Is the marginal cost of public funds equal to one?

Bjart Holtsmark*

Preliminary work

August 9, 2019

Abstract

Jacobs (2018) found that the marginal cost of public funds (MCF) is one in second best. An important premise for this result is the claim that there are certain shortcomings with the traditional definition of MCF, for example that the size and sign of the traditional MCF measure is sensitive to the choice of the untaxed good. A less frequently used definition of MCF is therefore applied instead. The contribution of the present paper is two-fold. First, it finds that the traditional MCF-measure is not sensitive to the choice of the untaxed good. Second, it finds that the proposed alternative definition has undesirable properties, for example that it is sensitive to the choice of the untaxed good also in cases where this does not make sense. The present paper therefore concludes that there is a weak basis for the conclusion that MCF is one in second best.

Keywords: Marginal cost of public funds, taxation, public goods.

JEL-codes: H20, H40, H50.

Acknowledgements: I am indebted to Agnar Sandmo and Erling Holmøy for constructive and helpful comments. I will also thank seminar participants at the Norwegian Ministry of Finance and at Statistics Norway for valuable comments and discussions.

*Research Department, Statistics Norway. E-mail: bjj@ssb.no T: +47 988 36 198 ORCID: 0000-0001-6154-4915.
1 Introduction

To take costs related to taxation into account, a number of countries have as a general recommendation that the funding costs of public projects should be multiplied with a certain factor greater than one. Jacobs (2018) argued against this practice because he found that the marginal cost of public funds (MCF) is one in second best. The present paper discusses Jacobs’ arguments and concludes that there is a weak basis for the conclusion that MCF is one.

Jacobs’ arguments include two main building blocks. First, he found that the traditional definition of MCF has three "undesirable" properties:

1. The traditional measure of MCF of lump-sum taxes is different from one even when taxes are optimized.

2. The traditional measure of MCF is not directly related to the marginal excess burden of taxation (MEB).

3. The traditional measure of MCF is found to be sensitive to the choice of the untaxed good with optimal taxation.

Second, due to the claimed undesirable properties of the traditional approach, another definition of MCF should be applied. With that approach, he found that MCF equals one in second best. Therefore, he concluded that "the modified Samuelson rule should not be corrected for the marginal cost of public funds."

The present paper has two main elements:

1. It argues against the critique of the traditional definition of MCF.

2. It analyses the proposed alternative definition of MCF and shows that it has some undesirable properties which makes it less useful than the traditional definition.

The conclusion of the present paper is that the Samuelson rule should be modified to take the marginal cost of public funds into account.

\[\text{Håkonsen (1998)}\] who also studied other alternative representations of MCF.
With regard to the first two points in Jacobs’ critique of the traditional approach, the present paper does not provide any thorough response, only the following brief comments.

The first point in Jacobs’ critique of the traditional MCF-measure regards cases where a lump-sum tax is imposed in addition to a distortionary tax. In their important and clarifying article, Ballard and Fullerton (1992, p. 125) argued that it is reasonable that MCF in such cases usually is less than one: "[T]he additional lump-sum tax also increases the revenue from the pre-existing wage tax. The combined increase in revenue exceeds the loss in utility, so that the "revenue effect" makes the marginal cost of public funds less than one." Other contributions provide similar arguments and conclusions, see for example Fullerton (1991) and Sandmo (1998). Hence, there are well established arguments why MCF of lump-sum taxes is less one, and it is difficult to see why this not should apply also to cases where taxes are optimized.

With regard to the second point in Jacobs’ critique, he finds it undesirable that the traditional MCF-measure of distortionary taxes "do not directly relate to the marginal excess burden of taxation, even though this relationship is suggested by, e.g., Pigou (1947), Harberger (1964), Browning (1976)." It could be difficult to see how the mentioned early contributions to the literature could undermine what has later been established as the traditional MCF measure. What today is considered the traditional MCF measure was far from clearly defined when the three mentioned works were published. A main point in for example Stiglitz and Dasgupta (1971) was that Pigou (1947) had overlooked the important result that MCF could be less than one, although they did not use the same terms.

Finally, Jacobs claimed that the traditional MCF-measure is sensitive to the choice of the untaxed good. The next section shows that with a precise definition of the traditional MCF measure, it is not sensitive to the choice of the untaxed good unless there are real asymmetries in the behavioural effects of the different tax instruments. The starting point is that if the costs and benefits of public goods are to be expressed in monetary terms, marginal utility has to be translated to monetary units through a normalization, see for example Slemrod and Yitzhaki (2001, p. 192). This has traditionally been done by dividing the shadow value of public revenue with the (average) marginal utility of a composite consumer good,
which in models without a consumption tax equals the marginal utility of income net of taxes (Atkinson & Stiglitz, 1980; Sandmo, 1998; Gahvari, 2006; Kleven & Kreiner, 2006; Kreiner & Verdelin, 2012).

However, when Jacobs (2018) included a consumption tax in the model, he followed Håkonsen (1998) and defined MCF as the shadow value of public revenue divided by the average of the individuals’ marginal utility of gross income, which is different from the marginal utility of the private good. That is the reason why they found that MCF is sensitive to the choice of the untaxed good. In practice two different definitions of MCF were applied. That explains why they found the traditional MCF to be sensitive to the choice of tax instruments in cases where this does not make sense. If MCF is defined as the shadow value of public revenue divided by the average marginal utility of private consumption, its size is sensitive to the choice of tax instrument only in cases where this makes sense.

The present paper also discusses the alternative MCF-measure proposed by Jacobs (2018), which is meant to meet his critique of the traditional measure and which is the basis for his conclusion that MCF is one.

The proposed MCF measure is defined as the shadow value of public revenue divided by the average social marginal value of income, as defined by Diamond (1975). With this approach, MCF is one with an optimal combination of either an income tax and a lump-sum tax or a consumption tax and a lump-sum tax.

The present paper studies some properties of the Diamond-based MCF measure, especially in a case where there is not access to lump-sum taxation. In this case there is full symmetry between income taxation and consumption taxation and any reasonable measure of the costs of taxation should be insensitive to the choice between the two tax instruments. The present paper therefore checks to what extent both the Diamond-based MCF measure and the traditional MCF measure has this property.

With regard to the traditional MCF measure, it is found that it has. Hence, it passes this test.

With regard to the proposed Diamond-based MCF-measure, it is found that without access to lump-sum taxation, the size of MCF is sensitive to the choice of tax instrument except for a few special cases. Even in second best, the Diamond-based MCF measure is sensitive to the choice of tax instrument in this case. Hence,
it is the Diamond-based MCF measure, not the traditional MCF measure, which is sensitive to the choice of tax instruments when this should not be the case. This is a serious weakness with the proposed MCF-measure.

