Environmental Pollution & the Political Economy of Public Debt

– preliminary draft – please do not circulate –

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8th August 2019

Abstract: This paper analyzes the political economy of government debt when elected politicians decide about the distribution of public funds between a clean and a polluting public good. When provision of the polluting good creates a stock of climate externalities, strategic incentives for the incumbent government arise from both a budget and emission interaction. In this framework, reelection uncertainty leads to inefficiently low public savings (or even debt) which are attenuated by the emission interaction, while first period pollution decreases regardless of the future government’s identity. If the incumbent government competes for office against an environmentalists’ party, the total welfare loss from emissions also decreases as a direct result of reelection uncertainty.

Keywords: emission externality, public debt, political economy

JEL classification: H23, H41, H63, Q54, Q58
1 Introduction

The political economy of public debt has received considerable attention in the past literature as sovereign debt creates a link between current and future political decisions even if today’s government will not remain in office. The reasons why current political decision makers would want to embrace public debt as a strategic instrument are manifold and range from the aim of minimizing the pork barrel’s contribution to debt stabilization (Alesina and Drazen, 1991) over concerns regarding interregional or intergenerational redistribution (see Cukierman and Meltzer, 1989 or Weingast et al., 1981) to binding future governments’ allocation of public funds. Likewise, the political economy of environmental policy has been in economists’ focus at least since Buchanan and Tullock (1975), who showed why existing firms in a polluting industry would prefer the introduction of quotas instead of an emission tax. The subsequent theoretical literature has put its primary emphasis on analyzing how interest groups can influence policy through lobbying (see Oates and Portney (2003) for an overview and Aidt (1998) in particular). However, to the best of my knowledge, a combined approach which intertwines the political economy of public debt and environmental policy has yet to be established.

Therefore, this paper proposes a model where political decision makers face uncertainty about reelection in the next period while allocating funds between two public goods. Provision of one good is clean (e.g. education or health services), provision of the other good (e.g. road infrastructure) creates emissions, thus, adding to a stock of environmental pollution. The political parties running for office in each period disagree about how much of the pollution externality should be internalized. Suppose politicians are either ‘environmentalists’ (E) who fully (or even over-) internalize pollution damages or ‘industrialists’ (I) who appreciate the externality only partially (or not at all) in their objective function. This setup is motivated by the recently proposed change to computation of the social costs of carbon (SCC) in the US. Under the Clean Air Act, the SCC used to internalize the global costs of carbon emission. Within the newly proposed framework, only ‘domestic’ benefits from avoided climate change are taken into account which considerably reduces the SCC (EPA, 2017).

1Alesina and Passalacqua (2016) provide a recent survey of the existing literature on the political economy of public debt.
By introducing an environmental externality to the voting economy, this paper contributes to the literature in two ways. First, since pollution does not depreciate instantaneously but accumulates as a stock over time, even the social planner will find it optimal to deviate from a zero debt rule. If pollution occurs in the form of green house gas (GHG) emissions such that pollution damages amount to the consequences of climate change, it would be socially optimal to generate public savings in the first period.\(^2\) Any politician who does not fully internalize pollution damages will prefer a smaller level of public savings. Thus, even in the case of certain reelection, government I would always accumulate fewer savings than party E regards as optimal. While I derive this result with an exogenously given endowment of public funds, it also extends to a scenario where the government endogenously generates tax revenues but collection is associated with a dead weight loss, e.g. when administrative costs occur (see Barro (1979) for the underlying model of tax smoothing). Hence, in the framework of this paper, public debt (or savings) is not merely a strategic instrument but also has a normative motivation.

Second, leading to the central insight from this model, debt no longer is the only channel through which the incumbent government affects future policy making. As in the standard model without pollution, public debt serves as a strategic measure to confine future governments’ spending capabilities and shift funds to the first period, where the incumbent can still allocate the public budget as they deem optimal. Suppose ‘industrialists’ are initially in office but will be superseded by ‘environmentalists’ in the next period. Since party I prefers a higher provision of the polluting good than party E will provide in the next period, the incumbent would accumulate debt and spend even more on the polluting good in the first period. However, since emissions remain in the atmosphere and decay slowly, party E will provide even less of the polluting good in the future, the higher provision in \(t = 1\) gets. Pollution, in a sense, disciplines the first period’s, environmentally unaware, government to pollute less. This effect becomes more pronounced when the atmospheric persistence of GHG emissions is very high. In contrast, if party E holds office in the first period but expects to be replaced by the industrialists, environmentalist will anticipate that too much of the polluting good will be provided in the next period. To prevent pollution damages from spiking

\(^2\)In a recent working paper, we establish normative rules for the optimal level of public debt in the presence of climate change. The social planner would optimally accumulate savings (debt) whenever the present value of cumulative marginal damages are higher (lower), the earlier a pollutant is emitted into the atmosphere (Kellner and Runkel, 2018).
in $t = 2$, they will cut spending on the polluting good in $t = 1$ and leave less than the socially optimal amount of savings. As a result, incumbent $E$ will over-provide the clean public good. Yet, while a high level of pollution persistence limits incentives to deviate from the socially optimal allocation of public funds, the present value of cumulative pollution will always exceed the first-best outcome. Furthermore, when introducing uncertainty about the future government’s identity, the effect on expected total pollution is path dependent. I.e., if party $E$ is in office initially, pollution shrinks while public savings rise when the reelection probability increases. The opposite holds if party $I$ initially holds power and the reelection probability increases.\(^3\)

This paper adapts the framework established by Tabellini and Alesina (1990) who analyze the effects on public debt when voters decide about the allocation of funds between two public goods. Since, in their approach, both goods are clean but parties represent different preferences, a government facing reelection uncertainty will choose inefficiently high debt while the social planner would aim for a balanced budget. In a comment, Peletier et al. (1999) expand the framework by modeling investments in a productive capital stock. These investments generate positive returns and increase the public budget in the next period. Their contribution proofs particularly interesting in comparison as it also provides a normative justification for non zero public debt and attenuates the negative consequences of political instability. Yet, their extension features a ‘positive’ interaction instead of welfare-decreasing pollution, all governments (as well as the social planner) would want to accumulate debt in the first period.

The remainder of the paper continues with an outline of the model in the next section. In the subsequent section, I will derive the social planner’s solution and the outcome with political stability as benchmark cases. Section 4 will provide the political economy results under reelection uncertainty. The paper will close with an numerical example and a concluding section.

