Tax Compliance and Tax Evasion
An Agent-Based Model Approach

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Abstract

A tax compliance Agent-Based Model, accounting for tax-morale and loss-aversion, was implemented over different network systems with social interactions at the local level. Ours is an innovative model which integrates endogenous characteristics of heterogeneous agents and proposes a realistic underlying fitness-function model structure. Audit rates, fines and tax morale non-linearly increase tax compliance, whereas tax rates have a non-linear negative impact on income disclosure. The most compelling result is the interpretation of a mechanism through which individuals may fully comply their tax obligations even when the probability of being audited is null. The second relevant finding is the non-linear channel through which the network architecture provides an opportunity to design an optimal audit policy form the social planner’s point of view.

JEL: H26, C63
Keywords: Tax Evasion, Agent-Based Models, Complex Networks.

The importance of tax collection for governments and public policy makers goes practically without saying. Ever since the seminal paper published by Becker (1968) regarding the economics of crime a vast congregation of literature has concentrated around the topic. Tax misreporting, by its illegal nature, conveys a high incentive for secrecy and, therefore, real data is not readily available in the national official statistics. Nevertheless, evidence may be used to assess the goodness of fit for different economic models which attempt to explain tax evasion process. Such evasive behavior is represented as a decision, or choice, made under uncertainty, which is itself dependent on a probability of being either caught or not. We provide a novel adaptation of the tax evasion model in an Agent-Based Model that considers an artificial society with social interactions. An absence of matchable incomes for the individuals allows for frequent opportunities of underreporting and income disguising. Agents update their subjective probability of being audited according to a weighted average of their own experience and the experiences of their “neighbors”.

Dwenger et al. (2016) published a fascinating finding: it was found in a field experiment in Germany how certain individuals fully complied their tax obligations even when the true probability of being audited was zero. Previously, Alm et al. (1992) had also found in a laboratory experiment how a fraction of subjects fully complied even under the absence of audit schemes. In order to study this phenomenon, we resort to a model which accounts for massive local interactions where agents, at the aggregate level, attempt to discover the true audit probability. We resort to simulation to analyze tax behavior in the presence of heterogeneous agents and see how compliance depends on individual values of tax morale and risk aversion as well as interactions with the perceived audit probability and structural parameters such as tax and fine rates. In particular, our research provides a micro-founded model and a network architecture which enables to computationally replicate the stylized facts found in the papers of Dwenger et al. (2016) and Alm et al. (1992).

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Section 1 of this paper offers a comprehensive literature review and details the background of our research. Further, Section 2 structures the optimization problem that individuals face whenever choosing to evade or comply with their tax duties. Moreover, this section offers several optimality conditions and closed-form solutions under specific scenarios. Section 3 explains the mechanism through which social influence and subjective beliefs regarding the enforcement policies are transmitted throughout the network of agents. Employing an Agent-Based Model, Section 4 presents results, robustness tests and a statistical analysis of the parameter estimations and their effects in the decision-making process. An extension of the paper is conferred in Section 5, where an optimal audit policy is specified from the social planner’s point of view. Finally, 6 summarizes and discusses the main results found in this paper.

1 Introduction

Following the influential paper published by Allingham and Sandmo (1972) exploring the rationale behind tax evasion phenomenon, a vast literature has further studied the modeling of tax compliance. A non-negligible portion of income-tax research has explored the mechanisms surrounding social interaction among the agents, particularly in agent-based modeling. Mittone and Patelli (2000) and Mittone (2006) delved into the psychological motives of tax compliance inherent to a society composed by three heterogeneous types of agents: full-compliers, imitators, and full-evaders (free-riders). The objective of their endeavor was bound to the analysis of aggregate tax behavior in function of the initial composition of the taxpaying population under two different audit schemes; uniform and tail auditing.

A comprehensive compilation of literature may include Davis et al. (2003) who find stable equilibria both under low and high enforcement schemes, linked by a non-linear and asymmetric transition; Hokamp and Pickhardt (2010) introduced an exponential utility function for agents in order to induce more realistic results; and Korobow, Johnson and Axtell (2007) introduced a network structure but also considered individuals who possessed heterogeneous characteristics and intrinsic perceptions about the enforcement regime. Moreover, tax morale has been a recurring matter in the models of tax compliance ever since Myles and Naylor (1996) asserted a social conformity framework in which agents attained an additional utility from conforming to the established social norms. Despite the complications to accurately define tax morale, hereafter tax morale will be understood as an umbrella term, in the sense of Luttmer and Singhal (2014), enclosing intrinsic motivation, reciprocity, culture, biases and social influences.

Stepping forward into a more contemporary literature review, Andrei et al. (2014) contributed an additional aspect to be taken into account for agent-based models of income tax evasion. The authors found that the network structure underlying the societal arrangement has a significant impact in the decision process dynamics; principally, individuals tended to disclose a larger fraction of their income whenever they embodied networks with higher levels of centrality. Amongst the number of social structures tested, the Erdos-Renyi random network and the Power Law (scale-free) networks incentive agents to comply the most given their larger capacity of propagating information and influence dissemination.

Alm, Bloomquist and McKee (2017) conducted a social experiment intended to learn about the burden of peer effects and social pressure in the context of tax compliance. The conclusions reached by the authors discuss how agents have a statistically significant positive effect in the tax disclosures of their ‘neighbors’ or ‘people with whom they frequently share information’: when an agent is surrounded by honest (cheating) individuals, the agent itself starts to behave in a more honest (cheating) manner.

2 A taxpayer’s decision to evade

Tax compliance decision-making is ordinarily modeled as a gamble or an investment opportunity involving one risky asset (undisclosed income) and a risk-free asset (disclosed income). The micro-founded expected utility to be optimized with respect to the fraction of income declared $d$ by each individual $i$ at time $t$, yet discarding the subindex for simplicity, may well be defined as:

$$EU[d] = p \cdot U(X) + (1 - p) \cdot U(Y),$$

(1)
where $X$ is the net income after taxes and penalties in case an audit takes place and $Y$ is the net income after taxes in case no audit takes place. Promptly substituting $X$ and $Y$ in terms of the gross earned income $I$, the penalty rate $\theta$ applied to the undisclosed fraction of income in case an audit occurs, and the tax rate applicable as a function of the income declared $\tau(i)$, $X$ and $Y$ are expressible as functions of $d$ as $X = I - \tau(d \cdot I) - \theta\tau(I - d \cdot I)$ and as $Y_{i,t} = I - \tau(d \cdot I)$. Thus, we can reformulate Equation 1 as:

$$EU[d] = p \cdot U[I - \tau(d \cdot I) - \theta\tau(I - d \cdot I)] + (1 - p) \cdot U[I - \tau(d \cdot I)].$$

(2)

Akin to the adjustment outlined by Hokamp and Pickhardt (2010), a power utility function is imputed into the model outlined in Equation 2. Again, considering for each agent $i$ and each time $t$, the utility function of every single agent is characterized in Equation 3.

$$U(d) = (1 + d)^{\kappa}W^{\rho(1 - \kappa)}$$

(3)

where the variables are denoted as: the fraction of declared income $d \in [0, 1]$, period-wealth $W = \{X, Y\}$ (which is, in fact, a function of $d$), risk-aversion $\rho \in (0, 1)$, and tax-morale $\kappa \in [0, 1]$. In this sense, a higher the tax morale yields a larger utility of complying; while a higher risk-aversion would yield a lower utility of wealth. Considering Equation 3, it might be advantageous to remark that the only parameter over which agents optimize is the fraction of income declared $d$, given that wealth $W$ is dependent on income, fine rates, tax rates and $d$, out of which the individual is bound to determine only the latter one, while the other variables remain exogenous. Furthermore, the utility function is concave from below with respect to both $(1 + d)$ and $W$.

