Heterogeneous Capital Tax Competition in a Federation with Tax Evasion

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Abstract

We consider a federal country with two regions. Individuals have to decide how to share their savings between two types of capital investments, and whether to invest in their region of residence or in the other one. We analyse how such decision is affected by the fact that one type of capital is taxed at a regional level while the other one is taxed at a federal level, and for the latter a different degree of tax evasion may arise across regions. We show how tax evasion arising at a federal level affects not only the federal tax policy but also the regional tax policies both directly and indirectly because of vertical tax competition. In particular, we show under which conditions a decrease in the level of tax compliance on the second type of capital can lead to a reduction in the federal tax rate on such type of capital investment, and simultaneously to an increase in the regional tax rate on the other type of capital investment.

Keywords: Fiscal federalism; Tax Competition, Tax evasion.
JEL Classification: H2; H41; H71; H77.

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1 Introduction

Capital is generally recognized as highly mobile, implying that capital flows across regions are significantly influenced also by tax policies chosen at a federal and local level. However, in their activity to impose taxes on different types of tax bases, federal and local governments’ decisions are constrained not only by the intensity of both horizontal and vertical tax competition which typically arise in federal set-ups, but also by dissimilar tax evasion opportunities across different regions, which can be due to several reasons. Examples include the fact that in regions that are dissimilar with respect to their historical and/or cultural background, citizens may have a different propensity to tax compliance or, when taxes are claimed at a local level, regional tax authorities may be differently efficient in raising taxes. Taxes can also be more easily evaded when they are issued by one level of government rather than another one (again for possible different levels of efficiency in raising taxes this time at one tier of government with respect to the other one). Further, taxes on some capital items can be more easily evaded with respect to others due to the different features of the diverse types of capital investments. Indeed, capital is not homogeneous in its nature: For example, capital investments may differ with respect to their degree of international mobility, and they can involve not only tangible capital, i.e. equipment, structures, and other material inputs, but also intangible capital, i.e. research and development, patents, copy rights, advertising, employee training and costumer relations. Such concerns, thus, give rise to an interesting question to be asked, namely how, in a federal country, the interaction between horizontal and vertical capital tax competition can be affected by the presence of two types of capital investment which are taxed by different tiers of government, but only the taxation of one of them can be evaded.

To answer to the above question we consider a simple model which describes a federal country divided into two identical regions where consumers can decide not only the region where to invest, but also the type of capital investment. In particular, we consider a set-up with two types of capital investment where one is taxed at a regional level (according to the source-based principle) while the other one is taxed at a federal level with a different degree of tax compliance across regions. Specifically, regional taxes on the first type of capital investments are used to finance a local public good, and no evasion may occur while a uniform federal tax on the second type of capital investment is used to finance a national public good, and it can be evaded by individuals living in the two regions at a different extent. Our main result shows how evasion on one type of capital taxation decided by the federal government affects both federal and regional tax policies, and consequently the return to savings for consumers. In particular, we show under which conditions a decrease in capital taxation decided by the federal government affects both federal and regional tax policies, and consequently the return to savings for consumers. In particular, we show under which conditions a decrease in
the level of tax compliance on the second type of capital can lead to a reduction in the federal tax rate on such type of capital investment and simultaneously to an increase in the regional tax rate on the other type of capital investment.

In the economic literature, the interplay between horizontal and vertical tax externalities has been analysed in order to understand whether, in federal countries, equilibrium tax rates tend to be inefficiently high or low. The seminal paper by Keen and Kotsogiannis (2002) considers a model with a federal country composed by identical states to examine under which conditions one type of externality dominates the other given that the two point in opposite directions. They show that, at equilibrium, inefficiently high or low state taxes arise depending on the relative elasticity of the supply of savings and the demand for capital, and on the extent to which the states tax rents. When two federal countries are analysed, Janeba and Wilson (2004) show that more vertical tax competition may counter the inefficiencies due to horizontal tax competition, in terms of public goods underprovision. When, instead, a federation consists of two countries, one of which is unitary while the other one has a federal structure, being divided into two identical regions, Grazzini and Petretto (2007) show that, from a social point of view, the federal country may still set an inefficiently low tax rate, while the unitary country may instead choose an inefficiently high tax rate, at equilibrium.

However, the incentives for countries to compete for capital tax bases may be affected by the possibility of tax evasion, and a specific line of research has concentrated its attention on the effects of tax evasion and auditing issues on the strategic behaviour of policy-makers. Cremer and Gavhari (2000) analyse the effectiveness of tax coordination policies in an economic union when there is the possibility of tax evasion, i.e. policy-makers can decide both the level of taxation and the audit probability (linked to the audit technology). In a similar vein, Stöwhase and Traxler (2005) analyse how regional governments may use audit rates strategically, and the decentralized choice of them may be affected both by fiscal competition among regions and the type of the fiscal equalization scheme (gross or net revenue sharing). The simultaneous presence of tax evasion and different specifications of an equalization scheme, vertical or horizontal, is examined by Grazzini and Petretto (2012) who show its impact on the overprovision of local public goods due to vertical

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2 For recent surveys on capital tax competition see, for example, Keen and Konrad (2013), Genschel and Schwarz (2011), and Zodrow (2010).
3 See also Keen and Kotsogiannis (2003) for a paper that analyses the same type of question, but in a set-up where policy-makers act as revenue-maximising Leviathans.
4 Also Flochel and Madies (2002) analyse such counterdistortionary role in a Leviathan setting to show that when tax competition is more intense, public subsidies are more efficiently supplied at a federal level rather than at a regional level. Still in a Leviathan setting, Wrede (1996) examines the interaction between horizontal and vertical tax externalities to illustrate that uncoordinated Leviathans will not generally position themselves on the downward-sloping side of the Laffer curve for total tax revenue.
5 A related line of research concerns the implications of tax avoidance for tax competition (especially for multinational enterprises). See, for example, the survey by Zodrow (2010) for useful references.
6 Notice that the effects of a lower audit probability are very similar to those due to a decrease in the statutory tax rate.
fiscal externality.

Of course, reality tends to be particularly complex and, in the tax competition literature, the models used are highly stylized. In particular, “capital” is generally considered as a homogeneous and divisible good and “can be read as a metaphor for anything that is mobile internationally and generates real output where it is applied” (Keen and Konrad (2013), p. 8). Capital, however, is heterogeneous in its nature, and from an optimal taxation perspective, in a model with heterogeneous capital and constant returns to scale, for example, Auerbach (1979) shows that it is generally nonoptimal to tax uniformly different types of capital.\footnote{For an analysis of optimal taxation in a set-up with two types of capital, i.e. tangible and intangible capital, see Hagen and Kanniainen (1995).} Within the literature on capital tax competition, the possibility to apply different tax rules to different types of capital tax base has been analysed with respect to preferential regimes that countries may use in order to discriminate usually in favour of those activities which are more mobile from an international viewpoint. Both at the OECD and EU level, such measures are considered socially undesirable because they are seen as harmful forms of tax competition which can make even worse the distortions due to tax competition. On such point of view there is not however an unanimous consensus. On the one hand, in a model with two different tax bases and two symmetric countries whose governments maximise tax revenue, Keen (2001) shows that, on the contrary, countries may use preferential regimes to compete for the most mobile tax bases, thus protecting less mobile tax bases from erosion due to a too fierce tax competition. In other words, if countries had to set a uniform tax rate on both tax bases, tax revenue in each country would be lower with respect to the set-up with preferential tax regimes. Such result has been confirmed also for the case of asymmetric countries with respect to their population size by Bucovetsky and Hauffer (2007). On the other hand, Janeba and Smart (2003) show that restrictions on preferential tax regimes need not be revenue decreasing when aggregate tax bases are elastic, and not exogenously given as in Keen (2001). The same type of result is also obtained by Haupt and Peters (2005) when aggregate tax bases are exogenously fixed, but in each country investors have a home bias.

