TAX COMPETITION WITH MOBILE LABOR, RESIDENTS, AND CAPITAL

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Abstract

We construct a unifying theoretical model of tax competition that includes mobile workers, mobile residents, and mobile capital – and is therefore appropriate for the study of local government policy. Local governments are atomistic with respect to the world capital market, but are linked by commuting patterns, the cost of which is endogenously determined by congestion. Tax competition involving capital creates fiscal externalities, which then alter decisions both about where to live and work. These fiscal externalities result from changing commuting patterns: when a jurisdiction increases its industrial capital tax rate, some workers change their work location to another jurisdiction, raising public good provision there. In this way, competition for workers affects competition for households. Commuting gives rise to “tax exporting,” with a higher capital tax in the (central) city partially borne by non-resident commuters from the suburbs, allowing the capital tax to remain optimal even when head taxes are available. We also consider taxes on labor, residents and a property tax. As well, we allow for the possibility of a “congestion” tax, a tax on commuters implemented by the city.

JEL: H2, H4, H7, R5

Keywords: tax competition, local governments, commuting, labor mobility

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1 Introduction

Theoretical models of tax competition often do not distinguish between international tax competition and tax competition within a metropolitan area.\footnote{In his review of tax competition, for example, Wilson (1999) writes, “I discuss models in which the governments may be interpreted as local, state, or provincial governments within a country, or as countries within the world economy. I often use the term ‘region’ to encompass these various interpretations.”} But there are important differences. Starting with Zodrow and Mieszkowski (1986) and Wilson (1986), models of tax competition have often assumed that capital is mobile across atomistic (price-taking) jurisdictions, but individuals do not choose where to work or live. Indeed, this type of model, with assumptions perhaps more reasonable at the international setting, is often used in empirical work using local tax rate data.\footnote{For his empirical study of tax competition in a metropolitan area, for example, Brueckner (2000) develops a tax competition model with Tiebout sorting of households, but household mobility occurs before taxes are chosen, and the model behaves similarly to models of asymmetric tax competition without household mobility. In fact, the total supply of capital is treated as fixed, although jurisdictions certainly have access to capital on national and international markets. See also Brueckner and Saavedra (2001).} These class of models differ from models in the spirit of Braid (1996), where governments set capital taxes in the presence of mobile labor within a metropolitan area, but where household residence is fixed. At the opposite extreme is Hoyt (1993), which analyzes local fiscal policies in a metropolitan area where households can change residences, but ignores labor market considerations by taking household incomes as exogenous. At the same time, the income tax competition literature (Lehmann, Simula and Trannoy 2014), has assumed that the place of employment and residence are the same, necessarily implying that tax-induced migration involves a change in both employment and residence. At the international, or perhaps even state level, it is reasonable to assume that labor mobility also involves a change of residence. This is not necessarily the case at the local level, where jobs and/or residence may change separately or simultaneously; but such a simultaneous decision of residence and employment is not present in the literature.

The purpose of the present paper is to construct a unifying theoretical model of capital and income tax competition that includes mobile workers, mobile residents, and mobile capital; and is therefore appropriate for empirical analysis using local tax rate data. As such, we create a general tax competition model that features the mobile capital component of Zodrow and Mieszkowski (1986)/Wilson (1986), the mobile labor component of Braid (1996)/Lehmann, Simula and Trannoy (2014), and add to it the novel possibility that residence may also be mobile, as in Hoyt (1993). Such a general model has broad applicability for the empirical literature on tax competition.

As such, our model draws inspiration from the empirical tax competition literature, summarized in Agrawal, Hoyt and Wilson (2019), which has found that governments exhibit
strong spatial interdependence in their tax rates despite being atomistic with respect to world capital markets.\(^3\) Such interdependence is often explained in theoretical models by assuming that governments have market power in the capital market (Keen and Konrad 2013), but such an observation is unsatisfying at the municipal level: indeed, no locality in the world, let alone no country in the world, has more than a small share of the global capital stock.

A primary goal of our model is to explain why empirical studies estimate that strategic interactions arise at the local level, even though jurisdictions are atomistic with respect to capital markets. To do this, inspired by Wildasin (2014),\(^4\) our model links local governments through locally mobile labor. In particular, if labor is locally mobile within a metropolitan area, then although governments are atomistic with respect to the capital market, they have some market power in the labor market. This translates to strategic policy interdependence in both labor and capital taxes, which in turn may influence residential decisions.

We are not the first to consider labor linkages across municipalities. Braid (1996) has done pioneering work on tax competition in a metropolitan area, and we follow this literature by assuming that the metropolitan area has access to capital at a fixed after-tax rate of return.\(^5\) This makes the model fundamentally different from standard models of capital tax competition, where tax competition is typically modeled as competition over a supply of capital that is completely inelastic. In those models, a reduction in one jurisdiction’s tax rate reduces the supply of capital to other jurisdictions. Here, jurisdictions do not directly compete for capital. Nevertheless, a source-based tax does affect the tax bases for other jurisdictions, because it alters worker decisions about where to work. Wildasin (2014) and Janeba and Osterloh (2013) have also made fundamental contributions relating to local tax competition.

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\(^3\)For example, Brueckner and Saavedra (2001) finds strong positive strategic interactions among municipalities competing over property tax rates. Although many studies find non-zero reaction functions at the local level, some recent studies find a tax reaction function with zero slope (Lyytikäinen 2012; Baskaran 2014). Non-zero reaction functions have also been found at the state and national level, though this primarily centers on corporate tax competition (Devereux, Lockwood and Redoano 2008). Of course, the empirical literature is not only confined to capital tax reaction functions and much of the literature has found significant interactions on local retail sales taxes and local income taxes (Agrawal 2015; Agrawal 2016; Eugster and Parchet 2018; Parchet 2019), but these are settings where local governments may have market power due to the mobility of the tax base.

\(^4\)Wildasin (2014) shares much of the same motivation as our paper. He seeks to explain strategic policy interdependence among local governments that are atomistic by capital markets but that are linked by commuting. In his model, capital and labor are mobile within the metropolitan area as well as the world. However, the jurisdiction of residence cannot be chosen and commuting costs are not endogenous. Because residence is fixed, he considers land value maximization while we consider welfare maximization. Furthermore, Wildasin (2014) considers only a tax on mobile capital. Nonetheless, Wildasin (2014) represents an extremely important contribution to the local tax competition literature.

\(^5\)See also Braid (2000), Braid (2002), Braid (2005), and Braid (2013). One issue on which Braid’s work has focused is the choice of tax instruments by local governments: a source-based capital tax, source-based wage tax, or head tax on residents. In different settings, he has found that the wage tax dominates the capital tax, and we also replicate this result.
capital tax competition.

As noted, our model departs from the prior literature by assuming that not only is labor interjurisdictionally mobile, but also residents choose where to live. Indeed, individuals in our model may make independent decisions on where to live and work: that is, the place of residence need not be the same at the place of work. Data from the American Community Survey and the German Socio-Economic Panel suggest that a high fraction of interjurisdictional job changes do not entail changing the municipality of residence. In this sense, we view our model as synthesizing several types of tax competition models: “regional models,” which focus on capital mobility; and “metropolitan models,” which focus on employment or resident location decisions; and “competitive income tax models,” which implicitly assume that the place of work and residence are the same. To model the location decisions of residents, we introduce a housing market with an endogenously-determined price of housing. Also, whereas the prior literature has assumed either no commuting costs or exogenously-fixed commuting costs, commuting costs are endogenously determined by congestion in our model. Such a model is consistent with the empirical literature demonstrating that state income taxes influence the commuting patterns of high-income households in cross-border metropolitan areas, but that these effects depend on whether the taxes are employment or residence based (Agrawal and Hoyt 2018; Coomes and Hoyt 2008).

The basic structure of our model is as follows. We consider a metropolitan area with two sub-metropolitan jurisdictions. Private consumption features two sectors: a tradeable industrial good produced from labor and industrial capital and a nontradeable good such as housing, produced from housing capital and land. To finance a public service, governments have access to taxes on industrial capital, employment-based wage taxes, residence-based income taxes, property taxes, and a tax on commuters. Individuals may live and work in different jurisdictions, but if commuting, the individual incurs a cost that is a function of the number of commuters.

One of our main insights is that, even though governments are atomistic with respect to capital markets, tax competition involving “industrial capital” creates fiscal externalities, which then alter decisions about where to live. These fiscal externalities result from changing commuting patterns: When a jurisdiction increases its industrial capital tax rate, some workers change their work location to another jurisdiction, which raises public service provision there. In this way, competition for workers affects competition for households.

We also show that commuting necessarily gives rise to “tax exporting.” In particular, a

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6Generalizations to include many local governments are discussed briefly.
7In the United States, local governments often select tax credits to make taxes either purely employment or residence based. Indeed, some municipalities tax both employment-based income and residence-based income such that an individual living and working in the city would pay both taxes.
higher capital tax in the city will be partially borne by non-resident commuters from the suburbs in the form of a lower wage, and this form of tax exporting allows the capital tax to remain optimal, even when a head tax is also available. The tax competition literature has struggled to explain why head taxes or equivalent tax instruments do not eliminate the use of capital taxes. Our model provides one explanation. We also show that tax exporting can arise through the use of the commuter tax, with the tax set at the revenue-maximizing level rather than at a level equal to the marginal congestion cost. As well as motivating the use of employment-based taxes, a labor or capital tax, our model makes it clear that these taxes are valuable instruments only when jurisdictions are net recipients of commuters, while jurisdictions that are net providers of commuters will choose to rely on the use of residential-based taxes, the head tax or property tax. However, either type of jurisdiction will choose to set a revenue-maximizing tax rate on commuters. If a suburb cannot directly tax commuters, then it will do so indirectly by subsidizing suburban workers. If such labor subsidies are not available, then the suburb will have an incentive to subsidize capital, thereby raising suburban wages.

In the next section, we describe the model. The equilibrium conditions and some comparative static results are presented in Section 3. In Section 4, we summarize the equilibrium policies chosen when the cities have access only to a single tax instrument: either the capital, property, labor or residential tax. Our analysis highlights the characteristics of the equilibrium policy; specifically, whether the public service is over- or underprovided, and the existence and sign of an externality. Section 5 characterizes equilibrium policies when the cities can employ multiple tax instruments. We begin with pairing of the residential tax, which is a head tax because labor supply is inelastic, with each of the other taxes, highlighting conditions under which a city will choose to tax another base in addition to residents. We then discuss the use of the commuter tax with the other tax instruments. Our final mix of tax instruments is the capital and property tax, highlighting the different impacts of an employment-linked tax (industrial capital) and a resident-based tax (residential property). As estimating the slopes of reaction functions for tax competition games has been a major focus of the empirical tax competition literature, we analyze these slopes in Section 6. Section 7 generalizes the results to multiple suburbs. Section 8 concludes.

2 Model and Notation

There are two jurisdictions, the central city (jurisdiction 1) and the suburb (jurisdiction 2), with $N$ identical individuals who can reside in either jurisdictions as well as be employed in either jurisdiction. Let $N_i$, $i = 1, 2$, and $L_i$, $i = 1, 2$, be the population and the labor
force in jurisdiction $i$. The labor force in each jurisdiction depends on its population and the number of commuters to or from the locality with $L_i = N_i - N_{ij}$ where $N_{ij} > 0$ signifies a net outflow of commuters from jurisdiction $i$ and $N_{ij} < 0$ signifies a net inflow to jurisdiction $i$. We assume that there are net commuter flows out of the suburbs ($N_{21} > 0$) and into the city ($N_{12} < 0$).\footnote{While we use the terminology “net” to refer to commuter flows, in fact, with the identical preferences commuting only occurs from the suburbs to the city in equilibrium.} Of course, $N_{12} = -N_{21}$. We denote the labor-to-population ratio in jurisdiction $i$ by $l_i = \frac{L_i}{N_i} = 1 - \frac{N_{ij}}{N_j}$, making the ratio of commuters out of the jurisdiction to population, $1 - l_i$.