A remark for the current tax decision model is the non-matchable income assumption, meaning that an auditor from the Tax Agency does not know beforehand the individuals’ incomes. If a society would happen to account for a non-negligible matching system for its labor market, the assumption may be relaxed to take into consideration only the non-matchable portion of the agents’ stipends without sacrificing any of the models’ intuitions and results.

Furthermore, in order for agents not to fully evade systematically, the fine rate must be strictly larger than 1, that is $\theta > 1$. Moreover, under the power utility function specified in Equation 3, this condition is necessary and sufficient for agents to fully comply whenever audits happen with certainty (see Appendix). Referring to the tax rate function $\tau(\cdot)$ the most common examples of such tax functions may be either a flat (constant) tax rate or a stepped-tax regimes, where agents are taxed progressively according to their income.

**Optimality under certain audits and null audits**

Considering the expected utility optimization problem specified in Equation 2, where the utility function is the one expressed in Equation 3, there are closed-form solutions for two scenarios: whenever tax investigations are nonexistent ($p = 0$) and whenever the audits occur with certainty ($p = 1$).

**Proposition 1** Whenever the probability of being audited at time $t$ is zero, individual $i$ might fully evade, partially evade or fully comply depending on its own willingness-to-pay taxes ($\gamma_{i,t}$).

$$[d_{i,t}^* | p_{i,t} = 0] = \begin{cases} 0 & \text{if } \gamma_{i,t} \leq \tau, \\ (0, 1) & \text{if } \tau < \gamma_{i,t} < \frac{2\tau}{1 - \tau}, \\ 1 & \text{if } \gamma_{i,t} \geq \frac{2\tau}{1 - \tau}. \end{cases}$$

where $\gamma_{i,t} = \frac{\kappa_i}{1 - p_{i,t}}$ is the willingness to pay taxes; understood as the individual’s tax morale corrected for its risk-aversion.

**Proposition 2** Whenever the probability of being audited at time $t$ is one, individual $i$ will fully comply.

$$[d_{i,t}^* | p_{i,t} = 1] = 1 \quad \forall \{i, t\}.$$
The implications arising from Proposition 1 are thought-provoking, in particular the notion of agents complying (and even fully-complying) under certain conditions even when the probability of being audited is zero and there is an absence of public game mechanisms. This proposition may offer a mathematically derived explanation for the startling results found by the field experiment of Dwenger et al. (2016) and by the experimental approach of Alm et al. (1992). Furthermore, the intuition behind Proposition 2 is quite straightforward. The proofs for Proposition 1 and Proposition 2 are available in the Appendix.

A crucial question arises whenever talking about the effects of parameters on evasion models: Do people like to pay taxes? In other words, is the expected utility function derivative with respect to the tax parameter ($\tau$) positive, negative or zero?

**Proposition 3** Assume the minimal net income an individual may receive after audits is zero, then in fact, people do not like to pay taxes. Moreover, the expected utility function is reduced whenever the tax rate $\tau$ increases.

$$\frac{\partial EU(d_{i,t})}{\partial \tau} < 0.$$ 

The assumption of having a minimal net income after audits of zero is analogous to establishing the notion that the maximal amount of money the Government or Tax Agency may collect for concept of taxes and fines is the entirety of the individual’s income. More precisely, a necessary condition for the income in case of an audit to become negative, is that $\theta$ must be strictly larger than one over $\tau$. However, this is hardly the case given that tax rates usually do not go over 50% and penalty rates frequently oscillate between 1.2 and 1.75 [see Hindriks and Myles (2006)]; making the necessary condition to be seldom met and providing a base to argue that our premise of non-negative incomes is not a strong assumption but rather a reasonable one. In several expected utility models there is a somewhat counterintuitive property of high tax rates promoting tax evasion, however this is a non-linear effect that arises given the fact that the penalty is set by $\theta\tau$; thus increasing the tax rate is in some sense affecting a pronounced enhancement of the fine rate. The proof for Proposition 3 is provided in the Appendix.

**Proposition 4** Proposition 4: Given the power utility functions are, in fact, iso-elastic, individuals have a Constant Relative Risk Aversion (CRRA). In particular, all agents have a CRRA coefficient of $R(W) = \rho_i \in (0,1)$.

**Remark 1:** A working assumption of the utility function is that $\rho$ cannot be zero nor one. This is, in fact, a considerably relaxed assumption which will be addressed in Section 3 through a particular network structure characterization. The proof for Proposition 4 which defines the CRRA coefficient is included in the Appendix.

**Remark 2:** Other working assumptions include:

- Income is strictly positive: in the contrary, there would be no problem to begin with.
- The tax rate $\tau$ is neither 0% nor 100%. In the first case, everyone would be fully compliant by definition; in the second case (given the lack of a public game re distributive mechanism) all agents would leave the labor market as the wage would be nominally zero (all your wage goes to taxes). Besides these aforementioned values, the tax function $\tau(\cdot)$ may be any non-regressive tax system.
- There are no retrospective audits, no possibility to send tax evaders to jail and the maximum penalty equals the entire salary of the individual.

3 Social influence and Network Effects

Subjective Audit Rate

The individuals’ subjective probability of being audited is updated based on their past experience. Moreover, their audit beliefs are likewise updated by the behaviors of their ‘neighbors’, defined as the
agents with whom they frequently exchange information, as in Alm, Bloomquist and McKee (2017). Hereafter, the subjective audit probability perceived by agent $i$ at time $t$ can be defined as a weighted average of the agent’s prior experience (temporal updating) and the perceived probability of its neighboring individuals (geographical updating) at the previous period.

The universe of agents coexists in a predefined network structure with (local) social interactions and each period agents exchange information with their neighbors, however they never get to know the entire situation nor the composition of the society in which they inhabit. Afterwards, agents update their own perceived audit probability by means of a weighted average of three possible channels: their subjective audit rate in the previous period (prior), their own recalling of past audits (memory), and the signals they received from their neighbors (social influence).

\[
\hat{p}_{i,t+1} = \lambda_1 \hat{p}_{i,t} + \lambda_2 \sum_{s=1}^{S_i} \frac{A_{i,t-s}}{S_i} + (1 - \lambda_1 - \lambda_2) \sum_{j \neq i}^{N_{i,t}} \frac{1}{N_{i,t}} \sum_{s=1}^{S_j} \frac{A_{j,t-s}}{S_j}
\]

where $\lambda_1$ and $\lambda_2$ are convex averaging weights, $A_{i,t-s}$ is valued one if the agent $i$ was audited in the period $(t - s)$ and zero otherwise, $S_j$ is the memory or number of audit periods that agent $j$ can recall in the past, and $N_{i,t}$ is the number of neighbors of agent $i$ at time $t$.

The effects imprinted by each channel of subjective updating in Equation 4 can be inferred from Figures 1 and 2. Figure 1 shows the mean perceived probability of being audited (z-axis) as a function of the weight given to the prior and memory components. Figure 2 is the analogous representation for the prior and social influence elements. It is immediate to see, for both charts, that whenever the weight invested in the prior component ($\lambda_1$) increases, then the mean subjective audit rate boost disproportionately. One and the other images present a graphical evidence to support the notion of conferring weighting power to the memory and social influence elements at the expense of the prior component. Consequently, $\lambda_1$ is strictly smaller than one in order for the model to account for social influences; otherwise the individuals would disregard the signals received and would never update their subjective probabilities of being audited.
compliance, a simplistic modification is made on the Tax Agency’s audit strategy. This alteration, however, will be crucial for the game-theoretic analysis of Section 5. It would not be a surprise that a Tax Agency would be more inclined into targeting individuals whose eye-catching income stands out from the sample. Ergo, proceeding for each agent, the endogenous probability of being audited is set in accordance to its income level, where individuals with higher salaries have larger probabilities of being audited. Following, the endogenous audit rate \( q \) for agent \( i \) is the true audit rate \( p \) multiplied by the ratio of the agent’s income over the average income of the population.

\[
q_i = \frac{I_i}{\sum_{j=1}^{N} I_j} \cdot Np
\]  

In this fashion, the true probability of an agent to be audited is proportional to its income level. Notwithstanding the adjustment implemented for the endogenous audit rates, the mean value of \( q \) remains equal to the true audit rate \( p \); allowing for consistent testing of parameters.

