Motivated Prospects of Upward Mobility∗

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Abstract

The prospect of upward mobility (POUM) hypothesis conjectures that the reason why the poor do not expropriate the rich and sometimes seem to vote against their self-interest is that they expect upward mobility and fear that high redistribution may negatively affect them in the future. This work formalizes the POUM hypothesis by explicitly modeling the voters’ beliefs about their prospective incomes. When belief formation and voting are strategic, the poor will form overly optimistic beliefs and vote for low taxes if the value of anticipation is high enough and if their optimism does not cause too drastic a change in tax policy. If belief formation and voting are non-strategic, the poor will always indulge in optimism and may even vote against their best interest. Furthermore, a striking result is that if the incomes of the rich increase as the transfers to the poor stagnate, the poor may demand less redistribution.

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1 Introduction

Why don’t the poor, as a majority, vote for extensive redistribution if it is in their economic interest to do so? Why does it sometimes seem that the poor vote against their own self-interest? The prospect of upward mobility (POUM) hypothesis conjectures that the reason why the poor do not expropriate the rich and sometimes seem to vote against their self-interest is that they expect to move upward on the income ladder and fear that the higher redistribution may negatively affect them in the future. This paper formalizes the POUM hypothesis by explicitly modeling the voters’ subjective beliefs about their prospective incomes and examines why and when these beliefs support low redistribution.

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The POUM hypothesis was introduced to the public choice literature by Bénabou and Ok (2001). They show that under favorable income dynamics, it is possible that more than half of the voters rationally expect above average future income. As a result, more than half of the voters prefer low redistribution and vote accordingly. While, according to the empirical evidence, both perceived upward mobility (Ravallion & Lokshin, 2000; Rainer & Siedler, 2008; Cojocaru, 2014) and actual upward mobility (Alesina & La Ferrara, 2005; Alesina & Giuliano, 2011; Checchi & Filippin, 2004; Bénabou & Ok, 2001) seem to decrease voters’ demand for redistribution, it also seems that perceived mobility and actual mobility do not necessarily correlate (Fischer, 2009; Alesina, Glaeser, & Sacerdote, 2001; Gottschalk & Spolaore, 2002; Kaufman, 2009). The puzzle then, and what the model in Bénabou and Ok (2001) fails to explain, is why prospects of upward mobility decrease the demand for redistribution even in the absence of actual upward mobility. For instance, in the US, the perceived upward mobility is higher than in Europe, producing a higher POUM effect while there does not seem to be much difference in actual upward mobility across the Atlantic (Alesina et al., 2001; Gottschalk & Spolaore, 2002; Checchi, Ichino, & Rustichini, 1999; Lefranc & Trannoy, 2005; Björklund & Jäntti, 1997; Couch & Dunn, 1997). In addition, as noted by Alesina and Giuliano (2011) and Minozzi (2013), the assumptions underlying the model of Bénabou and Ok (2001) are restrictive and empirically implausible. Therefore, Alesina and Giuliano (2011) suggest that a more plausible mechanism for the POUM effect could be overoptimism.

A vast literature in experimental psychology on overconfidence supports Alesina and Giuliano’s (2011) suggestion. Weinstein (1980) records the unrealistic optimism people have about their future life events: students tend to overestimate the likelihood of positive events such as getting a good job offer, earning a high salary and making good housing investments. In contrast, they underestimate the likelihood of health problems, unemployment and other negative events. Seaward and Kemp (2000) report how students overestimate their future incomes and job prospects and underestimate the time they need to pay off their student loans. This better-than-average-effect (Alicke & Govorun, 2005) is also not restricted to students and life events as demonstrated by Cross (1977) who find that 94% university faculty members assess their teaching ability above average. The inflated self-assessments are also linked to well-being (Taylor & Brown, 1988) and it is suggested that it is indeed this desirability of certain beliefs that lead to biases (Kunda, 1990).

This work models this overoptimism in the context of voting for redistribution. In the model, agents collectively choose a linear tax rate under uncertainty over their future incomes. After voting, and before the realization and redistribution of their incomes, they anticipate their future consumption. In addition to the utility from consumption, agents derive utility from the anticipation of their future consumption. This anticipation incen-
tives them to form overly optimistic beliefs. Given the technology for belief distortion, the motivated prospects of upward mobility emerge endogenously as a result of agents’ trade-off between anticipation and consumption. When belief formation and voting are strategic, the poor will form overly optimistic beliefs and vote for low taxes if the value of anticipation is high enough and if their optimism does not cause too drastic a change in tax policy. If belief formation and voting are non-strategic, the poor will always indulge in optimism and may even vote against their best interest. Furthermore, a striking result is that if the incomes of the rich increase as the transfers to the poor stagnate, the poor may demand less redistribution.

The rest of the work is organized as follows. In section 2, we briefly position the current work into the existing literature in political economy and psychological economics. Section 3 presents the model and derives the conditions for the POUM effect. Section 4 extends the analysis of the model by studying the effects of changes in the underlying income distribution, presents welfare analysis and considers the case of nonstrategic belief formation and voting. Section 5 concludes. All proofs of the lemmas and propositions are collected in the appendix.

2 Relations to the literature

2.1 Political Economy and Redistribution

If the rational choice model with narrowly defined utility together with the Median Voter Theorem cannot be corroborated by empirical observations, at least one of these underlying assumptions, rational choice or median voter’s power, must be the culprit. It might be the case that modeling voters as income maximizers does not capture all the relevant aspects of their decision-making or that the outcome that the electoral system provides does not reflect the preferences of the median voter.

In this work, the policy outcome is assumed to be the median voter’s bliss point and the focus is on the former of these possible caveats.\(^1\) Hence, this work can be positioned into the strand of literature initiated by Romer (1975) and Meltzer and Richard (1981), which aims to explain the extent of redistribution in democratic economies by studying what determines the voters’ demand for redistributive policies. The question asked is, how does the median voter decide on her vote?\(^2\)

\(^1\)Reasons for the latter could be, for instance, unequal political participation (Bénabou, 2000; Mahler, 2008), the political influence of the rich (Gilens, 2005), campaign contributions (Karakaroubounis, 2011), economic inequality (Lupu & Pontusson, 2011; Solt, 2008), electoral systems (Iversen & Soskice, 2006; Cukierman & Spiegel, 2003; Austen-Smith, 2000), and interest groups (Dixit & Londregan, 1998).

\(^2\)The obvious starting point is the voter’s current income, but preferences so narrowly defined have been unsatisfactory in explaining real-world tax policies (Bénabou, 1996; Borck, 2007; Luebker, 2014). Other factors explaining the demand for redistribution proposed in the literature are, for instance, efficiency costs of taxation (Meltzer & Richard, 1981), different individual (Piketty, 1995) and cultural (Corneo &
One line of literature focuses on mobility. First, income mobility, broadly speaking, refers to both upward and downward mobility. The premise is that instead of or in addition to current income, the policy preferences depend on future income. When voters are worried that their incomes might decrease relative to others, they could use redistribution as an insurance against downward mobility. This would increase the demand for redistribution. The POUM, on the other hand, focuses on the possibility of upward mobility, having the opposite effect: When voters expect their incomes to increase relative to others, they vote for less redistribution.

However, income mobility is also often connected to the roles of chance, circumstances, and effort in determining income. If voters perceive that the effort one exerts determines one’s prospects, they believe in a mobile society, but if they believe that the circumstances have a major role in determining one’s prospects, they believe in immobile society. Piketty (1995) studies how the interaction of income mobility and beliefs about determinants of income affects voting. In the present work, incomes are exogenous and, in the spirit of the POUM hypothesis, beliefs about social mobility refer solely to beliefs about the levels of future incomes.

The first characterization of the POUM effect is perhaps Hirschman’s (1973) tunnel effect which conjectures that people’s demand for redistribution decreases when they see the incomes of relatable people in their environment increase. This increases the expectations of their own incomes and they, therefore, tolerate more inequality. Hirschman and Rothschild (1973) are, however, agnostic to how these income expectations are formed.

The first formalization of the POUM effect was provided by Bénabou and Ok (2001). Their approach is to maintain rational expectations and show that favorable income dynamics can allow more than half of the voters to expect above-average incomes. The agents vote for a redistribution policy, which will be in place for a predetermined time, and expect their incomes to evolve according to a stochastic transition function. The deterministic part of this transition function is concave, which allows the majority of voters to believe that they will receive an above average income in the future. The stochastic part consists of skewed income shocks, which ensure that the original income distribution is preserved. This combination of skewed shocks and concave prospects lets the expected incomes and realized incomes diverge and makes the POUM effect possible with rational

Grüner, 2002; Alesina & Glaeser, 2004) histories and experiences, social preferences, such as altruism, inequality aversion and fairness considerations (Alesina & Angeletos, 2005; Alesina, Cozzi, & Mantovan, 2012; Alesina & Glaeser, 2004; Fong, 2001), status concerns (Gallice & Grillo, forthcoming), structure and organization of the family (Todd, 1985; Esping-Andersen, 1999; Alesina & Giuliano, 2010), and social mobility (Piketty, 1995; Hirschman & Rothschild, 1973; Bénabou & Ok, 2001). (See Alesina and Giuliano (2011) for a review on preferences for redistribution.) In addition to widening the scope of preferences, the literature has also studied the role of beliefs (Piketty, 1995; Alesina & Angeletos, 2005) and biased beliefs (Minozzi, 2013; Bénabou and Tirole, 2006; Bénabou, 2008). Given this rich set of explanations for the extent of redistribution, a parsimonious model seems unlikely, and a single factor should be interpreted as a part of the story, complementing and rivaling the other explanations.
expectations. However, as discussed in the Introduction, this approach is not without problems.

Minozzi (2013) proposes an explanation for the POUM effect by abandoning rational expectations and letting voters form overly optimistic prospects about their future income. His model relies on a game theoretic multi-self approach, where each citizen has, without their knowledge, an “agent” who controls their beliefs and optimizes the trade-off between optimistic beliefs and material payoffs. Citizens receive anticipatory flow utility in period 1, and a flow utility called outcome utility in period 2, when they receive their stochastic and exogenous incomes. The agent’s objective function for belief formation consists of these two sources of utility. In choosing the optimal beliefs by solving the trade-off between anticipatory and outcome utility, the agent knows the prior prospects of the citizen and how the tax policy is dependent on the chosen beliefs. If the poor citizens value anticipation enough, they will end up with optimistic beliefs and vote for low redistribution.

The departure of the current work from Minozzi’s (2013) model is most notably in the technology that agents use to distort their beliefs. The cognitive technology for belief distortion is adopted from Bénabou and Tirole (2002, 2006) and generalized such that we are able to analyze a whole continuum of cognitive technologies varying in the constraints they impose on belief distortion allowing full and partial Bayesian rationality but also nesting the non-Bayesian technology used by Minozzi (2013). The conditions for the POUM effect are derived for each of these cognitive technologies, and it is shown that for a set of cognitive technologies the poor prefer optimism and low taxes over realism and high taxes. Also, it is demonstrated how the results of Minozzi’s (2013) model are not robust to fully Bayesian updating of beliefs. Furthermore, in addition to strategic belief formation and voting, the only case considered by Minozzi (2013), here the case of sincere belief formation and voting is studied as well.

The POUM effect also emerges in the model of Bénabou and Tirole (2006). In their model, agents have overly optimistic beliefs about their productive ability and, hence, future income. When they believe themselves to be abler than others, they prefer less redistribution. Although their model, as the present work, derives the POUM effect by letting agents optimize over their beliefs, their work differs from the current one in its mechanism for the belief distortion. Specifically, what incentivizes the agents to hold biased beliefs differs. In their work, agents suffer from deficient willpower and form overly optimistic beliefs about their abilities in order to motivate themselves, and in this way to compensate for the imperfect willpower. That is, belief distortion works as a commitment device. In the current work, on the other hand, the beliefs are distorted because beliefs can be consumed and overly optimistic beliefs bring high anticipatory utility. However, these different incentives are not mutually exclusive, and probably both are at work. The
explanation for the POUM effect in Bénabou and Tirole (2006) should, therefore, be seen as complementary to the current work.

2.2 Psychological Economics and Motivated Beliefs

The current work accommodates psychological factors in two ways. First, we widen the scope of preferences by including anticipation of future consumption. Second, we let agents make optimal decisions about their beliefs.

Anticipatory utility is perhaps little used but certainly not a new idea in economics, dating back at least to Löwenstein (1987) and Schelling (1987). In Schelling’s words, our mind is both an information processing machine by which we make our decisions and a consuming organ deriving satisfaction from our emotions (Schelling, 1987, page 193). That is, we use our beliefs to predict the consequences of our actions, but we also consume them. Due to this latter function of beliefs, we derive utility or incur disutility simply by believing certain things. As experiments have shown, this consumption value of beliefs has consequences for our information processing (Kunda, 1990; Averill & Rosenn, 1972; Lerman et al., 1998) and our behavior (Cook & Barnes Jr, 1964; Löwenstein, 1987).

Anticipatory utility is modeled usually by letting the utility function have a term which is a linear (Minozzi, 2013; Bénabou, 2008, 2012; Brunnermeier & Parker, 2005) or a general (Caplin & Leahy, 2001; Köszegi, 2010; Bernheim & Thomadsen, 2005) function of the expectation of a later period utility flow. Anticipatory (dis)utility may also depend directly on the subjective probability of the occurrence of a certain event. In Akerlof and Dickens (1982), agents incur psychic costs of fear modeled as a “fear cost function” which depends on the perceived probability of an accident in their hazardous job whereas in Kopczuk and Slemrod (2005), utility depends on a fear index function which is increasing in the subjective probability of death.