## 2 Model

The model closely follows Tabellini and Alesina (1990) where a group of voters decides about the allocation of public funds between two different public

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3In the preliminary version, it is assumed that the reelection probability is exogenously given. Hence, environmental policy does not affect the median voters decision to vote for either party.
goods, \( g_t \) and \( f_t \). In the specification at hand, the voters’ preferences are represented by one of two parties, either ‘environmentalists’, \( E \), or ‘industrialists’, \( I \). The innovation of this paper is that the provision of one public good is also associated with environmental pollution. Pollution is generated at a constant ratio to the provision of \( g_t \) and causes damages of the form

\[
D_t(\gamma g_{t-1} + g_t), \quad \text{with} \quad D'_t > 0, \ D''_t \geq 0,
\]

such that \( \gamma \) captures the persistence of pollutants and total pollution is a stock which accumulates over time. Pollution could be seen as green house gas (GHG) emissions created in the production process of \( g_t \). Hence, damages amount to the detrimental effect of climate change on welfare. Without loss of generality, pollution from \( g_0 \) is normalized to zero. In the context of emissions, damages can be interpreted as the climate change caused by increasing the atmospheric GHG concentration above preindustrial levels.

**Assumption A1** *Pollution from the provision of \( g_t \) is assumed to decay slowly over time or persist entirely, i.e. \( \gamma \in (0, 1] \).*

Assumption 1 ensures that pollution actually is a stock variable (\( \gamma > 0 \)). If pollution decays immediately and does not accumulate over time, strategic interactions between different decision makers would only arise from the level of public debt but not the history of provision of the polluting good. In the basic model by Tabellini and Alesina (1990), political economy incentives to over-accumulate debt arise from variations in voters’ valuation of the two public goods. In the specification with a pollution externality, it is possible to assume that individuals preferences for either good are constant. Thus, for simplicity and in order to restrict attention solely to variations in damage internalization, individual \( i \)’s intertemporal welfare is given by

\[
W^i = E\left\{ \sum_{t=1}^{2} u(g_t) + u(f_t) - \theta_i D_t(\gamma g_{t-1} + g_t) \right\},
\]

with \( u'(x) > 0 \) and \( u''(x) \leq 0 \). Future utility is not discounted to avoid confounding debt accumulation due to ‘consumption’ smoothing with the political economy mechanism of strategic debt. Here, \( \theta_i \) indicates the degree to which individuals (and political parties) internalize pollution damages.
E.g., $\theta_i$ represents the awareness of how much man-made emissions affect climate change.\footnote{\text{This interpretation of $\theta_i$ gives rise to a more comprehensive specification of eq. (2), where all individuals $i$ experience the same level of pollution damages but the share $(1 - \theta_i)D_t(g)$ is believed to be a natural constant and not under the control of man.}}

**Assumption A2** The degree of internalization is on the interval $\theta_i \in [0, 1]$. All individuals $i$ identify with either party $E$ or $I$ and $\theta_E > \theta_I$.

The economy exists for two periods and will be endowed with exogenously given public funds of 1 in each period. Since public debt, $b$, has to be repaid at the end of the second period, the per-period budget constraints are given by

\begin{align*}
g_1 + f_1 & \leq 1 + b, \quad (3a) \\
g_2 + f_2 & \leq 1 - b. \quad (3b)
\end{align*}

The public budget constraints, equations (3a) and (3b), implicitly bind the government in $t = 2$ to fully repay public debt inherited from the previous period.

Decisions about the vector of provisions $(g_t, f_t)$ takes place at the beginning of each period $t$ and cannot pre-commit subsequent governments to provide a specific bundle in future periods. Since all individuals identify with one of the political parties, total welfare is the sum of individual levels given by (2). In this framework, political economy incentives arise from uncertainty about the identity of the median voter in the second period. Hence, for re-election uncertainty to arise, there must be some exogenous factor which can change the median voter’s alignment with either party. For the subsequent analysis, it is crucial that all individuals live in both periods such that the population is constant. Tabellini and Alesina (1990) argue that the median voter’s identity will not remain the same when either the (perceived) costs of participation or the eligibility to participate in elections change. In the context of environmental pollution, especially the former reason may be relevant. For instance, environmental catastrophes caused by climate change could be the catalyst for people, who previously abstained to cast their vote in future elections. If the median voter’s preferences diverge over time, i.e. $\theta_1 \neq \theta_2$, the incumbent government will be replaced in the second period which will also affect the combination of public goods and level of pollution in $t = 2$. The next section will ignore the possibility of reelection uncertainty.
and establish the social planner’s first-best allocation and the outcome if either party remains in office with certainty as a benchmark.

3 Social planner’s problem and certain reelection

The established literature on the political economy of public debt acknowledges that the ruling party in $t = 1$ can strategically employ public debt to transfer spending capabilities from the second period – where, from their perspective, funds will not be used optimally – and increase current spending. Thus, the incumbent confines any future government’s ability to act. If instead, the median voter’s identity remains unchanged, the government would be certain of reelection in the next period and maximize the median voter’s welfare by smoothing provision of the public goods across periods. For this reason, in an economy with non-durable, clean public goods, public debt would be zero in the absence of political uncertainty (see Persson and Svensson, 1989, Alesina and Tabellini, 1990 or Tabellini and Alesina, 1990).

However, if provision of one public good also creates pollution, the result with regard to optimal public debt changes. When, $\theta_t = \text{const.}$, the decision maker can maximize welfare over all variables in advance such that the optimization problem amounts to

$$\max_{g, b} W^i = u(g_1) + u(1 + b - g_1) + u(g_2) + u(1 - b - g_2)$$

$$- \theta_i \left( D_1(g_1) + D_2(\gamma g_1 + g_2) \right),$$

where the public budget constraints (3a) and (3b) were employed to eliminate $f_t$. The first-order conditions are given by

$$\partial W^i / \partial g_1 = u'(g_1) - u'(f_1) - \theta_i \left( D'_1(g_1) + \gamma D'_2(\gamma g_1 + g_2) \right) = 0,$$  \hspace{1cm} (5a)

$$\partial W^i / \partial g_2 = u'(g_2) - u'(f_2) - \theta_i D'_2(\gamma g_1 + g_2) = 0,$$  \hspace{1cm} (5b)

$$\partial W^i / \partial b = u'(f_1) - u'(f_2) = 0.$$  \hspace{1cm} (5c)

\(^5\)The results also apply to scenarios where both public goods cause pollution, but the pollution intensity of $g_t$ is higher.
Proposition 1 Under certain reelection, any politician ignoring the climate externality ($\theta_i = 0$) will keep a balanced budget. As the internalization preference $\theta_i$ increases, the decision maker will cut total emissions and accumulate savings (debt) if the cumulative marginal damages from pollution are higher (lower) in the first period.