Although the present paper argues against Jacobs’ critique of the traditional MCF-measure, it should be emphasized that does not mean that it defends the traditional measure in all other respects. I agree with Jacobs that there is a need for a critical view on the traditional MCF-measure. Valseth, Holtsmark, and Holtsmark (2019) is a closely related paper where it is argued that there is a serious weakness with the traditional MCF measure, namely that it implicitly measures costs and benefits of taxation using the regressive, uniform lump-sum tax as a benchmark. Valseth et al. (2019) find that in the presence of inequality, and if a distribution neutral benchmark is used instead, the estimated costs of taxation are found to be higher than with the traditional MCF measure.

The present paper is organized as follows. First, the traditional MCF-measure is analysed and it is shown that in second best it is not sensitive to the choice between the two considered tax instruments. Next, it explores the reason why Jacobs nevertheless found this to be a weakness with the traditional MCF measure. Third, some desirable properties of the traditional MCF-measure are demonstrated. The paper then turns to the alternative MCF-measure proposed by Jacobs and shows that this measure has some serious weaknesses. A section with numerical examples illustrates the results. Finally, there is a concluding section.

2 Some properties of MCF with the traditional definition

Let there be a total of $n$ individuals, with subscript $i \in \{1, \ldots, n\}$ denoting individual $i$. Individual $i$’s utility is given by:

$$u_i = u(c_i, l_i, G),$$

where $c_i$ is consumption of a (composite) private good, $l_i$ is leisure and $G$ is consumption of a pure public good. Individuals provide labour supply $h_i$ subject to
the time constraint $h_i + l_i = T$, where $T$ is total time available. Following Mirrlees (1971), the labour productivity of individual $i$ is given by $w_i$, and varies across individuals. This variation in labour productivity is the original source of inequality across individuals in the model, while different labour supply and taxes might reduce or increase inequality in income.

Labour is used in the production of both the private consumption good and the public good and a simple, linear production function is assumed. Let $q$ be the unit cost of producing the public good in terms of the private good. We then have:

$$
\sum_i c_i + qG = \sum_i w_i h_i.
$$

For our notation to be in line with the previous literature, we let $\lambda_i \equiv u_{ic}$ denote the marginal utility of private consumption for individual $i$ and let $\bar{\lambda}$ be its average. Furthermore, for notational simplicity, we define

$$m_i \equiv \frac{u_iG}{\lambda_i},$$

individual $i$’s marginal willingness to pay for the public good.

### 2.1 The first-best allocation

We define the first-best allocation in this economy as the maximum of a utilitarian welfare function $W = \sum_i u_i$. Given this welfare function, the first-best allocation is defined by the following $2n$ first-order conditions:

$$
\sum_j u_{jG} = \lambda_i q \quad \forall i
$$

$$
\sum_j u_{jG} = \frac{u_{iG}}{w_i} q \quad \forall i.
$$

---

2It is common in this literature on the MCF to let the government maximize a more general welfare function. The less general, additive welfare function applied here does not drive the main results but makes the introduction to our approach significantly more readable.
This leads to the rule for first-best provision of the public good, as formulated by Samuelson (1954):

\[ \sum_i m_i = q. \] (2)

The Samuelson rule ensures optimal provision of the public good, while the welfare loss from inequality is removed by redistribution of income to the point where the marginal utility of money is the same across all individuals.

### 2.2 The second-best allocation and the definition of the traditional MCF

To provide the public good the government must raise public revenue. In the model applied in this paper, the government can tax wage income, but can also tax consumption. Let \( t \in [0, 1] \) represent a linear income tax while \( \tau \geq 0 \) represents a linear consumption tax. In addition there is a uniform lump-sum transfer \( a \in (-w_jT, \infty) \) where \( w_j = \min(w_1, \ldots, w_n) \). The lower limit of the lump-sum transfer rules out cases in which the model has no solution. Net public revenue is then

\[ R = \tau \sum_i c_i + t \sum_i w_i h_i - na. \]

Individual \( i \) maximizes \( u_i \) with respect to \( c_i \) and \( l_i \), taking \( G \) as given, subject to the time constraint and the budget constraint:

\[ (1 + \tau)c_i = (1 - t)w_i h_i + a. \] (3)

When formulating the maximization problem of the \( n \) individuals and the corresponding Lagrange-functions, one should have in mind that how individuals’ budget constraint are included in the Lagrange functions determines the interpretation of the Lagrange multiplier and its size. The budget constraint is therefore normalized as follows:

\[ c_i = \frac{1 - t}{1 + \tau} w_i h_i + \frac{a}{1 + \tau}. \] (4)

The Lagrange function related to the individuals’ utility maximization could then be written:

\[ L_i(c_i, h_i, G, w_i, t, \tau, a) = u(c_i, T - h_i, G) - \lambda_i \left( c_i - \frac{1 - t}{1 + \tau} w_i h_i - \frac{a}{1 + \tau} \right). \] (5)
The following first-order conditions together with the budget constraint solve individual $i$’s problem:

$$u_c(c_i, l_i, G) = \lambda_i,$$  \hspace{1cm} (6)

$$u_l(c_i, l_i, G) = \lambda_i \frac{1-t}{1+\tau} w_i,$$  \hspace{1cm} (7)

defining $c_i(t, \tau, a, w_i, G)$, $h_i(t, \tau, a, w_i, G)$ and $l_i(t, \tau, a, w_i, G)$.

Let $v_i = v(t, \tau, a, w_i, G)$ be the indirect utility function. The envelope theorem gives that:

$$\frac{\partial v_i}{\partial t} = -\lambda_i \frac{1}{1+\tau} w_i h_i,$$  \hspace{1cm} (8)

$$\frac{\partial v_i}{\partial \tau} = -\lambda_i \frac{1}{1+\tau} c_i,$$  \hspace{1cm} (9)

$$\frac{\partial v_i}{\partial a} = \lambda_i \frac{1}{1+\tau},$$  \hspace{1cm} (10)

$$\frac{\partial v_i}{\partial G} = \frac{\partial u_i}{\partial G}.$$  \hspace{1cm} (11)

The government maximizes $W = \sum_i v_i$ subject to the public budget constraint:

$$\tau \sum_i c_i + t \sum_i w_i h_i - na = qG.$$  \hspace{1cm} (12)

The corresponding Lagrange function is:

$$L_g = \sum_i v(\cdot) + \mu \left( \tau \sum_i c_i(\cdot) + t \sum_i w_i h_i(\cdot) - qG - na \right).$$  \hspace{1cm} (13)

