Taxation and Regulation in a Market of Sin Goods with Persuasive Advertising

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Abstract

We introduce persuasive advertising in a duopoly market of differentiated harmful goods, where firms compete on prices and advertising. Since advertising artificially inflates consumers’ demand, there is over-consumption of sin goods. We then show how taxation increases aggregate surplus by improving consumers’ welfare and reducing firms’ profits. Adding regulation of advertising increases aggregate surplus further, but by increasing firms’ profits and creating a conflict between consumers not very sensitive to advertising — who experience welfare reductions — and consumers highly sensitive to it — who experience welfare gains.

Keywords: Harmful consumption, Corrective taxation, Regulation of advertising, Taxation under imperfect competition.

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1 Introduction

Product taxation and regulatory interventions by governments are becoming more and more important in those markets where there are public health concerns. This is the case of harmful goods, like tobacco, alcoholic drinks and junk food. Although there are many ways in which a public authority may try to shape or limit the consumption of harmful goods (including, for instance, prevention programs), two types of policy instruments that have been widely used are indirect taxes and regulatory restrictions. The latter can take a variety of forms, depending also on the type of goods under regulation: e.g., restrictions on youth access to the purchase of tobacco or alcoholic drinks, hours limits on the purchase of alcoholic drinks in pubs, and so on.¹

Excise taxes are aimed at increasing the price of the good in such a way to decrease expected consumption. Empirical studies (see, e.g., Clements et al., 1997, and Nelson, 1997) seem to confirm that the price elasticity for alcoholic beverages is significantly large, implying that price increases lead to substantial decreases in consumption, even if there are significant differences between the type of alcoholic beverage and the subgroups of consumers taken into consideration.² The intensity of public intervention through taxation differs across countries and sectors. As reported by Cnossen and Smart (2005) and by the World Health Organization (see, e.g., WHO, 2004), taxes on tobacco were on average equal to 60% of the retail price in the EU member states in 2003, whereas they were 19% for beer, 14% for wine and 39% for spirits. Concerning junk food, in 2003 the WHO advocated their taxation in order to limit obesity and related diseases. In 2013, the Mexican government introduced a tax on sugary drinks of about one peso per liter of soft drinks, and an 8% sales tax on high-calories foods.³ In 2010 Australia introduced a 10% tax on soft drinks, confectionery, biscuits and bakery

¹Taxation and/or regulatory policies aimed at limiting the consumption of harmful goods have been usually motivated by the argument that there are societal health costs connected with the consumption of harmful goods (see, e.g., Cremer et al., 2012). Other authors employ the intertemporal choice setting under hyperbolic-discounting to show that limiting the consumption of sin goods is justified in terms of social welfare because consumers behave in a time inconsistent manner, due to present-biased preferences (O’Donoghue and Rabin, 2003 and 2006, Gruber and Köszegi, 2004 and 2008).

²For example, on the one hand, distilled spirits consumption is more responsive to price variations than wine consumption, which in turn is more sensitive to price variations than beer consumption (Leung and Phelps, 1993). On the other hand, the consumption of alcoholic drinks by young people is less responsive to price variations than the consumption by older people (Chaloupka and Wechsler, 1996).

products.\textsuperscript{4}

With regard to regulatory policies, there is a growing use of health warnings against the consumption of tobacco and alcoholic drinks, restrictions on where one can consume the harmful good, limitations around the permissible levels of tar, nicotine or alcohol, and so on. In this paper, we consider a specific regulatory policy, namely limitations to the permitted advertising of sin goods. Indeed, governments are increasingly restricting the possibility for firms to advertise harmful goods. For example, in the US there is a ban for tobacco companies from sponsoring sporting and entertainment events. Similar limitations can be found, among others, in India, New Zealand and Taiwan. In Thailand, starting from 2013, new regulations require graphic health warnings to cover at least 85\% of cigarettes packages. From 2012, in the UK, supermarkets are forced to hide cigarettes under the counter or behind shutters. Australia introduced packaging rules in 2012, imposing a combination of warning photos and no branding or logos. The rationale for such advertisement restrictions is rooted in the idea that advertising tends to artificially increase the demand for harmful goods (Saffer and Chaloupka, 2000, Saffer and Dave, 2006, Dave and Saffer, 2013). Therefore, if governments aim at reducing tobacco or alcoholic drinks consumption, they can reach the goal also by restricting advertising on these goods (Saffer and Dave, 2002).

In this paper, we develop a theoretical model in which a public authority intervenes in a duopoly market of harmful goods by using an excise tax and regulatory measures on firms’ advertising. The specific novelty of our approach is precisely to add the usage of regulation to taxation as a policy instrument. Our setup allows for strategic interactions between producers along two dimensions. The first is the degree of substitutability/complementarity of sin goods. On the one hand, if goods are close substitutes (like, for instance, two brands of beer) then firms have little market power. On the other hand, if goods are complements (like, for instance, tobacco and alcoholic drinks for some consumers) then firms have strong market power.\textsuperscript{5} The second, and somewhat linked, strategic dimension is advertising. On the one hand, advertising can

\textsuperscript{4}Other examples can be found in Denmark, France and Hungary. Norway has had high duties on sweetened drinks and chocolate from 1981, and Samoa has taxed sugary drinks since 1984. Peru and Ireland are prompt to add levies on junk foods and there is a consistent debate in the US about this kind of taxation (see, for example, The New York Times, \textit{Bad Food? Tax It, and Subsidize Vegetables}, July 23, 2013).

\textsuperscript{5}If sin goods are neither complements nor substitutes we have the special case of two separate monopolies.
be predatory, meaning that advertising by one firm increases its own sales at the expenses of the rival firm and vice versa (clearly, this is more likely to be the case when the advertised goods are close substitutes). On the other hand, advertising by one firm can complement advertising by the other firm (like, for instance, when it promotes the type of lifestyle associated to a particular consumption habit).

Our theoretical framework builds on the literature on persuasive advertising. The latter generally refers to marketing policies that enhance the value of a product at the eye of the consumer; that is, they increase the consumers’ willingness to pay but not his or her welfare. This practice induces overconsumption of harmful goods, which in turn provides a rationale for public intervention.

Our primary goal is to characterize the optimal structure of policy intervention — taxation and restrictions on the advertising of sin goods — assuming that the goal of the public authority is to maximize a social welfare function defined as a weighted sum of consumers’ and producers’ surplus. With a given unit weight assigned to consumers’ surplus, we allow the weight assigned to producers’ surplus to range from one — an efficiency-oriented approach — to zero — an entirely consumer-oriented approach.

By first focusing only on taxation, our analysis shows that, irrespective of the type social welfare function, the optimal policy increases the welfare of all consumers but reduces firms’ profits. All consumers take advantage of taxation but not all for the same reason. Specifically, taxation in itself harms consumers whose consumption is not highly influenced by advertising, for the simple reason that their consumption is not severely distorted upwards and therefore higher prices because of taxation make them worse off. Overall, however, they benefit from taxation because the tax revenues they cash (tax revenues are distributed back uniformly on all consumers) more than

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6 In broad terms, advertising can be divided into two categories: informative advertising and persuasive advertising (Dixit and Norman, 1978; see Bagwell, 2007, for a comprehensive review of the literature). The former refers to situations in which advertising informs consumers about the existence of products or about prices (see, e.g., Grossman and Shapiro, 1984). This paper does not consider this type of advertising.

7 Alternative definitions of ‘persuasive advertising’ refer to different marketing practices. For example, it may refer to the firm’s efforts to convince the consumers that what they really want is a particular variety. That is, advertisement is used to change the ideal product variety. Furthermore, advertisement may be ‘persuasive’ in the sense that it leads consumers to perceive a higher differentiation between products with respect to the real difference between the products. That is, advertisement is used to increase the perceived difference among similar products. For a complete discussion of these issues, see von der Fehr and Stevik (1998).
outweigh the welfare loss due to consumption distortions. On the contrary, consumers
whose consumption is highly influenced by advertising take advantage both because
the tax corrects downward their consumption of sin goods and because they cash tax
revenues. Note that our findings are fully consistent with those of O’Donoghue and
Rabin (2006) but for the fact that the source of the bias in consumers’ preferences lies
in persuasive advertising rather than on hyperbolic discounting.\textsuperscript{8}

More interestingly, when regulation of advertising is brought into the picture, we
find that the optimal combination of the two instruments (taxation and regulation)
increases producers’ surplus as well as the welfare of consumers that are highly sensitive
to advertising, while it reduces that of consumers that are not highly influenced by
advertising. The increase in profits is due to the fact that regulation of advertising calls
for lower taxation, and the latter effect — that increases profits — dominates the former
— that reduces them. Lower taxation, combined with regulation, makes also all types
of consumers better off, but only if the loss in tax revenues is not accounted for. If the
latter is accounted for, we find that consumers that are not very sensitive to advertising
would be better off under a tax-only policy than under a tax-plus-regulation policy. The
intuition is simple. In the absence of regulation, the optimal tax on sin goods — that
is uniform since personalized taxes are unfeasible — is too high (low) for individuals
characterized by low (high) sensitivity to advertising. Since the tax is optimally set with
reference to the ’average’ consumer, individuals showing low sensitivity to advertising
end up under-consuming sin goods, while those that are highly sensitive to advertising
over-consume them. Introducing advertising regulation and reducing taxation has the
effect of reducing both under-consumption and over-consumption, since the impact of
regulation on the two types of consumers is more differentiated than that of taxation.