Proof. From the first-order condition with respect to public debt, equation (5c), it becomes apparent that any government will want to choose a constant supply of the clean good, $f_1 = f_2$, regardless of its willingness to internalize damages, $\theta_i$. Hence, if $\theta_i = 0$, conditions (5a) and (5b) reduce to $u'(g_t) = u'(f_t)$ which implies that provision of the polluting good, $g_t$, will also be constant. As the exogenous endowment with public funds is the same in both periods, there will be no incentive to accumulate public debt or savings. To prove the second part of Proposition 1, combine the first-order conditions with regard to $g_1$ and $g_2$ and employ (5c) to obtain

$$u'(g_1) - u'(g_2) = \theta_i \left( D'_1(g_1) + \gamma D'_2(\gamma g_1 + g_2) - D'_2(\gamma g_1 + g_2) \right). \tag{6}$$

Apparently, the sign of the RHS of expression (6) will be positive if $D'_1 + \gamma D'_2 > D'_2$, i.e. whenever the present value of cumulative marginal damages from pollutants emitted in $t = 1$ is larger than from production in the second period. Since marginal utility is decreasing in the quantity of the public good, any politician with preferences $\theta_i > 0$ will choose $g_1 < g_2$. Combining budget constraints (3a) and (3b) and solving for $b$ yields the optimal level of public debt (or savings) under certain reelection as

$$b = \frac{g_1 - g_2}{2} + \frac{f_1 - f_2}{2}. \tag{7}$$

With the clean public good always being provided in constant amounts, $b$ will be negative if the present value of cumulative marginal damages from pollution is decreasing over time. The higher the internalization preference $\theta_i$, the larger becomes the intertemporal gap between provision of $g_1$ and $g_2$. Thus, $b$ is decreasing in $\theta_i$ and politicians accumulate more public savings as their respective internalization preference rises. Together with equation (5b), this results in $g_1 < g_2 < f_t$, with the last inequality also increasing in $\theta_i$. Consequently, total pollution is decreasing in the internalization rate.

If the cumulative marginal damages from pollution are instead increasing over time, the RHS of equation (6) becomes negative. Thus, the politician
Corollary 1 Considering that $\theta_i$ is restricted to the interval $[0, 1]$, public savings (debt) under certain reelection are always (weakly) below the first-best level of savings (debt) implemented by the social planner, while emissions are excessively high.

Proposition 1 suggests that whether politicians facing reelection certainty (or the social planner, respectively) accumulate public debt or savings, crucially depends on whether cumulative marginal damages from pollution increase or decrease over time. In the context of climate change with a specific emphasis on GHG emissions, the latter case appears to be quite relevant. Since the atmospheric persistence of one of the major anthropogenetic drivers of climate change, CO$_2$, is very high, the respective persistence parameter, $\gamma$, is close to one, implying that

$$\lim_{\gamma \to 1} D'_1(g_1) + \gamma D'_2(\gamma g_1 + g_2) - D'_2(\gamma g_1 + g_2) \to D'_1(g_1) > 0,$$

regardless of the exact functional specification of the damage functions. In this scenario, politicians with preferences $\theta_i \in (0, 1]$ acknowledge that emissions are more harmful, the earlier they are released into the atmosphere. For this reason, the government will postpone the bulk of spending on the polluting good to the later period, where the associated welfare damage is less severe. In order to fund higher spending in $t = 2$, the public budget balance has to be negative (since the definition of $b$ requires that negative values represent savings) at the end of the first period. An internalizing politician will also divert funds from the polluting good to increase provision of the clean public good and thereby decrease total emissions. However, this shift is constant across periods and, thus, balance neutral. Both, the effect on the public budget balance and the reduction of total emissions, are most pronounced when damages are fully internalized, $\theta^* = 1$, in the first-best solution. However, as the politicians’ internalization preference declines, they will trend towards a higher emissions level and a more balanced budget.

Essentially, the benchmark scenario under certainty recreates the previous findings by Kellner and Runkel (2018), where we show that introducing

\[\text{Estimations suggest that 65-80\% of CO}_2\text{ emissions from fossil fuel burning decompose over the duration of 200 to 2000 years (Archer et al., 2009).}\]

pollution externalities in a tax smoothing framework create incentives for the social planner to deviate from a balanced budget rule, in a model with exogenous public endowments and partial internalization. The next section will now introduce reelection uncertainty.

4 Recursive solution under reelection uncertainty

4.1 Voting in the final period

To answer the central question of this paper – how reelection uncertainty affects the level of public debt and total pollution – I first solve the problem of the second period’s decision maker. At the beginning of the second and final period, public debt (or savings), \( b \), as well as past emissions from provision of \( g_1 \) are already predetermined through previous governments’ decisions. Hence, the second period’s government, subsequently referred to as government 2, maximizes welfare in \( t = 2 \) for their respective internalization preference, \( \theta_2 \), by choosing the public goods bundle \((g_2, f_2)\) such that

\[
\max_{g_2, f_2} W_2 = u(g_2) + u(f_2) - \theta_2 D_2(\gamma g_1 + g_2),
\]

(9)

where provision of the clean good, \( f_2 \), can be substituted for \((1 - b - g_2)\) which results from rearranging budget constraint (3b). From the corresponding first-order condition

\[
u'(g_2) - u'(1 - b - g_2) - \theta_2 D_2'(\gamma g_1 + g_2) = 0,
\]

(10)

the optimal provision of the public goods under voting is implicitly given as

\[
g^v_2 = g_2(b, \theta_2, \gamma g_1) \quad \text{and} \quad f^v_2 = 1 - b - g^v_2 = f_2(b, \theta_2, \gamma g_1).
\]

(11)

It becomes apparent that the decision maker’s choice in the second period is determined by their respective pollution awareness (i.e. the internalization rate \( \theta_i \)) but also ‘inherited’ variables, namely public debt and the stock of emissions remaining from first period provision. If \( \theta_2 = 0 \), government 2 ignores the externality and chooses provision of \( g_2 \) and \( f_2 \) such that marginal utilities equate (which simplifies to \( g_2 = f_2 \) if utility functions are the same). In this case, the stock of pollutants also becomes irrelevant for the allocation in \( t = 2 \) and the expressions are identical to the reaction functions defined
by Tabellini and Alesina (1990) for a politician who values both public goods equally.

Applying the implicit function theorem to equation (10), the marginal effect of the stock of emissions on the provision of the polluting good in $t = 2$ may be derived as

$$\frac{\partial g_2^v}{\partial g_1} = \frac{\gamma \theta_2 D_2''(\gamma g_1 + g_2^v)}{u''(g_2^v) + u''(f_2^v) - \theta_2 D_2''(\gamma g_1 + g_2^v)}. \tag{12}$$

Since $u''(x) \leq 0$ and $D_2''(x) \geq 0 \forall x$, the marginal effect of $g_1$ on the provision of $g_2$ will be on the interval $(-1, 0]$. To determine whether this expression increases or decreases with respect to the persistence rate $\gamma$ would require further assumptions about the third derivative of $D_2$. Analogously, application of the implicit function theorem to equation (3b) allows to derive the partial effect of $g_1$ on the demand for the clean good as

$$\frac{\partial f_2^v}{\partial g_1} = -\frac{\partial g_2^v}{\partial g_1} \geq 0. \tag{13}$$

Ceteris paribus, higher provision of the polluting good in the first period will also increase marginal damages in the second period which creates an incentive for government 2 to shift funds away from $g_2$ and instead increase provision of the clean good. The internalization parameter $\theta_2$ defines how elastic this reaction will be. The implicit function theorem can be applied to equations (10) and (3b) once more to also obtain the marginal effect of public debt on the second period variables as

$$\frac{\partial g_2^v}{\partial b} = \frac{u''(f_2^v)}{u''(g_2^v)} \cap \frac{\partial f_2^v}{\partial b} = -\left(1 + \frac{\partial g_2^v}{\partial b}\right).$$

The marginal effect of public debt is negative and on the interval $(-1, 0)$ for both good since higher debt reduces the overall budget available in $t = 2$.

