} \right)} X_{1,t}$$

$$S_{1,t} = \frac{\left(\frac{1}{(1-\pi)(\theta-(1+r))} \frac{1}{(\pi\star(1+r))} \frac{1}{1-\beta} \right)}{\left(\frac{1}{(1-\pi)(\theta-(1+r))} \frac{1}{(\pi\star(1+r))} \frac{1}{1-\beta} \right)}$$

As $X_{1,t}$ increases, the saving will decrease.

When $X_{1,t} = \frac{h_{1,t}}{1+\frac{1}{\pi^{1+\rho}} \left(\frac{1}{(1-\pi)(\theta-(1+r))} \frac{1}{(\pi\star(1+r))} \frac{1}{1-\beta} \right)}$, saving will be 0.

At that time, when the HI is enough to eliminate private saving, we have:

$$c_{1,t} + e_{1,t} + X_{1,t} = i_{1,t} = h_{1,t}$$

$$E[u(c_{1,t}, e_{1,t})] = \frac{c_{1,t}}{\sigma} + \frac{1}{1+\beta} \frac{1}{\pi} \left[(1-\pi)(\theta e_{1,t})^\sigma \right] + \frac{1}{1+\beta} \frac{1}{\pi} \left[\pi \left(\frac{1+r}{\pi} X_{1,t} \right)^\sigma \right]$$

From $\frac{\partial E}{\partial c_{1,t}} = \frac{\partial E}{\partial e_{1,t}}$, we get $\frac{c_{1,t}}{e_{1,t}} = \left(\frac{\theta(1-\pi)}{1+\beta}\right)^{\sigma-1} \theta$.

Also, we know $c_{1,t} + e_{1,t} = h_{1,t} - X_{1,t}$. As a result: $e_{1,t} = \frac{h_{1,t} - X_{1,t}}{(\theta(1-\pi))^{1/\sigma} \theta + 1}$.

So, the increase of HI will reduce the education investment.

In a word, the growth-optimal HI should be just enough to reduce saving to 0, no more and no less.

Now we want to find whether the growth-optimal HI is higher or lower than freely chosen HI.

According to Lemma 1, when individuals can choose freely,

$$\frac{\partial E}{\partial s_{1,t}} < (1-\pi) \frac{\partial E}{\partial e_{1,t}} + \pi \frac{\partial E}{\partial X_{1,t}}$$

At the individual optimal point, $\frac{\partial E}{\partial e_{1,t}} = \frac{\partial E}{\partial c_{1,t}} = \frac{\partial E}{\partial X_{1,t}}$

So, $\frac{\partial E}{\partial s_{1,t}} < (1-\pi) \frac{\partial E}{\partial e_{1,t}} + \pi \frac{\partial E}{\partial e_{1,t}}$.

However, at the growth-optimal point, $\frac{\partial E}{\partial s_{1,t}} = \frac{\partial E}{\partial e_{1,t}}$

Also, we know $\frac{\partial E}{\partial s_{1,t}} = \frac{\pi}{1+\beta} \left(\theta e_{1,t} \right)^{\sigma-1} (1+r) + \frac{\pi}{1+\beta} \left(\frac{1+r}{\pi} X_{1,t} \right)^{\sigma-1}$.

$$\frac{\partial E}{\partial e_{1,t}} = \frac{(1-\pi)}{1+\beta} \left(\theta e_{1,t} \right)^{\sigma-1} \theta$$

$$\frac{\partial E}{\partial e_{1,t}} - \frac{\partial E}{\partial s_{1,t}} = \frac{(1-\pi)}{1+\beta} \left(\theta e_{1,t} \right)^{\sigma-1} (\theta - (1+r)) - \pi \left(\frac{1+r}{\pi} X_{1,t} \right)^{\sigma-1} (1+r)$$
When the HI is at the growth optimal point, the increase of $X_{1,t}$ will crowd out $e_{1,t}$, so to make \[
\frac{\partial E}{\partial e_{1,t}} - \frac{\partial E}{\partial s_{1,t}} \bigg|_{s_{1,t}=0} > 0,
\] we must allow higher $X_{1,t}$ than the growth-optimal level.

So, freely-chosen HI is higher than growth-optimal HI, $X_{1,t}^{HI*} < X_{1,t}^{free}$, and the education investment at the freely-chosen point will also be lower, $e_{1,t}^{free} < e_{1,t}^{HI*}$. ■

In a word, a proper HI benefit limit will maximize education investment.

To see that more clearly, we can refer to Figure 01.