The rest of the paper is organized as follows. In Section 2 we characterize con-
sumers’ preferences for sin goods in the presence of advertising and derive the demand
functions. In Section 3 we derive the market equilibrium and then examine the impact
of policy instruments on prices, advertising and consumption of sin goods. Government
intervention in the sin goods market is examined in Section 4. Section 5 concludes.

\textsuperscript{8}A further difference with O’Donoghue and Rabin (2006) is that, while they assume that the market
for sin goods is perfectly competitive, we introduce imperfect competition.
2 The model

We consider a population of individuals that consume two types of goods: harmful goods (e.g., tobacco, alcoholic drinks, junk food) and ‘standard’ goods. Sin goods are exchanged in a duopoly market, with firms competing in prices and persuasive advertising (see, e.g., Dixit and Norman, 1978). The markets for standard goods are instead perfectly competitive.

In order to sharply focus on public intervention to correct for market distortions induced by advertising, we abstract from other reasons for public intervention in sin goods markets, such as overconsumption due to present-biased preferences. This motivation for public intervention has been extensively analyzed within the framework of hyperbolic discounting (see, e.g., Kőszegi 2005, O’Donoghue and Rabin, 2006). This means that, in our setup, consumers behave rationally when comparing the hedonic pleasure with the health harms of sin goods consumption (Becker and Murphy, 1988), while they are induced by advertising to over-consume such goods. As we discuss in Section 5, it is straightforward to extend our model to account for both sources of market failure, without affecting the main conclusions.

2.1 Advertising and consumers’ preferences

Let \( x_i \) denote the quantity consumed of the harmful good sold by firm \( i, i = a, b \), and let \( z \) denote the consumption of standard goods. Consumers’ preferences are represented by the utility function

\[
    u(x_a, x_b, z) = \sum_{i \in \{a, b\}} \left( \rho_i - h - \frac{x_i}{2} \right) x_i - \gamma x_a x_b + z. \tag{1}
\]

The variable \( \rho_i > 0 \) represents the marginal utility of sin good \( i \) at the zero consumption level. It is therefore a measure of the intensity of preferences for good \( i \), which we assume to be affected by firms’ advertising (see below). The parameter \( h \geq 0 \) represents the per-unit-of-consumption health harm. Finally, the parameter \( \gamma \) captures the degree of product substitutability/complementarity between goods \( a \) and \( b \); \( \gamma \in (-1, 1) \) is required to ensure strict concavity of the utility function in \((x_a, x_b)\). For \( \gamma > 0 \), the larger is \( \gamma \) the higher is the degree of substitutability in consumption, since an increase in consumption of good \( a \) reduces the marginal utility of good \( b \), and vice versa. For \( \gamma < 0 \), the larger is \( |\gamma| \) in absolute value the higher is the degree of complementarity in consumption, since an increase in consumption of good \( a \) increases the marginal utility.
of good $b$, and vice versa. Marginal utilities are instead independent for $\gamma = 0$. As shown by the last term of Eq. (1), the utility of standard consumption goods is linearly increasing in consumption $z$.

Let the parameter $\rho > 0$ represent the pre-advertising marginal utility (at zero consumption levels) of both goods. The post-advertising marginal utilities, $\rho_i$, $i = a, b$, (again, at zero consumption levels), are assumed to be determined by the following function of advertising levels $\eta_a$ and $\eta_b$ by firms (for a similar modelling approach of advertising, see Friedman, 1983):

$$\rho_i = \rho + \varphi(r)s(\eta_i - k\eta_j), \quad i, j = a, b; \ i \neq j. \tag{2}$$

Eq. (2) shows how persuasive advertising can impact the preferences for sin goods. In particular, $s \geq 0$ is a parameter capturing to which extent a consumer is influenced by advertising, while the term $\varphi(r) \geq 0$ expresses the impact of government regulation on advertising effectiveness in influencing consumers (regulation is described in more details below). Eq. (2) then shows that, for given $\varphi(r)s > 0$, preferences for sin good $i$ become more (less) intense with respect to the pre-advertising level if $\eta_i - k\eta_j$ takes positive (negative) values. Note that $\rho_i$ is — as it is natural in the context of persuasive advertising — an increasing function of advertising $\eta_i$ by firm $i$, while it can be either an increasing or a decreasing function of advertising $\eta_j$ by the other firm, depending on the sign taken by the parameter $k \in [-1, 1]$. A positive value of $k$ implies that advertisement is ‘predatory’, since if firm $i$ increases the advertisement of its product the marginal utility of buying the product of the rival firm decreases. In other words, firm $a$ benefits from investing in advertising not only by making its product more valuable but also by making the product of firm $b$ less valuable to consumers (and similarly for firm $b$). In contrast, a negative value of $k$ implies that advertisement is cooperative, since advertisement by one firm makes the products sold by both firms more valuable to consumers. This is the case in which firms’ advertising focuses more on

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9 In the given formulation of the utility function, positive values of the parameter $\gamma$ represent an inverse measure of the degree of horizontal product differentiation between two types or brands of the same harmful good (e.g., two types or brands of alcoholic drinks). For $\gamma = 0$, product differentiation is maximal; for $\gamma \to 1$, goods become homogenous, since they are perfect substitutes in consumption. When $\gamma < 0$ (complementarity in consumption), the natural interpretation is that goods $a$ and $b$ are of different type; for instance, joint consumption of cigarettes and alcoholic drinks.

10 A positive value of $k$ encompasses also the case of ‘comparative’ advertising (Chakrabarti and Haller, 2011, Anderson and Renault, 2009).
marketing the ‘product’ rather than individual brands, so that each firm’s advertising benefits its product as well as the competing one.\footnote{This occurs, for instance, when advertising promotes primarily the kind of life-styles which are usually associated with the consumption of the harmful good.}

In interpreting Eq. (2), it is also important to realize the possibility of a link between the values taken by the parameters $k$ and $\gamma$. In fact, if the two harmful products are perceived as weak substitutes by consumers, then advertising by firm $j$ should not impact much on consumers’ preferences for the good produced by firm $i$, and vice versa (i.e., if $\gamma$ is close to zero, then also $k$ should be close to zero). The same for complementary sin goods (i.e., if $\gamma < 0$, then $k$ should be close to zero). On the contrary, if products are perceived as highly substitutable in consumption, then advertising by firm $j$ is likely to have a negative impact on consumers’ preferences for good $i$, and vice versa (i.e., if $\gamma$ is close to one, then $k$ should be positive).

Turning finally to government regulation, the variable $r \geq 0$ in Eq. (2) represents a regulatory instrument (e.g., a ban on certain types of advertising contents, or a ban on certain types of media) that the government can use to dampen the impact of advertising on consumers’ preferences. We assume that $\varphi(r)$ is equal to one in the absence of regulation ($r = 0$) and then decreases as $r$ increases; i.e., $\varphi(0) = 1$, $\varphi' < 0$. Stricter regulation $r$, by reducing $\varphi(r)$, implies that the firm has to increase its advertising level $\eta_i$ to obtain the same impact on consumers’ preferences. This formulation implies that regulation is imperfect, consistently with the pertinent empirical literature.\footnote{Imperfect regulation may be due to strategic behavior of firms. See e.g. Federal Trade Commission (2007) and Assunta and Chapman (2014).}

### 2.2 The demand for sin goods

The consumer’s budget constraint is given by

$$z \leq I + \ell - \sum_{i \in \{a,b\}} p_i x_i,$$

where $p_i$ is the price of the good produced by firm $i$, $I$ is consumer’s income (exogenously given), and $\ell$ is a lump sum transfer from the government, equal for all consumers. The price of good $z$ is normalized to unity, since we assume that firms selling good $z$ under perfect competition operate at constant marginal costs, whose level is normalized to unity.
A consumer type is identified by a triple \((\rho, s, h)\) of attributes: the pre-advertising preferences for sinful consumption \(\rho\), the sensitivity to persuasive advertising \(s\), the per-unit-of-consumption health costs \(h\). We assume instead, for analytical simplicity, that the degree \(\gamma\) of product substitutability/complementarity, as well as the sign and degree \(k\) of advertising spillovers, are identical for all consumers and thus single-valued. Individuals’ attributes are distributed in the population, whose size is normalized to unity, according to the cumulative distribution \(F(\rho, s, h)\). A ‘bar’ over a parameter or a variable denotes its average or expected value over the distribution \(F(\cdot)\).

By maximizing Eq. (1), subject to Eqs. (2) and (3), and taking prices and advertising levels as given, we obtain the individual demands for the harmful goods by a type-\((\rho, s, h)\) consumer, \(x_i(p_a, p_b, \eta_a, \eta_b), i = a, b\). Aggregate demands are then equal to \(\bar{x}_i(\cdot) = \mathbb{E}[x_i(\cdot)], i = a, b\) (see Appendix A.1 for the analytical details).

### 3 Advertising and price competition

In this section, we first characterize the market equilibrium and then illustrate how policy instruments impact firms’ advertising and prices, and sin goods consumption.

#### 3.1 Market equilibrium

In the market for harmful goods, firms maximize profits by first competing on advertising and then on prices. We assume that firms take also the degree \(\gamma\) of substitutability/complementarity of their products, and the degree \(k\) of advertising spillovers, as given.\(^{14}\) Firms’ profits are defined as

\[
\pi_i(p_a, p_b, \eta_a, \eta_b) = (p_i - c - t)\bar{x}_i(p_a, p_b, \eta_a, \eta_b) - \frac{\alpha}{2}\eta_i^2, \quad i = a, b, \tag{4}
\]

where \(c \geq 0\) is the marginal (and average) production cost, \(t\) is a per unit tax on sales of harmful goods,\(^{15}\) and \((\alpha/2)\eta_i^2, \alpha > 0\), is the advertising cost function, convex (quadratic) in the advertising \(\eta_i\) by firm \(i\). Note that \(\eta_i\) represents advertising expressed

\(^{13}\)We drop the arguments \((\rho, s, h)\) to simplify notation whenever possible. Also, we assume that \(x_i(.) > 0\) for all types at all equilibrium prices.