The plan of the paper is as follows. Section 2 describes the model, and Section 3 analyses the individual decision on consumption and saving taking into account the possibility to choose whether to invest in one type of capital or in the other one, and the location of the capital investment, i.e. in the region of residence or in the other one. Section 4 examines how tax evasion can affect both the federal and the regional tax policy, and finally Section 5 contains some concluding remarks.

\section{The model}

Consider a federal country divided into two identical regions $i = A, B$, and suppose that in each region there is one individual $i = A, B$, with the following preferences:

\begin{equation}
U_i = U(C_i^1) + C_i^2 + g_i + \frac{G}{2}, \quad i = A, B,
\end{equation}
where $U(.)$ is a well-behaved utility function, $C^1_i$, $C^2_i$ denote individual consumption in period 1 and 2, respectively, and $g_i$, $G$ denote a local public good provided by the local government $i$ and a national public good provided by the national government, respectively.\(^8\) In the first period, in each region, every individual owns the same fixed endowment $E$ of income, and she decides how much to consume, how much to invest, the type of investment, i.e. in tangible capital and/or in intangible capital, and where to invest. The individual first and second period budget constraint obtains as

$$E = C^1_i + \sum_{j=A,B} k^j_i + \sum_{j=A,B} d^j_i, \quad i = A, B, \tag{2}$$

and

$$C^2_i = \sum_{j=A,B} (1 + r^j - t^j) k^j_i + \sum_{j=A,B} [1 + s^j - (1 - \alpha^j)\tau]d^j_i, \quad i = A, B, \tag{3}$$

where $k^A_i$ ($d^A_i$), $k^B_i$ ($d^B_i$) denote the investment in the first (second) type of capital made by an individual living in region $i$ in region $A$, $B$, respectively.\(^9\) In each region $i = A$, $B$, $r^i$ ($s^i$) denotes the gross remuneration of the first (second) type of capital earned by the capital investor, and $t^i$ ($\tau$) represents the regional (national) tax rate on the first (second) type of capital investment. In particular, we assume that the first type of capital is taxed at a regional level according to the source based principle while the second type of capital is taxed at a federal level. Contrary to the first type of capital investments, those in the second type of capital are characterized by the possibility to evade taxation, i.e. there exists a tax-gap between the tax base and the effective amount assessed by a national Fiscal Agency. Specifically, $\alpha^i, 0 \leq \alpha^i \leq 1, i = A, B,$ denotes the rate of revenue loss due to tax evasion on the second type of capital in each region.\(^10\) To simplify, $\alpha^i$ will be treated as an exogenous parameter but, of course, in an expected utility framework, it would be the result of a consumer choice on the optimal amount of revenue to be evaded given the probability to be discovered and the consequent fine. Our assumption of a positive value of evaded revenue implicitly refers to a set-up where, from the consumer’s point of view, tax evasion turns out to be convenient on the basis of the above calculus.

To sum up, in our model, the tax treatment of both types of capital depends on the location of the investment: In the case of the first type of capital, different tax treatments across regions are due to different tax rates while in the case of the second type of capital, different effective tax treatments across regions are due to different tax evasion opportunities, notwithstanding a common federal tax rate.\(^11\)

\(^8\)The model could easily be rephrased in terms of a confederation made of two countries.

\(^9\)Each type of capital is treated as an homogenous and divisible good without the possibility to distinguish foreign- and domestically-owned variants (Keen and Konrad (2013)).

\(^10\)In case our simple model were interpreted in terms of a confederation of states, it would describe in a highly stylized way the dissimilar propensity to evade capital taxation with respect to a much more complicated real set-up because of the existence of bilateral tax treaties between countries, and accordingly different methods which can be used to avoid taxation.

\(^11\)The model abstracts from the possibility that horizontal and/or vertical intergovernmental transfers could be implemented to offset the distortions due to both horizontal and vertical tax competition (for such an analysis, see...
In each region, the same consumption good $y^i$, $i = A, B$, is produced by adopting the same technology which uses the total amount of both types of capital invested within its borders as inputs, $K^i = \Sigma_{j=A,B} K^i_j$, and $D^i = \Sigma_{j=A,B} D^i_j$, $i = A, B$. More specifically, in each region $i$, we assume the following decreasing returns to scale production function for the final good $y^i$:

\[ y^i = f \left( K^i, D^i \right), \quad i = A, B, \] (4)

and, thus, profits obtain as

\[ \pi^i = y^i - r^i K^i - s^i D^i, \quad i = A, B, \] (5)

where the price of the consumption good $y^i$ is normalized to 1. The FOCs of this profit maximisation problem with respect to $K^i$ and $D^i$ obtains as

\[ f_{K^i}(.) = r^i, \]
\[ f_{D^i}(.) = s^i, \]

and accordingly, the demand for the first and the second type of capital is given by $K^i = K^i(r^i, s^i)$ and $D^i = D^i(s^i, r^i)$, respectively.

Finally, rents (5) arising in region $i$ from the production of the consumption good $y^i$ are supposed to be fully taxed at the local level,\(^1\) so that, for each region, the local public budget constraint obtains as

\[ g_i = t_i K^i + \pi^i, \quad i = A, B, \]

while the national public budget constraint obtains as

\[ G = \tau \sum_{j=A,B} (1 - \alpha^j) D^j. \]

Individual and public decisions are taken according to a three-stage game. At the first stage of the game, the federal government decides the optimal national tax rate on the second type of capital. At the second stage, in each region, the local government chooses its tax rate on the first type of capital invested within its border, taking as given the tax rate chosen by the other region and the federal tax rate on the second type of capital investments. At the third stage of the game, individuals in each region take their consumption and investment decisions.

3 Individual decision on consumption and saving

Let us solve the game by backward induction, and firstly consider the third stage of the game where each agent $i = A, B$, maximises (1) subject to (2) and (3). The FOCs with respect to $C^1_i$, $C^2_i$, $k^A_i$,

\(^1\)See Keen and Kotsogiannis (2002) and Grazzini and Petretto (2007) for analogous assumption.
\( k_i^B, d_i^A, \) and \( d_i^B \) obtain as

\[
\begin{align*}
C_i^1 : & \quad \frac{\partial U}{\partial C_i^1} = \mu, \\
C_i^2 : & \quad \gamma = 1, \\
k_i^j : & \quad \gamma (1 + r^j - t^j) = \mu, \\
d_i^j : & \quad \gamma [1 + s^j - (1 - \alpha^j)\tau] = \mu, \quad i,j = A,B,
\end{align*}
\]

where \( \mu \) and \( \gamma \) denote the Lagrangean multiplier associated to the first and second period individual budget constraint, respectively. Thus, the net return to savings denoted by \( \rho \), which differs from the cost of capital for firms because of taxation, obtains as follows

\[
\rho = r^i - t^i = s^i - (1 - \alpha^i)\tau, \quad i = A,B.
\]

This is a standard arbitrage condition which, however, refers to an asymmetric tax competition set-up given the different level of tax evasion in the two regions \( A \) and \( B \).\(^{13}\) Assuming full employment of capital allows us to obtain the market clearing condition:

\[
\sum_{j=A,B} K^j (\rho + t^j, \rho + (1 - \alpha^j)\tau) + \sum_{j=A,B} D^j (\rho + (1 - \alpha^j)\tau, \rho + t^j) = \Gamma(\rho),
\]

where \( \Gamma(\rho) \) denotes total savings in both types of capital, with \( \Gamma'(\rho) \geq 0 \). Accordingly, the previous equation determines the net return to savings as a function of the two regional tax rates on the first type of capital, the federal tax rate on the second type of capital, and the different degree of tax evasion in the two regions:

\[
\rho = \rho(t^A, t^B, (1 - \alpha^A)\tau, (1 - \alpha^B)\tau).
\]

