Parental altruism and estate taxation

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Abstract

We derive optimal nonlinear inheritance and linear income tax rates in a two generations model of inheritance where parents differ by their preferences for bequeathing. We provide simulations that show that results depend crucially on the degree of the so called “double counting” of the heirs’ utility in the social welfare function. We allow for income effects and characterize the entire tax schedule. Detailed simulations illustrate the implications of parental preferences interacting with social discounting. Optimal inheritance tax rates are negative and progressive unless double counting is minimal. The reasons are an intergenerational redistributive motive due to double counting and an intragenerational redistributive motive for the generation of heirs. For some parameterizations with limited double counting, an intragenerational redistributive motive for the generation of parents can lead to an optimal regressive inheritance tax.

Keywords Inheritance Taxation · Optimal Nonlinear Taxation

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1 Research Question

In the wake of an increasing interest in questions about economic inequality and social mobility, the taxation of intergenerational transfers has recently received much attention in the economic literature as inequality in inheritances is an important driver of life-time income inequality. Inheritance or bequest taxation is a widely contentious topic, as it relates to personal notions of the societal role of the family on the one hand, and the meritocratic principles of western societies on the other.\(^3\) The design of the inheritance tax depends on the trade-off between parental incentives to provide bequests and equity concerns in the childrens’ generation. This paper contributes to the debate on the properties of an optimal inheritance tax. In particular, it demonstrates how heterogeneity in parental altruism affects the optimal tax schedule. As will be seen, this depends crucially on how the social planner valuates the welfare of heirs.

Most analysis in the context of the optimal taxation of intergenerational transfers departs from the zero tax results provided by Atkinson and Stiglitz (1976), Chamley (1986) and Judd (1985) and the literature currently agrees that taxing or subsidizing transfers to some degree is optimal. In a prominent paper Farhi and Werning (2010) make a strong case for progressive, but negative marginal tax rates. A key characteristic of the paper is that the utility of children enters the maximization problem of the social planner twice, directly by the consideration of the childrens utility in the social welfare function and indirectly through the altruism of parents. This feature has recently found multiple applications and has proven to drive inheritance tax models towards

\(^1\)The national distributions of intergenerational transfers are typically very uneven, as e.g. Piketty and Saez (2013) point out. Empirical evidence does nevertheless not indicate that wealth inequality among children increases through transfers. Adermon et al. (2016) even show that inheritance taxation may in itself increase the inequality in wealth among heirs. See also Wolff and Gittleman (2014) for a concise discussion of the topic.

\(^2\)A prominent argument against the taxation of inheritances is the “double taxation” argument. Its proponents argue that taxing inheritances would mean to tax wealth that already has been taxed once it was accumulated (see for example Boadway et al. (2010)). This argument bases on the notion of a dynasty as economic unit. Compare also the results of the quantitative vignette study by Gross et al. (2017).

\(^3\)This discussion is particularly addressed in the literature on the equality of opportunity. The totality of transfers that parents pass on to their children throughout their lives differs strongly and may in itself give reason for bequest taxation.
negative tax rates in a variety of settings. At the same time, many scholars oppose this “double counting” of utility. In the present paper, we trace the sensitivity of optimal inheritance tax models with respect to this characteristic and we are particularly interested in interactions with another key characteristic of optimal inheritance tax models, namely the varying degree of preferences for bequests in the generation of parents.

Models widely vary by the sources of inequality, i.e. the characteristics over which individuals, particularly the parents and thus the bequest levels, differ. While most papers allow for differences in productivity some introduce further dimensions: Brunner and Pech (2012a) endow individuals with different levels of initial wealth. Piketty and Saez (2013), Boadway and Cuff (2015) and Farhi and Werning (2013) let parents differ by their taste for bequests. Similar to variations in productivity, preferences for inheritances are private information and require the social planner to take the incentive compatibility of the tax scheme into account. Both productivity and preferences can be inherited to some degree. Nevertheless, while being similar in nature, they might entail other reactions to inheritance taxation: In our simplest model specification, parents are equally productive but vary by their taste for bequeathing. The tax system here concerns only the parents’ trade-off between own consumption and their childrens’ consumption and thereby abstracts from the scenario in which more productive parents not only bequeath but also (potentially) consume more than less productive parents. This scenario adds to the discussion in Farhi and Werning (2013) on the question to what extend differences in preferences in the parents generation justify bequest taxation on normative grounds.

In order to assess the interaction of variations in parental preferences for transfers and the public valuation of the decedent’s utility, we resort to a simple two generations model: All parents have an altruistic bequest motive, but vary by how much they value their childrens’ future utility.

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4 Double counting occurs in some sense for instance in the recent contributions by Piketty and Saez (2013), Brunner and Pech (2012a), Brunner and Pech (2012b), Kopczuk (2013) but was already discussed in detail before. See Boadway and Cuff (2015) for an overview.

5 E.g. the contributions by Farhi and Werning (2010), Piketty and Saez (2013), Brunner and Pech (2012a), Saez and Stantcheva (2018), Kopczuk (2013)

6 Differences in productivity are mainly addressed by income taxation, whereas differences in preferences for bequeathing require an inheritance taxation.

7 We apply the altruistic bequest motives to preclude a somewhat odd implication of the also commonly used warm-glow motive: If parents draw utility from their net-bequest (and not their childrens’ utility), they might prefer bequest subsidies over other transfers even if this led to decrease in the lifetime income of their children.
The social planner furthermore discounts the utility of the second generation to some degree. For the sake of clarity, we restrict our main model to parents that are equally productive but relax this assumption later. Our model thus balances the equity of the second generation against the incentive compatibility of the parental generation. We derive first order conditions of the social planners problem. Instead of solving the system of equations, obtaining very complex expressions for the optimal tax system, we present a number of illustrative simulations: We consider the effects of the entire range from “no double counting” to full “double counting” and cover numerous variations in parental altruism. We extend our main model and also take the effect of alternative instruments for redistribution towards children and variations in the productivity of parents on the optimal inheritance tax system into account.

Many papers have for instance focused on rigorous mathematical derivations in complex settings and while they are able to mostly provide intuitive and illustrative analytical solutions for marginal tax rates, they often do not characterize the complete tax system (Farhi and Werning 2010, 2013). Other scholars identified the tax system completely by e.g. providing linear tax rates but often at the cost of assuming linear utility from consumption and thereby eliminating income effects from their models (Piketty and Saez 2013; Saez and Stantcheva 2018; Boadway and Cuff 2015). Simplifying the models facilitates outlining the specific effects at play, but also limits the applicability of these models. Our model relaxes the assumption of no income effects and allows to characterize the entire tax system. This effort comes at the cost of presenting only numerical solutions to our model.

Our approach to assess the interaction of the positive externalities of giving and variations in the altruism of parents is closely related to the contributions by Farhi and Werning (2013) and Boadway and Cuff (2015). In contrast to Farhi and Werning (2013) we characterize the entire tax system and provide a more detailed simulation of results. Other than Boadway and Cuff (2015), we do allow for decreasing marginal utility in consumption. Our results are mostly in line with the literature: The taxation of inheritances is, for most parts, progressive and, in line with Farhi and Werning (2013), our model suggests negative tax rates for inheriting. In contrast to the literature, our model however also shows that under certain, and not uncommon parameterizations of the model, even a regressive taxation of transfers can be optimal. The reason is that there is a trade-off between different redistributive aims: On the one hand, the social planner wants to redistribute to children, who receive a relatively small amount of inheritances. On the other hand she wants to redistribute to children whose parents obtain high utility from their children's consumption. As is
always the case when a social planner trades off consumption for one person against consumption for another, these results hinge on the social planners approach of how to value the individuals’ utilities. We show how equally conceivable valuation strategies of the social planer affect our results. Different valuations of individual utility can lead to regressive or progressive inheritance schedules.