**Network Structure**

A cornerstone of the ongoing Agent-Based model with social pressure is the network formation process. Extending the work of Andrei et al. (2014), a selection of nine different underlying network structures were tested ensuing comparisons and contrasts among one another: two types of random graphs, small words, large world, Watts-Strogatz, ring, wheel and two types of scale-free networks. Figure 3 and Figure 4 display an Erdos-Renyi random graph and a random geometric graph, respectively, in a toroidal world which ‘wraps up’ both vertically and horizontally. Albeit both graphs having the exact number of agents and the same amount of links, its interesting how the physical arrangement appears to be strikingly different, yet the density and degree distribution its the same. Hence, in order to visually exemplify the closeness centralities corresponding to each network, agents will be randomly re-arranged in a circular formation and the link density, potentially, shall proxy the centrality measures among individuals.
Figure 5 depicts a preferential attachment graph on a toroidal universe, and Figure 6 presents a fitness function model in a Cartesian environment; both are particular characterizations of scale-free networks. The former comprises agents with a diverse degree of connections, yet no attention is paid to the individual characteristics of agents for link allocation; the latter, however, follows a mathematical specification to determine or not an association between any two given individuals. Modeling the social relationships as a fitness-function network, individuals are constrained to link only with ‘relevant others’ within their reach in terms of age and social position. Attending to this notion, an artificial society was emulated on a Cartesian plane where the x-axis features the agents’ age whilst the y-axis represents the individuals’ income; agents slide horizontally until they reach a sufficiently advanced age, in which case they ‘retire’ from the labor market and are replaced by an offspring endowed with a fraction of the exiting agent’s wealth. Figure 6 depicts a pyramidal society where the top-right corner positions are occupied by old, wealthy individuals, the top-left corner is void (juvenile millionaires) and a dense bottom-left edge reveals a large amount of young agents with low or middle incomes. Evidently, it would be plausible to speculate that in the real world a person would discuss his or her fiscal matters rather exclusively with people of their own age and income level (social status).

Recalling Remark 1 of Proposition 4, a CRRA coefficient implies a decreasing absolute risk aversion coefficient (DRRA). In order to avoid values of $\rho$ very close to zero or one, it is possible to provide agents with endogenous DRRA parameters. It follows that a local and network-based risk-aversion parameter ($\tilde{\rho}$) may well be defined as the relative income position held by the individual with respect to its neighbors divided by the neighborhood size $|N|$ (including the agent) plus one, where the lowest absolute risk-aversion measures pertain the most affluent agents. Notice how this manner accordingly allows the global distribution of individual loss-aversion of $\rho$ to be symmetric around 0.50 both for the endogenous and the exogenous scenarios. The rank function is valued one for the most and $N$ for the least affluent agents.

\[ \tilde{\rho}_i = \frac{\text{rank}(I_i)}{|N|+1} \]  

Figure 7 offers a visual comparison between the distribution of the endogenous risk-aversion parameter for an Erdos-Renyi random-graph (see Figure 3) and a fitness function model network (see Figure 6). The endogenous risk-aversion parameter in the Fitness function model tends to acquire a bell-shaped distribution at a notoriously faster speed of convergence than the Erdos-Renyi random-graph. Examining the intuition behind this development is rather simple. For the fitness function arrangement, most individuals have neighbors both below and above their y-axis, which represents the income level; thus, a majority of agents are neither the richest nor the poorest person in their neighborhood. Moreover, on average, individuals here tend to be rather close to the median value of their relative income position. Exceptions may be the agents living in the upper (lower) borders of the structure, whom would be rich (poor) relative to their neighborhoods and attain a smaller (larger) endogenous risk-aversion.

![Individual Endogenous Loss-Aversion](image)

Figure 7: Endogenous risk-aversion levels

As mentioned, the model was tested on nine different underlying network structures. In order to better visualize the density and connectedness of these architectures, all of them are presented in a
circular representation below. Figure 8 and Figure 9 portray, respectively, the same networks as in Figure 3 and Figure 4 in a closed and bounded space, fashioning both images to turn out rather identical. Both Figure 10, using preferential attachment, and Figure 11, using a fitness function, are particular cases of the scale-free family of networks, analogous to the Power Law network employed by Andrei et al. (2014); nonetheless a word or two have been argued in favor of the fitness function model over other generic power law structures. Figure 12 exhibits a small world graph, know for its high clustering and small average shortest paths; Figure 13 accounts for a Watts-Strogatz model, understood as in Barthelemy and Amaral (1999) as a cross-over between a small and large world, the latter reported in Figure 14 as a lattice with a low disorder (re-wiring) degree. Figure 15 illustrates a Ring network, where a continuous pathway between nodes is accomplished by attaching each agent to exactly two other agents. Lastly, Figure 16 shows a wheel graph which arises from a regular network modified to host a randomly selected individual linked to the entire agent population.
4 Agent-Based Model Implementation

The main objective of the following section is understanding the effects of different regressors on the fraction of income declared at the society level. Korobow, Johnson and Axtell (2007) enlisted key distinctions that differed ABM’s from the initial mathematical models. Arguably one the most germane attributes of any Agent-Based Model is its intrinsic capability of embodying abounding heterogeneous agents, all possessing unique aspects and personalities. Accordingly, each agent is heterogeneous by acquiring specific built-in characteristics. Secondly, individuals base their decision process in subjective probabilities of being audited and not in true audit rates. Lastly, agents coexist in neighborhoods inside a larger societal structure which allows for information exchange at a local level, predominantly the frequency of audits perceived by themselves and their immediate neighbors. Similarly to Korobow et al., this model includes the three listed attributes, however there is a key difference in the assumptions about shared information. Korobow et al. implemented their model under the hypothesis that individuals shared their own, personal ‘payoffs’ for tax evasion among their neighborhoods, they acknowledged the unlikelihood of such delicate information becoming public in real-life scenarios. In order to outplay this limitation, now agents communicate their memory on previous audits and update their subjective probability of being audited in conjunction with their own memory and prior beliefs.

Stepping into the realm of the current work, agents are constituted as individuals embroidered by personal traits: tax-morale, loss-aversion, income, age and an initial subjective probability of being caught evading taxes. Moreover, the tax-morale and loss-aversion parameters are dynamic, increasing stochastically with respect to age. On any occasion in which an agent would happen to grow into an age above 65, it would be removed from the network analogous to a retired individual would exit the labor market; to replace the empty node left inside the network, a new individual, aged 18 and with its own particularities, would replace the available position. Tax morale is initialized as a common society-level parameter for the entire network, whereas loss-aversion is sampled from a uniform distribution U(0,1) for each individual. Thereafter, the willingness-to-pay taxes may be reconstructed from the tax-morale and risk-aversion as previously defined; which may be treated as an endogenous variable or remain exogenously given to the agent. For a society level tax-morale of $\kappa_S \leq 0.5$, we will model $\kappa \sim U(0, 2\kappa_S)$.