The first order conditions for the government’s maximization problem with respect to the lump-sum transfer $a$, the income tax, $t$, the consumption tax $\tau$, and the size
of the public sector, \( G \), are are:

\[
\frac{1}{1 + \tau} \sum_i \lambda_i = \mu \left( n - \tau \sum_i \frac{\partial c_i}{\partial a} - t \sum_i w_i \frac{\partial h_i}{\partial a} \right),
\]

(14)

\[
\frac{1}{1 + \tau} \sum_i \lambda_i w_i h_i = \mu \sum_i \left( \tau \frac{\partial c_i}{\partial t} + w_i h_i + t w_i \frac{\partial h_i}{\partial t} \right),
\]

(15)

\[
\frac{1}{1 + \tau} \sum_i \lambda_i c_i = \mu \sum_i \left( c_i + \tau \frac{\partial c_i}{\partial \tau} + t w_i \frac{\partial h_i}{\partial \tau} \right),
\]

(16)

\[
\sum_i \frac{\partial u_i}{\partial G} = \mu \left( q - \tau \frac{\partial c_i}{\partial G} - t w_i \frac{\partial h_i}{\partial G} \right).
\]

(17)

When the consumption tax is assumed to be zero, i.e. \( \tau = 0 \), the f.o.c. with respect to public consumption, \( G \), given in (17), together with the first order conditions (14) and (15) as well as the public budget constraint (12), give the second-best levels of the shadow value of public revenue, \( \mu \), public consumption, \( G \), the uniform lump-sum transfer, \( a \), and the income tax rate \( t \). If an consumption tax, \( \tau \), isappliedinsteadofan income tax \( t = 0 \), the the f.o.c. (16) applies instead of (15).

The first order condition for supply of the public good given in (17) gives the following modified Samuelson Rule:

\[
(1 + \delta_z) \sum_i m_i = \frac{\mu}{\lambda} \left[ q - \frac{\partial R}{\partial G} \right],
\]

(18)

where

\[
\delta_z \equiv \frac{cov(\lambda_i, m_i)}{\lambda \tilde{m}},
\]

and where we have used that

\[
cov(\lambda_i, m_i) = \frac{1}{n} \sum_i \lambda_i m_i - \bar{\lambda} \tilde{m}.
\]

(19)

The expression in square brackets on the right hand side of (18) represents the net
unit costs of the public good taking into account how supply of the public good influences public revenue through changes of either labour supply or consumption. The net unit cost is multiplied by the shadow value of public revenue, $\mu$ normalized with the average marginal utility of money, $\bar{\lambda}$. This fraction represents an operative corrective factor that could be used in cost-benefit analyses and represents the traditional MCF measure:

Definition 1 (The traditional MCF) The traditional marginal cost of public funds is defined as

$$MCF \equiv \frac{\mu}{\bar{\lambda}}. \quad (20)$$

As pointed out by Valseth et al. (2019), $\bar{\lambda}$ represents the aggregate welfare loss to individuals of raising one public dollar using a uniform lump-sum tax, which means that each consumer pays the same tax, in this case $1/n$ dollar. A uniform lump-sum tax is regressive. Hence, normalizing $\mu$ with $\bar{\lambda}$ means that the costs of taxation are evaluated using a regressive tax as the benchmark. Valseth et al. (2019) therefore find that it is more reasonable to evaluate the distributional effects of taxes using a distribution-neutral tax scheme as the benchmark. They define a MCF-measure based on that.

The main purpose of the present paper, however, is to discuss Jacobs’ critique of the traditional MCF-concept as well as his proposed alternative measure. The starting point for the present paper is therefore the traditional normalization procedure using $\bar{\lambda}$.

The left hand side of equation (18) includes the aggregate willingness to pay for the public good multiplied with a corrective factor $(1 + \delta_z)$, where $\delta_z$ in the literature has been interpreted as a variable that represents the distributional properties of the public good (Atkinson & Stiglitz, 1980; Wilson, 1991; Sandmo, 1998). However, it is useful to have in mind that Valseth et al. (2019) show that $\delta_z$ actually represents a mix of distributional properties of the public good and the tax.
2.3 Is the traditional MCF measure sensitive to the choice of the untaxed good with optimal taxation?

First, consider the case where a lump-sum transfer is combined with an income tax while the consumption tax is set to zero. Define \( y_i \equiv w_i h_i, \bar{y} \equiv (1/n) \sum_i y_i, \) and \( Y \equiv \sum_i y_i. \) From the f.o.c. (14) and (15) and assuming that \( \tau = 0, \) we have that:

\[
MCF_a = \frac{\mu}{\lambda} = \frac{1}{1 - \frac{t}{n} \sum_i \frac{\partial y_i}{\partial a}}, \tag{21}
\]

\[
MCF_t = \frac{\mu}{\lambda} = \frac{1}{1 + \frac{\delta_{\lambda y}}{1 + \theta_t}}, \tag{22}
\]

where the subscripts \( a \) and \( t \) indicate that MCF of a lump-sum transfer and an income tax, respectively, are considered and where:

\[
\delta_{\lambda y} = \frac{\text{cov}(\lambda_i, y_i)}{\bar{y}}, \tag{23}
\]

\[
\theta_t = \sum_i y_i \frac{\varepsilon_{h_t}}{Y}, \tag{24}
\]

where \( \varepsilon_{h_t} \equiv (t/h_i)(\partial h_i/\partial t) \) is the elasticity of individual \( i \)'s labour supply with respect to the tax rate \( t. \)

Next, consider the case where a consumption tax is combined with a lump-sum tax. Using the f.o.c. in (14) and (16) and assuming that \( t = 0, \) we have:

\[
MCF_a = \frac{1}{1 + \tau} \frac{1}{1 - \frac{\tau}{n} \sum_i \frac{\partial c_i}{\partial a}}, \tag{25}
\]

\[
MCF_{\tau} = \frac{1}{1 + \tau} \frac{1 + \delta_{\lambda c}}{1 + \theta_\tau}, \tag{26}
\]

---

3See corresponding expressions in for example Dahlby (1998); Sandmo (1998); Mayshar and Yitzhaki (1995); Slemrod and Yitzhaki (2001) for cases with sets of heterogenous individuals and in Mayshar (1990, 1991) for cases with a single representative individual.
where the subscript $a$ and $\tau$ indicate that MCF of a lump-sum transfer and a consumption tax are considered, respectively, and where

\[ \delta_{\lambda c} = \frac{\text{cov}(\lambda_i, c_i)}{\bar{\lambda} \bar{c}}, \]  
\[ \theta_\tau = \sum_i \frac{c_i}{c} \varepsilon_{c_i, \tau}. \]

(27)

(28)

where $\varepsilon_{c_i, \tau} \equiv (\tau/c_i)(\partial c_i/\partial \tau)$ is the elasticity of individual $i$’s private consumption with respect to the tax rate $\tau$. The first result follows:

Proposition 1 (MCF with optimal taxation) Assume that the government implements a lump-sum tax in combination with either an income tax or a consumption tax, in both cases such that second best is achieved. Then MCF is less than one and not sensitive to whether the solution is a result of a combination of a lump-sum tax and an income tax, or a combination of a lump-sum tax and a consumption tax.