\(^{14}\)In a more general setting, firms would engage in marketing strategies aimed at impacting on the above parameters to gain market power.

\(^{15}\)We focus on specific or per unit taxation since harmful goods like tobacco and alcoholic products are taxed in this form in most countries. However, we recognize that in some countries these goods are taxed also in ad valorem terms, and that a well established result in the theory of tax incidence is that the two forms of taxes, while equivalent under perfect competition, are not equivalent under imperfect
in **efficiency units**. Since (i) advertising expenditure, \( E_i = (\alpha/2)\eta_i^2 \), is convex in \( \eta_i \), and (ii) the consumers’ willingness to pay for sin good \( i \) is linearly increasing in \( \eta_i \), there are decreasing returns of advertising (in terms of influencing consumers’ behavior).\(^{16}\)

The market equilibrium is solved by backward induction (see Appendix A.1 for the analytical details). In stage 2, for given \((\eta_a, \eta_b)\), each firm maximizes its profit function \( (4) \) with respect its own price, taking as given the price of the other firm. Denote with \( p_i^{**}(\eta_a, \eta_b), i = a, b \), the (unique) Nash equilibrium at stage 2.\(^{17}\) By substituting the equilibrium prices \( p_i^{**} \) into Eq. \( (4) \), we obtain profits as a function of advertising:

\[
\pi_i^{**}(\eta_a, \eta_b) = (p_i^{**} - c - t)\bar{x}_i(p_i^{**}, p_i^{**}, \eta_a, \eta_b) - \frac{\alpha}{2}\eta_i^2, \quad i = a, b. \tag{5}
\]

In stage 1, each firm maximizes its profit function \( (5) \) with respect to own advertising, taking as given advertising by the other firm. The unique Nash equilibrium with positive advertising by firms is given by\(^{18}\)

\[
\eta^*(t, r) = \frac{2(\bar{p} - \bar{h} - c - t)(2 + k\gamma - \gamma^2)\varphi(r)\bar{s}}{\Psi}, \tag{6}
\]

The condition for \( \eta^*(t, r) > 0 \) is

\[
\bar{p} - \bar{h} - c - t > 0, \tag{7}
\]

while the condition for a unique and stable Nash equilibrium is (see Appendix A.1)

\[
\Psi \equiv \alpha(2 - \gamma)^2(2 + \gamma)(1 + \gamma) - 2(1 - k)(2 + k\gamma - \gamma^2)\left[\varphi(r)\bar{s}\right]^2 > 0. \tag{8}
\]

By substituting \( \eta^* \) from Eq. \( (6) \) into second-stage equilibrium prices \( p_i^{**}(\eta_a, \eta_b) \), we finally obtain the equilibrium price of sin goods,

\[
p^*(t, r) = c + t + \frac{(1 - \gamma)\left[\bar{p} - \bar{h} - c - t + \varphi(r)\bar{s}(1 - k)\eta^*(t, r)\right]}{2 - \gamma}, \tag{9}
\]

which is the same for goods \( a \) and \( b \) by symmetry between firms.

\(^{16}\)The chosen formulation is formally equivalent, but analytically more convenient, than the alternative one of expressing advertising in terms of expenditure \( E_i \). In this case, advertising costs are \( E_i \) in Eq. \( (4) \), while Eq. \( (2) \) should be written as \( \rho_i = \rho + \varphi(r)s[\zeta(E_i) - k\zeta(E_j)] \), with \( \zeta(.) \) a concave function of advertising expenditure.

\(^{17}\)Second-stage equilibrium variables are denoted with double stars.

\(^{18}\)Whenever equilibrium variables are symmetric in the index \( i \), we suppress it to simplify the notation.
The aggregate consumption of each harmful good is equal to (see Appendix A.1)

\[ \bar{x}^* (t, r) = \frac{\hat{p} - \hat{h} + \varphi(r) \bar{s}(1 - k) \eta^* - p^*}{1 + \gamma}. \]  

(10)

After substituting \( \eta^* \) from Eq. (6) and \( \bar{x}^* \) from Eq. (10) into Eq. (5), we get the equilibrium profits of each firm:

\[ \pi^* (t, r) = (p^* - c - t) \bar{x}^* - \frac{\alpha}{2} (\eta^*)^2. \]  

(11)

In Appendix A.1, we show that \( \pi^* (t, r) > 0 \), provided that the profit function \( \pi_{i}^{**} \) defined in Eq. (5) is strictly concave in own advertising \( \eta_i \).

We briefly compare, albeit only informally, the market outcomes under advertising and price competition with those under price competition only, with firms abstaining from advertising their products.\footnote{See Appendix A.3 for equilibrium prices and quantities in a market without advertising.} Advertising, by inflating consumers’ demand, allow firms to increase prices and total sales of sin goods. However, firms’ profits increase only if the increase in revenues outweighs the costs of advertising. It is straightforward to see that firms engage in a prisoners’ dilemma, with advertising competition resulting in a reduction of their profits, provided that advertising is predatory (i.e., values of \( k \) close to one) and/or firms products are close substitutes (i.e., values of \( \gamma \) close to one). Product and advertising complementarity (i.e., negative values of \( \gamma \) and \( k \), respectively) are instead associated to positive profit gains from advertising. Note also that advertising, by increasing the price of sin goods, reduces consumption of individuals that are not (too) influenced by advertising, whereas it increases consumption of those that are highly influenced by it.

### 3.2 Comparative statics on policy instruments

It is important to investigate how taxation and regulation of advertising impact on firms’ advertising, prices and sin goods consumption (all analytical details are in Appendix A.2).

By differentiating Eq. (6), it is easy to see that both higher taxation of harmful consumption and stricter regulation of its advertising reduce the equilibrium level of advertising; i.e.

\[ \frac{\partial \eta^*}{\partial t} = -\frac{\eta^*}{\hat{p} - \hat{h} - c - t} < 0, \]  

(12)

\[ \frac{\partial \eta^*}{\partial r} = \left( \frac{1}{\varphi} \frac{\partial \varphi}{\partial r} - \frac{1}{\Psi} \frac{\partial \Psi}{\partial r} \right) \eta^* < 0, \]  

(13)
where it is $\partial \Psi / \partial r > 0$.

By differentiating Eq. (9), we see that while stricter regulation always reduces the price of sin goods, the effect of higher taxation is ambiguous; i.e.

$$\frac{\partial p^*}{\partial t} = \frac{1}{2 - \gamma} + \frac{1}{2 - \gamma} \varphi(r)(s(1 - k) \frac{\partial \eta^*}{\partial t}), \quad (14)$$

$$\frac{\partial p^*}{\partial r} = \frac{1 - \gamma}{2 - \gamma} s(1 - k) \left( \eta^* \frac{\partial \varphi}{\partial r} + \varphi \frac{\partial \eta^*}{\partial r} \right) < 0. \quad (15)$$

Stricter regulation, by reducing advertising and hence consumers’ demand, reduces the market price of sin goods. This effect is also present when taxation is increased. However, higher taxation, by increasing firms’ marginal costs, tends also to determine an increase in prices. The net effect is ambiguous.

Both higher taxation and stricter regulation reduce aggregate consumption of harmful goods:

$$\frac{\partial x^*}{\partial t} = \frac{1}{1 + \gamma} \left( \varphi \tilde{s}(1 - k) \frac{\partial \eta^*}{\partial t} - \frac{\partial p^*}{\partial t} \right) = -\frac{\alpha(4 - \gamma^2)}{\Psi} < 0, \quad (16)$$

$$\frac{\partial x^*}{\partial r} = \frac{1}{1 + \gamma} \left[ \tilde{s}(1 - k) \left( \eta^* \frac{\partial \varphi}{\partial r} + \varphi \frac{\partial \eta^*}{\partial r} \right) - \frac{\partial p^*}{\partial r} \right] = -\frac{1}{\Psi} \frac{\partial \Psi}{\partial r} \bar{x}^* < 0. \quad (17)$$

As for individual consumption, by substituting the equilibrium prices and advertising levels, defined respectively in Eqs. (9) and (6), into the individual demand functions, defined in Eq. (A.1), we obtain that the equilibrium consumption level of an harmful good by a type-$(\rho, s, h)$ consumer is given by

$$x^*(t, r) = \frac{\rho - h + \varphi(r)s(1 - k)\eta^* - p^*}{1 + \gamma}. \quad (18)$$

By differentiating of Eq. (18), and by using Eqs. (14)-(15), the impact of policy instruments on individual consumption levels can be expressed as

$$\frac{\partial x^*}{\partial t} = -\frac{1}{(1 + \gamma)(2 - \gamma)} + \frac{\varphi(r)(1 - k)}{1 + \gamma} (s - \tilde{s}) \frac{\partial \eta^*}{\partial t} \leq 0 \quad \text{if} \quad s \geq \tilde{s}, \quad (19)$$

$$\frac{\partial x^*}{\partial r} = \frac{1 - k}{1 + \gamma} (s - \tilde{s}) \left( \eta^* \frac{\partial \varphi}{\partial r} + \varphi \frac{\partial \eta^*}{\partial r} \right) \leq 0 \quad \text{if} \quad s \geq \tilde{s}, \quad (20)$$

where

$$s \leq \tilde{s} = \max \left[ \tilde{s} + \frac{1}{(2 - \gamma)\varphi(r)(1 - k)(\partial \eta^*/\partial t)}, 0 \right] < \bar{s}, \quad (21)$$

$$\tilde{s} = \frac{1 - \gamma}{2 - \gamma} \tilde{s} < \bar{s}. \quad (22)$$

The insights that can be gained from Eqs. (19)-(20) are summarized by the following proposition.
Proposition 1  

