Pareto Efficient Income Taxation and
Self-financing Tax Cuts

Thomas Gaube*

February 15, 2019

Abstract

The paper compares two criteria for evaluating income tax schedules, namely absence of a self-financing tax cut and Pareto efficiency. It is first pointed out that both criteria are equivalent under nonlinear taxation but not under linear taxation. Then the equivalence is used for showing that (i) the effect of a tax reform on total revenue is a sufficient statistic for Pareto efficiency in the presence of tax evasion, that (ii) the criterion of a self-financing tax cut also applies to the analysis of work related expenses and that (iii) objectives that either maximize statutory disposable income or minimize statutory tax rates lead to a Pareto efficient outcome.

Keywords: nonlinear income taxation, Pareto efficiency, Laffer curve

JEL-Classification: H21, H24, D63

Preliminary Draft.

*Department of Economics, University of Osnabrück, D-49069 Osnabrück, Germany, thomas.gaube@uos.de
1 Introduction

The economic analysis of a nonlinear income tax in the tradition of Mirrlees (1971) is about the efficiency effects and the equity effects of a redistributive tax and transfer system. Within this literature, some work has dealt with the attempt of characterizing Pareto efficient tax schedules or, more generally, Pareto efficient fiscal policy. Prominent findings are for example the separability-results of Atkinson and Stiglitz (1976) on the nonlinear-linear tax mix or of Christiansen (1981) on the provision of public goods. Recent work on the subject builds on the idea that a test for Pareto efficiency is basically a test for a self-financing tax cut. This point is explored in Werning (2007) and is closely related to the concept of nonnegative implicit welfare weights.

The present paper aims at contributing to this literature. Two topics are investigated: First, it is asked under which assumptions a Pareto test and a Laffer test of fiscal policy coincide. It is shown that the equivalence rests on three assumptions - rational behavior of the taxpayers, preferences that are monotonic in consumption, and a fully flexible tax schedule. This means that the Laffer test is suitable under much weaker assumptions than are commonly made in the income tax literature; but it also implies that functional constraints, like the constraint of having only linear taxes, can make the Laffer test uninformative. The second topic deals with applications and extensions of the equivalence between Pareto efficiency and nonexistence of a self-financing tax cut. It is shown the equivalence has consequences (i) for the question of sufficient statistics when tax evasion is taken into account, (ii) for the implementation of efficient tax schedules when earnings also depend on work-related expenses, and (iii) for the type of objective functions that implement Pareto efficient outcomes.

The idea that the efficiency or inefficiency of a tax system is somehow related to the question whether a self-financing tax cut exists, has a long tradition in public economics. In particular, the identification of a self-financing tax cut has often been used for proving the inefficiency of some policy measure. Some examples are the no-distortion-at-the-top-property of a nonlinear income tax (Seade, 1982, Stiglitz (1982, 1987), and Brito et al (1990).

1First and still important contributions are Guesnerie and Seade (1982), Stiglitz (1982, 1987), and Brito et al (1990).

2See also Jacobs and Bovenberg (2011) on education expenditures and Gauthier and Laroque (2009) on the separability assumption in general.
1977), the derivation of an upper bound on the slope of a linear income tax (Hellwig, 1986), or the derivation of an upper bound on the top marginal tax with an unbounded income distribution (Saez, 2001; Piketty, Saez, and Stantcheva, 2014).

These findings all rely on the idea that being on the downward sloping part of the Laffer curve is an indicator of inefficiency. The other direction of that relationship, namely the consequences of being on the upward sloping part of the curve, does not play such a prominent role in the literature: In particular, optimal taxation in the Ramsey-Boiteux tradition is always about inefficiencies that arise because one tax is too high or too low relative to other taxes. An efficiency enhancing reform then typically takes place by decreasing some and increasing other taxes. Some work on nonlinear income taxation, however, indicates that this logic does not extend to efficiency enhancing reforms of a nonlinear schedule: Laroque (2005) considers a model with labor supply at the extensive margin and shows that all tax rates below the Laffer bound are Pareto efficient. Hence, being on the upward sloping part of the Laffer curve is necessary and sufficient for Pareto efficiency. Werning (2007) also argues that absence of a self-financing tax cut can be used for characterizing Pareto efficiency. Bourguignon and Spadaro (2012), Lorenz and Sachs (2016), and the literature on implicit welfare weights (e.g. Hendren, 2017) are closely related to that approach.

The remainder of this paper is organized as follows: Section 2 presents the basic version of the Mirrleesian income tax model and the main assumptions used in this paper. This model is used in Section 3 first for introducing the notion of a revenue-efficient income tax schedule. Then it is shown that revenue-efficiency is necessary and sufficient for Pareto efficiency in the model with nonlinear taxation, but insufficient for Pareto efficiency in the model with linear income taxation. Section 4 presents extensions and applications of these findings for variants of the basic model where (i) tax avoidance and tax evasion (ii) work-related expenses and (iii) non-welfarist objectives are taken into account.

2 The basic model

I consider an economy with a finite or infinite set $I$ of individuals. Population size is normalized to unity. The earnings and the consumption of an individual
i ∈ I are denoted y_i and c_i. The government imposes a potentially nonlinear tax and transfer schedule T(y) which transforms earnings into disposable income c(y) := y - T(y). The individuals make a rational decision about y_i and c_i by solving
\[
(c_i, y_i) \in \arg \max_{c_i, y_i} \{ v_i(c_i, y_i) \text{ s.t. } c_i \leq c(y_i) \}
\]
where utility functions v_i(c_i, y_i) represent the individuals’ preferences. The only assumption that is made with respect to v_i(c_i, y_i) is that it is strictly increasing in c_i. To summarize, we thus have the following

**Assumption:** The individuals’ preferences are strictly increasing in disposable income c_i. They behave according to these preferences.