The paper is structured as follows: Section 2 gives a more detailed overview of the recent contributions in this field. Section 3 derives our model. In Section 4 we provide our main results in a detailed simulation study and test the robustness of our model with some extensions. We introduce alternative ways of valuating individual utility in Section 5. Section 6 summarizes and discusses our results, Section 7 concludes.

2 Literature

The literature on the optimal taxation of intergenerational transfers usually departs from the famous zero tax results. Atkinson and Stiglitz (1976) show that given weak separability of leisure and consumption choices, a non-linear labor income taxation is optimal and no further taxation of capital or wealth (transfers) is necessary.\(^8\) The contributions by Chamley (1986) and Judd (1985) come to similar results: Based on an infinite horizon model, the authors show that capital taxes result in huge efficiency losses in the longer term, which make a zero tax rate on capital desirable.

Most of the recent contributions however find that some kind of tax or subsidy is reasonable. A famous example is the paper by Piketty and Saez (2013): This contribution bases on sufficient statistics formulas for optimal inheritance and labor income taxation in a steady state framework. Conceptually, this setting permits concrete applications of the tax formulas to the data of specific countries and thereby draws the literature in a more policy oriented direction. The authors use inheritances to model bi-dimensional inequality in life-cycle resources. The Atkinson-Stiglitz result collapses in this context as inheritances occur not only as a kind of consumption good in the utility of the testators, but in the form of income also as a component in the utility of the recipients. Inequality results from both heterogeneity in labor income and heterogeneity in received transfers. The equity-efficiency trade-off is thus not sufficiently addressed by only levying an income tax.

\(^8\) Taxation of capital and wealth transfers can be shown to be equivalent. Compare Cremer and Pestieau (2009) for further explanations.
tax. The Chamley-Judd result is however nested when the supply side elasticity of capital with respect to the net-of-tax return is infinite. The paper also derives the optimal long term tax rates for \( n \) generations and, in an earlier version (Piketty and Saez 2012), even endogenizes the wealth distribution that will ceteris paribus emerge under the given characteristics. The paper derives the above mentioned result of a progressive and negative tax rate by Farhi and Werning (2010) as a nested solution. The results as parameterized with data for France and the US suggest comparably high linear tax rates when the social planner primarily cares for the utility of the least privileged households.

The sufficient statistics approach has also been deployed by Saez and Stantcheva (2018) who derive, in distinction to Piketty and Saez (2013), non-linear tax rates for a capital and income taxation. The model builds on the model in Saez (2001) and relies on the applicability of the pareto distribution for income and capital. The Atkinson-Stiglitz result does not apply as capital income, in addition to productivity, is a source of inequality, Chamley-Judd again is nested depending on the infinity of elasticities. While these two contributions point to a more policy oriented research in optimal taxation by deriving their formulas from estimable parameters (elasticities and distributional parameters), both models hinge on the absence of income effects: Both models assume linear utility from consumption, which facilitates modeling considerably but limits the applicability of the model severely. Moreover, the desirability of redistribution under linear utility is doubtful.

Generally, employing multiple sources of inequality is not a prerequisite for the derivation of non-zero optimal inheritance tax rates: The presence of externalities in the context of intergenerational transfers warrants the implementation of tax instruments that seek to manipulate individual behavior in order to correct for otherwise inefficient allocations (pigouvian taxes). As mentioned in the introduction, the results in some recent papers were driven by a positive externality that results from giving: Both donor and recipient usually draw utility from the same act of transferring. A social planner who takes the utility of both parents and children from inheriting into account will rather tend to subsidize intergenerational transfers. In their simplest model specification Farhi and Werning (2010) consider a two generations model in which parents work, consume and bequeath

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9The sole effect on income of an intergenerational transfer which occurs just once would not warrant an inheritance tax.

10Interestingly, Saez and Stantcheva (2018) implement both consumption and wealth in the utility function of the individual.
and children only consume the bequest. Parental productivity is the only source of inequality as preferences for bequests are uniform and thus earnings and transfers correlate perfectly. Parents are altruistic, i.e. value the utility from consumption of their children, and the social planner considers both the utility of parents and children. In this setting, the optimal inheritance tax is progressive and negative, driven by the “double counting” of the children’s utility. If the social planner did not take the children’s utility into account, the Atkinson-Stiglitz result would apply. Farhi and Werning (2010) extend their model in some ways, their main result however maintains.

The somewhat counter-intuitive result of subsidizing intergenerational transfers\textsuperscript{11} has provoked opposition: Some scholars argue that this “double counting” of transfers is misleading as transfers are rather a zero-sum redistribution of resources between generations and do not leverage a pareto improvement. In a sense, the externality from giving, opponents say, does not really qualify as an externality that requires correction. A number of prominent papers have however adopted the double counting logic to different settings: Brunner and Pech (2012a) test the effect of double counting in a two generations model in which parents differ by their productivity and their initial wealth endowment. Correlations between productivity and wealth endowment counteract the tendency to subsidize bequests so that total effects are ambiguous. In a related paper, Brunner and Pech (2012b) use a joy-of-giving bequest motive, differences between individuals in productivity and initial wealth and introduce, similar to us, a parameter between 0 and 1 that allows to flexibly vary the impact of “double counting” on the optimal tax scheme. The results resemble those already mentioned. Kopczuk (2013) also resorts to a two generations model by considering the so called \textit{carnegie effect} as an example for a negative externality: The income effect of inheriting limits the work incentives of heirs. While this is not a problem in itself, it reduces the income tax revenues of the state, Kopczuk (2013) infers a \textit{negative fiscal externality}. This adds to the tendency to tax transfers, the sign of the total effect is thus again ambiguous. Piketty and Saez (2013) use a model in which the utility of children also occurs twice. The model bases on a warm-glow bequest motive, solves the long-run optimum and nests the Farhi-Werning result.

The purpose of our paper is closely related to the contributions by Boadway and Cuff (2015) and Farhi and Werning (2013), which both assess double counting in the context of parents with differing bequest motives. The former paper explicitly criticizes the practice of double counting,\textsuperscript{11}Farhi and Werning (2010) actually argue that some very common institutions in western societies subtly reflect the tendency of subsidizing transfers: Particularly free public schooling and the possibility for heirs to reject negative bequests which implicitly creates progressive and marginal negative tax rates.
provides some arguments against and tests its effect in a two generations model with two types of parents: A share of the parents has a warm-glow bequest motive and bequeaths, the remaining share does not. The authors find that when not putting weight on childrens’ utility, taxing bequests is both equity and efficiency enhancing. With weight on children, a trade-off emerges. Our paper is conceptually similar but deviates in some respects: In contrast to Boadway and Cuff (2015), we allow for income effects. We also present numerical simulations and characterize the entire tax system.

Farhi and Werning (2013) also allow for income effects and provides simulations. In their two generations model, parents have altruistic bequest motives but vary on a continuous scale in their degree of altruism. In contrast to Farhi and Werning (2013), we characterize the entire tax system. We also provide more detailed simulations over the full range of double counting childrens’ utility not at all or fully. In addition we show how results vary depending on how utilities are aggregated by the social planner. In sum, we believe that the interaction between double counting and bequest motives has not yet sufficiently been studied. Particularly, as the collective and individual valuation interact.