Differentiating (7) with respect to \( t^i \) and \( \rho \) yields

\[
\frac{\partial \rho}{\partial t^i} = \frac{K^i_\rho + D^i_\rho}{\Gamma' - \sum_{j=A,B} (K^j_\rho + K^j_\mu) - \sum_{j=A,B} (D^j_\mu + D^j_\rho)}, \quad i = A,B,
\]

where \( \Gamma' \geq 0, K^i_\mu, D^i_\mu < 0, \) and \( K^i_\mu = D^i_\mu > 0, \) \( i = A,B, \) being the substitution matrix a symmetric, negative definite matrix from the SOC for strict profit maximization. In order to analyse the effects due to the existence of two types of capital investment, the taxation of one of which can be partly evaded, in what follows we make the following plausible assumptions on the magnitude of the response of the demand for each type of capital to changes in their gross remuneration:

Assumption 1: \( |K^i_\rho| > D^i_\rho \) and \( |D^i_\mu| > K^i_\mu, \) \( i = A,B, \) i.e. the direct effect of the gross remuneration of each type of capital on its demand is higher than the indirect effect of it on the demand for the other type of capital;

Assumption 2: \( |K^i_\mu| > K^i_\mu, \) and \( |D^i_\mu| > D^i_\mu, \) \( i = A, B, \) i.e. the direct effect of the gross remuneration of the first (second) type of capital on its demand is higher than the indirect effect

\(^{13}\) For a treatment of symmetric vs. asymmetric horizontal tax competition see Keen and Konrad (2013).
of the gross remuneration of the second (first) type of capital on the demand for the first (second) type of capital.

From (9), it is easy to check that an increase in the regional taxation of the first type of capital has a negative effect on the net return to savings:

\[-1 < \frac{\partial \rho}{\partial t_i} < 0, \quad i = A, B.\]

Similarly, differentiating (7) with respect to \( \tau \), and \( \rho \) yields

\[
\frac{\partial \rho}{\partial \tau} = \frac{\sum_{j=A,B} (1 - \alpha^j) K_{s_j}^j + \sum_{j=A,B} (1 - \alpha^j) D_{s_j}^j}{\Gamma - \sum_{j=A,B} (K_{r_j}^j + K_{s_j}^j) - \sum_{j=A,B} (D_{r_j}^j + D_{s_j}^j)},
\]

where it is easy to check that

\[-1 < \frac{\partial \rho}{\partial \tau} < 0,\]

i.e. an increase in the federal taxation of the second type of capital has a negative effect on the net return to savings.

To investigate the effects of tax evasion, we now concentrate our attention on how the net remuneration to savings depends on the different degree of tax evasion in the two regions. In this respect, we can state the following

**Lemma 1.** \( 0 < \frac{\partial \rho}{\partial \alpha_i} < \tau, \quad i = A, B. \)

**Proof.** See the Appendix. \( \Box \)

In line with intuition, an increase in the degree of tax evasion has a positive effect on the net remuneration of all sources of capital. However, notice that this result does not take into account how the regional governments can react to different degrees of tax evasion when choosing their tax policy on the first type of capital. As we show in what follows, even if tax evasion can only arise at a federal level on the second type of capital investments, it also affects regional tax choices on the first type of capital investments, and accordingly the net return to savings for consumers at the end of the game.

Further, from (10), it is easy to check that

\[
\frac{\partial r^i}{\partial t_i} > 0, \quad \frac{\partial r^i}{\partial t_i} < 0, \quad i = A, B,
\]

and from (12), it follows that

\[
-\alpha^i < \frac{\partial s^i}{\partial \tau} = \frac{\partial \rho}{\partial \tau} + 1 - \alpha^i < 1 - \alpha^i, \quad i = A, B.
\]

While results in (13) are standard, those in (14) are novel, because, for example, an increase in the federal capital tax rate does not necessarily lead to an increase in the cost of capital as it is the case for regional capital taxation, for which \( \frac{\partial r^i}{\partial \tau} > 0, \quad i = A, B. \) In the case of federal capital taxation, an increase in the tax rate can lead to an increase or a decrease in the cost of the second capital.
type of capital depending on the different degree $\alpha^i, i = A, B$, of tax evasion in the two regions. In particular, $\frac{\partial s^i}{\partial t^i}, i = A, B$, tends to be positive (negative), the lower (higher) the value of the degree of tax evasion, $\alpha^i$. In the extreme cases, when $\alpha^i = 0$ then $\frac{\partial s^i}{\partial t^i} > 0$, and when $\alpha^i = 1$ then $\frac{\partial s^i}{\partial t^i} < 0$, $i = A, B$.

In each region, demand for each type of capital depends on both the regional capital tax rates and the levels of regional tax evasion on the second type of capital: $K^i = K^i(r^i, s^i) = (r + t^i, \rho + (1 - \alpha^i)\tau)$ and $D^i(\rho + (1 - \alpha^i)\tau, \rho + t^i)$. By deriving such demands for capital with respect to the regional tax rates, $t^i$, we obtain the following

$$\frac{\partial K^i}{\partial t^i} = K^i_{r^i} \left( \frac{\partial \rho}{\partial t^i} + 1 \right) + K^i_{s^i} \frac{\partial \rho}{\partial t^i} < 0, \quad \frac{\partial K^{-i}}{\partial t^i} = (K^{-i}_{r^{-i}} + K^{-i}_{s^{-i}}) \frac{\partial \rho}{\partial t^i} > 0, \quad i = A, B,$$

and

$$\frac{\partial D^i}{\partial t^i} = D^i_s \frac{\partial \rho}{\partial t^i} + D^i_r \left( \frac{\partial \rho}{\partial t^i} + 1 \right) > 0, \quad \frac{\partial D^{-i}}{\partial t^i} = (D^{-i}_s + D^{-i}_r) \frac{\partial \rho}{\partial t^i} > 0, \quad i = A, B.$$  

In words, an increase in the regional tax rate in one region leads to a decrease (increase) in the amount of the first (second) type of capital invested in that region, and simultaneously to an increase in the amount of both types of capital invested in the other region.

Further, by deriving the demand for capital $K^i(\cdot)$ and $D^i(\cdot)$ with respect to the federal tax rate, $\tau$, we obtain the following

$$\frac{\partial K^i}{\partial \tau} = (K^i_{r^i} + K^i_{s^i}) \frac{\partial \rho}{\partial \tau} + (1 - \alpha^i)K^i_{s^i} > 0, \quad i = A, B,$$

and

$$\frac{\partial D^i}{\partial \tau} = D^i_s \frac{\partial \rho}{\partial \tau} + D^i_r \left( 1 - \alpha^i \right) > 0, \quad i = A, B.$$  

Because of assumption 2, $\frac{\partial K^i}{\partial \tau} > 0$, i.e. an increase in the national tax rate on the second type of capital leads to an increase in the first type of capital invested in each region of the same country. Instead, the sign of $\frac{\partial D^i}{\partial \tau}$ can be either positive or negative. In particular, $\frac{\partial D^i}{\partial \tau} \gtrless 0 \iff \alpha^i \gtrless 1 + \left( 1 + \frac{\partial D^i}{\partial \tau} \right) \frac{\partial \rho}{\partial \tau}$. In other words, when the degree of tax evasion on the second type of capital is sufficiently low, an increase in its taxation at a federal level leads to a reduction in its capital investment. On the contrary, when the degree of tax evasion is sufficiently high, an increase in the tax rate on the second type of capital leads to an increase in its capital investment.