### 4.2 Voting in the first period

The political decision maker in the first period will choose $(g_1, f_1, b)$ in order to maximize their electorate’s utility across both periods. At the time

\footnote{To be precise, the marginal effect of the stock of emissions would be obtained by differentiating with regard to $\gamma g_1$. However, $\partial g_2^v/\partial g_1$ is monotonous in $\gamma$ and more expedient to employ later on.}
of voting in $t = 1$, the identity of the second period’s median voter (and consequently $\theta_2$) is yet unknown. However, since reaction functions $g^v_2$ and $f^v_2$ also depend on the internalization parameter $\theta_2$, the politician maximizes the expected utility

$$\max_{g_1, b} \mathbb{E}\{W_1\} = u(g_1) + u(1 + b - g_1) - \theta_1 D_1(g_1)$$

$$+ \mathbb{E}\left\{ u(g^v_2) + u(f^v_2) - \theta_1 D_2(\gamma g_1 + g^v_2) \right\} \quad (14)$$

If entrance into the political ‘market’ is restricted to parties $E$ and $I$ (i.e. either one of them will take office in the second period) and election probabilities are exogenous, the maximization problem in (14) can be expressed without the expectations operator as

$$\max_{g_1, b} \mathbb{E}\{W_1\} = u(g_1) + u(1 + b - g_1) - \theta_1 D_1(g_1)$$

$$+ \pi \left[ u(g_2(b, \theta_1, \gamma g_1)) + u(f_2(\cdot)) - \theta_1 D_2(\gamma g_1 + g_2(\cdot)) \right]$$

$$+ (1 - \pi) \left[ u(g_2(b, \theta_2, \gamma g_1)) + u(f_2(\cdot)) - \theta_1 D_2(\gamma g_1 + g_2(\cdot)) \right], \quad (15)$$

where $\theta_1 \neq \theta_2$. Therefore, the outcome with exogenous reelection probabilities $\pi \in (0, 1)$ will merely be a weighted average between the solutions under certain reelection ($\pi = 1$) and certain ‘loss of office’ ($\pi = 0$).

**Assumption A3** It is ex-ante known that $\theta_1 \neq \theta_2$, i.e. the incumbent politician will be superseded with certainty in the second period.

Assumption 3 may appear restrictive at first. Yet, as long as reelection probabilities are not endogenous on the parties’ first period decisions, this assumption allows to save considerably on notation without abandoning any intuition about the possible outcomes. When the reelection probability is high, the initial government’s decisions will more closely resemble the results under certain reelection which were derived in the previous section and vice versa. The first-order condition of the simplified maximization problem with regard to provision of the polluting good is then given by

$$u'(g_1) - u'(1 + b - g_1) - \theta_1 \left( D_1'(g_1) + \gamma D_2'(\gamma g_1 + g^v_2) \right)$$

$$+ \frac{\partial g^v_2}{\partial g_1} \left( u'(g^v_2) - u'(f^v_2) - \theta_1 D_2'(\gamma g_1 + g^v_2) \right) = 0. \quad (16)$$

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Whereas the respective condition for public debt follows as

\[ u'(1 + b - g_1) - u'(f_2^v) + \frac{\partial g_2^v}{\partial b} \left( u'(g_2^v) - u'(f_2^v) - \theta_1 D_2' (\gamma g_1 + g_2^v) \right) = 0. \]  \hspace{1cm} (17)

Substituting (16) into (17) yields

\[ u'(g_1) - u'(f_2^v) - \theta_1 \left( D_1'(g_1) + \gamma D_2'(\gamma g_1 + g_2^v) \right) + \left( \frac{\partial g_2^v}{\partial g_1} + \frac{\partial g_2^v}{\partial b} \right) \left( u'(g_2^v) - u'(f_2^v) - \theta_1 D_2' (\gamma g_1 + g_2^v) \right) = 0. \]  \hspace{1cm} (18)

The first term on the LHS of equation (18) represents the marginal benefit from increasing debt in order to fund higher spendings on the polluting good at the beginning of \( t = 1 \). Interpretation for the clean good follows analogously from condition (17). The sign of the last term in parentheses on the LHS of conditions (16) through (18) can be identified by considering equation (10) and when it is known whether \( \theta_1 \geq \theta_2 \). The term in parentheses would become zero for \( \theta_1 = \theta_2 \) due to the first-order condition from the second period. Thus, if \( \theta_1 > \theta_2 \) the term in parentheses is smaller than zero (and vice versa). This becomes more readily apparent, when employing the second period’s first-order condition (10) to substitute

\[ u'(g_2^v) - u'(f_2^v) - \theta_1 D_2'(\gamma g_1 + g_2^v) = (\theta_2 - \theta_1) D_2'(\gamma g_1 + g_2^v). \]  \hspace{1cm} (19)

Since both, \( \partial g_2^v/\partial g_1 \) and \( \partial g_2^v/\partial b \), are always negative, the last summand will reduce the spread between the marginal utility from \( g_1 \) (\( f_1 \)) and \( f_2^v \) (or increase the difference if \( \theta_1 < \theta_2 \), respectively). The additional term in condition (18) captures the perceives marginal damages from provision of \( g_1 \) and will always increase the spread between \( g_1 \) and \( f_2^v \) if government 1 internalizes at least part of the emission damages.

### 4.3 Strategic interactions and their impact on public debt

The interpretation of conditions (17) and (18) reveals the strategic incentives from reelection uncertainty (or as is the case here, from a certain change of office). The former condition defines the marginal value of debt in order to fund additional spending on the clean good in the first period. For a constant rate of pollution internalization, strategic considerations are of no concern.
such that provision of $f$ would neither be associated with an incentive to go into debt nor to accumulate savings. Yet, if $\theta_1 > \theta_2$, the incumbent anticipates that the future government will overspend on the polluting good while providing too little of the clean good. This creates an incentive to reduce the amount of public savings (or even to accumulate debt) and spend more on the clean good today. The same intuition holds for condition (18).