3 The Model

We present a model with two generations, in which altruistic parents receive uniformly exogenous labor income. We assume two types of parents that differ in $\delta$ which describes their taste for bequeathing to their children or more generally their degree of altruism. While parents decide on how much to consume now and how much to bequeath, children just consume. We explicitly model heterogeneity in the strength of the bequest motive by varying the rate at which parents discount their children’s utility. We abstract from interest payments. Parents’ direct utility function is given by

$$u_{i=l,h} = \alpha_i v(c_{i,t}) + \beta_i \tilde{v}(c_{i,t+1})$$

and the individual budget constraint is

$$I_i - T_{i,L}(I) - b_i = c_{i,t}$$
$$b_i - T_{i,b}(b_i) = c_{i,t+1}.$$
model we set $\alpha = 1$ and $\beta_i = \delta_{i=1,h}$ with $\delta_h \geq \delta_i$. Ordinal utility and thus the choice of parents is only determined by the ratio of $\alpha$ and $\beta$, but the absolute magnitudes of these parameters impact cardinal utility, which matters for the optimization problem of the utilitarian social planner.\(^{12}\) The utility of parents with respect to consumption is determined by $v(c_{i,t})$, where $v$ is increasing and concave in $c_{i,t}$. Children consume $c_{i,t+1}$, from which they derive utility $\tilde{v}(c_{i,t+1})$ which is increasing and concave as well. $I_i$ is exogenous labor income (which in the simplest calibration of the model we assume to be the same for all parents, such that the only heterogeneity is in the degree of altruism of parents) and $T_{i,L}$ and $T_{i,b}$ denote income and inheritance tax liabilities respectively. Below we will use the marginal tax rates $\tau_{1,b}$, $\tau_{2,b}$, and $\tau_L$ to denote the actual tax rates for the nonlinear inheritance and the linear income taxes.

We consider a tax system that is characterized by a linear income tax and an inheritance tax with two brackets. That is, the income tax is given by

$$T_{i,L} = \tau_L \times I_i.$$  

The inheritance tax schedule is defined by:

$$T_{i,b} = \begin{cases} 
\tau_{1,b} \times b_i & \text{if } b_i \leq k \\
\tau_{1,b} \times k + \tau_{2,b} \times (b_i - k) & \text{if } b_i > k,
\end{cases}$$

where $k$ is the inheritance amount above which the tax rate $\tau_{2,b}$ applies. Figure 1 gives a stylized example for the marginal tax rate on inheritances as function of the gross inheritance $b_i$.

Parents’ maximization problem and corresponding indirect utility function $\psi_i(\tau_L, \tau_{1,b}, \tau_{2,b}, I_i, k)$ are

$$\psi_i(\tau_L, \tau_{1,b}, \tau_{2,b}, I_i, k) = \max_{b_{i,t}} u_{i=1,h} \text{ s.t. } c_{i,t} = I_i(1 - \tau_L) - b_i$$

$$c_{i,t+1} = \begin{cases} b_i - \tau_{1,b} \times b_i & \text{if } b_i \leq k \\
 b_i - \tau_{1,b} \times k + \tau_{2,b} \times (b_i - k) & \text{if } b_i > k.
\end{cases}$$

Denote by $\phi_i(\tau_L, \tau_{1,b}, \tau_{2,b}, I_i, k)$ the utility of children given the optimal choice of parents. At the optimum the Euler equation

$$\frac{\partial \psi_i}{\partial c_{i,t+1}} = \frac{\alpha_i}{\beta_i(1 - T_{b}'(b_i))}$$

Farhi and Werning (2013) similarly introduce this kind of altruism heterogeneity, but assume that without redistribution marginal utility of consumption is the same for different values of $\delta$. In some specifications they additionally introduce social weights that decline with the strength of altruism. In subsection 5.1 we analyze a related case.
Figure 1: Stylized example of marginal tax rates.

holds (see the derivation in appendix section A.1).\textsuperscript{13} Allowing for potentially negative inheritances, this first order condition generally characterizes the individual optimum. In practice, parents can die with debts.\textsuperscript{14} Interpreting $b_i$ more generally as a transfer from parents to children, including \textit{inter vivo} transfers, a negative value of $b_i$ for instance occurs when children have to pay for their parents’ care as it may be the case in Germany.\textsuperscript{15}

The social planner sets tax rates $\tau_{1,b}$, $\tau_{2,b}$, and $\tau_L$ to maximize

$$
\max_{\tau_{1,b}, \tau_{2,b}, \tau_L} \sum_{i=1,h} (\psi_i(\tau_L, \tau_{1,b}, \tau_{2,b}, I_i, k) + \gamma \phi_i(\tau_L, \tau_{1,b}, \tau_{2,b}, I_i, k))
$$

$$
\text{s.t. } \sum_{i=1,h} (T_{i,L} + T_{i,b}) = G,
$$

$$
u_t(c_{l,t}, c_{l,t+1}) \geq u_t(c_{h,t}, c_{h,t+1}),
$$

$$
u_h(c_{h,t}, c_{h,t+1}) \geq u_h(c_{l,t}, c_{l,t+1}),
$$

where $\gamma$ denotes the social discount factor. Note that $T_{i,L}$ and $T_{i,b}$ are defined by equations (3) and (4). If $\gamma = 0$, the social planner does not directly take the utility of the children into account. Their utility then only enters the optimization indirectly through the (varying degrees of) altruism in the

\textsuperscript{13}Note that the Euler equation as it is given here implies that the tax is paid by the children, i.e. it is specified as inheritance tax. Specifying a bequest tax instead, would yield a slightly changed formula which would however be equivalent in expressing the same trade-off.

\textsuperscript{14}In many countries children can refuse negative inheritances, which can be interpreted as an inheritance tax of -100\%, see Farhi and Werning (2010).

\textsuperscript{15}Children may have to step in when parents cannot afford the costs of their care, i.e. pension and insurance payments do not suffice and children can afford to pay. The so called \textit{Elternunterhalt} results from BGB §1601 - 1603.
parents’ utility function. \( \gamma = 1 \) in contrast denotes the case of full “double counting”. \( G \) is a public good that needs to be financed through taxes. Equations (9) and (10) are the incentive compatibility constraints. As parents of the high type c.p. bequeath more than parents with a low preference for bequeathing, the first constraint needs to hold if the optimal inheritance tax redistributes to the high type and the second constraint needs to hold if the resulting inheritance tax redistributes to the low type. In the simulations, we assume that the constraints hold and check whether this is the case. For the simulations we assume that \( k = b_I \). We provide the corresponding first order conditions with non-binding incentive compatibility constraints in appendix section A.2.