Finally, by deriving the demand for capital $K^i(\cdot)$ and $D^i(\cdot)$ with respect to the regional degree of tax evasion on national taxation, $\alpha^i$, we obtain the following

$$\frac{\partial K^i}{\partial \alpha^i} = (K^i_{r^i} + K^i_{s^i}) \frac{\partial \rho}{\partial \alpha^i} - \tau K^i_{s^i} < 0, \quad (15)$$

$$\frac{\partial K^{-i}}{\partial \alpha^i} = \frac{\partial \rho}{\partial \alpha^i} (K^{-i}_{r^{-i}} + K^{-i}_{s^{-i}}) < 0, \quad i = A, B,$$

and

$$\frac{\partial D^i}{\partial \alpha^i} = D^i_s \left( \frac{\partial \rho}{\partial \alpha^i} - \tau \right) + D^i_r \frac{\partial \rho}{\partial \alpha^i} > 0, \quad (17)$$

$$\frac{\partial D^{-i}}{\partial \alpha^i} = \frac{\partial \rho}{\partial \alpha^i} (D^{-i}_s + D^{-i}_r) < 0, \quad i = A, B,$$  

(18)
where $\frac{\partial D^i}{\partial t^i} > 0$ because of Lemma 1. This means that an increase in the degree of tax evasion on the second type of capital in a region leads to a decrease (increase) in the first (second) type of capital invested in the same region while it leads to a decrease in the amount of both types of capital invested in the other region.

4 The effects of tax evasion on regional and federal tax policies

Let us now consider the second stage of the game. Suppose that inside each region $i$, $i = A, B$, a local government has to choose the tax rate on the first type of capital by taking as given the choice made by the other region, and the federal tax rate on the second type of capital. Accordingly, in each region $i$, the local government maximizes the indirect utility function of a representative agent with respect to the regional tax rate on the first type of capital:

$$\max_{t^i} V_i = U(1 + \rho)S_i(\rho) + (1 + \rho)S_i(\rho) + t^i \rho + \frac{1}{2} \tau \sum_{j=A,B} (1 - \alpha^j) D^j(\rho + (1 - \alpha^j)\tau, \rho + t^j), \quad i = A, B,$$

(19) where $S_i(\rho) = \sum_{j=A,B} k^j_i + \sum_{j=A,B} d^j_i$ denotes the individual savings of an agent living in region $i$, and recall that $\rho = \rho(t^A, t^B, (1 - \alpha^A)\tau, (1 - \alpha^B)\tau)$ from (8). The FOCs of this problem obtain as

$$F^i(t^i, t^{-i}, \alpha^i, \alpha^{-i}, \tau) = \frac{\partial \rho}{\partial t^i} S_i(\rho) + \left\{K^i \frac{\partial \rho}{\partial t^i} + K^i_s \frac{\partial \rho}{\partial t^s} \right\} - \left\{K^i \frac{\partial \rho}{\partial t^i} + 1\right\} + D^i \frac{\partial \rho}{\partial t^i} + \frac{1}{2} \tau \left\{(1 - \alpha^i) \left[D^i_s \frac{\partial \rho}{\partial t^s} + D^i_s \frac{\partial \rho}{\partial t^s} \right] + (1 - \alpha^{-i}) \frac{\partial \rho}{\partial t^s} \right\} = 0, \quad i = A, B.$$

(20) Each term in (21) has a straightforward interpretation as in the standard literature in capital tax competition, with the difference that, in our set-up, taxation on the first type of capital affects individual decisions not only with reference on how much to invest and in which region to invest, but also on whether to invest in the first or in the second type of capital whose taxation can evaded at a federal level. Accordingly, the tax rate on the first type of capital chosen at a regional level affects such individual decisions not only through the channel of the gross remuneration on the first type of capital, $r^i$, but also via the gross remuneration on the second type of capital, $s^i$.

More specifically, the first term in (21) represents the negative effect on the net remuneration to individual savings due to an infinitesimal increase in the regional tax rate on the first type of capital, $t^i$. The second term describes the sum of the direct and the indirect impact on regional tax revenue of an infinitesimal increase in $t^i$. As expected, the direct effect is positive while the indirect effect is negative when $t^i > 0$. This latter term captures the effect on the total amount of the first type of capital invested in region $i$ of an increase in $t^i$ both via the gross remuneration of the first type of capital, $K^i \frac{\partial \rho}{\partial t^i} + 1$, and via the gross remuneration of the second type of capital, $K^i_s \frac{\partial \rho}{\partial t^s}$. Together both effects describe the capital flight arising from region $i$ toward the other region following an infinitesimal increase in $t^i$. Accordingly, the other region benefit in terms
of an increase in its tax revenue, i.e. a positive horizontal externality. However, since such positive externality is not taken into account by region $i$, the latter perceives a negative indirect effect. The third term represents the change in terms of rent tax revenue due to an infinitesimal increase in $t_i$ which increases the cost of the first type of capital, but simultaneously it decreases the cost of the second type of capital in region $i$. From the point of view of the region $i$, the first part of such an effect is negative, i.e. $-K^i \left( \frac{\partial \rho}{\partial t^i} + 1 \right) < 0$, while the second part is positive, i.e. $-D^i \frac{\partial \rho}{\partial t^i} > 0$. Thus, the total effect will be negative (positive) when the effect through the first type of capital is higher (lower) than the effect through the second type of capital. For example, in case the total effect is negative, it also describes a horizontal externality given by the fact that the reduction in rent tax revenue in region $i$ corresponds to an increase in rent tax revenue in the other region. Finally, the fourth term describes a vertical externality that arises on the federal tax revenue from the second type of capital taxation following an infinitesimal change in the regional tax rate on the first type of capital. This term is given by the sum of two terms that are both positive: The first (second) one describes the increase in the federal tax revenue following a shift from investments in the first type of capital towards investments in the second type of capital in region $i$ ($-i$) due to an infinitesimal increase in $t^i$.

Conditions (21) define each region’s reaction function:

$$t^i = t^i(t^i(t^-i), \alpha^i, \alpha^{-i}, \tau), \quad i = A, B,$$

(22)

so that each regional tax rate on the first type of capital depends on the tax rate chosen by the other region, the federal tax rate, and the degree of tax evasion arising in both regions for the second type of capital. A Nash equilibrium of the game played by the regions is given by the solution to the system of the above reaction functions. Accordingly, by taking into account (22) into (8), at the Nash equilibrium, the net remuneration of capital obtains as

$$\rho = \rho(t^i(.), t^{-i}(.), \alpha^i, \alpha^{-i}, \tau) \quad i = A, B.$$

(23)

We are now in a position to analyse how tax evasion on federal taxation may affect regional taxation, at the second stage of the game. In order to pursue such an aim, let us firstly examine how the regional tax rate chosen in one region affects the one chosen in the other region. In this respect, we can state the following\textsuperscript{14}

**Proposition 1.** $\frac{\partial t^{-i}}{\partial t^i} > 0, \quad i = A, B.$

**Proof.** See the Appendix. \qed

Proposition 1 provides an unambiguous result: The standard result on strategic complementarity between regional capital tax rates also holds in the present framework where individuals can invest in two types of capital, and tax evasion is possible on the second one.

To proceed in our analysis on how tax evasion on the second type of capital may affect regional capital tax rates on the first type of capital, rewrite the optimality condition (21) as follows

\textsuperscript{14}For the sake of tractability, second cross derivatives are assumed negligible in the proofs where they appear.
At the Nash equilibrium, by taking into account (22), let us differentiate both first order conditions in (24) with respect to $\alpha^i$:

\[
\frac{\partial F^{-i}}{\partial \alpha^i} + \frac{\partial F^{-i}}{\partial t^i} \frac{dt^i}{d\alpha^i} + \frac{\partial F^{-i}}{\partial t^{-i}} \frac{dt^{-i}}{d\alpha^i} = 0, \\
\frac{\partial F^i}{\partial \alpha^i} + \frac{\partial F^i}{\partial t^i} \frac{dt^i}{d\alpha^i} + \frac{\partial F^i}{\partial t^{-i}} \frac{dt^{-i}}{d\alpha^i} = 0, \quad i = A, B.
\]