In addition to preferences, an important element of decisions in an uncertain world is beliefs. Hence, to understand decisions, it is crucial to understand beliefs. The departure from rational expectations is motivated by a vast literature in psychology (Alicke & Govorun, 2005; Moore & Healy, 2008; Tversky & Kahneman, 1974; Weinstein, 1980) and behavioral economics (Bénabou, 2015; Bénabou & Tirole, 2016). In addition to challenging the objectivity of beliefs, the literature on motivated reasoning directs us towards alternative options: Biases in human information processing, memories and beliefs are not random but they rather seem to be incentivized and partly determined by desires (Kunda, 1990; Braman & Nelson, 2007; Redlawsk, 2002; Taber & Lodge, 2006). In addition to accuracy goals, reasoning can be motivated by directional goals, that is, by desires and preferences.

The literature on motivated reasoning has inspired models of biased beliefs where the beliefs are a result of optimizing the trade-off between accuracy goals and directional
goals.\footnote{Minozzi (2013) calls the nonstandard beliefs that emerge in his model endogenous beliefs whereas Bénabou (2015) refer to these beliefs as motivated beliefs. In this work, these terms are used interchangeably. However, the term motivated beliefs is more informative. After all, all beliefs that are determined within a model, can be called endogenous. For instance, in this sense, the usual rational expectations are endogenous beliefs as well. Brunnermeier and Parker (2005) use the term optimal beliefs.} Anticipatory utility is one way to model such a directional goal for reasoning, but a complete model also requires the means for belief distortion. We call a cognitive technology a framework which provides the agents with the ways and constraints of distorting their beliefs. There are roughly two kinds of cognitive technologies used in the literature. In the first of these which we will call naive cognitive technologies, the beliefs can be simply chosen, and they do not need to depend on the prior beliefs or the objective probability distributions of reality. For instance, Minozzi (2013), Brunnermeier and Parker (2005), Kopczuk and Slemrod (2005), and Akerlof and Dickens (1982) use a naive cognitive technology. We call the second kind of cognitive technology a sophisticated cognitive technology. If the cognitive technology is sophisticated, agents are aware that they have incentives to bias their beliefs and assess their beliefs accordingly. Hence, the emerging beliefs are influenced by the prior beliefs and are anchored in reality. This second type of cognitive technology is used in Bénabou and Tirole (2002, 2006), Bénabou (2008, 2012), Bernheim and Thomadsen (2005), and Kopczuk and Slemrod (2005), and reviewed in Bénabou (2015) and Bénabou and Tirole (2016). The names for these two types of cognitive technologies follow from their different assumptions on the agents’ degree of Bayesian sophistication.\footnote{Kopczuk and Slemrod (2005) use terms backward naivete and backward sophistication.}

To sum up, a model of endogenous belief distortion has two crucial elements. First, agents must have an incentive to hold biased beliefs. Using the language of Bénabou and Tirole (2002), this can be called the demand for distorted beliefs. In the current work, agents are incentivized to hold biased beliefs by letting them derive utility from their high hopes. Second, agents must be able to influence their beliefs. This can be called the supply of distorted beliefs. The supply of distorted beliefs depends on the cognitive technology which sets the possibilities and limits for the belief distortion. The current paper considers the whole continuum of cognitive technologies from the completely naive to the fully sophisticated. Given the incentives and the technology of belief formation, biased subjective beliefs emerge as a result of optimization. This optimization involves trading-off the benefits of holding biased beliefs against the costs of inferior decisions due to inaccurate information and is subject to the constraints of the cognitive technology.\footnote{The emergence of non-standard beliefs as a result of optimization and purposeful actions distinguishes the motivated beliefs framework from the mechanical failures of rationality or bounded rationality, which leave the motivations of actions intact and only impose constraints on reasoning (see Bénabou and Tirole (2016)).}
3 The Model

3.1 The Economy and the Timing of the Model

The economy consists of a unitary continuum \( i \in [0, 1] \) of risk neutral agents who collectively decide on an income tax policy under uncertainty about their exogenous future incomes.\(^6\) In period 0, agents receive a signal conveying information about their prospective future incomes. In period 0, they also engage in various conscious and unconscious psychological processes of belief distortion, reality denial, and information avoidance which determine the signal they will remember in period 1.\(^7\) In the beginning of period 1, agents recall a signal and form beliefs about their future incomes based on their recollection. Then they vote for redistribution. They get to know the policy outcome immediately after the vote, and in the rest of period 1, they experience anticipatory utility as they anticipate their consumption which occurs in period 2, right after the incomes have been realized and redistributed. The timeline is given in Figure 1.

\(^6\)Risk aversion would induce a demand for redistribution in the form of insurance against downward mobility. This effect counters the effect of POUM. To not confound these effects, risk neutrality is assumed. By having uncertainty with respect to future incomes, the model remains agnostic to whether income expectations are result of beliefs over ability or life events (or over the deterministic or stochastic term of the production function that generates income).

\(^7\)Agents have imperfect recall in the sense that they forget information. The underlying game theoretical construct to model this inconsistency is to model agents consisting of two players, their two temporal selves (see Bénabou and Tirole (2002)). Also, the parallel interpretation throughout the paper is that the parents have influence over what their offsprings belief when the offsprings are making voting decisions.
3.2 Information and Beliefs

In period 0, each agent receives a noisy signal \( \sigma_i \in \mathcal{F} = \{F_H, F_L\} \). These signals are identical and independent draws from the following probability mass function:

\[
g(\sigma) = \begin{cases} 
q & \text{if } \sigma = F_H \\
1 - q & \text{if } \sigma = F_L 
\end{cases}
\]

where \( F_H \) and \( F_L \) are cumulative distributions over the future income levels such that \( \int_y y dF_H(y) > \int_y y dF_L(y) \) and \( y \geq 0 \).\(^8\) Using the language of Minozzi (2013), we call the agents who receive signal \( \sigma = F_H \) the likely rich and the agents who receive signal \( \sigma = F_L \) the likely poor. With a large number of agents, a fraction \( q \) of the population is likely rich and a fraction \( 1 - q \) likely poor. Furthermore, we assume that the likely poor agents constitute a majority, that is, \( q < \frac{1}{2} \). As agents are risk-neutral, a sufficient statistics for the analysis are the means of the distributions \( F_H \) and \( F_L \): \( y_H = \int_y y dF_H(y) \) and \( y_L = \int_y y dF_L(y) \), the incomes that the likely rich and the likely poor, respectively, can expect to earn in period 2. In the following, we refer to these distributions by their means and let the signal set be \( \{y_L, y_H\} \).\(^9\)

The possibility for belief distortion arises in the period-0 actions. After receiving a signal, each agent decides which of the two signals she will recall in period 1. As we will see, a likely poor agent has an incentive not to recall her true prospects. On the other hand, we make a sensible assumption that the likely rich agents will always choose to remember the signal they received and they, therefore, have no interesting decision to analyze. After all, if they underestimate their income, they lose anticipatory utility.\(^10\) Hence, we focus mainly on the more interesting decisions of the likely poor agents. Formally, in period 0,

\(^8\)Here the signals are independent for simplicity and to induce heterogeneity in the resulting income distribution. In general, the signals may be correlated. The special case of perfectly correlated types and signals can be used if the unknown variable is more common to agents in the sense that it reflects some general workings of the economy, like return to effort as in Bénabou and Tirole (2006), government efficiency as in Bénabou (2008) or expected value of a joint project as in Bénabou (2012).

\(^9\)We use a simplifying shortcut here. The underlying formal process is that Nature draws a state of the world, which determines the incomes of each agent. Agents receive some information about the state of the world via a signal determined by a signal function which lets them know a set of states of the world. Using the prior belief and the signal they then form a posterior belief. The posterior belief is, therefore, a function of the signal and fixed prior beliefs, so it is straightforward to associate a signal with a posterior belief and let the outputs of the signal function be the posterior beliefs agents have immediately after receiving the signal. Moreover, as the signal is a deterministic function of the state of the world, which Nature draws, we can simply let the received signal have the given distribution.

\(^10\)This seems a very plausible conjecture but technically this is not that simple. Depending on the off-equilibrium path beliefs, an agent sending a low signal might end up with higher beliefs than when sending a high signal. In the appendix, we make an assumption about these off-equilibrium path beliefs to exclude this peculiar theoretical possibility.
a likely poor agent $i$ chooses a recall rate $\lambda_i \in [0, 1]$ defined as

$$\lambda_i \equiv \Pr[\hat{\sigma}_i = y_L|\sigma_i = y_L],$$

(2)

where $\hat{\sigma}_i$ denotes both the action she chooses in period 0 and the signal agent $i$ recalls in period 1.\footnote{In the jargon of game theory, in period 0, an agent $i$ plays a mixed strategy $\left(\begin{array}{c} y_L \\ \lambda_i \\ y_H \\ 1 - \lambda_i \end{array}\right)$.}

In period 1, agent $i$'s information is based on a recalled signal $\hat{\sigma}_i \in \{y_L, y_H\}$. The memory of agents is probabilistic and their actions in period 0 determine the probability of each recollection. With probability $\lambda_i$, a likely poor agent will correctly recall $\hat{\sigma}_i = y_L$ and with probability $1 - \lambda_i$, she will recall $\hat{\sigma}_i = y_H$. By assumption, the likely rich agents always recall $\hat{\sigma}_i = y_H$. Of course, it is not claimed that people literally choose exact probabilities for the occurrences of their future memories. The choices in period 0 should be interpreted as all sorts of unconscious and conscious processes and actions that affect the availability of certain recollections. In equilibrium, agents act as if they were choosing optimal recall rates.

However, agents may not be completely in control of their beliefs. They may know that they have a tendency to forget bad news and remember good news. Therefore, they may not fully trust their recollections. If an agent $i$ recalls $\hat{\sigma}_i = y_H$ in the second period, she will assign a reliability $r(\lambda_i)$ to this signal:

$$r(\lambda_i|\chi) = \Pr[\sigma_i = y_H|\hat{\sigma}_i = y_H] = \frac{q}{q + \chi(1-q)(1-\lambda_i)},$$

(3)

where $\lambda_i$ is given by the period-0 strategy of agent $i$. $\chi$ is the naivete parameter measuring the degree of Bayesian sophistication. $\chi = 1$ corresponds to the full Bayesian rationality which is usually assumed in game theory.\footnote{Bayesian rationality refers to the use of Bayes rule in updating beliefs.} In the other extreme, $\chi = 0$, and the reliability of received signal is always 1. This means that in period 1, agents will completely trust their recollections and that in period 0, they are completely in control of their period-1 beliefs. The role of $\chi$ will be analyzed extensively later. Note that the reliability in (3) is defined only for the signal $\hat{\sigma}_i = y_H$. By assumption, only the likely poor might send a signal $\hat{\sigma} = y_L$, so the reliability of this signal is always 1.

With probability $1 - \lambda_i$, a likely poor agent recalls $\hat{\sigma}_i = y_H$ and is an **optimist**. In period 1, an optimist expects a gross income

$$E[y_i|F_{1,i}] = r(\lambda_i)y_H + (1 - r(\lambda_i))y_L,$$

(4)

where $F_{1,i}$ is the information of agent $i$ in period 1. Note how a decrease in $\lambda_i$ increases the probability of being an optimist and, as we will see, the expected anticipatory utility.
However, the effect is nonlinear for $\chi > 0$ since the reliability decreases as $\lambda_i$ increases. The more likely it is that a likely poor agent $i$ memorizes a false signal, the less reliable signal $\hat{\sigma}_i = y_H$ becomes. The more agents try to distort their beliefs, the more cautious they are when they are forming their beliefs.

With probability $\lambda_i$, a likely poor agent recalls $\hat{\sigma}_i = y_L$ and is a realist. As the reliability of signal $\hat{\sigma}_i = y_L$ is always 1, in period 1, a realist expects a gross income

$$E[y_i|F_{1,i}] = y_L.$$  \hfill (5)

The likely rich will recall $\hat{\sigma}_i = y_H$, and as they also do not know whether they truly are likely rich or likely poor, their expected income will coincide with the expected income of optimistic likely poor.

### 3.3 Preferences

In period 2, agents receive their incomes, pay taxes, and consume their disposable income. The government’s budget is balanced, and all tax revenue collected via a linear income tax is transferred in equal lump-sums to agents. There is no wastage in redistribution. Agents derive utility linearly from their consumption:

$$u_{2,i}(c_i) = c_i(\sigma_i, \tau) = (1 - \tau)y_i + \tau \bar{y},$$  \hfill (6)

where $c_i$ denotes consumption, $\tau$ is the income tax rate, and $\bar{y}$ is the average income:

$$\bar{y} = qy_H + (1 - q)y_L.$$  \hfill (7)

In period 1, agents do not yet know their income, but given their beliefs, they form expectations and experience a flow utility due to anticipation. The intertemporal preferences of agents from the perspective of period 1 are given by

$$u_{1,i}(\hat{\sigma}_i, \tau) = sE[u_{2,i}|F_{1,i}] + \delta E[u_{2,i}|F_{1,i}] = (s + \delta)E[(1 - \tau)y_i + \tau \bar{y}|F_{1,i}],$$  \hfill (8)

where the expectations are conditioned on the period-1 information $F_{1,i}$, $\delta \in [0, 1]$ is the standard discount factor and $s \geq 0$ is the “savoring” parameter measuring the importance of anticipation. The anticipatory utility is proportional to agent’s expectations: The higher expectations she has, the higher is the utility flow. This gives agents an incentive to distort their beliefs. Setting $s = 0$ yields the standard case with no anticipatory utility and, thus, no incentive to distort beliefs. The discount factor and the savoring parameter are common to all agents.
The intertemporal utility from the period-0 perspective is

\[
u_{0,i}(\sigma, \hat{\sigma}, \tau) = \delta E[sE[u_{2,i}|F_{1,i}]]F_{0,i}] + \delta^2 E[u_{2,i}|F_{0,i}] \\
= \delta sE[(1 - \tau) y_i + \tau \bar{y}|F_{1,i}] + \delta^2 E[(1 - \tau) y_i + \tau \bar{y}|F_{0,i}].
\]

(9)

The expected period-1 flow utility depends on the information in period 1 and the expected period-2 flow utility depends on the information in the period 0.\(^\text{13}\) That is, in period 0, agents know the true objective expectation of their incomes in period 2, but they also know that with upward biased beliefs they will receive higher utility in the period 1. The trade-off, which the optimal period-0 actions optimize, can be seen clearly here. Agents gain more utility if they have high hopes, but as we will see, with high hopes they will vote for low taxation, which then lowers their consumption in the last period.

