Abstract

I develop a theory of bounded rationality that I call lifecycle horizon learning to explore the welfare cost of Social Security policy uncertainty. Agents use adaptive expectations to forecast future aggregates, which introduces cyclical transition dynamics. This magnifies the welfare cost of policy uncertainty, compared to a rational expectations model. The ex-ante welfare cost of policy uncertainty (where policy is either a tax or benefit change in 2030 or 2040) is equivalent to 1.99 percent of period consumption for the cohort of agents most harmed in the life-cycle horizon learning model compared to 1.49 percent in a rational expectations framework.

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Key words: Learning; Bounded Rationality; Policy Uncertainty; Social Security; Fiscal Sustainability

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1 Introduction

“Uncertainty about Social Security’s future magnifies the anxiety that many Americans experience as they plan and prepare for retirement.”

- Commission on Retirement Security and Personal Savings
  Bipartisan Policy Center, June 2016

The aging of developed nations is straining unfunded pension systems like the U.S. Social Security system. The Old Age Survivor and Disability Insurance program (OASDI), commonly referred to as Social Security, is one of the largest transfer programs in the world, with total expenditures of nearly $952 billion in the most recent year. Social security provides benefits to 62 million people, the majority of whom receive retirement benefits. Social security’s cost has exceeded its tax income in each year since 2010 and the Social Security Administration (SSA) projects this relationship to extend through the long-term. The SSA Board of Trustees project that an immediate and permanent payroll tax rate increase of 2.78 percentage points (12.4 percent to 15.18 percent) or an immediate and permanent benefit cut of 17 percent applied to all current and future beneficiaries would be needed to insure sufficient tax revenue to cover promised benefits over the long-term (SSA, 2018).

Given the projected shortfall between Social Security tax revenues and promised benefits, reform seems likely. However, it is not clear if or when the government will act. The uncertainty regarding when or how Social Security will be reformed is interesting and economically important. A survey conducted by the American Life Panel in 2011 found that 56 percent of respondents are “not too confident” or “not confident at all” that the Social Security system will be able to provide them the level of benefits currently promised. Social security benefits are a major source of retirement income for many Americans, and uncertainty regarding future benefits could impact savings decisions. Luttmer and Samwick (2018) find that not knowing “exactly how much you will get in Social Security benefits” matters “very much” for 47.5 percent of respondents in their survey. The economy-wide uncertainty regarding Social Security deficits and debt accumulation is also interesting, as rapid debt growth could strain the economy (see, for example, Davig et al. (2010)).

I explore the macroeconomic consequences of Social Security policy uncertainty in a general equilibrium lifecycle model. Agents’ expectations about future Social Security policy

\[1\]The RAND American Life Panel is a nationally representative, probability-based panel of over 6000 members ages 18 and older who are regularly interviewed over the Internet for research purposes. This question comes from the 2011 survey Well-being ms179. More information about the ALP is available at alpdata.rand.org.
drive their decision making. These decisions, combined with realized government policy, determine macroeconomic output and prices. The way in which agents form expectations has a large impact on the short-run responses to changes in policy (or possible changes in policy). I consider rational expectations and adaptive expectations. My paper contributes to a growing literature that examines the response to anticipated fiscal policy in models of adaptive learning (see Evans et al. (2009), Mitra et al. (2013), Gasteiger and Zhang (2014), and Caprioli (2015)).

The rational expectations hypothesis is standard in macroeconomics and I develop a rational expectations model as the baseline for my analysis. I also relax the assumption of rational expectations and explore the consequences of agents using adaptive expectations in a model with an aging society and Social Security reform. The use of adaptive expectations is one type of adaptive learning model. A criticism of the rational expectations hypothesis is that it requires agents to possess more knowledge about the structure of the economy than any econometrician possesses about the actual economy. Models of adaptive learning relax that requirement, and allow for boundedly rational agents (Sargent (1993), Evans and Honkapohja (2001)).

Adaptive learning has several benefits. Models of adaptive learning produce more realistic dynamics that better fit observed data. Eusepi and Preston (2011) demonstrate that infinite-horizon learning amplifies technology shocks in a real business cycle model and creates more realistic dynamics. Branch and McGough (2011) show that a calibrated model with heterogeneous beliefs (some agents are fully rational, others use N-step ahead adaptive learning) provides a closer fit to business cycle data than the rational expectations baseline. Models of adaptive learning are also better able to match survey data on expectations (Branch (2004)), asset price volatility (Bullard and Duffy (2001) and Adam et al. (2016)), and experimental data on expectation formation (Adam (2007) and Pfajfar and Santoro (2010)), than rational expectations models. Additionally, adaptive learning can be viewed as a robustness check to examine the stability properties of a rational expectations equilibrium (Evans and Honkapohja (2001) and (2009)).

In this paper, I introduce a new framework for modeling bounded rationality that I call lifecycle horizon learning. In the lifecycle horizon learning model, households use an adaptive rule to forecast future wages, interest rates, and government debt levels. The households are fully-forward looking, and solve a standard optimization problem to choose their consumption and savings. The learning model differs from a traditional rational expectations model
only in that agents form expectations about endogenous variables adaptively. Households still respond to announced policy, since they are forward looking, and similarly still respond to policy uncertainty.

Lifecycle horizon learning is similar to “infinite horizon learning” or “optimal learning” in models with infinitely lived agents. For example, in Eusepi and Preston (2011), infinitely lived agents forecast wages, interest rates, and the capital stock for all periods into the infinite future. The agents form these forecasts using recursive least squares. The agents embed the forecasts into their lifetime budget constraint, and make optimal decisions, conditional on their forecasts. (See also Preston (2005) and (2006) for models with infinite horizon learning.) Lifecycle horizon learning is similar to infinite horizon learning in that agents forecast endogenous variables looking forward over their entire lifecycle, and make optimal decisions based on their forecasts. The main difference is that in the lifecycle horizon learning model, agents live for a finite number of periods, and thus they only forecast a finite number of periods into the future.

Lifecycle horizon learning is also similar to “finite horizon learning” developed by Branch, Evans, and McGough 2013. In both cases, households make decisions based on forecasts of endogenous macroeconomic variables, and decisions are conditioned on expected future savings. The main difference between the two modeling frameworks is that finite horizon learning takes place in a model with an infinitely-lived, representative agent, while lifecycle horizon learning takes place in a lifecycle model with finitely-lived agents. In a companion paper, I develop a model of finite-horizon lifecycle learning, in which finitely lived households make decisions based on finite planning horizon that is shorter than the length of their lifecycle (Cottle Hunt (2019a)).

The learning model I develop in this paper is related to the overlapping generations model developed in Bullard and Duffy (2001). Their model includes capital and an unbacked asset issued by the government that grows at an exogenous rate (money). Households in Bullard and Duffy’s model forecast inflation using least squares learning (that is, the agent uses recursive least squares to estimate inflation given the data they observe from the previous periods), looking forward over the entire life cycle. The households then embed their inflation forecasts into their lifetime budget constraint, and make optimal decisions, conditional on their forecasts. The learning model presented in this paper differs slightly from Bullard and Duffy (2001) in the way that households forecast endogenous variables. In Bullard

\[\text{The learning model developed in this paper is similar to “N-step Optimal Learning,” the second finite horizon learning model presented in Branch et al. (2013).}\]
and Duffy, households use recessive least squares to update their estimate of the regression coefficient (to forecast inflation). In this paper, households use adaptive expectations to forecast endogenous variables. Adaptive expectations are a special case of least-squares learning with a constant gain in which the only regressor is an intercept.\(^3\)

I use the lifecycle learning model developed in this paper, as well as a rational expectations model, to simulate the underlying demographic changes and policy reforms that the SSA suggests would be sufficient to eliminate the gap between promised benefits and promised tax revenue. I explore the consequences of policy uncertainty by modeling reform that can take place in one of two different dates and can take the form of a tax increase or a benefit cut.

I measure the welfare cost of policy uncertainty using both an ex-ante and an ex-post consumption equivalent variation technique. Three recent papers make similar welfare comparisons. The welfare metric that I present in this paper is most similar to Nelson (2017). Nelson and I both compare a given reform (for example, Social Security taxes increased in the year 2030) to the realization of that same reform as the result of an uncertain process. I find that the welfare effects, both positive and negative, are larger in the learning model compared to the rational expectations baseline. The welfare effects in my rational excitations model are similar in magnitude to the effects found by Nelson.

Recent work by Kitao (2018) also computes the welfare effects of Social Security policy uncertainty in a quantitative lifecycle model. Kitao takes a slightly different approach, by comparing any reform that results from the uncertain process to the same baseline reform. For example, a benefit reduction that occurs in 2040 as the result of an uncertain process is compared to a benefit reduction that happens in 2020 with certainty. Kitao’s analysis is calibrated to the Japanese economy, and the welfare effects that she finds are similar in magnitude to Nelson (2017) and my rational expectations analysis.

Caliendo, Gorry, and Slavov (2015) use a different measurement of uncertainty that isolates the effect of uncertainty from the desirability of the various policy reforms. They find smaller welfare effects. This is because the consumption equivalent variation techniques used in the other papers (mine included) capture both the cost to households of not knowing their future Social Security benefits (and/or taxes), and the cost of (correctly) believing there is a chance that their benefits will be cut and their consumption will be lower. Caliendo et al. isolate the cost of the uncertainty alone, controlling for the desirability of a given reform.

\(^3\)See Evans and Honkapohja (2001) section 3.3 for a discussion on the relationship between adaptive expectations and constant gain learning.
Finally, a recent empirical paper also examines the welfare cost of Social Security policy uncertainty. Luttmer and Samwick (2018) use survey data to measure the welfare cost of policy uncertainty. The median risk premium from their survey data suggest households are willing to give up 7 percent of their Social Security benefits to avoid uncertainty regarding the level of benefits they will receive in retirement.

This paper compliments the existing literature by showing the welfare effects of policy uncertainty are larger in a lifecycle horizon learning model. For example, the minimum ex-ante consumption equivalent variation that makes an agent indifferent between an announced policy reform or policy uncertainty (between four different options, tax increases or benefit cuts possible in two different periods) in the rational expectations model is -1.49 percent of period consumption for the most harmed cohort, meaning a household in the most harmed cohort would be willing to give up 1.49 percent of period consumption to avoid policy uncertainty. In the lifecycle horizon learning model, the ex-ante consumption equivalent variation for the same comparisons is -1.99 percent for the most harmed cohort. From an ex-post consumption equivalent variation perspective, the contrast between rational expectations and learning is even more pronounced. The most harmed cohort from an ex-post perspective would only need to give up 0.25 percent of period consumption to be indifferent between announced policy and uncertain policy under rational expectations; under adaptive learning the most harmed cohort would give up 1.98 percent of period consumption to avoid uncertainty.