By solving this system of equations with respect to $\frac{dt^i}{d\alpha^i}$ and $\frac{dt^{-i}}{d\alpha^i}$, the following two equations obtain

\[
\frac{dt^i}{d\alpha^i} = \frac{1}{\Omega^i} \left( \frac{\partial F^i}{\partial t^i} \frac{\partial F^{-i}}{\partial \alpha^i} - \frac{\partial F^{-i}}{\partial t^{-i}} \frac{\partial F^i}{\partial \alpha^i} \right), \quad i = A, B, \\
\frac{dt^{-i}}{d\alpha^i} = -\frac{1}{\Omega^i} \left( \frac{\partial F^i}{\partial t^i} \frac{\partial F^{-i}}{\partial \alpha^i} - \frac{\partial F^{-i}}{\partial t^{-i}} \frac{\partial F^i}{\partial \alpha^i} \right), \quad i = A, B.
\]

where

\[
\Omega^i = \frac{\partial F^i}{\partial t^i} \frac{\partial F^{-i}}{\partial t^{-i}} - \frac{\partial F^i}{\partial t^{-i}} \frac{\partial F^{-i}}{\partial t^i}.
\]

To perform our analysis, first we sign $\Omega^i$ with the following

**Lemma 2.** $\Omega^i < 0$, $i = A, B$.

**Proof.** See the Appendix.$\square$

We are now able to examine how national tax evasion on the second type of capital affects both regional taxation on the first type of capital, and the net remuneration of all types of capital. With respect to the first point, we can state the following

**Proposition 2.** If $\frac{D_s^i}{D^i} > \left| \frac{1}{1 + \frac{\partial F^i}{\partial t^i}} \right|$ then $\frac{dt^i}{d\alpha^i} > 0$ and $\frac{dt^{-i}}{d\alpha^i} > 0$, $i = A, B$.

**Proof.** See the Appendix.$\square$

Proposition 2 shows under which condition an increase in the degree of tax evasion on the second type of capital investments leads to an increase in the regional tax rates on the first type of capital. The reason behind this result is a consequence of the horizontal tax competition that
arises between regions which, however, contrary to the standard literature, refers not only to the
decision on where to allocate capital investments but also on whether to invest in the first or in
the second type of capital.

In particular, notice that the condition underlying Proposition 2 deserves some comments.
Given that it must be \( \frac{D^i_i}{D^i_s} < 1 \) because of assumption 2, it must also be that \( \left| \frac{\partial \rho}{\partial t} + 1 \right| < 1 \) to
guarantee that \( \frac{D^i_i}{D^i_s} > \left| \frac{1}{1 + \frac{\partial \rho}{\partial t}} \right| \). It is easy to check that \( \left| \frac{\partial \rho}{\partial t} + 1 \right| < 1 \) holds iff \( \frac{\partial \rho}{\partial t} < \frac{1}{2} \), i.e. the
effect of an increase in the regional tax rate on the net remuneration of capital has to be not too
large in absolute value. This implies that a reduced interval, i.e. \( -\frac{1}{2} \leq \frac{\partial \rho}{\partial t} < 0 \), needs to hold
instead of the one in (10).

To deep our understanding of the channels through which tax evasion on the second type of
capital may affect the regional tax policies, we can now analyse the second point, i.e. its effect
on the net remuneration of capital at the Nash equilibrium of the second stage of the game. By
differentiating (23) with respect to \( \alpha^i \), the total effect of tax evasion on the net remuneration of
capital obtains as

\[
\frac{d\rho}{d\alpha^i} = \frac{\partial \rho}{\partial \alpha^i} + \sum_{j=A,B} \frac{\partial \rho}{\partial t^j} \frac{\partial t^j}{\partial \alpha^i}, \quad i = A, B. \tag{28}
\]

Notice that \( \frac{d\rho}{d\alpha^i} \) describes the effect of an infinitesimal increase in the second type of capital tax
evasion of region \( i, i = A, B \), on the net remuneration of capital in (23) both directly, i.e. \( \frac{d\rho}{d\alpha^i} \) (see
Lemma 1), and indirectly via the change in both regional tax rates on the first type of capital, i.e.
\( \sum_{j=A,B} \frac{\partial \rho}{\partial t^j} \frac{\partial t^j}{\partial \alpha^i}, \quad i = A, B \), where \( \frac{\partial \rho}{\partial t^j} \) obtains from (9) and \( \frac{\partial t^j}{\partial \alpha^i} \), \( i, j = A, B \) obtain from (25) and
(26).

In this respect, we can state the following

**Corollary 1.** If \( \frac{D^i_i}{D^i_s} > \left| \frac{1}{1 + \frac{\partial \rho}{\partial t}} \right| \) then \( 0 < \frac{d\rho}{d\alpha^i} < \frac{\partial \rho}{\partial \alpha^i}, \quad i = A, B. \)

**Proof.** See the Appendix. \( \square \)

Corollary 1 shows that the result on the positive effect of the second type of capital tax evasion
on the net remuneration of capital obtained at the third stage of the game (Lemma 1) is confirmed
at the Nash equilibrium of the second stage of the game under the condition enlightened in Corollary
1. However, such Corollary also shows that the magnitude of this effect is lower at the second stage
of the game with respect to the first stage because of the counter negative indirect effect of tax
evasion on the net remuneration of capital through the change in regional tax rates.

To conclude on the second stage of the game, now we examine the effect on the regional tax
rate of an infinitesimal increase in the federal tax rate. The result in the following Lemma will be
useful for the subsequent analysis:

**Lemma 3.** If \( \left| \frac{\partial \rho}{\partial t} \right| > 1 - \alpha^i \) then \( \frac{\partial \rho}{\partial t} > 0, \quad i = A, B. \)
Proof. See the Appendix. □

Lemma 3 shows that there is strategic complementarity between the federal tax rate on the second type of capital and the regional tax rate on the first type of capital under the condition that $|\frac{\partial \rho}{\partial \tau}| > 1 - \alpha^i$. Notice that the latter requires that the absolute value of the effect of the federal tax rate on the net remuneration of capital needs to be sufficiently large. In other words, similarly to what noticed above with respect to the condition underlying both Proposition 2 and Corollary 1, a reduced interval $-1 < \frac{\partial \rho}{\partial \tau} < -(1 - \alpha^i)$ needs to hold instead of the one in (12).

Let us now move to the first stage of the game where the federal government maximises a social welfare function which is given by the sum of the indirect utility functions of the residents of the two regions. In particular, its problem obtains as

$$\max_{\tau} \sum_{j=A,B} \left[ U \left( E - S_j(\rho) \right) + (1 + \rho)S_j(\rho) + t^j K^j(\rho + t^j, \rho + (1 - \alpha^j)\tau) + \pi^j(\rho + t^j, \rho + (1 - \alpha^j)\tau) \right] +$$

$$+ \tau \sum_{j=A,B} (1 - \alpha^j)D^j(\rho + (1 - \alpha^j)\tau, \rho + t^j),$$

where recall that $\rho = \rho(t^i(.), t^{-i}(.), \alpha^i, \alpha^{-i}, \tau), i = A, B$, from (23). The FOC of this problem with respect to $\tau$ is given by

$$G(t^i, t^{-i}, \alpha^i, \alpha^{-i}, \tau) \equiv \frac{d\rho}{d\tau}S_j(\rho) + \sum_{j=A,B} \frac{\partial t^j}{\partial \tau}K^j + \sum_{j=A,B} t^j \left[ K^j_{ij} \Theta^i + K^j_{si} \Gamma^i \right] +$$

$$- \left[ \Theta^i \sum_{j=A,B} K^j + \Gamma^i \sum_{j=A,B} D^j \right] + \sum_{j=A,B} (1 - \alpha^j)D^j + \tau \sum_{j=A,B} (1 - \alpha^j) \left[ D^j_{si} \Gamma^i + D^j_{rj} \Theta^i \right] = 0,$$

where

$$\Theta^i \equiv \frac{d\rho}{d\tau} + \frac{\partial t^i}{\partial \tau}, \quad i = A, B,$$

$$\Gamma^i \equiv \frac{d\rho}{d\tau} + 1 - \alpha^i, \quad i = A, B,$$

and

$$\frac{d\rho}{d\tau} = \frac{\partial \rho}{\partial \tau} + \sum_{j=A,B} \frac{\partial \rho}{\partial \tau} \frac{\partial t^j}{\partial \tau} < 0,$$

because of (12), (10), and Lemma 2.