3.4 The Polity and Voting Decisions

The agents vote for tax rate \(\tau \in [\underline{\tau}, \overline{\tau}]\) in the beginning of period 1. Their policy preferences are given by (8), and they depend on the subjective beliefs they have in period 1. Maximization with respect to the tax rate leads to the following voting rule:\(^\text{14}\)

\[
\tau_i^* = \begin{cases} 
\underline{\tau} & \text{if } E[y_i|F_{1,i}] \geq \bar{y} \\
\overline{\tau} & \text{if } E[y_i|F_{1,i}] < \bar{y}
\end{cases}
\]

(10)

where \(\tau_i^*\) is the preferred tax rate of agent \(i\). If an agent expects in period 1 to earn an above average income in the period 2, she will vote for the minimum redistribution, and if she expects to earn a below average income, she will vote for the maximum redistribution. This parallels the classic result of Meltzer and Richard (1981). The linearity of the policy preferences leads to corner solutions, which simplifies the analysis here. As we will see, setting \(\overline{\tau} < 1\) and \(\underline{\tau} > 0\) allows us to exogenously restrict the set of feasible tax policies.

As the policy preferences given by (8) are single-peaked, the Median Voter Theorem (Black, 1948; Downs, 1957) applies and the tax policy will be the tax rate preferred by the median voter. With two policy platforms voted for, the median voter’s opinion will be the opinion of the majority.

If agents could not manipulate their expectations or if they did not have any incentives

\(^{13}\)Note that since information is lost between periods 0 and 1 the law of iterated expectations does not hold and \(E[sE[u_{2,i}|F_{1,i}]]F_{0,i}] \neq sE[u_{2,i}|F_{0,i}],\) but the smaller information set wins and \(E[sE[u_{2,i}|F_{1,i}]]F_{0,i}] = sE[u_{2,i}|F_{1,i}].\)

\(^{14}\)We assume that an indifferent agent votes for low taxes. This assumption turns out to be quite crucial as it determines the tax policy in the low tax equilibrium of the model in the case of \(\chi = 1\). We could, however, suppose, that there is an arbitrarily small amount of wastage involved in taxation, or that the voters deviate an arbitrarily small amount from the full Bayesian rationality. Both of these would make the agent with mean income strictly prefer low taxation.
to distort their beliefs (e.g., $s = 0$), they would vote according to their objective prospects, and the unique equilibrium would be the likely poor voting for high taxes and the likely rich voting for low taxes. The median voter would be among the likely poor, and the policy in the unique equilibrium would be high taxes. We will see how the possibility of subjective beliefs that differ from the objective standard allow additional equilibria with other policy outcomes.

Throughout the analysis, we focus on type-symmetric decisions within the two groups of voters. All of the likely rich choose $\hat{\sigma} = y_H$ and all of the likely poor choose the same $\lambda$. An optimist will always vote for $\tau = \bar{\tau}$ as seen from (10) and (4) and noting that $r(\lambda) \geq q$ for all $\lambda \in [0, 1]$. A realist will always vote for $\tau = \bar{\tau}$ by (5). Also, the likely rich will always vote for $\tau = \bar{\tau}$, similarly to the the optimistic likely poor. Putting all this together, the policy outcome can be derived as a function of $\lambda$. The total share of agents expecting above average income is $q + (1 - q)(1 - \lambda)$. The policy outcome $\tau^*$ depends on whether this share exceeds $\frac{1}{2}$ or not:

$$\tau^* = \begin{cases} \bar{\tau} & \text{if } \lambda < \frac{1}{2(1-q)} \\ \bar{\tau} & \text{if } \lambda \geq \frac{1}{2(1-q)} \end{cases} \quad (11)$$

In line with Minozzi’s (2013) model, we first let agents vote strategically.\textsuperscript{15} That is, they take account that their vote might be pivotal. As will be shown later, if agents voted sincerely, the trivial outcome would be everyone maximizing the anticipatory utility.\textsuperscript{16}

3.5 Conditions for the POUM effect, $\tau \in [0, 1]$

To gain some intuition and to analyze an interesting special case, we first set $\bar{\tau} = 1$ and $\bar{\tau} = 0$. The more general and more realistic case of $\bar{\tau} < 1$ and $\bar{\tau} > 0$ is analyzed in the next section.

Now that we know the voting decisions in period 1, we turn to the likely poor’s choice of $\lambda$ in period 0. Due to the discontinuity of the policy outcome, the likely poor really have only two options to choose from. They either form optimal beliefs among those which support high taxation or optimal beliefs among those which support low taxation. We now derive the conditions under which the likely poor choose optimism and low taxation over realism and high taxation. In other words, we derive the conditions under which the

\textsuperscript{15}Or rather we let agents form their beliefs strategically taking account how it affects the policy outcome. Technically speaking the voting here is sincere but agents can affect their policy preferences via their beliefs. The assumption that the policy outcome is $\tau$ in case of $\lambda = \frac{1}{2(1-q)}$ ensures that an optimal choice of $\lambda$ exists for all $s > 0$.

\textsuperscript{16}In contrast to models of Bénaub and Tirole (2006) and Bénaub (2008), where voting is sincere, here the possibility of losing income due to less redistribution is the only thing that restricts the optimism of voters. This lets us focus on the trade-off between anticipation and redistribution. Sincere voting is studied in section 4.3.
prospects of upward mobility support a low tax regime.

Let $\lambda$ be the optimal recall rate given $\lambda \geq \frac{1}{2(1-q)}$ and $\bar{\lambda}$ the optimal recall rate given $\lambda < \frac{1}{2(1-q)}$. If the likely poor choose $\lambda$, the tax rate will be $\tau^* = 1$. The expected utility then is

$$U_{\lambda,0,i} = \lambda u_{0,i}(y_L, y_L, 1) + (1 - \lambda)u_{0,i}(y_L, y_H, 1) = \delta s y + \delta^2 y.$$  \hfill (12)

Whether they end up being optimists or realists does not matter since in both cases they expect the redistribution to equalize all incomes. If they, on the other hand, choose $\bar{\lambda}$, the tax rate will be $\tau^* = 0$. The expected utility is

$$U_{\bar{\lambda},0,i} = \bar{\lambda} u_{0,i}(y_L, y_L, 0) + (1 - \bar{\lambda})u_{0,i}(y_L, y_H, 0) = \bar{\lambda} [\delta s y_L + \delta^2 y_L] + (1 - \bar{\lambda}) [\delta s [r(\bar{\lambda})y_H + (1 - r(\bar{\lambda}))y_L] + \delta^2 y_L].$$ \hfill (13)

With probability $\lambda$, a likely poor agent recalls $\hat{\sigma}_i = y_L$ and forms realistic beliefs. With probability $1 - \lambda$, a likely poor agent recalls $\hat{\sigma}_i = y_H$ and forms optimistic beliefs weighted by the reliability of the signal. In both cases she still ends up consuming $y_L$ in period 2.

The comparison of (12) and (13) tells us if the likely poor would rather choose high anticipatory utility in period 1 and low taxation with low consumption in period 2 over low anticipatory utility and high taxation with high consumption. The utility difference between these two choices, or the incentive to optimism, can be written as:

$$U_{\lambda,0,i} - U_{\bar{\lambda},0,i} = -\delta^2 (\bar{y} - y_L) + s \delta [\lambda y_L + (1 - \lambda) [r(\bar{\lambda})y_H + (1 - r(\bar{\lambda}))y_L] - \bar{y}].$$ \hfill (14)

The first term tells what a likely poor agent loses in income and consumption if the tax rate is $\tau^* = 0$ instead of $\tau^* = 1$. The second term tells what she expects to gain in anticipatory utility if she chooses $\bar{\lambda}$ instead of $\lambda$. Whether the likely poor have high or low recall rate depends on whether the incentive to optimism is positive and this crucially depends on the value of anticipation.

**Lemma 1 (Awareness choices of the likely poor, $\tau \in [0, 1]$).** When $\tau \in [0, 1]$, the likely poor choose $\lambda = \bar{\lambda} = 0$ if

$$s \geq s^*(\chi) \equiv \frac{\delta q + \chi (1-q)}{(1-\chi)(1-q)}.$$

Otherwise they choose $\lambda = \bar{\lambda} \in [\frac{1}{2(1-q)}, 1]$.

We have defined $s^*$ to be a threshold such that if $s \geq s^*$, then agents value anticipation enough for the gain in anticipatory utility to outweigh the loss of income, and the likely poor will be optimistic enough to vote for a low tax rate. If, on the other hand, $s < s^*$, then the anticipation is not enough to compensate for the lost income and the likely poor will remain realistic enough to vote for a high tax rate.
Lemma 2 (Equilibria, \( \tau \in [0,1] \)). An equilibrium is a 4-tuple \((y_H, \lambda^*, r(\lambda^*|\chi), \tau^*)\).\(^{17}\)

(i) If \( s \geq s^* \), there is an equilibrium in which the likely poor choose \( \lambda^* = \bar{\lambda} = 0 \), the likely rich choose \( \hat{\sigma} = y_H \), and the policy outcome is \( \tau^* = 0 \).

(ii) If \( s < s^* \), there are equilibria in which the likely poor choose \( \lambda^* = \bar{\lambda} \in [\frac{1}{2(1-q)}, 1] \), the likely rich choose \( \hat{\sigma} = y_H \), and the policy outcome is \( \tau^* = 1 \).

The POUM effect occurs in the equilibrium (i), so the condition for the possibility of the POUM effect is equivalent to the condition of the equilibrium (i).

Proposition 1 (The condition for the POUM effect, \( \tau \in [0,1] \)). When \( \tau \in [0,1] \), the condition for the POUM effect is \( U_{0,i}^\Delta - U_{0,i}^\lambda \geq 0 \iff s \geq s^* \).

If agents value anticipatory utility enough the prospects of upward mobility lead to low taxes. How much is enough depends on the threshold \( s^* \). The higher \( s^* \) is, the less likely the POUM effect is, and conversely, the lower \( s^* \) is, the more likely we will observe low taxation. This threshold varies with the parameters of the model. First, the POUM effect becomes more likely with discounting. Myopic preferences put more weight on anticipation which occurs before consumption.\(^{18}\) Second, the effects of changes in the income distribution are left for section 4.1. Third, the threshold depends on the degree of Bayesian sophistication \( \chi \), which we study more closely now.

Consider first the special case of completely naive inference. Setting \( \chi = 0 \), we get

\[
s^*(0) = \delta \frac{q}{1-q}. \tag{16}\]

This special case corresponds to Minozzi’s (2013) model.\(^{19}\) If, on the other hand, we let agents’ inference approach Bayesian rationality, we find:

\[
\lim_{\chi \to 1} s^*(\chi) = \infty. \tag{17}\]

The threshold required for the POUM effect to occur approaches infinity as the inference of agents approaches full Bayesian rationality. This means that with full Bayesian rationality the importance of anticipation \( s \) can never be above \( s^* \) and it can never be optimal for the likely poor to form beliefs that support low taxation. That is, on contrary to the special case of Minozzi’s (2013) model, where \( \chi = 0 \), if we acknowledge that the people cannot

---

\(^{17}\)There is a third type of symmetric equilibrium, where all agents choose \( \hat{\sigma}_i = y_H \) and the policy outcome is \( \tau^* = 0 \) even if \( s < s^* \). There would be no unilateral incentive to deviate.

\(^{18}\)Interestingly, in the model of Bénabou and Ok (2001), discounting makes the POUM effect less likely. This result in their model is, however, derived in a multiperiod setting and is not directly comparable.

\(^{19}\)Minozzi’s (2013) model which abstracts from discounting derives \( \delta^* = \frac{n-m}{m} \), where \( \delta^* \) is the threshold of the savoring parameter, \( n \) is the (finite) number of agents, and \( m \) is the number of the likely poor.
simply choose their beliefs and let $\chi > 0$, the threshold $s^*$ increases dramatically in $\chi$ and in the extreme case of full Bayesian rationality, the POUM effect can never occur.

Figure 2 tracks the threshold $s^*$ as a function of $\chi$. To give some concreteness to the results here, we note from the period-0 utility in (9) that if $s = \delta$, then agents value anticipatory utility as much as consumption. The dashed line in Figure 2, denoted by $\delta$, depicts this value of $s$. For the threshold values $s^* > \delta$, the anticipation of consumption must bring more utility to agents than the consumption itself to make the POUM effect possible. We see that $s^*$ is below $\delta$ only for very small values of $\chi$.

