Optimal income taxation and unemployment benefit with involuntary unemployment*

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Abstract
We consider optimal income taxation and search-contingent unemployment benefits in a model where there are both voluntary and involuntary unemployed households, where households' labor productivity and labor disutilities are heterogeneous, and households choose whether to participate and seek jobs in the labor market. We derive an optimal employment tax rule. We investigate the relationship between employment tax rates and the size of the search-contingent unemployment benefit. Our numerical simulations suggest that optimal employment tax rates for some income groups with low consumption are less likely to be negative and the government need not to employ both search-contingent unemployment benefits and in-work credits at the same time.

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Keywords: Optimal income taxation; Unemployment benefit; Involuntary unemployment; Extensive margin; Employment tax

1 Introduction

There are several types of income support policies for low income households. Wage subsidies or in-work credits (tax credits or benefits) are income transfers for low to medium income households who engaged in work. (Search-contingent) unemployment benefits (social assistance benefits) are income supports for job seeking households. Actual policy differs in countries. Some countries adopt one type. For example, Germany and Italy adopt unemployment benefits only, while U.S. adopts in-work credits only. Some countries (like Canada) adopt the two policies, while other countries (like Japan) adopt neither.

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Designing the income support policies for low income households remains an unsettled question.

With regard to in-work credits, recent literature has emphasized the role of an extensive margin of labor. With an extensive margin of labor, households choose whether to participate in the labor force with fixed hours of work. In this setting, it can be optimal to adopt wage subsidies or in-work credits with a negative participation tax for low-income households (see, e.g., Diamond (1980), Saez (2002), Choné and Laroque (2005, 2011), Jacquet et al. (2013), and Christiansen (2015)).

The most simple setting for the extensive margin does not consider the demand side of the labor market. This means that all households who choose to participate in the labor force can find employment, and there is no involuntary unemployment. Some studies develop optimal income taxation with involuntary employment. First, Boone and Bovenberg (2004, 2006, 2013) consider that households bear a search cost when they enter the labor market and seek jobs, and households choose work effort as well as search effort with quasi-linear preferences for leisure. Boone and Bovenberg (2004) consider the welfare benefit for (both voluntary and involuntary) unemployed households as exogenous and show that the in-work benefit (negative income tax) and welfare benefit have a U-shaped relationship. Additionally, negative employment taxes are likely to be optimal if the welfare benefit and the government’s required revenue is low while search costs are high. Second, Hungerbühler et al. (2006), Lehmann et al. (2011), and Jacquet et al. (2014) consider a search-matching framework in which the wage is determined by a bargaining process. Jacquet et al. (2014) consider the case Kalai bargaining wage setting and show that this matching environment induces significantly lower employment tax rates than the usual competitive model with endogenous participation only.

Although these studies have provided influential findings with both theoretical and policy implications, a large component of the literature does not focus on the role of wage subsidies (or in-work credits) and unemployment benefits. In the real world, governments employ wage subsidies for low-income workers as well as unemployment benefits (social assistance benefits) for both voluntary and involuntary unemployed households. Governments often restrict the qualifications for receiving benefits to those who engage in a job search. With involuntary unemployment, the search-contingent unemployment benefit could be a more efficient policy tool to encourage labor market participation than wage subsidies or in-work credits. A recent paper by Boadway and Cuff (2018) only investigates the joint design of a piecewise linear income tax/transfer and unemployment benefits with a search-matching framework. Their model adopts the extensive margin under which households choose whether to participate in job search and the intensity of that search. The authors analytically find that the optimal employment tax depends on the relative size of the transfer for the involuntarily unemployed and the voluntarily unemployed. Instead, we adopt a simple search cost model in which households are heterogeneous about labor productivities, labor disutilities, and search costs, and the search-contingent unemployment benefit is endogenous. This model is a simple extension of the original extensive
margin model used in Diamond (1980) and Saez (2002) (hereafter referred to as the full employment model) in which households are heterogeneous about labor productivities and disutilities. We use this model to consider involuntary unemployment and to compare both a full employment model and an extensive model with search and involuntary unemployment.

Thus, we examine the relation and the role of wage subsidies and unemployment benefit in the extensive model with search and involuntary unemployment both analytically and numerically. We derive the following results. First, in the case of involuntary unemployment, we identify the optimal employment tax rule, which depends on the optimal search-contingent unemployment benefit. In contrast to the full employment model, negative employment tax rates for households with lower consumption are less likely to occur. Second, the optimal search-contingent unemployment benefit depends on the effect of the benefit on the population of working households, but the total effect is ambiguous. Finally, we numerically solve the optimal tax policies for example economies. We confirm that employment tax rates and search-contingent unemployment benefit have a positive correlation. Negative employment tax rates (in-work credits or wage subsidies) could be valid only when the employment probability is close to unity or the search cost is close to zero. This suggests that the government need not to employ both search-contingent unemployment benefits and in-work credits at the same time.

The remainder of the paper is organized as follows. In Section 2, we present a model with an extensive margin of labor and involuntary unemployment. In Section 3, we investigate the optimal income tax problem with involuntary unemployment and derive optimal employment tax rates in the case of partial optimal taxation and full optimal taxation. In the former case, the government only employs labor income tax/transfers. In the latter case, the government employs labor income tax/transfers as well as unemployment benefit. In Section 4, we consider an example economy with various parameters and numerically determine optimal income tax policies and optimal search-contingent unemployment benefits. In Section 5, we present our conclusions.