The tradeable commodity is produced using capital ($K$) and labor ($L$), as described by the production function, $I_i = F_i(K^I_i, L_i)$. Constant returns to scale are assumed, implying zero profits:

$$1 = a^L_i w_i + a^k_i \left(\rho + t^I_i\right)$$

where the unit price of the industrial good is fixed at one, $w_i$ is the wage rate, $\rho$ is the net price of capital, $t^I_i$ is the tax on (industrial) capital used in the production of the tradeable commodity. Capital is globally traded at the exogenous price $\rho$. A nontradeable commodity (housing) is produced according to the constant-returns production function, $H_i = H_i(K^H_i, Q_i)$ where $K^H_i$ is capital and $Q_i$ is the (fixed) quantity of land used in the production of housing in jurisdiction $i$. The supply of housing is given by $H_i(p_i)$ where $p_i$ is the net-of-tax price of housing in jurisdiction $i$.

$$U_{ij} = x_{ij} + U^H(h_i) + V(G_i), \quad i, j = 1, 2$$

The utility function for a resident of jurisdiction $i$ working in jurisdiction $j$ is given by where $x_{ij}$ is consumption of a tradeable commodity; $h_i$ is consumption of a nontradeable commodity (housing) and $G_i$ is the level of public service provided to residents of jurisdiction $i$. The indirect utility function can be expressed as

$$U_{ij} = Y_{ij} + S(q_i) + V(G_i)$$

where $Y_{ij}$ is income net of commuting costs and taxes for resident of $i$ working in $j$, and $q_i$ is the gross-of-tax price of housing: $q_i = p_i \left(1 + t^H_i\right)$, where $t^H_i$ is the ad valorem tax on housing. The function, $S(p_i(1 + t^H_i)) = U(h(p_i(1 + t^H_i))) - (p_i(1 + t^H_i))h(p_i(1 + t^H_i))$, is consumer surplus from housing, where $h(p_i + t^H_i)$ is the demand for housing. We assume that all housing is owner-occupied. Although this assumption eliminates standard incentives to export tax burdens to non-residents, we shall see that both tax-exporting and tax-importing...
incentives remain. Individuals choose a residential location and purchase houses for an initial price, \( p_i^0 \) for a house of size \( h_i^0 \) in jurisdiction \( i \), and then choose the jurisdiction’s tax and expenditure policies. Under perfect foresight, the resulting value of the house is \( p_i = p_i^0 \) in equilibrium. For notational simplicity, we treat \( p_i \) as also the rent on the house, but we could also specify a future use of the land and housing capital, so that the current rent would be less than the house value \( p_i \).

\[ Y_{ij} = \text{income net of commuting and taxes for resident of } i \text{ working in } j; Y_{22} = w_2 - t_L^2 - t_R^2 + (p_2 - p_0^2) h_2^0; Y_{21} = w_1 - t_L^1 - t_R^1 + (p_2 - p_0^2) h_2^0 - A(N_{21}); Y_{11} = w_1 - t_L^1 - t_R^1 + (p_2 - p_0^2) h_2^0. \]

The term \( A(N_{21}) \) is the commuting cost from the suburb to the city, with \( A' > 0 \) and \( A'' > 0 \) denoting increasing marginal congestion costs.

We assume that there are constant costs with respect to both the level of the public service and population, that is \( C_i(N_i,G_i) = N_iG_i \). Then, the government budget constraint is given by

\[ (tI^i_k + t^L_i) l_i + t^C_i (l_1 - 1) + [t^P_i p_i h_i (q_i) + t^R_i] = G_i \]

(4)

where \( k_i \) denotes the capital-labor ratio in the production of the tradeable good and the term \( t^C_i (l_1 - 1) \) is the tax revenue from the commuter tax, available to both the suburbs and the city.

3 Equilibrium and Comparative Statics

Three equal utility conditions apply: 1) equal utility for a resident of the suburb who works there and for a suburban resident who works in the central city; 2) equal utility for a resident/worker in the suburb and a resident/worker in the city; and 3) equal utility for a resident/commuter of the suburb and a resident/worker in the city. Of course, if two of the equal utilities are met, then by transitivity, the third will be met.

The first equal-utility condition requires that the after-tax wage in a suburb equals the after-tax wage in the city less commuting costs:

\[ w_2 - t_L^2 = w_1 - t_L^1 - t_C^1 - t_C^2 - A(N_{21}). \]

(5a)

Note that the commuting taxes imposed by both the suburb and the city affect the return to the commuters working in the city.

9We omit possible capital endowments as they do not affect the analysis.
The second equal-utility condition may be stated

\[
w_2 + (p_i - p_i^0) h_i^0 - t_2^L - t_2^R + S \left(p_2(1 + t_2^P)\right) + V (G_2) = \]
\[
w_1 + (p_i - p_i^0) h_i^0 - t_1^L - t_1^R + S \left(p_1(1 + t_1^P)\right) + V (G_1), \ i = 1, 2. \tag{5b}
\]

This condition states that a resident owning a house in jurisdiction \(i = 1, 2\) must be indifferent between living in 1 or 2, regardless of the current (net) value of the house.\(^\text{10}\)

The final equilibrium condition is for the housing market. The fact that demand for housing is the product of housing demand per resident, \(h(p_i(1 + t_i^P))\), and population, \(N_i\), implies that \(N_i = \frac{H(p_i)}{h(p_i(1 + t_i^P))}\), giving the equilibrium condition,

\[
\frac{H (p_1)}{h (p_1(1 + t_1^P))} + \frac{H (p_2)}{h (p_2(1 + t_2^P))} = N. \tag{5c}
\]

Equations (5b) and (5c) determine the housing prices \((p_1, p_2)\) as functions of the taxes and public services.

### 3.1 Comparative Statics

Critical to understanding the policy choices of the governments is how these policies affect wage rates, population, the labor-population ratio, and housing prices. In this section we employ the equilibrium conditions to outline the comparative statics on these endogenous variables; that is, how balanced-budget changes in tax rates and public service levels affect them.

We begin by considering how wage rates are affected by changes in tax rates. Constant returns to scale ensures that the wage rate in a jurisdiction is affected only by the tax on capital in that jurisdiction. Differentiating (1) with respect to \(t_i^I\) gives

\[
w_{ti}^I = -\frac{a_i^k}{a_i^L} = -k_i \tag{6}
\]

where \(k_i\) is the capital-labor ratio in jurisdiction \(i\) and the notation \(z_{ij}^x\) refers to the derivative of a variable \(z\) in jurisdiction \(j\) with respect to a tax \(t^x\), \(x = I, L, R, P, C\) in jurisdiction \(i\). As the tax on industrial capital is the only tax that affects the wage rate, the capital-labor ratio is strictly a function of the relative prices of capital and labor, \(k_i = k_i \left(\frac{a_i^L}{a_i^k}\right)\), \(k_i' < 0\).

An increase in \(t_i^I\) increases this factor-price ratio, causing \(k_i\) to increase.

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\(^{10}\)As utility is quasilinear with the linear component being private consumption, there are no income effects for housing and the public service meaning the “bids” for housing in either jurisdiction are independent of net income.
As can be seen from (5a) and (6) only the taxes on industrial capital, labor, and commuters affect the number of commuters. Differentiating (5a) with respect to \( t_I^i \), \( t_L^i \), and \( t_C^i \) gives
\[
N_{ij}^i t_I^i = \frac{k_i}{A'} > 0, \quad N_{ij}^i t_L^i = \frac{1}{A'} > 0, \quad \text{and} \quad N_{ij}^i t_C^i = \frac{1}{A'} > 0. \quad (7)
\]
The taxes on capital and labor in the suburb increase the outflow of commuters to the city while either tax in the city reduces the inflow of commuters from the suburbs. Of course, the tax on commuters by either the city or the suburbs reduces the inflow of commuters to the city.

As seen from the government budget constraint, (4), the capital and labor tax bases depend on the labor-population ratio, \( l_i = 1 - \frac{N_{ij}}{N_i} \). Then the impacts of tax policies on the government budget and public services depend critically on how \( l_i \) changes with taxes. Differentiating \( l_i \) with respect to tax \( t_x^i \) gives
\[
l_i^i t_x^i = -\frac{1}{N_i} N_{ij}^i t_x^i + (1 - l_i) \hat{N}_{i}^i \quad (8)
\]
where \( \hat{N}_{i}^i = \frac{N_{i}^i}{N_i} \) is the “percentage” change in population. Other terms with a hat (\( \hat{\text{\text{\_\_}}} \)) defined analogously. As seen in (8), the impact of a tax on the labor-population ratio, \( l_i^i t_x^i \), consists of two components: the impact of the tax on the number of commuters, and the impact of the tax on population. The effect of an increase in commuters from the jurisdiction has an unambiguously negative effect on the \( l_i \) but the effect of an increase in the population has an ambiguous effect: an increase in population in the suburb \((1 - l_i > 0)\) will increase \( l_i \) but will decrease it in the city \((1 - l_i < 0)\).

The impact of an increase in any of the taxes on population can be decomposed into three effects: the direct effect of the tax on population; the (balanced-budget) effect of the associated increase in \( G_i \); and the effect of any adjustment in the public service in jurisdiction \( j \) \((G_j)\) needed to ensure a balanced budget, the fiscal externality. All three influence populations through changes in housing prices.

For the taxes on capital, residents, commuters, and labor, changes in populations are entirely determined by changes in housing prices, with
\[
\hat{N}_{i}^i = (\eta_i + \varepsilon_i) \hat{p}_{t_x^i}^i \quad \text{and} \quad \hat{N}_{ij}^i = -\frac{N_i}{N_j} \hat{N}_{i}^i, \quad x = I, R, L, C; \quad j \neq i; \quad (9a)
\]
and for the tax on property,
\[
\hat{N}_{i}^p = (\eta_i + \varepsilon_i) \hat{p}_{t_x^i}^p + \frac{\varepsilon_i}{(1 + t_x^i)} = \eta_i \hat{p}_{t_x^i}^p + \varepsilon_i \hat{q}_{t_x^i}^p \quad \text{and} \quad \hat{N}_{ij}^p = -\frac{N_i}{N_j} \hat{N}_{i}^p, \quad j \neq i; \quad (9b)
\]
where \( \eta_i \) is the elasticity of housing supply in jurisdiction \( i \), \( \varepsilon_i = -\frac{\partial h_i}{\partial q_i} h_i > 0 \) is the elasticity of housing demand, with \( q_i = p_i (1 + t_i^{e}) \).

To determine the effect of taxes on housing prices, we differentiate equilibrium conditions (5b) and (5c) with respect to each of the taxes. As we are interested in the equilibrium impacts on housing prices, we also employ the first-order condition for utility maximization of a resident/worker with respect to each of the taxes, which is

\[
U_i^x + p_i (h_i^e - h_i) \hat{p}_i^x + V' (G_i^e) G_i^x = 0, \quad x = I, L, R, P. \tag{10}
\]

The term \( U_i^x \) denotes the “direct” effect of an increase in the tax \( t_i^x \) absent any effect of the tax on public service levels: \( U_i^x = -k_i, U_i^L = U_i^R = -1 \) and \( U_i^P = -p_i h_i \left(1 + t_i^p \hat{p}_i^p\right) \). The terms \( G_i^e \) and \( G_i^x \) refer to the balanced-budget changes in the public service levels in jurisdiction \( i \) and jurisdiction \( j \) with a change in \( t_i^x \). Then differentiating (5b) and applying (10) gives

\[
p_i h_i \hat{p}_i^x - p_j h_j \hat{p}_j^i + V' (G_j) G_j^i = 0, \quad i, j = 1, 2; \ i \neq j; \ x = I, L, R. \tag{11a}
\]

For the property tax we have

\[
p_i h_i \hat{p}_i^x - \left(1 + t_i^p\right) p_j h_j \hat{p}_j^i + V' (G_j) G_j^i = 0, \quad i, j = 1, 2; \ i \neq j. \tag{11b}
\]

Differentiating (5c) and simplifying yields

\[
\hat{p}_i^x = -\frac{N_i (\eta_i + \varepsilon_i)}{N_j (\eta_j + \varepsilon_j)} \hat{p}_i^x, \quad i, j = 1, 2; \ i \neq j; \ x = I, L, R \tag{12a}
\]

and

\[
\hat{p}_j^i = -\frac{N_i (\eta_i + \varepsilon_i)}{N_j (\eta_j + \varepsilon_j)} \left[ \hat{p}_i^x + \frac{\varepsilon_i}{(\eta_i + \varepsilon_i)} \frac{1}{(1 + t_i^p)} \right], \quad i \neq j. \tag{12b}
\]

Then, as shown in Appendix A.1, we can use (11a) and (12a) to solve for \( \hat{p}_i^x \), \( x = I, L, R \):

\[
\hat{p}_i^x = -\frac{N_j (\eta_j + \varepsilon_j) V' (G_j) G_j^i}{D}, \quad x = I, L, R, \tag{13a}
\]

where \( D = N_2 (\eta_2 + \varepsilon_2) p_1 h_1 + N_1 (\eta_1 + \varepsilon_1) p_2 h_2 > 0 \). Analogously, using (12b) in (11b) to obtain \( \hat{p}_i^p \) gives

\[
\hat{p}_i^p = -\frac{\left[N_j (\eta_j + \varepsilon_j) V' (G_j) G_j^i + N_i \frac{(1 + t_i^p)\varepsilon_i p_j h_j}{(1 + t_i^p)}\right]}{D t_i^p}. \tag{13b}
\]
where \( D_{it} = N_j (\eta_j + \varepsilon_j) p_i h_i + N_i (\eta_i + \varepsilon_i) (1 + t^P_j) p_j h_j > 0 \). Whether, for the taxes on industrial capital, residents, and labor, \( \hat{p}_{it}^\lambda \) is positive or negative depends entirely on whether the fiscal externality, \( G_{ij}^\lambda \), is positive or negative, with a positive (negative) fiscal externality resulting in \( \hat{p}_{it}^\lambda < (>) 0 \). That the sign of \( \hat{p}_{it}^\lambda \) depends on the fiscal externality follows from directly from the fact that equilibrium conditions (5b) and (5c) are evaluated at the optimal (utility-maximizing) policies. At the optimal tax policy in jurisdiction \( i \), it must be the case that \( \frac{dU_i}{dt^x_i} = 0 \), meaning utility is maximized for the resident-worker. As a result, a positive fiscal externality in \( j \) will cause some residents of \( i \) to move to \( j \), which will raise housing prices in \( j \) and lower them in \( i \). We later discuss the effects of the property tax on housing prices.