Recent experimental evidence suggests that taxpayers do not always behave rationally (see, e.g., Farhi and Gabaix (2017) for a discussion). Nevertheless, the assumption is crucial for the present analysis because otherwise the observation of an expansion or contraction of the individuals’ budget set cannot readily be used as an indicator of a change in their (unobserved) well-being.

The individuals’ per-capita tax payment E[y_i - c_i] must be sufficient for financing an exogenous revenue requirement g ≥ 0. The government’s budget constraint thus reads
\[
E[y_i - c_i] \geq g.
\]

Since neither the individuals’ budget set nor their preferences need to be convex, problem (1) can have multiple solutions; see Figure 1 below for an illustration. Hence, there can also be multiple tax equilibria and an infinitesimal tax reform can have large effects on earnings both along the intensive as the extensive margin.

A tax schedule is usually evaluated by evaluating the allocation that it implements. Since a schedule can have multiple equilibria in the present framework, some clarifying definitions are required: An allocation \{c_i, y_i\}_{i \in I} that fulfills (1) and (2) under tax schedule T(y) is an equilibrium of that schedule. A tax schedule is feasible if it has at least one equilibrium. If it has multiple equilibria, each equilibrium must be, as a consequence of (1), associated with the same utility profile \{v_i(T)\}_{i \in I}, but not necessarily with the same tax revenue. Let r(T) denote
the highest tax revenue among all equilibria of schedule $T(y)$. Of course, a tax schedule can only be feasible if $r(T) \geq g$. It is budget balancing if $r(T) = g$.

An allocation is tax implementable if it is an equilibrium for some schedule $T(y)$. As long as the latter can take any functional form, the set of tax implementable allocations is equivalent to the set of allocations that fulfill (2) and the incentive-compatibility constraints

$$v_i(c_i, y_i) \geq v_i(c_j, y_j), \quad \forall i, j \in I.$$  \hspace{1cm} (3)

Hence, a potentially nonlinear tax schedule can be labelled Pareto efficient if it implements an allocation that is Pareto efficient subject to (2) and (3).

### 3 Pareto efficient and revenue efficient taxation

The aim of this section is to show that (i) any Pareto efficient nonlinear income tax schedule can be constructed by successive elimination of feasible tax cuts and that (ii) this property does not extend to linear schedules. Since the term tax cut is not well-defined in the context of a potentially progressive tax system (see Fullerton (2008) for a brief discussion), I will start with some definitions.

**Definition (Tax cut):** A tax reform that replaces schedule $T(y)$ with another schedule $\tilde{T}(y)$ is a tax cut if (i) $\tilde{T}(y) \leq T(y)$ holds for all $y$ with strict inequality for some $y$ and if (ii) a tax equilibrium under schedule $T(y)$ is no longer a tax equilibrium under schedule $\tilde{T}(y)$. The tax cut is feasible when the new schedule $\tilde{T}(y)$ is feasible.

Hence, a reform only qualifies for a tax cut if the tax is nowhere increased but somewhere decreased and if the reduction is economically relevant in the sense that it affects the behavior of some individual.

**Definition (Revenue efficiency):** A tax schedule $T(y)$ is revenue efficient if no feasible tax cut exists.

A schedule is thus revenue efficient if the individuals’ statutory cost of taxation $T(y)$ have been pushed down as far as one can without violating the government’s budget constraint. If the reduction primarily takes place at low incomes, where $T(y)$ might be negative, or at higher incomes has distributional consequences but does not affect the definition.
Note that a tax cut is equivalent to an expansion of the individual’s budget set \( B := \{(c_i, y_i) \mid c_i \leq c(y_i)\} \). The following proposition thus basically says that any Pareto superior allocation can be implemented by expanding this budget set and that any economically relevant expansion of that budget set leads to a Pareto superior outcome. The second claim is obvious and must hold for any system of linear and nonlinear taxation. The first claim does not hold in general, at least not for linear taxes.

**Proposition 1:** A potentially nonlinear tax schedule \( T(y) \) is Pareto efficient if and only if it is revenue efficient.

The proof of this result is straightforward: Consider the budget constraint \( c(y) \) implied by tax schedule \( T(y) \) and the (lower) envelope \( l(y) \) of the individuals’ indifference curves in an equilibrium under \( T(y) \). Both are illustrated in Figure 1 for a situation with two individuals \( i \) and \( j \). Let \( S(T) \) denote the support of the income distribution in one of the equilibria under \( T(y) \). Then, as illustrated in Figure 1, we must have

\[
c(y) \leq l(y) \quad \text{for all } y \geq 0 \quad \text{and} \quad c(y) = l(y) \quad \text{for all } y \in S(T).
\]

During a tax cut, \( c(y) \) is shifted upwards in such a way that the initial equilibrium is replaced by a new one. Hence, no individual can become worse off and some individual must become strictly better off. Conversely, note that the lower envelope of a new, Pareto-superior, allocation must lie strictly above \( l(y) \).
for some $y$ and can nowhere fall below $l(y)$. This implies that one can always implement such an allocation by shifting the budget constraint $c(y)$ upwards for some earnings without any need to shift it downwards for others.