To see why fully Bayesian likely poor agents can never be better off with low taxes, consider again the incentive to optimism given in (14). Plugging in the optimal recall rate $\lambda = 0$, the incentive to optimism can be written as

$$U_{0,i}^\lambda - U_{0,i}^\chi = -\delta^2(y - y_L) + s\delta[r(0|\chi) - q]\Delta y,$$

(18)

where $\Delta y \equiv y_H - y_L$. The second term in the right hand side is the gain in anticipation if an agent chooses $\lambda$ over $\chi$. Noting that $r(0|\chi) \to 1$ as $\chi \to 0$ and $r(0|\chi) \to q$ as $\chi \to 1$, it is easy to see how the value of the second term goes to zero as $\chi \to 1$ and why it does not when $\chi = 0$. The incentive to optimism is at its maximum when $\chi = 0$ and as agents’ inference approaches full Bayesian rationality the utility gain from anticipation vanishes.

The reliability which the agents use to weight the information of their recollection plays a crucial role here. For $\chi = 1$, the reliability $r(\lambda|\chi)$ is an increasing function of $\lambda$. The more realistic the likely poor are, the more reliable signal $\hat{\sigma}_i = y_H$ is. On the other hand, when the likely poor systematically memorize and recall $\hat{\sigma}_i = y_H$, they know that no matter what is their true signal, they recall $\hat{\sigma}_i = y_H$. In this case, the signal
does not carry any information, and agents’ beliefs correspond to the prior distribution, $r(0|\chi) = q$. However, when the degree of Bayesian sophistication decreases, the reliability becomes less dependent on $\lambda$, and the optimistic poor put more weight on their pleasant recollection. When $\chi = 0$, the reliability is independent of $\lambda$ and no matter how optimistic the likely poor are, they always fully trust their recollections.

It is instructive to see how the period-0 expectation of expected period-2 income in period 1, and expected anticipatory utility which is proportional to the expected income, varies with $\lambda$ and $\chi$. For this, we shortly abstract from taxation to see how the choice of $\lambda$ and the sophistication of agents’ inference interact in forming the belief about their future gross income. The expectation of expected gross income of a likely poor agent in period 1 from the point of view of period 0 as a function of $\lambda$ is\(^{20}\)

$$
\iota_{\text{gross}}(\lambda|\chi) = E[E[y_i|F_{1,i}]|\lambda, \chi, F_{0,i}] = (1 - \lambda)[r(\lambda)y_H + (1 - r(\lambda))y_L] + \lambda y_L. 
$$

This function is plotted in Figure 3 for different values of $\chi$. The lowest curve corresponds to the case $\chi = 1$. As agents put more and more weight on signal $\sigma_i = y_H$ in their period-0 strategy, that is, as they become more and more likely to remember $\sigma_i = y_H$, the expected income approaches the average income. In the case of $\lambda = 0$, each of the likely poor and each of the likely rich always recall signal $\sigma_i = y_H$. As everyone is pooling

\(^{20}\)See the discussion in section 3.2
on the same signal, receiving this signal does not give any information, and agents rely on the prior information when assessing their future income. In the case of full Bayesian rationality, it is, therefore, not possible for agents to achieve above average expectations. As they expect average income in the fully expropriating high tax regime, they cannot possibly improve their utility by distorting their beliefs and voting for low taxes.

On the contrary, when $\chi < 1$, agents can achieve above average expectations, and they, therefore, can have a gain in anticipatory utility to trade off against the lost income in the low tax regime. For agents with $\chi < 1$, a decrease in $\lambda$ does not affect the reliability of the signal as much as it affects for the Bayesian rational agents. In the limiting case of $\chi = 0$, represented by the linear curve in Figure 3, the reliability is independent of $\lambda$, and all agents can believe to be of type $y_H$. The expectations of naive agents are not as constrained as the expectations of Bayesian agents. The naive agents can, therefore, achieve higher hopes and higher anticipatory utility than their Bayesian counterparts.

What values of $\chi$ are feasible then? Do people have the introspection to realize that they might have a self-serving tendency to remember positive news and forget bad news or are they always able to deceive themselves into believing what they desire? Minozzi (2013) justifies his assumption of full naivete by arguing that belief formation is an automatic and unconscious process and, therefore, the agents cannot recall the process itself. They then completely trust their recollections, since they have forgotten the action of their past self or rather since they never even knew about the action of their unconscious self. On the other hand, Bénabou and Tirole (2002) argues that if a person consistently memorizes good news and ignores bad news, she will likely become aware of this tendency and, thus, will not fully rely on her recollections. So even if the belief formation is an automatic, unconscious process and people cannot, therefore, recall it happening, they, by learning from their past mistakes, will internalize the existence of this process and start adjusting their reliance on their memories accordingly.

Framed in other words, the implausible consequence of assuming $\chi = 0$ is that people are able to choose their beliefs without them in any way depending on the objective reality. To be clear, the beliefs supplied by a naive cognitive technology are usually restricted to the support of the outcome and can be further constrained to a subset of the support. Also, a naive cognitive technology does take the reality into account, when the beliefs are traded against their adverse consequences. However, in principle, it does not need to. Naive cognitive technologies are also nevertheless insensitive to the prior distribution of outcomes. When $\chi = 0$, an agent can believe to be likely rich no matter how small the prior probability of being rich is given that this prior probability is positive.

Even if the belief formation mechanism is an automatic and unconscious process, it seems implausible that this process does not need in any way to take account the infor-

\[21\text{See the footnote 2 in the appendix to Minozzi (2013).}\]
mation that the reality inevitably provides. Indeed, Kunda (1990) strongly argues that people do not seem to be completely free to believe what they want to believe. According to him, people can bias their beliefs only to the extent that they can justify their new beliefs. The main mechanism for the justification of the new beliefs is a biased memory search which implies that prior beliefs do play a role in determining the new beliefs. Also, according to evidence, changes in beliefs seem to be constrained by pre-existing beliefs (Kunda, 1990). Therefore, a belief formation technology with some Bayesian sophistication, which anchors the beliefs to the prior distribution, and, thus, to the reality in our model, would seem more plausible a representation of these psychological processes than a belief formation technology with none Bayesian sophistication.

However, assuming $\chi = 1$ is rather extreme as well and $\chi \in (0, 1)$ would most likely best reflect the reasoning of real people. Bénabou and Tirole (2002) present their model in the context of people distorting their beliefs to motivate themselves when facing time inconsistency problems. This might be a context in which people get enough feedback to learn about their unconscious information processing. In the context of the present work, where people form beliefs about their future incomes and vote for redistribution, the feedback mechanism may not facilitate this learning. The actions taken are long-lasting, there are not that many chances of learning, and the real-life mechanisms that transform votes to redistributive policies are noisy and complicated. It is thus plausible that the sophistication in the belief formation process depends on the context and that, indeed, in the context of forming beliefs about future income, people might be less sophisticated as in the context of motivating oneself in the everyday activities.

In the case of full Bayesian rationality, the wishful beliefs of the likely poor are bounded above to the average income, which is what they expect to receive if $\tau^* = 1$ as well. They cannot, therefore, increase their anticipatory utility by distorting their beliefs. However, what if they could not expect the incomes to be fully equalized under the high tax policy. Then they might be able to increase their anticipatory utility by distorting their beliefs even if they still ended up with expectations of average income. The case of $\tau = 1$ and $\tau = 0$ is too unrealistic a simplification and we therefore turn now to the more general case of $\tau \in [\tau, \bar{\tau}]$.

### 3.6 Conditions for the POUM effect, $\tau \in [\bar{\tau}, \tau]$

The weakness of the previous setting is that if agents vote for full expropriation, they know that their period-2 incomes will be the average income. Therefore, no matter what they believe, they will expect average income. On the other hand, if the likely poor choose optimism, they will lose all redistribution, which is a very high cost for optimism. To address these problems, we now consider the general case of our model and impose lower and upper limits on the tax rate. That is, we now set $\tau \in [\bar{\tau}, \tau]$, and require $\bar{\tau} < \tau$. 
so that the set of tax policies is always nonempty.

If we set \( \tau < 1 \), the anticipated consumption and the consumption of the likely poor realists will now be below average in the high tax regime. This makes the high tax equilibrium less attractive compared to the case of \( \tau = 1 \). The increase in payoff when choosing optimism over realism is now greater, and the condition for the POUM effect becomes looser.

At the other extreme, a full laissez-faire is not an innocuous simplification either. The likely poor have to trade optimism against losing all redistribution. Imposing a lower limit for redistribution makes this trade-off less drastic. If \( \tau > 0 \), there will be some taxation in the low tax regime as well, and the consequences of optimism are less severe for the likely poor. By setting \( \tau > 0 \), we make the decrease in period-2 consumption of the likely poor smaller in case they choose \( \lambda \) over \( \bar{\lambda} \). Again, this makes optimism more attractive and the POUM effect more likely. Of course, an increase in \( \tau \) decreases the anticipatory utility of the optimists, but the effect on period-2 consumption dominates for the likely poor.

To put these effects together, by restricting the set of available tax policies, we make the POUM effect more feasible in two ways. First, by decreasing the attractiveness of realism by having lower taxes and, therefore, lower anticipation and consumption in the high tax regime. Second, by increasing the attractiveness of optimism by having higher taxes in the low tax regime and, thus, higher consumption but possibly lower anticipation. The effects on period-2 consumption and realists’ anticipation dominate the effect on optimists’ anticipation so that the smaller is the range of allowed tax policies, the more likely the POUM effect occurs.

As before, the apparently continuous choice reduces to a binary choice, and the likely poor choose between \( \bar{\lambda} \) and \( \lambda \) knowing that choosing the former leads to high taxation and choosing the latter leads to low taxation. If they choose the former, the tax rate will be \( \tau^* = \tau \) and their expected payoffs are

\[
U_{0,i}^{\bar{\lambda}} = \bar{\lambda}u_{0,i}(y_L, y_L, \tau) + (1 - \bar{\lambda})u_{0,i}(y_L, y_H, \tau)
\]

\[
= \bar{\lambda}\left[\delta s[(1 - \tau)y_L + \tau \bar{y}] + \delta^2[(1 - \tau)y_L + \tau \bar{y}]\right]
+ (1 - \bar{\lambda})\left[\delta s[(1 - \tau)[r(\bar{\lambda})y_H + (1 - r(\bar{\lambda}))y_L] + \tau \bar{y}] + \delta^2[(1 - \tau)y_L + \tau \bar{y}]\right].
\]

This differs from (12) in that the tax does not fully equalize the incomes. When all incomes are not equalized, different expectations lead to different utilities from anticipation. This allows the anticipatory utility of optimists and realists to diverge also in the high tax regime. Note especially how a realist derives anticipatory utility from an expectation of below average income.

If the likely poor agents choose the latter, the tax rate will be \( \tau^* = \tau \) and their
expected payoffs are

\[ U_{\lambda, \tau}^{\lambda} = \lambda u_{0,i}(y_L, y_H, \tau) + (1 - \lambda) u_{0,i}(y_L, y_H, \tau) = \lambda \left[ \delta_s(1 - \tau) y_L + \tau y_L \right] +\]

\[ + \left[ (1 - \lambda) \left[ \delta_s(1 - \tau) y_L + \tau y_L \right] + \delta_2 [(1 - \tau) y_L + \tau y_L] \right]. \tag{21} \]

With probability \( \lambda \), a likely poor agent ends up being a realist and anticipates low consumption. With probability \( 1 - \lambda \), she ends up being an optimist and anticipates high consumption. In both cases, the period-2 consumption is low. However, in comparison to (13), the period-2 consumption is now higher, and the economic consequences of optimism are now less severe for the likely poor.

The incentive to optimism is

\[ U_{\lambda, \tau}^{\lambda} - U_{\lambda, \tau}^{\lambda} = -\delta_s(\tau - \tau)(y_L - y_L) + \delta_s \left[ (1 - \lambda)(1 - \tau)r(\lambda)\Delta y + (1 - \tau) y_L + \tau y_L \right] \]

\[ - \delta_s \left[ (1 - \lambda)(1 - \tau)r(\lambda)\Delta y + (1 - \tau) y_L + \tau y_L \right]. \tag{22} \]

To not clutter the page with notation, the incentive to optimism is written in a still interpretable, but different and a more compact form than (14). The first term tells the loss of income due to less redistribution. The second term measures the expected anticipatory utility if the likely poor choose \( \lambda = \lambda \). With probability \( 1 - \lambda \), there is an increase of \((1 - \tau)r(\lambda)\Delta y\) from the ”base level” of \((1 - \tau) y_L + \tau y_L\) in anticipatory utility. The third term similarly measures the expected anticipatory utility if the likely poor choose \( \lambda = \lambda \). If the incentive to optimism is positive, the likely poor will prefer to be optimists and choose \( \lambda = \lambda \).

**Lemma 3 (Awareness choices of the likely poor, \( \tau \in [\bar{\tau}, \bar{\tau}] \)).** When \( \tau \in [\bar{\tau}, \bar{\tau}] \), the likely poor choose \( \lambda = \lambda = 0 \) if

\[ s \geq s^{**}(\chi) = \frac{\delta(\tau - \bar{\tau})q}{(1 - \lambda)(1 - \tau)r(\lambda) - (1 - \lambda) - (1 - \tau)q}. \tag{23} \]

Otherwise they choose \( \lambda = \bar{\lambda} = \frac{1}{q(1 - q)} \).

As before, whether the savoring parameter is above or below the threshold \( s^{**} \), the likely poor will either prefer high anticipation with low redistribution or low anticipation with high redistribution. The choice of the likely poor determines the tax rate.