As we see in the following section that the impacts of policies on housing prices, as well as on wages and population, will be key to determining the optimal tax policies of the two governments and how they might differ from each other. While the impacts on housing prices matter, unlike the case with “atomistic” jurisdictions, in this setting in which each jurisdiction is assumed to have a nonzero share of the metropolis population, utility and “rent” maximization are not equivalent.

4 Optimal Policy with Taxes on a Single Base

Here we consider only the use of a single tax to finance the public service, beginning with the case of financing with an industrial capital, then examining property taxation as the sole tax instrument, and finally considering taxes on residents and workers.

4.1 Capital Tax

While numerous studies of local policy determination have assumed that the government objective is to maximize property values, a frequent justification for this objective is that utility and property value maximization are equivalent (Sonstelie and Portney (1978)). However the two objectives are only equivalent, in general, when jurisdictions are extremely small or “atomistic” meaning that their policies will have no impacts on prices or welfare in other jurisdictions. As we are assuming that both our jurisdictions have significant shares of the metropolis population, each jurisdiction’s tax policies may affect housing prices and welfare in the other jurisdiction. Then property value and utility maximization are not equivalent. Here we choose to focus on the more general objective of utility maximization of resident/owners:

\[
\max_{t^I_i} \quad w_i + (p_i - p_i^0) h_i^0 + S(p_i) + V(G_i). \tag{14}
\]
At an equilibrium, where \( p_i = p_i^0 \), the first-order condition simplifies to

\[-k_i + V'(G_i)G_i^{\prime} = 0 \tag{15} \]

To find \( G_i^{\prime} \), differentiate (4) and employ (7) - (9a) to obtain

\[ G_i^{\prime} = k_i l_i \left[ 1 + t_i' \left( \dot{k}_{iI}^{\prime} + \dot{l}_{iI}^{\prime} \right) \right] = k_i \left[ l_i + t_i' \left( \frac{k_i'}{k_i} + \left( -\frac{1}{N_i A'} + (1 - l_i)(\eta_i + \varepsilon_i) \hat{p}_{iI}^{\prime} \right) \right) \right] \tag{16} \]

Using (16) in (15) enables us to express the first-order condition as

\[ V'(G_i) = \frac{1}{l_i \left[ 1 + t_i' \left( \dot{k}_{iI}^{\prime} + \dot{l}_{iI}^{\prime} \right) \right]} = \frac{1}{l_i + t_i' \left( \frac{k_i'}{k_i} + \left( -\frac{1}{N_i A'} + (1 - l_i)(\eta_i + \varepsilon_i) \hat{p}_{iI}^{\prime} \right) \right)} \tag{17} \]

where we refer to \[ \frac{1}{l_i \left[ 1 + t_i' \left( \dot{k}_{iI}^{\prime} + \dot{l}_{iI}^{\prime} \right) \right]} \] as the marginal cost of funds \( (MCF_{iI}) \).

Absent of any effects of the taxes on the capital-labor or the labor-population ratios \( \left( \dot{k}_{iI}^{\prime} = \dot{l}_{iI}^{\prime} = 0 \right) \) whether the public service is over- or under-provided relative to the efficient level \( (V'(G_i) = 1) \) depends on whether the worker-population ratio is greater or less than one. In the city, with \( l_1 > 1 \) we have overprovision \( (V'(G_1) < 1) \) as the capital tax effectively exports some of the tax cost of public services in the city to residents of the suburbs through the wages received by commuters. \(^{11}\) In the suburb, the inequality \( l_2 < 1 \) implies underprovision \( (V'(G_2) > 1) \)

Of course, both the capital-labor and the labor-population ratios are likely to respond to the tax increase. While \( k_i \) unambiguously falls, raising the \( MCF \), the impact on the labor-population ratio is ambiguous. In both the suburb and the city, the impact of the tax on commuters reduces \( l_i \). But as seen in (16), the impact of the tax on jurisdiction population and therefore \( l_i \) depends on changes in housing prices and whether \( l_i \) is greater or less than one. In the city, with \( l_i > 1 \), a decrease [increase] in housing prices and therefore population will act to increase [decrease] \( l_i \); the reverse applies for the suburb, where \( l_i < 1 \).

### 4.1.1 Fiscal Externality and Effect on Property Values

The tax on industrial capital in jurisdiction \( i \) is chosen to maximize the utility of the resident/worker in jurisdiction \( i \). Thus, a marginal increase in the tax will not affect the worker’s utility in \( i \); nor will it affect his utility in \( j \), since the free mobility of households implies that initial \( i \) residents are indifferent about where to live, before and after the tax

\(^{11}\) Tax exporting is common in the tax competition literature. For example, see Haufler (1996).
change. However, the initial residents of \( j \) are affected by \( i \)'s tax change, since the location of their housing ownership is in \( j \), rather than \( i \). Specifically, the impact of an increase in \( t_i \) on the utility of a resident/worker in jurisdiction \( j \) is

\[
p_j \left( h_j^0 - h_j \right) \hat{p}_j^i + V' \left( G_j \right) G_{ji}^i = V' \left( G_j \right) G_{ji}^i. \tag{18}
\]

We use the fact that \( h_j = h_j^0 \) in equilibrium. Thus, any increase or decrease in the values of the houses owned by the residents is entirely offset by increases or decreases in the cost of housing in which they reside. Of course, these residents will respond to a price change by adjusting their consumption of housing, but the envelope theorem implies that such adjustments have no first-order impact on utilities. As shown in the Appendix A.1 the fiscal externality associated with an increase in the capital tax in jurisdiction \( i \) is measured by the change in jurisdiction \( j \)'s per capita tax revenue:

\[
G_{ji}^I = t_i^J k_j \hat{p}_i^j = t_i^J k_j \left[ \frac{k_i}{N_i} + (1 - l_j) (\eta_j + \epsilon_j) \hat{p}_i^j \right], \tag{19}
\]

where the impact of \( t_i^J \) on the labor-population ratio in jurisdiction \( j \), \( l_i^J \), depends on its effect on both commuting and population.

Consider first the change in the division of workers between the two jurisdictions. A higher tax rate in jurisdiction \( i \) reduces the wage rate there, which causes some workers to move to the other jurisdiction until changes in commuting costs make workers indifferent between jurisdictions again. As a result, \( l_i \) falls and \( l_j \) rises. By itself, the rise in \( l_j \) increases per capita tax revenue in \( j \), and the resulting rise in \( G_j \) makes \( j \) more attractive to the residents of jurisdiction \( i \), who start migrating to jurisdiction \( j \). As a result, housing demand falls in \( i \) and rises in \( j \), causing housing prices to fall in \( i \) and rise in \( j \).

Starting from the equilibrium, where (17) holds, we may substitute (19), first using (12a) to replace \( \hat{p}_i^j \) with \( \hat{p}_i^j \), into (13a) to obtain an expression for the fall in \( p_i \):

\[
\hat{p}_i^j = -\frac{(\eta_j + \epsilon_j) V' \left( G_j \right) t_i^J k_j k_i}{E_{ti}^j} < 0, \tag{20}
\]

where \( E_{ti}^j > 0 \) to ensure stability.\(^{12}\) It follows that the utility-maximizing tax rate for jurisdiction \( i \), \( t_i^* \), will exceed the rate that maximizes housing prices (property values). Note also that since \( p_i \) is falling only because households are moving to \( j \) to take advantage of a positive fiscal externality, we may conclude that starting from the equilibrium, the fiscal

\[^{12}E_{ti}^j = D + N_j (\eta_j + \epsilon_j) V' \left( G_j \right) t_i^J k_j (1 - l_j) (\eta_i + \epsilon_i)\]
externality created by a rise in either jurisdiction’s tax rate is positive. More formally, use (13a) to obtain $\sign(\hat{p}_{i,t}^j) = -\sign \left( G_{i,t}^j \right)$. Since $\hat{p}_{i,t}^j < 0$, we then $G_{i,t}^j > 0$ for a tax increase in either jurisdiction.

These results and other implications of these policies are summarized in the proposition below.

**Proposition 1.** Let $\{t_1^*, t_2^*\}$ be the equilibrium values of capital taxes, and let $N_i \left( t_1^*, t_2^* \right) \equiv N_i^*$, $l_i \left( t_1^*, t_2^* \right) \equiv l_i^*$, $p_i \left( t_1^*, t_2^* \right) \equiv p_i^*$, and $G_i \left( t_1^*, t_2^* \right) \equiv G_i^*$ be the associated equilibrium values of population, labor/population ratio, housing price, and the public service for $i, j = 1, 2; i \neq j$. Then at $\{t_1^*, t_2^*\}$:

1. In the suburb, where $l_2 < 1$, the marginal cost of funds, $MCF_{t_2} = V'(G_2) > 1$, and the public service level is below the efficient level at which $V'(G_i) = 1$.

2. In the city, if $l_1 > \frac{1}{1+t_1^* \left[ \frac{A_{k_1}}{N_1} + (1-l_2)N_2 (\eta_1+\varepsilon_1) (\eta_2+\varepsilon_2) V'(G_2) t_2^* k_2 \frac{k_1}{A_{k_1}} \right]} > 1$, then $MCF_{t_1} < 1$ and $V'(G_1) < 1$, implying that the public service level exceeds the efficient value.

3. Given $t_2^*$, the tax rate $t_1^*$ exceeds the rate that maximizes property values; that is, $\hat{p}_{i,t}^j = -\frac{N_j (\eta_j+\varepsilon_j) V'(G_j) t_1^* k_1}{E_{t_1}} < 0$.

4. The population of jurisdiction $i$, $N_i \left( t_1^*, t_2^* \right)$, will be below the efficient population (given $t_2^*$), with $N_i \left( t_1^*, t_2^* \right) = -\frac{N_j (\eta_j+\varepsilon_j) V'(G_j) t_1^* k_1}{E_{t_1}} < 0$.

5. As $\hat{p}_{i,t}^j < 0$, $i = 1, 2$, an increase in the tax in the suburb will unambiguously decrease the worker-population ratio $l_2^* > 0$, but have an ambiguous effect on the worker-population ratio in the city, $l_1^*$. 

6. In both the city and the suburb, the industrial capital tax will generate a positive fiscal externality $\left( G_{i,t}^j > 0, i = 1, 2 \right)$.

Note that the unambiguous positive sign of $l_2^*$ can be understood by writing, $l_2 = 1 - \frac{N_2}{N_2}$. As the tax rises in the suburb, the number of commuters, $N_2$, rises, and the residential population, $N_2$, falls, implying that $l_2$ falls. In contrast, a rise in the city tax has ambiguous effects on $l_1$ because its effects on the worker and residential population have opposite signs.

We can also say that the public service level is below the efficient value in the suburb, but this conclusion does not always extend to the city, where a sizable level of tax exporting may result in an inefficiently high public service level.
4.2 Property Taxation

When the public service is financed by a property tax, the government budget becomes

\[ t_i^P p_i h(q_i) = G_i \]  

(21)

To find the optimal property tax rate, we can solve for the optimal unit tax, \( \tau_i^P = t_i^P p_i \), and then find the equivalent ad valorem tax.\(^{13}\) The first-order condition for this unit tax is

\[-h_i + V'(G_i) G_{\tau_i^P} = 0.\]  

(22)

Differentiating the government budget constraint to solve for \( G_{\tau_i^P} \), we obtain

\[ V'(G_i) = \frac{1}{1 - t_i^P p_i \varepsilon_i q_i^\tau_i^P} > 1.\]  

(23)

The extent that the marginal cost of funds (\( MCF_t^P \)) exceeds one depends on the tax rate, the price elasticity of demand for housing, and the extent that the tax increase raises the gross price of housing. It is easily shown that \( \hat{p}_i^P \geq -\frac{1}{1+t_i^P} \), from which it follows that the property tax raises the gross price of housing: \( \hat{q}_i^P = \hat{p}_i^P + \frac{1}{1+t_i^P} > 0 \). A higher housing demand elasticity tends to increase the marginal cost of funds, since the higher \( q_i \) then has a greater negative effect on per capita housing, which lowers the tax base. To conclude, the impacts the property tax and the determination of its optimal rate are qualitatively the same for both jurisdictions.