The last point is true only if $c(y)$ can flexibly be amended. This is why Proposition 1 relies on the assumption of a nonlinear income tax. In order to substantiate this claim, consider now the case of a linear income tax

$$T(y) = ty - b$$

with marginal tax rate $t$ and demogrant $b$. The definitions of revenue efficient and Pareto efficient nonlinear tax schedules also apply to linear schedules. In the linear case, we can express them in terms of $t$ and $b$: A linear schedule $T(y)$ with parameters $t$ and $b$ is **revenue inefficient** if another linear schedule $\tilde{T}(y)$ with parameters $\tilde{t} \leq t$ and $\tilde{b} \geq b$ exists that is feasible and implements a different allocation than $T(y)$. Otherwise, $T(y)$ is **revenue efficient**. Similarly, $T(y)$ is **Pareto efficient** if it implements an allocation that is not Pareto dominated by another allocation that can also be implemented by a linear schedule.

**Proposition 2:** Assume that the tax schedule $T(y)$ must be linear. Then revenue efficiency of $T(y)$ is necessary for Pareto efficiency, but not sufficient.

**Proof:** Necessity: Assume that a feasible schedule $\hat{T}(y)$ with parameters $\hat{t}$ and $\hat{b}$ is revenue inefficient. Then another feasible schedule $\tilde{T}(y)$ with parameters $\tilde{t} \leq \hat{t}$ and $\tilde{b} \geq \hat{b}$ exists that implements a feasible allocation which differs from the equilibrium under $\hat{T}(y)$. This implies that at least one of the inequalities $\tilde{t} \leq \hat{t}$ and $\tilde{b} \geq \hat{b}$ must be strict. Hence, $\tilde{c}(y) \leq \hat{c}(y)$ holds for all $y$. Each individual thus has a strictly larger budget set under $\tilde{T}(y)$ than under $\hat{T}(y)$. This leads to a Pareto superior outcome except if the initial $y_i$ is zero for all individuals. In that case, however, the initial equilibrium remains unaffected.

Insufficiency: For a proof that revenue efficiency is not sufficient for Pareto efficiency, consider Figure 2 which depicts an example with two individuals. Their preferences are such that indifference curves are of the form $v_1$ and $v_2$. The arrows indicate how the kinks of the curves change when utility is increased. I assume $g = 0$ such that average tax revenue must be nonnegative. Hence, the budget constraint $\hat{c}(y) = 1 + y/3$ implements a feasible allocation, namely $(y_1, c_1) = (0, 1)$, $(y_2, c_2) = (3, 2)$. It can easily be verified by means of Figure 2 that no alternative constraint $c(y) = b + (1 - t)y$ with $b \geq 1$ and $(1 - t) \geq 1/3$ is feasible. Therefore,
Figure 2: With a linear tax, non-existence of a self-financing tax cut does not imply Pareto efficiency

\( \hat{c}(y) \) is revenue efficient. However, \( \tilde{c}(y) = y \) is also feasible and implements a Pareto superior allocation. Therefore, \( \hat{c}(y) \) is not Pareto efficient.

4 Extensions and Applications

4.1 Sufficient statistics when tax avoidance and tax evasion are taken into account

In practice, an individual’s true earnings \( y_i \) are rarely identical to her taxed earnings \( z_i \) and the gap can be significant. There are many reasons for this gap: Some of them are intended or at least legal, like deductions that have been designed for incentive reasons; others are illegal or at least unintended, like undeclared earnings or the abuse of loopholes in the tax code.

What are the consequences of avoidance and evasion for the evaluation of tax policy when neither the size and the distribution of the gap nor the size and the distribution of the monetary and psychic costs that are associated with the gap are known? Feldstein (1999) argued that the welfare cost of taxation only depend on the elasticity of taxable income \( z_i \). Hence, neither the elasticity of real income \( y_i \) (the gap) nor the cost of evasion play a role. Chetty (2009) has shown that this conclusion rests on the assumption that avoidance activities have no indirect effect on the government budget. Otherwise, the effects of tax policy on both \( z_i \) and \( y_i \) are relevant for measuring the welfare cost of taxation. Hendren (2016)
extends this argument by considering a flexible framework where a change in tax rates can lead to variety of behavioral effects. Each of these effects is associated with some elasticity and knowledge of all these elasticities would thus be sufficient for measuring the cost of taxation. However, Hendren (2016) shows that one does not need to know all these elasticities because the only thing that matters is the individual’s overall contribution to government revenue. The channel through which this contribution is made is irrelevant.

The abovementioned literature attempts to measure the welfare cost imposed on a single individual by a linear tax. The common aspect of these analyses is the question whether the different reactions of the taxpayers can be summarized by a simple sufficient statistic. The aim of this section is to do the same for the problem of identifying Pareto efficient nonlinear tax schedules. It is shown that the crucial information is the same as in Hendren (2016), namely the effect of a tax reform on total government revenue. Neither the decomposition of this effect into several behavioral channels nor the size or the distribution of the hidden cost of avoidance and evasion are relevant.

This finding also complements previous work on the consequences of avoidance and evasion for fiscal policy with a nonlinear income tax. This literature has shown, for example, that the income tax should be supplemented by indirect taxes (Boadway, Marchand and Pestieau, 1994) and an income-dependent audit policy (Cremer and Ghavari, 1995) when tax evasion plays some role. Piketty, Saez and Stantcheva (2014) investigate the consequences of tax evasion on the revenue maximizing top income tax rate. They show that this tax rate depends asymmetrically on the elasticities of declared and undeclared earnings, but only if a change in undeclared earnings has a different effect on government revenue than a change in declared earnings. Huang and Rios (2016) extend this analysis by characterizing the optimal nonlinear tax in conjunction with a linear consumption tax that affects both declared and undeclared earnings. Again, two elasticities enter the tax formula and their relative role depends on the importance of the consumption tax rate relative to the marginal income tax rate.