The analysis of the linear and quadratic coefficients found in Table 1 along with their respective significance codes for 58,320 simulations ran over 5,832 different possible parameter combinations. The tax function was set to be a flat-tax rate applicable universally, allowing for a richer statistical analysis on the effects of tax rates on the aggregate level of tax compliance. Keeping in mind the nature of the outcome variable $d$ to be bounded between zero and one, a specific statistical model must be applied for data analysis. The results produced by the simulation were tested under a Tobit model censored for minimum and maximum values of zero and one, bounds, respectively, boundaries included.

Table 1 delineates the different linear and non-linear effects that each parameter imposes in the fraction of income disclosed. Column (1) and Column (2) both deal with the effects of tax rates, audit rates, fines, tax-morale and endogenous variables. The last regressors in Column (1) take into consideration the average closeness-centrality of all individuals in the underlying network where agents coexist; the closeness-centrality is interpreted as ‘how easily a node may be reached from all other nodes’. Hereby and after closeness will be understood as the inverse of the average distance of a node to all other nodes; where the distance between two nodes is the shortest path in which one node may reach the other. Consequently, a node who requires few steps to reach other nodes will have a lower average distance, implying a larger closeness within the network. On the other hand, Column (2) controls for each network structure by adding one dummy variable for each type while keeping the Erdos-Renyi random-graph as the baseline case, allowing for comparisons with the results by Andrei et al. (2014).
Table 1: Dependent variable: Aggregate [d]

<table>
<thead>
<tr>
<th>Tobit Model</th>
<th>Closeness</th>
<th>Dummy variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regressors</td>
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<td>(2)</td>
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<tr>
<td>Intercept</td>
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<td>-2.146 ***</td>
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<tr>
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<td>-3.392 ***</td>
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<td>$\tau^2$: tax_rate$^2$</td>
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<td>2.347 ***</td>
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<td>4.060 ***</td>
</tr>
<tr>
<td>$p^2$: audit_rate$^2$</td>
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<td>-12.550 ***</td>
</tr>
<tr>
<td>$\theta$: fines</td>
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<td>0.055 ***</td>
</tr>
<tr>
<td>$\theta^2$: fines$^2$</td>
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<td>-0.003 ***</td>
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<tr>
<td>$\kappa$: tax_morale</td>
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<tr>
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<tr>
<td>Wheel</td>
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</table>

Signif. codes: '***' 0.001 '**' 0.01 '*' 0.05

Opening the analysis of the Tobit model, tax rates seem to impose a negative effect on tax compliance, yet such impact seems to marginally increase for very large levels of tax duties. This effect, which can be interpreted from a positive estimated coefficient for $\tau$ and a negative estimated coefficient for $\tau^2$, as seen in Table 1, is represented in Figure 17 where tax evasion (the complement of one minus the fraction of tax compliance) increases non-linearly with respect to the tax rate. A basal development of any taxation model is the understanding of how tax rates behavior reflect an impact on the collected revenues from the Tax Agency’s point of view. The Laffer Curve is the representation of tax revenues as a function of the tax rate. Governments cannot over-raise the tax rate as it would incentive agents to evade taxes, reducing the governmental revenue. Figure 18 details the corresponding Laffer Curve for the simulated society. Given that incomes may differ among the agents, tax evasion is computed as one minus the ratio of declared earnings over the true income.

Figure 17: Tax evasion as a function of $\tau$  
Figure 18: Laffer Curve
An outcome from Table 1 that falls in line with common sense is the positive coefficient for audit rates; as the true probability of being audited increases, a larger proportion of agents experience audits, which in turn communicate the event to their neighbors and the information about a harsher enforcement environment becomes public knowledge, ensuing higher tax compliance among individuals. The quadratic term of the audit rate suggests that, despite the notion of larger audit rates implying higher tax compliance, this policy tool will tend to lose effect as the audit probabilities start turning ‘too high’. The resulting non-linear effects of audit rates on tax evasion are of obvious relevance for public policy, shedding light on a possibility that raising audit rates and strengthening enforcement schemes will yield higher returns on collected tax payments up to some point in which increasing the audit rates will stop having a significant deterrence effect on tax evasion. Given a hypothesis that a Tax Agency would manage to gain awareness of the heterogeneous characteristics of the population, it would be theoretically possible for them to compute an optimal audit rate taking into account their expected added collections given a cost for augmenting such enforcement efforts.

Fines (or penalties), represented by the parameter $\theta$ in this model, retain a somewhat secondary role at inhibiting tax evasion. Analogous to findings in the tax policy literature, see for example Alm, Jackson and McKee (1992), fine rates have statistically significant effects to deter evasion even though their estimated coefficients are rather low. Fines help deter evasion only up to some degree given that in the model specification they only appear interactively with the true audit rate $p$ forming the enforcement criterion $\theta p$. Ergo, for relatively high values of $p$, parameter $\theta$ loses its strength as the final income after audits cannot be negative. Side by side with these results, Kirchler et al. (2008) point out that policy makers should concentrate less in penalties and enforcement, and instead, focus on policies aiming to heighten voluntary compliance.

Tax morale, understood as the umbrella term that encloses any intrinsic, moral or social reason which compels individuals to abide the fiscal rules to a greater extent than a rational agent would have done, is a central component for this analysis. It may be wise to commence the discernment of tax morale by acknowledging how fundamentally difficult it is to measure a society’s morale, moreover, the pragmatic predicament it would impose to specify a public policy directed at boosting an abstract concept which cannot be properly identified nor measure to begin with. Miscellaneous interpretations could typify a government’s control over a society’s (tax) morale through a larger participation and political inclusion of citizens, by means of a larger ‘return’ (as in a public investment game) on levied taxes, or even by generating a feeling over how well the budget is being spent. Relying on citizen’s perception of their trust on institutional authorities may be, as well, a proxy conjecture about societal morale. Adopting the assumption, however, that tax-morale is not only measurable but mutable, has a positive and statistically significant effect on tax compliance, that is, individuals endowed with a higher tax-morale would be more inclined to disclose their true incomes and diminish in this way their fiscal evasion. Notwithstanding, there is a non-linear effect in which the tax-morale loses its power to convey people to comply as it becomes too large, just as the true audit rate. Consequently, societies whose citizen’s tax-morale is low should be more concerned in establishing an agenda which would encourage taxpayers’ involvement with society and policies targeting the promotion of an efficient resource allocations in their social programs.

Table 1 incorporates the effects of including two additional dummy variables, for both endogenous audits and willingness-to-pay taxes. Inferring the effect of $q$ or endogenous audits is slightly more intricate than it seems. From Equation 5 it is trivial that the average endogenous audit probability $\bar{q}$ is equal to the true audit rate $p$, and so the average perceived audit rate should be the same regardless of whether it is endogenous or not. Moreover, given that declared income is expressed as a fraction, the variable does not record any income level effects, i.e. if high-income agents would pay more and their low-income counterparts would pay less the coefficient should remain immutable even if the revenues have risen. Be that as it may, it so happens that the distribution of individually perceived audit rates has drastically changed, having a considerably fatter right-tail under the endogenous audit rate scenarios as seen in Figure 19. Following, it may seem that targeting key individuals (high-income agents, for example) may indeed exacerbate the deterrence effect of audits on tax evasion throughout the linkages in a societal network. A game-theoretic exploration of this phenomenon is detailed in Section 5 of this paper.
The conception of an endogenous risk-aversion implies on its own that the \( \text{williness-to-pay} \) taxes \( \gamma \) is now, in fact, endogenous as well; at least to some extent given it is also anchored by the society’s tax-morale. Figure 20 graphs the power-law distribution followed by the individual \text{williness-to-pay} taxes of the agents: where the majority are very reluctant to pay taxes and a decreasing share of people are more willing to pay their dues.