4 Simulation

The simulations base on the log-utility specification, i.e. \( v(c_{i,t}) = \log(c_{i,t}) \) and \( \tilde{v}(c_{i,t+1}) \) alike.\(^{16}\)

Denote by \( b'_1 \) the inheritance amount an individual would choose if tax bracket 1 expanded beyond \( k \). Denote by \( b'_2 \) the inheritance amount an individual would choose if tax bracket 2 expanded beyond \( k \). In the case of a progressive inheritance tax the latter case would imply a ”virtual income” that is higher than actual net earnings. The terms are given by

\[
\begin{align*}
\frac{b'_1}{b'_2} &= \frac{\beta_i/\alpha_i(1-\tau_L)I_i}{1+\beta_i/\alpha_i} \\
\beta_i/\alpha_i(1-\tau_L)(1-\tau_{2,b})I_i &= k(\tau_{2,b} - \tau_{1,b}) \\
\end{align*}
\]

(11)

Denote by \( U(b'_1) \) and \( U(b'_2) \) utility that parents would derive if the respective hypothetical inheritance amount was chosen. Three cases can be distinguished for the optimal inheritance from the individuals’ perspective \( b^*_i \):

\[
\begin{align*}
I & : \quad b^*_i = b'_1 \quad \text{if} \quad (b'_1 \leq k \land b'_2 < k) \lor (b'_1 \leq k \land b'_2 \geq k \land (U(b'_1) > U(b'_2))) \\
II & : \quad b^*_i = k \quad \text{if} \quad b'_1 \geq k \land b'_2 \leq k \\
III & : \quad b^*_i = b'_2 \quad \text{if} \quad (b'_1 \geq k \land b'_2 > k) \lor (b'_1 \leq k \land b'_2 \geq k \land (U(b'_1) < U(b'_2)))
\end{align*}
\]

(12)

Note that in the case of equality of the respective first condition of the first and the third cases, the respective terms for \( b^*_i \) equal \( k \). Moreover, the first term in parentheses for cases I and III

\(^{16}\)A key characteristic of log-utility is that income and substitution effect offset each other.
is relevant in a progressive tax system, while the second terms in parentheses are relevant in a regressive tax system. 17

Figure 2 illustrates the three cases for the case of a progressive inheritance tax. In subfigure 2a the point of the optimal inheritance, where the indifference curve is tangent to the budget line, lies inside the first tax bracket. Note that negative optimal inheritances fall under this case. Subfigure 2b shows the case of a corner solution. There is no solution, where the indifference curve is tangent to the budget line and thus the corner solution \( k \) is chosen. Point \( b'_1 \) illustrates the first condition for Case II and \( b'_2 \) illustrates the second condition. Subfigure 2c illustrates the case where the optimum lies inside the second tax bracket.

\[ \psi_i(\tau_L, \tau_{1,b}, \tau_{2,b}, I_i, k) \] and \[ \phi_i(\tau_L, \tau_{1,b}, \tau_{2,b}, I_i, k) \] are given by

\[
\psi_i(\tau_L, \tau_{1,b}, \tau_{2,b}, I_i, k) = \alpha_i \log \left( \frac{(1 - \tau_L)I_i}{1 + \beta_i/\alpha_i} \right) + \beta_i \log \left( \frac{(1 - \tau_{1,b})\beta_i/\alpha_i(1 - \tau_L)I_i}{1 + \beta_i/\alpha_i} \right), \tag{13}
\]

\[
\phi_i(\tau_L, \tau_{1,b}, \tau_{2,b}, I_i, k) = \log \left( \frac{(1 - \tau_{1,b})I_i - k}{1 + \beta_i/\alpha_i} \right)
\]

if \( \frac{\beta_i/\alpha_i(1 - \tau_L)I_i}{1 + \beta_i/\alpha_i} \leq k \), i.e., \( b_i^* \) is below the tax threshold. The first term in logs represents consumption of parents and the second term in logs the net inheritance, i.e. consumption of children.

Clearly, the inheritance increases with \( \beta_i/\alpha_i \) and consumption of parents and children decreases with \( \tau_L \). The second case occurs if \( b_i^* = k \) and is a corner solution:

\[
\psi_i(\tau_L, \tau_{1,b}, \tau_{2,b}, I_i, k) = \alpha_i \log \left( (1 - \tau_L)I_i - k \right) + \beta_i \log \left( (1 - \tau_{1,b})k \right), \tag{14}
\]

\[
\phi_i(\tau_L, \tau_{1,b}, \tau_{2,b}, I_i, k) = \log \left( (1 - \tau_{1,b})k \right)
\]

if \( \frac{\beta_i/\alpha_i(1 - \tau_L)I_i}{1 + \beta_i/\alpha_i} > k \) and \( \frac{\beta_i/\alpha_i(1 - \tau_L)(1 - \tau_{2,b})I_i - k(\tau_{2,b} - \tau_{1,b})}{(1 + \beta_i/\alpha_i)(1 - \tau_{2,b})} < k \). The first term indicates that the individual optimum is not on the first tax bracket and the second term indicates that the individual optimum is not on the second tax bracket.

\[
\psi_i(\tau_L, \tau_{1,b}, \tau_{2,b}, I_i, k) = \alpha_i \log \left( \frac{-\tau_{2,b}(I_i\tau_L - I_i + k) + I_i\tau_L - I_i + k\tau_{1,b}}{\beta_i/\alpha_i + 1}(\tau_{2,b} - 1) \right)
\]

\[
+ \beta_i \log \left( \frac{\beta_i/\alpha_i((k - I_i)\tau_{2,b} + I_i\tau_L(\tau_{2,b} - 1) + I_i - k\tau_{1,b})}{\beta_i/\alpha_i + 1} \right), \tag{15}
\]

\[
\phi_i(\tau_L, \tau_{1,b}, \tau_{2,b}, I_i, k) = \log \left( \frac{\beta_i/\alpha_i((k - I_i)\tau_{2,b} + I_i\tau_L(\tau_{2,b} - 1) + I_i - k\tau_{1,b})}{\beta_i/\alpha_i + 1} \right)
\]

\[17\text{A theoretically possible case is that } U(b'_1) = U(b'_2), \text{ in which the individual chooses } b'_1 \text{ or } b'_2 \text{ with equal probability.} \]
Figure 2: Optimal individual solutions

(a) Case I a
(b) Case II
(c) Case III a
(d) Case I b
(e) Case III b

Note: Optimal solutions of individual under a progressive inheritance tax system. $b^*$ denote the optimal...
if \( \frac{\beta_i}{\alpha_i (1 - \tau_1)(1 - \tau_2) I_i - k (\tau_2 - \tau_1, b)} > k \), i.e., \( b^*_i \) is above the tax threshold.

The derivative of the net inheritance with respect to \( \tau_2, b \) is \( \frac{\beta_i}{\alpha_i (1 - \tau_2) I_i - k (\tau_2 - \tau_1, b)} + \frac{1}{\beta_i} \). This is negative as long as \( I_i > k \), which holds as long as individuals do not bequeath more than their gross earnings. The derivative of net inheritance with respect to \( \tau_1, b \) is \( -(k/(1 + \beta_i/\alpha_i)(-1 + \tau_2, b))) \), which is positive as long as \( \tau_2, b < 1 \). The reason is that \( \tau_2, b \) only has the effect of an income effect. It decreases the consumption level of children, but has no substitution effect.

The tax bracket that applies to higher inheritances starts slightly above the optimal inheritance of the low type, i.e. we set \( k = b_l + 0.02 \). We set the public good \( G = 0 \).

### 4.1 Benchmark case

For the benchmark case we set

\[
\alpha_i = 1, \quad \beta_i = \delta_i. \quad (16)
\]

Figure 3 displays different combinations of altruism for parents with low and high preference for bequeathing and the resulting optimal tax rates on the left hand side and the corresponding gross and net inheritances on the right hand side. The abscissa always displays different values of \( \gamma \), i.e. the degree of social discounting. Note that the figures in the main part of the paper show results for the range \( 0 \leq \gamma \leq 0.1 \). We provide the same graphs with the full range of double counting, i.e. \( 0 \leq \gamma \leq 1 \), in appendix section A.3. We normalize exogenous labor income \( I = 1 \) for both types.