*Increasing taxation and regulation increases (decreases) the optimal level of consumption of those consumers whose choices are less (more) affected by advertising. The fraction of consumers whose consumption decreases is larger under regulation than under taxation.*

In the case of regulation, Eq. (20) shows that individuals characterized by a value of $s$ above the threshold $\tilde{s}$ reduce consumption in response to stricter regulation, while those with a value below the threshold increase consumption. The explanation is simple. Stricter regulation reduces firms’ advertising and therefore aggregate consumption of harmful goods. However, by reducing their price, it also stimulates individual consumption. For consumers that are not very sensitive to advertising (i.e., those with $s < \tilde{s}$), the latter effect prevails over the former, while the converse holds true for consumers that are highly influenced by advertising. The same channel operates in the case of taxation, which also reduces firms’ advertising, as it is shown by the second term on the right hand side of Eq. (19). However, since taxation tends also to increase the price of harmful goods, it bears a negative impact on the consumption of all individual types, as shown by the first term in Eq. (19). Increased consumption by consumers that are not very sensitive to advertising (those for which $s < \tilde{s}$) is therefore possible also after a tax increase. However, since $\tilde{s} < \tilde{s}$, more individuals reduce their consumption following an increase in regulation than an increase in taxation.

Note finally that increased individual consumption by consumers that are not very sensitive to advertising, as a result of stricter regulation or higher taxation, is possible only when firms have market power. Under perfect competition ($\gamma \rightarrow 1$), all consumers reduce their sin goods consumption when either $r$ or $t$ are increased. The reason is that when firms price at marginal cost regulation does not impact on market price and taxation is always shifted 100% on consumers price.

4 Government intervention

We now examine government policies aimed at limiting sin goods consumption. We first define the social welfare functional representing the objective function of the public authority. To set the benchmark, we derive the optimal tax policy in a market in which firms do not advertise sin goods. Finally, we characterize the optimal tax and regulation policy in the presence of advertising.
4.1 Social welfare

To define social welfare, we appeal to the literature on the normative analysis of persuasive advertising (e.g., Dixit and Norman, 1978), which takes the pre-advertising consumers’ preferences as an unbiased measure of the utility enjoyed by consumers of sin goods. Welfare of a \((\rho, s, h)\)-type consumer is thus defined as

\[
w^*(t, r) = 2 \left( \rho - h - \frac{\pi^*}{2} \right) x^* - \gamma(x^*)^2 - 2\pi^*x^* + \ell^* + I,
\]

where \(\ell^*\) — the per capita lump sum subsidy that the government distributes to consumers — is equal to the average tax revenue, \(2t\bar{x}^*\), minus the administrative costs, \(\xi(r)\), that the government sustains to enforce the regulatory measures on advertising, with \(\xi(.)\) a convex function. In line with most of the literature on optimal taxation, it is instead assumed that taxation is costless to administer. Hence, we have

\[
\ell^*(t, r) = 2t\bar{x}^*(t, r) - \xi(r) \geq 0.
\]

Note that we impose the constraint that net tax revenues must be non-negative, which implies that sin goods cannot be subsidized. The reason for imposing such a constraint, however, is not an ad hoc restriction on the tax instrument. Rather, while positive revenues can be distributed lump sum to consumers (in the form, for instance, of public transfers or public goods uniformly distributed), negative revenues require lump sum taxation of consumers, and lump sum taxes — although fully efficient — are unfeasible because of imperfect information.

Aggregate social welfare can be defined as

\[
\Omega^*(t, r) = W^*(t, r) + \theta \Pi^*(t, r),
\]

where \(W^* = \mathbb{E}_F[w^*]\) and \(\Pi^* = 2\pi^*\), with individual equilibrium profits \(\pi^*\) defined in Eq. (11), are aggregate consumers’ welfare and aggregate firms’ profits, respectively. The parameter \(\theta \in [0, 1]\) represents the relative weight the public authority assigns to producers’ surplus with respect to consumers’ surplus. If \(\theta = 1\), then \(\Omega^*\) is a measure of the aggregate surplus generated by the market, which means that the goal of the public authority is market efficiency. Instead, if \(\theta = 0\), only consumers’ surplus and tax revenues — which are distributed to consumers — count in social welfare.\(^{20}\)

\(^{20}\)The policy maker may want to exclude, or give little weight, to firms’ profits for various reasons. For instance, when firms are foreign multinationals whose profits are not distributed to resident consumers, or when profits are distributed to resident individuals but are highly concentrated in the hands of few wealthy shareholders, or simply for ideological motivations.
4.2 Optimal tax policy in a market without advertising

In order to highlight how advertising calls for public intervention, it is useful to set the benchmark by characterizing the optimal tax policy in a market without advertising. In this situation, a policy maker maximizing $\Omega^*$ would set the tax rate on the sales of sin goods to (see Appendix A.3)

$$t^\text{noadv} = \frac{(1 - 2\theta)(1 - \gamma)(\rho - h - c)}{3 - 2\theta - 2\gamma(1 - \theta)}. \quad (26)$$

If the goal of public policy is efficiency ($\theta = 1$), then it is optimal to subsidize sin goods with a negative tax rate,

$$t^\text{noadv}_{\theta=1} = -(1 - \gamma)(\rho - h - c), \quad (27)$$

in order to correct for firms’ with market power pricing above marginal cost. In fact, the subsidy $t^\text{noadv}_{\theta=1}$ would drive the market price at the marginal production cost $c$, and the aggregate consumption of sin goods — that is below its efficient level because of imperfect competition — would be driven to its efficient level. Since the only market distortion is due to the price-cost mark-up (recall that consumers are rational, so they fully account for the health costs of sin goods consumption), taxation targets this market failure by subsidizing firms. However, since the subsidy would be paid by consumers, the above policy would maximize the aggregate surplus of the economy by reducing consumers’ surplus and by augmenting producers’ surplus. Moreover, it would require lump sum taxation of consumers, a situation we have ruled out above by setting the restriction that net revenues must be non-negative.

It is interesting to study the feasible tax policies in the absence of advertising. From Eq. (26), it is immediate to see that $t^\text{noadv}_{\theta=1} \leq 0$ if $\theta \geq \frac{1}{2}$. The optimal tax is zero if $\theta \geq \frac{1}{2}$, while it is positive if $\theta < \frac{1}{2}$. For instance, if $\theta = 0$, we have that

$$t^\text{noadv}_{\theta=0} = \frac{1 - \gamma}{3 - 2\gamma}(\rho - h - c). \quad (28)$$

Under this policy, although consumers are worse off because taxation reduces their consumption of sin goods (which is already inefficiently low because of price-cost mark-up), overall they end up being better off because they cash tax revenues. Firms are instead worse off, as their profits decrease.

4.3 Optimal tax policy in a market with advertising

Matters become more complicated when turning to the characterization of the optimal tax policy in the presence of persuasive advertising. To start with, we take the level $r$
of regulation of advertising as given (regulation is examined in the next section). By
differentiating the social welfare function (25) with respect to the tax rate, we obtain
(see Appendix A.4)
\[
\frac{1}{2} \frac{\partial \Omega^*}{\partial t} = -\varphi(r)(1 - k)\eta^* E \left[ \frac{\partial x^*}{\partial t} \right] + \\
+ \theta(p^* - c) \frac{\partial x^*}{\partial t} - \theta \alpha \eta^* \frac{\partial \eta^*}{\partial t} + \\
+ (1 - \theta) \frac{1}{2} \frac{\partial \ell^*}{\partial t} - (1 - \theta) \bar{x}^* \frac{\partial p^*}{\partial t}.
\] (29)

This derivative shows the various types of marginal benefits and marginal costs of
a small increase in the tax rate \( t \), for given regulation \( r \). The term in the first row
is positive and represents the marginal welfare gains accruing to consumers because
taxation reduces over-consumption of sin goods whose demand is inflated by firms’
advertising. Note that these marginal benefits depend both on taxation impacting on
the price \( p^* \) and on taxation impacting on advertising \( \eta^* \) (see the expression for \( \partial x^*/\partial t \)
in Eq. 19). The latter effect cancels out by the envelope theorem in the consumers’
maximization problem (see Appendix A.4). Note also that these marginal welfare gains
do not depend on the type of social welfare function, since consumers’ welfare has a
fixed unit weight in Eq. (25). All the other elements appearing in Eq. (29) depend
instead on the type of social welfare function.

The first term in the second row of Eq. (29) is negative and represents a marginal
cost of increasing taxation, namely the fact that higher taxation increases the price-
cost margin \( p^* - c \), which is positive because of imperfect competition. This term calls
for a subsidy component in the determination of the optimal tax, as already discussed
in the previous sub-section. The second term is instead positive and represents the
marginal benefits brought about by taxation by reducing firms’ costs on advertising,
which are a waste of resources for society. Note finally that both the marginal cost
and the marginal benefit in the second row are proportional to \( \theta \), the weight given to
profits in social welfare.