2 The model

The economy is populated by a government and one unit of continuous households with an indivisible labor supply. Each household is heterogeneous about the two-dimensional characteristic \( \theta = (w, \delta) \) where \( w \in \mathbb{R}_+ \) and \( \delta \in \mathbb{R} \), respectively, represent labor productivity and labor disutility. There are \( I \in \mathbb{N} \) possible levels of labor productivity. The domain of labor productivity is denoted by \( \Omega = \{w_1, w_2, \ldots, w_I\} \in \mathbb{R}^{I+}_+ \) where \((w_1 < w_2 < \cdots < w_I)\). Households’ characteristics are private information and the government cannot observe \( \theta \). Let \( F : \Omega \times \mathbb{R} \to \mathbb{R}_+ \) denote the cumulative probability distribution function (c.d.f) of the population. The c.d.f. is public information.

There are \( I \) labor markets for each labor productivity. A household with productivity \( w_i \) cannot mimic another productivity \( w_j \) (\( i \neq j \)). A household chooses labor participation
status \( s \in \{0, 1\} \) given its \( \theta \). If a household with productivity \( w_i \) decides to participate in the labor market \( (s = 1) \), then household suffers the utility cost of job search, \( \Psi_i \), and has the probability of job matches, \( p_i \in [0, 1] \). If the household receives a job offer, then the household works fixed hours and earns \( w_i \) units of (before-tax) labor income and suffers the utility cost of labor, \( \delta \). If the household does not find a job, then the household does not get \( w_i \) and does not pay \( \delta \). If the household does not participate \( (s = 0) \), their labor income is zero, and the household suffers no search cost and disutility of labor. The labor productivity is known to the public if and only if the household participates. The government observe household search activity, employment status and the productivity if employed. Note that nonworking households in this economy are composed of both the involuntary unemployed and the voluntary unemployed.

There is a divisible consumption good. Let \( U(c) \) denote the household’s utility from consumption, \( c \). \( U \) is a twice continuously differentiable utility function satisfying \( U' > 0 \) and \( U'' < 0 \). All households have no real assets at the beginning of the economy.

The government’s policy is a tax on labor income and unemployment benefits conditional on the job search. Let \( T(w_i) \) and \( -T(0) \) denote the labor income tax of working households with productivity \( w_i \) and the uniform subsistence income for a nonworking household, respectively. \( T : \{0\} \cup \Omega \rightarrow \mathbb{R} \) represents a labor income tax schedule. In addition, the government provides unemployment benefits conditional on a job search, \( b \). This unemployment benefit is paid only if a household searched for a job in the labor market and was not employed.\(^1\) We assume that the government cannot monitor households’ job offer, that is, the government cannot distinguish the case household searched and did not get an offer and the case household searched, received a job offer, but rejected the offer. If \( U(-T(0) + b) - \Psi_i > U(-T(0)) \), some low-productivity workers pretend to be involuntary unemployed. Thus, we assume the following incentive compatible conditions hold for all \( i \):

\[
U(-T(0) + b) - \Psi_i \leq U(-T(0)).
\]

Given \( T(0) \), (1) determines an upper bound of \( b \). The condition (1) also implies no rejection of a job offer.

Given labor income tax schedule \( T \), search-contingent unemployment benefit \( b \), and individual characteristic \( \theta \), the household chooses labor participation status \( s \in \{0, 1\} \) to maximize the expected utility,

\[
s[p_i[U(w_i - T(w_i)) - \Psi_i - \delta] + [1 - p_i][U(-T(0) + b) - \Psi_i]] + [1 - s]U(-T(0)).
\]

Let \( V_i \) denote the difference of expected utility between a household that participates in labor market \( i \) except labor disutility, \( \delta \), and a household that does not participate divided

\(^1\)One can obtain same result by the policy that a search benefit for everyone who searched for a job and income tax raised by the amount of the benefit.
by \( p_i \) as follows:

\[
V_i(p_i, T, b) = U(w_i - T(w_i)) + \frac{[1 - p_i]}{p_i} U(-T(0) + b) - \frac{\Psi_i + U(-T(0))}{p_i}.
\]

Suppose a household has a \( \delta \) smaller than \( V_i \). The household’s expected utility is greater at \( s = 1 \). Thus, the optimal labor participation plan \( s \) is \( 1 \) (resp. \( 0 \)) if \( \delta \) is smaller (resp. larger) than \( V_i \).