**Lemma 4 (Equilibria, \( \tau \in [\bar{\tau}, \bar{\tau}] \)).** An equilibrium is a 4-tuple \( (y_H, \lambda^*, r(\lambda^*|\chi), \tau^*) \).

(i) If \( s \geq s^{**} \), there is an equilibrium in which the likely poor choose \( \lambda^* = \lambda = 0 \), the likely rich choose \( \hat{\sigma} = y_H \), and the policy outcome is \( \tau^* = \bar{\tau} \).\footnote{There is a third type of symmetric equilibrium, where all agents choose \( \hat{\sigma} = y_H \) and the policy outcome is \( \tau = 0 \) even if \( s < s^{**} \) as there would be no unilateral incentive to deviate.}
(ii) If \( s < s^{**} \), there is an equilibrium in which the likely poor choose \( \lambda^* = \bar{\lambda} = \frac{1}{2(1-q)} \), the likely rich choose \( \hat{\sigma} = y_H \), and the policy outcome is \( \tau^* = \tau \).

The POUM effect occurs in the equilibrium (i) and the conditions for the POUM effect are the same as the conditions for this equilibrium.

**Proposition 2 (The condition for the POUM effect, \( \tau \in [\tau, \overline{\tau}] \)).** The condition for the POUM effect is \( U_{\lambda,\delta}^\lambda - U_{\bar{\lambda},\delta}^{\bar{\lambda}} \geq 0 \iff s \geq s^{**} \).

Interestingly, \( s^{**} \) is now finite for all \( \chi \in [0,1] \). In contrast to the setting in the previous section, the POUM effect becomes possible even if the agents are fully Bayesian information processors. Figure 4 depicts \( s^{**} \) as a function of \( \chi \). We see that the threshold \( s^{**} \) does not increase in \( \chi \) as sharply as \( s^* \) does. As before, to ease the interpretation, the dashed line depicts the values of \( s \) for which the agents derive as much utility from the anticipation of consumption as from consumption itself. The parameter values for the allowed tax policies used in Figure 4 are \( \tau = 0.25 \) and \( \overline{\tau} = 0.45 \), and they represent roughly the total tax revenues as a percentage of the gross domestic product in the US and in the Nordic Countries, respectively (OECD, 2018).\(^{23}\) These values and countries are chosen to represent the extremes of taxation among the developed countries and serve only as an example. As we will see, the bounds of allowed tax policies have a clear effect on \( s^{**} \).

Proposition 3 formalizes the effect of the naivete parameter \( \chi \), visible in Figure 4.

\(^{23}\)q = 0.3 and \( \delta \) is normalized to 1. Note that the curve is independent of the values of \( y_L \) and \( y_H \).
Proposition 3 (Effect of change in the degree of Bayesian sophistication). The partial derivative of $s^{**}$ with respect to $\chi$ is positive, that is, $\frac{\partial s^{**}}{\partial \chi} > 0$ for all parameter values. The more sophisticated the cognitive technology is, the less likely is the POUM effect.

Even if the POUM effect is now possible for all $\chi \in [0, 1]$, it can still be questioned whether it is feasible for all $\chi \in [0, 1]$. Again, the agents may have to value anticipation more than consumption to prefer low taxes if the range of the feasible tax rates is big enough. To see this, consider the threshold value $s^{**}$ when $\chi = 1$:

$$s^{**}(1) = \delta \frac{(\tau - \tau)(1 - q)}{(1 - \tau)q}.$$  \hfill (24)

Now $s^{**}(1) > \delta$, for all pairs $(\tau, \tau)$, such that $\tau > (1 - q)\tau + q$. We could argue that within a jurisdiction, the range of feasible tax rates is small and, hence, the POUM effect is feasible also for a sophisticated cognitive technology. On the other hand, as discussed, fully Bayesian sophistication may not be the correct specification in the belief distortion technology to represent people’s beliefs about their future incomes and their voting behavior. Certainly, the set of values of $\chi$ for which the POUM effect is feasible has now increased in comparison to the case in the previous section.

To understand how the conditions of the POUM effect depends on the maximum and minimum taxes, consider first what happens when we set an upper limit on the tax rate. The upper limit of the tax is relevant when the likely poor choose $\lambda = \bar{\lambda}$, since then the resulting policy is high taxes. By imposing a restriction on how much of the income can be redistributed we make the prospects of choosing $\lambda = \bar{\lambda}$ worse. Consider the effects on the period-2 consumption and period-1 anticipatory utility separately. First, a decrease in the upper limit of the tax rate decreases the period-2 consumption of the likely poor in the high tax regime, which makes voting for high taxes less rewarding. Second, for those of the likely poor who end up being realists, the lower consumption in period 2 implies lower anticipation in period 1. Those of the likely poor who end up being optimists will expect above-average incomes, and they will, therefore, gain in anticipatory utility as the upper limit of the tax decreases. However, the overall effect unambiguously decreases the attractiveness of choosing $\lambda = \bar{\lambda}$. Proposition 4 formalizes this total effect of the upper limit of the tax rate.

Proposition 4 (Effect of upper limit of tax rate). The partial derivative of $s^{**}$ with respect to $\tau$ is positive, that is, $\frac{\partial s^{**}}{\partial \tau} > 0$ for all parameter values. The POUM effect becomes more likely as $\tau$ decreases.

Consider next what happens when we set a positive lower limit for the allowed tax rate. The prospects of choosing $\lambda = \underline{\lambda}$, are now better. The likely poor choosing $\lambda = \underline{\lambda}$
leads to low taxes, so it is here where the lower limit of the tax rate is interesting. Again, there is an effect on the period-2 consumption and on the period-1 anticipation. First, even if the likely poor vote for low taxation, redistribution does not vanish altogether. Since they are trading their optimism against redistribution, the cost of optimism is now lower. The reduction in their period-2 consumption is not as big as with the possibility of complete laissez-faire. This makes choosing high anticipation and low taxes more attractive. Second, when choosing \( \lambda = \lambda \), all of the likely poor end up being optimists. If they then anticipate above average income, that is, if \( \chi < 1 \), then an increase in the lower limit of the tax rate will decrease their anticipatory utility. The less sophisticated the agents are, the more they expect to earn, and the higher is the decrease in their anticipation. The effect on anticipatory utility is opposite to the effect on consumption. The effect on consumption, however, dominates. Proposition 5 formalizes this.

**Proposition 5 (Effect of lower limit of tax rate).** The partial derivative of \( s^{**} \) with respect to \( \tau \) is negative, that is, \( \frac{\partial s^{**}}{\partial \tau} < 0 \) for all parameter values. The POUM effect becomes more likely as \( \tau \) increases.

To summarize these effects, the utility from choosing \( \lambda = \lambda \) increases with the lower bound of the tax rate and the utility from choosing \( \lambda = \lambda \) decreases when we impose an upper bound for the tax rate. This means that the utility cap between choosing \( \lambda = \lambda \) and \( \lambda = \lambda \) increases as the range of allowed tax policies decreases. This utility cap is, by definition, the incentive to optimism. An increase in the incentive to optimism then leads to less stringent conditions for the POUM effect.

To gain further intuition on the conditions for the POUM effect, write \( s^{**} \) as

\[
\begin{align*}
s^{**} & = \frac{\delta(\tau - \tau)q\Delta y}{\tau_{\text{net}}(\lambda, \tau) - \tau_{\text{net}}(\lambda, \tau)} \\
& = \frac{\delta(\tau - \tau)q\Delta y}{\tau_{\text{net}}(\lambda, \tau) - \tau_{\text{net}}(\lambda, \tau)}
\end{align*}
\]

where

\[
\tau_{\text{net}}(\lambda, \tau) \equiv \lambda[(1 - \tau)y_L + \tau \bar{y}] + (1 - \lambda)[(1 - \tau)(r(\lambda)y_H + (1 - r(\lambda))y_L) + \tau \bar{y}]
\]

is the ex ante expectation of the expected consumption of the likely poor in period 1 given the choice of \( \lambda \) and tax policy \( \tau \), and where \( \lambda = 0 \), and \( \lambda = \lambda \). The nominator of (25) represents the difference in period-2 consumptions in the two different tax regimes. Clearly, when \( \tau \) decreases or \( \tau \) increases, this difference becomes smaller. As discussed, when this difference becomes smaller the loss in the period-2 consumption when choosing

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24From this expression it is simple to derive Minozzi’s (2013) result in another form. By setting \( \tau = 0 \), \( \tau = 1 \), \( \delta = 1 \), and \( \chi = 0 \), we get \( s^{**} = \frac{y_p - y}{y + y - y} \). Minozzi’s (2013) condition for the POUM effect is \( \delta > \delta^* = \frac{y - y_p}{y + y - y} \), where \( y_p \) is the income of the likely poor, \( y_r \) income of the likely rich, and \( \delta^* \) the threshold in the savoring parameter \( \delta \).
$\lambda = \overline{\lambda}$ over $\lambda = \overline{\lambda}$ decreases. If the nominator decreases, $s^{**}$ decreases proportionally and the POUUM effect becomes more likely. The denominator of (25) is proportional to the difference in expected anticipatory utility of the likely poor between their choices of low or high recall rate. When this difference increases, the likely poor have more to gain in anticipation and belief distortion becomes more attractive. If the denominator increases, $s^{**}$ decreases and the POUUM effect becomes more likely.

In choosing their awareness rate, the likely poor agents make a trade-off between anticipation and consumption. By imposing limits on possible tax rates, we alter this trade-off such that they have less to lose in consumption. The stakes of wrong decisions due to biased beliefs are now smaller, and optimism is, thus, more attractive.

4 Further Analysis

4.1 Effects of changes in the income distribution

As already seen, given the value that agents put on anticipation $s$, the threshold $s^{**}$ determines whether the POUUM effect occurs. The comparative statics of $s^{**}$, therefore, reveal how the conditions for the POUUM effect vary as the parameters of the model change. In this section we consider the effects of changes in $y_L$, $y_H$, $\overline{y}$, and $q$.

Following Minozzi’s (2013) analysis, we first examine the changes in $y_L$ and $y_H$ holding the average income constant. Proposition 6 collects these results.

**Proposition 6 (Effects of changes in $y_L$ and $y_H$ holding $\overline{y}$ constant).** Holding the average income constant, the threshold $s^{**}$ decreases in $y_L$ and $y_H$, that is, the POUUM effect becomes more feasible when $y_L$ or $y_H$ increase.\(^{25}\)

If the incomes of the likely rich increase such that the average income stays constant, the conditions for the POUUM effect become looser. Similarly, if the incomes of the likely poor increase such that the average income stays constant, the conditions for the POUUM effect become again looser. We insist on holding the average income constant because it makes the effects interesting. The average income $\overline{y}$ is a function of both $y_L$ and $y_H$ and taking this into account gives us $\frac{\partial s^{**}}{\partial y_H} = \frac{\partial s^{**}}{\partial y_L} = 0$ as can easily be seen by noting that $s^{**}$ in (23) is independent of both $y_H$ and $y_L$. So by letting the average income adapt to the changes in the incomes of the likely poor or the likely rich, the condition for the POUUM effect would not change.

Looking at the incentive to optimism given in (22) gives us an idea of how to interpret Proposition 6. For the agents, changes in the average income imply changes in the transfers they receive, whereas changes in either $y_L$ or $y_H$ imply changes in the expectations of their

\(^{25}\)All the results of this section are similar for $s^{*}$.
pre-tax income. That is, holding average income constant means holding the tax revenue and transfers constant, whereas increases in the high and low levels of income mean increased expectations of gross income. Increased prospects of gross income, when the transfers are expected to stagnate, make optimism more rewarding. This kind of change in the income distribution could occur, for instance, if the income tax is regressive such that the increase in the incomes of the likely rich leads to less than a proportional increase in the tax revenue. We could also interpret the income levels \( y_L \) and \( y_H \) more loosely as what the likely poor perceive these income levels to be. The perceived income of the likely rich could change without affecting the tax revenue, for instance, if the incomes in other jurisdictions change and the likely poor observe this or if the consumption habits of the likely rich change towards more conspicuous consumption. "In 1972, a storm of protests from blue-collar workers greeted Senator McGovern’s proposal for confiscatory estate taxes. They apparently wanted some big prizes maintained in the game. The silent majority did not want the yacht clubs closed forever to their children and grandchildren while those who had already become members kept sailing along." writes Okun (2015, p. 47).

Similarly, the change in the average income has no effect as such, \( \frac{\partial s^{**}}{\partial \bar{y}} = 0 \), but holding \( y_L \) and \( y_H \) constant and letting \( \bar{y} \) change gives us

**Proposition 7 (Effect of change in \( \bar{y} \) holding \( y_L \) and \( y_H \) constant).** Holding \( y_L \) and \( y_H \) constant, the threshold \( s^{**} \) increases in \( \bar{y} \), that is, the POUM effect becomes less feasible when the average income increases.

The case of holding \( y_L \) and \( y_H \) constant and letting \( \bar{y} \) change mirrors the previous discussion. If the likely poor expect increased transfers but the prospects of gross income stay the same, then realism becomes more attractive.

The changes in the fraction of the likely rich produce more complicated effects mainly because the reliability and the optimal recall rate \( \bar{\lambda} \) are functions of \( q \). We only characterize the effects. Consider a change in the income distribution where the proportion of the likely rich becomes smaller. A decrease in \( q \) has three effects. First, it decreases the average income and the tax revenue and, therefore, makes realism less attractive. Second, it decreases \( \bar{\lambda} \), the optimal choice if the likely poor opt for high taxes. When there are fewer likely rich agents voting for low taxes, it allows the likely poor to be more optimistic even if they opt for high redistribution. This makes realism more attractive. Third, as \( q \) and, hence, \( \bar{\lambda} \) decrease, they both contribute to decreasing the reliability of the signal \( \hat{\sigma_i} = y_H \) and, therefore, make the anticipated income lower and optimism less attractive.

