Bailout Addiction: Does Bailout Anticipation Induce Adverse Selection?*

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Abstract

The anticipation of a future bailout worsens adverse selection at present, causing a market freeze now and inviting government intervention ("bailout trap"). When firms raise financing, high-quality firms are willing to bear adverse selection costs because they anticipate profitable future opportunities to buy assets of failed lower-quality firms. But search frictions in private trading impede efficiency, inviting a bailout. This reduces buyers' *ex post* trading profits, dissuading *ex ante* financing participation by high-quality firms, possibly causing a freeze, and leading to market-unfreezing bailouts ("bailout addiction"). We analyze a "capital assistance fund" as a distortion-reducing regulatory response.

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Keywords: Adverse selection, Market freeze, Government bailout

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1 Introduction

As with public pronouncements after bailouts during the 2007-09 crisis, governments like to assert that there will be no more future bailouts of failing firms. Bailouts are costly for taxpayers and are also viewed as worsening moral hazard, so they entail both political and economic costs. However, during financial crises, firms in distress often have difficulty in raising external funding, as we witnessed during the 2007-09 crisis. This makes it difficult for the government to maintain a hands-off approach, and it often steps in with a bailout of some sort – loan guarantees, subsidized lending, toxic asset purchases, or direct equity injections – to provide these firms financing they cannot otherwise obtain. It is believed that, absent the intervention, socially valuable investments would be foregone. Philippon and Skreta (2012) and Tirole (2012) examine the optimal design of government intervention to thaw markets frozen by adverse selection in static settings. In their models, the degree of adverse selection that freezes the market is taken as exogenous. The worse the adverse selection, the larger and more costly is the intervention. The only cost of intervention in these models is the shadow cost of the investment made by the government in the bailout.[1]

This paper develops a dynamic model to highlight a novel cost of bailouts, namely that the anticipation of a future bailout worsens *ex ante* adverse selection, i.e., adverse selection worsens *endogenously.* To see our main point, note that there is heterogeneity in the financial conditions of firms during a crisis, so some firms experience a funding freeze and some do not (see, e.g., Pérignon, Thesmar, and Vuillemey (2018) for empirical evidence). Absent government intervention, financially-strong firms acquire assets of weak firms, probably at depressed prices, thereby making profits; see Berger and Bouwman (2013) for empirical evidence. The anticipation of such future trading profits can induce some high-quality firms to participate in raising financing at prior dates, despite being pooled with lower-quality firms. This raises the average quality of firms seeking financing. But when the government steps in to bail out weak firms *ex post* in a crisis, it competes with strong (high-quality) firms. This reduces profitable asset acquisition opportunities for strong firms, diminishing their *ex ante* incentives to bear adverse selection costs in raising financing. Thus, the pool of firms seeking financing worsens in quality *ex ante*. This raises two questions: how does bailout anticipation *endogenously* determine *ex ante* adverse selection, and what are its consequences, including welfare implications?

Preview of Model and Results: To address these questions, we consider a two-period (three-date) adverse selection setting with a continuum of firm types. Each firm needs financing at date 0 (initial

[1] These papers do not examine moral hazard engendered by bailouts, such as excessive risk taking, high leverage and other inefficiencies (e.g., Chaney and Thakor (1985) and Rosas and Jensen (2010)).

investment) and date 1 (continuation investment) for a project that yields a non-pledgeable payoff at date 2. Being penniless at date 0, a firm must sell its legacy asset to raise the date-0 financing. The legacy asset may produce a cash flow at date 1 that is partially pledgeable and partially non-pledgeable. Firms differ in the likelihood of realizing this cash flow, and the probability, θ, of this cash flow being realized with the legacy asset is privately known to each firm; we refer to θ as the quality (i.e., type) of the firm. As in Akerlof (1970), legacy asset sales occur at a pooling price reflecting the average asset quality in the market, so firms with the best assets stay out, eschewing their projects, and only those with $\theta \leq \theta^*$ participate in sales, where θ^* is endogenously determined. Among participating firms, those with higher qualities bear adverse selection costs in financing. If a participating firm's legacy asset pays off at date 1, it can finance its date-1 continuation project investment with the legacy asset's (non-pledgeable) cash flow. If not, it may trade with a successful firm (whose legacy asset pays off) in an over-the-counter market involving searching and matching to sell its project; selling the whole project is necessary because its date-2 terminal payoff is non-pledgeable to financiers.