The upper panel shows the results of the simulation when assuming moderate and comparably close values for parental altruism with \( \delta_l = 0.6 \) and \( \delta_h = 0.8 \). Both types receives bequest subsidies for the entire range of \( \gamma \). The state finances these inheritance subsidies with positive tax rates on labor income. Note that the low type receives subsidies that are at least as big as those of the high type for most values of \( \gamma \) (compare Figure 9 in the appendix). As more altruistic parents always bequeath more, this range displays the well known result of a progressive and negative inheritance tax rate as e.g. derived by Farhi and Werning (2010). Nonetheless, for values of roughly \( \gamma < 0.025 \) we observe the more altruistic type to receive higher subsidies. For this range of comparably high social discounting, the optimal tax system thus is regressive. The reason is that marginal utility of
Figure 3: Tax rates and income of children as a function of $\gamma$ (double counting).

(a) Marginal tax rates:

Panel A: $\delta_l=0.6$ and $\delta_h=0.8$

(b) Inheritance amounts:

Panel B: $\delta_l=0.4$ and $\delta_h=0.8$

Panel C: $\delta_l=0.2$ and $\delta_h=0.8$

Note: $a_l = 1, b_l = \delta_l, \text{ income is normalized to } 1.$

consumption for the high type is higher than for the low type, implying an intragenerational redistributive motive for the parent generation. As $\gamma$ increases, the intragenerational redistribution between children becomes more important, leading to a progressive tax schedule. The differences in negative tax rates are very small in the beginning, behave generally smoothly and remain relatively small. The figure however shows the strong sensitivity of the model to the chosen degree of social discounting: Only for small values of $\gamma$ tax rates remain reasonable in size. Full “double
counting” with $\gamma = 1$ would even result in inheritance subsidies of around 150\% as shown in the appendix. Such values appear drastic and question whether the implications of “double counting” have been sufficiently taken into consideration in the recent literature. The stark dependency of results on the value of $\gamma$ is illustrative. Farhi and Werning (2013) limit their simulation to values of up to $\gamma = 0.02$, i.e., the social planner puts a very low weight on heirs. Numerous papers, however, implement full double counting in their models, which, in this setting would yield tremendous optimal subsidies.

The upper right panel shows the corresponding gross and net inheritances of the offspring of low and high type as function of values of $\gamma$. Note that the gross labor income is normalized to 1. The child of the more altruistic type always receives a higher gross inheritance and a higher net inheritance than the child of the type with less pronounced preferences for bequeathing. Looking again at the range of $\gamma$ that implies a regressive tax system, the regressiveness means that even though the gross inheritance of the offspring of the more altruistic type exceeds the inheritance of its less altruistic peer, the optimal subsidy further redistributes from the less altruistic parents to the children of the more altruistic type by using the revenues of the income tax levied in the parents’ generation. This form of redistribution is optimal as the marginal utility of consumption of the parents with high preference for bequeathing exceeds, given the chosen parameters, the marginal utility of parents with a low degree of altruism. After all, while the given choice of parameter values yields that redistribution from the high type to the low type is optimal for low values of social discounting, this result is not unambiguously true for the entire range of values for social discounting.

The middle panel displays the same simulation exercise with $\delta_l = 0.4$ and $\delta_h = 0.8$. The results differ slightly and particularly reveal that the model may yield optimal regressive inheritance tax rates for a far bigger range of values for $\gamma$. Namely, for values up to $\gamma = 0.07$ the bequest subsidies of the more altruistic type exceed those of the less altruistic type. Hence, while the regressive tendencies in panel A may seem almost negligible, panel B suggests that a surprisingly regressive tax scheme may be optimal for values of $\gamma$ that well exceed those used in the simulation by Farhi and Werning (2013).

The lower panel completes the picture by providing results for rather close values of $\delta_i$. The range of regressive tax rates does not further increase. Both tax rates however increase and reach very significant rates already for small values of gamma. Moreover, the low type actually pays a small but positive tax for gamma $= 0$ and thus contributes to the regressive redistribution by earn-
ings and inheritance tax.\textsuperscript{18} Summing up, while progressive taxation is optimal for the most part of values for $\gamma$, the result of clearly regressive inheritance tax schemes for low levels of “double counting” remains. This result is interesting, given that the literature primarily discusses progressive optimal inheritance tax schemes. It results from the assumption that more altruistic parents have higher cardinal utility of bequeathing than less altruistic parents. Also, the results suggest that, given this model specification, all differences in $\delta_i$ will translate in some sort of ambiguity in the direction of the redistribution. That is, differences in the taste for bequeathing will in this specification always entail a regressive tax scheme for respectively low values of $\gamma$ and specifically for $\gamma = 0$. Hence, even in the absence of the positive externality from giving and the corresponding pigouvian correction, a redistributive purpose exists that justifies some sort of bequest taxation.

### 4.2 Lump-sum transfers

It may however be possible that our results of negative inheritance tax rates are an artefact: So far, the inheritance tax is the only means by which the social planner can support children. In fact, in reality intergenerational transfers by the social planner are common, e.g. in the form of the educational system. In order to test this concern, we extend the model slightly by introducing a lump sum transfer to the children, such that the consumption level of children is given by

$$c_{i,t+1} = b_t - T_{i,b} + TR$$

and the budget constraint of the social planner is

$$\sum_{i=l,h} (T_L + T_{i,b} - TR) = G.$$ \textsuperscript{(18)}

The transfer is uniform and exogenously set.

Figure 4 shows the results for the scenario of $\delta_l = 0.4$ and $\delta_h = 0.8$ and is thus comparable to the middle panel of Figure 3. The lump sum transfer is here set to $TR = 0.2$. The left panel again plots the inheritance tax rate for the two types as a function of the social discount factor $\gamma$. The introduction of the transfer does not alter the inheritance tax rates (note that scales differ slightly). However, the labor income tax, $\tau_L$, is shifted upwards in order to levy the revenue required to finance the transfers to children. The fact that the inheritance tax rates are unaffected by the lump sum transfer meets the concern that a transfer could, to some degree, render the inheritance tax

\textsuperscript{18}The further apart the $\delta$, the stronger are the motives for a positive inheritance tax.
Figure 4: Tax rates and income of children as a function of $\gamma$ with lump sum transfer.

(a) Marginal tax rates:

\[ \delta_l = 0.4 \text{ and } \delta_h = 0.8; \ TR = 0.2 \]

(b) Inheritance amounts:

\[ \beta_l = T_l, \ \beta_h = T_h \]

Note: $\alpha_i = 1, \beta_i = \delta_i$, income is normalized to 1.

Obsolete. The right panel plots gross and net inheritances of the children. Compared to Figure 3, the gross bequests of both types have decreased while the disposable income of children remains unaffected. The lump sum transfer here represents a second income for the children, thus a second source to finance consumption, and thereby reduces the incentives for parents to bequeath. Higher transfer values even can drive the gross bequests negative, so that parents in practice fault against their children.\footnote{The figures here do not display this case. Figure 6 panel A however is an example for such a scenario. Farhi and Werning (2010) describe that most countries allow children to decline negative inheritances and read this as example for a negative, progressive inheritance tax.}

4.3 Variations in productivity

So far, the earnings of the two types were equal and fixed. In order to introduce a second source of inequality, we assign different degrees of productivity to the parental generation. Hence, incomes are still fixed so that no further incentive compatibility constraint is introduced, but vary between types.