On the other hand, if the first government’s internalization preference is comparably low, they would divert funds to the first period as they anticipate that spending in $t = 2$ will be biased towards (subjective) overprovision of the clean good when environmentalists take office. Thus, any government would want to reduce public savings or accumulate debt, knowing that the future decision maker will allocate funds inefficiently from the incumbent’s point of view. For means of reference, I refer to this as the budget interaction. This effect has been well established by Tabellini and Alesina (1990). However, the stock externality now attenuates the incentive to distortively shift funds to the first period as it creates an additional emission interaction.

First, consider again the implications if $\theta_1 > \theta_2$. Government 1 now is aware that a higher provision of $g_1$ will not just limit potential emissions in $t = 2$ by draining funds from the second period, but also cause pollution damage in both periods. Knowing that their successor will always produce too much pollution, the incumbent might even decide to reduce provision of $g_1$ compared to the outcome under certain reelection. This effect attenuates the incentives to shift funds to the first period whenever the incumbent is ‘greener’ than its successor. Conversely, if an environmentally unaware politician ($\theta_1 < \theta_2$) is initially in office, they are aware that their successor, who already provides too little of the polluting good, will provide even less when the stock of emissions from $t = 1$ is high. By abstaining from excessive spending on the polluting good in the first period and, thus, leaving more funds and fewer emissions, the incumbent may ensure that the second period’s government does not cut provision of $g_2$ too severely. As a result, even a politician who prefers a low internalization rate might decide to provide less of the polluting good in $t = 1$ than when reelection is certain.

In order to systematically assess these effects, I define the quadratic utility

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8The political economy effect for the clean good is thus very similar to the incentives derived by Tabellini and Alesina (1990) when government 1 has a higher preference for good $f$. 

function
\[ u(x) = \alpha x - \frac{\beta}{2} x^2, \quad \text{with} \quad \alpha \geq 1, \beta > 0, \quad (20) \]
for both public goods \( g_t \) and \( f_t \), as well as the quadratic damage function
\[ D_t(x) = \frac{\delta_t}{2} x^2, \quad \text{with} \quad \delta_t > 0 \forall t. \quad (21) \]

Applying these explicit functional forms to the problem facilitates deriving closed-form expressions for the impact of reelection uncertainty on debt and public good provision over the whole interval of internalization rates. This property of the quadratic model is desirable as it ensures that the subsequent findings still hold if preferences vary heavily between parties, e.g. when environmentalists prefer close to full internalization \((\theta_E \to 1)\) while industrialists tend to ignore the externality \((\theta_I \to 0)\).\(^9\)

**Proposition 2** Under reelection uncertainty, any incumbent government, regardless of their identity, \( \theta_1 \), will accumulate more public debt (or decrease savings) at the end of the first period, the more their successor’s expected internalization preference, \( \theta_2 \), deviates from \( \theta_1 \).

**Proof.** To prove Proposition 2, it is expedient to introduce the distance parameter \( \Delta = (\theta_2 - \theta_1) \) measuring how much preferred internalization rates differ between periods. Since \( \theta_1 \) is given at the beginning of the first period, when the identity of government 1 is revealed, \( \Delta \) can be varied by either increasing or decreasing \( \theta_2 \). Hence, the distance parameter can take values on the interval \( \Delta \in [-1, 1] \). In the appendix, I show that Cramer’s rule may be applied to a system of equations, which consists of the first-order conditions \((10), (16)\) and \((17)\) from periods one and two, in order to obtain the marginal effect
\[ \frac{\partial b}{\partial \Delta} = \frac{|J_b|}{|J|}, \quad \text{(22)} \]

\(^9\)It is worth noticing that the subsequent findings also apply to the general model with an implicit utility function for marginal differences between internalization preferences, i.e for \( \theta_2 - \theta_1 \to 0 \).
where $J$ is the Jacobian matrix and equivalent to the problem’s Hessian, the determinant of which is always negative in the Welfare maximum. The numerator can be derived as

$$|J_b| = 2D'_2D''_2 \frac{\partial g''_2}{\partial b} \left(2\beta(1 - \gamma) + \theta_1\delta_1\right)\Delta$$  \hspace{1cm} (23)

Combining equation (23) with $|J| < 0$ and recalling that $D'_2, D''_2 > 0, \gamma \leq 1$ and $\frac{\partial g''_2}{\partial b} < 0$, it becomes straightforward to show that the marginal effect in (22) will be zero for $\Delta = 0$, i.e. when the the internalization preference does not change such that $\theta_t = const.$ as under certain reelection. Analogously, $\frac{\partial b}{\partial \Delta}$ will be negative (positive) when the second period government prefers a lower (higher) internalization rate than the incumbent, i.e. for $\Delta < (>)0$. Consequently, the function of public debt over the distance parameter, $b(\Delta)$, is $u$-shaped with its minimum at $\Delta = 0$. Hence, the incumbent government will accumulate more debt, the further apart internalization preferences are. This effect occurs regardless of whether the political challenger prefers a higher or lower rate of pollution internalization. 

Note that Proposition 2 is not affected by how cumulative marginal damages from pollution evolve over time. The reason is that the underlying mechanisms influencing the budget decision are not the same. Whether optimal debt (or debt under certainty, respectively) should be negative or positive is determined by the pollution externality. The effect of reelection uncertainty, however, is driven by the strategic interaction which outweighs the emission interaction. Thus, the first period’s politician will always accumulate more debt when reelection becomes less certain and when preferences differ from the expected second period internalization rate. In conjunction with Corollary 1, this may have a varying impact on where debt under uncertainty is located in comparison to the outcome under certainty and the first-best solution.

First, assume that cumulative marginal damages decrease over time, which has been argued to be the potentially more relevant scenario when analyzing GHG emissions. The social planner would optimally accumulate public savings, $b^* < 0$, while all politicians who do not fully internalize the externality will only retain an inefficiently low amount of savings on the interval $b \in (b^*, 0)$ when certain of reelection. Since, according to Proposition 2, uncertain election outcomes encourage politicians to save less than under certainty (and potentially accumulate debt), the public budget balance will deviate even more from its first-best level.
In contrast, the effect is no longer unambiguous if the cumulative marginal
damages from pollution are higher in the second period such that the first-
best solution demands for a positive level of public debt. Under certain
reelection, politicians would seek a more balanced budget than socially opti-
mal with public debt shrinking to zero as the preferred internalization rate
decreases. However, as in the case with decreasing cumulative marginal dam-
ages, competition for reelection (i.e. uncertainty) bears strategic incentives
for any incumbent to accumulate more debt in the first period. Thus, two
results can emerge. First, when debt under certain reelection, \( b \), was very
small, an increase due to uncertainty might reduce the distance to optimal
debt, \( b^* \), and increase the efficiency of the public budget balance. Second,
considering that for a high internalization rate, \( \theta_1 \), \( b \) already approaches \( b^* \)
under certain reelection, an additional increase in \( b \) due to uncertainty could
cause public debt to ‘overshoot’. This results in excessively high debt and
the intertemporal allocation of funds being less efficient than under certainty.