Nash bargaining between the buyer and the seller determines the price at which trade occurs in the market, leaving the buyer with a profit at date 1. For high-θ firms, which are more likely to become buyers, the anticipation of this date-1 trading profit (partially) compensates for their date-0 adverse selection loss in the legacy asset market. This increases θ^* and hence the average quality of firms raising financing at date 0. Consequently, a freeze in the legacy asset market is avoided – such a freeze would occur in a static setting without trading.[2] As market trading efficiency improves, the positive impact of trading on θ^* gets stronger, so more firms raise financing at date 0 and the average quality of those participating firms rises.

Private market trading improves welfare because it: (i) unfreezes the legacy asset market, so a subset of firms can raise financing at date 0, and (ii) allows some failing firms' (whose legacy assets fail to pay off) projects to be transferred to successful firms and hence continued at date 1. But trading also has an inefficiency – each seller faces uncertainty about whether it will be able to find a buyer for its project due to search frictions in the market, and project surplus is lost if trade does not occur.

This market inefficiency invites a bailout to improve welfare. If the government bails out some failing firms at date 1, it guarantees that these firms are able to continue their projects. In our model, the bailout is an equity injection. Meanwhile, since the private market continues operating while the bailout is conducted, the bailout both influences market trading and is influenced by it. In particular, the bailout reduces the

[2] We focus on cases in which θ^* is not high enough absent trading, so legacy asset sale proceeds are insufficient for date-0 financing; consequently, without trading, no firm invests in its project (and none sells its legacy asset) at date 0: a market freeze.

number of failing firms in the market. Consequently, for each of these failing firms *outside* the bailout, its probability of finding a buyer for its project in the market also improves. That is, search frictions in the private market both motivate the bailout and are themselves *endogenously* affected by the bailout. *Both* firms inside and outside the bailout benefit from the bailout – these efficiency gains rationalize the bailout. We solve for the number of firms bailed out and the bailout price.

However, this bailout has a pernicious effect on date-0 financing. The bailout, by increasing the post-bailout buyer/seller ratio in the market, heightens competition for project acquisitions faced by private buyers, reducing their trading profits. Since high-θ firms are more likely to become buyers at date 1, this lowers θ^* and hence the average quality of firms seeking financing at date 0. This, in turn, increases the odds and size of a bailout at date 1, setting in motion a "bailout trap" that may end in *all* firms avoiding market financing at date 0. Thus, a market that dynamically resolves adverse selection (through trading) and is not frozen at the outset ends up being frozen when a future bailout is anticipated. This freeze then invites a date-0 intervention to jump-start the market. We show that the anticipation of the date-1 bailout also makes the date-0 intervention more expensive.

One way to think about this analysis is that low-quality firms would like high-quality firms to participate in the market at date 0 so their legacy assets can be sold at a high enough price to enable their initial project investments. High types are willing to participate despite the price discount in legacy assets they sell at date 0 because they anticipate opportunities to acquire low types' projects at discount at date 1. Interestingly, low types are able to initiate their projects at date 0, which are then available for trade at date 1, only if high types bear adverse selection costs in the date-0 legacy asset market. That is, there is a laissez-faire, *intertemporal* reciprocity or insurance mechanism across types. A government bailout breaks that intertemporal across-types insurance.

Essentially, we show that the very possibility of a future bailout eliminates the counterfactual of a self-correcting market mechanism that would have come into play absent bailout anticipation. This highlights a novel cost of bailouts, besides the usual moral hazard consequence. Ironically, if the date-0 intervention does unfreeze the market at that date, it will be viewed as a "success" because the cost of the disappearance of the market self-correction counterfactual remains unseen. We analyze both the date-1 bailout and the date-0 jump-start intervention induced by bailout anticipation. We show that the larger the anticipated date-1 bailout, the more expensive is the date-0 intervention. That is, the anticipation of the date-1 bailout not only necessitates the date-0 intervention, but also makes it more costly.