The population of households who participate in labor market \( i \), denoted by \( k_i \), is,

\[
k_i = \int_{\theta \in \Theta_i} dF(\theta), \quad i = 1, 2, \ldots, I,
\]

where \( \Theta_i \) denotes the region of \( \theta \) defined by

\[
\Theta_i \equiv \{(w, \delta) | w = w_i \text{ and } \delta < V_i\}.
\]

Thus, the population of households with earning (before-tax) labor income \( w_i \), denoted by \( n_i \), and the population of nonworking households, denoted by \( n_0 \), are,

\[
n_i = p_i \times k_i, \quad i = 1, 2, \ldots, I,
\]

\[
n_0 = 1 - \sum_{i=1}^{I} n_i.
\]

The following symmetric relations hold:

\[
\frac{\partial n_i}{\partial T(w_i)} = -\frac{\partial n_0}{\partial T(w_i)} \leq 0, \quad i = 1, 2, \ldots, I,
\]

\[
\frac{\partial n_0}{\partial T(0)} = -\sum_{i=1}^{I} \frac{\partial n_i}{\partial T(0)} \geq 0.
\]

Let \( E U_{ij} \) and \( E U_0 \) respectively denote the average of the utility of the households with productivity \( w_i \) who decided to participate in the labor market and get their employment status \( j \), \( (j = e \text{ (employed) or } u \text{ (unemployed)}) \) and the utility of the households who does not participate as follows:

\[
E U_{ij} = \begin{cases} 
U(w_i - T(w_i)) - E_i(\delta) - \Psi_i & \text{ (if } j = e) \\
U(-T(0) + b) - \Psi_i & \text{ (if } j = u),
\end{cases}
\]

\[
E U_0 = U(-T(0)),
\]

where \( E_i(\delta) = E(\delta|w = w_i \text{ and } \delta \leq V_i) \). Correspondingly, let \( f_{ij} \) and \( f_0 \) respectively
denote the population of the households who obtain $EU_{ij}$ and $EU_0$ as follows:

$$f_{ij} = \begin{cases} k_ip_i & \text{(if } j = e) \\ k_i[1 - p_i] & \text{(if } j = u) \end{cases},$$

$$f_0 = \sum_{i=1}^I [q_i - k_i],$$

where $q_i$ denotes that the household population obtains productivity $w_i$.

The government’s objective function is the general social welfare function that aggregates household’s (average) utilities, $SW = \sum_j \sum_i W(EU_{ij})f_{ij} + W(EU_0)f_0$, where $W$ is increasing and concave function ($W' > 0$ and $W'' \leq 0$). The government must finance an exogenous expenditure $G$. The government’s budget constraint is given by,

$$\sum_{i=1}^I k_i [p_iT(w_i) - [1 - p_i]b] + n_0T(0) \geq G. \quad (4)$$

The optimal taxation problem for the government is to find the labor income tax schedule $T$ and the job search-contingent unemployment benefit $b$ that maximizes the social welfare function, $SW$, subject to the budget constraint (4) and IC condition (1).

We consider two types of optimal taxation problems: the full optimal taxation problem and the partial optimal taxation problem. The former corresponds to the pair of the labor income tax schedule $T$ and the job search-contingent unemployment benefit $b$ that maximize $SW$ subject to (4) and IC condition (1). However, the decision process of the unemployment benefit could be independent of the decision of income tax by the tax authority sometime. Thus, we also investigate the latter problem, which solves the same maximization problem under a given level of $b$. In the latter problem, we set $b = 0$.

3 Employment tax rate

The Lagrangian of the government problem in the case of full optimal taxation is

$$\mathcal{L} = SW + \lambda \left[ \sum_{i=1}^I k_i [p_iT(w_i) - [1 - p_i]b] + n_0T(0) - G \right],$$

where $\lambda$ is the Lagrange multiplier associated with the government budget constraint. Assuming the interior solution, the first-order conditions of the Lagrangian with respect

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2In the above Lagrangian, we omitted the Kuhn-Tucker constraint about IC conditions (1). Alternatively, we solved the problem without IC conditions and checked the solution satisfying the IC conditions.
to $T(w_i)$, $T(0)$ and $b$ are
\[ \frac{\partial L}{\partial T(w_i)} = \frac{\partial SW}{\partial T(w_i)} + \lambda \left[ \frac{\partial k_i}{\partial T(w_i)} [p_i T(w_i) - (1 - p_i) b] + p_i k_i + \frac{\partial n_0}{\partial T(w_i)} T(0) \right] = 0, \text{ for } i \neq 0, \]
\[ \frac{\partial L}{\partial T(0)} = \frac{\partial SW}{\partial T(0)} + \lambda \left[ \sum_{i=1}^I \frac{\partial k_i}{\partial T(0)} [p_i T(w_i) - (1 - p_i) b] + n_0 + \frac{\partial n_0}{\partial T(0)} T(0) \right] = 0, \]
\[ \frac{\partial L}{\partial b} = \frac{\partial SW}{\partial b} + \lambda \left[ \sum_{i=1}^I \left( \frac{\partial k_i}{\partial b} [p_i T(w_i) - (1 - p_i) b] - k_i [1 - p_i] \right) \right] = 0. \]

With involuntary unemployment, households with productivity $w_i$ that participate in the labor market and find a job forgo subsistence income $-T(0)$ and pay income tax $T(w_i)$. Thus, $T(w_i) - T(0)$ is the employment tax for households with productivity $w_i$. We also define the employment tax rates faced by households with productivity $w_i$ by
\[ \tau_i = \frac{T(w_i) - T(0)}{w_i}, \text{ for } i \neq 0. \]

Before deriving the optimal employment tax rate rules, let $g_i$ and $\eta_i$ respectively denote the average marginal social weight of consumption for working households with productivity $w_i$ expressed in terms of public funds and the participation elasticity as follows:
\[ g_i = -\frac{1}{\lambda p_i k_i} \frac{\partial SW}{\partial T(w_i)}, \text{ for } i = 1, 2, \ldots, I, \]
\[ \eta_i = -\frac{\partial k_i}{k_i} \left( w_i - T(w_i) + T(0) \right) \frac{\partial T(w_i)}{\partial T(w_i)}, \text{ for } i = 1, 2, \ldots, I. \]

### 3.1 Partial optimal taxation

We first consider the case of partial optimal taxation. Then, we examine the full optimal taxation problem.