### 4.2.1 Impacts on Population and Fiscal Externalities

We now argue that if jurisdiction \( i \) raises its tax rate from its equilibrium level, with the public service level increasing to balance the government budget, then residents from other jurisdictions are attracted to it. By the optimality of the initial policy, there is no first-order change in the utilities of jurisdiction \( i \) residents. But because the net price of housing falls, residents of other jurisdictions can now buy housing in \( i \) more cheaply than before, whereas existing residents of \( i \) already own their homes and therefore do not directly benefit from this fall in the net price. As a result, new residents are attracted to \( i \), which lowers \( p_j \).

What does this movement in residents from jurisdiction \( j \) to \( i \) imply about fiscal externalities? As both the property tax and the cost of the public service are on a per capita

\(^{13}\)Note that the jurisdictions are still playing a Nash game in ad valorem taxes. The Nash equilibrium still depends on whether unit taxes or ad valorem taxes are the strategy variables.
basis, changes in population have no effect on the budget constraint. Following Section 4.1.1, we consider the fiscal externality associated with an increase in \( t_i^p \). Differentiating the utility of a resident/worker in jurisdiction \( j \) gives

\[
\frac{dU_j}{d t_i^p} = p_j \left( h_j^0 - (1 + t_j^p) h_j \right) \hat{p}_i^j + V' \left( G_j \right) G_i^j = -t_j^p p_j h_j \hat{p}_i^j + V' \left( G_j \right) G_i^j. \tag{24}
\]

Because the tax is on the value of housing, changes in the price of housing affect tax payments. In this case, the decrease in housing prices will increase welfare. Differentiating the government budget constraint in jurisdiction \( j \) yields

\[
G_i^j = t_j^p p_j h_j (1 - \varepsilon_j) \hat{p}_i^j. \tag{25}
\]

Substituting for \( G_i^j \) in (24), and for \( V' \left( G_j \right) \) from (23) gives

\[
\frac{dU_j}{d t_i^p} = t_j^p p_j h_j \left[ \frac{1 - \varepsilon_j}{1 - t_j^p p_j \varepsilon_j \hat{q}_i^j} - 1 \right] \hat{p}_i^j, \tag{26}
\]

where utility maximization requires that \( 1 - t_j^p p_j \varepsilon_j \hat{q}_i^j > 0 \). Note that \( \hat{q}_i^j = (1 + dp)/q < 1/q \). Thus, the term in the square brackets is negative, implying that the tax change in \( i \) increases the utilities of residents in \( j \). The fiscal externality is again positive.

We summarize these findings in the proposition below:

**Proposition 2.** Let \( \{t_1^p, t_2^p\} \) be the equilibrium values of the property taxes that satisfy (17) \( N_i \left( t_i^p, t_j^p \right) \equiv N_i^*, l_i \left( t_i^p, t_j^p \right) \equiv l_i^*, p_i \left( t_i^p, t_j^p \right) \equiv p_i^*, \) and \( G_i \left( t_i^p, t_j^p \right) \equiv G_i^* \) be the associated equilibrium values of population, labor/population ratio, housing price, and the public service for \( i, j = 1, 2; i \neq j \). Then at \( \{t_1^p, t_2^p\} \):

1. The marginal cost of funds for both jurisdictions is positive: \( MCF_i^p = V' \left( G_i \right) > 1 \), implying that the public service level is below the efficient level.
2. The tax rate \( t_i^p \) exceeds the rate that maximizes property values \( (p_i) \), with an increase in the tax rate decreases property values in both jurisdiction \( i \left( \hat{p}_i^j < 0 \right) \) and jurisdiction \( j \left( \hat{p}_i^j < 0 \right) \).
3. An increase in \( t_i^p \) increases jurisdiction \( i \)’s population \( (N_i^j > 0) \) and decreases jurisdiction \( j \)’s population \( (N_i^j < 0) \).
4. The fiscal externality associate with increases in \( t_i^p \) from the equilibrium level is positive.
5. An increase in suburb’s property tax from the equilibrium level will increase its labor-population ratio \( \left( I_{P}^{2} l_{l}^{2} < 0 \right) \), while an increase in the cities property tax will reduce its labor-population ratio \( \left( I_{P}^{1} l_{l}^{1} < 0 \right) \).

4.3 Residential and Labor Taxation

Subnational income taxes around the world differ in the rules governing which jurisdiction has taxing authority over personal income. For example, in the United States, state income taxes may be either residence-based or employment-based. The default option is to tax the income at its source, but some states sign reciprocal agreements so that individuals need only pay taxes to the state of residence (Agrawal and Hoyt 2018). Even at the local level, municipal and city income taxes differ in their sourcing rule. For example, in the state of Ohio, taxes are potentially due in both the town of employment and residence. This is because municipalities may set a tax credit for their residents that reduces their residential tax by the amount of taxes paid to the town of employment. While some municipalities offer a full credit (employment-based taxation) other municipalities offer no credit (resulting in both employment and residence-based taxation). In other states, the rules at the municipal level are either residence or employment based. Given this diversity, and given the large amount of commuting across local jurisdictions, we model both residence and employment based local income taxes.

Because of the similarity in the residential and labor tax results, particularly in their effects on housing markets, much of our discussion of them can be combined. As we shall see, the distinction between the two taxes is in their effects on governments budgets as the labor tax affects commuting while the residential tax does not. Then for each tax, we can express the first-order condition for utility maximization for a resident/owner by

\[
-1 + V'(G_i)G_{i,t}^x = 0, \quad i = 1, 2; \quad x = R, L.
\]

The impact of an increase in \( t_{l}^{i} \) on the public service level in jurisdiction \( i \) is

\[
G_{i,t}^l = l_i + t_L l_{l}^L = l_i + t_{l}^{i} \left[ -\frac{1}{N_i} \frac{1}{A} + (1 - l_i) (\eta_i + \epsilon_i) \hat{p}_{i,t}^{L} \right], \quad \text{(28a)}
\]

and for the residential tax,

\[
G_{i,t}^R = 1. \quad \text{(28b)}
\]

The impacts on tax revenues in jurisdiction \( j \) from an increase in the tax in jurisdiction \( i \)
for the labor tax is

$$G_{tL}^i = t_LL^i_{tL} = t_L^L \left( \frac{1}{Nj} + (1 - l_j) (\eta_j + \varepsilon_j) \hat{p}_{tL}^j \right)$$  \hspace{0.5cm} (29a)$$

and for the residential tax by

$$G_{tR}^i = 0.$$  \hspace{0.5cm} (29b)

### 4.3.1 Optimal Residential Tax

Evaluating (27) and \(G_{tR}^i = 1\), it is apparent that at the optimal residential tax, \(t^*_L\), the public service is efficiently provided, that is, \(V'(G^*_i) = 1\). Then as there is no fiscal externality \(G_{tR}^i = 0\), using (29a) in (13a) gives \(\hat{p}_{tR}^i = 0\). Thus, the tax rate also maximizes property values (housing prices).

### 4.3.2 Optimal Labor Tax

Using (28a) in (27) allows us to express the optimality condition for the labor tax as

$$V'(G_i) = \frac{1}{l_i \left[1 + t_L^i h_L^i \right]} = \frac{1}{l_i + t_L^i \left[ - \frac{1}{Nj} + (1 - l_i) (\eta_i + \varepsilon_i) \hat{p}_{tL}^i \right]}.$$  \hspace{0.5cm} (30)$$

The first expression for the \(MCF_{tL}\) shows that like the capital tax, the \(MCF_{tL}\) depends on the labor-population ratio. An increase in \(l\) reduces the \(MCF\) but a large reduction in \(l \left(\hat{h}_{tL}^i\right)\), increases the \(MCF\). The second expression is obtained by substituting for \(l_L^i\) and shows that the reduction [increase] in commuters to [from] jurisdiction \(i\) will increase \(l\) while an decrease [increase] in the housing prices will increase [decrease] \(l_i\) if \(l_i > \langle < \rangle 1\). In contrast to the residential tax, using (29a) in (13a) gives the equilibrium price gradients for the labor tax,

$$\hat{p}_{tL}^i = \frac{-(\eta_j + \varepsilon_j) V'(G_j) t^L_j}{E_{tL}} \frac{1}{A'} < 0$$  \hspace{0.5cm} (31a)$$

and

$$\hat{p}_{tL}^i = \frac{N_i (\eta_i + \varepsilon_i) V'(G_i) t^L_i}{E_{tL}} \frac{1}{A'} > 0$$  \hspace{0.5cm} (31b)$$

where \(E_{tL} > 0\).\(^{14}\)

These price changes reflect the fiscal externality created by the rise in \(i\)'s labor tax. When jurisdiction \(i\) raises its labor tax rate, some labor moves to jurisdiction \(j\), which increases the per capita tax base there, creating the positive fiscal externality. This fiscal externality

\(^{14}\)The term \(E_{tL} = D + N_i (\eta_j - \varepsilon_j)^2 V'(G_j) t^L_j p_j h_j (1 - l_j) > 0\).
then causes some households in \( i \) to move to \( j \), causing housing prices to rise in \( j \) and fall in \( i \).

If \( l_i < 1 \), which is the case in the suburb, then both the increase in commuters and the decline in population lead to a decline in the labor-population ratio: \( l'_i < 0 \). We can then conclude that \( V'(G_i) = MCF_{tL} > 1 \) – the public service is less than the efficient level. If \( l_i > 1 \), which is the city case, then the decrease in the population acts to increase the labor-population ratio, and the public service may be underprovided or overprovided. If

\[
 l_i > 1 + t'^{L*}_{i} \frac{\beta_i}{1-(\alpha_i+\varepsilon_i)p'^{L*}_{i}}, \text{then } V'(G_i) = MCF_{tL} < 1, \text{implying that the public service is overprovided.}
\]

Summarizing these results:

**Proposition 3.** Let \( t_i^{R*} \) and \( t_i^{L*} \) be the utility-maximizing residential and labor taxes when each tax is the sole source of revenue for jurisdiction \( i \). Then:

1. At \( t_i^{R*}, i = 1,2 \) the public service level is efficient, that is, \( V'(G_i^{*}) = 1 \), property values are maximized, and no fiscal externality is generated \( (G_i^{tR} = 0) \).

2. a) At \( t_i^{L*}, i = 1,2 \), an increase in \( t_i^{L} \) decreases property values \( (p'^{L}_{iL} < 0) \). The public service is unambiguously underprovided in the suburb \((V'(G_2^{*}) > 1)\), In the city, if \( l_1 > [\ll 1 + t_i^{L*} \frac{\beta_i}{1-(\alpha_i+\varepsilon_i)p'^{L*}_{iL}}, \text{the public service is overprovided [underprovided]: } V'(G_1^{*}) > [\ll 1.\)

b) The fiscal externalities generated by both jurisdictions are positive \( (G_i^{tL} > 0, i, j = 1,2; i \neq j) \), as the increases in the labor tax in one jurisdiction increases the labor-population ratio in the other jurisdiction.

## 5 Policies with Multiple Tax Instruments

Local governments employ multiple taxes to raise revenue. Despite this empirical fact, much of the fiscal competition literature focuses on a single tax instrument.\(^{15}\) We now consider the mix of policies the jurisdictions will choose when given the option to use multiple tax instruments. As the tax on residents yields an efficient public service level, an obvious starting point might be to consider when and why the jurisdictions might choose to deviate from taxing something other than residents. With multiple tax instruments, it is possible to have “negative” taxes, that is, subsidies that are financed, along with the public service.

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\(^{15}\)Some exceptions with multiple instruments include Bucovetsky and Wilson (1991), Braid (2013), Braid (2005), Braid (2002), Braid (2000), and Braid (1996). However, as noted in the introduction, these papers do not share the defining features of our model.
expenditures, by other taxes. We consider this possibility and find, in fact, that there are cases in which subsidies are, in fact, optimal. After considering the use of the residence tax with the other tax bases we then consider the use of a commuter tax along with the other tax instruments and, finally, perhaps a more policy relevant issue: the joint taxation of industrial capital and residential property.

5.1 Residential Taxes in Combination with Other Tax Instruments

5.1.1 Optimal Mix of Residential and Labor Tax

As modeled, both the residence and labor tax are head taxes, at least conditional on choice of residence or the location of employment. A more policy relevant comparison would be a tax on income of residents, in contrast to a tax on earnings of those employed in the jurisdiction. As our focus is on the impact of mobility on tax bases and policies rather than labor supply elasticity, our head tax may be thought of as income tax with an assumption of an inelastic labor supply. Further, as both taxes have the same impacts on individual labor supply, differences between the use of the two taxes should not depend on differences in the effects on individual labor supply, but on their differences on labor-population ratios through mobility. More generally, the comparison of the residential and labor tax provides a contrast between resident-based taxes, such as residential property, and employment-based taxes, including taxes on industrial capital.

With taxes on both labor and residents, the government budget constraint is now

\[ G_i = t_i^L l_i + t_i^R. \]  

As we have seen, the residential tax does not affect the allocation of labor and population across jurisdictions, and therefore does not affect \( l_i \). Thus, \( G_{t_i^R} = 1 \), which gives \( V'(G_i) = 1 \).