Evidence: There is empirical evidence consistent with our theory, although we are not aware of a direct test. Dokupil (2003) notes that the airline bailouts after the 9/11 attacks that obviated the need for U.S. carriers to go through bankruptcy made the industry weaker. Caballero, Hoshi, and Kashyap (2008) provide evidence on the adverse effect that zombie lending in Japan generated in terms of discouraging entry by healthy firms. See also Giannetti and Simonov (2013) for related evidence that poorly designed Japanese bank bailouts encourage zombie lending.[3]

Policy Implication: The usual objection to bailouts is that they engender risk-shifting moral hazard. The anticipation of a future bailout can make a future bailout more likely because of the actions of firms that increase the likelihood of failure. However, for regulated firms like banks, this moral hazard can be restrained by higher capital requirements and regulatory monitoring. In addition, there may also be ways to prevent moral hazard through carefully designed bank resolution mechanisms. Philippon and Wang (2023) provide a theoretical analysis in which the *distribution* of bailouts provides incentives. Specifically, bailing out the best-performing banks while letting the worst ones fail creates a tournament that may eliminate moral hazard. Thus, there are various approaches to coping with bailout-induced moral hazard.

Our analysis highlights a different and perhaps more vexing problem – the *ex ante* increase in adverse selection that not only makes the future bailout more likely and more expensive, but also potentially freezes the market even *before* that. This is not a problem readily solvable by the usual tools of prudential regulation. Thus, our analysis sheds light on the manner in which prudential regulation needs to adapt in an environment in which governments cannot credibly precommit to not bail out firms for which the financing market is frozen.

Normative Analysis of Possible Solution: Based on this analysis, we ask: could there be regulatory mechanism that would accommodate bailouts at date 1 without precipitating a market freeze at date 0? In Section 4, we propose one such mechanism, which we call the "capital assistance fund." With this fund, the regulator collects a contribution/fee at date 0 that participating firms finance out of their post-investment asset sale proceeds. The regulator then uses the fund at date 1 to both bail out losers as well as assist winners, but the optimal bailout is necessarily partial.

Related Literature: Our paper is related to the literature on government intervention in markets afflicted with adverse selection. In Philippon and Skreta (2012), the intervention involves direct lending or debt guarantees to banks to reduce how much they have to borrow in markets. They examine how the

[3]While these findings are generally supportive of our model, they do not represent a direct empirical test. Future research will need to provide a direct test.

government should design its mechanism when private markets are open, so participation decisions convey information about private types. In Tirole (2012), the government buys the weakest assets to jump-start a frozen market.[4] In these papers, the government's mechanism design involves dealing with endogenous reservation utilities determined by the mechanism itself. Our model shares this feature in that the bailout affects not only the payoffs of firms receiving the bailout, but also those operating outside the bailout in the market. In Camargo, Kim, and Lester (2016), government intervention restarts trade, but too much intervention depletes trade of its information content. In contrast to these papers, we focus on how the anticipation of a future bailout worsens the quality of firms participating in the market *ex ante*. That is, while they design the optimal intervention taking the pool of firms – and hence the degree of adverse selection – as exogenously given, we show that the chosen intervention endogenously affects the types of firms comprising the initial pool.[5]

Also related is a literature on dynamic adverse selection and its interaction with government intervention.[6] In Camargo and Lester (2014) and Fuchs and Skrzypacz (2015), a market frozen by adverse selection may thaw by itself over time as the worst assets are sold first and both the quality of traded assets and their market prices gradually improve. They show that interventions may induce owners of bad assets to delay trade to wait for the market recovery and future subsidies, thereby slowing down the recovery.[7] There are key differences between this literature and our paper. While in this literature the anticipation of future interventions delays the exit of bad assets, in our model the anticipation hastens the exit of good assets. This mechanism difference leads to very different policy implications. Policy interventions in this literature focus on removing bad assets (as many and as fast as possible),[8] whereas in our model the appropriate policy interventions focus on rewarding asset buyers when the government intervenes.