For the partial optimal taxation problem, the government does not employ the unemployment benefit ($b = 0$). Thus, we only consider the two first-order conditions of the Lagrangian with respect to $T(w_i)$ and $T(0)$ with $b = 0$. Using (5), we rewrite the first-order conditions and obtain the optimal employment tax rule for partial optimal taxation as follows (see Appendix A.1):
\[ \frac{\tau_i^P}{1 - \tau_i^P} = \frac{1 - g_i^P}{\eta_i^P}, \text{ for } i = 1, 2, \ldots, I, \]

where superscript $P$ shows the case of partial optimal taxation. Equation (6) is similar to the standard optimal participation tax rule in the extensive margin with full employment (see Diamond (1980), Saez (2002), Choné and Laroque (2005, 2011), Jacquet et al. (2013), and Christiansen (2015)). We consider the full employment case as a special case of partial optimal taxation. In the full employment case, there is no involuntary unemployment and no utility costs for a job search, so $p_i = 1$ and $\Psi_i = 0$ for all $i$. Thus, the government need
not employ the unemployment benefit \((b = 0)\).

Although the arguments of the social welfare are different between the partial optimal taxation and full employment case, the introduction of involuntary unemployment does not affect the optimal tax rule as long as \(b = 0\). We can apply the same interpretation with both rules. If the right-hand side of (6) is negative for some working group with lower consumption, then the employment (or participation) tax rate for such an income group must be negative.

### 3.2 Full optimal taxation

In the full optimal taxation case, there is involuntary unemployment, and the government employs labor income tax schedule \(T\) as well as the job search-contingent unemployment benefit \(b\).

Using (5), we rewrite the first-order conditions and the optimal employment tax rate rule in the case of full optimal taxation as follows (See Appendix A.1):

\[
\frac{\tau_i^F}{1 - \tau_i^F} = \frac{1}{\eta_i^F} \left[ 1 - g_i^F - \frac{1}{p_i k_i} \frac{\partial k_i}{\partial T(w_i)} [1 - p_i] b \right], \quad \text{for } i = 1, 2, \ldots, I, \tag{7}
\]

where superscript \(F\) shows the case of full optimal taxation. Equation (7) is the optimal employment tax rule with involuntary unemployment and search-contingent unemployment benefit. Equation (7) shows that the optimal employment tax rate depends on the search-contingent unemployment benefit (the third term of the right-hand side of (7)). Since \(\frac{\partial k_i}{\partial T(w_i)} \leq 0\), if \(b \geq 0\), this term would be non-negative in total. Thus, even when \(g_i^F > 1\) and \(1 - g_i^F < 0\), the third term cancels it out, and the negative employment tax rate is less likely to occur.

The search-contingent unemployment benefit is endogenous in full optimal taxation. We examine the effect of the search-contingent unemployment benefit on optimal employment tax rates. To do so, we rewrite the first-order condition with respect to \(b\) (see Appendix A.2), and we obtain the optimal search-contingent unemployment benefit as follows:

\[
b^* = g_b - 1 + \frac{\sum_{i=1}^I \frac{\partial k_i}{\partial b} p_i [T(w_i) - T(0)]}{\sum_{i=1}^I k_i [1 - p_i]} h, \tag{8}
\]

where \(g_b\) is the average marginal social weight of unemployment benefits for the involuntary unemployed expressed in terms of public funds, and \(h\) is the rate of the increase in the involuntary unemployed due to a marginal increase in unemployment benefit as follows:

\[
g_b = \frac{1}{\lambda \sum_{i=1}^I k_i [1 - p_i]} \frac{\partial SW}{\partial b}, \quad h = \frac{\sum_{i=1}^I \frac{\partial k_i}{\partial b} [1 - p_i]}{\sum_{i=1}^I k_i [1 - p_i]}. \]

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The denominator on the right-hand side of (8) shows that when the unemployment benefit results in a higher number of involuntary unemployed, the benefit should be decreased. The third term of the numerator on the right-hand side of (8) is positive and shows the marginal increase in tax revenue from an increase in the number of working households due to the unemployment benefit divided by the number of involuntary unemployed. $b^*$ depends on the partial derivative of $k_i$ with respect to $b$, $\frac{\partial k_i}{\partial b}$. If it increases, both of the third term in the numerator and the denominator on the right-hand side of (8) increase so that the total effect is ambiguous. Numerical simulations are required to investigate the effect of the search-contingent unemployment benefit on the optimal employment tax rates.

4 Numerical simulations

The previous section shows that employment tax rates for some working households with low consumption are less likely to be negative when there is involuntary unemployment and search-contingent unemployment benefits. This section numerically solves for optimal tax policies using an example economy. We suggest that there is a positive correlation between employment tax rates for the least productive workers and search-contingent unemployment benefits, which implies that the government need not to employ both search-contingent unemployment benefits and in-work credits at the same time.