All effects that work via the reliability of recalled signal depend crucially on \( \chi \). Hence, for low values of \( \chi \), the reliability does not depend that much on the prior distribution or \( \bar{\lambda} \) and the first effect dominates. In this case, POUM effect becomes more likely as the prospects of choosing \( \lambda = \bar{\lambda} \) are now worse. For high values of \( \chi \), the reliability is
highly dependent on the prior and $\lambda$ and the second and third effect dominate. In this case, a decrease in $q$ makes POUM less likely. For intermediate values of $\chi$, the relative dominance of these effects varies, and the total effect is nonmonotonic.

4.2 Welfare

In the simple model of the current paper, utilities are linear in period-2 consumption, meaning that the aggregate utility is not sensitive to the distribution of consumption. Therefore, the aggregate utility is trivially maximized by maximizing the anticipation, no matter what the distribution of consumption is. Hence, the aggregate utility as a measure of welfare is not very informative. This section, therefore, after a brief discussion on the distribution and aggregation of consumption and anticipation, focuses on the welfare of the likely poor and the likely rich separately.

Thanks to the additivity of consumption utility and anticipatory utility, the aggregate levels of these two components can be studied separately. Furthermore, as the utilities with respect to consumption and anticipation are both linear, the welfare consists of aggregate consumption and aggregate anticipation.

Since redistribution does not produce any wastage here, the aggregate consumption stays constant at the average consumption throughout the analysis. Only the distribution of consumption between the likely poor and the likely rich varies depending on the chosen tax policy. The higher is the tax rate, the more equally the aggregate consumption is distributed among the likely rich and the likely poor.

The more novel component of welfare is the aggregate anticipation. This is the sum of anticipation of those agents who recalled $\hat{\sigma}_i = y_L$ and of those who recalled $\hat{\sigma}_i = y_H$. A fraction $(1 - q)\lambda$ of agents recalls $\hat{\sigma}_i = y_L$ and they anticipate a gross income of $y_L$. A fraction $q + (1 - q)(1 - \lambda)$ of agents recalls $\hat{\sigma}_i = y_H$ and they anticipate a gross income of $r(\lambda)y_H + (1 - r(\lambda))y_L$. Note especially that those who truly belong to the likely rich anticipate the same gross income as those of the likely poor who recall signal $\hat{\sigma}_i = y_H$. The aggregate anticipatory utility derived from the anticipation of gross income is

$$ (1 - q)\lambda sy_L + [(1 - q)(1 - \lambda) + q]s[r(\lambda)y_H + (1 - r(\lambda))y_L]. $$

The aggregate anticipation of net income is simply

$$ (1 - \tau)[(1 - q)\lambda sy_L + [(1 - q)(1 - \lambda) + q]s[r(\lambda)y_H + (1 - r(\lambda))y_L]] + s\tau y' $$

and it behaves similarly to the aggregate anticipation of gross income.

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26 The aggregate anticipation of net income is simply

$$ (1 - \tau)[(1 - q)\lambda sy_L + [(1 - q)(1 - \lambda) + q]s[r(\lambda)y_H + (1 - r(\lambda))y_L]] + s\tau y' $$
agents expect average income. Therefore, for the special case of $\chi = 1$, the aggregate anticipation is similar to the aggregate consumption in the sense that only the distribution of the anticipation varies. As the Bayesian constraint is relaxed and values of $\chi < 1$ are allowed, the aggregate anticipation can exceed the anticipation of average income, and it is no more independent of $\lambda$. In this case, the aggregate anticipation is maximized at $\lambda = 0$.

The counterintuitive consequence of the assessment of the reliability of recollections is that for all $\chi > 0$, the likely rich will underestimate their future income. If all of the likely poor choose to memorize the signal $\hat{\sigma}_i = y_H$, then all agents, the likely rich and the likely poor, will recall this signal in period 1. When the likely rich assess the reliabilities of their recollections, they know that no matter which signal they receive in period 0, they will always recall $\hat{\sigma}_i = y_H$. In the case of full Bayesian rationality, this means that the likely rich use their prior information to form their expectations and, hence, underestimate their future income.27 If, on the other hand, the likely poor choose to memorize the signal they received, then the likely rich, after recalling $\hat{\sigma}_i = y_H$ know that the only way to recall this signal is to be likely rich. In this case, they form accurate expectations.

This dependence of the anticipation of the rich on the awareness rates of the likely poor can be thought of as a negative externality. As $\lambda$ decreases, the likely poor are more and more optimistic and the likely rich more and more pessimistic. That is, when the likely poor engage in optimism, they redistribute anticipation. If $\chi = 1$, and the likely poor choose $\lambda = 0$, they equalize all anticipation. In this case, the average anticipation is constant, and the gain in anticipatory utility of the likely poor is exactly offset by the loss in the anticipatory utility of the likely rich. The strength of this externality and its redistributive effect increase in $\chi$. For completely naive agents, the reliability of recollection is independent of $\lambda$, and there is no externality.

This externality should, however, not be thought of as a causal relationship between the cognitive processes of different agents, but as an externality across information states, as Bénabou and Tirole (2002, p. 907) put it. The likely rich do not underestimate their prospects because the likely poor overestimate theirs, but because they know that had they themselves been likely poor, they might still have memorized the signal $\hat{\sigma}_i = y_H$. The negative externality for the likely rich is, therefore, caused by their own information processing strategy, that is, by their own hypothetical action in an alternative history.

If the likely poor choose the low tax equilibrium with high expectations, they are

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27Interestingly, Cruces, Perez-Truglia, and Tetaz (2013) find evidence, that in addition to the poor overestimating their position in the income distribution, the rich tend to underestimate theirs. However, their proposed mechanism is different: Agents estimate the overall income distribution by extrapolating from the incomes of their reference group. If the reference group does not well represent the overall income distribution, the estimates will be biased. Also, underconfidence is a well-documented phenomenon in the literature of psychology and tends to concern those with the best prospects. See, for instance, Moore and Healy (2008).
clearly better off in this equilibrium. The pessimism of the rich, however, raises the
rather surprising question of whether the likely rich are worse or better off in the low
tax equilibrium. In the standard case, without anticipatory utility, the rich are clearly
better off in the low tax equilibrium. When we take the anticipation into the analysis, the
rich still have higher period-2 consumption in the low tax equilibrium, but the negative
externalities across information states erode their anticipation in period 1. We now see,
which of these effects dominates.

In the low tax equilibrium, the utility of the likely rich from the viewpoint of period
0 is

\[
\begin{align*}
    u_{0,i}(y_H, y_H, \tau) &= \delta s \left[ (1 - \tau) [r(\lambda)y_H + (1 - r(\lambda)y_L) + \tau \bar{y}] + \delta^2 [(1 - \tau)y_H + \tau \bar{y}] \right], \quad (28)
\end{align*}
\]

and in the high tax equilibrium the utility of the likely rich is

\[
\begin{align*}
    u_{0,i}(y_H, y_H, \tau) &= \delta s \left[ (1 - \tau) [r(\lambda)y_H + (1 - r(\lambda)y_L) + \tau \bar{y}] + \delta^2 [(1 - \tau)y_H + \tau \bar{y}] \right]. \quad (29)
\end{align*}
\]

Again, whether the anticipation effect dominates depends on the value of anticipation.
The likely poor choosing optimism and low taxes makes the likely rich worse off if (29) is
greater than (28). If \((1 - \tau)r(\lambda) - (1 - \tau)r(\lambda) - (\tau - \tau)q > 0\), the condition for this reads:

\[
    s < \frac{\delta(\tau - \tau)(1 - q)}{(1 - \tau)r(\lambda) - (1 - \tau)r(\lambda) - (\tau - \tau)q}. \quad (30)
\]

Since the denominator is positive and the nominator negative, the right-hand side of (30) is
negative. As \(s \geq 0\), the condition is never satisfied, and the likely rich are always better off
in the low tax equilibrium.\(^{28}\) If, on the other hand, \((1 - \tau)r(\lambda) - (1 - \tau)r(\lambda) - (\tau - \tau)q < 0\), the condition for the likely rich to be worse off in the low tax equilibrium reads:

\[
    s > \frac{\delta(\tau - \tau)(1 - q)}{(\tau - \tau)q + (1 - \tau)r(\lambda) - (1 - \tau)r(\lambda)} \equiv s^{***}(\chi). \quad (31)
\]

Obviously, whether the likely rich are worse off in the low tax equilibrium is an inter-
esting question only when the low tax equilibrium is possible. Figure 5 depicts \(s^{**}\) and
\(s^{***}\) as a functions of \(\chi\). As we have seen, the low tax equilibrium occurs if \(s \geq s^{**}\). By
definition of \(s^{***}\), the rich are worse off in the low tax equilibrium if \(s > s^{***}\). For \(\chi = 1\) the thresholds \(s^{**}\) and \(s^{***}\) coincide, and for \(\chi \in [0, 1)\) \(s^{***}\) is greater than \(s^{**}\). Therefore,
only for the fully Bayesian agents the optimism of the likely poor necessarily makes the
rich worse off. For \(\chi < 1\) this does not need to be the case.

\(^{28}\) The condition is also never satisfied when \((1 - \tau)r(\lambda) - (1 - \tau)r(\lambda) - (\tau - \tau)q = 0\), since the right-hand
side of (30) approaches infinity as the denominator approaches zero from the right and the condition then
would require \(s\) to be greater than infinity. When the denominator approaches zero from the left, the
right-hand side approaches negative infinity and the argument in the text applies.
Proposition 8 (The welfare of the likely rich). Whether the likely rich are worse off in the low tax equilibrium depends on the degree of the Bayesian sophistication and the value of anticipation.

(i) For $\chi = 1$, the likely rich are worse off in the low tax equilibrium than in the high tax equilibrium.

(ii) For $\chi < 1$, the likely rich are worse off in the low tax equilibrium only if $s > s^***(\chi)$.

In Figure 5, for parameter pairs $(s, \chi)$ below the lower curve, the POUM effect does not occur. Between the two curves, the POUM effect occurs and it makes the likely rich better off. Above the upper curve, the POUM effect occurs, and it makes the likely rich worse off.

Interestingly, an implication of the model is that the fully Bayesian likely rich are worse off with low taxes if the value of anticipation is high enough for likely poor to choose optimism and low taxes. Again, however, completely sophisticated cognitive technology might be of only theoretical interest. The threshold value $s^***$ goes up fairly rapidly for $\chi < 1$, which makes this result less relevant.

4.3 Sincere Voting

The beliefs are most likely to be distorted by desires if the individual cost of holding biased beliefs is small, as is the case in voting if the probability of being pivotal is negligible (Bénabou & Tirole, 2016; Caplan, 2011). Here we consider sincere voting and belief
formation. That is, voters do not consider themselves to be pivotal in the determination of the tax policy and, therefore, form their beliefs without taking into account how it affects their policy preferences and voting.

In the model with strategic behavior, agents trade their optimism against redistribution. If we let agents ignore this trade-off by nonstrategic behavior, the only thing restricting the optimism of agents are the constraints of the cognitive technology. Therefore, taking \( \tau^* \) as given, the dominant action for the likely poor is to choose \( \lambda = 0 \) for all \( s > 0 \): The lower \( \lambda \) they choose, the higher anticipatory utility they can expect. The loss of income and consumption in period 2 due to less extensive redistribution does not enter the trade-off since the agents do not think they can in any way influence the policy outcome. In the unique equilibrium, all agents recall \( \hat{\sigma} = y_H \), they expect at least average income, and the tax policy is \( \tau^* = \bar{\tau} \). This is curiously the equilibrium even if the likely poor do not value anticipation much and are worse off than if they had all been realists and voted for high taxes.

Interestingly, another way to motivate sincere voting is to derive it as a limiting case of the model with strategic behavior. When the range of the feasible tax rates goes to zero, the threshold \( s^{**} \) goes to zero as well: \( \tau - \bar{\tau} = 0 \) implies \( s^{**}(\chi) = 0 \), and choosing \( \lambda = \bar{\lambda} \) over \( \lambda = \overline{\lambda} \) is optimal for all \( s > 0 \). When the upper and lower bounds of the tax policy coincide, the likely poor cannot affect the tax rate by voting, and it is optimal for them to indulge in optimism.

The likely poor take \( \tau \) as given and choose \( \lambda \) to maximize

\[
U_{0,i}(\lambda) \equiv \lambda u_{0,i}(y_L, y_L, \tau) + (1 - \lambda) u_{0,i}(y_L, y_H, \tau)
+ \lambda [(\delta s + \delta^2][(1 - \tau)y_L + \tau \bar{y}]]
= (1 - \lambda) [\delta s[(1 - \tau)[r(\lambda) y_H + (1 - r(\lambda)) y_L] + \tau \bar{y}]
+ \delta^2[(1 - \tau)y_L + \tau \bar{y}]]. \quad (32)
\]

The best response, independently of the choices of others, is \( \lambda = 0 \). This implies an equilibrium tax rate of \( \tau^* = \bar{\tau} \).

**Proposition 9 (Equilibrium, sincere voting).** If the likely poor do not condition their belief and voting choices on the tax policy, then, for all \( s > 0 \), there is a unique equilibrium, where \( \lambda^* = 0 \), the likely rich recall \( \hat{\sigma}_i = y_H \), and \( \tau^* = \bar{\tau} \).