While the three elements in the first and second row of Eq. (29) illustrated above
are linked to efficiency (aggregate surplus) the two terms in the third row are linked to
the distribution of the surplus between consumers and firms. Indeed, both factors have
weight \( 1 - \theta \). The first term represents the difference between the marginal benefits for
consumers and the marginal costs for firms associated to tax collection. Finally, the
second term in the third row of Eq. (29) captures the effects on social welfare of the
impact of taxation on the price of sin goods. If the latter is positive (negative), a net social welfare cost (gain) is observed at the margin.

We next use the derivative shown in Eq. (29) to obtain the expression of the optimal tax rate in the two extreme cases in which firms’ profits have the same weight as consumers’ welfare (θ = 1) and firms’ profits have zero weight (θ = 0).

Setting θ = 1, from the first order condition obtained from letting Eq. (29) equal to zero, we get the following (implicit) expression for the optimal tax rate (see Appendix A.4):

\[
t(r)_{\theta=1} = \varphi(r)\bar{s}(1 - k)\eta^e + (2 - \gamma)\varphi(r)(1 - k)\eta^e \cdot \text{cov} [s, \partial x^*/\partial t] + (2 - \gamma)\alpha\eta^e \cdot \frac{\partial \eta^e}{\partial t} - (1 - \gamma)(\bar{o} - \bar{h} - c).
\]

Eq. (30) shows that the optimal tax is made up of three positive components and a negative one (the last one). The first term of the optimal tax rate is equal to the average bias in consumers marginal utilities for sin goods (at zero consumption levels) that are due to advertising. Note that this term is zero if k = 1, i.e. if advertising by the two competing firms is ‘fully predatory’, so that they neutralize each other. Since the covariance term is negative, also the the second component of the optimal tax is positive. That cov [s, \partial x^*/\partial t] < 0 can be immediately seen from Eq. (19): the more a consumer is influenced by advertising, the more she responds to taxation by reducing her consumption, since taxation reduces advertising. Therefore, the greater the covariance in absolute value, the larger the optimal tax rate, since taxation targets more intensively the consumers that more need to be discouraged from consuming sin goods. The first term in the second row is positive and depends on the marginal cost, \alpha, of advertising by firms. As already illustrated above, taxation is useful for correcting the wasteful costs sustained by firms for advertising. Note, indeed, that this term of the optimal tax is present also if k = 1. That is, even if advertising does not distort preferences, taxation is useful as a corrective device for the ‘waste’ of resources that firms devote to it. The last term, which is negative, is the subsidy component of the optimal tax, already discussed at length above.

Next, by setting θ = 0, we obtain the following expression (again, an implicit equation) for the optimal tax rate (see Appendix A.4):

\[
t(r)_{\theta=0} = \varphi(r)\bar{s}(1 - k)\eta^e + \varphi(r)(1 - k)\eta^e \cdot \text{cov} [s, \partial x^*/\partial t] - 1 - \frac{\partial \eta^e}{\partial t} \cdot \bar{x}^*.
\]
The first term is identical to the first term in the expression for $\theta = 1$, while the second one differs only for the factor $(2 - \gamma)$, which is present in Eq. (30) but not in Eq. (31). The third term is instead different and it is positive. It represents the marginal benefits accruing to consumers when receiving tax revenues, net of the impact of taxation on the prices of sin goods. The term $1 - \frac{\partial p^*}{\partial t}$ is always positive even if $\frac{\partial p^*}{\partial t} > 0$, since with linear demand taxation never causes price overshifting and thus $\frac{\partial p^*}{\partial t} < 1$.

The key insights of the above discussion are summarized by the following proposition.

**Proposition 2** Assume that the level of regulation is exogenously given. Regardless of the specific social welfare function chosen by the policy maker, optimal taxation corrects for the bias in consumers’ demands induced by persuasive advertising. When the policy maker cares for both consumer and producer surplus, the optimal tax accounts also for firms’ advertising costs and for the price-cost mark-up.

### 4.4 Optimal regulation of advertising

We now allow the policy maker to choose the level of advertising regulation. By differentiating the social welfare function with respect to $r$, we obtain (see Appendix A.5)

$$
\frac{1}{2} \frac{\partial \Omega^*}{\partial r} = -\varphi(r)(1 - k)q^*E \left[ s \frac{\partial x^*}{\partial r} \right] - \frac{1}{2} \frac{\partial \xi}{\partial r} + \theta(p^* - c) \frac{\partial \bar{x}^*}{\partial r} - \theta\alpha p^* \frac{\partial \eta^*}{\partial r} + (1 - \theta)t \frac{\partial p^*}{\partial r}. \quad (32)
$$

All terms in Eq. (32), but for the second one in the first row (representing the marginal cost to enforce stricter regulation of advertising), are similar to the terms appearing in Eq. (29), with derivatives with respect to $r$ instead of $t$. Their interpretation is similar and hence is omitted. It is instead interesting to work out in more details Eq. (32) in the two specific instances of the social welfare function considered above for taxation. By setting $\theta = 1$, and letting the tax rate to be optimally determined as
a function of \( r \) according to Eq. (30), the derivative in Eq. (32) can be written as

\[
\frac{1}{2} \frac{\partial \Omega^*}{\partial r} \bigg|_{t=t(r)\theta=0} = \varphi(r)(1-k)\eta^* \left\{ \frac{\text{cov} \left[ s, \frac{\partial x^*}{\partial t} \right]}{\frac{\partial x^*}{\partial r}} - \frac{\text{cov} \left[ s, \frac{\partial x^*}{\partial r} \right]}{\frac{\partial x^*}{\partial r}} \right\} \frac{\partial x^*}{\partial r} + \\
+ \alpha \eta^* \left\{ \frac{\partial \eta^*}{\partial t} - \frac{\partial \eta^*}{\partial r} \right\} \frac{\partial x^*}{\partial r} - \frac{1}{2} \frac{\partial \xi}{\partial r}. \tag{33}
\]

The right hand side of Eq. (33) consists of three terms, the last one of which represents the marginal costs of enforcing stricter regulation. Hence, for regulation of advertising to be desirable, it must be that at least one of the first two terms in Eq. (33) is positive and greater than marginal costs.

We examine these two terms in turn. The first term in Eq. (33) refers to regulation reducing sin goods consumption. Since both covariances are negative (see Eqs. 19 and 20), this term is positive if and only if

\[
\frac{\text{cov} \left[ s, \frac{\partial x^*}{\partial r} \right]}{\frac{\partial x^*}{\partial r}} > \frac{\text{cov} \left[ s, \frac{\partial x^*}{\partial t} \right]}{\frac{\partial x^*}{\partial t}} > 0. \tag{34}
\]

If this is the case, stricter regulation of advertising is more effective than higher taxation in determining diversified reductions in sin goods consumption between individuals with different sensitivities to advertising. Then, even if taxation is already optimally set, introducing regulation of advertising improves social welfare. Indeed, applying a uniform tax on heterogenous agents entails that those having low (high) sensitivity to advertising end up under- (over-) consuming sin goods. Enriching the set of policy instruments by adding regulation allows the policy maker to reduce the tax rate, at the same time discouraging more (less) the agents characterized by high (low) sensitivity to advertising (see inequality 34).

A similar argument applies to the second term of Eq. (33, which refers to the reduction in advertising costs sustained by firms. This term is positive if and only if

\[
\frac{\partial \eta^*}{\partial r} > \frac{\partial \eta^*}{\partial t} > 0, \tag{35}
\]

that is, if and only if regulation is relatively more effective than taxation at curbing advertising.

If \( \theta = 0 \), again letting the tax rate be optimally set as a function of \( r \) according to Eq. (31), the derivative in Eq. (32) can be written as

\[
\frac{1}{2} \frac{\partial \Omega^*}{\partial r} \bigg|_{t=t(r)\theta=0} = \varphi(r)(1-k)\eta^* \left\{ \frac{\text{cov} \left[ s, \frac{\partial x^*}{\partial t} \right]}{\frac{\partial x^*}{\partial r}} - \frac{\text{cov} \left[ s, \frac{\partial x^*}{\partial r} \right]}{\frac{\partial x^*}{\partial r}} \right\} \frac{\partial x^*}{\partial r} + \\
+ \bar{x} \left\{ \frac{\partial \eta^*}{\partial t} - \frac{\partial \eta^*}{\partial r} \right\} \frac{\partial x^*}{\partial r} - \bar{x} \left[ \frac{\partial x^*}{\partial r} \right] \left[ \frac{1}{2} \frac{\partial \xi}{\partial r} \right]. \tag{36}
\]
The first and the last term are identical to the first and the third one in Eq. (33). The second term in Eq. (36) is positive if and only if
\[
\frac{\partial p^*}{\partial t} - \frac{\partial \bar{x}^*}{\partial \bar{x}} < 0.
\] (37)
Since \(\partial p^*/\partial r < 0\), \(\partial \bar{x}^*/\partial r < 0\) and \(\partial \bar{x}^*/\partial t < 0\), a sufficient condition for inequality (37) to be met is that \(\partial p^*/\partial t > 0\). If taxation increases the consumers’ price, then regulation is beneficial, as it pushes in the opposite direction reducing the consumers’ price. Finally, the third term in Eq. (36) is negative, representing the marginal loss in the tax revenues received by consumers when regulation becomes stricter.

The following proposition summarizes our main findings on the optimal regulation of advertising.

**Proposition 3** Regardless of the specific social welfare function chosen by the policy maker, adding regulation to taxation enhances social welfare by impacting relatively more (less) than taxation on the consumption of individuals that are more (less) sensitive to advertising.

### 4.5 Optimal policies: numerical results

The analysis of the derivatives of the social welfare function with respect to the policy instruments highlights the driving forces behind public intervention in the sin goods market. However, it does not allow a clear characterization of the optimal structure of government policy. To this end, we rely on numerical simulations that are reported in Table 1 (see Appendix A.6 for the details).