4.1 Parameter setting

(1) Utility function

We assume the utility function $U$ is

$$U(c) = c^\alpha,$$

where $\alpha \in \mathbb{R}_+$ determines the curvature of $U$. We assume that the range of $\alpha$ is $\alpha \in [0.75, 0.99]$.

(2) Labor productivity

There are three possible labor productivity levels, ($I = 3$ and $w_1 < w_2 < w_3$). We assume the domain of labor productivity $(w_1, w_2, w_3) = (1, 2, 4)$. This means that the labor productivity distribution is skewed.

Let $q = (q_1, q_2, q_3)$ denote the vector of the population where $q_i$ denotes that the household population obtains productivity $w_i$. We assume that the range $q_1$ and $q_3$ is $q_1 \in [0.2, 0.45]$ and $q_3 \in [0.2, 0.45]$. Since the total population is unity, $\sum q_i = 1$, the range $q_2$ is $q_2 \in [0.1, 0.6]$. 

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(3) Participation ratio at the laissez-faire equilibrium

Let \( \rho = (\rho_1, \rho_2, \rho_3) \) denote the vector of the participation ratio where \( \rho_i \) is the participation ratio of households who have labor productivity \( w_i \) when there is no income tax and no unemployment benefits. We assume \( \rho_i \in [0.8, 0.99] \) for all the productivity groups.

(4) Employment probability

Let \( p = (p_1, p_2, p_3) \) denote the vector of the employment probability where \( p_i \) is the employment probability in labor market \( i \). We assume \( p_i \in [0.7, 1] \) for all the productivity groups.

(5) Search cost

Let \( \Psi = (\Psi_1, \Psi_2, \Psi_3) \) denote the vector of the utility cost of job search. We assume the utility cost of job search in labor market \( i \) is specified by \( \Psi_i = [1 - p_i] \gamma w_i^\alpha \) where \( \gamma \in [0, 1] \) is a common parameter, and \( w_i^\alpha \) is the utility from earnings when there is no income tax and no unemployment benefits in market \( i \). Our specification assumes that the magnitude of the search costs in market \( i \) are some ratio of the utility from earnings in the laissez-faire equilibrium. The ratio is the same for all markets. This means that highly productive jobs tend to be more difficult to find. The search cost is higher when the employment probability in the market is smaller.

(6) Labor disutility

The labor disutility \( \delta_i \) is uniformly distributed over the interval \( [\delta_i, \delta_i] \subset \mathbb{R} \). Thus, the density of the labor disutility \( \delta_i \) is \( \frac{q_i}{\delta_i - \delta_i} \). Let \( \delta_i^* \) denote the labor disutility, which is equal to the difference of expected utility between participation in the labor market and nonparticipation divided by \( p_i \) such that \( \delta = V_i \) when there is no income tax and no unemployment benefits divided by \( p_i \) such that \( \delta = V_i \) when there is no income tax and no unemployment benefits. With uniform distributions, the population of households that participate in labor market \( i \), \( k_i \) becomes \( k_i = \frac{q_i \delta_i^*}{\delta_i - \delta_i} \), and the participation ratio, \( \rho_i \), is \( \rho_i = \frac{k_i}{q_i} = \frac{\delta_i^*}{\delta_i - \delta_i} \).

We assume the labor participation elasticity with respect to labor productivity when there is no income tax and no unemployment benefits, \( \frac{\partial k_i / k_i}{\partial w_i / w_i} \), is common over \( i \). Let \( e \) denote the common elasticity. We obtain the following equation:

\[
\delta_i = \delta_i^* - \frac{\alpha [w_i]^\alpha}{e} \left( \frac{1}{\rho_i} - 1 \right)
\]

We rewrite \( e \) as follows:

\[
e = \frac{\partial k_i / k_i}{\partial V_i / \partial w_i} \frac{w_i}{k_i} = \frac{q_i \alpha [w_i]^\alpha - 1}{\delta_i - \delta_i} \frac{w_i}{k_i} = \frac{\alpha [w_i]^\alpha}{\rho_i (\delta_i - \delta_i)}.
\]

Thus, using the participation ratio, \( \delta_i^* - \delta_i = \frac{\alpha [w_i]^\alpha}{e} \) and \( \delta_i - \delta_i^* = [1 - \rho_i] \delta_i - \delta_i = \frac{\alpha [w_i]^\alpha}{e} \left( \frac{1}{\rho_i} - 1 \right) \).
where $\delta^*_i = w_i^\alpha - \frac{\Psi_i}{p_i}$. Given $\alpha$, $e$ and $w_i$, $\rho_i$, $p_i$, and $\Psi_i$ determine the domains of the labor disutility for households with productivity $w_i$, $\delta_i$, and $\delta^*_i$, respectively. We consider $e \in [0.01, 1]$.

(7) **Social welfare function**

We specify the social welfare function $SW$ as the constant relative inequality aversion type as follows:

$$SW = \begin{cases} 
\sum_{i,j} \ln(EU_{ij}) f_{ij} + \ln(EU_0) f_0 & \text{(if } \beta = 1) \\
\sum_{i,j} \frac{EU_{ij}^{1-\beta}}{1-\beta} f_{ij} + \frac{EU_0^{1-\beta}}{1-\beta} f_0 & \text{(otherwise)}
\end{cases}$$

where $\beta$ is a constant elasticity of relative inequality aversion. The utilitarian case is $\beta = 0$ and the Rawlsian case is $\beta \to \infty$.