As the labor tax has no effect on the (per-capita) revenue from the residential tax, the first-order condition is the same as when it was the only source of revenue:

\[ \frac{dU^i}{dt_i^L} \bigg|_{t_i^R=t_i^{R_*}} = -1 + V'(G_i) \left[ l_i + t_i^L l_i^L \right] = 0 \]  

where \( l_i^L = -\frac{1}{N_i L_i} + (1 - l_i) (\eta_i + \varepsilon_i) \hat{p}_i^L \), which is negative. The notation \( t_i^{R_*} \) refers to the residential tax rate that ensures \( V'(G_i) = 1 \). After rearranging and using the optimality of the residential tax \( (V'(G_i) = 1) \), we have
As \( l_{iL} < 0 \), the sign of the second term is the negative of the sign of \( t_i^L \). If \( l_i < 1 \), as is the case in the suburb, the first term of (31e) is negative, requiring \( t_i^L < 0 \); that is, labor is subsidized. In contrast, if \( l_i > 1 \), which is the case in the city, then the first term of (31e) is positive, so \( t_i^L > 0 \); labor is taxed.

In the absence of commuting, with \( l_i = 1 \), welfare cannot be improved by adding a labor tax or subsidy to the residential tax. Note that the tax policies of both jurisdictions have the effect of reducing commuting to the city and, by doing so, have reduced the negative externality associated with commuting. We later address whether this externality has been reduced too much or too little from the viewpoint of efficiency.

### 5.1.2 Residential Tax and Capital Tax

We can proceed in an analogous fashion to consider the joint use of the residential and capital taxes. Once again, it is optimal to set the tax on residents such that the efficient public service level is obtained (\( V''(G_i) = 1 \)). The first-order condition for the capital tax also remains the same, since the capital tax has no effect on the per capita revenue from the residential tax. We express this first-order condition as follows:

\[
\frac{dU^i}{dt_i}\bigg|_{t_i^R=t_i^R^*} = V''(G_i) (l_i - 1) + V''(G_i) t_i^L l_i = 0 \quad (31f)
\]

where \( k_i \hat{l}_i < 0 \) is the (percentage) change in the per-capita capital tax base. Rearranging and using the optimality of the residential tax (\( V''(G_i) = 1 \)) gives

\[
\frac{dU^i}{dt_i}\bigg|_{t_i^R=t_i^R^*} = k_i V''(G_i) (l_i - 1) + V''(G_i) t_i^L k_i \hat{l}_i = 0 \quad (31g)
\]

Following the analysis of the residential tax and labor tax, the sign of the first term depends on whether \( l_i > (> ) 1 \), and the sign of the second term is the negative of the sign of \( t_i^L \). Thus, we now conclude that capital is subsidized in the suburb (\( t_i^L < 0 \)), where \( l_2 < 1 \), whereas capital is taxed in the city (\( t_i^1 > 0 \)), where \( l_1 > 1 \).

Again, capital is either taxed or subsidized only because \( l_i \neq 1 \), \( i = 1, 2 \), as result of commuting. In the absence of commuting, neither jurisdiction would tax or subsidize capital and, like the labor tax, the subsidization of capital by the suburbs and the taxation of it by the city both act to reduce commuting and the un-priced externality associated with it. The suburb’s capital subsidy raises wages there, giving residents a greater incentive to work in the
suburb, whereas the city’s capital tax lowers wages there, which also induces more suburban residents to work in the suburb. Unlike the labor tax/subsidy policy, however, distorting capital investments with taxes and subsidies is an inefficient way to lower commuting.

5.1.3 Residential and Property Taxation

Finally, we consider the use of both a residential tax and a property tax, where the latter may be viewed as another form of a tax on residents. Again, the residential tax will be set so that the public service is efficiently provided \( V'(G_i) = 1 \) and, as was the case with the labor and capital taxes, changes in the property tax do not affect per-capita residential tax revenue. For simplification, we consider a unit tax, rather than an ad valorem tax on housing. The first-order condition for the property tax is

\[
\left. \frac{dU_i}{dt_i} \right|_{t_i^R=t_i^{R^*}} = -h_i + V'(G_i) \left[ h_i + \tau_P^i h_i^P \right] = 0. \tag{31h}
\]

It should be apparent from (31h) that with the residential tax set so that \( V'(G_i) = 1 \), the only value of the tax on housing that satisfies (31h) is \( \tau_P^i = 0 \). Thus, the property tax will not be employed when the residential tax is available.

The critical difference between the property tax and the residential and labor tax/subsidy policies is that the latter affect commuting, whereas the property tax does not. Thus, the property tax does not serve as a means of reducing the externality associated with commuting.

We summarize our findings on the use of residential taxation with the other taxes in the following proposition.

Proposition 4.

1. When jurisdictions have only the option to tax either labor or residents:
   a) The suburb will tax residents and subsidize labor. It is always optimal for the city to employ a tax on labor.
   b) In both jurisdictions, the public service is efficiently provided in equilibrium \( V'(G_i^*) = 1, \ i = 1, 2 \).

2. When jurisdictions have only the option to tax either industrial capital or residents:
   a) The suburb will tax only residents and subsidize industrial capital. It is always optimal for the central city to employ a tax on industrial capital.
   b) In both jurisdictions, the public service is efficiently provided in equilibrium \( V'(G_i^*) = 1, \ i = 1, 2 \).
3. When jurisdictions only have the option to tax either property or residents:
   a) Neither jurisdiction will employ the property tax.
   b) In both jurisdictions, the public service is efficiently provided in equilibrium \( (V' (G_i) = 1, i = 1, 2) \), and neither tax generates a fiscal externality \( (G_{ir} = 0, i, j = 1, 2) \).

5.2 The Commuter Tax in Combination with Other Taxes

We will argue that the commuter tax is part of a jurisdiction’s optimal tax system, though it would not be the sole source of revenue in reasonable cases. We begin by considering the use of the commuter tax jointly with the residential tax. The rationale for commuter taxation in this case will apply to its joint use with our other tax instructions.

5.2.1 Residential and Commuter Taxation

We have essentially already considered this case when we analyzed the use of a residential and labor tax. A positive commuter tax for the suburb is equivalent to taxing residents who work in the suburb at a lower rate than residents who commute, which can be obtained with a positive residential tax and negative labor tax. Thus, we already know that the equilibrium commuter tax is positive. Similarly, a positive commuter tax for the city is equivalent to employing positive residential and labor taxes, which we also found to be optimal for the city.

However, the optimality condition for the positive commuter tax yields additional insights. The commuter tax is offset by changes in commuting costs, so the utility of the resident/worker changes only through the effect of the tax on government revenue. Consequently, the optimal commuter tax for a jurisdiction maximizes per-capita tax revenue. For both the city and suburb, the first-order condition for this revenue-maximization problem is

\[
N_{21} - t_i \frac{1}{A'} = 0, \tag{31i}
\]

where \(1/A'\) is the change in the number of commuters.

Note that the commuter tax is serving the role of an optimal congestion fee. Indeed, (31i) can be rearranged to obtain the classic formula for this tax: \(t_i^C = A'N_{21}\), which says that the tax equals the marginal externality cost imposed by an additional commuter. However, commuters are optimally taxed only if one of the jurisdictions is imposing the tax. When both impose the tax, then one jurisdiction’s tax creates a negative externality by reducing the revenue from the other jurisdiction’s tax. As a result, the sum of the two taxes is too high from the viewpoint of efficient road use.
Whether to also use the residential tax depends entirely on whether the revenue-maximizing
tax rate on commuters, \( t^*_C \), results in over- or underprovision of the public service, which
cannot be determined by inspection of the first-order condition for the commuter tax, (31i). If
taxation of commuters is not sufficient to raise enough revenue, then the residential tax will
be used, and, in equilibrium, set so that \( V'(G_i) = 1 \). If \( V'(G_i) < 1 \) when the public service
is financed only by the tax on commuters, the residential tax is not used. Ideally, however,
the government would redistribute the excess revenue from the commuter tax through a
residential subsidy, resulting in efficient public service provision.

5.2.2 Commuter taxes with other tax policies

Our results about the use of the commuter tax in combination with the residential tax
suggest that the condition necessary for employing the commuter tax is simply whether
implementation of the commuter tax increases tax revenue. Of course, with multiple tax
instruments, the effects of the commuter tax on tax revenue are more complicated. But it
remains the case that the commuter tax will be employed whenever it increases per capita
tax revenue:

\[
\left. \frac{dU}{dt} \right|_{t^*_C=0} = V'(G_i) \left. G_i^C \right|_{t^*_C=0} > 0, \tag{31j}
\]

This condition is satisfied with the property tax, and likely also holds for the tax on industrial
capital and labor.

In the presence of a tax on industrial capital or labor, the commuter tax creates a positive
fiscal externality. Because jurisdiction \( i \)'s commuter tax reduces commuting, it raises labor
in jurisdiction \( j \), which raises \( j \)'s labor-population ratio. This change raises the tax bases for
both the capital tax and the labor tax, creating a positive fiscal externality in jurisdiction
\( j \), which causes some residents of \( i \) to move to \( j \). If \( i = 1 \) (the city), we then have the
surprising result that the city’s tax on commuters makes the commuters better off (along
with all other residents of the suburb). As explained above, any expansion of labor in the
suburb is accompanied by an expansion in the capital stock, keeping wages fixed. But now
the additional labor increases per capita tax revenue, which benefits suburban residents.

For the remaining discussion of commuter taxes, we focus on the city’s commuter tax.

5.2.3 Commuter and Capital Taxation

Assume that it is optimal for the central city to tax commuters, in addition to capital. Then
the commuter tax is optimally set at the revenue-maximizing rate. But at this rate, the
tax revenue collected from the tax on capital is independent of changes in the number of
commuters—the tax no longer serves as a “proxy” for tax exporting to commuters, as this tax exporting is now done directly by the commuter tax. Thus, the marginal effect of the capital tax on revenue becomes

\[ G^1_{\ell I t} = k_1 \left[ 1 + t_1^I \left( \frac{k_1'}{k_1} + (1 - l_1) (\eta_1 + \varepsilon_1) \hat{p}^{1}_{\ell t} \right) \right], \tag{31k} \]

and we can express the first-order condition for the capital tax as

\[ V'(G_1) = \frac{1}{1 + t_1^I \left( \frac{k_1'}{k_1} + (1 - l_1) (\eta_1 + \varepsilon_1) \hat{p}^{1}_{\ell t} \right)}. \tag{31l} \]

Whether the public service is under- or overprovided is ambiguous. The impact of a tax increase on the capital stock is negative \((t_1^I l_1 k_1' < 0)\), but the tax once again creates a positive fiscal externality, causing some city residents to move to the suburb, as evidenced by the depressed housing prices in the city \((\hat{p}^{1}_{\ell t} < 0)\). The decrease in the city population increases the labor-population ratio \(((1 - l_1) (\eta_1 + \varepsilon_1) \hat{p}^{1}_{\ell t} > 0)\).

When comparing (31l) to the first-order condition for capital taxation in the absence of the commuter tax, (17), we see that the denominator on the right side differs by \(l_1 - 1 - t_1^I \frac{k_1}{k_1} \frac{1}{N_t}\), the additional net revenue collected from commuters via the capital tax. But this difference can be positive or negative.

### 5.2.4 Commuter and Labor Tax Policy

The logic behind the optimal mix of a commuter and labor tax is almost identical to that of the optimal mix of the capital and commuter tax. Specifically, when the commuter tax is set at its revenue-maximizing rate, and the marginal impact of the labor tax on revenue is

\[ G^1_{t L t} = \left[ 1 + t_1^L (1 - l_1) (\eta_1 + \varepsilon_1) \hat{p}^{1}_{t L t} \right]. \tag{31m} \]

Again, the number of commuters or changes in their number do not affect marginal tax revenue, except through changes in the population \(((1 - l_1) (\eta_1 + \varepsilon_1) \hat{p}^{1}_{t L t})\). In particular, the city population declines, since the fiscal externality created by the labor tax attracts some city residents to the suburb, and this decline leads to a reduction in city housing prices. Using (31m), the first-order condition for the labor tax can be expressed as

---

\(^{16}\)As is the case with the tax only on capital, at the optimal tax policies, \(\hat{p}^{1}_{\ell t}\) is determined by (13a) with the sign of \(\hat{p}^{1}_{\ell t}\) opposite of the sign of the fiscal externality, \(G^2_{\ell t}\). As the suburb has no commuter tax, the sign of the fiscal externality with respect to the capital tax remains positive.
With $\hat{p}_{i}^{1} < 0$ in equilibrium, the marginal cost of funds from the labor tax is less than one, meaning that the public service will be overprovided. Without the commuter tax, we previously saw that there could be underprovision.