The table is divided in three parts in order to see whether and how public policy is affected by the type of strategic interactions occurring between firms in the sin goods market. In the first part (rows 0a–5a), the demands for sin goods \(a\) and \(b\) are independent \((\gamma = 0)\) and there are no advertising spillovers \((k = 0)\). This amounts to consider the market for sin goods as consisting of two separated monopolies, not interacting with each other. In the second part of the table (rows 0b–5b), sin goods are instead substitutes in consumption \((\gamma = 0.5)\) and advertising is predatory, since demand spillovers are negative \((k\) positive, equal to 0.25). Finally, in the third part (rows 0c–5c) sin goods are complements in consumption \((\gamma = -0.5)\), as it is advertising by firms, since demand spillovers are positive \((k\) negative, equal to \(-0.25)\). This because we want to see whether and how public policy is affected by the type of strategic interactions occurring between firms in the sin goods market.
Consider the two-monopoly case. Row 0a shows the equilibrium prices ($p$), quantities ($x, x_j$), consumers’ welfare ($W, w_j$) and firms’ profits (II) in markets without advertising. The simulation considers four consumers types, indexed by $j = 1, 2, 3, 4$, depicting increasing sensitivity to advertising (the parameter $s$ in the model). In particular, type 1 does not care about advertising, while type 3 and 4 are two and three times more sensitive to advertising than type 2. Apart from this, consumers are identical. Clearly, in the absence of advertising, all types of individuals consume the same amount of sin goods and enjoy the same level of welfare, the value of which has been normalized to zero to serve as a reference point.

Row 1a shows the equilibrium with advertising ($\eta$) but without government intervention. As expected, with firms’ advertising, the market price increases, as well as aggregate consumption and — in this case — firms’ profits. For types 2, 3 and 4, advertising causes a welfare loss, because it distorts their demands upwards. Also type 1 experiences a welfare loss, but for a different reason: since her demand is not distorted by advertising, it is the price increase due to advertising that makes her worse-off.

The remaining four rows report the optimal policies, first for an efficiency based social welfare function ($\theta = 1$, rows 2a and 3a), then for a consumer-based social welfare function ($\theta = 0$, rows 4a and 5a). In both cases, it is first computed the optimal tax rate with no regulation of advertising, and then the optimal mix of policy instruments.

If the public authority uses only taxation, then the optimal tax makes all types of consumers better off and firms worse off, no matter the type of social welfare function and no matter the type of market structure (for $\theta = 1$, compare row 1 with 2; for $\theta = 1$, compare row 1 with 4; in both cases, for a, b, c). Column $t$ reports the unit tax rate, while column $t\%$ reports the tax $t$ as a percentage of producers’ price ($p - t$). Different consumers benefit from taxation for different reasons. Taxation in itself has a negative impact on consumers that do not respond, or respond weakly, to advertising, since they are not overconsuming sin goods and therefore the price increase caused by taxation makes them worse off. They benefit from taxation only once the tax revenue they cash is accounted for. Consumers that are very sensitive to advertising, instead, benefit for two reasons: first because taxation reduces overconsumption of sin goods; second because they cash tax revenues.

The picture is different if also regulation of advertising is employed. By comparing row 2 with 3 (for $\theta = 1$), or row 4 with 5 (for $\theta = 0$), we see that adding regulation to taxation increases firms’ profits and the welfare of consumers that are highly sensitive to

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advertising, whereas it reduces the welfare of consumers that are not highly influenced by advertising. Profits increase because introducing regulation reduces the optimal tax rate: dramatically, when the goal of the authority is efficiency ($\theta = 1$), not so much if the goal is consumers’ welfare ($\theta = 0$). Actually, the combined tax-regulation policy makes in itself all types of consumers better off. It is only because of the drop in tax revenues that the consumers not very sensitive to advertising end up being worse off.

The above numerical analysis, which is robust to different parameters’ settings, allows us to state (with a slight abuse of terminology) the following result as a proposition.

**Proposition 4** Adding regulation of advertising to taxation increases both aggregate consumers’ surplus and firms’ profits. Nonetheless, at the individual level, regulation determines a conflict of interest between consumers that do not respond to advertising and consumers that do.

This finding suggests that firms may have an interest in ‘welcoming’ the introduction of advertising regulatory mechanisms to complement sin goods taxation, which has interesting implications in a political economy perspective.

## 5 Concluding remarks

In this paper we examine how a public authority can intervene in sin goods markets to correct for the overconsumption induced by firms’ persuasive advertising. We show how regulation of advertising can complement the traditional instrument of taxing sin goods. We find that by introducing regulation of advertising, and accordingly adjusting the tax rate optimally, brings benefits not only to the consumers that are more sensitive to advertising — because they are those that more overconsume sin goods — but also to firms, which see their profits increase — because regulation calls for lower taxation. Consumers that are not influenced by advertising, or who are not very sensitive to it, are instead damaged by regulation — because the benefits of a lower tax on their consumption choices are more than offset by the loss of tax revenues.

Our results are robust to the relaxation of the simplifying assumption of fully rational consumers with respect to the health harms caused by sin goods consumption. If, as in O’Donoghue and Rabin (2006), we consider consumers that have present biased preferences, with $\beta \in [0, 1]$ the hyperbolic discounting parameter expressing the degree
of the bias (with \( \beta = 1 \) there is no bias), then our results carry through with the addition of the term \( h - \bar{h} \) to the formula of the optimal tax rate, representing the so-called (aggregate) marginal ‘internality’; that is, the share of health harms that consumers fail to internalize in their consumption decisions because of biased preferences for sin goods consumption.

Given the wide array of policies that governments employ to cope with unhealthy consumption habits, our work can be extended in several directions. Among these, two are more directly linked to the issue addressed in this paper. The first is to consider alternative, or complementary, policy instruments to impact on the advertising of sin goods. Two means of making more costly for firms to advertise sin goods are partial deductibility of expenditures on advertising to compute profits, or taxation of advertising services. The second extension is to add to the picture ‘advertising’ by the government, in the form of public campaigns to inform the population about the hazards of consuming sin goods. Again, the key research question is to investigate how the various policy instruments interact to fulfill the goals of the public authority.
Table 1: Optimal policies in markets with persuasive advertising.

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Mathematical Appendix

A.1 Derivation of the market equilibrium

By maximizing Eq. (1) subject to Eqs. (2) and (3), we obtain the individual demands for the harmful goods by a type \((\rho, s, h)\) consumer; i.e.

\[
x_i(p_a, p_b, \eta_a, \eta_b) = \frac{\rho - h}{1 + \gamma} + \frac{\varphi(r)s[(1 + k\gamma)\eta_i - (k + \gamma)\eta_j] + \gamma p_j - p_i}{1 - \gamma^2}, \quad i, j = a, b; i \neq j. \tag{A.1}
\]

Aggregate demands are then equal to

\[
x_i(p_a, p_b, \eta_a, \eta_b) = \frac{\rho - h}{1 + \gamma} + \frac{\varphi(r)s[(1 + k\gamma)\eta_i - (k + \gamma)\eta_j] + \gamma p_j - p_i}{1 - \gamma^2}, \quad i, j = a, b; i \neq j. \tag{A.2}
\]

From the first order conditions for maximizing the profit functions \(\pi_i\) defined in Eq. (4) with respect to the own price \(p_i\), we obtain the unique Nash equilibrium

\[
p_i^{**}(\eta_a, \eta_b) = c + t + \frac{(1 - \gamma) [\tilde{\rho} - \tilde{h} - c - t + \varphi(r)s(\eta_i - k\eta_j)]}{2 - \gamma} + \frac{\gamma \varphi(r)s(1 + k)(\eta_i - \eta_j)}{(2 - \gamma)(2 + \gamma)}, \quad i, j = a, b; i \neq j. \tag{A.3}
\]

The second order conditions for a maximum are satisfied, since the profit functions (4) are strictly concave in own prices, \(\partial^2 \pi_i/\partial p_i^2 = -2/(1 - \gamma^2) < 0\). Moreover, the Nash equilibrium is stable, since \(1 - (\partial \tilde{p}_a/\partial p_b) (\partial \tilde{p}_b/\partial p_a) = 1 - \gamma^2/4 > 0\), where \(\tilde{p}_i\) is the best-response function of firm \(i\) to price \(p_j\) set by the other firm.

By substituting the equilibrium prices \(p_i^{**}\) into the profit functions (4), we obtain the profits \(\pi_i^{**}(\eta_a, \eta_b)\) defined in Eq. (5) as a function of advertising by firms. We assume that the profit function \(\pi_i^{**}(\cdot)\) is strictly concave in advertising \(\eta_i\); i.e.

\[
\frac{\partial^2 \pi_i^{**}}{\partial \eta_i^2} = \frac{2(2 + k\gamma - \gamma^2)^2 [\varphi(r)s]^2}{(2 - \gamma)^2(2 + \gamma)^2(1 - \gamma^2)} - \alpha < 0, \quad i = a, b. \tag{A.4}
\]

Since \(2 + k\gamma - \gamma^2 > 0\) for \(\gamma \in (-1, 1), k \in [-1, 1]\), the first term in Eq. (A.4) is positive and reflects the fact that profits, gross of expenditure on advertising, are increasing and convex in \(\eta_i\), as advertising augments consumers’ willingness to pay for sin goods. The second term is negative and depends the degree of convexity of the expenditure on advertising. Hence, for the profit function to be concave, it is necessary that the parameter \(\alpha\) is sufficiently large.