**Figure 01 Freely-chosen HI and growth-optimal HI**

As IPI binds the education investment and insurance together, we can regard the insurance payment as a part of the expense of education, as the coverage proportion $\delta$ increases, the education investment will be more expensive but less risky. Then we can get the following budget constraint and utility function:

\[
\begin{align*}
    &c_{1,t} + \left(1 + \frac{\pi}{1 + r} \delta \theta\right) e_{1,t} + s_{1,t} = h_{1,t} \quad (24) \\
    &E\left[c_{1,t}, e_{1,t}, s_{1,t}\right] = \frac{c_{1,t}^\sigma}{\sigma} + \frac{1}{1 + \beta \sigma} \left[\left(1 - \pi\right)\left(\theta e_{1,t} + (1 + r)s_{1,t}\right)\right] + \frac{1}{1 + \beta \sigma} \left[\pi\left(\theta e_{1,t} + (1 + r)s_{1,t}\right)\right] \quad (25)
\end{align*}
\]

\[
\begin{align*}
    &c_{1,t} + \left(1 + \frac{\pi}{1 + r} \delta \theta\right) e_{1,t} = h_{1,t} \quad (26) \\
    &E\left[c_{1,t}, e_{1,t}, s_{1,t}\right] = \frac{c_{1,t}^\sigma}{\sigma} + \frac{1}{1 + \beta \sigma} \left[\left(1 - \pi\right)\left(\theta e_{1,t}\right)\right] + \frac{1}{1 + \beta \sigma} \left[\pi\left(\theta e_{1,t}\right)\right] \quad (27)
\end{align*}
\]

Equation (24) and (25) are the budget constraint and utility function when the saving is not 0. Equation (26) and (27) are the budget constraint and utility function when the saving is 0.

**Proposition 4**: With proper benefit limit, IPI can be better in enhancing the human capital accumulation and economic growth than HI.

Proof:

We have got the growth-optimal HI in the previous analysis $X_{1,t}^*$. We know \[
\frac{\partial E}{\partial X_{1,t}} > \frac{\partial E}{\partial e_{1,t}} = \frac{\partial E}{\partial c_{1,t}}. \]
We denote the education investment at that time as $e_{1,t}^{HI*}$.

Then we have $\delta^{HI} = \frac{X_{1,t}^*(1+r)}{\pi \theta e_{1,t}}$ and $\delta^{HI,\pi \theta} = \frac{X_{1,t}^*}{e_{1,t}}$. \[
\begin{align*}
    &\frac{\partial E}{\partial X_{1,t}} \delta^{HI,\pi \theta} + \frac{\partial E}{\partial e_{1,t}} > \frac{\partial E}{\partial c_{1,t}} \quad (26) \\
    &\frac{\partial E}{\partial e_{1,t}} \delta^{HI,\pi \theta} + \frac{\partial E}{\partial e_{1,t}} \delta^{HI,\pi \theta} + \frac{\partial E}{\partial e_{1,t}} \frac{\partial E}{\partial c_{1,t}} = \frac{\partial E}{\partial c_{1,t}} \quad (27)
\end{align*}
\]

IPI allows individuals to increase insurance and education investment at a fixed proportion. Imagine an individual draws 1 dollar from current consumption and add $\frac{1}{\delta^{HI,\pi \theta} (1+r) + 1}$ dollar to his education investment and $\frac{\delta^{HI,\pi \theta}}{(1+r)} \frac{1}{\delta^{HI,\pi \theta} (1+r) + 1}$ dollar to his insurance. The coverage proportion will still be $\delta^{HI}$, but his utility will increase. So, individuals will continue to increase
insurance and education investment at that proportion until \(\frac{\partial E}{\partial\delta}\frac{\partial H}{\partial\delta} + \frac{\partial E}{\partial c_{1,t}}\). At that time, we denote the education investment as \(e^{HI}\).

So, even if \(\delta^{HI}\) is not the coverage that can maximize human capital accumulation, IPI will be better than HI in enhancing human capital accumulation.

We assume the optimal IPI coverage for enhancing human capital accumulation is \(\delta^{PI^*}\), and the corresponding education investment is \(e_{1,t}^{\delta^{PI^*}}\). Of course, according to the definition, \(e_{1,t}^{\delta^{PI^*}} > e^{HI}\).

**Proposition 5:** As the coverage proportion \(\delta\) increase from 0, the amount of saving decreases, and the investment in education increases. When \(\delta = \delta^{NS}\), the saving will become 0. The growth-optimal IPI coverage is \(\delta^{PI^*} = \max\{\delta^{NS}, \delta^{*}\}\). \(\delta^{NS}\) and \(\delta^{*}\) are defined in the following proof.