4.2 **Benchmark case**

Table 1 shows optimal income taxation for full employment, optimal partial taxation, and the optimal full taxation in the benchmark case where all parameter settings are listed at the top of the table, which are set at the almost center of the specified range and the government is utilitarian.

We first focus on full employment (see Table 1.A.). In this case, the population of households who participate in labor market $i$ corresponds to the population of households who work in labor market $i$, (i.e., $p_i = 1$). The first part, “No tax,” shows the allocation when no labor income tax is imposed; that is, disposable income $y_i$ is equal to labor productivity $w_i$. The resulting GDP (total production) is 2.07. The last part, “Optimal income taxation,” is the optimal income taxation for full employment. The redistribution from more productive workers to less productive workers improves the welfare ($EU$ increases from 1.609 to 1.622) but has a negative impact on GDP (GDP decreases from 2.07 to 1.906). As shown in the previous section, the participation tax rate for some working group with lower consumption could be negative. This example shows that the participation tax rate for the least productive workers is negative ($\tau_1 = -0.047$), and in-work credits are desirable.

Next, we focus on involuntary unemployment (see Table 1.B.). Even with no tax, the population of households that participate in labor market $i$ is different from the population of households that work in labor market $i$ due to involuntary unemployment, (i.e., $p_i < 1$). The second part, “Partial optimal taxation,” shows the case where the government does not employ the unemployment benefit ($b = 0$). In this case, the employment tax rate for the least productive worker is positive (i.e., $\tau^F_1 = 0.021$). The third part, “Full optimal taxation,” shows the case where the government employs labor income tax as well as the search-contingent unemployment benefit. In comparison to partial optimal taxation, the employment tax rate for the least productive workers is higher (i.e., $\tau^F_1 = 0.043$), and
Table 1: Optimal taxation for the benchmark case ($\beta = 0$)

### A. Full employment case

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<thead>
<tr>
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<td>3</td>
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### B. Involuntary unemployment case

<table>
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<th>$\alpha = 0.875$, $q = (0.3, 0.4, 0.3)$, $p = (0.9, 0.9, 0.9)$, $\gamma = 0.5$, $e = 0.5$ $w = (1, 2, 4)$, $\Psi = (0.075, 0.138, 0.252)$, $G = 0$</th>
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the search-contingent unemployment benefit is useful ($b = 0.253$). Full optimal taxation improves GDP (GDP increases from 1.571 to 1.59) as well as welfare ($EU$ increases from 1.391 to 1.397). The search-contingent unemployment benefit reduces the uniform subsistence income for a nonworking household ($-T(0)$ decreases from 0.249 to 0.229) so that a fine-tuned income tax/transfer is available.\(^4\)

Finally, we focus on the case where the government has the redistributive taste ($\beta = 0.5$). With full employment (see Table 2.A.), the participation tax rate for the least productive worker is negative ($\tau_1 = -0.250$), so that in-work credits are desirable. Table 2.B. shows the involuntary unemployment case. From the second part, “Partial optimal taxation,” the employment tax rate for the least productive worker is positive (i.e., $\tau_1^p = 0.129$). From the third part, “Full optimal taxation,” the employment tax rate for the least productive worker is positive (i.e., $\tau_1^F = 0.070$). In comparison to the utilitarian case (Table 1), the employment tax rate for the least productive workers in the case of full optimal taxation is smaller but still positive, and the search-contingent unemployment

\(^4\)In this example, the difference in welfare between full optimal taxation and partial optimal taxation, expressed in terms of public funds, amounts to 0.443% of GDP.
Table 2: Optimal taxation for the benchmark case with the redistributive taste ($\beta = 0.5$)

**A. Full employment case**

Parameter setting

$\alpha = 0.875$, $q = (0.3, 0.4, 0.3)$, $\rho = (0.9, 0.9, 0.9)$, $p = (1, 1, 1)$, $e = 0.5$

$w = (1, 2, 4)$, $\Psi = (0, 0, 0)$, $G = 0$

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Optimal income taxation

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**B. Involuntary unemployment case**

Parameter setting

$\alpha = 0.875$, $q = (0.3, 0.4, 0.3)$, $\rho = (0.9, 0.9, 0.9)$, $p = (0.85, 0.85, 0.85)$, $\gamma = 0.5$, $e = 0.5$

$w = (1, 2, 4)$, $\Psi = (0.075, 0.138, 0.252)$, $G = 0$

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Partial optimal taxation

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Full optimal taxation

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<th>GDP</th>
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benefit is higher ($b = 0.269$). With concave social welfare function, the government desire to redistribute to nonworking households and least productive workers more so that the uniform subsistence income for a nonworking household become higher. This policy results in the higher number of nonparticipant. Thus, the search-contingent unemployment benefit has a role of increasing the incentive to participate and search in the labor market.