The rest of the paper is organized as follows. I present the model in section 2 and introduce Lifecycle Horizon Learning in section 3. The model is calibrated in section 4. Social security reform is introduced in section 5. Welfare analysis is in section 6. Section 7 includes alternative modeling assumptions, and section 8 concludes.

2 Model

2.1 Households

Households live for $J$ periods, choose asset allocation ($a^j$ for $j = 1, \cdots, J-1$) in the first $J-1$ periods of life, and consumption ($c^j$ for $j = 1, \cdots, J$) in all $J$ periods of life, to maximize utility, taking price, and government Social Security policy (tax rate $\tau$, and benefit $z$) as given. Agents receive wage $w_t$ for labor provided in period $t$, and retire exogenously at date $T \leq J$. The gross real return on savings in period $t$ is given by $R_{t+1}$. Superscripts on
variable indicate lifecycle stage (i.e., age), and subscripts indicate time period.\footnote{As an example, $a^2_{t+1}$ is age 2 savings (savings in the second stage of life), in time period $t+1$.}

Households maximize discounted expected lifetime utility (1) subject to period budget constraints (2). Here $E^*_t(x)$ indicates the time $t$ expectation of $x$. The star indicates that the expectations need not be rational.

$$\max_{a^j_{t+j-1}} E^*_t \sum_{j=1}^{J} \beta^{j-1} u(c^j_{t+j-1})$$

$$c^j_{t+j-1} + a^j_{t+j-1} \leq R_{t+j-1} a^{j-1}_{t+j-2} + y^j_{t+j-1} \quad \text{for } j = 1, \ldots, J$$

Here $y^j$ indicates period labor income during working life and Social Security income after retirement:

$$y^j_{t+j-1} = (1 - \tau_{t+j-1})w_{t+j-1} \quad \text{for } j < T$$

$$y^j_{t+j-1} = z^j_{t+j-1} \quad \text{for } j \geq T.$$

The lifespan is certain and agents do not have a bequest motive, so they exhaust all of their resources in the final period of life

$$a^J_{t+J-1} = 0.$$

The household first order conditions are:

$$u'(c^j_{t+j-1}) = \beta E^*_t [R_{t+j}u'(c^{j+1}_{t+j})] \quad \text{for } j = 1, \ldots, J - 1.$$

The agent (trivially) chooses to consume all of her resources in the final period of life, that is

$$c^j_{t+j-1} = R_{t+j-1} a^{j-1}_{t+j-2} + z^j_{t+j-1}.$$

Labor is supplied inelastically and preferences are given as the standard constant elasticity of substitution function:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \quad \text{if } \sigma \neq 1$$

$$u(c) = \ln(c) \quad \text{if } \sigma = 1.$$
2.2 Demographics

To start off the economy, I assume that in period zero, there are \( J \) cohorts who enter the economy with given asset holdings according to their age. I assume that the initial young enter the economy with zero assets, and all other cohorts enter with \( a^j \) for \( j = 2, \ldots, J \). Successive cohorts enter the model with zero assets when they are young.

\( N_t \) indicates generation \( t \) and is given by number of young (i.e., the generation born) at time \( t \). The population grows at rate \( n_t \) such that

\[
N_t = (1 + n_t)N_{t-1}.
\]

The population at any time \( t \) is the sum of all living cohorts. Labor is supplied inelastically, thus the labor force (denoted \( H_t \)) is simply the working age population:

\[
H_t = \sum_{j=1}^{T-1} N_{t+1-j}.
\]

2.3 Production

The consumption good in the economy (\( Y_t \)) is produced by a single firm (or equivalently many small firms) using a constant elasticity of substitution technology that takes aggregate capital (\( K_t \)) and labor (\( H_t \)) as inputs and produces the consumption good according to:

\[
Y_t = F(K_t, H_t) = AK_t^\alpha H_t^{1-\alpha}
\]

The parameter \( \alpha \) measures the intensity of use of capital in production, and \( A \) represents technology or total factor productivity.\(^5\) Factor markets are competitive and capital and labor (hours worked) are paid their marginal products. The gross real interest rate \( R_t \) is given by:

\[
R_t = F_K(K_t, H_t) + 1 - \delta
\]

where \( \delta \) is the rate of depreciation. The wage rate \( w_t \) is given by

\[
w_t = F_H(K_t, H_t).
\]

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\(^5\) I abstract from technological growth, although it is straightforward to incorporate this into the model. If technological process was included in the model, aggregate variables would grow at the rate \((1 + n)(1 + g)\) along the balanced growth path (if \( g \) indicates the growth rate of technology).
2.4 Government

The government runs a modified pay-as-you-go Social Security system. The government pays retirement benefits to the retired generations by taxing the working generations and by (possibly) issuing debt.

The pay-roll tax rate is $\tau_t$. The tax has two components, a baseline tax rate $\tau^0_t$, and a Leeper tax $\tau^1_t$ that responds to the level of government debt. Although actual Social Security taxes do not change based on government debt, a Leeper tax can be thought of as a way to capture legislative unease with increasing debt. I consider a Leeper tax of the following form:

$$\tau_t = \tau^0_t + \tau^1_t \left( \frac{B_t}{H_t} \right).$$

where $\tau^0 \in [0, 1]$ is the baseline tax rate when government debt is zero, $\tau^1 \geq 0$ is the incremental tax, and $B_t/H_t$ is government debt per labor hours. Notationally, a tax rate without a superscript ($\tau_t$) will refer to the entire pay-roll tax $\tau_t = \tau^0_t + \tau^1_t \left( \frac{B_t}{H_t} \right)$.

Social security benefits $z^j_t$ are paid according to a benefit earning rule:

$$z^j_t = \phi_t AIPE^j_t$$

where $z^j_t$ represents the benefit paid to a retiree of age $j$ in time $t$ who initially retired at age $T$. The parameter $\phi_t$ is the replacement rate and shows how much of a worker’s earnings are replaced by Social Security benefits. The term $AIPE^j_t$ represents the average indexed period earnings of an agent who is currently age $j$ in time $t$ (who retired exogenously at age $T$). This term is defined as:

$$AIPE^j_t = \sum_{i=1}^{T} \frac{I_i T w_t - T + i}{T}$$

for $j \geq T$

where $I_{i,T}$ indicates the wage-index for time period $i$ relative to retirement date $T$ (the ratio of average wages in year $T$ to average wages in year $i$). In the United States, Social Security benefits are a function of average indexed monthly earnings (or AIME) from the

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6The Leeper tax leaves open the possibility of a total tax rate greater than one hundred percent if bond levels are high or if $\tau^1$ is large (i.e., $\tau^0 + \tau^1 \left( \frac{B_t}{H_t} \right) > 1$, for large $B_t/H_t$ and/or large $\tau^1$). I avoid this by imposing the restriction $\tau^1$ is either the exogenous parameter value chosen by the economist, or the (smaller) parameter value such that $\tau^0 + \tau^1 \left( \frac{B_t}{H_t} \right) = 1$. In the policy experiments that follow, the majority of the tax burden will come from the baseline tax rate $\tau^0$, and the Leeper tax $\tau^1$ will be used only to calibrate a dynamically efficient steady state with positive bonds.

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8
highest 35 years of wage earnings for a given individual.\textsuperscript{7} I simplify slightly, and assume that benefits depend on average indexed earnings from all working years. I make one additional simplification by assuming the benefit earning rule is simply a fraction ($\phi$) of average earnings rather than a piece-wise linear progressive function of earnings. This simplification is trivial since there is no within-cohort heterogeneity in this model, so all members of a given cohort would receive the same benefit even if a more complicated benefit earning rule was used. Note finally that benefits are constant during the retirement phase of life for any particular cohort.\textsuperscript{8}

The government is not required to balance the Social Security budget. If Social Security taxes are less than Social Security benefits, the Social Security program runs a deficit. The time $t$ deficit is defined as the time $t$ Social Security benefits less the pay-roll tax revenue:

\[ \text{Deficit}_t = \sum_{j=T}^{J} N_{t+1-j} \phi_t AIPE_j^t - H_t(\tau^0_t + \tau^1_t (B_t/H_t))w_t. \]

Recall $H_t$ indicates the working age population since each worker exogenously supplies one unit of labor.

The government issuance of bonds is equal to the gross interest on outstanding debt plus the Social Security deficit from the previous period

\[ B_{t+1} = R_t B_t + \sum_{j=T}^{J} N_{t+1-j} \phi_t AIPE_j^t - H_t(\tau^0_t + \tau^1_t (B_t/H_t))w_t. \]

\subsection*{2.5 Markets}

There are four markets: labor, capital, bonds, and goods. Prices adjust in equilibrium to ensure all markets clear.

Labor market clearing requires the total number of hours worked equal the labor input of the representative firm:

\textsuperscript{7}For more information on how Social Security benefits are calculated see https://www.ssa.gov/oact/cola/Benefits.html.

\textsuperscript{8}In the United States, Social Security benefits grow according to a cost of living adjustment, that ensures retirees’ purchasing power is not eroded by inflation or wage growth. There is no inflation or wage growth (i.e. technology or productivity growth) in this model, so constant benefits are consistent with the U.S. cost of living adjustment.
Each worker supplies one unit (or hour) of labor exogenously in each working period of life, so $H_t$ can be thought both as the total labor supplied in period $t$, and also as the number of workers in period $t$.

Asset market clearing requires that aggregate capital and bonds next period are equal to the total savings of each cohort:

$$K_{t+1} + B_{t+1} = \sum_{j=1}^{J} N_{t+1-j} a_j^t. \quad (6)$$

Bond market clearing is ensured by the government’s flow budget constraint, reprinted below:

$$B_{t+1} = R_t B_t + \sum_{j=T}^{J} N_{t+1-j} \phi_t A I P E_t^j - H_t (\tau_t^0 + \tau_t^1 (B_t/H_t)) w_t. \quad (7)$$

Goods market clearing requires that aggregate output is equal to aggregate consumption and aggregate investment. The goods market clears by Walras law. The goods market clearing equation is printed below for reference:

$$F(K_t, H_t) = \sum_{j=1}^{J} N_{t+1-j} c_j^t + K_{t+1} - (1 - \delta) K_t. \quad (8)$$

Along the balanced growth path, cohort size ($N_t$), labor hours worked ($H_t$), output ($Y_t$), capital ($K_t$), and bonds ($B_t$) all grow at rate $n$. Therefore, it will be convenient to rewrite the market clearing equations (7) and (6) in per-hours terms by defining $b_t = B_t/H_t$, and $k_t = K_t/H_t$. I will refer to variables in per-hours terms as “efficient.”