Proof:

According to (24) and (25),

\[
e = \left(1 - \frac{\pi}{1 + \beta}\right) (\theta + (1 + r)B)^{\sigma - 1} (1 + r) + \frac{\pi}{1 + \beta} (\delta \theta + (1 + r)B)^{\sigma - 1} (1 + r) \right) \frac{\partial^\theta}{\partial\delta} + (1 + \frac{\theta}{1 + r}) \delta \pi + B)
\]

\[
s = Be, \text{where } B = \frac{\theta}{1 - \beta}\frac{(A - \beta)}{(1 - \delta)} > 0 \left(\frac{\pi}{1 - \beta}\frac{(1 + r - \delta \theta)}{(1 - \delta)}\right)^{1 - \sigma} = A < 1.
\]

Through calculation, we have \(\frac{\partial e}{\partial \delta} > 0, \frac{\partial s}{\partial \delta} < 0\).

As shown in lemma 1, as we increase \(\delta\), which means higher level of insurance is allowed. individuals will find it optimal to use the combination of investment in education and insurance to replace saving. When \(B = \frac{\theta}{1 + r} (\frac{A - \beta}{1 - \delta}) = 0\), there will be no saving at all, which is equivalent to \(\frac{1 - \pi}{1 + \beta}\left(\frac{\pi}{1 - \beta}\frac{(1 + r - \delta \theta)}{(1 - \delta)}\right)^{1 - \sigma} = \delta\), we get \(\delta = \delta^{NS}\).

If we increase \(\delta\) above \(\delta^{NS}\), there will be no saving, according to (26) (27), we get:

\[
e = \frac{h}{\overline{\delta}} \left(\frac{1 + \frac{\pi}{1 + r} \delta}{1 - \pi} + \frac{\pi}{1 + r}\right)^{1 - \sigma} + \left(\frac{1 + \frac{\pi}{1 + r} \delta}{1 - \pi} + \frac{\pi}{1 + r}\right)^{1 - \sigma - 1} (1 + \beta) \left(\frac{1 + \frac{\pi}{1 + r} \delta}{1 - \pi} + \frac{\pi}{1 + r}\right)^{1 - \sigma - 1} + \frac{\pi}{1 + r}
\]

We define \(G = \left(\frac{1 + \frac{\pi}{1 + r} \delta}{1 - \pi} + \pi \delta \sigma (1 + \beta)\right)^{1 - \sigma} + \left(\frac{1 + \frac{\pi}{1 + r} \delta}{1 - \pi} + \pi \delta \sigma (1 + \beta)\right)^{1 - \sigma - 1} (1 + \beta) \left(\frac{1 + \frac{\pi}{1 + r} \delta}{1 - \pi} + \pi \delta \sigma (1 + \beta)\right)^{1 - \sigma - 1} + \frac{\pi}{1 + r}
\]

\[
\lim_{\delta \to 0} \frac{\partial G}{\delta} = -\infty; \quad \lim_{\delta \to 1} \frac{\partial G}{\delta} = \pi - \frac{1 - \pi}{1 + r} \left(1 + \beta\right) \left(\frac{1 + \frac{\pi}{1 + r} \delta}{1 - \pi} + \pi \delta \sigma (1 + \beta)\right)^{1 - \sigma - 1} (1 + \beta) \left(\frac{1 + \frac{\pi}{1 + r} \delta}{1 - \pi} + \pi \delta \sigma (1 + \beta)\right)^{1 - \sigma - 1} + \frac{\pi}{1 + r}
\]

As \(\frac{\partial G}{\delta}\left(\frac{1 + \frac{\pi}{1 + r} \delta}{1 - \pi} + \pi \delta \sigma (1 + \beta)\right) = \frac{\theta - (1 + r + \theta \pi) \sigma}{(1 + r) \theta} \geq \frac{\theta - ((1 + r) + \theta \pi)}{(1 + r) \theta} > 0\), we have \(\lim_{\delta \to 1} \frac{\partial G}{\delta} > 0\).

\(^1\)The subscription of \(i_{1,t}, e_{1,0}, c_{1,t}\) and \(s_{1,t}\) are omitted for simplification.
Also, through calculation, we find \( \frac{\partial^2 \theta}{\partial \delta^2} > 0 \). So, there is a unique \( \delta = \delta^* \) to make \( e \) get the highest value.

When \( \delta^* > \delta^{NS} \), \( \delta^* \) will be the growth-optimal point. When \( \delta^* \leq \delta^{NS} \), \( \delta^{NS} \) will be the growth-optimal point. To see that more clearly, we can refer to Figure 2.

Obviously, the freely-chosen IPI coverage proportion \( \delta^{free} \) is higher than both \( \delta^* \) and \( \delta^{NS} \). The dashed curve portraits the situation when there is no access to savings. We can see if the peak of the dashed curve \( \delta^* \) falls between 0 and \( \delta^{NS} \), just like the situation in Figure 2 (a), it can’t be achieved, and the optimal coverage will be \( \delta^{NS} \). If \( \delta^* \) is above \( \delta^{NS} \), just like the situation in Figure 2 (b), \( \delta^* \) will be the optimal point.