The condition (A.4) for concavity of the profit function \(\pi_i^{**}(\eta_a, \eta_b)\) in \(\eta_i\) can be expressed as follows:

\[
\Delta \equiv \alpha(2 - \gamma)^2(2 + \gamma)^2(1 - \gamma^2) - 2(2 + k\gamma - \gamma^2)^2 [\varphi(r)s]^2 > 0. \tag{A.5}
\]

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From the first order conditions for maximizing the profit functions $\pi^*_i(\eta_a, \eta_b)$ with respect to own advertising, we obtain the best-response $\tilde{\eta}_i$ by firm $i$ as a linear function of advertising $\eta_j$ by firm $j$ (the best response $\tilde{\eta}_j(\eta_i)$ is defined in a similar way); i.e.

$$\tilde{\eta}_i(\eta_j) = \frac{2(2 + \gamma)(1 - \gamma)(\bar{\rho} - \bar{h} - c - t) - (\gamma + 2k - k\gamma^2) \varphi(r) \bar{s} \eta_j}{\Delta}, \quad i, j = a, b; i \neq j,$$  

(A.6)

where $\Delta > 0$ from condition (A.5).

We assume that $\bar{\rho} - \bar{h} - c - t > 0$ (see Eq. 7 in the text), so that the intercept $\tilde{\eta}_i(0)$ of the best response function shown in Eq. (A.6) is positive. Moreover, we assume that its slope is less than unity; i.e.

$$\frac{\partial \tilde{\eta}_i}{\partial \eta_j} = -\frac{2(\gamma + 2k - k\gamma^2)(2 + k\gamma - \gamma^2) [\varphi(r) \bar{s}]^2}{\Delta} < 1, \quad i, j = a, b; i \neq j,$$  

(A.7)

which implies that the condition $\Psi > 0$ defined in Eq. (8) in the text holds true.

$$\Psi = \alpha(2 - \gamma)^2(2 + \gamma)(1 + \gamma) - 2(1 - k)(2 + k\gamma - \gamma^2) [\varphi(r) \bar{s}]^2 > 0.$$  

(A.8)

By linearity of the best response functions shown in Eq. (A.6), under the conditions shown in Eqs. (A.5), (8) and (7), there exists a unique, stable, and symmetric Nash equilibrium, with positive advertising levels, which is defined in Eq. (6) in the text.

In terms of the model parameters, equilibrium prices, quantities and profits are given by

$$p^*(t, r) = c + t + \frac{\alpha(\bar{\rho} - \bar{h} - c - t)(4 - \gamma^2)(1 - \gamma^2)}{\Psi}, \quad (A.9)$$

$$x^*(t, r) = \frac{\alpha(\bar{\rho} - \bar{h} - c - t)(4 - \gamma^2)}{\Psi}, \quad (A.10)$$

$$\pi^*(t, r) = \frac{\alpha(\bar{\rho} - \bar{h} - c - t)^2 \Delta}{\Psi^2}. \quad (A.11)$$

A.2 Taxation and regulation: comparative statics

By differentiating Eq. (6), we get the expressions shown in Eqs. (12) and (13) in the main text; i.e.

$$\frac{\partial \eta^*_r}{\partial t} = -\frac{2(2 + k\gamma - \gamma^2) \varphi(r) \bar{s}}{\Psi} = -\frac{\eta^*_r}{\bar{\rho} - \bar{h} - c - t} < 0,$$

$$\frac{\partial \eta^*_r}{\partial r} = \frac{2(\bar{\rho} - \bar{h} - c - t)(2 + k\gamma - \gamma^2) \bar{s}}{\Psi^2} \left( \Psi \frac{\partial \varphi}{\partial r} - \varphi \frac{\partial \Psi}{\partial r} \right) = \left( \frac{1}{\varphi} \frac{\partial \varphi}{\partial r} - \frac{1}{\Psi} \frac{\partial \Psi}{\partial r} \right) \eta^*_r < 0,$$

where, by differentiating Eq. (8), we have

$$\frac{\partial \Psi}{\partial r} = -4(1 - k)(2 + k\gamma - \gamma^2) \varphi \bar{s} \frac{\partial \varphi}{\partial r} > 0.$$

Footnote: If $\gamma + 2k - k\gamma > 0$ ($< 0$), then $\frac{\partial \tilde{\eta}_i}{\partial \eta_j} > 0$ ($< 0$), and competition in advertising shows strategic complementarity (substitutability).
By differentiating Eq. (A.9), we get
\[
\frac{\partial p^*}{\partial t} = 1 - \alpha(4 - \gamma^2)(1 - \gamma^2),
\]
\[
\frac{\partial p^*}{\partial r} = -\frac{\alpha(\bar{\rho} - \bar{h} - c - t)(4 - \gamma^2)(1 - \gamma^2)}{\Psi^2} \frac{\partial \Psi}{\partial r} = -(p^* - c - t) \frac{1}{\Psi} \frac{\partial \Psi}{\partial r} < 0.
\]

Finally, by differentiating Eq. (A.10), we obtain
\[
\frac{\partial x^*}{\partial t} = -\frac{\alpha(4 - \gamma^2)}{\Psi} < 0,
\]
\[
\frac{\partial x^*}{\partial r} = -\frac{\alpha(\bar{\rho} - \bar{h} - c - t)(4 - \gamma^2)}{\Psi^2} \frac{\partial \Psi}{\partial r} = -\frac{1}{\Psi} \frac{\partial \Psi}{\partial r} x^* < 0.
\]

### A.3 Optimal tax in a market without advertising

In the absence of advertising, firms compete only in prices. Equilibrium prices and quantities are given by
\[
p^{\text{noadv}} = c + t + \frac{(1 - \gamma)(\bar{\rho} - \bar{h} - c - t)}{2 - \gamma}, \quad (A.12)
\]
\[
x^{\text{noadv}} = \frac{\bar{\rho} - \bar{h} - c - t}{(2 - \gamma)(1 + \gamma)}. \quad (A.13)
\]

By substituting Eqs. (A.12) and (A.13) into Eq. (11), (23) and (24), and then maximizing the social welfare function defined in Eq. (25), one gets the optimal tax rate shown in Eq. (26).

### A.4 Optimal tax in a market with advertising

By adding and subtracting \(2\varphi(r)s(1 - k)\eta^*x^*\) to an individual consumer welfare (see Eq. 23), the latter can be written as
\[
w^*(t, r) = u^*(t, r) - 2\varphi(r)s(1 - k)\eta^*x^*, \quad (A.14)
\]

where
\[
u^*(t, r) = 2 \left( p + \varphi(r)s(1 - k)\eta^*x^* - h - \frac{x^*}{2} \right) x^* - \gamma(x^*)^2 - 2p^*x^* + \ell^* + I \quad (A.15)
\]
is the indirect utility function.

By differentiating Eq. (A.14) with respect to \(t\) (and applying the envelope theorem when differentiating \(u^*\), we get
\[
\frac{\partial w^*}{\partial t} = -2x^* \frac{\partial p^*}{\partial t} + \frac{\partial \ell^*}{\partial t} - 2\varphi(r)s(1 - k)\eta^* \frac{\partial x^*}{\partial t} \quad (A.16)
\]
and, aggregating over consumers,
\[
\frac{\partial W^*}{\partial t} = E \left[ \frac{\partial w^*}{\partial t} \right] = -2x^* \frac{\partial p^*}{\partial t} + \frac{\partial \ell^*}{\partial t} - 2\varphi(r)(1 - k)\eta^*E \left[ s \frac{\partial x^*}{\partial t} \right]. \quad (A.17)
\]
As for aggregate profits, by differentiating Eq. (11) with respect to $t$, we have
\[
\frac{\partial \Pi^*}{\partial t} = 2 \frac{\partial x^*}{\partial t} = 2(p^* - c) \frac{\partial x^*}{\partial t} + 2x^* \frac{\partial p^*}{\partial t} - \frac{\partial \ell^*}{\partial t} - 2\alpha \eta^* \frac{\partial \eta^*}{\partial t}. \tag{A.18}
\]

By substituting for Eqs. (A.17) and (A.18) into the derivative
\[
\frac{\partial \Omega^*}{\partial t} = \frac{\partial W^*}{\partial t} + \theta \frac{\partial \Pi^*}{\partial t},
\]
we get the derivative shown in Eq. (29).

Letting $\theta = 1$, from Eq. (29) the first order necessary condition for maximizing social welfare can be written as
\[
\frac{1}{2} \frac{\partial \Omega^*}{\partial t} \bigg|_{\theta=1} = -\varphi(r)(1-k)\eta^* E \left[ s \frac{\partial x^*}{\partial t} \right] + (p^* - c) \frac{\partial x^*}{\partial t} - \alpha \eta^* \frac{\partial \eta^*}{\partial t} = 0. \tag{A.19}
\]

By substituting for
\[
p^* - c = \frac{t}{2 - \gamma} + \frac{1 - \gamma}{2 - \gamma} [\bar{p} - \bar{h} - c + \varphi(r)\bar{s}(1-k)\eta^*]
\]
into Eq. (A.19) and rearranging, we obtain
\[
\frac{t}{2 - \gamma} \frac{\partial x^*}{\partial t} = \varphi(r)(1-k)\eta^* E \left[ s \frac{\partial x^*}{\partial t} \right] - \frac{1 - \gamma}{2 - \gamma} [\bar{p} - \bar{h} - c + \varphi(r)\bar{s}(1-k)\eta^*] \frac{\partial x^*}{\partial t} + \alpha \eta^* \frac{\partial \eta^*}{\partial t}
\]
that can finally be written as
\[
t = (2-\gamma)\varphi(r)(1-k)\eta^* \frac{E \left[ s \frac{\partial x^*}{\partial t} \right]}{\partial x^* / \partial t} - (1-\gamma) \bar{x} \left[ \frac{t}{2 - \gamma} + \frac{1 - \gamma}{2 - \gamma} \frac{\partial \ell^*}{\partial t} - \frac{\partial \ell^*}{\partial t} \right] + (2-\gamma)\alpha \eta^* \frac{\partial \eta^* / \partial t}{\partial x^* / \partial t}.
\]

Since
\[
E \left[ s \frac{\partial x^*}{\partial t} \right] = \bar{s} \frac{\partial x^*}{\partial t} + \text{cov} \left[ s, \frac{\partial x^*}{\partial t} \right], \tag{A.20}
\]
the expression for the optimal tax rate can be written as in Eq. (30).