One interesting aspect of these formulas is that marginal tax rates depend on these two elasticities whereas other aspects like the size and the distribution of evasion cost are irrelevant. However, this might be a consequence of the assumptions that have been made, in particular the absence of income effects and
identical, additively separable evasion cost. Proposition xx below shows this is not the case. As long as the preferences are monotonic in consumption, all one needs to know is the aggregate fiscal effect of a tax cut. Whether evasion is part of this effect or whether plays a different role in different income brackets is irrelevant.

In the following, I will distinguish between the individuals’ true earnings $y_i$ and taxed earnings $z_i$. The difference between them is denoted by $e_i := y_i - z_i$ and it is assumed that $e_i$ is nonnegative. In order to capture a variety of reasons for the deviation between earnings and taxed earnings, I assume that $e_i$ does not only affect the individuals’ income tax $T(y_i - e_i)$, but can also induce other financial costs $k_i(e_i)$ or can have a direct effect on utility $v_i(c_i, y_i, e_i)$. Likewise, I allow for an indirect effect $\tau_i(e_i)$ on the government’s budget, for example a fine or additional revenues from other taxes or other taxpayers.

As before, I assume that $v_i(c_i, y_i, e_i)$ is monotonically increasing in $c_i$. Evasion $e_i$ can either have a positive, a negative, or no effect on the individuals’ utility. It is also assumed that the individuals behave rationally by solving

$$\max_{c_i, z_i, e_i} v_i(c_i, y_i, e_i) \quad \text{s.t.} \quad c_i \leq y_i - T(z_i) - k_i(e_i), \quad y_i = z_i + e_i$$

(5)

for a given tax schedule $T(z)$. A tax schedule is feasible if it induces $z_i$ and $e_i$ such that the government’s budget constraint

$$E[T(z_i)] + E[\tau_i(e_i)] \geq g$$

(6)

is satisfied. A feasible schedule $\hat{T}(z)$ is revenue inefficient if another feasible schedule $\tilde{T}(z)$ exists such that (i) $\tilde{T}(z) \leq \hat{T}(z)$ holds for all $z$ and (ii) the initial equilibrium under $\hat{T}(z)$ is not an equilibrium under $\tilde{T}(z)$. Otherwise, $\hat{T}(z)$ is revenue efficient.

Like problem (1) of the basic model, problem (5) can have multiple solutions; however, each solution must be associated with the same utility $v_i(c_i, y_i, e_i)$. A feasible tax schedule thus always implements a unique utility profile. Hence, the initial definition of a Pareto-efficient tax schedule applies here as well.

**Proposition 3:** Assume that the individuals’ taxed earnings $z_i = y_i - e_i$ can differ from their true earnings $y_i$ due to tax avoidance and/or tax evasion $e_i$. Then a tax schedule $T(z)$ is Pareto efficient if and only if it is revenue efficient.
Proof: Consider a tax schedule $\hat{T}(z)$ with corresponding equilibrium allocation $\hat{a} := \{(c_i, \hat{z}_i, \hat{e}_i)\}_{i \in I}$. Let $\hat{Z} := \{\hat{z}_i\}_{i \in I}$ denote the support of the distribution of taxable income in $\hat{a}$. I will prove the result by showing that Pareto inefficiency of $\hat{T}(z)$ implies revenue inefficiency of $\hat{T}(z)$ and vice versa.

Pareto inefficiency implies revenue inefficiency: Assume that $\hat{T}(z)$ is Pareto inefficient. Then a feasible schedule $\tilde{T}(z)$ exists that implements a Pareto superior allocation $\tilde{a} := \{(\tilde{c}_i, \tilde{z}_i, \tilde{e}_i)\}_{i \in I}$. Let $\tilde{Z} := \{\tilde{z}_i\}_{i \in I}$ denote the support of the distribution of taxable income in $\tilde{a}$. This allows us to define a third tax schedule, namely

$$\bar{T}(z) := \begin{cases} \hat{T}(z) & \text{for } z \in \hat{Z} \\ \tilde{T}(z) & \text{for } z \notin \hat{Z}. \end{cases}$$

In the following, it will be shown that (i) $\bar{T}(z)$ is feasible, (ii) $\bar{T}(z) \leq \hat{T}(z)$ holds for all $z$, and (iii) $\hat{a}$ is not an equilibrium under $\bar{T}(z)$. This implies that $\hat{T}(z)$ is revenue inefficient.

(i) Since $\hat{a}$ Pareto dominates $\tilde{a}$, we must have $\hat{T}(z) \leq \tilde{T}(z)$ for all $z \in \hat{Z}$ because otherwise an individual of type $i$ could choose $(\tilde{z}_i, \tilde{e}_i)$ under schedule $\hat{T}(z)$ and be better off than under schedule $\tilde{T}(z)$. Due to the definition of $\bar{T}(z)$, we thus also have $\bar{T}(z) \leq \hat{T}(z)$ for all $z$.

(ii) $\tilde{z}_i$ maximizes (together with $\tilde{e}_i$) utility of type $i$ under schedule $\tilde{T}(z)$ and this utility is the same or higher than what the individuals can attain under schedule $\hat{T}(z)$. Therefore $\tilde{z}_i$ is also a best choice of $i$ under schedule $\hat{T}(z)$. This means that $\bar{T}(z)$ implements $\hat{a}$. Since $\tilde{a}$ is feasible, $\bar{T}(z)$ is also feasible.