![Figure 02 Saving and education investment under different IPI coverage](image)

We can verify our above analysis by a numerical example. Through calculation, we get the following table (GO refers to growth-optimal, NI refers to no insurance case).

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( c_{1,t} )</th>
<th>( e_{1,t} )</th>
<th>( s_{1,t} )</th>
<th>( X_{1,t} )</th>
<th>( \delta )</th>
<th>( h_{2,t+1} )</th>
<th>( g_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>NI</td>
<td>58.69</td>
<td>23.79</td>
<td>17.53</td>
<td>0</td>
<td>0</td>
<td>35.69</td>
</tr>
<tr>
<td>Free HI</td>
<td>57.18</td>
<td>35.10</td>
<td>0</td>
<td>7.72</td>
<td>77%</td>
<td>52.65</td>
<td>5.29%</td>
</tr>
<tr>
<td>GO HI</td>
<td>59.73</td>
<td>36.70</td>
<td>0</td>
<td>3.57</td>
<td>34%</td>
<td>55.05</td>
<td>10.1%</td>
</tr>
<tr>
<td>Free IPI</td>
<td>57.18</td>
<td>35.10</td>
<td>0</td>
<td>7.72</td>
<td>77%</td>
<td>52.65</td>
<td>5.29%</td>
</tr>
<tr>
<td>GO IPI</td>
<td>57.95</td>
<td>39.25</td>
<td>0</td>
<td>2.80</td>
<td>25%</td>
<td>58.88</td>
<td>17.75%</td>
</tr>
<tr>
<td>5</td>
<td>NI</td>
<td>32.45</td>
<td>66.16</td>
<td>140</td>
<td>0</td>
<td>0</td>
<td>330.78</td>
</tr>
<tr>
<td>Free HI</td>
<td>31.42</td>
<td>64.35</td>
<td>0</td>
<td>4.22</td>
<td>6.89%</td>
<td>321.77</td>
<td>543.54%</td>
</tr>
<tr>
<td>GO HI</td>
<td>32.72</td>
<td>67.00</td>
<td>0</td>
<td>0.28</td>
<td>0.44%</td>
<td>335.01</td>
<td>570.02%</td>
</tr>
<tr>
<td>Free IPI</td>
<td>31.42</td>
<td>64.35</td>
<td>0</td>
<td>4.22</td>
<td>6.89%</td>
<td>321.77</td>
<td>543.54%</td>
</tr>
<tr>
<td>GO IPI</td>
<td>32.19</td>
<td>67.54</td>
<td>0</td>
<td>0.276</td>
<td>0.4282%</td>
<td>337.69</td>
<td>575.39%</td>
</tr>
</tbody>
</table>

When the return of education is moderate (\( \theta = 1.5 \)), insurance is very important for economic growth. We can find that without insurance, the growth rate is negative, which means that the human capital will continue to shrink, and the economy cannot escape from the poverty trap. If individuals have access to HI or IPI without limits, HI or IPI will replace saving and spare more money for education investment, which can guarantee positive economic growth. At that time, benefit limits can make things even better. In our example, with proper limitation, GO HI can double the Free HI growth rate and GO IPI can triple the Free IPI growth rate.

When the return of education is very high (\( \theta = 5 \)), insurance will not be very important for economic growth. Free HI or
Free IPI can deter economic growth, while the GO HI and GO IPI can boost economic growth. However, all those effects are very limited, considering the scale of growth rate.

In a word, both HI and IPI are powerful tools for boosting small open economies’ taking off; proper limits can make them more powerful, and limited GO IPO is the most powerful one. When the return of education is moderate, HI and IPI can be very important. At that time, proper benefit limitations can greatly improve the outcome.

IV. Conclusion

According to the above analysis, we can get three main conclusions.

First, Individuals prefer insurance instead of saving to defense against risks, the freely chosen HI and IPI will yield the same result. Free insurance can either enhance or deter human capital accumulation, depending on the relative return to education and physical capital.

Second, the government can enhance human capital accumulation by limiting benefits. With proper benefit limits, both HI and IPI can enhance human capital accumulation. In other words, the government needs to curb individuals’ appetite for safety to guarantee higher economic growth. And the growth-optimal IPI can be better in enhancing the human capital accumulation and economic growth than HI.

Finally, when the return to education is moderate, HI and IPI can be critical for a small open economy, proper benefit limitations are also very important in that situation. When the return to education is extremely high, HI and IPI may not be very important to human capital accumulation and economic growth rate.

References