The capital and bond market clearing equations can be written in efficient terms as:

$$\left( k_{t+1} + b_{t+1} \right) (1 + n_t) = \frac{\sum_{j=1}^{J} N_{t+1-j} a_j^t}{H_t}$$

$$\left( 1 + n_t \right) b_{t+1} = R_t b_t + \frac{\sum_{j=T}^{J} N_{t+1-j} \phi_t A I P E_t^j}{H_t} - (\tau_t^0 + \tau_t^1 (B_t/H_t)) w_t.$$

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2.6 Rational Expectations Equilibrium

Given initial conditions \( k_0, b_0, a_{-1}^1, \cdots a_{-1}^{J-1} \), and an initial population \( \sum_{j=1}^{J} (1 + n)^{1-j} N_0 \) (where \( N_0 \) is the the initial cohort of young and \( n \) is the population growth rate), a competitive equilibrium is a sequences of functions for the household savings \( \{a_t^1, a_t^2, \cdots, a_t^J\}_{t=0}^{\infty} \), production plans for the firm, \( \{k_t\}_{t=1}^{\infty} \), government bonds \( \{b_t\}_{t=1}^{\infty} \), factor prices \( \{R_t, w_t\}_{t=0}^{\infty} \), and government policy variables \( \{\tau_t^0, \tau_t^1, \phi_t\}_{t=0}^{\infty} \), that satisfy the following conditions:

1. Given factor prices and government policy variables, individuals’ decisions solve the household optimization problem (1) and (2)
2. Factor prices are derived competitively according to (3) and (4), and
3. All markets clear according to (5), (6), and (7).

There are \( J + 1 \) equilibrium equations, which hold for time periods \( t = 0, \cdots \infty \). Written in efficient terms, the equilibrium equations include the capital clearing equation, the bond market clearing equation, and \( J - 1 \) household first order conditions. These are reprinted below:

\[
(k_{t+1} + b_{t+1})(1 + n_t) = \frac{\sum_{j=1}^{J} N_{t+1-j} a_t^j}{H_t}
\]

\[
(1 + n_t)b_{t+1} = R_t b_t + \frac{\sum_{j=T}^{J} N_{t+1-j} \phi_t w_{t+T-j}}{H_t} - (\tau_t^0 + \tau_t^1 (B_t/H_t)) w_t
\]

\[
(R_t a_{t-1}^{j-1} + y_t^j - a_t^j)^{-\sigma} = \beta E_t[R_{t+1}(R_{t+1} a_{t+1}^j + y_{t+1}^{j+1} - a_{t+1}^{j+1})^{-\sigma}] \quad \text{for } j = 1, \cdots, J - 1.
\]

Here \( y^j \) indicates period labor income during working life and Social Security income after retirement. Note that \( a^0 = 0 \), and factor prices are given by (3) and (4).

The equilibrium definition above accommodates a time-varying population growth rate \( n_t \). In a steady state, the growth rate of the population is constant. When the growth rate is constant, several terms can be written concisely by defining \( \nu = N_t/H_t = (\sum_{j=1}^{T-1} (1 + \cdots) \)
The steady state is a collection \( \{k, b, a_1, \cdots, a_J\} \) that solves:

\[
(k + b)(1 + n) = \nu \sum_{j=1}^{J} (1 + n)^{1-j} a^j
\]

\[
(1 + n)b = R(k)b + \nu \sum_{j=T}^{J} (1 + n)^{1-j} \phi w(k) - (\tau^0 + \tau^1 b)w(k)
\]

\[
(R(k)a^{j-1} + y^j - a^j)^{-\sigma} = \beta \left[ R(k)(R(k)a^j + y^{j+1} - a^{j+1})^{-\sigma} \right]
\]

for \( j = 1, \cdots, J - 1 \)

with \( y^j = (1 - (\tau^0 + \tau^1 b))w(k) \) for \( j < T \), \( y^j = \phi w(k) \) for \( j \geq T \), \( a^J = 0 \), and factor prices \( R(k) \) and \( w(k) \) given by (3) and (4). Note in steady state \( w_t = w(k) \forall t \), so the average indexed period earnings for all cohorts are simply equal to the wage thus steady state Social Security benefits are simply \( \phi w(k) \).

The equilibrium of the model is not unique; for many parameter combinations there are two steady states. The number of steady states depends on the model parameters and can be characterized by a saddle-node bifurcation. Chalk (2000) discusses the saddle-node bifurcation in a similar OLG model with government debt and physical capital.\(^9\)

## 3 Lifecycle Horizon Learning

Under the rational expectations hypothesis (RE), households make optimal savings and consumption choices given their (rational) forecasts of future prices and policy. Households make decisions at age zero looking forward over their entire life cycle. These rational agents are fully forward looking, and consider every stage of the lifecycle when making decisions. Under RE, agents know the underlying equations that govern the economy and form expectations of future variables using the mathematical expected value.

This paper backs away from RE and proposes a learning model in which agents combine limited knowledge about the structure of the economy with adaptive forecasts for future macroeconomic aggregates. I consider agents who do not have perfect foresight over factor prices. Thus, agents do not know the future path of wages or interest rates over their lifecycle. Agents use adaptive expectations to forecast future prices, but behave optimally in all other ways. Agents update their forecasts as more information becomes available. I

\(^9\)See Azariadis (1993) (Chapter 8 and Chapter 20) for a discussion of saddle-node bifurcations in planar OLG models.
call this behavioral assumption lifecycle horizon learning (LCH learning).

Under LCH learning, households are forward looking and make optimal savings and consumption choices given their (adaptive) forecasts of future macroeconomic aggregates. Adaptive expectations may be viewed as a special case of adaptive learning suitable for non-stochastic models. The adaptive expectations assumption in this paper is analogous to constant gain least squares learning in a model with random shocks (like a productivity shock). LCH learning is similar to infinite-horizon learning, developed in Marcet and Sargent (1989) and emphasized by Preston (2005 and 2006). The key difference is that infinite-horizon learning models are based on a representative agent who lives for an infinite number of periods. LCH learning applies the same forward looking behavior to finitely lived agents in an overlapping generations lifecycle model.\(^\text{10}\) Throughout this paper, I assume homogeneous expectations across agents.

Agents in the LCH learning model forecast wages and interest rates using adaptive expectations of the following form:

\[
\begin{align*}
    w^e_{t+1} &= \gamma w_t + (1-\gamma)w^e_t \\
    R^e_{t+1} &= \gamma R_t + (1-\gamma)R^e_t
\end{align*}
\]

with \(\gamma \in (0,1)\). Here \(w^e\) indicates expected wage, and \(R^e_t\) indicates expected interest rate. Agents also form expectations at time \(t\) of prices in period \(t+j\) for \(j > 1\):

\[
\begin{align*}
    w^e_{t+j} &= w^e_{t,t+j} = w^e_{t+1} \\
    R^e_{t+j} &= R^e_{t,t+j} = R^e_{t+1}
\end{align*}
\]

When the Leeper tax is non-zero (\(\tau^1 > 0\)), then agents need to know future bond levels in order to estimate their after-tax pay. For simplicity, I assume that agents forecast bonds using the same adaptive learning rule they use to forecast prices.

\[
\begin{align*}
    b^e_{t+1} &= \gamma b_t + (1-\gamma)b^e_t \\
    b^e_{t,t+j} &= b^e_{t,t+1} \quad \text{for } j > 1
\end{align*}
\]

Agents enter the model with knowledge of the previous period’s prices, bonds, and expecta-

\(^{10}\)As explained in the introduction, LCH learning is also similar to the model developed in Bullard and Duffy (2001), in which agents forecast inflation over a finite lifecycle and make optimal choices based on those forecasts.
tions. At any moment in time, all agents in the model have the same expectation for future prices and bonds. This assumption is equivalent to assuming agents inherit expectations from the previous generation.

A young agent in the LCH learning model chooses first period savings and consumption and plans future savings and consumption to satisfy her \( J - 1 \) first order equations and her lifetime budget constraint. The young agent’s time \( t \) plan of second period savings is denoted \( a_{t,t+1}^2 \). In the following period, her time \( t + 1 \) actual choice of second period savings is given by \( a_{t+1}^2 \). Her time \( t + 1 \) plan does not have to be consistent with her time \( t \) choice. She can update her savings decision based on the new information she receives in period \( t + 1 \).

Abstracting from policy uncertainty\(^\text{11}\), the young agent in time \( t \) solves:

\[
(R_{t+j-1}a_{t+j-2}^j + y_{t+j-1}^j - a_{t+j-1}^j)^{-\sigma} = \beta R_{t}^{e}(R_{t+1}^{e}a_{t+1}^{j-1} + y_{t+1}^{j-1} - a_{t+1}^{j-1})^{-\sigma} \tag{14}
\]

for \( j = 1, \cdots, J - 1 \)

with \( y_{t+j}^{j+1} \) indicating expected period labor income or Social Security income. That is:

\[
y_{t+j}^{j+1} = (1 - \tau_{t+j})w_{t+j}^{e} \quad \text{for} \quad j < T
\]

\[
y_{t+j}^{j+1} = \phi_{t+j}w_{t+j+T-j}^{e} \quad \text{for} \quad j \geq T.
\]

Similarly, an agent of age two solve \( J - 2 \) first order equations, for the remaining \( J - 2 \) periods of her lifecycle. In total, the \( J \) cohorts alive in any period solve \( \sum_{j=1}^{J-1} j = \frac{(J-1)J}{2} \) first order conditions. Together, the decisions of households of all ages (14), asset market clearing (6), and government bonds (7), and the expectation equations (8) through (12), create a recursive system that governs the dynamics of the economy.