(iii) Some individual $i$ must be strictly better off in $\tilde{a}$ than in $\hat{a}$. This individual would thus never choose $\hat{a}_i$ under $\hat{T}(z)$. Hence, $\hat{a}$ cannot be an equilibrium under $\hat{T}(z)$.

Revenue inefficiency implies Pareto inefficiency: Assume that $\hat{T}(z)$ is revenue inefficient. Then a feasible schedule $\tilde{T}(z)$ exists such that (i) $\tilde{T}(z) \leq \hat{T}(z)$ holds for all $z$ and (ii) the initial equilibrium $\hat{a}$ is not an equilibrium under $\tilde{T}(z)$. Condition (i) implies that no individual can be worse off under $\tilde{T}(z)$ than under $\hat{T}(z)$. Condition (ii) implies that some individual refuses to choose the initial $(c_i, \hat{z}_i, \hat{e}_i)$ under schedule $\hat{T}(z)$. Hence, he or she must be strictly better off under $\tilde{T}(z)$ than under $\hat{T}(z)$.

Proposition 3 shows that the logic of Proposition 1 also applies if tax avoidance
or tax evasion are included in the analysis: A budget-balancing nonlinear tax schedule is Pareto efficient if and only if a self-financing tax cut does not exist; thereby, however, the effect of the tax cut on all sources of government revenue must be taken into account. Because of the latter point, Proposition 3 is closely related to the findings of Feldstein (1999), Chetty (2009), and Hendren (2016): Under the assumption $\tau(e_i) = 0$ of Feldstein (1999), the income tax $T(z)$ is the only source of government revenue. Hence, the efficiency of the tax schedule only depends on the reaction of taxable income $z_i$ on a tax cut. Under the assumption $\tau_i(e_i) \neq 0$ of Chetty (2009), a change in evasion $e_i$ has an indirect effect on government revenue due for example to fines for tax evaders or higher (declared) earnings of tax consultants. In this case, the efficiency of the tax depends on both the reaction of taxable income $z_i$ and earned income $y_i$ on a tax cut. To some extent, however, the distinction between the two elasticities is irrelevant because the only thing that matters is total government revenue $T(z) + \tau(e)$. This is in line with Hendren (2016) and implies that information about the fiscal effect of a tax cut on a macro-level is as valuable as a microeconomic decomposition of this effect into several behavioral elasticities.

4.2 Deductability of work-related expenses

There seems to be a broad consensus in tax theory and tax policy that the tax base of the income tax should be an economic surplus, which implies that work-related expenses should be fully deductible (see, e.g. Mirrlees et al. (2011), p. 63 for a discussion). As long as such expenses do only have an effect on earnings (and not directly on utility), they can be interpreted as an input to a private production function. In this case, the question whether full deductability is desirable boils down to the question whether production efficiency should hold in second best.

Previous work on wage related expenses within the context of a nonlinear income tax has shown that production efficiency (i.e. full deductability) can be constrained Pareto efficient, but not necessarily so. Critical is a separability assumption which was first discussed by Jacobs and Bovenberg (2011) and Gauthier and Laroque (2009). In the following, I will complement their findings in two ways: First, it is shown that Pareto efficiency is again equivalent to nonexistence of a self-financing tax cut. This finding does not hinge on the separability
assumption. Second, it is shown that the local optimality of full deduction implies that implementation is possible with a schedule that depends only on the economic surplus.

In practice, the precise definition of work-related expenses can be extremely difficult. Even in theory, the concept is not easily defined. Most of the relevant literature, for example, discusses the topic in the context of education costs. In this case, costs can take many forms, for example opportunity costs of foregone earnings, monetary costs like tuition fees, and utility cost of reduced leisure time when work and education are not substitutable. In the following, I will only consider the case of purely monetary expenses because this is the easiest case in practice and also the most transparent case when defining production efficiency.

Like in the basic model, I assume that the individuals’ preferences can be represented by utility functions $v_i(c_i, y_i)$ which are strictly increasing in disposable income $c_i$. The individuals’ earnings are now denoted $z_i$ and are the outcome of an earnings function $z_i(y_i, e_i)$ where $y_i$ stands for the individual’s efficiency units of labor and $e_i$ denote their work-related expenses. Hence, contrary to the basic model, I now distinguish between earnings $z_i$ and efficient labor $y_i$ because the former now also depends on monetary expenses $e_i$. Still, since $e_i$ can be set to zero and the normalization $z_i(y_i, 0) = y_i$ can be employed, the basic model is just a special case of the present one.

The expenses $e$ are observable and can therefore be taxed together with the earnings $z$. The individuals know the schedule $T(z, e)$ when making their earnings decision

$$\max_{c_i, y_i, e_i} v_i(c_i, y_i) \text{ s.t. } c_i \leq z_i - e_i - T(z_i, e_i), \quad z_i = z_i(y_i, e_i). \tag{7}$$

As before, this problem can have multiple solutions for each individual. A profile of such solutions is an equilibrium if it also feasible, i.e if

$$E[T(z_i, e_i)] \geq g. \tag{8}$$

The definition of a Pareto efficient tax schedule is the same as before. The definition of a tax cut does also not change, except that some schedule $T(z, e)$ is now replaced with another schedule $\tilde{T}(z, e)$ such that $\tilde{T}(z, e) \leq T(z, e)$ holds for all $(z, e)$ with strict inequality for some $(z, e)$.