![Figure 1: Public budget balance at the end of the first period](image)

The results with regard to increasing cumulative marginal damages seem
mostly relevant in the case of environmental ‘tipping points’ when pollution
damages are fairly low until the stock of emissions has reached a critical
threshold causing a substantial amplification of damages. Therefore, the
central insight from Proposition 2 in the context of this paper is that reelection
uncertainty encourages the incumbent government to only accumulate insufficient savings or go into debt if public savings are called for in the first-
best solution. Figure 1 is based on numerical results for the specification
\((\alpha, \beta, \delta_1, \delta_2) = (1, 1, 1, 1)\) of the quadratic model defined by functions (20) and (21) in order to illustrate these findings. Each panel depicts public debt at the end of \(t = 1\) over the possible second period internalization rates, \(\theta_2\), for a given preference \(\theta_1\) and various emission persistence rates. In all cases, public debt becomes minimal when \(\theta_1 = \theta_2\) which corresponds to certain reelection. In addition, the curves\(^{10}\) \(b(\theta_2)\) become flatter as the atmospheric persistence of emissions increases.

4.4 Impact on first period provision and emissions

As already hinted at above, upon developing the general intuition for the dynamics caused by reelection uncertainty, provision of the polluting public good is also affected by both, budget and emission interactions. In contrast to the political economy effects on public debt, the emission interaction can not just attenuate the former but even overturn the direction of the total effect on public good provision in the first period.

**Proposition 3** Under reelection uncertainty, if the atmospheric persistence of emissions is above (below) a certain threshold \(\tilde{\gamma}\), any incumbent government, regardless of their identity, \(\theta_1\), will reduce (increase) provision of the polluting good, \(g_1\), the more their successor’s expected internalization preference, \(\theta_2\), deviates from \(\theta_1\).

**Proof.** The derivation of Proposition 3 is conducted analogously to the proof of Proposition 2. Hence, Cramer’s rule is applied to the system of equations of (10), (16) and (17) in order to obtain

\[
\frac{\partial g_1}{\partial \Delta} = \frac{|J_{g_1}|}{|J|},
\]

(24)

where

\[
|J_{g_1}| = \beta D_2' D_1^\theta \frac{\partial g_2}{\partial b} \left( 1 + 2\beta + \theta_2 \delta_2 - 6\gamma \right) \Delta.
\]

(25)

Since \(|J|\) remains negative, the direction of the marginal effect in (24) is determined by the sign of \(|J_{g_1}|\). From (25), it becomes immediately apparent

\(^{10}\)Since \(\theta_1\) is held constant, \(b(\theta_2)\) corresponds to merely a vertical shift of function \(b(\Delta)\) by \(\theta_1\).
that provision of the polluting good in $t = 1$ also reaches an extremum when reelection is certain, i.e. $\Delta = 0$. Whether this extreme point is a maximum or minimum, depends on whether the term in parentheses is negative or positive. The parentheses, in turn, are negative, whenever the persistence parameter $\gamma$ is larger than a critical threshold $\tilde{\gamma}$ and vice versa. Conditional on the parameters $\beta$ and $\delta_2$, such a critical value $\tilde{\gamma}$ can exist on the interval $\gamma \in [0, 1]$. Therefore, the function of $g_1$ over $\Delta$ is (inversely) $u$-shaped if $\gamma < (>) \tilde{\gamma}$. This implies that for $\gamma$ large (small) enough, provision of the polluting good will be highest (lowest) under certain reelection and decrease (increase) as the politicians disagree more over the subjectively optimal internalization rate.\footnote{Again, for marginal variations between internalization preferences, $(\theta_2 - \theta_1) \rightarrow 0$, the results from Proposition 3 also extend to general, concave utility functions.}

**Corollary 2** Since emissions are linear in provision of the polluting public good, reelection uncertainty leads to a decrease in the stock of emissions, $\gamma g_1$, for any $\gamma > \tilde{\gamma}$.

Figure 2: Provision of polluting public good in the first period

The specification of the damage function implicitly assumes that emissions are one to one with the quantity of $g_t$ provided by the government. Thus, figure 2 depicts both, first period provision of the polluting good and first period emissions, as a function of the expected internalization rate $\theta_2$ for given $\theta_1$. When computing the numerical model for a small atmospheric persistence, here $\gamma = 0.2$, provision of the polluting good in $t = 1$ indeed
increases if $\theta_2$ diverges from $\theta_1$. Since just a small fraction of first period pollution carries over to the second period, the emission interaction is relatively weak. Hence, the incumbent government can transfer funds to the first period in order to increase provision of both public goods without having to fear that, if $\theta_2 > \theta_1$, their successor will severely cut spending on $g_2$ as the inherited stock of emissions, $\gamma g_1$ remains small. On the other hand, if $\theta_2 < \theta_1$, the incumbent is aware that abstaining from the polluting good in $t = 1$ will not significantly reduce damages from pollution in the second period as total emissions in $t = 2$ depend primarily on the successor’s decision over $g_2$. As a result, for persistence rates below the threshold $\tilde{\gamma}$, the budget interaction dominates the strategic considerations affecting provision of $g_1$.

The insight from Proposition 3 is rather striking. Intuitively, we might expected that competition from a ‘greener’ party would discipline an incumbent politician to reduce provision of the polluting good and first period emissions. Quite surprisingly, tough, this effect can be provoked by any political competitor on the spectrum, whether they prefer a higher internalization rate than the incumbent or not. In conclusion, this means that (granted that $\gamma > \tilde{\gamma}$) reelection uncertainty will not just create a public budget inefficiency but also effectively reduce pollution in the first period. As stated before, this effect becomes more pronounced when the reelection probability shrinks or the magnitude of the preference distance, $|\Delta|$, increases.

4.5 Total emissions and welfare damages under uncertainty

Thus far, we have learned that reelection uncertainty through a competing party of any ideology $\theta_2 \neq \theta_1$ will always increase public debt on the one hand, while first period provision of the polluting public good will decrease, on the other hand, if the persistence rate is high enough. Yet, in order to assess the overall environmental implications of uncertainty, it is also necessary to determine the net effect on the stock of emissions at the end of the second period. From the social planner’s perspective (i.e. $\theta^* = 1$), the total (discounted) intertemporal welfare effect of emissions is given by

$$WL = D_1(g_1) + D_2(\gamma g_1 + g_2).$$

(26)

As turns out, the political economy effect on $WL$ can differ depending on whether the future government’s expected internalization preference is higher or lower than the incumbent’s.
Proposition 4 If the atmospheric persistence of emissions is high enough \( (\gamma > \bar{\gamma}) \), competition from a party with ‘greener’ preferences will always reduce the total welfare loss from emissions in comparison to the outcome under certain reelection. If the political challenger prefers a lower internalization rate, the total welfare loss initially increase as preferences diverge but could top out at some \( \Delta \in (-1, 0) \).