The concluding parameter in Column (1), the closeness centrality of the network as previously defined, yielded a statistically significant positive effect in the aggregate tax compliance and a negative coefficient on its squared transformation. There exists, therefore, a non-linear channel through which closeness in a network may stimulate tax compliance at the aggregate level. Perchance these enhancing peer-effects gain impetus from the spread of information and the availability of knowledge regarding the audit frequency.

### Table 2: Closeness centrality measures

<table>
<thead>
<tr>
<th>Network</th>
<th>Closeness</th>
<th>Network</th>
<th>Closeness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Erdos-Renyi</td>
<td>0.3734</td>
<td>Small_Worlds</td>
<td>0.2362</td>
</tr>
<tr>
<td>Random_Geom</td>
<td>0.1714</td>
<td>Watts_Strogatz</td>
<td>0.2418</td>
</tr>
<tr>
<td>Pref_Attach</td>
<td>0.3806</td>
<td>Ring</td>
<td>0.0113</td>
</tr>
<tr>
<td>Fitness_Model</td>
<td>0.3631</td>
<td>Lattice</td>
<td>0.0808</td>
</tr>
<tr>
<td>Wheel</td>
<td>0.5035</td>
<td>\text{Average}</td>
<td>0.2625</td>
</tr>
</tbody>
</table>

Column (2) provides a deeper look into the dynamics of closeness centrality by recurring to dummy variables in the Tobit model. Reminiscent of the Erdos-Renyi graph as the baseline case, the random geometric and the preferential attachment networks do not have a statistically significant different repercussion regarding tax behavior. Lattice, ring, small worlds and wheel structures impose a curtailed effect on tax evasion, statistically significant lower than the benchmark; these effects may be seen in the statistically significant negative coefficients that these dummies attained in Table 1. Watts-Strogatz worlds seem to retain a statistically significant reduced effect with respect to the benchmark, nonetheless the estimated coefficient is relatively modest in comparison to the alternative networks. Ultimately, the fitness function model conveys a small increase in the aggregate tax levels, nevertheless limited to a minor statistical significance. As a final word, the closeness of wheel networks, as seen in Table 2 is markedly high, however it is not efficient in the sense of enhancing tax compliance; consequently, a large centrality is no guarantee for discouraging tax evasion.

A conclusive robustness check takes into account the results provided by the previous literature surrounding ABM’s of tax compliance. Korobow, Johnson and Axtell (2007), approached the tax evasion problem through the usage of social networks in an ABM setup where gents live in a Moore-neighborhood
network with limited knowledge about the true enforcement parameters and can merely perceive the
audit probability. Following, agents can take three actions: fully evade, fully comply, or under-report
and then locally communicate their payoffs within their neighborhood. The authors found that whenever
individuals internalized the information into their own decisions it enhanced a larger tax compliance,
attributing the increase of income disclosure to social interactions. They called, however, for further
research into the development of a more realistically precise model. Figure 21 and Figure 22 illustrate
the emergence of a bottom-up compliance behavior, both in the fraction of aggregate declared income and
for the average perceived audit probability. In the former, even if the agents act independently and do
not know the exact declared income of others, they converge to an aggregate level of tax compliance; the
oscillating convergence at three different steady states is presented with respect to their respective levels
of societal tax morale, having \( \kappa = \{10\%, 25\%, 40\%\} \). In the latter, in spite of the seemingly accurate
mean perceived probability, a t-test proved that agents overestimated the true audit rate for \( p = 2\% \) and
\( p = 6\% \) yet they underestimated it at the \( p = 10\% \) level. Accordingly, agents consistently fail to discover
the true audit rate both individually and collectively.

Alm, McClelland and Schulze (1992) studied individual tax evasion by means of economic experimen-
tation and discovered that over two thirds of individuals either fully-evade or fully-comply, generating
a dichotomous distribution of the share of income disclosed: a behavior which is not supported by the
expected utility theory. Figure 23 depicts the last idea that the power utility model has to offer by
reproducing the dichotomous behavior of individual taxpayers found in economic experiments for dif-
ferent levels of tax morale. Intriguingly, a power utility model with social interactions may be able to
replicate not only the aggregate, but the individual level findings presented by preceding experimental
and agent-based models.

Figure 21: Aggregate tax compliance

Figure 22: Mean subjective probability

Figure 23: Histogram of individually declared income varying tax-morale
5 Extension: Optimal audit strategies

Ensuing the derivations computed in the previous section, it is germane to recall how the subjective audit rate, defined in Equation 4, resembles a local-average game\(^1\). The perceived audit rate is updated in function of the agent’s prior, memory and the information received from its neighbors. The last term in Equation 4 is analogous to the empirical mean audit rate experienced by \(i\)'s neighborhood \((N_i)\) over all the periods of time for which the agents keep memory. Redefining this last term as:

\[
\bar{A}_{i,t} = \sum_{j \neq i} \frac{1}{N_{i,t}} \sum_{s=1}^{N_i} A_{j,t-s},
\]

The updating mechanism may be reinterpreted as:

\[
\hat{p}_{i,t+1} = f(\text{prior}_t) + g(\text{memory}_t) + \psi \bar{A}_{i,t},
\]

where \(\bar{A}_{i,t}\) is the sample estimate for the ‘social norm\(^2\)’ of neighborhood \(N_i\) at time \(t\), multiplied by an influence parameter \(\psi\). Accordingly, the dynamics of subjective audit rates may be understood and studied as local-average repeated games.

Ushchev and Zenou (2018) derived important properties of local-average games. In particular, it is important to acknowledge that network-based policies have no effect whenever all agents have are homogeneous, i.e., whenever there is no agent heterogeneity with respect to the true audit rate. Agents thus converge to a Nash Equilibrium which is roughly the true audit rate, as seen in Figure 22. A social planner who is aware of this, might then attempt to add heterogeneity to the network. Assume a social planner, or Tax Agency, is capable of executing a credible message such that all individuals believe the news are true and binding. Aforesaid official communication would read something akin to ‘From now on, the probability of being audited will be directly proportional to the income level of each individual.’

Subsequently, individuals would update their subjective probability of being audited in an endogenous fashion, as established in Equation 5. Ensuing:

\[
\hat{q}_{i,t+1} = \hat{p}_{i,t+1} \cdot \frac{I_i}{\sum_{j=1}^{N_i} I_j} \cdot \bar{A}_{i,t},
\]

It is easy to realize how the modification specified in Equation 5 if essentially the same as the one endogenous probability employed in Equation 8 in the Agent-Based Model of Section 4.

Who should the social planner audit?

From a game-theoretic point of view, a social planner who wishes to maximize the aggregate value of the actions exerted by the society, should subsidize agents who maximize the following equation [see Ushchev and Zenou (2018)]:

\[
S_i = \psi \sum_{j \neq i} g_{ij} (x_j - \bar{x}_j),
\]

where \(\psi\) is the social influence effect of the neighbor’s signals into the subjective audit rate updating mechanism of Equation 8. The coefficient \(g_{i,j} = \{0, 1\}\) comes from the adjacency matrix representation of the network; \(g_{i,j} = 1\) if agent \(i\) ‘listens’ to agent \(j\) and zero otherwise. The action chosen by agent \(j\) is denoted as \(x_j\), while \(\bar{x}_j\) defines the average value of the actions chosen by \(j\)'s neighbors.