The decision problem (7) can be split up into the problem of finding optimal
expenses \( e_i \)

\[
\max_{e_i} \quad z_i - e_i - T(z_i, e_i) \quad \text{s.t.} \quad z_i = z(y_i, e_i). \tag{9}
\]

conditional on efficiency units of labor \( y_i \), and then solving for \( y_i \) and \( e_i \). The first-order conditions of this problem can be written in the form

\[
\frac{\partial z_i(y_i, e_i)}{\partial e_i} = 1 + \frac{T_e(y_i, e_i)}{1 - T_z(y_i, e_i)}. \tag{10}
\]

Production \( z_i(y_i, e_i) \) is efficient if the marginal return of \( e_i \) equals its marginal cost for all individuals, i.e. if

\[
\frac{\partial z_i(y_i, e_i)}{\partial e_i} = 1 \quad \text{for all } i. \tag{11}
\]

It is clear that this condition only holds in equilibrium if \( T_e(y_i, e_i) = -T_z(y_i, e_i) \) holds in equilibrium. This means that the tax schedule \( T(z, e) \) must allow for full deduction of expenses \( e_i \) at each point in \( (z_i, e_i) \in \mathcal{I} \).

Jacobs and Bovenberg (2011) and Gauthier and Laroque (2009) argue that \( (11) \) holds in second best if a deviation from production efficiency has no effect on incentive compatibility and this in turn is the case if the assumption

\[
z_i(y, e) = z(y, e) \quad \text{for all } i \quad \text{and} \quad z_y(y, e) > 0 \tag{12}
\]

is made. Jacobs and Bovenberg (2011) establish the result by investigating the first-order conditions of a welfare maximization problem, Gauthier and Laroque (2009) by showing that any non-satiated allocation that violates \( (11) \) can be replaced by another, Pareto superior allocation. Part (b) of the following Proposition basically replicates the finding of Gauthier and Laroque (2009), but there are two differences. First, the assumption of a non-satiated allocation is replaced by the assumption that preferences are monotonic in \( c_i \). Second, it it shown that \( T_z(\cdot) = -T_e(\cdot) \) not just holds locally in an efficient allocation, but that schedules that have \( T_z(\cdot) = -T_e(\cdot) \) everywhere are sufficient for implementing any constrained Pareto efficient allocation.

Part (a) of Proposition 4 points out that Proposition 1 extends to nonlinear schedules with more than a single observable. In the present context, this means that an inefficient treatment of work-related expenditures can always be cured by (weakly) reducing \( T(z, e) \) everywhere in \( z - e \)-space. This is not obvious for the
following reason: Consider a situation where 50% of work-related expenditures can be deducted but it turns out that 100% would be efficient. Then the tax base would shrink from \((z - e/2)\) to \((z - e)\) and one might conclude that statutory tax rates must be increased for keeping tax revenue constant. If this were true, individuals without work related expenses would loose. The result, however, shows that a schedule \(\tilde{T}(z, e)\) must exist, that has the same or a lower tax burden everywhere in \(z - e\)-space. This implies \(\tilde{T}(z, 0) \leq T(z, 0)\) such that individuals without such expenditures would also either gain or remain unaffected.

**Proposition 4:** Assume that the individuals’ earnings \(z_i(y, e_i)\) depend on their efficiency units of labor \(y_i\) and their work-related expenses \(e_i\).

(a) A schedule \(T(z, e)\) is Pareto efficient if and only if it is revenue efficient.

(b) Assume (12). Then any Pareto efficient allocation can be implemented by a tax schedule of the form \(T(z - e)\).

**Proof:** (a) The proof is analogous to the proof of Proposition 3 and is therefore omitted.

(b) Under the assumption (12), the first-order conditions (10) and (11) are the same for all individuals. Let \(e_T(y)\) denote the solution to (10) under tax schedule \(T(z, e)\) and let \(e^*(y)\) denote the solution to (11). This allows us to define \(z_T(y) := z(y, e_T(y))\) and \(z^*(y) := z(y, e^*(y))\). Due to the assumption that \(z(y, e)\) is strictly increasing in \(y\), one obtains that (i) \(z_T(y)\) and \(z^*(y)\) are strictly increasing in \(y\) and that (ii) \(S^*(y) := z^*(y) - e^*(y)\) is also strictly increasing in \(y\). In the following, I will first explain why (11) must hold in a Pareto efficient allocation. In the second step, it is explained why a schedule of the form \(T(z, e) = T(z - e)\) can always be used for implementing such an allocation.

(Step 1) Consider a tax schedule \(\hat{T}(z, e)\) with corresponding allocation \((\hat{c}_i, \hat{y}_i, \hat{z}_i, \hat{e}_i)\) such that \((\hat{c}_i, \hat{z}_i) = (e_T(\hat{y}_i), z_T(\hat{y}_i)) \neq (e_i^*(\hat{y}_i), z_i^*(\hat{y}_i))\). Then we can construct a new schedule

\[
\bar{T}(z, e) := \begin{cases} 
\hat{T}(z, e) & \text{for } (z, e) = (e^*(\hat{y}_i), z^*(\hat{y}_i)) \\
\hat{T}(z, e) & \text{for } (z, e) \neq (e^*(\hat{y}_i), z^*(\hat{y}_i)).
\end{cases}
\]

where \(\bar{T}(z, e)\) is chosen such that

\[
z^*(\hat{y}_i) - e^*(\hat{y}_i) - \bar{T}(z^*(\hat{y}_i), e^*(\hat{y}_i)) = z_T(\hat{y}_i) - e_T(\hat{y}_i) - \hat{T}(z_T(\hat{y}_i), e_T(\hat{y}_i))
\]
holds for all \((z_T(\tilde{y}_i), e_T(\tilde{y}_i))\). It can easily be verified that this schedule is incentive compatible and that a reform from \(\tilde{T}(z, e)\) to \(\tilde{T}(z, e)\) is a tax cut with additional revenues for the government. Therefore, \(\tilde{T}(z, e)\) is Pareto inefficient.