4.3 Sensitivity analysis: Monte Carlo simulations

To investigate the robustness of this property, we conducted a Monte Carlo simulation from 10,000 samples where all the parameters (i.e., $\alpha$, $q$, $\rho$, $p$, $\gamma$, and $e$) are uniformly distributed over the specified range and the government is utilitarian. Figure 1 shows the average optimal participation tax rate for each productivity level in the full employment case, the average optimal employment tax rate for each productivity level for partial optimal taxation and full optimal taxation, and the average optimal unemployment benefit. Average optimal employment (participation) tax rates and optimal unemployment benefits in Figure 1 have almost the same patterns as the benchmark case. Additionally, we
(Notes) This figure shows the average optimal participation tax rate (PTR) for each productivity level (red line), the average optimal employment tax rate (ETR) for each productivity level (green and blue lines), and the average optimal unemployment benefit (UB) for all productivity levels (see black diamond point). The black lines show the range for the first and third quartiles of the optimal PTR, ETR for each productivity level, and UB for all productivity levels. The cross points show the medians.

calculate the first and third quartiles of these cases to determine the statistical dispersion of the result. The results show that, in many cases, the pattern of the optimal employment (participation) tax rates are similar to the benchmark case. These statistics (i.e. average, median and the first and third quartiles) for optimal participation tax rates for the least productive working households are negative while those of optimal employment tax rates for the least productive working households are positive, and the statistics are higher for full optimal taxation. We also examine the distribution of optimal employment tax rates for the least productive working households for both partial optimal taxation and full optimal taxation, $\tau_1^P$ and $\tau_1^F$ (see Figure 2). The mean, median and mode of optimal employment tax rates for the least productive working households for full optimal taxation are higher than the values for partial optimal taxation. The distribution of employment tax rates for full optimal taxation is more dispersed and has a thicker right tail than that of partial optimal taxation.\footnote{We count the sign pattern about $\tau_1^P$ and $\tau_1^F$. In our Monte Carlo simulation, 77.6\% of the samples show that $\tau_1^F$ is greater than $\tau_1^P$ with the same parameter settings.}

Next, we examine the relationships between the optimal employment tax rate of the least productive working households and the optimal unemployment benefit with various
(Notes) This figure shows the histogram of the optimal employment tax rate for the lowest productivity level.

employment probabilities and search costs. To do so, we derive several fitted curves from the samples. We first focus on the employment probabilities (See Figure 3.A.). When the employment probability is equal to unity, all households that participate in the labor market find a job, and there is no involuntary unemployment. From equation (7) above, the higher the \( p_1 \), the closer to zero is the third term on the right-hand side of (7), and the optimal employment tax rate rule becomes similar in form to the optimal participation tax rule. Figure 3.A. indicates that this intuition is valid. If the employment probability \( p_1 \) is higher, then the optimal employment tax rate for full optimal taxation is close to zero or negative. On the other hand, parameter \( p_1 \) does not seem to affect the level of unemployment benefit. Second, we examine the effect of search costs (see Figure 3.B.). If the utility cost of job search \( \Psi_1 \) is smaller, then the unemployment benefit is rapidly decreasing and close to zero. Following this move in unemployment benefits, the optimal employment tax rate for full optimal taxation is also decreasing. This implies a positive correlation between unemployment benefits and the optimal employment tax rate for full optimal taxation.

Finally, we examine the relationships between the optimal employment tax rates of various types of working households, uniform subsistence income for nonworking households, and the optimal unemployment benefit (see Figure 4). Note again that these policy instruments are all endogenous. Figure 4 shows that the optimal employment tax rates for various type of working households and the optimal search-contingent unemployment benefit have positive correlations, and the optimal uniform subsistence income for nonworking households and the optimal search-contingent unemployment benefit have negative correlations. Particularly, the slope of the fitted curve of the employment tax rate is steepest.
for the least productive working households and the correlation coefficient of $\tau_1^F$ and $b$ is about 0.471, so that when the optimal unemployment benefit is small and close to zero, the optimal employment tax rates for the least productive working households should also be small. Thus, the government need not to employ both the search-contingent unemployment benefit and in-work credits (negative employment tax rates) for the least productive working households at the same time. In our Monte Carlo simulation, only 11.98% of the samples show that $\tau_1^F < 0$ and $b > 0$. This suggests that when there are fewer search frictions in the labor market or the employment probability is high, it is more desirable
to employ in-work credits than the search-contingent unemployment benefit. In contrast, when there are many search frictions and households cannot easily obtain a job offer, the government should employ the search-contingent unemployment benefit rather than in-work credits as a redistribution policy. This is because there is a need to raise incentives among households to participate and search in the labor market.

5 Conclusion

In this study, we investigated optimal income taxation and optimal unemployment benefits in a model with an extensive margin of labor and involuntary unemployment. In the case of involuntary unemployment, the government can employ labor income tax/transfers as well as a search-contingent unemployment benefit. We derive optimal employment tax rules, which depend on the average marginal social weight, the participation elasticity, the tax effect on the participant population in labor markets, and the optimal search-contingent unemployment benefit. Due to the existence of the search-contingent unemployment benefit, a negative employment tax rate for some working households with low consumption is less likely to occur.