---

\(^1\)A local-average game is one in which an agent’s action is affected by the average value of its neighbors’ actions; if we understand the subjective probability of an individual as an action, then we are dealing with a local-average game (also called Spatial Auto-Regression in some cases). For a very detailed explanation, see Ushchev and Zenou (2018)

\(^2\)The social norm in a local-average game is understood as the ‘generally accepted value’ of a parameter inside the neighborhood; the value of the subjective probability of being audited for this particular case.
Substituting the social influence $\psi$ by its $(1 - \lambda_1 - \lambda_2)$ value from Equation 4 and considering the heterogeneity correction from Equation 9; and substituting the action $x_j$ for the endogenous, subjective audit rate of the agent, $x_j := \hat{q}_j$, then Equation 10 becomes:

$$S_i = (1 - \lambda_1 - \lambda_2) \frac{I_i}{\sum_{j=1}^{n} I_j} \sum_{j \neq i} g_{ij} (\hat{q}_j - \overline{q}_j).$$

(11)

By removing the values that do not depend on $i$, maximizing Equation 11 is analogous to maximize:

$$S_i = I_i \sum_{j \neq i} g_{ij} (\hat{q}_j - \overline{q}_j) \approx I_o i \sum_{j \neq i} g_{ij} (\hat{q}_j - \overline{q}_j)$$

(12)

There are two fundamental considerations for Equation 12. The first one, is that the actual income of the agent, $I_i$, is non-observable for the social planner; if it were, then there would be no need to model tax evasion since the social planner would have perfect ability to match all income reports. Therefore, a proxy variable $I_o$ is applied, where this income proxy may be understood as an individuals consumption, social status, mortgage, car value, and any other observable variables that may hint the income level of an agent. Second, the term on the right hand side, $\hat{q}_j - \overline{q}_j$, does not depend on $i$, but it depends on $j$. Thus, in a local–average model the key group of players are those whose neighbors’ heterogeneity is above their ‘social norm’. Consequently, the group of agents which should be subsidized, or in this case audited, are those who attain maximal values of the product between observable income level times the deviation of its neighbor’s heterogeneity from their social norm. A remark must be made from Equation 12: one should not audit the most well-off agents $j$, but audit their neighbors $i$ instead.

Figure 24 employs the fitness-function network from Figure 6 to offer a simple interpretation of this last remark. Agents living in the green sector have an heterogeneity factor above their social norm: $\hat{q}_j > \overline{q}_j \iff q_j > \overline{q}_j$. This is due the fact that their true endogenous audit rates are larger than the average true endogenous audit rate of their neighbors: by living in the upper border, they have more neighbors with lower income (with respect to them) than neighbors with higher income than them. Meanwhile, agents living in the blue sector have an heterogeneity factor below their social norm: $\hat{q}_j < \overline{q}_j \iff q_j < \overline{q}_j$. Analogously, they have more neighbors with higher incomes than neighbors with lower incomes than them. Conclusively, the game-theoretic solution for the social planner is to audit the agents which live in the upper border of the network architecture, giving a preferential weight to those who possess a higher observable income.

Figure 24: Targeted individuals (red) live in the upper border and then to be ‘richer’.

Making use of the Agent-Based Model implementation discussed in Section 4 of this paper, we ran a collection of audit strategies on the tax compliance model to see whether or not the simulations attained
different results. Table 3 shows the outcomes obtained by seven contrasting audit schemes: random audits, cut-off (audit whoever declared the lowest amount), four centrality measures\(^3\), and the optimal game-theoretic solution derived from the subsidy policy of Ushchev and Zenou (2018). Column Average \(\hat{q}\) denotes the mean-perceived subjective audit rate by the society as a whole; Column \(\Delta \hat{q}\) represents the change in the aggregate optimal fraction of income declared by agents; Column \(\Delta -Revenue\) shows the change in the tax revenue of the social planner; Column \(p\text{-value}\) defines whether or not the distribution of subjective audit rates is similar to the one obtained under a random auditing selection process.

Table 3: Comparing* strategies under heterogeneous true–audit rates: \(\hat{q}\)

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Average (\hat{q})</th>
<th>(\Delta \hat{q})</th>
<th>(\Delta -Revenue)</th>
<th>(p\text{-value})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random</td>
<td>6.89%</td>
<td>0%</td>
<td>0%</td>
<td>0.000</td>
</tr>
<tr>
<td>Cut-off</td>
<td>4.24%</td>
<td>-5%</td>
<td>-9%</td>
<td>0.000</td>
</tr>
<tr>
<td>Degree</td>
<td>6.40%</td>
<td>3%</td>
<td>0%</td>
<td>0.169</td>
</tr>
<tr>
<td>Betweenness</td>
<td>6.01%</td>
<td>6%</td>
<td>2%</td>
<td>0.000</td>
</tr>
<tr>
<td>Closeness</td>
<td>5.25%</td>
<td>5%</td>
<td>-1%</td>
<td>0.000</td>
</tr>
<tr>
<td>Intercentrality</td>
<td>5.71%</td>
<td>-1%</td>
<td>-7%</td>
<td>0.000</td>
</tr>
<tr>
<td>Ushchev &amp; Zenou</td>
<td>12.88%</td>
<td>13%</td>
<td>16%</td>
<td>0.000</td>
</tr>
</tbody>
</table>

*Comparisons made versus the Random audits scenario

Table 3 shows how, under heterogeneous and endogenous audit rates, the social planner may devise an optimal audit policy to enhance aggregate tax compliance. It is important to notice how the audit rates do not directly affect the revenues nor the aggregate fraction of income declared by the society. The optimal audits affect the average perceived subjective rate; this has a direct effect on the aggregate optimal declarations as explained in Table 1 of Section 4; afterwards, the revenue is naturally intensified once the society experiences a surge in the fraction of income disclosed. So it is true that a social planner can devise network-based policies to boost tax collections.

6 Conclusions

A micro-founded expected utility theory tax evasion model was presented under a power utility function specification and implemented through an Agent-Based model with heterogeneous agents and local social interactions; simulated over different underlying network structures. Agents have limited knowledge about their surroundings yet may acquire endogenous parameters of loss-aversion and audit rates, depending not only on their income levels but also in the corresponding ones from their neighbors. An exploratory setup shed light in the possibility of choosing specific network structures which may yield a more realistic result, particularly a fitness-function model that accounts for age and social status (income) was questioned and deemed to be appropriate for modeling taxpayers’ behavior. In top of that, the assumption on the information exchanged is relaxed from communicating evasion rates and payoffs to simply sharing their past memories about the occurrence or not of former audits.

There is a large area of action for public policy makers in the further study of how audit rates and fines non-linearly increase tax compliance, yet both tools tend to lose effect whenever over-enforced. Whilst tax rates have a non-linear negative impact on income disclosure, tax morale offers an opportunity for governments with an unreceptive image among their citizens to call for a larger voluntary tax contribution by attending for a better public image of government spending or by strengthening their political inclusion. Moreover, a potentially compelling property of the model specification is its capability of reproducing both individual behavioral patterns and aggregate convergence levels of tax compliance as the encountered in economic experiments and antecedent agent-based models. In particular, our ABM adaptation was able to replicate the presence of fully compliant agents even in the scenario where audits are not enforceable, as previously found in an existent field experiment. Furthermore, the dichotomous behavior of

\(^3\)Degree: agent with the highest number of neighbors. Betweenness: agent with the highest control of information flow. Closeness: agent with the highest efficiency of information flow. Intercentrality: agent with the largest weighted count of paths initiating from itself, as defined by Ballester et al. (2006).
the distribution of income disclosures prevailing in laboratory experiments was found as well in our paper.

Ultimately, a possible game-theoretic extension of this paper was introduced and it shed light on the possibility for a social planner to devise and design specific audit schemes to enhance tax compliance by boosting the level of perceived audit rates among the society. Building up the game on top of a simple fitness-function model, a social planner should audit individuals who are well-off and whose neighbors are even more affluent than itself. Following this strategy, the society-level subjective audit probability is maximized, directly optimizing the fractions of income disclosed and indirectly maximizing the tax revenues. Notwithstanding, the authors of this paper exhort for more research regarding optimal audit policies for tax evasion deterrence.