(Step 2) Consider an incentive compatible allocation that is production efficient, i.e. where \(z_i = z^*(y_i)\) and \(e_i = e^*(y_i)\) holds for all individuals. Let \(T^*_i := T(z^*_i, e^*_i)\) denote the corresponding tax payment. We can now choose \(\tilde{T}(z, e) = T^*_i\) for all \((z, e)\) with surplus \(z - e = z^*_i - e^*_i\). This is possible because of property (ii). Note that tax schedule \(\tilde{T}(z, e)\) implements the allocation \((z^*_i, e^*_i)\) because a deviation from \((z^*_i, e^*_i)\) without changing the surplus \((z - e)\) reduces utility and a deviation from \((z^*_i, e^*_i)\) with a change in \((z - e)\) is not worthwhile because \((z^*_i, e^*_i)\) is incentive compatible.

4.3 Non-welfarist objectives that lead to Pareto-efficient outcomes

The literature on optimal income taxation typically presumes that tax policy aims at redistributing some measure of utility among individuals. Outside the economics profession, progressive taxation is usually discussed in terms of redistributing money. As long as earnings are exogenous and impose no cost on the individuals, the difference between these concepts is more or less semantic. With endogenous earnings and a presumed disutility of labor, however, the distribution and redistribution of some measure of utility is only loosely related to the distribution and redistribution of disposable income.

A straightforward consequence of Proposition 1 is that one can have an objective function that does not make use of any notion of utility, but nevertheless leads to a Pareto efficient outcome. Due to the latter property, it is disputable to what extent this alternative approach actually differs from the standard approach. Therefore, I will first consider a standard welfare function with Pareto weights \(\gamma_i\):

\[
W := \int \left[ \gamma_i \cdot v_i(c_i, y_i) \right] \, di \quad \text{where} \quad \gamma_i > 0
\]

(13)

When such an objective is employed, results always depend on the welfare effect of marginally increasing some individual’s disposable income, i.e. \(\gamma_i \cdot (\partial v_i(\cdot)/\partial c_i)\). This is the main variable for the characterization of distributional preferences.
Saez and Stantcheva (2016) argue that this term measures is all one needs to know and that it can be replaced by some function $g_i(c_i, y_i)$. A tax reform with a change $\Delta T(y_i)$ of the individuals’ tax payments is then beneficial if

$$ \int_i [g_i(c_i, y_i) \cdot \Delta T(y_i)] \, di > 0. \quad (14) $$

Saez and Stantcheva (2016) show that this approach is flexible enough for capturing many normative concepts of fairness and that the endogeneity of weights $g_i(c_i, y_i)$ is unproblematic at least if the criterion is used for evaluating a local tax reform.

One interesting aspect of the weights $g_i(c_i, y_i)$ is that they lift the dichotomy between notions of a fair distribution of utilities $v_i(c_i, y_i)$ and notions of a fair distribution of (disposable) incomes $(y_i, c_i)$. In the following, I will consider objective functions that only depend on $(y_i, c_i)$ and still obtain Pareto efficient outcomes. They are either based on the maximization of some weighted average

$$ C(y) := \int_y [\alpha(c, y) \cdot c(y)] \, dy \quad \text{where} \quad \frac{\partial[\alpha(c, y) \cdot c]}{\partial c} > 0 \quad (15) $$

of the individuals’ statutory disposable incomes $c(y)$ or the minimization of some weighted average

$$ T(y) := \int_y [\beta(T, y) \cdot T(y)] \, dy \quad \text{where} \quad \frac{\partial[\beta(T, y) \cdot T]}{\partial T} > 0 \quad (16) $$

of their statutory tax burden $T(y)$. These objectives can be labelled non-welfaristic because they neither depend on the individuals’ utilities $v_i(c_i, y_i)$ nor on the relative frequency of $i$ in the economy. I will first discuss how these objectives are related to the Pareto criterion and then how they are related to welfare functions of the form $W$.

Note that $C(y)$ basically maximizes the size of the individuals’ budget set. Hence, it has some resemblance to an indirect utility function, but it is no such function because otherwise information about the individuals’ preferences would

---

3Saez and Stantcheva (2016) also allow for additional characteristics $x_i^u, x_i^b$, and $x_i^s$ of an individual among which $x_i^u, x_i^b$ can enter the utility function $v_i(\cdot)$ and $x_i^u, x_i^s$ can enter the welfare weight $g(\cdot)$. Such characteristics could be introduced here as well. Since they are not relevant for a comparison between maximization of $W$ and $C$ or $T$, they are omitted.

4See Fleurbaey and Maniquet (2018) for a discussion.
have been used. Still, if one accepts the assumption made in Section 2 that preferences are monotonic in $c_i$, an expansion of the budget set cannot make an individual worse-off. In other words, due to the monotonicity assumption, the critique of Kaplow and Shavell (2001) does not apply to objectives $C(y)$ and $T(y)$.

**Proposition 5:** If a nonlinear schedule $T(y)$ is designed either

(i) by maximizing some weighted average $C(y)$ of the statutory disposable income $c(y)$ or

(ii) by minimizing some weighted average $T(y)$ of the statutory tax $T(y)$

subject to constraints (1) and (2), a Pareto efficient outcome is obtained.