LCH learning can be viewed as a small deviation from rational expectations in the sense that agents are still forward looking and only use a different rule to forecast future aggregates. Adaptive expectations are plausible in a world in which the true data generating process is complex. Adaptive expectations are optimal if agents think that wages, interest rates, and bonds follow IMA(1,1) processes. That is, expectations of the form given in Equation (8) are *rational* if the change in the variable of interest has the following form \( \Delta x_t = \epsilon_t + \theta \epsilon_{t-1} \) for variable \( x \in \{w, R, b\} \), shocks \( \epsilon \), and parameter \( \theta \in (0, 1) \). The use of adaptive

\(^{11}\)When policy uncertainty is included, the household first order condition equates the utility of consumption in time \( t \) with the discounted, expected utility of consumption in period \( t + 1 \), where the expectation is over the possible realizations of policy.
expectations is equivalent to agents believing that the variable they are forecasting has a mixture of permanent and transitory shocks (Muth (1960)). The gain parameter, $\gamma$, has a natural interpretation in this context. If all shocks are transitory, the optimal forecasting rule is constant, i.e., $\gamma=0$. In contrast, if all shocks are permanent, then the best forecasting rule is a random walk, i.e., $\gamma = 1$. In this particular model, all shocks are permanent, but agents are not endowed with this knowledge. I choose the gain parameter used in simulations to minimize the welfare cost of agents inaccurately forecasting along the transition path. I explore alternative gain parameters as robustness checks.\textsuperscript{12}

In the LCH learning model, agents have full knowledge of government policy. Agents know the future path of social taxes and the benefit replacement rate if there is no policy uncertainty. If future policy is stochastic, agents form expectations of future policy parameters using rational expectations. Agents know the finite set of policy parameters (and/or reform dates) and the relative probability of each.

### 3.1 Stability of REE under Learning

Determinacy is often used as an equilibrium selection device in rational expectations models with multiple equilibria. A complementary approach to selecting equilibria is to conduct stability analysis under learning. If agents’ (non-rational) expectations and forecasts converge to a rational expectations equilibrium (REE) in a model with learning, the REE is said to be stable under learning (see Evans and Honkapohja (2001)). I numerically verify that determinate REE in my model is stable under LCH learning.

Given constant (potentially incorrect) expectations $p^e = (R^e, w^e, b^e)'$, the learning dynamics of the LCH learning model asymptotically converge to $p = (R, w, b)'$. This convergence from beliefs to actual prices is called a T-map.

$$ T : \mathbb{R}^3 \rightarrow \mathbb{R}^3 $$

(15)

A fixed point of the $T$ map is E-stable if it locally stable under the ordinary differential equation

$$ \frac{dp}{d\tau} = T(p) - p. $$

(16)

\textsuperscript{12}Evans and Ramey show that by appropriately tuning the free parameters of the forecast rule agents can obtain the best forecast rule within a given class of under-parameterized learning rules (Evans and Ramey (2006)).
Parameter Value
\( \alpha \) Capital share of income \( \frac{1}{3} \)
\( \beta \) Discount factor \( \left( \frac{1 + 0.05}{1} \right)^{10} \)
\( \sigma \) Inverse elasticity of substitution 1
\( \delta \) Depreciation \( 1 - (1 - 0.10)^{10} \)
\( n \) Population growth rate 0.01
\( A \) TFP factor 10

Table 1: Baseline parameters for 6 period model

E-stability requires the real parts the eigenvalues of the derivative matrix \( dT < 1 \). I have numerically verified the determinate (high capital) steady state is E-stable under LCH learning. Additionally, I numerically verified that the low capital steady state (which is a saddle) is not stable under learning.\(^{13}\)

4 Parameterization

Agents enter the model at age 25 and live for six periods \(( J = 6 )\). Each period is 10 years, and agents die with certainty at age 85. The capital share of income, discount factor, inverse elasticity of substitution, and depreciation rate are all set to standard parameter values.\(^{14}\) The total factor productivity parameter \( A \) is chosen to ensure that two-steady states exist for reasonable Social Security parameters. The parameter values are listed in Table 1.

The Social Security administration estimates the current ratio of Social Security beneficiaries to workers to be 0.35. This ratio is expected to increase to 0.46 by 2035, and to 0.5 by 2095 under the medium cost assumptions (SSA (2017)). The increase in the ratio of beneficiaries to workers is driven by both increasing lifespans for retirees, and declining birthrates for working generations. I abstract from these details in the model, and capture all population changes using the growth rate \( n \). For the exercises in this paper, I will choose \( n \) such that the ratio of retirees to workers is 0.3 and then increases gradually to 0.485, as illustrated in Figure 1.

\(^{13}\)As discussed in the main text, the underlying model does not always have two steady states. The number of steady states depends on the parametrization of the model. I verified the E-stability of the steady states associated with many different parameterizations of the model. I searched across the parameter space \( \beta \in (0, 1], \alpha \in (0, 1), \sigma \in (0.5, 4), \delta \in [0, 1], n \in [0, 5] \) and across different Social Security parameterizations (such as no Social Security, lower taxes, higher taxes, etc.).

\(^{14}\)See, for example, Branch et al. (2013). My discount factor \( \beta \) is slightly closer to one and the depreciation rate is slightly higher. I choose the higher discount factor and depreciation rate to better match the size of tax increase necessary to ensure long-run solvency of Social Security.
In the learning model, I set the gain parameter $\gamma = 0.93$. This gain parameter minimizes the maximum welfare cost to an agent of using adaptive forecasts in the LCH model along the transition path that includes a demographic shock and a change to Social Security. I construct a consumption equivalent variation that compares the utility of consumption of each cohort of agents in the LCH model to the utility of consumption for “infinitesimally rational” cohorts of agents. An infinitesimally rational agent is able to predict future prices with perfect foresight in the LCH world, but is such a small part of the market that she does not change prices. I compute the consumption equivalent variation that makes an infinitesimally rational agent indifferent between her own consumption and the consumption of the lifecycle horizon learners. I chose the gain parameter $\gamma$ to minimize the maximum welfare cost. It is unsurprising that the gain parameter is close to $\gamma = 1$, since all shocks in the model are permanent. The simulation results are sensitive to the gain parameter. I explore that relationship in section 7.3.

5 Announced Social Security Reform

The following section illustrates the mechanics of the learning models, in the absence of policy uncertainty, in order to build intuition of how the learning models differ from rational expectations.

The aging of society puts pressure on the Social Security system. In the absence of reform, the government would be required to fund Social Security benefits by issuing debt. However,
there is a limit to how much the government can borrow. I illustrate these dynamics in the following example.\footnote{The government could also fund Social Security benefits using tax revenue from some other existing tax. I have abstracted from all government spending and taxing in this paper, which is equivalent to assuming that the government cannot (or will not) change other taxes or spending to finance Social Security. This assumption could be relaxed in future work.}

Suppose that the government runs a Social Security program that generates a small surplus (government assets); however, as the population ages, the operating surplus changes to a deficit. The government continues to operate the same Social Security system which depletes the government assets and causes the government to accrue debt. After a number of periods, the government reforms Social Security by cutting benefits or raising taxes.

I calibrate this example to correspond to the U.S. system. The initial population growth rate is \( n = 0.1802 \) and falls to \( n = 0.01 \) in 1980. This generates a smooth transition in the ratio of retirees to workers from 0.3 to 0.485 after six periods (which is near SSA projections). The initial policy mimics the current U.S. system: the payroll tax rate is set to \( \tau^0 = 0.124 \) and the benefit replacement rate is \( \phi = 0.4 \). Reform is calibrated as either a tax increase to \( \tau^0 = 0.1516 \), or a benefit cut to \( \phi = 0.332 \). The SSA estimates these reforms would eliminate the funding shortfall in the Social Security system. In order to have a dynamically efficient economy with positive government debt, the Leeper tax rate is set to \( \tau^1 = 0.045 \) before and after reform.

Figure 2 plots the path of capital and bonds for this tax increase reform. The plot includes paths for an economy with Rational Expectations (in green), and for an economy with LCH Learning. Government debt increases before the reform takes place and then converges to the new higher steady state value. Debt is increasing since the government is running a deficit in the Social Security system. Debt would become explosive, were it not for the reform in the year 2030. The reform is announced, so agents adjust their behavior before the reform date. Capital increases initially as savings go up, and then falls to the steady state.

Agents in the learning model overestimate the interest rate and underestimate the wage in the first few periods following the change in the population growth rate (which moves the economy away from the initial steady state). Because agents are expecting a higher interest rate and a lower wage, they save (relatively) more. This increases the capital stock relative to the rational expectations model. The increased capital stock decreases the interest rate and increases wages, which causes agents to save less, which eventually drives down the capital stock. This oscillatory pattern continues along the convergence path to the new steady state.
Figure 2: Time paths for capital and bonds for RE (green) and LCH (yellow) for an announced tax increase. The initial population growth rate $n$ is 0.1802 and falls to 0.01 in 1980. Demographic changes drive increasing debt until tax increase in 2030. Initial government policy is $\tau_0, \tau_1, \phi$ equal to $(0.124, 0.045, 0.4)'$. The policy change increases $\tau_0$ to 0.1516. The gain parameter $\gamma = 0.93$ for this example.

The expectations of LCH learners are depicted in Figure 3. This figure plots agents’ time $t$ expectation of interest rates ($R_{t+1}^e$), wages ($w_{t+1}^e$), and bonds ($b_{t+1}^e$), against the realized interest rates ($R_{t+1}$), wages ($w_{t+1}$), and bonds ($b_{t+1}$). In the initial steady state, agents’ expectations match the data. Following the demographic shock, agents expect the interest rate, wages, and bond per worker to stay the same—but they are wrong. They observe that the interest rate is lower than expected and wages are higher than expected, and they update their forecast for the next period accordingly. The gain parameter $\gamma$ is set to 0.93 in this example, which means agents are placing 93 percent weight on the observed variable (interest rate, wage, bond), and only 7 percent weight on their expectation from the previous period. This large update causes the expectations to adjust quickly.

6 Policy Uncertainty

The policy change of the previous section is non-stochastic and announced. However, the future of the U.S. Social Security system seems difficult to anticipate. A growing body of research focuses on the welfare cost of this type of uncertainty.\textsuperscript{16} I calculate the welfare cost

\textsuperscript{16}See, for example, Kitao (2018), and Caliendo et al. (2015), Nelson (2017). I follow Caliendo et al. (2015) and use the words “uncertainty” and “risk” interchangeably in this paper. All of the examples I will consider have known probabilities, and thus might be called “risky.”
Figure 3: Time paths for expected interest rates, wages, and government debt levels (blue lines), and realized interest rates, wages, and debt (yellow lines) in the LCH model with an announced tax increase. Agents overestimate the interest rate and underestimate the wage in the initial periods as they learn that capital and debt are increasing. The initial population growth rate is 0.1802 and falls to 0.01 in 1980. Demographic changes drive increasing debt until a tax increase in 2030. Initial government policy is $\tau^0, \tau^1, \phi$ equal to $(0.124, 0.045, 0.4)'$. The policy change increases $\tau^0$ to 0.1516. The gain parameter $\gamma = 0.93$. 
of policy uncertainty under RE and under LCH learning.