Proof. See appendix A. ■

2.4 Why did Jacobs (2018) find that MCF with optimal taxation is sensitive to the choice of tax instrument?

The question is then why Jacobs (2018) found that MCF is greater than one with an optimal combination of a consumption tax and a lump-sum transfer, while it is less than one with an optimal combination of an income tax and a lump-sum transfer.

This apparent shortcoming with the traditional definition of MCF has a simple explanation. In construction of the Lagrangian related to individual maximization Jacobs (2018) looked to the budget constraint (3) instead of the normalized budget constraint (4). That gave him the Lagrangian:

\[ L_i'(\cdot) = u(c_i, T - h_i, G) - \lambda_i'((1 + \tau)c_i - (1 - t)w_i h_i - a), \]  
\[ \lambda_i' \]  

(29)

where $\lambda_i'$ is the Lagrange-multiplier. Let $\bar{\lambda}'$ be its average. Jacobs defined MCF to be $\mu/\bar{\lambda}'$. 

14
The crucial point here is that with the use of the Lagrangian in (29), the f.o.c. in (6) now is replaced with:

$$\frac{u_c(c_i, h_i, G, w_i)}{1 + \tau} = \lambda_i'.$$

(30)

It follows that in the case with a consumption tax (and \( t = 0 \)), Jacobs’ Lagrange-multiplier \( \lambda'_i \) represents the marginal utility of income \textit{before} tax payments (\textit{gross} income). In the case with an income tax (and \( \tau = 0 \)), \( \lambda'_i \) represents the marginal utility of income \textit{after} tax payments (\textit{net} income), which equals the marginal utility of private consumption. Hence, when Jacobs, as Håkonsen (1998), defined MCF to be \( \mu/\bar{\lambda}' \), he is dealing with two different definitions of MCF, one in the case with an income tax and another in the case with a consumption tax. That explains why the size of MCF differ in the two cases.

The point could also be illustrated by considering the first order condition (18) in the case with a consumption tax. Dividing both sides with \( \bar{\lambda}' \) instead of \( \bar{\lambda}_g \) gives the following modified Samuelson Rule:

$$(1 + \tau)(1 + \delta_z) \sum_i m_i = \frac{\mu}{\bar{\lambda}'} \left[ q - \frac{\partial R}{\partial G} \right]$$

(31)

which obviously has the same solution as (18), because both sides are multiplied by \( (1 + \tau) \). The higher corrective factor on the right hand side of (31) compared to (18) corresponds to the additional corrective factor on the left hand side. In other words, if \( \bar{\lambda}' \) is used for normalization instead of \( \bar{\lambda} \), operative rules for making cost-benefit comparisons become unnecessary complicated, as the corrective factor on the left hand side must be adjusted in relation to what tax instrument is used. This breaks with the basic idea of the modified Samuelson Rules, where the left hand side should take care of costs and benefits related to the public project under consideration, while the right hand side should take care of costs related to production of the public good and tax distortions as well as possible distributional effects related to tax funding. Thus, it could be concluded that it is less meaningful to normalize the shadow value of public revenue with the average marginal utility of gross income.

With the chosen formulation of the Lagrange-function in Jacobs (2018), see
(29), a better choice would have been to define MCF as:

\[ MCF \equiv \frac{1}{1 + \tau \lambda'}. \]  

(32)

Then he would not have found MCF to be sensitive to the choice of the untaxed good.

### 2.5 Some desirable properties of an MCF measure

Jacobs (2018) introduced an alternative MCF measure. The next section analyzes some properties of that measure. In preparation for this, corresponding properties of the traditional MCF-measure are analyzed in the following.

The individual budget constraints have two versions depending on the choice of tax instruments. With the consumption tax set to zero, they could be written:

\[ c_i = (1 - t)w_i h_i + a. \]  

(33)

With the income tax set to zero:

\[ (1 + \tau)c_i = w_i h_i + a. \]  

(34)

The latter could also be written:

\[ c_i = (1 - t_c)(w_i h_i + a), \]  

(35)

where

\[ t_c \equiv \frac{\tau}{1 + \tau}. \]  

(36)

A comparison of the equations (33) and (35) could be useful. It shows an important difference between the two tax instruments. While transfers are not subject to income taxation, a consumption tax implicitly applies to transfers also. Thus, if \( a \neq 0 \), there is an asymmetry with regard to effects of changing the income tax and the consumption tax. While increasing the consumption tax implicitly reduces the transfers received by individuals, increasing the income tax leaves the transfers
unchanged. Thus, if $a \neq 0$, changing the consumption tax will have different effects on both labour supply and income distribution compared to changing the income tax. It should therefore be emphasized that when $a \neq 0$, one can not always expect MCF to be insensitive to the choice of the untaxed good.

With the assumption that $a = 0$, this is different. Equations (33) and (35) are then reduced to:

$$c_i = (1 - t)w_i h_i,$$

$$c_i = (1 - t_c)w_i h_i,$$

(37)

(38)

The symmetry between equations (37) and (38), makes it clear that when $a = 0$, then the effects of a consumption tax and an income tax are identical both with respect to resource allocation and income distribution, and therefore also welfare. It follows that when $a = 0$, (marginal) costs related to public funding are always the same with the two tax instruments. This should to be reflected in an appropriate MCF-measure. Thus, if $a = 0$, and an income tax $t$ is replaced with a consumption tax $\tau = t/(1 - t)$, MCF should not change. Proposition 2, which follows below, shows that this is the case with the traditional definition of MCF. In contrast, in Section 3 is is shown that this is not always the case with the Diamond-based MCF-measure, see also the numerical examples.

In preparation for Proposition 2, note that if $a = 0$, it follows from (22) and (26) that:

$$MCF_t = \frac{1 + \delta_\lambda y}{\varepsilon_{Rt}},$$

$$MCF_\tau = \frac{1 + \delta_\lambda c}{1 + \tau \frac{1}{\varepsilon_{R\tau}}},$$

(39)

(40)

where $\varepsilon_{Rt}$ and $\varepsilon_{R\tau}$ are the elasticities of public revenue $R$ with respect to the income tax $t$ and the consumption tax $\tau$, respectively.