References


Appendix

**Proposition 1:** Whenever the perceived probability is zero

Assume that the (perceived) audit probability tends to zero such that the expected utility form can be expressed as:

\[ p_{i,t} = 0 \] such that

\[ \text{EU}(d_{i,t}) = \arg \max_{\{d_{i,t}\}} (1 + d_{i,t})^{\kappa_{i,t}}[I_{i,t}(1 - \tau d_{i,t})]^{1 - \rho_{i,t}} \]

\[ \frac{\partial \text{EU}(d_{i,t})}{\partial d_{i,t}} = (1 + d_{i,t})^{\kappa_{i,t}}(1 - \rho_{i,t})[I_{i,t}(1 - \tau d_{i,t})]^{-\rho_{i,t}}(\tau I_{i,t}) + \kappa_{i,t}(1 + d_{i,t})^{\kappa_{i,t} - 1}[I_{i,t}(1 - \tau d_{i,t})]^{1 - \rho_{i,t}} \]

Equalizing the partial derivative to zero and dividing both sides by \((1 + d_{i,t})^{\kappa_{i,t} - 1}[I_{i,t}(1 - \tau d_{i,t})]^{1 - \rho_{i,t}}\) we get:

\[ (1 + d_{i,t})(1 - \rho_{i,t})(-\tau I_{i,t}) + \kappa_{i,t}[I_{i,t}(1 - \tau d_{i,t})] = 0 \]

Solving for \(d_{i,t}\) we can derive the optimal declared income whenever the perceived probability of being audited is assumed to be null as a function of \(\tau, \rho_{i,t}\), and \(\kappa_{i,t}\) as follows:

\[ \left[ d^*_{i,t} \right]_{p_{i,t}=0} = 1 - \frac{(1-\rho_{i,t})\tau}{\frac{\kappa_{i,t}(1-\rho_{i,t})}{\tau + \frac{\kappa_{i,t}(1-\rho_{i,t})}{\rho_{i,t}}}} \]  \hspace{1cm} (13)

where \(d^*_{i,t} \in [0,1]\).

Plenty of interesting conclusions can be derived from Equation (13). Noticing that the denominator is strictly positive, then the boundary conditions of full-evasion and full-compliance may be computed from the numerator term.

An agent will be a full-evader whenever the numerator is smaller or equal than zero.

\[\frac{1 - \rho_{i,t}}{\kappa_{i,t}} \geq \frac{1}{\tau} \Rightarrow \frac{\kappa_{i,t}}{1 - \rho_{i,t}} \leq \tau \]

Following, a new parameter \(\gamma_{i,t}\) is defined as the willingness to pay of agent \(i\) and may be understood as its tax-morale corrected by its own risk-aversion: \(\gamma_{i,t} = \frac{\kappa_{i,t}}{1 - \rho_{i,t}}\).

The willingness to pay is increasing with respect to both tax morale and risk aversion. In continuation, the last equation can be restated as: \(\gamma_{i,t} \leq \tau\).

Which implies that whenever the agents’ willingness to pay is smaller or equal to the tax rate, and the perceived audit probability is zero, full-evasion will take place.

\[\therefore \text{if } \gamma_{i,t} \leq \tau \Rightarrow \left[ d^*_{i,t} \right]_{p_{i,t}=0} = 0. \]  \hspace{1cm} (14)

Notwithstanding, it is also interesting to explore the cases, if any, when agents are fully-compliant even in scenarios where the perceived audit probability is zero, i.e., whenever:

\[\frac{1 - \frac{1 - \rho_{i,t}}{\kappa_{i,t}} \tau}{\tau + \frac{1 - \rho_{i,t}}{\kappa_{i,t}} \tau} \geq 1 \Rightarrow \frac{1 - \tau}{\gamma_{i,t}} \geq 1 \]  \hspace{1cm} (15)

Applying basic algebra it is straightforward to derive the condition: \(\gamma_{i,t} \geq \frac{2\tau}{1-\tau}\).

Consequently, if the willingness to pay is high enough with respect to the tax rate, then agents may be full-compliers even in the absence of audits. The term on the right is increasing with respect to \(\tau\);
meaning that if taxes increase, less individuals would be fully-compliant whenever the perceived audit probability is zero.

From Equation (13) and Results (14) and (15) it is straightforward to derive in the following expression:

\[
[d^*_i,t|p_{i,t}=0] = \begin{cases} 
0 & \text{if } \gamma_{i,t} \leq \tau \\
(0, 1) & \text{if } \tau < \gamma_{i,t} < \frac{2\tau}{1-\gamma}
\end{cases}
\]

Where \(\gamma_{i,t} = \frac{\kappa_{i,t}}{1-\rho_{i,t}}\) is the willingness to pay taxes; understood as the tax morale corrected for risk-aversion.

Perhaps it would be of some interest to study the distribution of the willingness to pay taxes. The notion that a majority of people is not very keen to pay taxes, whereas a small population is highly law-abiding is quite intuitive. For a fixed society-level value of tax morale (\(\kappa\)) and a uniformly distributed risk-aversion level between zero and one (\(\rho\)) a generic distribution of the willingness to pay taxes (\(\gamma\)) can be portrayed in the following figure. Figure 25 displays how the willingness to pay taxes (WTP), for a society-level tax morale of 0.20, follows a power law distribution, where the frequency of people decreases as a function of the WTP taxes.

Figure 25: Willingness-to-pay-taxes distribution

**Proof of Proposition 2:** Whenever the perceived probability is one

In a similar fashion to the previous *Proposition*, the assumption that agents may have a perceived audit probability of one is studied and properties are derived in order to assess which agents, if any, would evade even when facing a certain audit.

\[ p_{i,t} = 1 \text{ such that } EU(d_{i,t}) = U(X_{i,t}) \]

\[ \Rightarrow \arg \max_{\{d_{i,t}\}} EU(d_{i,t}) = \arg \max_{\{d_{i,t}\}} (1 + d_{i,t})^{\kappa_{i,t}}[I_{i,t}(1 - \tau d_{i,t} - \theta \tau(1 - d_{i,t}))]^{1-\rho_{i,t}} \]

\[ \frac{\partial EU(d_{i,t})}{\partial d_{i,t}} = (1 + d_{i,t})^{\kappa_{i,t} - 1}[I_{i,t}(1 - \tau d_{i,t} - \theta \tau(1 - d_{i,t}))]^{-\rho_{i,t}}[I_{i,t}(\theta + \tau)] + \kappa_{i,t}(1 + d_{i,t})^{\kappa_{i,t} - 1}[I_{i,t}(1 - \tau d_{i,t} - \theta \tau(1 - d_{i,t}))]^{1-\rho_{i,t}} = 0 \]

After equalizing the partial derivative to zero and dividing both sides by \((1 + d_{i,t})^{\kappa_{i,t} - 1}[I_{i,t}(1 - \tau d_{i,t} - \theta \tau(1 - d_{i,t}))]^{-\rho_{i,t}}\) we can simplify to:

\[ (1 + d_{i,t})(1 - \rho_{i,t})(\theta + \tau) + \kappa_{i,t}(1 - \tau d_{i,t} - \theta \tau(1 - d_{i,t})) = 0 \]

\[ \Rightarrow (1 + d_{i,t})(1 - \rho_{i,t})(\theta - 1) = -\kappa_{i,t}(1 - \tau d_{i,t} - \theta \tau(1 - d_{i,t})) \]

\[ \Rightarrow (1 + d_{i,t})\frac{(1 - \rho_{i,t})}{\kappa_{i,t}}(\theta - 1) = d_{i,t}(1 - \theta) + \theta - \frac{1}{\tau} \]
Applying basic algebra the optimal value of declared income may be computed in an analogous way to the case where the perceived probability was null. Moreover, it is derived as:

\[
[d^*_{i,t}|p_{i,t}=1] = \frac{\theta \tau - 1}{\theta - 1} \cdot \frac{(1 - \rho_{i,t}) \tau}{\kappa_{i,t}} + \frac{(1 - \rho_{i,t}) \tau}{\kappa_{i,t}}
\]

which is very similar to Equation (13) where \( \hat{p}_{i,t} = 0 \); however the first numerator term became \( \phi \equiv \frac{\theta \tau - 1}{\theta - 1} \), which is a function of both \( \theta \) and \( \tau \) and is constant for all agents and all periods.