5.2.5 Commuter and Property Tax Policy

Unlike the capital and labor taxes, the commuter tax has no direct effect on tax revenue from the property tax, so the revenue-maximizing tax rate is identical to the optimal rate when the commuter tax is the sole source of revenue for the central city. However, the commuter tax rate affects the revenue collected from increases in the property tax. In particular, we argued previously that, starting from the equilibrium, a marginal increase in jurisdiction $i$'s property tax rate attracts new residents to $i$. As a result, per capita commuter tax revenue declines, and this additional cost of property taxation raises $MCF_{i}^{t}$, further contributing to underprovision of the public service.

5.2.6 Commuter and Residential and Labor Taxation

An exception to employing the commuter tax occurs when the jurisdictions can employ both the residential and labor taxes. In fact, we already observed that the suburb’s optimal commuter tax can be replicated with a positive residential tax and negative labor tax, whereas the city’s commuter tax can be replicated with positive residential and labor taxes. To see this more formally, we compare the government budget constraints with the commuter tax and the residential tax with the budget constraint when the labor tax and residential tax are available. With the commuter tax and residential tax, the budget constraint is

$$t_{i}^{C} \frac{N_{21}}{N_{i}} + t_{i}^{RC} = G_{i}, \quad (31o)$$

and with the labor tax and residential tax, it is

$$t_{i}^{L} \frac{N_{21}}{N_{i}} + t_{i}^{RL} + t_{i}^{L} = G_{1} ; \quad t_{2}^{L} \left(1 - \frac{N_{21}}{N_{2}}\right) + t_{2}^{RL} = G_{2} \quad (31p)$$

where the subscripts $C$ and $L$ on $t_{i}^{RC}$ and $t_{i}^{RL}$ identify the tax on residents that is accompanying the residential tax. Then for the city, the equivalence is obtained if $t_{1}^{C} = t_{1}^{L}$ and $t_{i}^{RC} = t_{i}^{RL} + t_{i}^{L}$. For the suburbs, equivalence is obtained when $t_{2}^{C} = -t_{2}^{L} \frac{N_{21}}{N_{2}}$ and $t_{2}^{RC} = t_{2}^{RL} + t_{2}^{L}$. As noted above, the city will employ a tax on labor, and the suburb will
subsidize labor to obtain equivalence to the commuter tax.

A jurisdiction’s optimal commuter tax cannot be replicated when it has access to a residential tax and either a capital tax or property tax. With the capital tax, in particular, the government budget constraints are

\[ t_1^I k_1 \left( 1 + \frac{N_{21}}{N_1} \right) + t_1^{R_{i_1}} = G_1; \quad t_2^I k_2 \left( 1 - \frac{N_{21}}{N_2} \right) + t_2^{R_{i_2}} = G_2. \]  

(31q)

The per capita tax revenue from the capital tax depends on both the capital-labor ratio and the number of commuters, whereas only the number of commuters matters for the commuter tax. Thus, we cannot replace the commuter tax with an equivalent capital tax.

### 5.3 Capital and Property Tax Policy

Our analysis of the use of the residential tax jointly with each of the other three taxes, particularly the capital and labor taxes, has given us some insights into how governments will choose the mix of residential-based taxes (the residential and property tax) and employment-related taxes (capital and labor). We now consider the use of both the capital and property tax.

Consider first whether it is optimal to tax property if the jurisdiction is taxing capital. In the city, where there is tax exporting, we have seen that financing the public service with a capital tax may result in underprovision or overprovision of the public service. For this reason alone, it may not be optimal to also tax property. In the suburb, we have seen that the public service is underprovided under a capital tax. This suggests that the property tax should also be used if it generates additional revenue per resident. However, we also found that at least under the equilibrium for an optimal property tax, a marginal increase in the property tax attracts additional residents to the jurisdiction, which lowers per capita tax revenue from the capital tax. Thus, we cannot immediately conclude that introducing the property tax would generate additional revenue. We conclude that it is not always desirable to supplement a capital tax with a property tax.

Suppose instead that jurisdictions are initially taxing property. Then should a capital tax also be used? We have found the public service is underprovided under the property tax, due to the housing market distortion. In other words, the marginal cost of funds exceeds one. If a jurisdiction introduces a small capital tax, there is no change in the property tax base, which does not depend on the labor-population ratio. In the city, where \( l_1 > 1 \), introducing a capital tax will certainly be desirable, since tax exporting lowers the marginal cost of funds below one for a sufficiently small capital tax. Suppose, however, that the suburb is already supplementing its property tax with a capital tax. Then a small capital tax in
the city creates a positive externality by raising the labor-capital ratio in the suburb, which causes some city residents to move to the suburb. As a result, housing prices decline in the city, which raises the property tax base associated with any unit tax on housing there. This is an additional benefit that the city receives from taxing capital.

We conclude that if the two jurisdictions are allowed to supplement their property taxes with capital taxes, then the city will always do so. But the suburb may choose not to also tax capital, because $l_i < 1$, implying that a portion of the wage reduction from the capital tax is borne by commuters in the form of higher commuting costs.

6 Reaction Functions

We now investigate the slopes of the reaction functions for the tax game with the industrial capital tax rates as the strategy variable. Our main finding is that the reaction functions need not both slope up, and they may differ in sign for the city and suburb. To show this, we treat residential location decisions as fixed and consider an example where the utility from the public service is given by the log function, $V(G_i) = \ln G_i$. Then first-order condition (17) becomes

$$\frac{1}{G_i} = \frac{1}{l_i \left[ 1 + t_i \left( \hat{k}_i t + \hat{l}_i t \right) \right]},$$

(31r)

Substituting for $G_i$ from the government budget constraint and rearranging gives

$$\frac{1}{t_i k_i} = \frac{1}{1 + t_i \left( \hat{k}_i t + \hat{l}_i t \right)}.$$

(31s)

Now consider a rise in the tax rate $t_j$, $j \neq i$. The only change in the optimality condition occurs in the semi-elasticity for work, $\hat{l}_i$. For the city $(i = 1)$, if the suburb’s tax rate rises, then this semi-elasticity must become less negative for two reasons. First, commuting rises, which raises labor in the city: $l_1$ increases. Second, the higher amount of commuting increases road congestion. Thus, $A$ rises, which also means that $A'$ rises under our assumptions. Since $\frac{d\mu_i}{dt_i} = A' \mu_i l_i l_i t$ to re-establish the equilibrium commuting level following a tax change, the higher $A'$ implies that the rise in $t_1$ makes $l_i l_i$ less negative. We may conclude that the semi-elasticity becomes less negative, lowering the right side of the optimality condition. The city then responds to the suburb’s tax increase by raising its tax rate: the city’s reaction function slopes up.

17 Other taxes are currently being investigated.
But matters differ for the suburb \((i = 2)\). If the city raises its tax rate, commuting falls, and so \(l_2\) rises in the suburb. But now the fall in commuting lowers \(A'\). When the suburb raises its tax rate, \(l_2\) falls to satisfy, \(\frac{dl_2}{dt_2} = A'N_2l_2^{P_2}\). Since the city’s tax increase has lowered \(A'\) in this condition, \(l_2^{P_2}\) becomes more negative, creating the possibility that the semi-elasticity \(\hat{l}_2^{P_2}\) rises in absolute value. For this reason, it is possible that the suburban reaction function slopes down. More specifically, it can be shown that the share of suburban residents who commute is sufficiently high, given \(A''/A'\), then the suburban reaction function will slope down.

The possibility that different jurisdictions have reaction functions with slopes that differ in sign creates problems for empirical studies that estimate the slopes of reaction curves by regressing one jurisdiction’s tax rate on some weighted average of tax rates in neighboring jurisdictions. Such estimates may provide evidence of a zero slope, although the slope is significantly positive for some jurisdictions and significantly negative for others.

7 Multiple Suburbs

We now sketch a simple modification to our model of a single suburb and city to allow for multiple suburbs and a single (central) city. Our framework is essentially the same, but with \(J\) jurisdictions, where jurisdiction 1 is the city and jurisdictions \(j = 2, ..., J\) are the suburbs. As before, commuting is in one direction – from the suburbs to the city, with \(N_{i1}\) being the number of commuters from suburb \(i\) to the city. We also assume that the cost of commuting from suburb \(i\) to the city is strictly a function of \(N_{i1}\) and not the number of commuters from the other suburbs to the city. As well, the commuting cost function can vary among suburbs and is given by \(A^i(N_{i1})\). All other assumptions regarding utility and housing production are the same.

The equilibrium conditions are analogous to those in the case of a single suburb and the city. For each suburb, equal utility for a resident/worker in suburb \(i\) and a resident/commuter of suburb \(i\) means that

\[
  w_i - t_i^L = w_1 - t_1^L - t_1^C - t_i^C - A^i(N_{i1}), \quad i = 2, ..., J
\]  

(31t)

The second equal utility condition requires that a resident/worker in the suburb has a utility equal to the utility of a resident/worker in the city:

\[
  w_i - t_i^L - t_1^R + S(p_i(1 + t_i^P)) + V(G_i) = w_1 - t_1^L - t_1^R + S(p_1(1 + t_1^P)) + V(G_1) \quad i = 2, ..., J
\]  

(31u)
By transitivity, the equal utility condition applies across suburbs as well:

$$w_i - t_i^L - t_i^R + S(p_i(1 + t_i^P)) + V(G_i) = w_j - t_j^L - t_j^R + S(p_j(1 + t_j^P)) + V(G_j) \quad i, j = 2, ..., J; \ i \neq j$$

(31v)

The housing equilibrium condition can be expressed as

$$\frac{H(p_1)}{h(p_1(1 + t_1^P))} + \sum_{j=2}^{J} \frac{H(p_j)}{h(p_j(1 + t_j^P))} = N.$$  

(31w)

The main difference with our previous analysis is that the externalities created by a suburb’s policies do not just affect the city, but also other suburbs. For example, consider again the industrial capital tax. We previously saw that at the Nash equilibrium, a rise in a suburb’s capital tax creates a positive fiscal externality for the city by expanding its tax base. With more than one suburb, the additional public service provision in the city created by a rise in one suburb’s capital tax attracts new residents from all suburbs, thereby benefiting the residents of the other suburbs. In the limit, when each suburb becomes negligibly small, each suburb treats public service provision in the city as fixed. As a result, it assumes that there are no incentives for its residents to move to the city when it raises its capital tax. Thus, its labor-population ratio does not fall as much as it would when there were one or only a few suburbs, implying that its tax base falls less. This observation suggests that the marginal cost of funds will be lower than it would be with one or only a few suburbs, implying greater public service provision. Still, the suburbs as a whole create positive fiscal externalities when they all raise their tax rates at the margin, and some suburban residents take advantage of these externalities by moving to the city.

For the city, behavior is largely unaffected, at least qualitatively, since the city remains large relative to the suburbs. But some interesting differences remain. Recall, in particular, that the city sets its commuter tax to maximize tax revenue. We previously saw that this tax also satisfies the rule for an optimal congestion tax. This result would remain in the case of identical suburbs. In general, however, congestion levels on the roads connecting suburbs to the city will vary across suburbs, so a single commuter tax will no longer represent on optimal congestion tax for each road. The tax will be inefficiently high for commuters from some suburbs, and inefficiently low for commuters from others. Conditions under which there is an overall improvement in efficiency are currently under investigation.
8 Conclusions

We present a simple model in which jurisdictions are atomistic with respect to capital markets, but because they are not with respect to labor or housing markets, capital taxes will generate fiscal externalities, and strategic behavior will be observed for jurisdictions within the same labor market, that is, the metropolis. We link jurisdictions by mobility of both labor and residents (population) and, importantly, allow the decisions of where to live and where to work to be independent of each other, that is, we allow for individuals to commute between jurisdictions. That the residential population and the labor force in a jurisdiction are not necessarily equal will mean that there is an important difference in the impacts of taxes based on residential population, such as a head tax or property tax, and employment-based taxes, including a tax on labor or industrial capital. We find that with commuting, employment-based taxes are desirable in jurisdictions who receive commuters, providing them the opportunity to “tax export.” This tax exporting also means there is a distinct possibility that public services could be overprovided. For the jurisdictions that have a net outflow of commuters, residence-based policies are desirable, including a head tax on residents and a residential property tax.