**Proposition 2 (MCF without lump-sum taxes)** Assume that $a = 0$. Let $MCF_t$ represent the traditional measure of the marginal cost of public funds when $\tau = 0$ and $t = \tilde{t} \geq 0$. Correspondingly, let $MCF_\tau$ represent the case where
\[ \hat{r} = \frac{\tilde{t}}{1 - \tilde{t}} \text{ and } t = 0. \text{ Then we have that} \]

\[ MCF_\hat{r} = MCF_\tilde{t} \]  \hspace{1cm} (41)

irrespective of whether the solution is second best or not.

**Proof.** See appendix A. ■

### 3 Some properties of MCF with the Diamond-based definition

Due to the alleged undesirable properties with the traditional MCF-measure, Jacobs (2018) recommends another definition. In the following some properties of the proposed MCF-measure is analyzed. Because Håkonsen (1998) and Jacobs (2018) are starting points for this, their setup for individuals’ utility maximization is followed. They applied an individual Lagrange function corresponding to the formulation in (29). The envelope theorem now gives that:

\[ \frac{\partial v_i}{\partial a} = \lambda_i, \]  \hspace{1cm} (42)

\[ \frac{\partial v_i}{\partial t} = -\lambda_i w_i h_i, \]  \hspace{1cm} (43)

\[ \frac{\partial v_i}{\partial \tau} = -\lambda_i c_i. \]  \hspace{1cm} (44)

The public budget constraint is given by (12) and the Lagrange-function in (13) is again applied to the government’s maximization problem. Using (42)-(44), the
first order conditions with respect to the three tax parameters then become:

\[
\mu \left( n - \tau \sum_i \frac{\partial c_i}{\partial a} - t \sum_i w_i \frac{\partial h_i}{\partial a} \right) = \sum_i \lambda_i', \quad (45)
\]

\[
\mu \sum_i \left( \frac{\tau \partial c_i}{\partial \tau} + w_i h_i + t w_i \frac{\partial h_i}{\partial t} \right) = \sum_i \lambda_i' w_i h_i, \quad (46)
\]

\[
\mu \sum_i \left( c_i + \tau \frac{\partial c_i}{\partial \tau} + t w_i \frac{\partial h_i}{\partial \tau} \right) = \sum_i \lambda_i' c_i. \quad (47)
\]

When the consumption tax is zero, i.e. \( \tau = 0 \), the f.o.c. with respect to public consumption, \( G \), given in (17), together with the first order conditions (45) and (46) as well as the public budget constraint, give the second-best levels of the shadow value of public revenue, \( \mu \), public consumption, \( G \), the uniform lump-sum transfer, \( a \), and the income tax rate \( t \). If an consumption tax, \( \tau \), is applied instead of an income tax, the the f.o.c. (47) applies instead of (46).

When the traditional MCF-measure was defined in the previous section, I followed Sandmo (1998) and normalized by dividing with the average marginal utility of net income. Håkonsen (1998) and Jacobs (2018) suggested to divide by the average social marginal value of income as defined by Diamond (1975):

**Definition 2 (Diamond-based MCF)** Based on Diamond (1975), define the social marginal value of income to individual \( i \) as:

\[
\alpha_i = \lambda_i' + \mu \left( t w_i \frac{\partial h_i}{\partial a} + \tau \frac{\partial c_i}{\partial a} \right). \quad (48)
\]

and let \( \bar{\alpha} \) be its average. The Diamond-based MCF-measure is defined as

\[
MCF^D \equiv \frac{\mu}{\bar{\alpha}}, \quad (49)
\]

where \( \mu \) is the shadow value of public revenue.

By inserting \( \bar{\alpha} \), as defined above, into (46) and (47), we obtain the proposed
MCF-measures:

\[ MCF^D_t = \frac{1 + \delta_{\alpha y}}{1 + \varepsilon_y + \rho_y}, \]  
\[ (50) \]
\[ MCF^D_\tau = \frac{1 + \delta_{\alpha c}}{1 + \varepsilon_\tau + \rho_c}, \]  
\[ (51) \]

where

\[ \delta_{\alpha y} = \frac{\text{cov}(\alpha_i, y_i)}{\bar{\alpha}y}, \]  
\[ (52) \]
\[ \delta_{\alpha c} = \frac{\text{cov}(\alpha_i, c_i)}{\bar{\alpha}c}, \]  
\[ (53) \]
\[ \rho_y = t \sum_i y_i w_i \frac{\partial h_i}{\partial a}, \]  
\[ (54) \]
\[ \rho_c = \tau \sum_i c_i \frac{\partial c_i}{\partial a}. \]  
\[ (55) \]

It should here be noted that the denominator in (50) could be written as in the following expression:

\[ MCF^D_t = \frac{1 + \delta_{\alpha y}}{1 + \theta^H_t} \]  
\[ (56) \]

where

\[ \theta^H_t = \sum_i \frac{y_i}{y} \varepsilon^H_{h_{i,t}} \]  
\[ (57) \]

and where \( \varepsilon^H_{h_{i,t}} \) is the elasticity of the Hicksian (compensated) labour supply with respect to the tax rate. As pointed out by Håkonsen (1998), this means that when \( \tau = 0 \), the Diamond-based definition of MCF is what Ballard and Fullerton (1992) labeled the Pigou-Harberger-Browning approach, see also Wildasin (1984). According to Ballard and Fullerton (1992, p. 119), the "Pigou-Harberger-Browning approach compares a distortionary tax with an equal-revenue lump-sum tax". They concluded that the concept has relevance and is on firm ground if the marginal
public "project" is a lump-sum rebate to the taxpayers.

With the assumption that \( a = 0 \), we could simplify (50) and (51) to:

\[
MCF^D_t = \frac{1 + \delta_{ay}}{\varepsilon_{Rt} + \rho_y},
\]

(58)

\[
MCF^D_\tau = \frac{1 + \delta_{ac}}{\varepsilon_{R\tau} + \rho_c}.
\]

(59)

First, consider (58). Because marginal utility of consumption is decreasing with income, it is reasonable to assume that \( \text{cov}(\alpha_i, y_i) \) is negative. Hence, the numerator is positive. With regard to the denominator, \( \varepsilon_{Rt} \) is zero at the top of the Laffer curve. The second term of the denominator, \( \rho_y \), is negative because leisure is assumed to be a normal good. This means that \( MCF^D_t \) approaches infinity at a level of \( t \) which is to the left of the top of the Laffer-curve.