Each term in (30) can be given a simple interpretation analogous to the ones already provided for the regional governments at the second stage of the game with the difference that of course now the maximisation problem is the one of the federal government with respect to the tax rate on the second type of capital, and the net remuneration of capital is (23). The first term is equivalent to the first term in (24) except that it is expressed with respect to an infinitesimal change in the federal tax rate $\tau$ instead of in the regional tax rate $t^i$. The second and the third term represent a vertical externality on regional tax revenue from the first type of capital taxation. In particular,
the second term describes the direct effect on regional tax revenue of an increase in the federal tax rate. This effect is positive because the federal and regional tax rates are strategic complements by Lemma 3. The third term represents the indirect effect on regional tax revenue of an increase in the federal tax rate. Such effect is negative (positive) when $\Theta^i > (\langle)0$ and $\Gamma^i < (\rangle)0$. The fourth term describes a vertical externality on regional rent taxation due to an infinitesimal increase in the federal tax rate. The first part of such an effect is negative (positive) while the second part is positive (negative) when $\Theta^i > (\langle)0$ and $\Gamma^i < (\rangle)0$, and thus the total effect will be positive or negative depending on which effect prevails on the other. Finally, the fifth and the sixth term represent the direct and the indirect effect on the federal tax revenue of an increase in the federal tax rate, respectively. When $\tau > 0$, the direct effect is always positive while the indirect effect is positive (negative) when $\Theta^i > (\langle)0$ and $\Gamma^i < (\rangle)0$.

We can now analyse how tax evasion on the second type of capital affects the federal tax rate. In this respect, we can state the following

**Proposition 3.** If i) $1 - \alpha^i < \left| \frac{\partial \rho}{\partial \tau} \right| < \frac{\partial \alpha^i}{\partial \tau}$ and ii) $1 - \tau < \left| \sum_{j=A,B} \frac{\partial \rho}{\partial \tau} \frac{\partial \alpha^j}{\partial \tau} \right| < 1$ then $\frac{\partial \tau}{\partial \sigma^i} < 0$, $i = A, B$.

**Proof.** See the Appendix. □

Proposition 3 shows that tax evasion negatively affects the federal tax rate under two conditions i) and ii) which concern the magnitude of the total effect of the federal tax rate on the net remuneration of capital, and the partial effect of the federal tax rate on the net remuneration of capital via the regional tax rates, respectively.

The result in Proposition 3 allows us also to analyse the effects of tax evasion on regional taxation by taking into account not only a direct effect arising at the second stage of the game and summarised in Proposition 2, but also an indirect effect that arises because of vertical tax competition and in particular via the strategic complementarity between the federal tax rate on the second type of capital and the regional tax rate on the first type of capital (see Lemma 3). What is particularly interesting for us is the fact that these two effects go in opposite direction. On the one hand, an indirect effect arises: An increase in tax evasion on the second type of capital leads to a decrease in the national tax rate which, in its turn, leads to a decrease in the regional tax rate on the first type of capital by Lemma 3. On the other hand, at the second stage of the game, Proposition 2 has shown a direct effect according to which an increase in tax evasion on the second type of capital leads to an increase in the regional tax rate on the first type of capital. To sum up, we can thus conclude that tax evasion on the second type of capital positively (negatively) affects the regional tax rate on the first type of capital when such direct effect dominates (is dominated by) the indirect effect.
5 Concluding remarks

The aim of this paper has been to analyse the implications of the existence of both heterogeneous capital and tax evasion for the interplay between horizontal and vertical tax competition in a federal country. In particular, we have analysed a federal country where consumers can decide both the region where to invest, and whether to invest in one or another type of capital. Each type of capital is assumed to be taxed by a different level of government with a different degree of tax compliance: The first type of capital is taxed at a regional level (according to the source-based principle) and no evasion may occur while the second type of capital is taxed at a federal level, and it can be evaded by individuals living in the two regions at a different extent. Our aim has been to study how, in a federal country with these two sources of asymmetry, tax evasion at a federal level affects both federal and regional tax policies. Our main result shows under which conditions an increase in tax evasion on the second type of capital arising at a federal level negatively affects the federal tax rate and positively affects the regional tax rate.

Finally, notice that our analysis has been performed in a simple model that describes the interaction between horizontal and vertical tax externalities with two types of capital investments and tax evasion. Such simple set-up could be extended in several directions. For example, extensions to broader set-ups could include asymmetric regions with respect to population, investors’ home bias, and different degrees of tax evasion between the two types of capital.

6 Appendix

Proof of Lemma 1.

Differentiating \( (7) \) with respect to \( \alpha^i, i = A, B, \) and \( \rho \) yields

\[
\frac{\partial \rho}{\partial \alpha^i} = \frac{-\tau (K_{s^i}^i + D_{s^i}^i)}{\Gamma' - \sum_{j=A,B}(K_{r^j}^j + K_{s^j}^j) - \sum_{j=A,B}(D_{s^j}^j + D_{r^j}^j)}, \quad i = A, B, \tag{32}
\]

where it is easy to check that \( 0 < \frac{\partial \rho}{\partial \alpha^i} < \tau, \quad i = A, B. \square \)

Proof of Proposition 1.

To evaluate how a change in \( t^i \) affects the tax rate in the other region, \( t^{-i} \), rewrite the optimality condition (21) for region \(-i\) as follows

\[
F^{-i}(t^{-i}, t^i, \alpha^{-i}, \alpha^i, \tau) \equiv \\
\equiv \frac{\partial \rho}{\partial t^{-i}} \left[ S^{-i}(\rho) - K^{-i}(\rho + t^{-i}, \rho + (1 - \alpha^{-i})\tau) - D^{-i}(\rho + (1 - \alpha^{-i})\tau, \rho + t^{-i}) \right] + \\
\quad + t^{-i} \left[ K_{r^{-i}}^{-i} \left( \frac{\partial \rho}{\partial t^{-i}} + 1 \right) + K_{s^{-i}}^{-i} \frac{\partial \rho}{\partial t^{-i}} \right] + \\
\quad + \frac{1}{2} \tau \left\{ (1 - \alpha^{-i}) \left( D_{s^{-i}}^{-i} \frac{\partial \rho}{\partial t^{-i}} + D_{r^{-i}}^{-i} \left( \frac{\partial \rho}{\partial t^{-i}} + 1 \right) \right) + (1 - \alpha^i) \left( D_{s^i}^i \frac{\partial \rho}{\partial t^i} + D_{r^i}^i \frac{\partial \rho}{\partial t^i} \right) \right\} = 0, \\
\quad i = A, B. \tag{33}
\]
Since equation (33) implicitly defines the reaction function \( t^{-i} = t^{-i}(t^i, \alpha^{-i}, \alpha^i, \tau), \) \( i = A, B, \) we can evaluate how a change in \( t^i \) affects the tax rate in the other region, \( t^{-i}, \) i.e. \( \frac{\partial t^{-i}}{\partial t^i} = -\frac{\partial F^{-i}/\partial t^i}{\partial F^{-i}/\partial t^{-i}}. \) By following a procedure familiar in the tax competition literature (Andersson et al. (2004)), i.e. assuming \( \frac{\partial F^{-i}/\partial t^{-i}}{\partial F^{-i}/\partial t^i} < 0 \) by the second order condition of the problem in (19), then \( \text{sign} \frac{\partial F^{-i}/\partial t^i}{\partial F^{-i}/\partial t^{-i}}. \) Thus, from (33), it is easy to check that
\[
\frac{\partial F^{-i}}{\partial t^i} = \frac{\partial \rho}{\partial t^i} \frac{\partial \rho}{\partial t^{-i}} \left[ \frac{dS^{-i}}{d\rho} - (K_{r^{-i}} - K_{s^{-i}}) - (D_{s^{-i}} + D_{r^{-i}}) \right] > 0, \ i = A, B. \tag{34}
\]
because of (10), and assumption 2. Thus, \( \frac{\partial t^{-i}}{\partial t^i} > 0. \square \)