Introducing aggregate uncertainty to the model raises some concerns; a steady state wealth distribution will not exist in general in an OLG model with aggregate uncertainty (see Krueger and Kubler (2004) for a discussion). I overcome this problem by modeling government policy as an exogenous, stochastic process. This allows me to abstract from the political economy concerns that actually influence policy reform. Although interesting, these concerns are beyond the scope of this paper and generally not included in this literature. The stochastic path of policy is exogenous from the perspective of households, and also from the perspective of the government. The expectations of households and the government are consistent with this exogenous, stochastic process.

More formally, let \( \omega_t = (\tau^0_t, \tau^1_t, \phi_t)' \) describe the government policy parameters at time \( t \). Suppose that the Social Security program will be reformed at some future date. The reform date falls within a known, finite set. Suppose that the realization of \( \omega_{t+1} \) depends on \( \omega_t \), and is contained in the finite set \( \Omega \). Suppose also that each possible reform converges to a steady state (the path is non-explosive).

Let the probability of realizing a particular value of \( \omega_{t+1} \) given \( \omega_t \) be described by \( \pi(\omega_{t+1}|\omega_t) \). Using this notation, the expected value in the household first order equations (2.1) can be written as:

\[
u'(c^j_{t+j-1}) = \beta \sum_{\omega_{t+j} \in \Omega} \pi(\omega_{t+j}|\omega_{t+j-1}) R_{t+j} u'(c^{j+1}_{t+j}) \quad \text{for } j = 1, \ldots, J - 1 \quad (17)\]

I assume that policy uncertainty takes the following form: reform is possible in either date \( S \), or in date \( S + 1 \). Within each period two reforms are possible, either a benefit cut or a tax increase. Thus, there are four possible paths for the economy. The probability of each path is \( p = 0.25 \).\(^{17}\) It is as if Congress flips a coin to decide if they should take action, and then they flip a coin again to see what type of action they should take. Obviously this is a dramatic simplification, but it provides a consistent framework in which to conduct welfare analysis.

All agents in the economy know the four possible reforms and their relatively probability. I calibrate this example to correspond to the U.S. system. The economy starts in a steady

\(^{17}\)Nelson (2017) uses a similar stochastic process, and presents results for different probabilities of a given reform. Changing the probability of a given reform can qualitatively change the transition dynamics leading up to and following the reform. For example, if the probability of a benefit cut is higher, this increases the precautionary saving of young households. I have examined different probabilities of reform in my analysis and only present the results for \( p = 0.25 \) for each reform in this paper.
state with a ratio of retirees to workers of 0.3 and with government policy: \( \tau^0 = 0.124 \), \( \tau^1 = 0.045 \), and \( \phi = 0.4 \). In 1980, the growth rate of the population falls to 0.01 leading the ratio of retirees to workers to grow, eventually reaching 0.485. The demographic change causes the Social Security system to run a deficit, which increases government debt.

Reform is calibrated as either a tax increase to \( \tau^0 = 0.1516 \), or a benefit cut to \( \phi = 0.332 \). The reform takes place in either date 2030 or 2040. The SSA estimates these reforms would eliminate the funding shortfall in the Social Security system. In order to have a dynamically efficient economy with positive government debt, the Leeper tax rate is set to \( \tau^1 = 0.045 \) before and after reform. The gain parameter for the learning model is set to \( \gamma = 0.93 \).

Intuitively, there is a trade-off in the timing and size of reform. If the government acts sooner, they can enact a smaller reform (cut benefits by less or raise taxes by less). If the government waits to reform, government debt will grow, which will ultimately require larger endogenous Social Security surpluses to service the debt. One approach to modeling Social Security policy reform uncertainty would be to computationally search for the minimum reform (the smallest reduction in benefits or increase in taxes) such that the economy converges to a steady state in the long-run. Because the model abstracts from several details of Social Security (including survivor benefits, spousal benefits, disability benefits, and all within-cohort heterogeneity), this approach results in policy changes of magnitudes that do not easily compare to SSA estimates. Thus, I’ve taken an alternative approach of using the same reforms for both decades, calibrated to be what the SSA estimates would be sufficient to fund Social Security for the long run: a tax increase from 12.4\% to 15.16\%, or a 17\% reduction in benefits. This second approach has the advantage that the tax reform converges to the same steady-state (and thus the same long-run amount of government debt, and the same Social Security deficits) regardless of when it is enacted. Similarly, the benefit reform converges to the same steady state (distinct from the tax-increase steady state) regardless of the implementation date. I explore an alternative calibration in section C that models smaller policy changes in the reform that takes place in 2030 rather than 2040. The main results of the paper are qualitatively similar under both specifications.

Before considering the welfare cost of reform, first consider the transition dynamics of the four possible equilibrium paths in the RE model. The four possible paths for the economy are identical up to 2020. Agents in the initial years face the same uncertainty about future Social Security policy. This uncertainty is either fully or partially resolved in 2030. Along the first two paths of the economy, agents observe that policy was reformed in 2030 (either
as a tax increase or benefit cut). Uncertainty is resolved for these two paths. Along the other two paths, agents observe that policy was not reformed in 2030, so they know reform will take place in 2040, but they do not know which reform until the policy is realized. In 2040, all uncertainty is resolved, and all four paths are (potentially) different. There are two possible steady states for this example, the tax increase steady state and the benefit cut steady state. Both tax increase paths converge to the tax increase steady state; similarly, both benefit cut paths converge to the benefit cut steady state. The asset paths for this example are presented in Figure 4.

Agents in the learning model do not know future prices. They make decisions based on their adaptive expectations. In the initial few periods following the change in the population growth rate, the LCH agents over-save compared to the rational agents because they are overestimating the interest rate. This drives up capital in the LCH model compared to the RE model. The LCH agents benefit from this higher capital stock (since the economy is dynamically efficient) and don’t have to save as much in the periods right before the policy reform. Following a policy reform, the paths of savings, capital, and bonds are cyclical in the LCH model since agents are slowly updating their forecasts of prices and bonds.

Note, that all of the agents in the RE and LCH model know the policy process. All agents know the possible dates of reform, the new policy parameters, and the probability of each reform. All agents are fully forward looking (to the end of their lifecycle). The only difference is that the LCH agents forecast future prices and bonds adaptively, while the rational agents make fully rational forecasts. The paths of capital and assets for the LCH and RE models are presented together in Figures 5 and 6.

6.1 Ex-ante Welfare Cost

The welfare cost of policy uncertainty is calculated in this section using an ex-ante Consumption Equivalent Variation (CEV) technique. The rational expectations model is considered separately from the learning model. In each, the expected lifetime utility of consumption for agents in the world with uncertainty is compared to the expected lifetime utility of an agents who experience the same policy realizations with certainty. Because there are four possible policy realizations, I make four welfare comparisons. Early tax reform that results from the uncertain process is compared to announced early tax reform, and so on.
Figure 4: Transition paths for asset holdings in the RE model with policy uncertainty. Four reforms are possible. Either taxes are increased in 2030 (purple line), benefits are cut in 2030 (red/brown line), taxes are increased in 2040 (light blue line), or benefits are cut in 2040 (gold line). All four paths are identical until period 2020. This is because all agents face the same policy uncertainty leading up to the first possible reform. The two possible reform dates are indicated with red and pink dots. The beginning of the demographic transition is indicated with a blue dot.
Figure 5: Transition paths for capital and bonds in the LCH model and RE model with policy uncertainty. Four reforms are possible in each model. Either taxes are increased in 2030 (LCH: blue, RE: purple line), benefits are cut in 2030 (LCH: yellow, RE: red/brown line), taxes are increased in 2040 (LCH: green, RE: light blue line), or benefits are cut in 2040 (LCH: red, RE: gold line). The LCH paths are cyclical, while the RE paths converge to the new steady states quickly. The dynamics before the reform are driven by demographic change in 1980 and the forward looking behavior of agents anticipating reform. The benefit cut paths converge to a steady state with higher capital.

The ex-ante CEV for policy uncertainty in the RE model is calculated as:

$$\sum_{j=1}^{J} \beta^{j-1} u(c_{t+j-1}^{\omega_{t+j-1}}(1 + \Delta)) = \sum_{\omega_{t+j-1} \in \Omega} \pi(\omega_{t+j-1}|\omega_{t+j-2}) \sum_{j=1}^{J} \beta^{j-1} u(c_{t+j-1}^{\omega_{t+j-1}, u})$$

where $c_{t+j-1}^{\omega_{t+j-1}}$ indicates age $j$ consumption in period $t+j-1$ in the model with policy $\omega_{t+j-1}$ and rational expectations and $c_{t+j-1}^{\omega_{t+j-1}, u}$ is consumption in the model with RE and policy uncertainty where policy $\omega_{t+j-1}$ was realized. Period utility is given by $u(c)$, and $\Delta$ is the consumption equivalent variation for policy uncertainty.

I consider four possible reforms: tax increase in 2030, tax increase in 2040, benefit cut in 2030, or benefit cut in 2040. The consumption equivalent variation compares the expected lifetime utility of agents along a particular path (for example, tax increase in 2030) in a world where all four policies were possible to the lifetime utility of agents where that same policy path was announced from the beginning (for example, announced tax increase in 2030). A similar metric is constructed for the LCH model.

Nelson (2017) constructs a similar ex-ante consumption equivalent variation in his paper. The ex-ante comparison used by Kitao (2018) is also related, although not identical. Kitao
Figure 6: Transition paths for asset holdings in the LCH model and RE model with policy uncertainty. Four reforms are possible in each model. Either taxes are increased in 2030 (LCH: blue, RE: purple line), benefits are cut in 2030 (LCH: yellow, RE: red/brown line), taxes are increased in 2040 (LCH: green, RE: light blue line), or benefits are cut in 2040 (LCH: red, RE: gold line). The benefit cut paths converge to a steady state with lower age 1 and 2 savings ($a^1$ and $a^2$) and higher age 3, 4 and 5 savings ($a^3$, $a^4$, $a^5$). The LCH paths are cyclical, while the RE paths converge to the new steady states quickly. The dynamics before the reform are driven by demographic change in 1980; the rational agents respond to the demographic changes while the learning agents do not. Both types of agents are forward looking and respond to looming reform.
Figure 7: Rational Expectations Ex-ante Consumption equivalent variation (CEV). Each panel plots the ex-ante CEV from the rational expectations model. The top left panel shows the fraction of period consumption that an agent would be willing to give up in the rational expectations world with announced tax increase in 2030 to avoid experiencing the early tax increase as the result of the uncertain process where all four reforms are equally likely. The top right shows the CEV comparisons for the early benefit cut, bottom left shows the late tax increase, and the bottom right shows the late benefit cut. A negative CEV indicates that agents would prefer the announced policy.

examines reform that takes place in either 2020, 2030, or 2040 in Japan. She compares the expected utility of consumption for a realized policy with the announced 2030 reform; that is, the uncertain path is always compared to announced reform in 2030. In contrast, I compare each uncertain path to the announced path for that same reform. The comparison in this paper illustrates how much consumption an agent would be willing to give up so that her lifetime utility in the world with certain policy would be the same as her expected lifetime utility in the world with uncertainty. I prefer this specification because the CEV converges to zero after a reform is enacted, because the certain and uncertain paths converge to the same steady state (for a particular policy change). This welfare metric includes the welfare of cost of uncertainty due to risk aversion, as well as the welfare cost (or benefit) of experiencing (or avoiding) a particular policy path that might be less desirable to a particular cohort. As a robustness check, I compare uncertain reforms to the same baseline (as in Kitao (2018)) in section B. Caliendo et al. (2015) develops a different metric to isolate the welfare costs of uncertainty alone.