Next consider (59), i.e. the case where \( t = 0 \) and a consumption tax is applied instead. It is still reasonable to assume that the numerator is positive while \( \varepsilon_{R\tau} \) is also zero at the top of the Laffer curve. The second term of the denominator \( (\rho_c) \) is positive because consumption is a normal good. This means that \( MCF^D_\tau \) is positive along the entire upward sloping part of the Laffer-curve. Moreover, \( MCF^D_\tau \) is positive also for tax levels where \( -\rho_y < \varepsilon_{R\tau} < 0 \), which is along the downward sloping part of the Laffer-curve. This reveals that \( MCF^D_\tau \) could be different from \( MCF^D_t \) for the same effective tax rate \( t^* \) also when \( a = 0 \). This is an undesirable property of the Diamond base MCF-measure, because tax-based funding of a public project in these two cases gives exactly the effective tax rates, and identical effects on all individuals' private consumption, labour supply, and utility. Hence, the MCF should have been identical in those two cases, see the discussion in Section 2.5.

3.1 The Diamond-based MCF-measure in a model with a single individual

The previous subsection analyzed properties of the Diamond-based MCF measure in the case where \( a = 0 \). It was found that also in that case, the Diamond-based MCF could be sensitive to the choice of the untaxed good. However, it was
not clarified whether this unfortunate property with the Diamond-based MCF measure also applies to second best, given that lump-sum taxes (or transfers) are not available. This questions is more easily analyzed for the case where \( n = 1 \).

With \( n = 1 \), the expressions in (50) and (51) could be simplified to:

\[
MCF_t^D = \frac{1}{1 + \frac{t}{h} \frac{\partial h}{\partial t} + tw \frac{\partial h}{\partial a}}, \tag{60}
\]

\[
MCF_\tau^D = \frac{1}{1 + \frac{\tau}{c} \frac{\partial c}{\partial \tau} + \tau \frac{\partial c}{\partial a}}. \tag{61}
\]

From (36) we have that \( t_c \equiv \frac{\tau}{1 + \tau} \). Assuming that \( a = 0 \), the latter expression could be written (for details, see Appendix C):

\[
MCF_\tau^D = \frac{1}{1 + (1 - t_c) \frac{t_c}{h} \frac{\partial h}{\partial t_c} + t_c w \frac{\partial h}{\partial a}}. \tag{62}
\]

Comparing (60) and (63) shows that if \( a = 0 \) and \( n = 1 \), then \( MCF_t^D = MCF_\tau^D \) if, and only if, \( t = \tau = 0 \). This makes it clear that when lump-sum taxes or transfers are unavailable as a marginal source for public funding, then the size of the Diamond-based MCF-measure could be sensitive to the choice between the untaxed good, also in second best.

### 4 Numerical illustrations

This section present simulations of a numerical model with two individuals were carried out. The purpose is to illustrate some of the theoretical results in the preceding sections, primarily the result that the traditional MCF-measure is not sensitive to the choice of the untaxed good in cases where that would have been undesirable, while the Diamond MCF-measure in contrast is sensitive to the choice of the untaxed good in some of these cases.

It should be emphasized that the model simulations could not be used to draw any conclusions about the size of MCF. For example, the result that MCF is 1.1
Figure 1: A numerical example with two individuals with different labour productivities \((w_1 = 1 \text{ and } w_2 = 2)\). The horizontal axis measures public consumption. Funding source is either an income tax or a consumption tax. The dotted line shows the aggregate utility of the two consumers (welfare). The green, double-lined curve shows the traditional MCF-measure, which is the same in both cases (consumption tax or income tax). The black, solid line and the blue, dashed line show MCF with the Diamond-based definition and an income tax and a consumption tax, respectively.
in a second-best case where lump-sum taxes are unavailable is highly sensitive to
different numerical assumptions made. For example, if the individuals’ marginal
utility of public consumption had been assumed to be lower, the optimal size of
the public sector would have been lower, with correspondingly lower tax level and
MCF in second best.

The model has the structure of the theoretical model of the preceding sections,
where the government maximizes the sum of the individual utilities. The util-
ity functions were specified as CES-functions that include private consumption,
leisure, and public consumption. The elasticity of substitution between leisure
and private consumption was assumed to be 1.25, which means that labour supply
is upward sloping. It was assumed that individual 2 has a productivity two times
as high as individual 1, i.e. \( w_1 = 1 \) and \( w_2 = 2 \). For further details about the
model, see Appendix B.

With the chosen parameters and assuming that the lump-sum transfer \( a \) could
be set at any preferred level, the second best solution is achieved with public
consumption \( G = 17.39 \) which is funded with a lump-sum tax (negative transfer)
\( a = -6.5 \) combined with an income tax \( t = 0.14 \). Table 1 gives the corresponding
rates in the case with a consumption tax combined with a lump-sum tax. In this
second best solution (in both cases, irrespective of the considered tax instrument),
and with the traditional definition it was found that \( MCF = 0.89 \), while with the
Diamond-based definition, \( MCF^D = 1 \).

Simulations were also carried out when the lump-sum transfer were assumed
to be zero \( (a = 0) \), see Figure 1. The horizontal axis measures consumption of
the public good. The left vertical axis measures MCF while the right vertical axis
measures welfare.

The green double-line represents the traditional MCF-measure with respect to
both an income tax and a consumption tax. The black solid line represents the
Diamond-based MCF-measure when an income tax is applied. The blue dashed
line represents the Diamond-based MCF-measure when a consumption tax is ap-
plicated. In accordance with the results in the previous section, the dashed blue
line does not consequently coincide with the black line, representing the Diamond-
based MCF-measure with an income tax. Welfare maximum is reached when
public consumption \( G = 14.7 \). At this point the traditional \( MCF = 1.11 \) irrespec-
Table 1: Simulation of second best in two cases with different restrictions on the tax system.

<table>
<thead>
<tr>
<th>Case with income tax:</th>
<th>Lump-sum taxes are available</th>
<th>Lump-sum taxes are not available</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lump-sum transfer</td>
<td>$a$</td>
<td>-6.50</td>
</tr>
<tr>
<td>Income tax</td>
<td>$t$</td>
<td>0.14</td>
</tr>
<tr>
<td>MCF (Diamond)</td>
<td></td>
<td>1.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case with consumption tax:</th>
<th>Lump-sum transfer</th>
<th>Consumption tax</th>
<th>MCF (Diamond)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a$</td>
<td>$\tau$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-7.58</td>
<td>0.17</td>
<td>1.04</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.00</td>
<td>1.77</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Both cases:</th>
<th>MCF (traditional)</th>
<th>Public consumption</th>
<th>Welfare</th>
<th>Distributional characteristic of $G$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$G$</td>
<td>$W$</td>
<td>$\delta_2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.89</td>
<td>17.39</td>
<td>45.65</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.11</td>
<td>9.01</td>
<td>42.41</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.11</td>
<td>-0.09</td>
<td></td>
</tr>
</tbody>
</table>

tive of whether an income tax or a consumption tax is applied. In contrast, the Diamond-based MCF-measure is sensitive to the choice of tax instrument also in this second best solution (given that $a = 0$): The Diamond MCF-measure when an income tax is applied is 8.71, while it is 1.77 when a consumption tax is applied.