Both types of jurisdictions have an incentive to employ a commuter tax, or an equivalent combination of residential and labor taxes. In the latter case, the optimal labor tax is negative for the suburb and positive for the city. With a capital tax used instead of the labor tax, the suburb also desires to subsidize capital, to reduce commuting, whereas the city would like to tax capital, again to reduce commuting. We have identified important efficiency properties of the commuter tax and shown that it does not actually harm commuters if levied by the city.
References


A Appendix

A.1 Comparative Statics on Equilibrium Price Gradients and Fiscal Externalities

A.1.1 Equilibrium Price Gradients

Differentiating with the equal utility condition, (5b), with respect to $t^x$, $x = I, R, L$

$$U^x_i + p_i h_i \hat{p}^i_t - p_i h_i \hat{p}^i_t + V'(G_i) G^i_t = p_i h_i \hat{p}^i_t - p_j h_j \hat{p}^j_t + V'(G_j) G^j_t, i, j = 1, 2, i \neq j, x = I, R, L$$  \hspace{1cm} (A.1)

and with respect to $t^P$ gives

$$U^x_i + p_i h_i \hat{p}^i_t - p_j h_j \hat{p}^j_t + V'(G_j) G^j_t = p_i h_i \hat{p}^i_t - p_j h_j \hat{p}^j_t + V'(G_j) G^j_t, i, j = 1, 2, i \neq j$$  \hspace{1cm} (A.2)

Applying (10) to evaluate at the utility maximizing tax rate and with equilibrium level of housing ($h_i^0 = h_i$) gives

$$p_i h_i \hat{p}^i_t - p_j h_j \hat{p}^j_t + V'(G_j) G^j_t = 0 \text{ or } \hat{p}^j_t = \frac{p_i h_i \hat{p}^i_t}{p_j h_j} + \frac{1}{p_j h_j} V'(G_j) G^j_t, i, j = 1, 2; x = I, L, R.$$  \hspace{1cm} (A.3)

For the property tax we have

$$p_i h_i \hat{p}^i_t - (1 + t^P_j) p_j h_j \hat{p}^j_t + V'(G_j) G^j_t = 0$$

$$\Rightarrow \hat{p}^j_t = \frac{p_i h_i \hat{p}^i_t}{(1 + t^P_j) p_j h_j} + \frac{1}{(1 + t^P_j) p_j h_j} V'(G_j) G^j_t, i, j = 1, 2.$$  \hspace{1cm} (A.4)

Differentiating the equilibrium condition for the housing market, (5c) with respect to $t^x$, $x = I, R, L$ gives

$$N_i (\eta_i + \varepsilon_i) \hat{p}^i_t + N_j (\eta_j + \varepsilon_j) \hat{p}^j_t = 0.$$  \hspace{1cm} (A.5)

and with respect to $t^P$

$$N_i (\eta_i + \varepsilon_i) \hat{p}^i_{t^P} + N_i \varepsilon_i \frac{1}{(1 + t^P)} + N_j (\eta_j + \varepsilon_j) \hat{p}^j_{t^P} = 0.$$  \hspace{1cm} (A.6)

Solving (A.5) and (A.6) for $\hat{p}^i_t$, $x = I, L, R$ and $\hat{p}^j_{t^P}$ respectively yields

$$\hat{p}^j_{t^P} = \frac{-N_i (\eta_i + \varepsilon_i) \hat{p}^i_t}{N_j (\eta_j + \varepsilon_j) \hat{p}^i_t}.$$  \hspace{1cm} (A.7)
and

\[
\hat{p}_{t_i}^j = -\frac{N_j (\eta_j + \varepsilon_j)}{N_j} \left[ \hat{p}_{t_i}^j + \frac{\varepsilon_j}{(\eta_j + \varepsilon_j) (1 + t_i^p)} \right] \\
= -\frac{N_j}{N_j} \left\{ [\eta_j \hat{p}_{t_i}^j + \varepsilon_j \hat{\theta}_{t_i}^j] \right\} 
\]  
(A.8)

Then solving for \( \hat{p}_{t_i}^j \), \( x = L, R, I \) using (A.3) and (A.7) gives

\[
\hat{p}_{t_i}^j = -\frac{N_j (\eta_j + \varepsilon_j) V' (G_j) G_{t_i}^j}{D}, \hspace{1em} x = L, R, I 
\]  
(A.9)

for \( t_i^x \), \( x = I, L, R \) where \( D = N_2 (\eta_2 + \varepsilon_2) p_1 h_1 + N_1 (\eta_1 + \varepsilon_1) p_2 h_2 > 0 \). Using (A.4) and (A.6) for the property tax we obtain

\[
\hat{p}_{t_i}^j = -\frac{N_j (\eta_j + \varepsilon_j) V' (G_j) G_{t_i}^j}{D_{t_i}^j} - \frac{N_i (1 + t_i^j) \varepsilon_i p_j h_j}{D_{t_i}^j} 
\]  
(A.10)

where \( D_{t_i}^j = N_i (\eta_i + \varepsilon_i) (1 + t_i^j) p_j h_j + N_j (\eta_j + \varepsilon_j) p_i h_i > 0 \)

Focusing on the cases with single tax instrument financing the public service, ((4)) with a single tax, to determine the equilibrium price changes it is necessary to find the fiscal externality, \( G_{t_i}^j \). To determine the impacts of these taxes recall that

\[
\hat{p}_{t_i}^j = -\frac{1}{N_j} N_{t_i}^{ji} + (1 - l_j) \frac{\hat{N}_{t_i}^j}{N_j}, 
\]  
(A.11)

\[ \hat{N}_{t_i}^{ji} = -\frac{k_j}{A' N_j} \hspace{1em} \text{and} \hspace{1em} \hat{N}_{t_i}^j = -\frac{1}{N_j A'}, \]
(A.12)

\[ \hat{N}_{t_i}^j = (\eta_j + \varepsilon_j) \hat{p}_{t_i}^j = -\frac{N_i}{N_j} (\eta_i + \varepsilon_i) \hat{p}_{t_i}^j, \hspace{1em} x = I, L, R \]  
(A.13)

and

\[
\hat{N}_{t_i}^j = (\eta_j + \varepsilon_j) \hat{p}_{t_i}^j = -\frac{N_i}{N_j} \left\{ (\eta_i + \varepsilon_i) \hat{p}_{t_i}^j + \frac{\varepsilon_i}{(1 + t_i^p)} \right\} . 
\]  
(A.14)

Then using (A.11) - (A.14) and differentiating the respective budget constraints give

\[
G_{t_i}^j = t_j^L k_j \frac{\varepsilon_j}{(\eta_j + \varepsilon_j) (1 + t_i^p)} \left[ \frac{k_j}{A' N_j} - (1 - l_j) \frac{N_i}{N_j} (\eta_i + \varepsilon_i) \frac{\varepsilon_i}{(1 + t_i^p)} \right] 
\]  
(A.15)

\[
G_{t_i}^j = t_j^L \hat{p}_{t_i}^j = t_j^L \left[ \frac{1}{A' N_j} - (1 - l_j) \frac{N_i}{N_j} (\eta_i + \varepsilon_i) \hat{p}_{t_i}^j \right] 
\]  
(A.16)

As the budget constraint with the residential tax is simply \( G_j = t_j^R \) we have
and for the property tax we have

\[ G^{j}_{tR} = 0, \quad (A.17) \]

Then using (A.15) - (A.17) in (A.9) gives

\[ \hat{p}^{i}_{tI} = \frac{-(\eta_j + \varepsilon_j) V'(G_j) t^I j k^i_j}{E^I_i} < 0, \quad (A.19) \]

\[ \hat{p}^{i}_{tL} = \frac{-(\eta_j + \varepsilon_j) V'(G_j) t^L j}{E^{L_i}} < 0, \quad (A.20) \]

and

\[ \hat{p}^{i}_{tR} = 0 \quad (A.21) \]

where \( E^I_i > 0 \) and \( E^{L_i} > 0 \) to ensure stability.\(^{18}\) For the property tax we use (A.18) in (13b) to obtain

\[ \hat{p}^{i}_{tP} = \frac{N_j (\eta_j + \varepsilon_j) V'(G_j) t^P j (1 - \varepsilon_j) p_j h_j}{D^P_{iP}} \left( \frac{N_i}{N_j} \left[ \frac{(\eta_i + \varepsilon_i)}{(\eta_j + \varepsilon_j)} \hat{p}^{i}_{tP} + \frac{\varepsilon_i}{(\eta_j + \varepsilon_j)} \frac{1}{1 + t^P_i} \right] \right) - \frac{N_i (1 + t^P_i)^{1/2}}{D^P_{iP}} \varepsilon_j p_j h_j \]

\[ N_j p_j h_j \left[ \frac{t^P (V'(G_j)(1 - \varepsilon_j) - 1)}{E^P_{iP}} \varepsilon_i \frac{(1 + t^P_i)}{1 + t^P_i} < 0 \quad (A.22) \right] \]

where \( E^P_{iP} > 0 \).\(^{19}\)

### A.2 Comparative Statics for the Property Tax and Industrial Capital Tax

#### A.2.1 Evaluating Price Gradients

To determine whether the optimal tax policy when the jurisdiction has the option to tax both property and capital, it is first necessary to determine what impact the tax will have

\(^{18}\)\( E^{I_i} = D + N_j (\eta_j + \varepsilon_j) V'(G_j) t^I j k^i_j (1 - \varepsilon_j) (\eta_i + \varepsilon_i) \) and \( E^{L_i} = D + N_j (\eta_j + \varepsilon_j) V'(G_j) t^L j (1 - \varepsilon_j) (\eta_i + \varepsilon_i). \)

\(^{19}\)\( E^{P_{iP}} = D^P_{iP} - N_j V'(G_j) t^P j (1 - \varepsilon_j) p_j h_j (\eta_i + \varepsilon_i). \)
on housing prices. The expression (A.10) still applies but as the capital tax is also used the fiscal externality, $G_{t_i}^j$ and $G_{t_i}^j$ will differ with

$$G_{t_i}^j = t_f^j (1 - \varepsilon_j) p_j h_j \hat{p}_{t_i}^j + t_f^j k_j l_{t_i}^j$$  \hspace{1cm} (A.23)

Then using (A.15) in (A.23) gives

$$G_{t_i}^j = t_f^j (1 - \varepsilon_j) p_j h_j \hat{p}_{t_i}^j + t_f^j k_j \left[ \frac{k_i}{N_i} \frac{1}{N_j} - (1 - l_j) \frac{N_i}{N_j} (\eta_i + \varepsilon_i) \right] \hat{p}_{t_i}^j$$  \hspace{1cm} (A.24)

Then using (A.8) to solve for $\hat{p}_{t_i}^j$ gives

$$G_{t_i}^j = - \left[ t_f^j (1 - \varepsilon_j) p_j h_j - t_f^j k_j \frac{N_i}{N_j} (\eta_i + \varepsilon_i) \right] \frac{N_i (\eta_i + \varepsilon_i)}{N_j (\eta_i + \varepsilon_i)} \left[ \frac{N_i}{N_j} (\eta_i + \varepsilon_i) \right] \frac{1}{1 + t_f^j} + t_f^j k_j \frac{k_i}{N_i} \frac{1}{N_j} \hat{p}_{t_i}^j$$  \hspace{1cm} (A.25)

### A.2.2 Evaluating the Price Gradient at $t_i^f = 0$

$$-p_i h_i \hat{p}_{t_i}^j - k_i + V'(G_i) \left[ k_i l_i + t_f^p p_i h_i (1 - \varepsilon_i) \hat{p}_{t_i}^j \right]$$

$$= \left\{ - (1 + t_f^p) p_j h_j + V'(G_j) \left[ t_f^p (1 - \varepsilon_j) p_j h_j + t_f^j k_j (1 - l_j) \right] (\eta_j - \varepsilon_j) \right\} \hat{p}_{t_i}^j$$

and

$$\hat{p}_{t_i}^j = \frac{-N_i (\eta_i + \varepsilon_i)}{N_j (\eta_i + \varepsilon_j)} \hat{p}_{t_i}^j.$$  \hspace{1cm} (A.26)

Then

$$\left\{ p_i h_i \left[ 1 - t_f^i p_i h_i (1 - \varepsilon_i) \right] + p_j h_j \frac{N_i (\eta_i + \varepsilon_i)}{N_j (\eta_j + \varepsilon_j)} \left[ (1 + t_f^p) - V'(G_j) \left[ t_f^p (1 - \varepsilon_j) + \frac{t_f^j k_j p_j h_j (1 - l_j)}{1 + t_f^j} \right] \right] \right\} \hat{p}_{t_i}^j$$

$$= k_i \left( V'(G_i) - 1 \right)$$

$$\hat{p}_{t_i}^j = \frac{N_j (\eta_j - \varepsilon_j)}{E_{t_i}^f} k_i \left( V'(G_i) l_i - 1 \right)$$  \hspace{1cm} (A.28)

$$\hat{p}_{t_i}^i = \frac{N_j (\eta_j - \varepsilon_j)}{E_{t_i}^f} k_i \left( V'(G_i) l_i - 1 \right)$$  \hspace{1cm} (A.29)
where

\[ E_{t_i'=0} = N_j (\eta_j + \varepsilon_j) p_i h_i \left[ 1 - t_i^P p_i h_i (1 - \varepsilon_i) \right] \]

\[ + p_j h_j \frac{N_i (\eta_i + \varepsilon_i)}{N_j (\eta_j + \varepsilon_j)} \left[ (1 + t_i^P) - V' (G_j) \left[ t_j^P (1 - \varepsilon_j) + \frac{t_j^P k_j}{p_j h_j} (1 - l_j) (\eta_j - \varepsilon_j) \right] \right] > 0 \]  

(A.30)