For our example economy, we simulate an optimal participation tax rate in the case of full employment and an optimal employment tax rate in the case of involuntary unemployment with both partial optimal taxation and full optimal taxation. We calculate the optimal participation and employment tax rate under various parameter specifications and find that the participation tax rates for low consumption households are likely to be negative in the case of full employment. However, this is not the case for involuntary unemployment and unemployment benefit. We confirm that employment tax rates and the search-contingent unemployment benefit have a positive correlation. Only when the employment probability is close to unity or the search cost is close to zero negative employment tax rates could be valid. When the search cost is high for households, the role of the search-contingent unemployment benefit is significant. The unemployment benefit that targets households who want to work and participate in the labor market have a greater benefit than in-work credits or wage subsidies when there are serious search frictions. These findings suggest that the government need not to employ both search-contingent unemployment benefits and in-work credits at the same time.

Although we show that the search-contingent unemployment benefit improves welfare with involuntary unemployment, the underlying assumption is that the government can completely verify whether households search for a job in labor markets. Previous studies on unemployment benefit shed light on the problem of monitoring beneficiaries (see, e.g., Boadway and Cuff (1999), Boadway et al. (2003), and Boone and Bovenberg (2013)). Thus, it is important that future studies clarify the role of in-work credits and unemployment benefits in the case of incomplete monitoring of job searches.
References


A Appendix

A.1 Derivation of equations (6) and (7)

The first-order conditions of the Lagrangian with respect to $T(w_i)$ in the full optimal taxation are

$$\frac{\partial SW}{\partial T(w_i)} + \lambda \left[ \frac{\partial k_i}{\partial T(w_i)} [p_i T(w_i) - (1 - p_i)b] + p_i k_i T(w_i) \right] = 0.$$  

We rewrite the first-order conditions with respect to $T(w_i)$ using $\frac{\partial n_0}{\partial T(w_i)} = -p_i \frac{\partial k_i}{T(w_i)}$ as follows:

$$\frac{\partial SW}{\partial T(w_i)} + \lambda \left[ \frac{\partial k_i}{\partial T(w_i)} [p_i T(w_i) - (1 - p_i)b] + p_i k_i - p_i \frac{\partial k_i}{T(w_i)} T(0) \right] = 0.$$  

Dividing both sides by $\lambda p_i k_i$ and using $g_i = -\frac{1}{\lambda p_i k_i} \frac{\partial SW}{\partial T(w_i)}$ and $\eta_i = -\frac{\partial k_i}{k_i} \frac{w_i - T(w_i) + T(0)}{T(w_i)}$, we rearrange the above equation as follows:

$$1 - g_i - \frac{1}{p_i k_i} \frac{\partial k_i}{\partial T(w_i)} [1 - p_i]b = -\eta_i \frac{T(w_i) - T(0)}{w_i - T(w_i) + T(0)}.$$  

Finally, using $\tau_i = \frac{T(w_i) - T(0)}{w_i}$, we obtain the following equation,

$$\frac{\tau_i}{1 - \tau_i} = \frac{1}{\eta_i} \left[ 1 - g_i - \frac{1}{p_i k_i} \frac{\partial k_i}{\partial T(w_i)} [1 - p_i]b \right].$$  

This equation corresponds to equation (7). With $b = 0$, we can obtain equation (6).

A.2 Derivation of equation (8)

The first-order condition of the Lagrangian with respect to $b$ for full optimal taxation is

$$\frac{\partial SW}{\partial b} + \lambda \left[ \sum_{i=1}^{I} \left[ \frac{\partial k_i}{\partial b} [p_i T(w_i) - (1 - p_i)b] - k_i [1 - p_i] \right] + \frac{\partial n_0}{\partial b} T(0) \right] = 0.$$  

Noting that $\frac{\partial n_0}{\partial b} = -\sum_{i=1}^{I} p_i \frac{\partial k_i}{\partial b}$, we rewrite the first-order condition as follows:

$$\frac{\partial SW}{\partial b} + \lambda \left[ \sum_{i=1}^{I} \frac{\partial k_i}{\partial b} [p_i T(w_i) - (1 - p_i)b] - \sum_{i=1}^{I} k_i [1 - p_i] - \sum_{i=1}^{I} p_i \frac{\partial k_i}{\partial b} T(0) \right] = 0.$$  


Dividing both sides by $\lambda \sum_{i=1}^{I} k_i [1 - p_i]$, we rearrange the above equation as follows:

$$
\frac{1}{\lambda \sum_{i=1}^{I} k_i [1 - p_i]} \frac{\partial SW}{\partial b} - 1 = -\frac{\sum_{i=1}^{I} \frac{\partial k_i}{\partial b} p_i [T(w_i) - T(0)]}{\sum_{i=1}^{I} k_i [1 - p_i]} + \frac{b \sum_{i=1}^{I} \frac{\partial k_i}{\partial b} [1 - p_i]}{\sum_{i=1}^{I} k_i [1 - p_i]}.
$$

Finally, using $g_b = \frac{1}{\lambda \sum_{i=1}^{I} k_i [1 - p_i]} \frac{\partial SW}{\partial b}$ and $h = \frac{\sum_{i=1}^{I} \frac{\partial k_i}{\partial b} [1 - p_i]}{\sum_{i=1}^{I} k_i [1 - p_i]}$, we obtain the following equation,

$$
b^* = g_b - 1 + \frac{\sum_{i=1}^{I} \frac{\partial k_i}{\partial b} p_i [T(w_i) - T(0)]}{h}.
$$

This equation corresponds to equation (8).