The utility of a representative likely poor agent is

\[
U_{0,i}(0) = \delta s[(1 - \bar{\tau})[r(0) y_H + (1 - r(0)) y_L] + \tau \bar{y}]
+ \delta^2[(1 - \bar{\tau})y_L + \tau \bar{y}], \quad (33)
\]
whereas if the likely poor had coordinated on choosing \( \lambda = \frac{1}{2(1-q)} \), a representative likely
poor agent would have enjoyed utility

\[ U_{0,i}(\lambda) = \lambda \left[ \delta s[(1 - \tau)y_L + \tau y_H] + \delta^2[(1 - \tau)y_L + \tau y_H] \right] 
+ (1 - \lambda) \left[ \delta s[(1 - \tau)(1 - \bar{\lambda})y_L + \tau y_H] + \tau y_H + \delta^2[(1 - \tau)y_L + \tau y_H] \right], \quad (34) \]

where \( \bar{\lambda} = \frac{1}{2(1 - q)} \). From Lemma 3, we know that if \( s < s^{**} \), (34) is greater than (33).

**Proposition 10 (Welfare of the likely poor).** If \( s < s^{**} \), the likely poor are worse off in the low tax equilibrium, than if they had coordinated on voting for high taxes.

A free-riding problem emerges among the likely poor: for each, it is individually rational to indulge in optimism, but with coordinated actions they could increase their payoffs. This is similar to the public goods game, where the individually rational agents do not contribute even if they would all be better off by contributing. Here the public good is the redistribution, and the cost of contribution is lower anticipatory utility.\(^{29}\)

When the likely poor do not think that their beliefs will influence the policy outcome, they maximize their utility by maximizing their optimism. In contrast to the case of strategic belief formation, which admittedly is a strong requirement on the behavior of the voters, sincere voting always leads to the POUM effect.

5 Conclusion

Overoptimism can provide a mechanism for the POUM hypothesis. This paper has formalized this mechanism by modeling the means and reasons for belief distortion and derived the conditions under which the poor majority of voters distort their beliefs enough to prefer low taxes in the time of voting. The poor do not expropriate the rich because they themselves believe to be rich someday, and they value these beliefs.

These motivated prospects of upward mobility emerge endogenously as a result of agents’ choices between anticipation and consumption. The crucial factors in these choices are the value of anticipation and the relative differences in anticipation and consumption between the potential equilibria.

First, the more the poor expect to gain in anticipation when forming biased beliefs, the more biased these beliefs will be. Specifically, if the incomes or perceived incomes of the rich increase while transfers stagnate, the poor will be more likely to indulge in optimism and vote for low taxes. Hence, the striking result is that contrary to the benchmark model of Meltzer and Richard (1981), where the increase in inequality always increases

\(^{29}\) However, the likely poor coordinating on realism to support high taxes is not a Pareto improvement when considering the whole electorate, as providing a public good in a public good game is. As seen in section 4.2, the likely rich are worse off in the high tax equilibrium if \( s < s^{***} \) which is always the case when \( s < s^{**} \).
the demand for redistribution, in the model proposed here, an increase in inequality can
decrease the demand for redistribution.

Second, the less the likely poor expect to lose in consumption when forming biased beliefs, the more biased these beliefs will be. How much the likely poor can expect to lose in consumption depends on the potential tax rates in different equilibria. Hence, the smaller is the difference in the potential policy outcomes, the more likely the POUM effect is. Specifically, if the voters do not think that their vote has an impact on the policy outcome, that is, if they do not act strategically, they always form the most optimistic beliefs possible and vote for low taxes. If the value of anticipation is low, individually and collectively rational choices diverge, and the poor voters are trapped in a bad equilibrium. By coordinating in voting for higher taxes, they could achieve higher welfare. In this case, the likely poor vote against their own self-interest.

The feasibility of the POUM effect also depends crucially on the specification of the cognitive technology, namely, on the naivete parameter \( \chi \). The less constraining the cognitive technology is, the more voters can bias their beliefs. Therefore, the POUM effect becomes more feasible as an explanation for the limited size of the government in democracies when we specify the cognitive technology with small values of \( \chi \). This can be clearly seen when comparing the results of Minozzi’s (2013) POUM model with our results. In Minozzi’s model agents are naive and can effectively choose their beliefs without the restrictions of prior beliefs or reality. When making the more conventional assumption about the voters forward-looking behavior and setting \( \chi = 1 \) corresponding to the standard Bayesian rationality in belief updating, the poor voters cannot bias their beliefs enough for the POUM effect to occur. This result, however, hinges on the simple specification with linear policy preferences and a policy choice between complete equalization and complete laissez-faire. By exogenously restricting the possible tax policies, it is shown that the POUM effect can be an important factor in voting behavior even if we endow the voters with a more realistic cognitive technology than in Minozzi (2013).

Appendix: Proofs of Lemmas and Propositions

Proof of Lemma 1. Solve first the optimal recall rate given the constraint \( \lambda < \frac{1}{2(1-q)} \). Note that here the constraint set is right-open. However the argument of the maximum
is the lower and closed bound of the set and, hence, the maximum exists.

\[
\lambda = \arg \max_{\lambda \in [0, \frac{1}{2(1-q)}]} \{ \lambda \delta s y_L + \delta^2 y_L \} + (1 - \lambda) [\delta s r(\lambda) y_H + (1 - r(\lambda)) y_L + \delta^2 y_L] \]

\[
\lambda = \arg \max_{\lambda \in [0, \frac{1}{2(1-q)}]} \{(1 - \lambda) r(\lambda)\}
\]

\[
\lambda = \arg \max_{\lambda \in [0, \frac{1}{2(1-q)}]} \left\{ \frac{(1 - \lambda) q}{q + \chi(1-q)(1-\lambda)} \right\} \quad (35)
\]

The derivative of the argument in (35) can be written as

\[
\frac{d}{d\lambda} \left( \frac{(1 - \lambda) q}{q + \chi(1-q)(1-\lambda)} \right) = -\frac{q^2}{[q + \chi(1-q)(1-\lambda)]^2} < 0 \quad (36)
\]

and is always negative since \([\chi(1-q) - q]^2 < [\chi(1-q)]^2\). Therefore, the optimal recall rate is the lower bound of the constraint, that is, \(\lambda = 0\).

The utility, given that the agent chooses \(\lambda \geq \frac{1}{2(1-q)}\), in (12) is independent of the choice of \(\lambda\). The best response is the interval \(\lambda \in [\frac{1}{2(1-q)}, 1]\). Plugging \(\lambda = 0\) into condition \(U_{0,i}^\lambda - U_{0,i}^\lambda \geq 0\) and solving for \(s\) yields (15).

**Proof of Lemma 2.** If \(s \geq s^*\) the likely poor will choose the awareness rate \(\lambda = 0\) and will not want to deviate by Lemma 1. In this equilibrium, no one ever chooses \(\sigma_i = y_L\), so the information set following this action is on off-equilibrium path and the beliefs in the information set following \(\sigma = y_L\) cannot be defined using Bayes rule or its variations. Define \(p \equiv Pr[\sigma_i = y_H | \sigma_i = y_L]\) and require \(p \leq q\) to rule out the possibility of players strategically memorizing a low signal in order to end up with higher expectations. As the profitability of a deviation depends on whether the agents are able to increase their anticipatory utility by deviating, with these off-equilibrium path beliefs the likely rich have no incentive to deviate either. Given the strategies of the likely rich and the likely poor, the policy outcome as function of \(\lambda\) given in (11) implies \(\tau^* = 0\).

If \(s < s^*\), the likely poor choose the awareness rate \(\lambda \in [\frac{1}{2(1-q)}, 1]\) and will not want to deviate by Lemma 1. Given the strategies of the likely poor and the likely rich, the belief in the information set following \(\sigma = y_L\) is \(Pr[\sigma = y_H | \sigma = y_L] = 1\). Therefore, by deviating, a likely rich agent would end up believing to be likely poor and lose anticipatory utility. Hence, the likely rich have no incentive to deviate. The policy outcome as function of \(\lambda\) given in (11) in this case implies \(\tau^* = 1\).

**Proof of Proposition 1.** By Lemma 2 there is an equilibrium with low taxes if \(U_{0,i}^\lambda - U_{0,i}^\lambda \geq 0 \iff s \geq s^*\).

**Proof of Lemma 3.** Solve first the optimal recall rate given the constraint \(\lambda < \frac{1}{2(1-q)}\). Note that here the constraint set is right-open. However the argument of the maximum
is the lower and closed bound of the set and, hence, the maximum exists.

\[
\lambda = \arg \max_{\lambda \in [0, \pi^{1-q}]} \left\{ \lambda \left[ \delta s[(1 - \tau)y_L + \tau y_H] + \delta^2[(1 - \tau)y_L + \tau y_H] \right] + (1 - \lambda) \left[ \delta s[(1 - \tau)[r(\lambda)y_H + (1 - r(\lambda))y_L + \tau y_H] + \delta^2[(1 - \tau)y_L + \tau y_H] \right] \right\}
\]

\[
= \arg \max_{\lambda \in [0, \pi^{1-q}]} \left\{ (1 - \lambda)r(\lambda) \right\}
\]

\[
= \arg \max_{\lambda \in [0, \pi^{1-q}]} \left\{ \frac{(1 - \lambda)q}{q + \chi(1 - q)(1 - \lambda)} \right\} \tag{37}
\]

The derivative of the argument in (37) can be written as

\[
\frac{d}{d\lambda} \left( \frac{(1 - \lambda)q}{q + \chi(1 - q)(1 - \lambda)} \right) = -\frac{q^2}{[q + \chi(1 - q)(1 - \lambda)]^2} < 0 \tag{38}
\]

and is always negative. The optimal recall rate is therefore given by the lower bound of the constraint, \( \bar{\lambda} = 0 \).

Solve the optimal recall rate given the constraint \( \lambda \geq \frac{1}{2(1-q)} \).

\[
\bar{\lambda} = \arg \max_{\lambda \in [\frac{1}{2(1-q)}, 1]} \left\{ \lambda \left[ \delta s[(1 - \tau)y_L + \tau y_H] + \delta^2[(1 - \tau)y_L + \tau y_H] \right] + (1 - \lambda) \left[ \delta s[(1 - \tau)[r(\lambda)y_H + (1 - r(\lambda))y_L + \tau y_H] + \delta^2[(1 - \tau)y_L + \tau y_H] \right] \right\}
\]

\[
= \arg \max_{\lambda \in [\frac{1}{2(1-q)}, 1]} \left\{ (1 - \lambda)r(\lambda) \right\}
\]

\[
= \arg \max_{\lambda \in [\frac{1}{2(1-q)}, 1]} \left\{ \frac{(1 - \lambda)q}{q + \chi(1 - q)(1 - \lambda)} \right\} \tag{39}
\]

The derivative of the argument can be written as

\[
\frac{d}{d\lambda} \left( \frac{(1 - \lambda)q}{q + \chi(1 - q)(1 - \lambda)} \right) = -\frac{q^2}{[q + \chi(1 - q)(1 - \lambda)]^2} < 0 \tag{40}
\]

and as before, is always negative. The optimal recall rate is therefore given by the lower bound of the constraint, \( \bar{\lambda} = \frac{1}{2(1-q)} \). Plugging in the optimal recall rates \( \lambda \) and \( \bar{\lambda} \) into condition \( U_{0,i}^{\lambda} - U_{0,i}^{\bar{\lambda}} \geq 0 \) and solving for \( s \) yields (23).

Proof of Lemma 4. If \( s \geq s^{**} \) the likely poor will choose the awareness rate \( \lambda = 0 \) and will not want to deviate by Lemma 3. In this equilibrium, no one ever chooses \( \hat{\sigma}_i = y_L \), so the information set following this action is on off-equilibrium path and the beliefs in the information set following \( \hat{\sigma}_i = y_L \) cannot be defined using Bayes rule or the variation of the Bayes rule or its variations. Define \( p \equiv Pr[\sigma_i = y_H|\hat{\sigma}_i = y_L] \) and require \( p \leq q \) to rule out the possibility of players strategically memorizing a low signal in order to end up with higher expectations. As the profitability of a deviation depends on
whether the agents are able to increase their anticipatory utility by deviating, with these off-equilibrium path beliefs the likely rich have no incentive to deviate either. Given the strategies of the likely rich and the likely poor, the policy outcome as function of \( \lambda \) given in (11) implies \( \tau^* = \tau \).

If \( s < s^{**} \), the likely poor choose the awareness rate \( \lambda = \frac{1}{2(1-q)} \) and will not want to deviate by Lemma 3. Given the strategies of the likely poor and the likely rich, the belief in the information set following \( \hat{\sigma} = y_\ell \) is \( Pr[\sigma = y_\ell | \hat{\sigma} = y_\ell] = 1 \). Therefore, by deviating, a likely rich agent would end up believing to be likely poor and lose anticipatory utility. The likely rich have no incentive to deviate. The policy outcome as a function of \( \lambda \) given in (11) in this case implies \( \tau^* = \pi \).

**Proof of Proposition 2.** By Lemma 4, there is an equilibrium with low taxes if \( U_{0,i}^\lambda - U_{0,i}^\pi \geq 0 \iff s \geq s^{**} \).