The ex-ante consumption equivalent variations for the RE model are plotted for each of the four possible policy paths in Figure 7. In this example, the economy begins in a steady state with a high population growth rate. In 1980, the growth rate falls and the Social
Security system begins to run a deficit. Bonds increase until reform is enacted and the economy converges to a new steady state associated with the particular reform. The four possible reforms are a benefit cut in 2030 or 2040 or a tax increase in 2030 or 2040. The CEV shows the fraction of consumption a cohort who experience announced policy would be willing to give up such that their expected lifetime utility with policy uncertainty would be the same. In all four comparisons, the CEV is largest in magnitude for the cohorts born in the six periods leading up to a reform. A negative CEV indicates that agents are worse off in expectation in the world with uncertainty.

The CEVs are negative for the several cohorts leading up to an uncertain tax increase. This means that in expectation, agents are worse off facing policy uncertainty and then having their taxes increased, rather than facing an announced tax increase. In contrast, the CEV is positive for several cohorts leading up to benefit cut. These agents are better off in expectation than the agents who face a benefit cut with certainty. The CEV is positive because the chance of experiencing a tax increase during old age is relatively desirable compared to facing a benefit reduction. In all four cases, the CEVs are relatively small in magnitude. The most harmed cohort are the agents born in 2000 who face policy uncertainty and then experience the tax increase in 2040. They would need to be compensated by 1.49 percent of their consumption to have the same utility as agents who experience the tax increase in a world with announced policy.

The relatively small welfare cost of policy uncertainty in the rational expectations framework that I find is consistent with work by other authors. Nelson (2017) finds the ex-ante welfare cost of policy uncertainty in the U.S. to be about 2 percent of consumption for the most-harmed cohort. Kitao (2018) finds the welfare cost of pension reform uncertainty in a model calibrated to the Japanese economy to be 0.8 percent-1.5 percent of period consumption. Bütler (1999) finds that the welfare cost of misperceived Social Security benefits is between 0-3.46 percent of period consumption in a partial equilibrium model.

Figure 8 plots the ex-ante consumption equivalent variation for the lifecycle horizon learning model on top of the CEV from the rational expectations model. The lifecycle horizon learning CEV compares the expected lifetime utility of agents in the model with lifecycle horizon learning with policy uncertainty to the lifetime utility of agents in the learning model with announced policy. Similar to the rational expectation model, the learning model shows

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18Recall that the metric used by Kitao is not identical to that used in this section of the paper or by Nelson.
19Bütler’s results are not easily comparable to other papers, because agent beliefs are not model consistent in her paper. See Phelan (1999) for a discussion of Bütler (1999).
Figure 8: Lifecycle Horizon Learning and rational expectations ex-ante consumption equivalent variations (CEV). Each panel plots the ex-ante CEV from the LCH model (in yellow) on top of the ex-ante CEV from the rational expectations model (in blue). The top left panel shows the fraction of period consumption that an agent would be willing to give up in either the rational expectations world or the lifecycle horizon learning world with announced tax increase in 2030 to avoid experiencing the early tax increase as the result of the uncertain process where all four reforms are equally likely. The top right shows the CEV comparisons for the early benefit cut, bottom left shows the late tax increase, and the bottom right shows the late benefit cut. A negative CEV indicates that agents would prefer the announced policy.
negative CEVs for cohorts born before a tax increase and positive CEVs for cohorts leading
up to a benefit cut. These agents are worse-off (or better-off) in expectation than agents who
experience announced policy. In contrast to the rational expectations world, the CEV has
a large dip for cohorts born right after a reform is enacted in 2030. Cohorts of agents born
after a reform do not face uncertainty directly; the CEV captures the general equilibrium
effects that they experience as a result of the savings decisions from the previous cohorts.
The agents born during or after an early reform experience lower wages and thus lower con-
sumption because their predecessors did not save as much as they would have if they had
know the reform was coming in 2030 rather than 2040. This dip is lowest following the 2030
tax increase. Agents born in 2040, after taxes were increased in 2030 with certainty would be
willing to give up 1.99 percent of their period consumption to avoid experiencing the same
policy (benefit cut in 2030) that was the result of the uncertain process. The minimum CEV
along the other three paths are also lower in the lifecycle horizon learning world than in the
rational expectations world. The greatest difference is for agents born immediately after the
2030 benefit cut.

The welfare effects of uncertainty are larger in the learning model compared to the ratio-
nal expectations model because agents in the learning model do not anticipate how Social
Security policy changes will impact the wage and interest rate. Even after a policy has been
implemented, it takes the agents several periods to learn the new prices. This analysis sug-
gests that looking at rational expectations alone underestimates the welfare cost of policy
uncertainty.

6.2 Ex-post Welfare Cost

The analysis in the previous section suggests that the general equilibrium feedback effects
of policy uncertainty are larger in a world where agents are using adaptive learning. This
section uses an ex-post consumption equivalent variation to examine the general equilibrium
feedback. This measurement also captures the experienced utility of each cohort.

The ex-post welfare cost of policy uncertainty is calculated in this section using con-
sumption equivalent variations. In the ex-post CEV, the utility of an agent who experiences
a given realization of the uncertain policy process is compared to the utility of an agent
who experiences the same policy realization with certainty. The ex-post CEV is backward
looking and calculates the amount of consumption that could be taken from an agent along
a particular policy path that would make their lifetime utility the same as an agent who
Figure 9: Ex-post consumption equivalent variation measure of the welfare cost of policy uncertainty in the RE model. The welfare cost is constructed by comparing the utility of realized policy parameters of a given path with the utility of that same policy change in a perfect foresight, RE model. The CEV is show pictured for all four possible paths: taxes are increased in 2030 (top left), benefits are cut in 2030 (top right), taxes are increased in 2040 (bottom left), or benefits are cut in 2040 (bottom right). A negative CEV indicates that an agent would have higher utility in a RE model without policy uncertainty. A positive CEV indicates that utility is higher in the uncertain model.

The welfare cost of Social Security policy uncertainty is illustrated below in figure 9. The figure depicts the CEV for each of the four possible paths in the rational expectations model. The calibration of this example is identical to example 5.

The welfare cost of Social Security policy uncertainty for the RE model is small. The cost

experienced the same policy as the result of the uncertain process.

The ex-post CEV for policy uncertainty in the RE model is calculated as:

$$\sum_{j=1}^{J} \beta^{j-1} u(c_{t+j-1}^{j} \omega^{j+1} (1 + \Delta)) = \sum_{j=1}^{J} \beta^{j-1} u(c_{t+j-1}^{j} \omega^{j+1} , \Delta)$$

(18)

where $c_{t+j-1}^{j} \omega^{j+1}$ indicates age $j$ consumption in period $t + j - 1$ in the model with policy parameters $\omega$ and rational expectations and $c_{t+j-1}^{j} \omega^{j+1} , \Delta$ is consumption in the model with RE and policy uncertainty where policy path $\omega$ was realized. As before, $u(c)$ indicates the period utility function, and $\Delta$ is the CEV for policy uncertainty. A similar CEV is constructed for the lifecycle horizon learning model which computes the fraction of period consumption an agent who experiences a particular policy path with certainty would have to give up to be indifferent to experience the same policy path in the world with uncertainty.

The welfare cost of Social Security policy uncertainty is illustrated below in figure 9. The figure depicts the CEV for each of the four possible paths in the rational expectations model. The calibration of this example is identical to example 5.

The welfare cost of Social Security policy uncertainty for the RE model is small. The cost
Figure 10: Ex-post consumption equivalent variation measure of the welfare cost of policy uncertainty in the lifecycle horizon learning model (in yellow), displayed on top of the ex-post CEV in the rational expectations model (in blue). The welfare cost is constructed by comparing the utility of realized policy parameters of a given path with the utility of that same policy change in either the lifecycle horizon learning or rational expectations model. The CEV is shown for all four possible paths: taxes are increased in 2030 (top left), benefits are cut in 2030 (top right), taxes are increased in 2040 (bottom left), or benefits are cut in 2040 (bottom right). A negative CEV indicates that an agent would have higher utility in a model without policy uncertainty. A positive CEV indicates that utility is higher in the uncertain model. The CEVs are larger in magnitude in the LCH model.

is the less than 0.25 percent of period consumption for cohorts along all four paths. The initial cohorts benefit if the reform is a benefit cut, since they do not have to pay higher taxes and do not experience the benefit reduction in their lifetime. The agents born a few periods before and after the benefit cut are harmed because they receive lower benefits, but are not alive to experience the feedback effect of higher wages that result in the new steady state. The most harmed cohort are agents born in 2030 if the realized policy is the benefit cut in 2030. These agents would give up 0.25 percent of consumption to avoid being born in a world with policy uncertainty (even though the uncertainty is resolved in the period in which they are born).

The ex-post lifecycle learning CEV is presented in Figure 10, on top of the ex-post rational expectations CEV. The ex-post lifecycle learning CEV is positive for several cohorts if taxes are increased in 2040. This is because the possibility of facing reform in 2030 increases agent savings and drives up the capital stock in the model with policy uncertainty, relative to the model with announced tax increases. The accumulation of extra capital raises agent welfare,
relative to the baseline. The maximum (positive) CEV is 1.11 percent and occurs in 2030. The cohort of agents born in 2030 in the LCH model with a certain tax increase in 2040 would need to have 1.11 percent added to their period consumption in order to be indifferent between their consumption, and the consumption of agents born in 2030 in a LCH model that had a 25 percent chance of each reform (where the tax increase in 2040 was realized).

The ex-post LCH CEV is negative for a few early cohorts who are harmed by increased precautionary saving, and for a few cohorts following the reform who experience the bottom of the swing in the capital stock. The swings in state variables are larger in the uncertain model, and thus the welfare cost is larger. The minimum CEV is -1.98 percent and occurs in 2050 for agents who are born after the tax increase in 2030. The minimum CEV along the other three paths is between -0.32 percent and -1.92 percent. The ex-post and ex-ante welfare comparisons are presented together in appendix A.