5 Conclusion

Jacobs (2018) challenges previous research with regard to the question of possible costs of taxation and optimal supply of tax-funded public goods. If the conclusions in Jacobs (2018) are confirmed by further research, they have important consequences. Accordingly, influenced by Jacobs’ analyses the Dutch government recently decided to set the marginal cost of public funds to one in cost benefit analyses of public projects.

Jacobs (2018) found that the traditional definition of MCF has some undesirable properties. First, it was considered as a shortcoming that MCF of lump-sum taxes are different from one even when they are optimized. Second, that MCF is not directly related to the marginal excess burden of taxation (MEB). Third,
that the traditional measure of MCF is found to be sensitive to the choice of the untaxed good.

In accordance with the first element of the critique of the traditional MCF, the right hand side of (21) shows that if the income tax is positive, MCF of the lump-sum tax is less than one. The reason is that the lump-sum tax has no distortionary substitution effects, but has an income effect which draws in the direction of increased labour supply. If there is a positive income tax, which has caused an inefficiently low labour supply, increased labour supply represents an efficiency gain. This is an essential point of both Stiglitz and Dasgupta (1971) and Atkinson and Stern (1974), and their critique of Pigou (1947). The same arguments apply to a lump-sum tax combined with a consumption tax, cf. (25) and Proposition 1. Hence, it might be argued that we are dealing with a strength with the traditional definition of MCF, not a shortcoming.

With regard to the second element of the critique, that the traditional MCF is not directly related to MEB, Jacobs (2018) argues that Pigou (1947), Harberger (1964), and Browning (1976) suggested such a relationship. It is outside the aspirations of this paper to discuss this question thoroughly. At this point I simply refer to the discussion in contributions such as Stiglitz and Dasgupta (1971), Atkinson and Stern (1974) and Ballard and Fullerton (1992).

The present paper analyzes in more detail the question of whether the traditional MCF-measure is sensitive to the choice of the untaxed good. That result in Jacobs (2018) was not confirmed. On the contrary, it was found that MCF is the same in second best irrespective of the choice of the untaxed good. The crucial point is simply that MCF must be defined as the shadow value of public revenue divided by the average marginal utility of net income in both cases. Jacobs (2018) found the traditional MCF to be sensitive to the choice of tax instrument in second best because he in the case with a consumption tax divided the shadow value of public revenue with the average marginal utility of gross income.

Because Jacobs (2018) found that the traditional MCF-measure was undesirable, another and less frequently used definition was suggested and applied. With this definition the shadow value of public revenue was normalized with what Diamond (1975, p. 338) defined as the social marginal utility of private income. The present paper studied some properties of both MCF concepts in a case where
the lump-sum transfer was set to zero. In that case public revenue collection with a consumption tax has exactly the same effects on distribution and resource allocation as revenue collection with an income tax. Hence, it does not make sense that an MCF-measure in this case is sensitive to the choice of tax instruments, irrespective of whether the solution is second best or not.

The traditional MCF passed the test as it is always insensitive to the choice of tax instrument when \( a = 0 \).

In contrast, with the Diamond-based MCF-measure, the size of MCF is sensitive to the choice of tax instrument when \( a = 0 \). It was shown that in the case where \( n = 1 \), this unfortunate property of the considered MCF-measure could also apply to second best.

Finally, it should be noted that Jacobs’ conclusion that the marginal cost of public funds is one in second best is based on an assumption that lump-sum-taxes could be a marginal source of public funding. This assumption is in contrast to many important contributions to the literature, where the starting point usually has been that the possibilities for lump-sum taxation is limited and therefore not should be considered as a marginal source of finance (Pigou, 1947; Stiglitz & Dasgupta, 1971; Diamond & Mirrlees, 1971; Atkinson & Stern, 1974). Jacobs argues that reducing for example tax credits could have the properties of lump-sum taxation. However, it is not specified what type of tax credits he has in mind. In practice it might be difficult to find good examples of tax credits with pure lump-sum properties. Hence, in practice the assumption that lump-sum taxation could be a marginal source for public funding might be too optimistic.
Appendices

A Proofs

Proof of Proposition 1. First, assume that the second best allocation of resources is achieved with a combination of a consumption tax $\hat{\tau}$ and a lump-sum transfer $\hat{a}$ (while $t = 0$). Then an income tax $\hat{\tilde{t}} = \hat{\tau}/(1 + \hat{\tau})$ which is combined with a lump-sum transfer $\tilde{a} = \hat{a}/(1 + \hat{\tau})$, will also lead to be the second best allocation because the effective tax rates and net transfers are the same in the two cases.

In the case where $t = 0$, while $\tau = \hat{\tau}$ and $a = \hat{a}$, we have from the budget constraint (4) that

$$\frac{\partial c_i}{\partial \hat{a}} = \frac{1}{1 + \hat{\tau}} \left( 1 + w_i \frac{\partial h_i}{\partial \hat{a}} \right). \quad (A.1)$$

Because $\tilde{a} = \hat{a}/(1 + \hat{\tau})$, we have that

$$\frac{\partial h_i}{\partial \tilde{a}} = \frac{1}{1 + \frac{1}{\hat{\tau}}} \frac{\partial h_i}{\partial \hat{a}}. \quad (A.2)$$

Plugging (A.2) into (A.1) gives that

$$\frac{\partial c_i}{\partial \tilde{a}} = \frac{1}{1 + \hat{\tau}} \left( 1 + w_i \frac{1}{1 + \frac{1}{\hat{\tau}}} \frac{\partial h_i}{\partial \tilde{a}} \right). \quad (A.3)$$

Inserting this expression into equation (25) gives that when $t = 0$ we have that

$$MCF_a = \frac{1}{1 - \frac{\hat{\tau}}{1 + \hat{\tau}} \frac{1}{n} \sum_i w_i \frac{\partial h_i}{\partial \tilde{a}}}, \quad (A.4)$$

which is equal to $MCF_a$ as defined in (21) if $t = \hat{\tau}/(1 + \hat{\tau})$.