**Proof:** Assume that a feasible schedule $T(y)$ with corresponding $c(y)$, $T(y)$, and $C(y)$ is not Pareto efficient. Then, by Proposition 1, another feasible schedule $\tilde{T}(y)$ exists such that $\tilde{T}(y) \leq T(y)$ for all $y$ with strict inequality for some $y$. This implies $\tilde{C}(y) > C(y)$ and $\tilde{T}(y) < T(y)$. Hence, if $T(y)$ implements an inefficient allocation, it can neither maximize $C(y)$ nor minimize $T(y)$. ■

The main difference between maximization of $C(y)$ and application of criterion (14) is that the former maximizes values of the function $c(y)$ whereas the latter maximizes the social value of redistributing income between individuals with different $(c_i, y_i)$. The weights $\alpha(c, y)$ attributed to some pair $(c, y)$, i.e. some value of function $c(y)$, are exogenous; the weights $g_i(c_i, y_i)$ attributed to some individual are potentially endogenous. This means that at least the interpretation of both objectives is different. From a technical point of view, however, they are closely connected because they can both be used for implementing any Pareto efficient allocation.

Next, I will explain how the logic of maximizing some weighted sum $C(y)$ or minimizing some weighted sum $T(y)$ is related to traditional normative conceptions of fair taxation. For an illustration, I employ the equal sacrifice principle and the equivalence principle.

**Equal sacrifice principle:** The normative concept of equal sacrifice proposes a tax schedule that imposes the same absolute, relative, or marginal cost of taxation on each individual. In the economics literature, this concept is typically discussed in a framework with exogenous earnings $y_i$ where the cost of taxation are measured
by comparing the individuals’ consumption utility \( V(c_i) \) in a situation with and without the tax. From a welfare perspective, this is problematic because \( y_i \) is not exogenous in reality and even if it were exogenous, its cost are not taken into account. From a practical perspective, it is attractive because it readily translates into ‘fair’ and ‘unfair’ tax schedules: Depending on whether absolute, relative, or marginal cost shall be equalized, a fair schedule must have the property that either

\[
V(y_i) - V(y_i - T_i) \quad \text{or} \quad \frac{V(y_i) - V(y_i - T_i)}{V(y_i)} \quad \text{or} \quad V'(y_i - T_i)
\]

is the same for all individuals. Of course, the functional form of that schedule also depends on the choice of function \( V(c_i) \). For example, equalization of absolute sacrifice under function \( V(c_i) = \ln c_i \) implies that only proportional schedules \( T(y) = t \cdot y \) should be employed. The equal sacrifice concept thus determines a class of schedules, among which one can choose that schedule that balances the government’s budget. Let \( \tilde{T}(y) \) denote that schedule and \( \tilde{c}(y) \) the corresponding budget constraint; see Figure 3 for an illustration.

It is important to note that an equal-sacrifice schedule is derived from a concept of fairness that interprets an increase of earnings \( y_i \) as a gain and an increase of the tax \( T_i \) as a loss. It is nonindividualistic because neither the number of individuals in a certain income bracket nor the individuals’ ‘true’ utility (including the disutility of earnings) plays a role. Therefore, the budget-balancing equal sacrifice schedule \( \tilde{T}(y) \) is not necessarily Pareto efficient. However, the following simple modification of the concept always leads to a Pareto efficient
outcome: In the first step, the equal-sacrifice budget-balancing schedule $\bar{T}(y)$ is calculated. In the second step, problem

$$\max_{c_i, \bar{y}_i} \mathcal{C}(y) \quad \text{s.t.} \quad (1), (2), \quad c_i \geq \bar{c}(y_i).$$

(17)
is solved. Since $\bar{c}(y)$ is feasible, Proposition 5 applies to this problem as well. One can now distinguish between two cases: When $\bar{T}(y)$ is Pareto efficient, $\bar{c}(y)$ is the only solution to problem (17) and the equal-sacrifice schedule will be implemented. When $\bar{T}(y)$ is not Pareto efficient, (17) leads to a Pareto superior outcome. This is readily visible to all individuals as only expansions of the budget set are possible. Hence, a deviation from equal sacrifice is accepted if and only if the the budget constraint can be shifted upwards in some income bracket without loosing tax revenue from those individuals who are positively affected. When the tax cut generates more revenue, the weights $\alpha(c, y)$ determine how this gain is redistributed among different income brackets.

**Equivalence principle:** According to this principle, the contribution $T(y)$ of each taxpayer should be proportional to her benefit from government services $g$ and it is usually assumed that this benefit $B(y)$ is increasing in $y$. Hence, a fair schedule must be of the form $T(y) = t \cdot B(y)$ where the proportionality factor $t$ determines whether tax revenue is sufficient for financing expenditures $g$. Let $\bar{t}$ denote the smallest $t$ that balances the budget and $\bar{T}(y) = \bar{t} \cdot B(y)$ the corresponding schedule.

Based on the equivalence principle, a Pareto efficient schedule can again be derived in two steps. In the first step, $\bar{T}(y)$ is calculated. In the second step, problem

$$\min_{c_i, \bar{y}_i} T(y) \quad \text{s.t.} \quad (1), (2), \quad T_i \leq \bar{T}(y_i).$$

(18)
is solved. It has the same properties as problem (17) above: When $\bar{T}(y)$ is Pareto efficient, it is the only solution to (18); when $\bar{T}(y)$ is not Pareto sufficient, a Pareto superior outcome is obtained that differs from $\bar{T}(y)$ only because the statutory tax can be reduced in some income bracket without actually loosing revenue from the taxpayers in that bracket.
References