My analysis suggests that rational expectations models may understate the welfare cost of Social Security policy uncertainty. The lifecycle horizon learning model I propose can be viewed as a small deviation from rational expectations in which agents maintain model consistent beliefs. Agents in the LCH model are still optimizing and they are still forward looking, yet the welfare cost of policy uncertainty is much larger. The welfare cost is driven mainly by the cyclical changes in capital stock that are introduced into the model by adaptive learning. The most harmed agents in the LCH model would be willing to give up nearly 2 percent of period consumption to avoid living in a world of policy uncertainty. In the RE model, the most harmed agents would only be willing to give up 0.25 percent of period consumption to avoid the policy uncertainty (from an ex-post perspective). The welfare cost in the learning model is nearly an order of magnitude larger than in the rational case.

7 Robustness

7.1 Welfare with one baseline reform

In the main specification of the paper, the welfare effects of policy uncertainty are modeled using a consumption equivalent variation that compares a given realization of an uncertain policy to a given policy as if it were enacted with certainty. In this section, I explore an alternative welfare metric. Instead of comparing each of the four possible uncertain policies to each of the four certain policies, all policies are compared to the same baseline reform. That is, all four uncertain paths are compared to the early tax reform (or all four uncertain
paths are compared to the early benefit reform). This is conceptually similar to the welfare analysis in Kitao (2018). This welfare measurement has the advantage of comparing all realizations of the uncertain process to the same baseline.

I construct a consumption equivalent variation (CEV) that compares the expected lifetime utility of agents along a particular path in the world with uncertainty to the expected lifetime utility of agents when taxes are increased with certainty in 2030. All four possible reforms are compared to the early tax reform (taxes cut in 2030). A positive CEV indicated that the expected lifetime utility is higher for agents in the world with uncertainty than in the world with a certain tax increase in 2030.

These CEVs for rational expectations and lifecycle horizon learning are presented in figure 11. The CEVs for the tax reforms converge to zero, since the steady-state is the same following a tax cut in 2030 or following the same tax cut in 2040. The CEV for the early and late benefit cut are negative leading up to a reform and then large and positive. This is because the benefit cut increases savings and converges to a steady state with higher capital (and thus higher wages, consumption, and lifetime utility) than the tax increase reform. The CEVs are identical in all four paths until a reform is enacted, since the expected lifetime utility is the same along all four paths until the policy uncertainty is resolved. This is depicted in figure 13, which shows the rational expectations CEVs on the same graph.

I also consider the early benefit reform as the baseline for the comparison, in both the RE and LCH models. When the early benefit is used as the baseline comparison, the benefit reforms are still relatively more attractive than the tax reforms in the long-run. The CEVs for the benefit cut converge to zero, while the tax increase CEVs converge to a negative value. The CEVs with the benefit cut as the baseline are presented in figure 12. Figure 14 plots the CEVs from the RE model on the same graph to highlight that the CEVs are identical until a policy is enacted.

The results are qualitatively similar from an ex-post perspective. I construct an ex-post consumption equivalent variation that compared realized utility along a path with uncertainty to a realized utility of a certain baseline reform (either a tax increase in 2030, or a benefit cut in 2030). The results are not displayed graphically, but are summarized in appendix table B.1, along with the ex-ante CEVs. In contrast to the ex-ante CEVs, the ex-post CEVs are not identical in the five periods leading up to an uncertain reform; since the ex-post CEV measured realized utility rather than expected utility.

The analysis of this section is broadly consistent with the baseline specification in sections
Figure 11: Ex-ante consumption equivalent variation measure of the welfare cost of policy uncertainty compared to an early tax increase in the lifecycle horizon learning model (in yellow), and rational expectations model (in blue). The welfare cost is constructed by comparing the expected lifetime utility along a path with policy uncertainty with the expected lifetime utility of a certain early tax increase in either the lifecycle horizon learning or rational expectations model (LCH compared to LCH, RE to RE). The CEV is shown for all four possible paths: taxes are increased in 2030 (top left), benefits are cut in 2030 (top right), taxes are increased in 2040 (bottom left), or benefits are cut in 2040 (bottom right). A positive CEV indicates that an agent would have higher utility facing policy uncertainty than facing a certain tax increase in 2030. The CEVs are larger in magnitude in the LCH model.

6.1 and 6.2. Specifically, the welfare effects of policy uncertainty are magnified in the lifecycle horizon learning model. The most harmed cohort would be willing to give up more of their consumption in the LCH world to avoid policy uncertainty than in the RE world. Similarly, the cohort that benefits from uncertainty the most does so in the LCH world. This is because the learning dynamics amplify the distortions from the policy uncertainty.

The most harmed cohort in the baseline specification is the cohort born in 2050 following an uncertain tax increase in 2030. The most harmed cohort in this section (when each realization is compared to the tax increase in 2030) is the cohort born in 2050 following an uncertain tax increase in 2040. In both cases, agents are harmed because the capital stock is lower than it would be as a result of the policy uncertainty and thus agents' consumption is lower. Using 2030 tax increase as the baseline reform for the CEV makes the 2040 tax increase seem relatively worse for agents born after the reform because capital increases more slowly if the reform does not take place until 2040.
Figure 12: Ex-ante consumption equivalent variation measure of the welfare cost of policy uncertainty compared to an early benefit cut in the lifecycle horizon learning model (in yellow), and rational expectations model (in blue). The welfare cost is constructed by comparing the expected lifetime utility along a path with policy uncertainty with the expected lifetime utility of a certain early benefit cut in either the lifecycle horizon learning or rational expectations model (LCH compared to LCH, RE to RE). The CEV is shown for all four possible paths: taxes are increased in 2030 (top left), benefits are cut in 2030 (top right), taxes are increased in 2040 (bottom left), or benefits are cut in 2040 (bottom right). A positive CEV indicates that an agent would have higher utility facing policy uncertainty than facing a certain tax increase in 2030. The CEVs are larger in magnitude in the LCH model.

Figure 13: Ex-ante consumption equivalent variation that compares expected lifetime utility of consumption along an uncertain path to a certain tax increase in 2030. There are four possible paths in the uncertain process; a tax increase in 2030, a benefit cut in 2030, a tax increase in 2030, and a benefit cut in 2040. All four reforms are equally likely. The tax cut is the same magnitude regardless of when it is enacted; the same is true for the two benefit reforms.
Figure 14: Ex-ante consumption equivalent variation that compares expected lifetime utility of consumption along an uncertain path to a certain benefit cut in 2030. There are four possible paths in the uncertain process; a tax increase in 2030, a benefit cut in 2030, a tax increase in 2030, and a benefit cut in 2040. All four reforms are equally likely. The tax cut is the same magnitude regardless of when it is enacted; the same is true for the two benefit reforms.

7.2 Smaller sooner reforms

As an alternative specification to the uncertainty outlined in section 6, suppose that if the government reforms Social Security in 2030, they enact smaller reforms (they cut benefits by less or raise taxes by less). If the government enacts reform sooner, the reform need not be as large. Less debt will accumulate in the lead up to reform, and thus smaller surpluses will be needed to service the debt. As an illustration, I consider early reforms that are half the magnitude of late reforms: taxes are raised to \( \tau^0 = 0.1378 \) if reform takes place in 2030 compared to \( \tau^0 = 0.1516 \) if reform takes place in 2040, and benefits are cut to \( \phi = 0.366 \) if reform takes place in 2030 compared to \( \phi = 0.332 \) if reform takes place in 2040. There are four possible reforms: tax increase in 2030 or 2040, or benefit cut in 2030 or 2040. Each reform is equally likely in the stochastic process.

The ex-ante consumption equivalent variation welfare metrics for this exercise are presented in Figure 15. This consumption equivalent variation compares the expected lifetime utility of consumption of each cohort in the world with uncertainty who experiences a particular reform (e.g., 2040 benefit cut) to the expected lifetime utility of consumption of a cohort who experiences the same reform (e.g., 2040 benefit cut) in a world with certainty. The CEV shows the fraction of period consumption an agent would be willing to give up to avoid policy uncertainty. The earlier reforms are smaller in magnitude than the later reforms, and thus, unsurprisingly, the CEVs are also smaller in magnitude for the earlier reforms.

\(^{20}\)I show the relationship between the size and timing of reform analytically in a simpler two-period model Cottle Hunt (2019b).
Figure 15: Lifecycle Horizon Learning and rational expectations ex-ante consumption equivalent variations (CEV), with smaller, sooner reforms. Each panel plots the ex-ante CEV from the LCH model (in yellow) on top of the ex-ante CEV from the rational expectations model (in blue). The top left panel shows the fraction of period consumption that an agent would be willing to give up in either the rational expectations world or the lifecycle horizon learning world with announced small tax increase in 2030 to avoid experiencing the early tax increase as the result of the uncertain process where all four reforms are equally likely. The top right shows the CEV comparisons for the small early benefit cut, bottom left shows the large late tax increase, and the bottom right shows the large late benefit cut. A negative CEV indicates that agents would prefer the announced policy.

Qualitatively, the ex-ante welfare effects are similar, but not identical when the 2030 reforms are smaller in magnitude than the 2040 reforms. The largest positive CEV occurs for the late benefit cut and the lowest negative CEV (the greatest welfare cost) occurs for the late tax increase in 3 out of 4 possible ex-ante measures (RE same size of reform; LCH same size of reform; RE sooner, smaller reform; and LCH sooner, smaller reform). From an ex-post welfare perspective, the largest welfare benefit (positive CEV) always occurs prior to the late tax reform, and the largest welfare cost (lowest CEV) occurs for the early benefit cut for two of the four possible measures. The CEVs are presented in appendix Table C.1.

As an additional robustness check, I have also combined the exercises of section B and C, and calculated the consumption equivalent variations that equate expected lifetime utility of consumption of facing policy uncertainty (when the 2030 reforms are smaller in magnitude than the 2040 reforms) to the expected lifetime utility of experiencing a 2030 tax increase with certainty. I have constructed a similar ex-post CEV. All of these robustness results are available upon request.
7.3 Gain parameter selection and sensitivity

The qualitative results of this paper are sensitive to choice of the gain parameter $\gamma$. Below I outline the process I used to select the gain parameter, and also provide details about how my results would differ if a smaller gain parameter were used.