Figure 5 summarizes the results following from this model variation in the known way: The upper panel is a benchmark case in which we set $\delta_l = \delta_h = 0.8$. There is thus no need for redistribution along the altruism of parents but, instead, with respect to the differing productivity levels that translate into differing gross bequests.\footnote{Typically, different levels of productivity would require non-linear labor taxes in order to permit redistribution within the generation.} The upper left panel shows that in this setting a pro-
gressive tax scheme is unambiguously optimal. The high type pays a positive tax for very low values of $\gamma$ and receives a bequest subsidy for higher values of $\gamma$ that is nonetheless always lower than the subsidy for the less altruistic type.

Figure 5: Tax rates and income of children as a function of $\gamma$ with differences in parental earnings.

(a) Marginal tax rates:

Panel A: $\delta_{\text{low}} = \delta_{\text{high}} = 0.8$; $I_{\text{low}} = 0.6$ and $I_{\text{high}} = 1$

(b) Inheritance amounts:

Panel B: $\delta_{\text{low}} = 0.4$ and $\delta_{\text{high}} = 0.8$; $I_{\text{low}} = 0.6$ and $I_{\text{high}} = 1$

Panel C: $\delta_{\text{low}} = 0.4$ and $\delta_{\text{high}} = 0.8$; $I_{\text{low}} = 0.9$ and $I_{\text{high}} = 1$

Note: $\alpha_i = 1, \beta_i = \delta_i$.

In panel B we reintroduce different degrees of altruism. We assign the higher income to the type with higher preference for bequeathing. Such a positive correlation between altruism and labor income could result, e.g., if the more altruistic type has also been more patient when investing
in his human capital. The other parameters remain as set, panel B of Figure 5 shows the corresponding results: The labor income tax is not affected, the tax rate for the low type is slightly shifted upwards, the function in general steeper, while the tax rate for the high type is shifted downwards. Compared to panel A, the differences due to the variations in $\delta_i$ are stark and overturn the direction of the redistribution. The resulting tax system is regressive up to values of $\gamma \approx 0.055$ and progressive thereafter. The type with lower preference for bequeathing even pays a positive bequest tax for very small values of $\gamma$. The pattern of the result is similar to the one in panel B in Figure 3. The differences in earnings however reduce the range of the social discounting parameter $\gamma$ for which a regressive scheme would be optimal. Given that equal productivity renders a regressive inheritance tax scheme optimal for low values of $\gamma$ (as shown in Figure 3, panel B) and given that the absence of differences in parental altruism leads to an unambiguously progressive scheme (Figure 5, panel A), panel B shows an exemplary parameterization in which the effect of the parental differences in altruism dominate. The significant impact of earnings differences is then stressed by panel C: The $\delta$ parameters are maintained, the difference in earnings between low and high type is however increased to $I_h = 1$ and $I_l = 0.6$. This again overturns the direction of the redistributive aim of the social planner and results in a progressive tax system for all values of $\gamma$: The low type receives bequest subsidies for the full range $\gamma$ takes here, the more altruistic type pays a positive bequest tax up to $\gamma = 0.08$.

Figure 6 resumes the discussion from above and introduces a lump sum transfer that is paid to the children. The figure reproduces the parameterization from Figure 5 panel C and adds a transfer of $TR = 0.2$ to the setting.

The transfers again render the parents to leave lower intergenerational gross transfers as children already receive an income. In contrast to the discussion in section 4.2, the introduction of transfers here however also affects the inheritance tax rates: As income is unequally distributed, the income tax also already reduces the proportional difference in parental disposable income and redistributes to the lower type. Less redistribution via the inheritance tax is needed, so that inheritance tax rates approach each other. However, the results here are fully in line with those from section 4.2 as the transfer still does not affect the disposable income of children (and thus parents), i.e. the sum of transfer and inheritance maintains over the introduction of the lump sum transfer.\footnote{This set up, however, would usually require a non-linear labor tax.}\footnote{Note as well, that the case here differs from the one in Figure 4 as the less altruistic type receives a lower income than before so that the aggregate income in this economy here is also lower.}

\pagebreak
Figure 6: Tax rates and income of children as a function of $\gamma$ with differences in parental earnings and lump sum transfer.

(a) Marginal tax rates:

$\delta_l = 0.4$ and $\delta_h = 0.8$; $I_l = 0.6$ and $I_h = 1$; $TR = 0.2$

(b) Inheritance amounts:

Note: $\alpha = 1, \beta = \delta_i$

5 Variations in the valuation of the social planner

So far, we have only used a limited scope of variations in the model. Particularly, from the individuals’ point of view, the social planner could take their utility in different ways into account. Farhi and Werning (2013) for instance resort to a variation of eq. 5 in their simulations such that $\alpha$ and $\beta$ are defined as

$$\alpha_i = 1 - \kappa_i$$

$$\beta_i = \kappa_i$$

where $\kappa$ has a uniform distribution on the interval $[0.1, 0.9]$. Their simulations basically replicate the results from Farhi and Werning (2010) by yielding negative and progressive inheritance tax rates (see first panel in Figure 1 in Farhi and Werning (2013)). The tax rates are progressive in that they are increasing with $b$, i.e. the more altruistic parents are, the lower are the optimal inheritance tax subsidies they receive. We denote this variation of the model the normalized case and seek to replicate it in our model by setting $\alpha$ and $\beta$ in eq. 5 as

$$\alpha_i = \frac{1}{1 + \delta_i}$$

$$\beta_i = 1 - \frac{1}{1 + \delta_i}$$

This definition ensures that the proportion between $\alpha$ and $\beta$ is the same in the normalized case for a given value of $\delta_i$ as in our initial model, i.e. ordinal utility is the same, which renders results
comparable. In addition $\alpha$ and $\beta$ add up to one for all individuals as in Farhi and Werning (2013). With log utility this implies that with equal earnings the marginal utility of consumption is the same for all parents in the laissez-faire.\textsuperscript{23}

We also consider a third approach of how the social planner can take the individuals’ utility into account, which we denote the inverse valuation (IV) case. Here, eq. 5 is transformed linearly by setting

$$\alpha_i = \frac{1}{\delta_i} \text{ and } \beta_i = 1.$$  \hspace{1cm} (22)

Both of these approaches to take individual utility into account do not alter the valuation of behavioral opportunities from the perspective of the individual but solely from the perspective of the social planner. Both of the alternatives affect our previous results and therefore require some further discussion.

5.1 Inverse valuation

Figure 7 presents the results for the simulation with the IV-type of valuation of individual utility. The parameterization otherwise corresponds to panel A and B of Figures 3 and 8. Comparing the A-panels of the three Figures clarifies the mechanism: Our initial model suggested an inheritance tax also for $\gamma = 0$ and implied regressive schemes for low and a progressive scheme for higher values of $\gamma$. The normalized valuation shifts the tax rate for the more altruistic type upwards so that a zero inheritance tax is optimal for $\gamma = 0$ or a progressive tax system for any degree of “double counting”. The IV-valuation here shifts $\tau_{2,b}$ even more upwards and thereby mirrors our initial findings: Even when the social planner does not explicitly take into account the utility of the second generation, a progressive inheritance tax scheme is optimal here because the social planner believes that marginal utility of more selfish parents is higher than that of more altruistic parents. For $\gamma < 0.001$ also the less altruistic type pays a positive inheritance tax, albeit much smaller than that of the high type. For values of $\gamma > 0.015$ both types receive inheritance tax subsidies, the tax system is progressive throughout. Panel B shows a similar, but much more pronounced pattern. The range of values for which positive tax rates occur grows, the difference in the taxation of the two types increases substantially.