**Proof of Lemma 2.**
To show that \( \Omega^i < 0, \) let us rewrites (24) and (34), respectively, as follows
\[
\frac{\partial F^i}{\partial t^i} = \gamma^i - \tilde{\gamma}^i, \ i = A, B, \tag{35}
\]
and
\[
\frac{\partial F^i}{\partial t^{-i}} = \frac{\partial \rho}{\partial t^i} \frac{\partial \rho}{\partial t^{-i}} \tilde{\gamma}^i, \ i = A, B, \tag{36}
\]
where
\[
\gamma^i = \left( \frac{\partial \rho}{\partial t^i} \right)^2 \tilde{\gamma}^i, \tag{37}
\]
\[
\tilde{\gamma}^i = \frac{dS_i}{d\rho} - (K_{r^i} + D_{r^i}) - (K_{s^i} + D_{s^i}), \tag{38}
\]
\[
\tilde{\gamma}^i = \frac{\partial \rho}{\partial t^i} (K_{r^i} + D_{r^i}) - K_{s^i} \left( \frac{\partial \rho}{\partial t^i} + 1 \right) - K_{s^i} \frac{\partial \rho}{\partial t^{-i}}. \tag{39}
\]
Then, by using (35) and (36), (27) can be rewritten as
\[
\Omega^i = \left( \gamma^i - \tilde{\gamma}^i \right) \left( \gamma^{-i} - \tilde{\gamma}^{-i} \right) - \tilde{\gamma}^i \gamma^{-i}, \ i = A, B. \tag{40}
\]
Suppose, by contradiction, that \( \Omega^i > 0. \) From (40), this implies that
\[
\frac{\gamma^i - \tilde{\gamma}^i}{\tilde{\gamma}^i} \frac{\gamma^{-i} - \tilde{\gamma}^{-i}}{\tilde{\gamma}^{-i}} > 1, \ i = A, B,
\]
or
\[
\left[ \left( \frac{\partial \rho}{\partial t^i} \right)^2 - \frac{\gamma^i}{\tilde{\gamma}^i} \right] \left[ \left( \frac{\partial \rho}{\partial t^{-i}} \right)^2 - \frac{\gamma^{-i}}{\tilde{\gamma}^{-i}} \right] > 1, \ i = A, B. \tag{41}
\]
In (41), \( \frac{\gamma^i}{\tilde{\gamma}^i} > 0, i = A, B, \) because Assumption 1 implies that \( \gamma^i > 0, \) and \( \tilde{\gamma}^i > 0 \) by taking also into account (10). Further, from Assumptions 1 and 2 and (10), it follows that
\[
\tilde{\gamma}^i - \gamma^i = (K_{r^i} + D_{r^i}) \left( 1 + \frac{dS_i}{d\rho} - (K_{r^i} + K_{s^i}) \frac{\partial \rho}{\partial t^i} - (K_{r^i} - K_{s^i}) + D_{s^i} \right) < 0, \ i = A, B,
\]
which implies that \( \frac{\tilde{\gamma}^i}{\gamma^i} \) \( < 1 \). Thus, \(-1 < \frac{\tilde{\gamma}^i}{\gamma^i} < 0 \). In (41), notice that \( \left( \frac{\partial F^i}{\partial t^i} \right)^2 - \frac{\tilde{\gamma}^i}{\gamma^i} \) must be negative because \( \left( \frac{\partial F^i}{\partial t^i} \right)^2 - \frac{\tilde{\gamma}^i}{\gamma^i} \) \( < 0 \) given that \( \tilde{\gamma}^i > 0 \) and \( \gamma^i - \tilde{\gamma}^i = \frac{\partial F^i}{\partial t^i} < 0 \) being the second order condition of the problem in (19). Further, \( \left( \frac{\partial F^i}{\partial t^i} \right)^2 - \frac{\tilde{\gamma}^i}{\gamma^i} \) \( < 0 \) together with \(-1 < \frac{\tilde{\gamma}^i}{\gamma^i} < 0 \) and \( 0 < \left( \frac{\partial F^i}{\partial t^i} \right)^2 < 1 \) from (10), imply that \(-1 < \left[ \left( \frac{\partial F^i}{\partial t^i} \right)^2 - \frac{\tilde{\gamma}^i}{\gamma^i} \right] < 0, \ i = A,B. \)

Accordingly, \( \left( \frac{\partial F^i}{\partial t^i} \right)^2 - \frac{\tilde{\gamma}^i}{\gamma^i} \) \( < 0 \) together with \(-1 < \frac{\tilde{\gamma}^i}{\gamma^i} < 0 \) and \( 0 < \left( \frac{\partial F^i}{\partial t^i} \right)^2 < 1 \) from (10), imply that \(-1 < \left[ \left( \frac{\partial F^i}{\partial t^i} \right)^2 - \frac{\tilde{\gamma}^i}{\gamma^i} \right] < 0, \ i = A,B, \) which contradicts (41). Thus, we have proved that \( \Omega^i < 0, \ i = A,B. \) □

**Proof of Proposition 2.**

First, let us prove that \( \frac{\partial t^i}{\partial \alpha^i} > 0 \). To sign (25), from (24) and taking into account Assumption 1, it is easy to check that the first term in parenthesis is positive:

\[
\frac{\partial F^i}{\partial \alpha^i} = \frac{\partial \rho}{\partial \alpha^i} \frac{\partial F}{\partial t^i} \quad \left( \frac{dS}{dt} - (K_{t^i} + D_{t^i}) - (K_{s^i} + D_{s^i}) \right) > 0, \quad i = A,B. \tag{42}
\]

By using (33), the second term in parenthesis obtains as

\[
\frac{\partial F^{-i}}{\partial \alpha^i} = \frac{\partial \rho}{\partial \alpha^i} \frac{\partial F^{-i}}{\partial t^i} \left( \frac{dS_i}{dt} - (K_{t^i}^{-i} + K_{s^i}^{-i}) - (D_{t^i}^{-i} + D_{s^i}^{-i}) \right) - \frac{1}{2} \frac{\partial \rho}{\partial t^i} \frac{\partial F}{\partial t^i} < 0, \quad i = A,B, \tag{43}
\]

which is negative because of (10), and assumption 2. Further, the third term in parenthesis is negative since \( \frac{\partial F^{-i}}{\partial t^i} < 0 \) by the second order condition of the problem in (19). Finally, from (24) the last term in parenthesis can be rewritten as

\[
\frac{\partial F^i}{\partial \alpha^i} = \Phi^i + \Psi^i, \quad i = A,B, \tag{44}
\]

where

\[
\Phi^i = \frac{\partial \rho}{\partial \alpha^i} \frac{\partial F^i}{\partial t^i} \tilde{\gamma}^i + \frac{\partial \rho}{\partial t^i} \tau K_{s^i}^i < 0, \tag{45}
\]

and

\[
\Psi^i = \frac{1}{2} \tau \left( D_{s^i}^i \frac{\partial \rho}{\partial t^i} - D_{t^i}^i \left( \frac{\partial \rho}{\partial t^i} + 1 \right) \right), \quad i = A,B. \tag{46}
\]