The gain parameter $\gamma = 0.93$ used through the paper was chosen to minimize the welfare cost of forecasting using adaptive expectations rather than rational expectations. Specifically, I compute the consumption of an infinitesimally rational agent who lives in a learning model and compare the lifetime utility of that agent to the utility of the agents using adaptive learning. I then search for the gain parameter $\gamma$ that minimizes this welfare cost.

Suppose a single rational agent is born in the LCH model. The rational agent understands that everyone else is using LCH learning, and she is also able to predict future prices with perfect foresight. She is such a small part of the market that her individual choices do not change prices. This agent is called an infinitesimally rational agent since she is fully rational, but she is an infinitesimally small part of the economy. I compute the CEV that equates the utility of the infinitesimally rational agent with the utility of the LCH agent. I selected a gain parameter to reduce the welfare cost (that is, to maximize the minimum CEV value). With a gain parameter of $\gamma = 0.93$, the minimum CEV for this comparison is -0.17 percent for an announced benefit cut reform (-0.30 percent for the tax increase reform). The CEV is less than 0.2 percent in magnitude for the first six periods (driven mainly by the demographic change), and then quickly falls to zero. The negative sign indicates that LCH agents are worse off than the rational agent living in a LCH world. This CEV is presented below in Figure 16. The relatively small CEV for most periods suggests that agents using adaptive expectations would (likely) not be tempted to try to use some other forecasting rule to plan their future consumption and savings. If the CEV were larger, and more persistent, this would suggest that the learning agents would have an incentive to find some other way to forecast future endogenous variables. For this reason, I choose a gain parameter that minimized the cost of making forecast errors, from the perspective of an agent.

The gain parameter effects the transition paths for the learning economy. As the size of the gain parameter falls, the learning agents make larger forecast errors (since they place more weight on their previous expectations and less weight on the new information they’re observing), which generates larger swings in the endogenous variables, such as the capital stock. The endogenous cycles are larger in amplitude and magnitude when the gain parameter is smaller. This is presented visually in figure 17. This figure plots the path of capital.
Figure 16: Consumption Equivalent Variation (CEV) measure of the welfare cost of using adaptive learning along the transition path of an announced Social Security benefit cut. The CEV compares an infinitesimally rational agent to a lifecycle horizon learner. The infinitesimally rational agent is effected by the general equilibrium effects of learning, but it able to predict future prices. The initial population growth rate is 0.1802 and falls to 0.01 in 1980. Initial government policy is \( \tau_0, \tau_1, \phi \) equal to \((0.124, 0.045, 0.4)\)’. In 2030, the benefit replacement rate falls to \( \phi = 0.332 \). The gain parameter is \( \gamma = 0.93 \).

Figure 17: Paths of capital and bonds for different gain parameters \( \gamma \) following a demographic change in 1980 and a Social Security tax increase in 2030. The path of the economy under rational expectations is the blue line, learning with a gain parameter of \( \gamma = 0.2 \) is the yellow line, \( \gamma = 0.4 \) is green, \( \gamma = 0.6 \) is red, and \( \gamma = 0.9 \) is purple.

An additional way to examine the impact of the gain parameter \( \gamma \) is to conduct the same welfare analysis of section 6.2 using different values for \( \gamma \). The minimum and maximum ex-post CEVs for policy uncertainty under the learning model is presented in table 2 for
<table>
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Table 2: Minimum and Maximum ex-post CEV and the birth year of the most harmed cohort for policy uncertainty for different gain parameters $\gamma$ in the LCH model.

different gain parameters. Note that the gain parameter $\gamma = 0.93$ was chosen to minimize the welfare cost of making forecast errors using adaptive expectations instead of rational expectations *in a world with certain policy*. Thus the gain parameter does not minimize the welfare cost of experiencing policy uncertainty.

### 7.4 Dynamic Efficiency

As a robustness check, I calibrated the model to be dynamically *inefficient* without a Leeper tax and ran similar experiments (raising taxes or lowering Social Security benefits). I parameterize a dynamically inefficient model by setting the discount factor $\beta$ greater than one. This increases agents desire to consume in old age and drives up savings, and thus the capital stock. I use some of the parameterization of Bullard and Russell (1999): $\sigma = 4.2$, annual $\beta = (\frac{1}{1-0.041})$, $\alpha = 0.25$, and annual $\delta = 0.1$. Under this specification, agents in learning models still suffer relative to RE agents when facing demographic changes, policy changes, and policy uncertainty.

### 8 Conclusion

Demographic changes in the United States make future Social Security reform likely, as more beneficiaries are supported by each worker paying taxes. If the program is left un-

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21My model does not compare directly to Bullard and Russell (1999) in two key ways; first I do not have stochastic survival probability, nor do I have endogenous labor choice. Thus, this calibration should be viewed as a robustness exercise only, and not an attempt to replicate Bullard and Russell (1999).
changed, benefits will exceed tax receipts and the Social Security trust fund will be depleted by around the year 2034. The uncertainty regarding the timing and type of Social Security reform influences the saving and consumption choices of cohorts who anticipate a reform in their lifetime. The welfare effects of policy uncertainty are not limited to cohorts alive before a reform; cohorts born during or after a reform can also be impacted by the general equilibrium feedback effects of policy uncertainty. I calculate these welfare effects using both ex-ante and ex-post consumption equivalent variation techniques in two modeling frameworks. First, I calculate the welfare effect in a standard, rational expectations multi-period overlapping generations models. Second, I relax the rational expectations assumption and model agents who forecast future interest rates and wages adaptively. I call this model lifecycle horizon learning. In the learning model, agents make optimal decisions over their lifecycle, given their (potentially imperfect) forecasts of wages, interest rates, and government debt.

Consistent with the existing literature, I find the welfare effects of Social Security policy uncertainty in a rational expectations framework to be in the range of 0.25 percent to 1.49 percent of period consumption. Forward looking, rational agents are able to save and partially self-insure against the aggregate risk of unfavorable policy change. The greatest ex-post welfare cost of policy uncertainty in the rational model is equivalent to less than 0.25 percent reduction in period consumption. By using the lifecycle horizon model, I am able to show that the welfare effects are larger for agents using adaptive learning. I find that the maximum ex-post welfare cost to agents in a model with Social Security policy uncertainty is about an order of magnitude larger in a model with adaptive learning. The maximum ex-post welfare cost of policy uncertainty in the LCH model (compared to the LCH model with announced policy) is 1.98 percent.

Policy makers who use rational expectations models to predict the welfare impacts of Social Security policy uncertainty will understate the effects. To the extent that the learning dynamics are a realistic depiction of agent level behavior, the welfare effects of uncertain Social Security reforms are larger than the standard rational expectations model suggests. If agents are unable to predict the general equilibrium effects of a change in government policy, they will not be able to respond optimally. The results of this paper suggest that policy makers can help agents by announcing policy and also by explaining how the policy will impact wages and interest rates.

The learning model I develop in the paper includes demographic changes and endogenous government debt. Several interesting questions that are beyond the scope of this paper
can be addressed in this framework. I plan to explore the relationship between delaying Social Security reform, growing deficits, and explosive government debt in future work. The learning model developed in this paper provides a framework to examine explosive debt that is not possible in a standard, rational expectations framework. The model could also be used to explore the relationship between recessions, public pensions, and lifecycle savings.
References


A  Ex-ante and Ex-post CEV

To compute the welfare cost of facing Social Security policy uncertainty, I use both an ex-ante and an ex-post consumption equivalent variation technique (see section 6.1 and 6.2). The ex-ante comparison uses the expected lifetime utility of consumption of an agent while the ex-post comparison uses the utility of the realized consumption path. These two measures are the same if expected lifetime utility is the same as realized lifetime utility. The only source of uncertainty in this model is from Social Security policy, thus the ex-ante and ex-post comparisons only differ for the five (or six) cohorts of agents who are alive before an uncertain reform is enacted. Figures A.1 and A.2 plot the ex-ante and ex-post consumption equivalent variations for each of the four possible reforms in the rational expectations model and in the lifecycle horizon model.

Rational Expectations

Figure A.1: Ex-ante and Ex-post Consumption equivalent variations for the rational expectations model and the lifecycle horizon learning model. Each panel plots the ex-ante and ex-post CEV for a particular reform in the RE model. The figures are organized such that the top left is the early tax reform, the top right shows the CEV comparisons for the early benefit cut, bottom left shows the late tax increase, and the bottom right shows the late benefit cut. A negative CEV indicates that agents would prefer the announced policy.

22Five cohorts face uncertainty if the reform takes place in 2030, since the life-span is six periods; the agents born up to five periods before the reform face uncertainty. Six cohorts face uncertainty if the reform takes place in 2040.
Figure A.2: Ex-ante and Ex-post Consumption equivalent variations for the rational expectations model and the lifecycle horizon learning model. Each panel plots the ex-ante and ex-post CEV for a particular reform in the LCH model. The figures are organized such that the top left is the early tax reform, the top right shows the CEV comparisons for the early benefit cut, bottom left shows the late tax increase, and the bottom right shows the late benefit cut. A negative CEV indicates that agents would prefer the announced policy.
## B CEV with one baseline reform

### Minimum CEV (cohort birth year; CEV)

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### Maximum CEV (cohort birth year; CEV)

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<td>0.0026</td>
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Table B.1: Minimum and Maximum CEV comparing uncertainty to an announced tax increase in 2030 (or an announced benefit cut in 2030). Ex-ante CEVs compare expected utility along a path with policy uncertainty to the expected utility along the baseline path (either 2030 tax increase or 2030 benefit cut). The ex-post CFVs compare realized utility along an uncertain path to realized utility of a certain tax increase in 2030 or a certain benefit cut in 2030. The RE CEVs compare rational expectations with uncertainty to rational expectations with certainty. The LCH CEVs compare lifecycle horizon learning. Ex-ante and ex-post CEVs are presented in the table.
## C Smaller sooner reforms

<table>
<thead>
<tr>
<th>Minimum CEV (cohort birth year; CEV)</th>
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<td>1990 -0.0024</td>
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<tr>
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Table C.1: Minimum and Maximum CEV under the baseline experiment (taxes increased to $\tau^1 = 0.1516$ in 2030 or 2040, or benefits cut to $\phi = 0.332$ in 2030 or 2040), and under the alternative experiment where the 2030 reforms are smaller in magnitude (taxes increased to $\tau^1 = 0.1378$ or benefits cut to $\phi = 0.366$) and the 2040 reforms are the same as the baseline. Each CEV compares the realization of a given policy as the result of an uncertain process where all four policies were equally likely with that same policy announced with certainty. The RE CEVs compare rational expectations with uncertainty to rational expectations with certainty. The LCH CEVs compare lifecycle horizon learning. Ex-ante and ex-post CEVs are presented in the table.