Our initial specification just as the inverse valuation case thus appear to be extreme cases that supplement the approach of Farhi and Werning (2010): Compared to our initial specification, the

\textsuperscript{23}With log utility $c_{ij}^* = \alpha_i I / (\alpha_i + \beta_i)$ and thus $\partial u_i^* / \partial c_{ij} = \alpha_i / (\alpha_i + \beta_i) = 1 / I$ for $\alpha_i + \beta_i = 1$.  

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Figure 7: Tax rates and income of children as function of $\gamma$ with inverse valuation of individual utility.

(a) Marginal tax rates:

Panel A: $\delta_l = 0.4$ and $\delta_h = 0.8$

(b) Inheritance amounts:

Panel B: $\delta_l = 0.6$ and $\delta_h = 0.8$; $I_l = 0.6$ and $I_h = 1$

Note: $\alpha_i = \frac{1}{\pi}$ and $\beta_i = 1$. income is normalized to 1.

IV-case deflates the cardinal utility of the more altruistic and inflates the utility of the less altruistic type. The initial scenario of a higher marginal utility of consumption of the more altruistic type is thus reversed.

After all, these three approaches of taking parental utility into account seem equally plausible. While both progressive and regressive tax schemes can be optimal for very low “double counting”,
all three approaches yield stark progressive inheritance taxes for most of the social discounting range. The simulation exercise nonetheless reveals that the interaction of social discounting and preferences for bequeathing are sensitive to the model specification which may yield contrasting results. In particular, the potentially considerable impact of “double counting” has to be documented by providing the full range of possible parameterizations. The pattern of effects of the variations in the valuation also apply to the variations of our initial model that we presented in Section 4 and are not reported here with inverse and normalized valuation.

5.2 Normalization of altruism

As noted, the simulation results by Farhi and Werning (2013) suggest that negative tax rates in a progressive tax scheme can be optimal. They derive these results by assuming a weighted utilitarian objective function and a limited degree of double counting, i.e. $\gamma = 0.02$ (in our notation).

Figure 8 presents the corresponding results of our model: We apply the normalization of altruism type of valuation of parental altruism and otherwise replicate the simulation as displayed in Figure 3, panel A and B. Namely, we set $\alpha_i$ and $\beta_i$ such that they correspond to $\delta_i = 0.4$ and $\delta_h = 0.5$ in panel A and to $\delta_i = 0.4$ and $\delta_h = 0.8$ in panel B. The results of this simulation are given in Figure 8.

First, the pattern of the resulting tax rates is in line with that in Farhi and Werning (2013). Panel A and B both show progressive tax systems with negative marginal tax rates for both types. The more altruistic parents are, the lower are the inheritance subsidies they receive. In contrast to Farhi and Werning (2013), our simulation covers the full range of social discounting (see Figure 13 in the appendix) and stresses that tax rates can be large for modest levels of “double counting”. These simulation results however deviate slightly from our previous results as presented in panel A and B in Figure 3. Particularly, the normalized type of weighting does not yield regressive tax schemes for low values of $\gamma$ anymore. While the differences seem almost negligible in panel A, the location of the $\tau_{2,h}$ line has clearly shifted that much that no regressive inheritance tax schemes maintain. The shift in $\tau_{2,h}$ is even more pronounced in panel B: While our initial specification yields a regressive scheme for values of $\gamma \leq 0.085$, the normalization of altruism valuation yields progressive schemes for $\gamma > 0$ and no inheritance taxes for $\gamma = 0$. Hence, in line with Farhi and Werning (2013).

\^{24}In the normalized case, for example $\delta_i = 0.4$ and $\delta_h = 0.5$ correspond to $\alpha_i = 0.71$ and $\beta_i = 0.29$ and $\alpha_h = 0.67$ and $\beta_h = 0.33$. 24
only the externality in giving arising from the “double counting” warrants an inheritance taxation in this model specification.

The higher the difference in preferences for bequeathing between parents, the more pronounced is the difference in inheritance tax subsidies. Apparently, the linear transformation of the utility function suffices to preclude regressive tax schemes. The normalization of cardinal utility does away with the distributive motive between parents. The social planner thus aims at equalizing the consumption prospects of the children.

6 Discussion of results

Our results stress the dependency of the optimal inheritance tax system on degrees of altruism and social discounting. The first set of results, as presented in Figure 3 shows that also in a restrictive setting, variations in altruism and social discounting can lead to a regressive tax system and very large negative optimal tax rates. Overall, most parameterizations yield progressive tax schemes, though. The results in Figure 5 describe mostly also progressive tax schemes, while proving that modest levels of inequality in earnings do not prevent regressive tax rates from occurring. However, few models disclose the impact of social discounting on their results, in particular as many scholars do not derive results for a variety of values of $\gamma$: Proponents of the direct and indirect consideration of childrens’ utility will probably implement full “double counting”, $\gamma = 1$. Our complete characterization of the tax systems suggests however that this practice can lead to implausibly high tax rates. Opponents of “double counting”, in contrast, will probably only indirectly consider the childrens’ utility (i.e. set $\gamma = 0$). Doing so draws the tax scheme, c.p., more or less pronounced towards regressive taxation but leaves the character of the tax system ambiguous when different dimensions of inequality overlap and depending on the valuation type. Depending on the inequality between individuals, even regressive tax rates can turn out to be optimal. We also present variations in the way the social planner values individual utility and show that equally plausible valuation approaches do not yield regressive tax schemes. It is not clear, why any of the presented valuation strategies should be preferred over the other. Our results thus complement the recent findings in the literature that primarily suggest progressive tax schemes to be optimal by deriving the conditions for a regressive exception.

The different valuation strategies also reveal the character of the forces driving the tax rates: The normalized valuation stresses that the positive externalities from giving evoke a pigouvian
Figure 8: Tax rates and income of children as function of $\gamma$ with normalized valuation of individual utility.

(a) Marginal tax rates:

$$\tau_L, \tau_1, \tau_2, \tau_b$$

(b) Inheritance amounts:

$$b_L, b_h, b_l - T_L, b_h - T_h$$

Panel A: $\delta_l = 0.4$ and $\delta_h = 0.8$; $l_l = 0.6$ and $l_h = 1$

Panel B: $\delta_l = 0.6$ and $\delta_h = 0.8$; $l_l = 0.6$ and $l_h = 1$

Panel B: $\delta_l = 0.4$ and $\delta_h = 0.8$; $l_l = 0.9$ and $l_h = 1$

Note: $\alpha_i = \frac{1}{1+\delta_i}$ and $\beta_i = 1 - \frac{1}{1+\delta_i}$, income is normalized to 1.

correction. The stark differences between benchmark and inverse valuation underline the redistributive purposes of positive tax rates that emerge from the specific valuation approach. Given heterogeneities in the parental preferences for bequeathing, bequest taxation may be optimal irrespective of externalities from giving, but crucially depend on the redistributive motive inherent in the beliefs of the social planner about cardinal utility of relatively selfish and altruistic parents.
After all, our model is still restrictive. We only implement a linear labor income tax, so that no redistribution within the parents’ generation is possible via the income tax. In particular the scenario in which productivities differ between parents would call for such a tool. The key trade-off in our model however concerns the inequality in the children’s generation and the incentives to bequeath in the parental generation. In the center of our interest is thus the redistribution between low and high types across generations. Endogenous labor supply would not necessarily alter this mechanism. Also, while our model is restrictive in the regard of labor supply and taxation, it takes income effects in the parental generation into account, a feature that is lacking in several recent publications and particularly in those, that fully characterize the tax system (Piketty and Saez 2013; Saez and Stantcheva 2018).