**Proof of Proposition 3.**

\[
\frac{\partial s^{**}}{\partial \chi} = \frac{-\delta(\tau - \tau)q[(1 - \tau)\frac{\partial r(0)}{\partial \chi} - (1 - \tau)(1 - \lambda)\frac{\partial r(\lambda)}{\partial \chi}]}{[(1 - \tau)r(0) - (1 - \tau)(1 - \lambda)r(\lambda) - (\tau - \tau)q]^2},
\]

where

\[
(1 - \tau)(1 - \lambda)\frac{\partial r(\lambda)}{\partial \chi} = (1 - \tau)(1 - \lambda)\frac{q(1 - q)(1 - \lambda)^2}{[q + \chi(1 - q)(1 - \lambda)]^2} - (1 - \tau)\frac{q(1 - q)}{[q + \chi(1 - q)]^2} < 0
\]

since

\[
(1 - \tau)(1 - \lambda)\frac{q(1 - q)(1 - \lambda)^2}{[q + \chi(1 - q)(1 - \lambda)]^2} < (1 - \tau)\frac{q(1 - q)}{[q + \chi(1 - q)]^2}
\]

\[
\iff (1 - \tau)[q^2(1 - \lambda)^2 + 2q\chi(1 - q)(1 - \lambda)^2 + \chi^2(1 - q)^2(1 - \lambda)^2]
\]

\[
< (1 - \tau)[q^2 + 2q\chi(1 - q)(1 - \lambda) + \chi^2(1 - q)^2(1 - \lambda)^2]
\]

which holds since

\[
q^2(1 - \lambda)^2 + 2q\chi(1 - q)(1 - \lambda)^2 < q^2 + 2q\chi(1 - q)(1 - \lambda)
\]

and \( 1 - \tau < 1 - \tau \). Therefore \( \frac{\partial s^{**}}{\partial \chi} > 0 \).

**Proof of Proposition 4.**

\[
\frac{\partial s^{**}}{\partial \tau} = \frac{\delta q[(1 - \tau)[r(\lambda) - (1 - \lambda)r(\lambda)]]}{[(1 - \tau)r(0) - (1 - \lambda)(1 - \tau)r(\lambda) - (\tau - \tau)q]^2}
\]

where

\[
r(\lambda) - (1 - \lambda)r(\lambda) = \frac{q^2}{[q + \chi(1 - q)]^2(1 - q)[q + \lambda^2(1 - 2q)]} > 0.
\]
Therefore $\frac{\partial s^{**}}{\partial \tau} > 0$.

**Proof of Proposition 5.**

\[
\frac{\partial s^{**}}{\partial \tau} = -\frac{\delta q[(1 - \tau)r(\lambda) - (1 - \lambda)r(\bar{\lambda})]}{[(1 - \tau)r(0) - (1 - \lambda)(1 - \tau)r(\bar{\lambda}) - (\tau - \bar{\tau})q]^2} \tag{47}
\]

where

\[
r(\lambda) - (1 - \lambda)r(\bar{\lambda}) = \frac{q^2}{[q + \chi(1 - q)]^2(1 - q)[q + \chi\frac{1}{2}(1 - 2q)]} > 0. \tag{48}
\]

Therefore $\frac{\partial s^{**}}{\partial \tau} < 0$.

Lemma 5 establishes a result that is useful in determining the sign of further partial derivatives of $s^{**}$.

**Lemma 5.** $(1 - \tau)r(0) - (1 - \lambda)(1 - \tau)r(\bar{\lambda}) > 0$

**Proof of Lemma 5.**

\[
(1 - \tau)r(0) - (1 - \lambda)(1 - \tau)r(\bar{\lambda}) = \frac{2(1 - q)(q + \chi\frac{1}{2}(1 - 2q))(1 - \tau) - (q + \chi(1 - q))(1 - 2q)(1 - \tau)q}{(q + \chi(1 - q))^2(1 - q)(q + \chi\frac{1}{2}(1 - 2q))}. \tag{49}
\]

Define

\[
a \equiv 2(1 - q)(q + \chi\frac{1}{2}(1 - 2q)), \tag{50}
\]

\[
b \equiv (q + \chi(1 - q))(1 - 2q), \tag{51}
\]

and write the numerator of (49) as

\[
[a(1 - \tau) - b(1 - \tau)]q \\
\iff [a - b - (a\tau - b\tau)]q. \tag{52}
\]

The numerator of (49) is positive if

\[
a\tau - b\tau < a - b \\
\iff a(1 - \tau) > b(1 - \tau) \tag{53}
\]

which holds since $a - b = q > 0$ and $\tau > \bar{\tau}$ implies $1 - \tau > 1 - \bar{\tau}$. The denominator of (49) is positive for all $q \in (0, 1)$. Since both the denominator and the numerator of (49) are positive, the expression is positive and this establishes the result.

**Proof of Proposition ???.** Write $s^{**}$ as

\[
s^{**} = \frac{\delta(\tau - \bar{\tau})(\bar{y} - y_L)}{t_{net}(\lambda, \bar{\tau}) - t_{net}(\lambda, \bar{\lambda})} \tag{54}
\]
Proof of Proposition 7. Write $s^{**}$ as

$$s^{**} = \frac{\delta(\bar{\tau} - \tau)(\bar{y} - y_L)}{t_{net}(\bar{\lambda}, \bar{\tau}) - t_{net}(\lambda, \tau)}$$

where

$$t_{net}(\lambda, \tau) := \lambda[(1 - \tau)y_L + \tau\bar{y}] + (1 - \lambda)[(1 - \tau)(r(\lambda)y_H + (1 - r(\lambda))y_L) + \tau\bar{y}]$$

is the ex ante expectation of the expected net income of the likely poor in period 1 given $\lambda$ and $\tau$, and where $\bar{\lambda} = 0$ and $\bar{\lambda} = \frac{1}{2(1-q)}$. Compute the partial derivative with respect to $y_H$ holding the average income $\bar{y}$ constant.

$$\frac{\partial s^{**}}{\partial y_H} = -\frac{\delta(\bar{\tau} - \tau)(\bar{y} - y_L)(1 - \bar{\lambda})(1 - \bar{\tau})r(\bar{\lambda})}{[(1 - \bar{\tau})r(0) - (1 - \bar{\lambda})(1 - \bar{\tau})r(\bar{\lambda}) - (\bar{\tau} - \tau)q]^2(\Delta y)^2} < 0.$$  

By lemma 5, the derivative is negative for all parameter values, which implies that $s^{**}$ decreases in $y_H$, when $\bar{y}$ is held constant.\(^{30}\) Compute the partial derivative with respect to $y_L$ holding the average income $\bar{y}$ constant.

$$\frac{\partial s^{**}}{\partial y_L} = -\frac{\delta(\bar{\tau} - \tau)(y_H - \bar{y})(1 - \bar{\lambda})(1 - \bar{\tau})r(\bar{\lambda})}{[(1 - \bar{\tau})r(0) - (1 - \bar{\lambda})(1 - \bar{\tau})r(\bar{\lambda}) - (\bar{\tau} - \tau)q]^2(\Delta y)^2} < 0$$

By lemma 5, the derivative is negative for all parameter values, which implies that $s^{**}$ decreases in $y_L$, when $\bar{y}$ is held constant.\(^{31}\)

**Proof of Proposition 7.** Write $s^{**}$ as

$$s^{**} = \frac{\delta(\bar{\tau} - \tau)(\bar{y} - y_L)}{t_{net}(\bar{\lambda}, \bar{\tau}) - t_{net}(\lambda, \tau)}$$

where

$$t_{net}(\lambda, \tau) := \lambda[(1 - \tau)y_L + \tau\bar{y}] + (1 - \lambda)[(1 - \tau)(r(\lambda)y_H + (1 - r(\lambda))y_L) + \tau\bar{y}]$$

is the ex ante expectation of the expected net income of the likely poor in period 1 given $\lambda$ and $\tau$, and where $\bar{\lambda} = 0$ and $\bar{\lambda} = \frac{1}{2(1-q)}$. Compute the partial derivative with respect to $\bar{y}$ holding $y_L$ and $y_H$ constant.

$$\frac{\partial s^{**}}{\partial \bar{y}} = \frac{\delta(\bar{\tau} - \tau)(1 - \bar{\lambda})(1 - \bar{\tau})r(\bar{\lambda})(y_H - y_L)}{[(1 - \bar{\tau})r(0) - (1 - \bar{\lambda})(1 - \bar{\tau})r(\bar{\lambda}) - (\bar{\tau} - \tau)q]^2(\Delta y)^2} > 0$$

By lemma 5, the derivative is positive for all parameter values, which implies that $s^{**}$ increases in $\bar{y}$, when $y_L$ and $y_H$ are held constant.\(^{32}\)

**Proof of Proposition 8.** First, consider part (i). If $s^{**}(\chi) \geq s^{**}(\chi)$, then always when there is a low tax equilibrium, the likely rich are worse off in it. This condition is satisfied

\(^{30}\)By letting $\tau = 1$, $\bar{\tau} = 0$, $\chi = 0$, and $\delta = 1$, we get $\frac{\partial s^{**}}{\partial y_H} = -\frac{\bar{y} - y_L}{(y_H - y)}$, which is the result in Minozzi (2013).

\(^{31}\)By letting $\tau = 1$, $\bar{\tau} = 0$, $\chi = 0$, and $\delta = 1$, we get $\frac{\partial s^{**}}{\partial y_L} = -\frac{1}{(y_H - y)}$, which is the result in Minozzi (2013).

\(^{32}\)By letting $\tau = 1$, $\bar{\tau} = 0$, $\chi = 0$, and $\delta = 1$, we get $\frac{\partial s^{**}}{\partial \bar{y}} = \frac{y_H - y_L}{(y_H - y)}$, which is the result in Minozzi (2013).
for $\chi = 1$ since $s^{**}(1) = s^{***}(1)$. This establishes part (i). Consider now part (ii). Show first that $s^{**}(\chi) < s^{***}(\chi) \forall \chi \in [0,1)$.

$$s^{**}(\chi) < s^{***}(\chi)$$
$$\iff s^{**}(\chi) - s^{***}(\chi) < 0$$
$$\iff -(1 - \tau)r(0) + \frac{1}{2}(1 - \tau)r \left( \frac{1}{2(1 - q)} \right) + q(\tau - \tau) < 0. \quad (61)$$

Now show that the left-hand side of (61) is increasing in $\chi$. The derivative of the left-hand side of (61) with respect to $\chi$ is

$$\left(1 - \tau\right) \frac{q(1 - q)}{[q + \chi(1 - q)]^2} - \frac{1}{4} \left(1 - \tau\right) \frac{q(1 - 2q)}{[q + \frac{1}{2} \chi(1 - 2q)]^2}$$

$$\geq \left(1 - \tau\right) \frac{q(1 - q)}{[q + \chi(1 - q)]^2} - \frac{1}{4} \left(1 - \tau\right) \frac{q(1 - q)}{[q + \frac{1}{2} \chi(1 - 2q)]^2}$$

$$= \left(1 - \tau\right) q(1 - q) \left[ \frac{1}{[q + \chi(1 - q)]^2} - \frac{1}{[2q + \chi(1 - 2q)]^2} \right], \quad (62)$$

where (62) is positive for all $\chi \in [0,1)$ since

$$[q + \chi(1 - q)]^2 < [2q + \chi(1 - 2q)]^2$$

$$\iff [q + \chi(1 - q)]^2 < [q + \chi(1 - q) + (1 - \chi)q]^2 \quad (63)$$

for all $\chi \in [0,1)$. Hence, the derivative of the left-hand side of (61) with respect to $\chi$ is positive so the left-hand side of (61) is increasing in $\chi$. Since $s^{**}(1) = s^{***}(1)$, the left-hand side of (61) is zero when $\chi = 1$. Hence, since the left-hand side of (61) is increasing in $\chi$, it has to be negative for $\chi \in [0,1)$. This establishes that $s^{**}(\chi) < s^{***}(\chi) \forall \chi \in [0,1)$.

Now since $s^{**}(\chi) < s^{***}(\chi) \forall \chi \in [0,1)$, an existence of a low tax equilibrium does not necessarily mean that $s > s^{**}$ and the likely rich are worse off only if $s > s^{***}$. This establishes part (ii).

**Proof of Proposition 9.** Denote the optimal choice of the likely poor by $\lambda^{*}$.

$$\lambda^{*} = \arg \max_{\lambda \in [0,1]} \left\{ (1 - \lambda) \left[ \delta s((1 - \tau)[r(\lambda)y_H + (1 - r(\lambda))y_L] + \tau \delta y + \delta^2[(1 - \tau)y_L + \tau \delta y] \right]ight.$$

$$\left. + \lambda \left[ (\delta s + \delta^2)(1 - \tau)y_L + \tau \delta y \right] \right\}$$

$$= \arg \max_{\lambda \in [0,1]} \left\{ (1 - \lambda)r(\lambda) \right\}$$

$$= \arg \max_{\lambda \in [0,1]} \left\{ \frac{(1 - \lambda)q}{q + \chi(1 - q)(1 - \lambda)} \right\} \quad (64)$$

The derivative of the argument can be written as

$$\frac{d}{d\lambda} \left( \frac{(1 - \lambda)q}{q + \chi(1 - q)(1 - \lambda)} \right) = -\frac{q^2}{[q + \chi(1 - q)(1 - \lambda)]^2} < 0 \quad (65)$$
and is always negative. The optimal recall rate is therefore given by the lower bound of the constraint, $\lambda^* = 0$. Since the maximum is unique, the choice $\lambda = \lambda^*$ strictly dominates all other choices of $\lambda$ and, hence, the unique equilibrium is all the likely poor choosing $\lambda^*$.

Proof of Proposition 10. By Lemma 3, if $s < s^{**}$, (34) is greater than (33).

References