Proof. First, the effect of diverging preferences, \( \Delta \), on the total welfare loss, \( WL \), through a marginal change in \( \theta_2 \) at \( \theta_1 = \text{const.} \) can be derived as

\[
\frac{\partial WL}{\partial \Delta} = \frac{\partial D_1(g_1)}{\partial \theta_2} + \frac{\partial D_2(\gamma g_1 + g^v_2)}{\partial \theta_2} = \left(D'_1 + \gamma D'_2\right)\frac{\partial g_1}{\partial \theta_2} + D'_2 \frac{\partial g^v_2}{\partial \theta_2} \tag{27}
\]

Corollary 2 implies that emissions and, consequently, pollution damages decline in the first period if \( \gamma < \bar{\gamma} \). This is represented by the first term in the rightmost equality of equation (27) which is inversely \( u \)-shaped over \( \theta_2 \). However, since

\[
\frac{\partial g^v_2}{\partial \theta_2} = \frac{D'_2(\gamma g_1 + g^v_2)}{u''(g^v_2) + u''(1 - b - g^v_2) - \theta_2 D''_2(\gamma g_1 + g^v_2)} < 0, \tag{28}
\]

as obtained from equation (10) via the implicit function theorem, the second term will always be decreasing in \( \theta_2 \). Therefore, if the future government is expected to be ‘greener’ than the incumbent government, \( \theta_2 > \theta_1 \), both terms are on the decreasing margin such that \( WL \) decreases as the preference distance increase. In the opposite case, if \( \theta_2 < \theta_1 \), the second term now increases while the first term still decreases as the distance parameter’s magnitude rises. Substituting for the explicit expressions in (27), it can be shown that for small differences \( \Delta < 0 \), the latter still outweighs the former effect in the quadratic case. Thus, total emission damages reach their maximum on the ‘left’ side of the certain reelection-outcome. However, it is generally unclear whether this maximum is located within the interval \( \Delta \in (-1, 0) \). If so, the welfare loss from emissions could be declining again for values of the distance parameter sufficiently close to \(-1\).

Proposition 4 remains rather opaque about the net welfare effect if the second period’s government is expected to internalize fewer emissions than the incumbent. While it is possible to derive a closed form expression, in the quadratic model which specifies whether \( \partial WL/\partial \Delta \geq 0 \) for \( \Delta < 0 \), this term is devoid of any interpretative value. Instead of descending into a
discussion of the conditions determining the sign of this expression for the sake of technical comprehensiveness, it is worthwhile to focus on the intuition to be gained from Proposition 4.

Unlike, public debt, $b$, and first period provision of the polluting good, $g_1$, which both reach their extreme points in the certain reelection-outcome ($\theta_1 = \theta_2$) and will unequivocally increase (or decrease depending on atmospheric persistence and the development of cumulative marginal damages over time) from competition on both sides of the political spectrum, emission damages, $WL$, will reach their maximum at some $\theta_2 < \theta_1$ for any given $\theta_1$ and continuously decrease as the second period internalization rate rises.

Consider again the scenario where atmospheric persistence of emissions is high and the cumulative marginal damages from pollution decrease over time. In this case, reelection uncertainty would always have a positive environmental impact in the first period by reducing emissions from a lower provision of $g_1$, regardless of whether $\theta_2 \gtrless \theta_1$. However, if the second period government has a lower environmental awareness, i.e. prefers a lower internalization rate in expectation, they will emit pollutants in excess, over-compensating the savings from $t = 1$ and causing and increase in total intertemporal damages (at least for small negative values $\Delta$ as explained in the proof of Proposition 4). In contrast, if the expected future decision maker chooses a higher internalization rate than the incumbent, they will provide less of the polluting good in $t = 2$ than the first period government would under certain reelection. Alongside a lower provision of $g_1$ due to uncertainty, the total welfare loss from emissions decreases under reelection uncertainty only if $\theta_2 > \theta_1$.

Figure 3: Total welfare loss from pollution
Figure 3 illustrates these findings in the numerical model lending base to three main insights. First, for the given parametrization of the model, the total welfare loss from emissions, $WL$; is largest when the second period government ignores the externality and, thus, decreases over the whole possible interval of $\Delta$. Second, intertemporal welfare damages increase in the atmospheric persistence as first period emissions become more harmful. Finally, if the incumbent government already completely internalizes pollution, i.e. when the certain reelection-outcome coincides with the social planner’s solution, all possible contestants for office in $t = 2$ would prefer a lower internalization rate such that elections would always be detrimental from a climate protection perspective. This result is depicted in panel 3 of figure 3. On the opposite, if the incumbent party ignores the environmental externality by large, fostering competition for office in the second period from a ‘greener’ contestant would benefit welfare by reducing expected emissions in both periods (as seen in the first panel).

5 Conclusion

In this paper, I build on the contribution by Tabellini and Alesina (1990) by introducing an environmental externality to their political economy model. Politicians allocate public funds in order to provide clean and polluting public goods which generate utility for their electorates but also create emissions which accumulate over time causing a welfare loss. I find that when politicians disagree on the optimal internalization rate of emissions, the strategic dependencies between incumbent and future government are no longer characterized by just an inefficient budget interaction but an additional emission interaction also occurs. The first interaction, already established in the political economy literature on public debt, provokes any incumbent government to run a third-best budget balance in an attempt to bind the future decision makers’ spending capabilities. Therefore, with regard to budget efficiency, the outcome under voting is always inferior to certain reelection of the incumbent. However, I also find that, if emissions are sufficiently persistent (as in the case of CO$_2$), competition for office in the next legislative period will incite the incumbent to reduce provision of the polluting good and, thus, cut emissions immediately. Furthermore, if the contesting party prefers a higher internalization rate than the incumbent, the expected total welfare loss from pollution will also decline due to reelection uncertainty. In contrast, if the
political challenger were even more ignorant of the environmental externality than the politician in office, expected intertemporal net emission damages would rise, while first period provision still decreases.

Since all these effects become more pronounced as either the initial government’s probability to win future elections decreases or the competitor’s preferences diverge more from the incumbent’s internalization rate, two main implications can be drawn from the model. First, there appears to be a trade-off between budget efficiency and environmental protection. If the reelection probability is low, the incumbent government has a strong incentive to accumulate a high level of strategic, inefficient debt (or savings) but will emit fewer pollutants at the same time. At first glance, this result might be discouraging as it seems to buttress the popular opinion that climate protection endeavors can only be realized at the cost of excessive debt. However, the budget inefficiency is not caused by costly spending on mitigation or adaption but rather is a result of strategic interactions which stem from disagreement on the optimal goods bundle and also occurs in the basic model without an environmental externality. In fact, the detrimental effect on public debt accumulation is even attenuated by the emission interaction. Second, the long term impact of reelection uncertainty on climate depends on the political status quo. If, as still seems to be the case in many countries, incumbent governments are rather reluctant to implement rigorous environmental protection laws and have a bias towards GHG intensive industries, the growing appeal of green parties, also to be observed especially in a number of European states, can ‘force’ current decision makers to consider a more climate friendly approach. To that end, it is already sufficient that green parties generally demand much more radical emission savings even if their chances to actually win elections still are comparably low. Returning to the introducing example of the EPA-proposal to adjust the SCC downwards, while the Obama administration might have decided to release fewer emissions than initially planned in the face of a much more (environmentally) conservative competitor,\(^{12}\) the model suggests that emissions the under current US administration will eventually overcompensate these savings.