Because leisure is assumed to be a normal good, $\partial h_i/\partial a < 0$. Then it follows from (A.4) that $MCF \leq 1$ irrespective of case considered. ■

Proof of Proposition 2. This proposition claims that if $a = 0$ and $\tau = t/(1 - t)$,
then always $MCF_t = MCF_{\tau}$. This is true only if

$$\frac{1 + \delta_{\lambda_v}}{\varepsilon_{Rt}} = \frac{1 + \delta_{\lambda_c}}{(1 + \tau)\varepsilon_{Rt}},$$

(A.5)

cf. (39) and (40).

First, define:

$$q = \frac{1 - t}{1 + \tau}.$$  

(A.6)

Starting with the denominator of the l.h.s. of equation (A.5) and the case where $a = \tau = 0$. Define $Y = \sum_i y_i$. We could then define the function

$$Y = Y(q, w_1, ...., w_n, G).$$  

(A.7)

We have that when $a = 0$, then $R = tY$ and it follows that

$$\frac{\partial R}{\partial t} = Y + \frac{\partial Y}{\partial q} \frac{\partial q}{\partial t} = Y - t \frac{\partial Y}{\partial q}. \tag{A.8}$$

Thus, the denominator of the l.h.s. of equation (A.5) could be written as follows:

$$\varepsilon_{Rt} = 1 - \frac{t \frac{\partial Y}{Y}}{\partial q}. \tag{A.9}$$

Next, consider the denominator of the r.h.s. of equation (A.5) and the case where $a = t = 0$. Then we have that

$$R = \tau c = \frac{\tau}{1 + \tau} y.$$  

(A.10)

We have that

$$\frac{\partial Y}{\partial \tau} = \frac{\partial Y}{\partial q} \frac{\partial q}{\partial \tau} = -\frac{\partial Y}{\partial q} \frac{1}{(1 + \tau)^2}, \tag{A.11}$$

and it follows that

$$\frac{\partial R}{\partial \tau} = \frac{1}{(1 + \tau)^2} Y - \frac{\tau}{(1 + \tau)^3} \frac{\partial Y}{\partial q}. \tag{A.12}$$
Thus, we have that
\[(1 + \tau)\varepsilon_R r = \left(1 - \frac{\tau}{1 + \tau} \frac{1}{Y} \frac{\partial Y}{\partial q}\right)\]  
(A.13)

Taking into account that \(\tau = t/(1 - t)\) and (A.9), it follows that the denominator of the l.h.s is equal to the denominator of the r.h.s. of equation (A.5).

Finally, consider the numerators of of equation (A.5). It follows that when \(a = 0\), then \(c_i = qy_i\) and

\[\text{cov}(\lambda_i, c_i) = q \cdot \text{cov}(\lambda_i, y_i).\]  
(A.14)

This means that \(\delta_{\lambda y} = \delta_{\lambda c}\) if \(a = 0\), cf. (23) and (27). Hence, the numerator of the l.h.s. is equal to the numerator of the r.h.s. of equation (A.5).

\[\Box\]

B Description of the model used in the numerical example

In the numerical model used for illustrative purposes in section 4, there are \(n = 2\) individuals with the following utility function:

\[u_i = x \left(\alpha^{1-\rho} c_i^{\rho} + \beta^{1-\rho} l_i^{\rho} + \gamma^{1-\rho} G^{\rho}\right)^{\frac{1}{\rho}}\]  
(B.1)

where \(x, \alpha, \beta, \gamma,\) and \(\rho\) are parameters. Define

\[s = \frac{1}{1 - \rho}.\]  
(B.2)

\(s\) is the elasticity of substitution between consumption and leisure and was set to 1.25. The parameters \(\alpha\) and \(\beta\) where calibrated such that \(\alpha^{1-\rho} = 0.3\) and \(\beta^{1-\rho} = 0.7\). The wage rates for the two individuals were 1 and 2, respectively.

The complete list of applied parameter values follows:
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>0.45</td>
<td>γ</td>
<td>0.25</td>
</tr>
<tr>
<td>α</td>
<td>0.22</td>
<td>β</td>
<td>0.64</td>
</tr>
<tr>
<td>ρ</td>
<td>0.20</td>
<td>s</td>
<td>1.25</td>
</tr>
<tr>
<td>T</td>
<td>24.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

C The Diamond-based MCF when $a=0$

We consider a one-person economy in two cases, one with an income tax and one with a consumption tax:

\[
c = (1 - t_c)wh + a, \quad (C.1)
\]

\[
(1 + \tau)c = wh + a. \quad (C.2)
\]

The latter equation could be written as follows:

\[
c = (1 - t_c)(wh + a), \quad (C.3)
\]

where

\[
t_c = \frac{\tau}{1 + \tau}, \quad (C.4)
\]

which means that

\[
\tau = \frac{t_c}{1 - t_c}, \quad (C.5)
\]

\[
\frac{\partial t_c}{\partial \tau} = (1 - t_c)^2. \quad (C.6)
\]
It follows that
\[
\frac{\partial c}{\partial \tau} \mid_{a=0} = \left( -y + (1 - t_c) \frac{\partial y}{\partial t_c} \right) (1 - t_c)^2, \tag{C.7}
\]
\[
\frac{\partial c}{\partial a} \mid_{a=0} = (1 - t_c)\left( w \frac{\partial h}{\partial a} + 1 \right). \tag{C.8}
\]

We have that
\[
MCF_{\tau}^D = \frac{1}{1 + \frac{\tau}{c} \frac{\partial c}{\partial \tau} + \frac{\partial c}{\partial a}}. \tag{C.9}
\]

Using both (C.6) and (C.7), the denominator in (C.9) could be written
\[
1 + \frac{\tau}{c} \frac{\partial c}{\partial \tau} + \frac{\partial c}{\partial a} = 1 + \frac{\tau}{c} \left( -y + (1 - t_c) \frac{\partial y}{\partial t_c} \right) (1 - t_c)^2 + \tau (1 - t_c)\left( w \frac{\partial h}{\partial a} + 1 \right)
\]
\[
= 1 + \tau \left( -y + (1 - t_c) \frac{\partial y}{\partial t_c} \right) \frac{(1 - t_c)^2}{(1 - t_c)wh} + (1 - t_c)\left( w \frac{\partial h}{\partial a} + 1 \right)
\]
\[
= 1 + t_c \left( -1 + \frac{1 - t_c}{y} \frac{\partial y}{\partial t_c} + \left( w \frac{\partial h}{\partial a} + 1 \right) \right)
\]
\[
= 1 + (1 - t_c)\frac{t_c \frac{\partial h}{h \partial t_c}}{h \partial t_c} + t_c w \frac{\partial h}{\partial a}.
\]
References