### A.2.3 Equilibrium Conditions with Both taxes

Using (13a) and (13b) with the fiscal externalities, \( G_{t_i'}^j \) and \( G_{t_i'}^j \) enables us to determine \( \hat{\phi}_{t_i}^j \) and \( \hat{\phi}_{t_i'}^j \). The fiscal externalities when both taxes are employed are

\[
G_{t_i}^j = t_j^P p_j h_j (1 - \varepsilon_j) \hat{\phi}_{t_i}^j + t_j^P k_j \left[ \frac{1}{N_j A'} + (1 - l_j) \frac{N_j^j t_i^j}{N_j} \right]
\]

\[
= t_j^P p_j h_j (1 - \varepsilon_j) \hat{\phi}_{t_i}^j + t_j^P k_j \left[ \frac{1}{N_j A'} + (1 - l_j) (\eta_j + \varepsilon_j) \hat{\phi}_{t_i}^j \right]
\]

\[
= t_j^P k_j \frac{1}{N_j A'} - \left[ t_j^P p_j h_j (1 - \varepsilon_j) \frac{(\eta_i + \varepsilon_i)}{(\eta_j + \varepsilon_j)} + (1 - l_j) (\eta_i + \varepsilon_i) \right] \frac{N_i}{N_j} \hat{\phi}_{t_i}^j
\]

and

\[
G_{t_i'}^j = t_j^P p_j h_j (1 - \varepsilon_j) \hat{\phi}_{t_i'}^j + t_j^P k_j (1 - l_j) \frac{N_j^j t_i^j}{N_j}
\]

\[
= t_j^P p_j h_j (1 - \varepsilon_j) \hat{\phi}_{t_i'}^j + t_j^P k_j (1 - l_j) (\eta_j + \varepsilon_j) \hat{\phi}_{t_i'}^j
\]

\[
= \left[ t_j^P p_j h_j (1 - \varepsilon_j) + (1 - l_j) (\eta_i + \varepsilon_i) \right] \frac{N_i}{N_j} \left[ -\frac{(\eta_i + \varepsilon_i)}{(\eta_j + \varepsilon_j)} \hat{\phi}_{t_i'}^j - \varepsilon_i \frac{1}{(\eta_j + \varepsilon_j) (1 + t_i^P)} \right]
\]

(A.32)

Then using (A.31) in (13a) gives

\[
\hat{\phi}_{t_i}^j = -\frac{(\eta_i + \varepsilon_i) V' (G_j) t_i^P k_j k_i}{E t_i^* t_i^P} < 0
\]

(A.33)

where \( E_{t_i't_i'} = D + N_i (\eta_j + \varepsilon_j) V' (G_j) \left[ t_j^P p_j h_j (1 - \varepsilon_j) \frac{(\eta_i + \varepsilon_i)}{(\eta_j + \varepsilon_j)} + (1 - l_j) (\eta_j + \varepsilon_j) \right] > 0 \). Evaluating \( \hat{\phi}_{t_i'}^j \) is more complicated; we have

\[
\hat{\phi}_{t_i'}^j = -\frac{N_i (1 + t_i^P) \varepsilon_i p_j h_j - N_j (\eta_j + \varepsilon_j) V' (G_j) G_{t_i'}^j}{D}
\]

(A.34)

and substituting for \( G_{t_i'}^j \) in (A.34) using (A.32) gives

38
\[
\hat{p}_{i}^{t} = \frac{-N_{i} \frac{(1 + t_{P}^{p})}{(1 + t_{P}^{p})} \varepsilon_{i} p_{j} h_{j} - N_{j} (\eta_{j} + \varepsilon_{j}) V'(G_{j}) \left[ t_{P}^{p} p_{j} h_{j} (1 - \varepsilon_{j}) + (1 - l_{j}) (\eta_{i} + \varepsilon_{i}) \right] N_{j} \left[ \frac{(\eta_{j} + \varepsilon_{j}) \hat{p}_{i}^{t} - \frac{\varepsilon_{i}}{\eta_{i} + \varepsilon_{j}}}{(1 + t_{P}^{p})} \right]}{D} \tag{A.35}
\]

\[
\hat{p}_{i}^{t} = \frac{-N_{i} \frac{\varepsilon_{i}}{1 + t_{P}^{p}} p_{j} h_{j} \left[ (1 + t_{P}^{p}) - V'(G_{j}) \left[ t_{P}^{p} (1 - \varepsilon_{j}) + \frac{(1 - l_{j}) (\eta_{i} + \varepsilon_{i})}{p_{j} h_{j}} \right] \right]}{E \hat{t}^{I} \hat{p}_{i}^{t}^{*} \hat{t}^{P_{i}}} \tag{A.36}
\]

### A.3 Optimal Centralized Policies

#### A.3.1 Equilibrium Wage, and Price Gradients and Population and Commuting Patterns

As with a capital tax by the individual governments, the capital tax by the centralized government will reduce wages with

\[
w_{i}^{t} = -k_{i}, \quad i = 1, 2 \tag{A.37}
\]

Differentiating (5a) with respect to each of the taxes gives

\[
N_{i}^{21} = -\frac{(k_{1} - k_{2})}{A'}, \quad N_{i}^{21} = 0, \quad \text{and} \quad N_{i}^{21} = -\frac{1}{A'} \tag{A.38}
\]

Assuming that there are equal housing demand and supply elasticities \((\eta_{1} = \eta_{2} = \eta; \varepsilon_{1} = \varepsilon_{2} = \varepsilon)\), differentiating the housing market equilibrium, (5c), with respect to \(t^{x}, x = I, L, R, C\) gives

\[
\hat{p}_{i}^{2} = -\frac{N_{1}}{N_{2}} \hat{p}_{i}^{1}, \quad x = L, R, C \tag{A.39}
\]

and for \(t^{p}\) we obtain

\[
\frac{N_{1}}{N} \hat{p}_{i}^{1} + \frac{N_{2}}{N} \hat{p}_{i}^{2} = -\frac{\varepsilon}{(\eta + \varepsilon)} \frac{1}{(1 + t^{p})}. \tag{A.40}
\]

Evaluating the equal utility condition, (5b), gives

\[
U_{i}^{1} - p_{1} h_{1} \hat{p}_{i}^{1} = U_{i}^{2} - p_{2} h_{2} \hat{p}_{i}^{2}, \quad x = I, L, R, C, P. \tag{A.41}
\]

Then for \(x = I\), the tax on industrial capital we have \(U_{i}^{1} = -k_{i}\). Then using (A.39) and (A.40) gives

\[
\hat{p}_{i}^{t} = -\frac{k_{i} - k_{j}}{N_{2} p_{1} h_{1} + N_{1} p_{2} h_{2}} \tag{A.42}
\]
For the residential and labor tax, \( x = L \) and \( x = R \), we have \( U^{i}_{tc} = -1 \), \( x = L, R \) then
\[
\hat{p}^{i}_{tc} = 0, \ x = L, R
\]  
(A.43)
and for commuter tax, \( x = C \), we have \( U^{i}_{tc} = 0 \) and
\[
\hat{p}^{i}_{tc} = 0.  
\]  
(A.44)
Finally, we have the impact of the property tax with \( U^{i}_{tp} = -p_i h_i \). Then using (A.40) and (A.41) we obtain
\[
\hat{p}^{i}_{tp} = -\left[ \frac{N_j (p_i h_i - p_j h_j) + N p_j h_j \frac{\varepsilon}{(\eta + \varepsilon)}}{N_2 p_1 h_1 + N_1 p_2 h_2} \right] \frac{1}{1 + t^P}  
\]  
(A.45)
Finally, we can consider the impact of the uniform taxes on the populations of the two jurisdictions. Using (9a) with (A.42) it follows that the centralized tax on industrial capital is given by
\[
\hat{N}^{i}_{tI} = (\eta + \varepsilon) N_j \frac{k_j - k_i}{N_2 p_1 h_1 + N_1 p_2 h_2}  
\]  
(A.46)
The tax on residents, labor or commuting have no affect on the populations of the two jurisdictions. The impact of an increase in the property tax on population can be seen by using (9b) with (A.45) to obtain
\[
\hat{N}^{i}_{tp} = -(\eta + \varepsilon) \left[ \frac{N_j (p_i h_i - p_j h_j) + N p_j h_j \frac{\varepsilon}{(\eta + \varepsilon)}}{N_2 p_1 h_1 + N_1 p_2 h_2} \right]  
\]  
(A.47)

A.3.2 Impact of Centralized Policies on the Government Budget.

The general budget constraint can be expressed as
\[
\overline{NG} = (t^I k_1 + t^L) (N_1 + N_{21}) + (t^I k_2 + t^L) (N_2 - N_{21}) + N_1 \left( t^R + t^P p_1 h_1 \right) + N_2 \left( t^R + t^P p_2 h_2 \right) + t^c N_{21}  
\]  
(A.48)
Differentiating with respect to \( t^I \) when there are no other tax instruments gives
\[
G_{tI} = \left[ k_1 l_1 \frac{N_1}{N} + k_2 l_2 \frac{N_2}{N} \right] \left[ 1 + t^I \frac{k_1}{k_2} \right] - \frac{t^I (k_1 - k_2)^2}{N} \left[ \frac{N_1 N_2}{N_1 p_1 h_1 + N_2 p_2 h_2} + \frac{1}{A^P} \right]  
\]  
(A.49)
With respect to the residential and labor taxes, we have
\[ G_{tx} = 1, \ x = R, \ L. \quad (A.50) \]

For the commuter taxes we have

\[ G_{tC} = N_{21} \left( 1 - t_C \frac{1}{N_{21}A'} \right) \quad (A.51) \]

### A.3.3 Optimal Centralized Policies

The general objective function of the centralized government is assumed to be

\[
(w_1 (t^I) - t^L) (N_1 + N_{21}) + (w_2 (t^I) - t^L) (N_2 - N_{21}) + N t^R + N_1 S (p_1 (1 + t^P)) + N_2 S (p_2 (1 + t^P)) + \overline{NV} (G (t^I, t^L, t^R, t^P, t^C))
\]

(A.52)

### A.4 Numerical Example

#### A.4.1 Outline of the Model

Let the quasilinear utility function be given by

\[ U_i = x_i + \alpha_1 h_i^{\beta_1} + \alpha_2 G_i^{\beta_2}, \ \alpha_i > 0, \ 0 < \beta_i < 1, \ i = 1, 2 \quad (A.53) \]

which gives

\[ h_i (p_i (1 + t^P)) = (\alpha_1 \beta_1) \frac{1}{\bar{\beta}_1} p_i^{1 - \frac{1}{\bar{\beta}_1}} \quad (A.54) \]

making the elasticity of demand \( \varepsilon = \frac{1}{1 - \beta_1} \). Housing production is given by

\[ H_i (p_i) = \gamma_i p_i^{\eta_i}, \ \gamma_i > 0, \ 0 < \eta_i < 1, \ i = 1, 2 \quad (A.55) \]

making the elasticity of supply \( \eta_i \). Production is given by

\[ x_i^* = \lambda_i L_i^{\nu_i} K_i^{1 - \nu_i} \quad (A.56) \]

Then the zero profit condition is satisfied when

\[ \frac{\lambda_i}{2} \frac{w_i^{\nu_i}}{(\rho + t_i^{lI})^{1 - \nu_i}} = 1 \quad (A.57) \]

with

\[ K_i = \left( \frac{w_i}{\rho + t_i^I} \right) L_i \ \text{or} \ k_i = \left( \frac{w_i}{\rho + t_i^I} \right) \quad (A.58) \]
Let the commuting costs be given by

\[ A(N_{21}) = \mu N_{21}^\sigma, \quad \sigma > 1 \]  \hspace{1cm} (A.59)

**A.4.2 Equilibrium Conditions**

Equilibrium in the housing market (population), (5c) requires that

\[ N_1 + N_2 = \frac{\gamma_1 p_1^\eta_1}{(\alpha_1 \beta_1)^{1-\beta_1} p_1^{1-\beta_1}} + \frac{\gamma_2 p_2^\eta_2}{(\alpha_1 \beta_1)^{1-\beta_1} p_2^{1-\beta_1}} = N \]  \hspace{1cm} (A.60)

Using (A.59) in (5a) gives the equal utility for commuters,

\[ N_{21} = \left[ \frac{w_1 - w_2 - (t_L^1 - t_L^2) - (t^r_1 + t^r_2)}{\mu} \right]^{\frac{1}{\sigma}} \]  \hspace{1cm} (A.61)

and the equal utility condition resident/worker in suburbs and for the resident/worker, (5b), in the central city is

\[

tag*{\begin{align*}
    w_2 - t_L^2 - t_R^2 + \alpha_1 h_2^{\beta_1} - p_2 (1 + t^p_2) h_2 + \alpha_2 G_{2}^{\sigma_2} = \\
    w_1 - t_L^1 - t_R^1 + \alpha_1 h_1^{\beta_1} - p_1 (1 + t^p_1) h_1 + \alpha_2 G_{1}^{\sigma_2}
\end{align*}}
\]  \hspace{1cm} (A.62)

where \( h_i \) is defined by (A.54)