Consequently, agents will be full-compliers whenever the latter value is equal or larger to one. Substituting for \( \phi \) and \( \gamma_{i,t} \) in Equation (16) we derive:

If \( \frac{\phi \tau}{\gamma_{i,t}} \geq 1 \Rightarrow [d^*_{i,t}|p_{i,t}=1] = 1 \)

Solving the left hand inequality the necessary condition is expressed as:

If \( \gamma_{i,t} \geq \frac{2 \tau}{\phi - \tau} \Rightarrow [d^*_{i,t}|p_{i,t}=1] = 1 \)

The denominator is non-positive whenever \( \phi \leq \tau \)

\( \phi \leq \tau \iff \frac{\theta \tau - 1}{\theta - 1} \leq \tau \iff \theta \tau - 1 \leq \theta \tau - \tau \iff \tau \leq 1 \)

Noticing that \( \gamma_{i,t} \) and \( \tau \) are non-negative and \( \phi - \tau \) is non-positive, then the former condition is void. In other words,

\[ [d^*_{i,t}|p_{i,t}=1] = 1 \ \forall \{i,t\} \]

A crucial question arises whenever talking about the effects of parameters on evasion models: Do people like to pay taxes? In other words, is the utility function derivative with respect to the tax parameter (\( \tau \)) positive, negative or zero?

There are two possible states, whether the individual is audited and attains an income \( X_{i,t} \), or whenever it is not audited and realizes an income \( Y_{i,t} \). First, on any occasion in which the agent is not audited:

\[
\frac{\partial U(Y_{i,t})}{\partial \tau} = \frac{\partial}{\partial \tau} \left( 1 + d_{i,t})^{\kappa_{i,t}} [I_{i,t}(1 - \tau d_{i,t})]^{1 - \rho_{i,t}} \right)
\]

\[
\Rightarrow \frac{\partial U(Y_{i,t})}{\partial \tau} = (1 + d_{i,t})^{\kappa_{i,t}} [I_{i,t}]^{1 - \rho_{i,t}} (1 - \rho_{i,t})[1 - \tau d_{i,t}]^{-\rho_{i,t}} [-d_{i,t}] \quad (17)
\]

**Proof of Proposition 3: Do people like to pay taxes?**

All the terms in the right-hand side of Equation (17) are non-negative, with the exception of the last term \([-d_{i,t}]\) which is non-positive. Therefore, the derivative of the utility function with respect to the tax rate is non-positive. Moreover, it attains the value of zero only if \( I_{i,t} = 0 \), or \( \rho_{i,t} = 1 \), or \( d_{i,t} = 0 \) or (\( \tau = 100\% \) and \( d_{i,t} = 1 \)).

\[ \Rightarrow \frac{\partial U(Y_{i,t})}{\partial \tau} \leq 0 \]

Whereas for the state in which the individual faces an audit by the Tax Agency, an assumption shall be made: the biggest penalty that an individual may face, is the totality of its income, i.e., the agent may not end the period with a negative income.

\[
\frac{\partial U(X_{i,t})}{\partial \tau} = \frac{\partial}{\partial \tau} \left( 1 + d_{i,t})^{\kappa_{i,t}} [I_{i,t}(1 - \tau d_{i,t} - \theta \tau(1 - d_{i,t}))]^{1 - \rho_{i,t}} \right)
\]
In Equation (18) the first three terms on the right-hand side are clearly non-negative and the fifth one is strictly negative as \( \theta \). Therefore, the utility function derivative with respect to the tax rate is non-positive. Moreover, it equals zero only if \( I_{i,t} = 0 \), or \( \rho_{i,t} = 1 \), or \( X_{i,t} = 0 \) at the end of the audit.

More precisely, a necessary condition for the income in case of an audit to become negative, is that \( \theta \) must be strictly larger than one over \( \tau \). However, this is hardly the case given that tax rates usually do not go over 50% and penalty rates frequently oscillate between 1.2 and 1.75 [see Hindriks and Myles (2006)]; making the necessary condition to be met and providing a base to argue that our premise of non-negative incomes is not a strong assumption but rather a reasonable one. In several expected utility models there is a somewhat counterintuitive property of high tax rates promoting tax evasion, however this is a non-linear effect that arises given the fact that the penalty is set by \( \theta \tau \) and thus increasing the tax rate is in some sense an analog of increasing the fine rate.

Lastly, it is important to see that even when the utility functions for both states \( X_{i,t} \) and \( Y_{i,t} \) are non-negative, there is a reasonable explanation to argue that the partial derivative of the expected utility with respect to the tax rate is strictly negative. From equations (17) and (18) we get:

\[
\Rightarrow \frac{\partial U(X_{i,t})}{\partial \tau} = (1 + d_{i,t})^{\kappa_{i,t}}[I_{i,t}]^{1-\rho_{i,t}}(1-\rho_{i,t})(1-\tau d_{i,t} - \theta \tau (1-d_{i,t}))^{-\rho_{i,t}}[-d_{i,t} - \theta + \theta d_{i,t}] \tag{18}
\]

\[
\Rightarrow \frac{\partial U(X_{i,t})}{\partial \tau} \leq 0
\]

It follows from the Equation (19) that the right-hand side of the equation is non-negative, and it equals zero only if either \( I_{i,t} = 0 \) or \( \rho_{i,t} = 1 \). Nevertheless, on one side \( \rho \in (0,1) \) by definition, and, in the other side, it would make no sense that the truly earned income is actually zero, given that the agent would not even be considered inside the set of tax-paying individuals. Accordingly, the derivative of the expected utility function with respect to the tax rate is strictly negative, or in vernacular words, individuals do not inherently like to pay taxes.

**Proof of Proposition 4: Absolute Risk-Aversion and Relative Risk-Aversion**

Recalling the power utility function specified in Equation 3, it is easy to derive the first and second derivatives with respect to \( W \).

\[
U(d) = (1 + d)^\kappa W^{(1-\rho)}
\]

\[
U_W(d) = (1 + d)^\kappa (1-\rho)\rho W^{(-\rho)}
\]

\[
U_W^\prime(d) = (1 + d)^\kappa (1-\rho)(-\rho)W^{(-\rho-1)}
\]

And so the coefficient of absolute risk-aversion \( A(W) \) is given by:

\[
A(W) = \frac{-u''}{u'} = \frac{-\frac{\kappa (1-\rho)(-\rho)W^{(-\rho-1)}}{(1 + d)^\kappa (1-\rho)W^{(-\rho)}}}{},
\]

which simplifies to:

\[
A(W) = \frac{-u''}{u'} = \frac{\rho}{W}.
\]

Following, the coefficient of relative risk-aversion \( R(W) \) is given by:

\[
R(W) = W \cdot A(W) = \rho.
\]