\(^{12}\)While it may be difficult to reliably confirm this notion with empirical data, there is some conclusive evidence which might point in the same direction. For instance, President Obama ‘last-minute’-declared the Bears Ears National Monument in December 2016, tying up numerous mining sites and considerable Uran deposits. The area covered by the National Monument has since been reduced by 85 percentage under the current administration.
 Needless to say, this paper is far from an encompassing treatment of the politico-economic interactions between environmental issues and public debt. Indeed, this particular direction of political economy remains yet rather unexplored, both theoretically and empirically. The most notable limitation of my contribution is that reelection probabilities remain exogenously fixed. Strategic incentives and interactions would certainly change if parties were able to influence the probabilities by committing to certain debt and climate policies. I also implicitly assume that all voters can identify with one of two parties which are perfectly identical except for their internalization preferences. Politicians see no value in winning office if they would have to deviate from their preferred internalization rates even marginally. Thus, it seems worthwhile to examine how endogenous probabilities affect the outcome. However, this does not appear to be feasible in the analytical framework with closed-form solutions employed here but rather calls for a calibrated simulation.

References


Appendix

Voting economy with exogenous probabilities

From the politicians’ optimization problems in periods one and two, the system of equations

\[ G_1 = u'(g_1) - u'(1 + b - g_1) - \theta_1 \left( D'_1(g_1) + \gamma D'_2(\gamma g_1 + g_2) \right) \]

\[ + \Delta \frac{\partial g_2}{g_1} D'_2(\gamma g_1 + g_2) = 0, \quad (29a) \]

\[ G_2 = u'(g_2) - u'(1 - b - g_2) - \theta_1 D'_2(\gamma g_1 + g_2) \]

\[ - \Delta D'_2(\gamma g_1 + g_2) = 0, \quad (29b) \]

\[ G_b = u'(1 + b - g_1) - u'(1 - b - g_2) + \Delta \frac{\partial g_2}{\partial b} D'_2(\gamma g_1 + g_2) = 0, \quad (29c) \]

can be derived to describe the equilibrium in the voting economy when re-election probabilities are ex-ante known (continue to assume \( \pi = 0 \)) and exogenous. The difference between voters’ internalization preferences is given by \( \Delta \equiv (\theta_2 - \theta_1) \). Hence, if \( \Delta = 0 \), the median voters’ and, consequently, the decision makers’ preferences are constant across time such that the system of equations above mirrors the first-order conditions (5a) - (5c) under reelection certainty. Per definition, the difference term \( \Delta \) is confined to the interval \([-1, 1]\).

Proposition 2 suggests that, for any given value of \( \theta_1 \), debt will be minimal when the incumbent party is sure to be reelected in the second period. Public debt increases whenever the expected future government’s preferences are more or less ‘green’ than the incumbent’s. Expressed in mathematical terms, the function \( b(\Delta) \) has to be \( u \)-shaped with its minimum at \( b(0) \) and \( b'(\Delta) \), \( b''(\Delta) > 0 \).

To proof these properties, we apply Cramer’s rule to find

\[ \frac{\partial b}{\partial \Delta} = \frac{|J_b|}{|J|}, \quad (30) \]

where

\[ |J| = \det \left( \begin{array}{ccc} \frac{\partial G^1}{\partial g_1} & \frac{\partial G^1}{\partial g_2} & \frac{\partial G^1}{\partial b} \\ \frac{\partial G^2}{\partial g_1} & \frac{\partial G^2}{\partial g_2} & \frac{\partial G^2}{\partial b} \\ \frac{\partial G^b}{\partial g_1} & \frac{\partial G^b}{\partial g_2} & \frac{\partial G^b}{\partial b} \end{array} \right), \quad (31) \]
and \(|J_b|\) is obtained by substituting the last column in \(|J|\) for the vector

\[
\psi = \left(-\partial G^1/\partial \Delta, -\partial G^2/\partial \Delta, -\partial G^3/\partial \Delta\right)'.
\]  

(32)

Since the closed form solution for the model with a general, implicit utility function only is unambiguous for marginal differences between internalization preferences (i.e. \(\Delta \to 0\)), I decide to derive the solution for the quadratic specification of the model with (20) and (21) instead. Thus, substituting for the explicit functions in equations (29a) through (29c) and after some tedious but straightforward algebra, the determinant of the Jacobian follows as

\[
|J| = \frac{\partial g^v}{\partial b} \beta \left(8\beta(\beta + \theta_2\delta_2) + 2\beta\theta_1\delta_2(1 + \gamma(3\gamma - 2)) + (\theta_2\delta_2)^2(3 + \gamma(3\gamma - 2))\right) \\
+ \frac{\partial g^v}{\partial b} \theta_1\delta_1 \left(6\beta(\beta + \theta_2\delta_2) + \delta_2(\beta\theta_1 + 2\theta_2^2\delta_2)\right),
\]  

(33)

which is negative for all possible values of \(\theta_1\) and \(\theta_2\), considering that \(1 + \gamma(3\gamma - 2) > 0\) for all \(\gamma \in [0, 1]\). Next, the determinant of \(|J_b|\) is given by

\[
|J_b| = 2D_2'\partial g^v_2 \left(2\beta(1 - \gamma) + \theta_1\delta_1\right)\Delta,
\]  

(34)

implying that \(b'(0) = 0\), \(b'\left(\Delta\right) > 0\) for \(\Delta > 0\) and \(b'\left(\Delta\right) < 0\) for \(\Delta < 0\), respectively, which is equivalent to \(b''\left(\Delta\right) > 0\).

Similarly, in order to obtain

\[
\frac{\partial g_1}{\partial \Delta} = \frac{|J_{g_1}|}{|J|},
\]  

(35)

the first column in \(J\) has to be substituted for the vector \(\varphi\) which, eventually, yields

\[
|J_{g_1}| = \beta D_2'\partial g^v_2 \left(1 + 2\beta + \theta_2\delta_2 - 6\gamma\right)\Delta,
\]  

(36)

such that \(g_1'(0) = 0\), while the curvature of function \(g_1(\Delta)\), i.e. whether \(g_1''(\Delta) \geq 0\), depends on the sign of the term in parentheses on the RHS of (36).