The term \( \Phi^i \) is negative because of (10), Lemma 1, and \( \tilde{\gamma}^i > 0 \) as it was shown in the proof of Lemma 2 while the term \( \Psi^i \) is negative if \( \frac{D_{t^i}^i}{D_{s^i}^i} > \left| \frac{1}{1 + \frac{\partial F^i}{\partial \alpha^i}} \right| \). Thus, if \( \frac{D_{t^i}^i}{D_{s^i}^i} > \left| \frac{1}{1 + \frac{\partial F^i}{\partial \alpha^i}} \right| \) then \( \frac{\partial F^i}{\partial \alpha^i} < 0, \)

and the difference in parenthesis in (25) is negative. Accordingly, from Lemma 2, if \( \frac{D_{t^i}^i}{D_{s^i}^i} > \left| \frac{1}{1 + \frac{\partial F^i}{\partial \alpha^i}} \right| \) then \( \frac{\partial t^i}{\partial \alpha^i} < 0. \)

Let us now prove that \( \frac{\partial t^{-i}}{\partial \alpha^i} > 0. \) In (26), the first term in parenthesis is negative being \( \frac{\partial F^{-i}}{\partial \alpha^i} \) the second order condition of the problem in (19), while from (43), the second term is negative. Further,
from (34) the third term in parenthesis is positive, and as we have shown above if 
\[
\frac{D^i_{si}}{D^i_{ri}} > \left| \frac{1}{1 + \frac{D^i_{ri}}{D^i_{si}}} \right|
\]
then the fourth term in parenthesis is negative, and the difference in parenthesis in (26) is positive. 
Thus, from Lemma 2, if 
\[
\frac{D^i_{ri}}{D^i_{si}} > 1 + \frac{1}{\frac{D^i_{ri}}{D^i_{si}}}
\]
then the fourth term in parenthesis is negative, and the difference in parenthesis in (26) is positive.

Proof of Corollary 1.
In (28), first it is easy to check that 
\[
\frac{\partial \rho}{\partial \alpha^i} > 0
\]
from Lemma 1, and 
\[
\frac{\partial \rho}{\partial \tau^i} < 0
\]
from (10). Further, 
\[
\frac{\partial \rho}{\partial \alpha^i} > 0, \quad i = A, B,
\]
if 
\[
\frac{D^i_{ri}}{D^i_{si}} > \left| \frac{1}{1 + \frac{D^i_{ri}}{D^i_{si}}} \right|
\]
from Proposition 2. This implies that if 
\[
\frac{D^i_{ri}}{D^i_{si}} > \left| \frac{1}{1 + \frac{D^i_{ri}}{D^i_{si}}} \right|
\]
then 
\[
\frac{\partial \rho}{\partial \alpha^i} > 0, \quad i = A, B.
\]
Second, it is also immediate to check that if 
\[
\frac{D^i_{ri}}{D^i_{si}} > \left| \frac{1}{1 + \frac{D^i_{ri}}{D^i_{si}}} \right|
\]
then 
\[
\frac{\partial \rho}{\partial \alpha^i} < \frac{\partial \rho}{\partial \alpha^i}
\]
because 
\[
\sum_{j=A,B} \frac{\partial \rho}{\partial \rho^j} \frac{d^j}{d\tau^j} < 0
\]
in (28).

Proof of Lemma 3.
At the Nash equilibrium, by taking into account (22), let us differentiate (33) and (24) with 
respect to \( \tau \):
\[
\frac{\partial F^{-i}}{\partial \tau} + \frac{\partial F^{-i} dt^{-i}}{d\tau} + \frac{\partial F^{-i} dt^i}{d\tau} = 0, \quad (47)
\]
\[
\frac{\partial F^i}{\partial \tau} + \frac{\partial F^i dt^i}{d\tau} + \frac{\partial F^i dt^{-i}}{d\tau} = 0, \quad i = A, B.
\]
By solving this system of equations, it is easy to check that 
\[
\frac{dt^i}{dt} = \frac{1}{\Omega^i} \left( \frac{\partial F^i}{\partial \alpha^i} - \frac{\partial F^{-i}}{\partial \alpha^{-i}} \right), \quad i = A, B. \quad (48)
\]
From (42), the first term in parenthesis is positive, and the third term is negative by the second 
order condition of the problem in (19). From (24), the second term in parenthesis obtains as (an 
equivalent expression obtains for the fourth term in parenthesis, *mutatis mutandis*)
\[
\frac{\partial F^i}{\partial \tau} = \frac{\partial \rho}{\partial \tau^i} \left( K^i_{ri} + D^i_{ri} \right) \frac{\partial \rho}{\partial \tau^i} - \left( K^i_{si} + D^i_{si} \right) \left( \frac{\partial \rho}{\partial \tau^i} + 1 - \alpha^i \right) \right) + \left( 1 - \alpha^i \right) \left( \frac{D^i_{ri}}{D^i_{si}} + \frac{D^i_{ri}}{D^i_{si}} \right) \frac{\partial \rho}{\partial \tau^i} \right) + \left( 1 - \alpha^i \right) \left( \frac{D^i_{ri}}{D^i_{si}} + \frac{D^i_{ri}}{D^i_{si}} \right) \frac{\partial \rho}{\partial \tau^i} \right) \right) \right), \quad i = A, B. \quad (49)
\]
From (12), and (10) if 
\[
\frac{\partial \rho}{\partial \tau^i} < -(1 - \alpha^i)
\]
then 
\[
\frac{\partial F^i}{\partial \tau} > 0, \quad i = A, B. \quad (1 - \alpha^i)
\]
and 
\[
\Omega^i \geq 0
\]
then 
\[
\frac{dt^i}{dt} \geq 0, \quad i = A, B. \quad (50)
\]
Proof of Proposition 3.
By following the same procedure underlying proposition 1, since equation (30) implicitly defines 
the reaction function \( \tau = \tau(t^i, t^{-i} \alpha^i, \alpha^{-i}), \quad i = A, B \), we can evaluate how a change in \( \alpha^i \) affects the
federal tax rate, i.e. \( \frac{\partial \tau}{\partial \alpha^i} = -\frac{\partial G}{\partial \tau} \). By assuming that \( \frac{\partial G}{\partial \tau} < 0 \) by the second order condition of the problem in (29), then \( \text{sign} \frac{\partial \tau}{\partial \alpha^i} = \text{sign} \frac{\partial G}{\partial \tau}, \; i = A, B. \) Thus, from (30), it is easy to check that

\[
\frac{\partial G}{\partial \alpha^i} = \Lambda \frac{d\rho}{d\tau} \frac{dS_i}{d\rho} + \Theta^i \sum_{j=A,B} \left( \frac{\partial j}{\partial \alpha^j} K^j_{r^j} - \tau D^j_{r^j} \right) + \Gamma^i \sum_{j=A,B} \left( \frac{\partial j}{\partial \alpha^j} K^j_{s^j} - \tau D^j_{s^j} \right) + \Lambda \sum_{j=A,B} \frac{d\rho}{d\tau} \left( K^j_{r^j} - D^j_{r^j} \right) - \Delta \sum_{j=A,B} \frac{d\rho}{d\tau} \left( K^j_{s^j} - D^j_{s^j} \right), \; i = A, B, \tag{51}
\]

where

\[
\Lambda \equiv \sum_{j=A,B} \frac{d\rho}{d\tau} \frac{\partial \theta^j}{\partial \tau} + 1, \\
\Delta \equiv \sum_{j=A,B} \frac{d\rho}{d\tau} \frac{\partial \theta^j}{\partial \tau} + 1 - \tau.
\]

Notice that condition i) \( 1 - \alpha^i < \left| \frac{d\rho}{d\tau} \right| < \frac{\partial \theta^i}{\partial \tau} \) implies \( \Theta^i > 0 \) and \( \Gamma^i < 0 \) while condition ii) \( 1 - \tau < \left| \sum_{j=A,B} \frac{d\rho}{d\tau} \frac{\partial \theta^j}{\partial \tau} \right| < 1 \) implies \( \Lambda > 0 \) and \( \Delta < 0 \). Accordingly, in (51), it is easy to check that \( \frac{\partial \tau}{\partial \alpha^i} < 0 \) because of Assumptions 1 and 2, and (31). □

References