Letting $\theta = 0$, from Eq. (29) the first order necessary condition for maximizing social welfare can be written as
\[
\frac{1}{2} \frac{\partial \Omega^*}{\partial t} \bigg|_{\theta=0} = -\varphi(r)(1-k)\eta^* E \left[ s \frac{\partial x^*}{\partial t} \right] + \frac{1}{2} \frac{\partial \ell^*}{\partial t} + \frac{1}{2} \frac{\partial \ell^*}{\partial t} = 0. \tag{A.21}
\]

Substituting for
\[
\frac{\partial \ell^*}{\partial t} = 2\bar{x}^* + 2 \frac{\partial \bar{x}^*}{\partial t},
\]
Eq. (A.21) can be written as
\[
-\varphi(r)(1-k)\eta^* E \left[ s \frac{\partial x^*}{\partial t} \right] + \bar{x}^* \left( 1 - \frac{\partial p^*}{\partial t} \right) + t \frac{\partial x^*}{\partial t} = 0,
\]
and this latter equation, using Eq. (A.20), can be finally rewritten as in Eq. (31).
A.5 Optimal regulation policy

By differentiating Eq. (A.14) with respect to $r$ (and applying the envelope theorem when differentiating $u^*$), we get

$$\frac{\partial w^*}{\partial r} = -2x^* \frac{\partial p^*}{\partial r} + \frac{\partial \ell^*}{\partial r} - 2\varphi(r)s(1-k)\eta^* \frac{\partial x^*}{\partial r}. \hspace{1cm} (A.22)$$

Substituting for

$$\frac{\partial \ell^*}{\partial r} = 2t \frac{\partial x^*}{\partial r} - \frac{\partial \xi}{\partial r},$$

and aggregating over consumers, we obtain

$$\frac{\partial W^*}{\partial r} = E \left[ \frac{\partial w^*}{\partial r} \right] = -2x^* \frac{\partial p^*}{\partial r} + 2t \frac{\partial x^*}{\partial r} - \frac{\partial \xi}{\partial r} - 2\varphi(r)(1-k)\eta^* E \left[ s \frac{\partial x^*}{\partial r} \right]. \hspace{1cm} (A.23)$$

By differentiating aggregate profits with respect to $r$, we get

$$\frac{\partial \Pi^*}{\partial r} = 2 \frac{\partial x^*}{\partial r} = 2(p^* - c) \frac{\partial x^*}{\partial r} + 2x^* \frac{\partial p^*}{\partial r} - 2t \frac{\partial x^*}{\partial r} - 2\alpha \eta^* \frac{\partial \eta^*}{\partial r}, \hspace{1cm} (A.24)$$

and, by substituting for Eqs. (A.23) and (A.24) into the derivative $\partial \Omega^*/\partial r$, we get the derivative shown in Eq. (32).

Letting $\theta = 1$, the derivative in Eq. (32) becomes

$$\frac{1}{2} \left. \frac{\partial \Omega^*}{\partial r} \right|_{\theta = 1} = -\varphi(r)(1-k)\eta^* E \left[ s \frac{\partial x^*}{\partial r} \right] - \frac{1}{2} \frac{\partial \xi}{\partial r} + (p^* - c) \frac{\partial x^*}{\partial r} - \alpha \eta^* \frac{\partial \eta^*}{\partial r}.$$  

Substituting for $E[s(\partial x^*/\partial r)] = s(\partial x^*/\partial r) + \text{cov}[s, (\partial x^*/\partial r)]$, the above derivative can be written as

$$\frac{1}{2} \left. \frac{\partial \Omega^*}{\partial r} \right|_{\theta = 1} = \{p^* - c - \varphi(r)s(1-k)\eta^*\} \frac{\partial x^*}{\partial r} - \varphi(r)(1-k)\eta^* \text{cov} \left[ s, \frac{\partial x^*}{\partial r} \right] - \frac{1}{2} \frac{\partial \xi}{\partial r} - \alpha \eta^* \frac{\partial \eta^*}{\partial r}. \hspace{1cm} (A.25)$$

The first order condition for $t$ in Eq. (A.19) can be manipulated in a similar way to obtain

$$\frac{1}{2} \left. \frac{\partial \Omega^*}{\partial t} \right|_{\theta = 1} = \{p^* - c - \varphi(r)s(1-k)\eta^*\} \frac{\partial x^*}{\partial t} - \varphi(r)(1-k)\eta^* \text{cov} \left[ s, \frac{\partial x^*}{\partial t} \right] - \alpha \eta^* \frac{\partial \eta^*}{\partial t} = 0. \hspace{1cm} (A.26)$$

Finally, by solving for $\{p^* - c - \varphi(r)s(1-k)\eta^*\}$ from Eq. (A.26), substituting into Eq. (A.25) and rearranging, we get the derivative shown in Eq. (33).

The analytical steps are similar for $\theta = 0$. The derivative in Eq. (32) is

$$\frac{1}{2} \left. \frac{\partial \Omega^*}{\partial r} \right|_{\theta = 0} = -\varphi(r)(1-k)\eta^* E \left[ s \frac{\partial x^*}{\partial r} \right] - \frac{1}{2} \frac{\partial \xi}{\partial r} + t \frac{\partial x^*}{\partial r} - \bar{x} \frac{\partial p^*}{\partial r},$$

which can be written as

$$\frac{1}{2} \left. \frac{\partial \Omega^*}{\partial r} \right|_{\theta = 0} = \{t - \varphi(r)s(1-k)\eta^*\} \frac{\partial x^*}{\partial r} - \varphi(r)(1-k)\eta^* \text{cov} \left[ s, \frac{\partial x^*}{\partial r} \right] - \frac{1}{2} \frac{\partial \xi}{\partial r} - \bar{x} \frac{\partial p^*}{\partial r}. \hspace{1cm} (A.27)$$

The first order condition for $t$ in Eq. (A.19) can be expressed as

$$\frac{1}{2} \left. \frac{\partial \Omega^*}{\partial t} \right|_{\theta = 0} = \{t - \varphi(r)s(1-k)\eta^*\} \frac{\partial x^*}{\partial t} - \varphi(r)(1-k)\eta^* \text{cov} \left[ s, \frac{\partial x^*}{\partial t} \right] - \bar{x} \frac{\partial p^*}{\partial t} + \bar{x}^* = 0. \hspace{1cm} (A.28)$$

By solving for $\{t - \varphi(r)s(1-k)\eta^*\}$ from Eq. (A.28), substituting into Eq. (A.27) and rearranging, we obtain the derivative shown in Eq. (36).
A.6 Numerical simulation

In order to allow for meaningful comparisons of optimal policies under different values of the parameters \((\gamma, k)\), we calibrate the model so that initial market conditions are the same under the three configurations of \((\gamma, k)\) considered in Table 1 in the main text: \((0, 0)\), \((.5, .25)\), \((- .5, -.25)\). To do so, in the numerical simulation we consider the following generalization of the consumer’s utility function:

\[
u(x_a, x_b, z) = \sum_{i \in \{u,b\}} \left( \rho_i - h - \frac{x_i}{2\lambda} \right) x_i - \frac{\gamma}{\lambda} x_a x_b + z, \tag{A.29}\]

which includes — with respect to the utility function (1) used in the main text — an extra parameter, \(\lambda > 0\), affecting the slope of the marginal utility of sin goods consumption. \(\lambda\) enters also in the advertising cost function of firms, that we now write as \(\lambda \eta^2\). The role of \(\lambda\) is that of ‘fixing’ initial market conditions when firms do not advertise sin goods. In particular, see Table 1, aggregate sin goods consumption \(x\) in the absence of advertising and no policy intervention is equal to 6.67 for all values of \((\gamma, k)\) considered. To make aggregate sin goods consumption in the presence of advertising and no policy intervention independent of \((\gamma, k)\) we instead adjust the consumers’ attribute \(s\). We consider four types of consumers, labelled \(i = 1, 2, 3, 4\), and we set \(\nu_1 = 0, \nu_2 = 5, \nu_3 = 10, \nu_4 = 15\), where the average is \(\bar{\nu} = 7.5\). For given \((\gamma, k)\), we then set \(s_i = \nu_i \bar{s}/\bar{\nu}\) with \(\bar{s}\) set such that aggregate sin goods consumption in the presence of advertising and no policy intervention is equal to 10.00.

The other parameters are set as follows: \(\alpha = 1, c = 1, n_i = .25, \rho_i = 8, h_i = 6, i = 1, 2, 3, 4\). As for functional forms, we specify \(\varphi(r) = 1/(1 + 0.1r), \xi(r) = 0.025r^2\). Computations of optimal policies are done with Maple (code available from the authors).