7 Conclusions

We present a simple two generations model in which parents have an altruistic bequest motive but vary in the degree of altruism. We test in how far this setting interacts with the common procedure to count the utility of children in the optimization problem directly and indirectly. Our results only partly confirm the results presented in Farhi and Werning (2013) and Boadway and Cuff (2015): While our model predicts negative and progressive inheritance taxation in most cases, it also shows that a regressive inheritance taxation can be optimal in several common settings that have so far not found sufficient attention in the optimal taxation literature. This is the case when the social planner believes that marginal utility of more selfish parents is higher than that of more altruistic parents. Our paper thereby adds to the discussion about the interaction of different specifications of bequest motives and the so called “double counting”.

References


A Appendix

A.1 Derivation of Euler equation (eq. 6)

Inserting the budget constraint into Equation (5) and taking the first derivative with respect to $b_i$, applying the chain rule, and rearranging yields the Euler equation.

$$\frac{\partial u_i}{\partial b_{i,t}} = -\alpha_i \frac{\partial v}{\partial c_{i,t}} + \beta_i \frac{\partial \tilde{v}}{\partial c_{i,t+1}} \times \left(1 - \frac{\partial T_{i,b}}{\partial b_{i,t}}\right) = 0 \quad (23)$$

A.2 First order conditions

The first order conditions of the optimization problem of the social planner are the budget constraint and the following:

$$\frac{\partial L}{\partial \tau_1} = \alpha_l \frac{\partial \psi_l}{\partial \tau_1} + \alpha_h \frac{\partial \psi_h}{\partial \tau_1} + (\gamma + \beta_l) \frac{\partial \phi_l}{\partial \tau_1} + (\gamma + \beta_h) \frac{\partial \phi_h}{\partial \tau_1} + \lambda \left(2\tau_1 \frac{\partial b_l}{\partial \tau_1} + \tau_2 \frac{\partial b_h}{\partial \tau_1} + 2b_l(\tau_1) - \tau_2 \frac{\partial b_l}{\partial \tau_1}\right) = 0 \quad (24)$$

$$\frac{\partial L}{\partial \tau_2} = \alpha_h \frac{\partial \psi_h}{\partial \tau_2} + (\gamma + \beta_h) \frac{\partial \phi_h}{\partial \tau_2} + \lambda \left(\tau_2 \frac{\partial b_h}{\partial \tau_2} + b_h(\tau_2) - b_l(\tau_2)\right) = 0 \quad (25)$$

$$\frac{\partial L}{\partial \tau_L} = \alpha_l \frac{\partial \psi_l}{\partial \tau_L} + \alpha_h \frac{\partial \psi_h}{\partial \tau_L} + (\gamma + \beta_l) \frac{\partial \phi_l}{\partial \tau_L} + (\gamma + \beta_h) \frac{\partial \phi_h}{\partial \tau_L} + \lambda \left(2l + 2\tau_1 \frac{\partial b_l}{\partial \tau_L} + \tau_2 \left(\frac{\partial b_h}{\partial \tau_L} - \frac{\partial b_l}{\partial \tau_L}\right)\right) = 0 \quad (26)$$
A.3 Simulation
Figure 9: Tax rates and income of children as a function of $\gamma$ (double counting).

(a) Marginal tax rates:

Panel A: $\beta_l=0.6$ and $\beta_h=0.8$

(b) Inheritance amounts:

Panel B: $\delta_l=0.4$ and $\delta_h=0.8$

Panel C: $\delta_l=0.2$ and $\delta_h=0.8$

Note: $\alpha_i = 1$, $\beta_i = \delta_i$, income is normalized to 1.

Figure 3 extended in the range of $\gamma$. 
Figure 10: Tax rates and income of children as a function of $\gamma$ with lump sum transfer.

(a) Marginal tax rates:

\[ \delta_l = 0.4 \text{ and } \delta_h = 0.8; TR = 0.2 \]

(b) Inheritance amounts:

Note: $\alpha_i = 1, \beta_i = \delta_i$, income is normalized to 1.

Figure 4 extended in the range of $\gamma$. 
Figure 11: Tax rates and income of children as a function of $\gamma$ with differences in parental earnings.

(a) Marginal tax rates:

Panel A: $\delta_{low} = 0.8$ and $\delta_{high} = 0.8$; $I_{low} = 0.6$ and $I_{high} = 1$

Panel B: $\delta_{low} = 0.4$ and $\delta_{high} = 0.8$; $I_{low} = 0.6$ and $I_{high} = 1$

Panel C: $\delta_{low} = 0.4$ and $\delta_{high} = 0.8$; $I_{low} = 0.9$ and $I_{high} = 1$

(b) Inheritance amounts:

Note: $\alpha_i = 1, \beta_i = \delta_i$, income is normalized to 1.

Figure 5 extended in the range of $\gamma$. 
Figure 12: Marginal tax rates and income of children as a function of $\gamma$ with differences in parental earnings and lump sum transfer.

(a) Marginal tax rates:

Panel A: $\delta_l = 0.4$ and $\delta_h = 0.8$; $I_l = 0.6$ and $I_h = 1$; $TR = 0.2$

Note: $\alpha_i = 1, \beta_i = \delta_i$, income is normalized to 1.

(b) Inheritance amounts:

Figure 6 extended in the range of $\gamma$. 
Figure 13: Tax rates and income of children with normalized valuation of individual utility.

(a) Marginal tax rates:

Panel A: $\delta_l = 0.4$ and $\delta_h = 0.8$; $I_l = 0.6$ and $I_h = 1$

Panel B: $\delta_l = 0.4$ and $\delta_h = 0.8$; $I_l = 0.9$ and $I_h = 1$

(b) Inheritance amounts:

Panel B: $\delta_l = 0.4$ and $\delta_h = 0.8$; $I_l = 0.9$ and $I_h = 1$

Note: $\alpha_i = \frac{1}{1+\delta_i}$ and $\beta_i = 1 - \frac{1}{1+\delta_i}$, income is normalized to 1.
Figure 14: Tax rates and income of children with inverse valuation of individual utility.

(a) Marginal tax rates:

Panel A: $\delta_l = 0.4$ and $\delta_h = 0.8$

Panel B: $\delta_l = 0.6$ and $\delta_h = 0.8$; $l_l = 0.6$ and $l_h = 1$

Panel B: $\delta_l = 0.6$ and $\delta_h = 0.8$; $l_l = 0.6$ and $l_h = 1$

(b) Inheritance amounts:

Panel B: $\delta_l = 0.6$ and $\delta_h = 0.8$; $l_l = 0.6$ and $l_h = 1$

Note: $\alpha_i = \frac{1}{\delta_i}$ and $\beta_i = 1$, income is normalized to 1.