Structure: Section 2 describes the model. Section 3 contains the analysis. Section 4 examines policy implications and solutions. Section 5 concludes. Proofs are in Appendix A.

[4]Some other papers have also analyzed optimal forms of government intervention. For example, in Philippon and Schnabl (2013), banks restrict lending to firms because of debt overhang. They show that the government's efficient recapitalization program injects capital against preferred stock plus warrants and requires sufficient bank participation.

[5]This is somewhat reminiscent of the Lucas critique that the model parameters based on which policies are designed are not policy-invariant.

[6]We only discuss a few studies closely related to our paper; for other contributions, see, for example, Chari, Shourideh, and Zetlin-Jones (2014) and Guerrieri and Shimer (2014).

[7]Daley and Green (2012) show that trade may be delayed when there is the potential for news to arrive in the market. They examine how a planner can control news quality to speed up trade and improve welfare. Chiu and Koeppl (2016) analyze a different setting in which the lemons problem does not diminish over time through trading. As a result, they find that an intervention is necessary for the market recovery and delaying the intervention is optimal.

[8]For instance, in Fuchs and Skrzypacz (2015) the government can set an initial tax-exempt trading window followed by high taxes to encourage early trade to prompt fast departure of bad assets.

2 Model

Types and Timing: The model has three "dates", $\{\text{dawn}, \text{noon}, \text{night}\}$, universal risk neutrality, and no discounting. There is a continuum of firms with unit mass. Each firm has a project at dawn. An initial investment of I at dawn yields the firm a non-pledgeable private benefit of B in the morning. With a continuation investment of δI at noon, the project further returns Y with certainty at night; Y is not available if δI is not invested.

Each firm has a legacy asset with a payoff $\tilde{x} \in \{X, 0\}$ realized at noon. Firms have no cash at dawn, so they rely on the sale of legacy assets to finance investments at dawn. The legacy asset's payoff X is partially pledgeable; specifically, $X = X_{\text{p}} + X_{\text{np}}$, where X_{p} is pledgeable while X_{np} is not. In contrast, the project's return Y is fully non-pledgeable to financiers, although another firm buying ownership in the project can have access to Y like the firm that is originally endowed with the project. Although X_{np} is not pledgeable to outside investors (while X_{p} can), it is a cash flow just as tangible as X_{p}.[9] So, the firm may use X_{np} at noon for additional investments.

Each firm's "type" is the success probability of its legacy asset: $\Pr(\tilde{x} = X) = \theta \in [\underline{\theta}, \bar{\theta}] \subset [0, 1]$. The mass of type-$\theta$ firms is $f(\theta)$. The total firm mass is one, so $\int_{\underline{\theta}}^{\bar{\theta}} f(\theta)d\theta = 1$. Thus, $f(\theta)$ is also the density of the θ distribution, with the corresponding cumulative distribution function $F(\theta)$. While each firm knows its θ privately at dawn, $f(\theta)$ and $F(\theta)$ are common knowledge. The unconditional mean of θ is $\mathbb{E}[\theta] = \int_{\underline{\theta}}^{\bar{\theta}} \theta f(\theta)d\theta$.

Suppose firms with $\theta \in [\underline{\theta}, \theta^*]$ sell legacy assets to raise financing at dawn; θ^* will be determined in the analysis. This specification postulates that if a type θ sells its legacy asset, then all types $\theta' < \theta$ also sell their legacy assets. This is known as the "skimming property" and we will verify it. It simplifies the equilibrium analysis because the set of firms engaging in asset sales is a truncation of the initial firm distribution. Denote $m(\underline{\theta}, \theta^*) \equiv \mathbb{E}[\theta|\underline{\theta} \le \theta \le \theta^*] = \int_{\underline{\theta}}^{\theta^*} \frac{\theta f(\theta)}{F(\theta^*)} d\theta$ the average type among those asset sellers.

Trading: At noon, among legacy asset sellers, call firms with $\tilde{x} = X$ "winners," and those with $\tilde{x} = 0$ "losers." The realization of $\tilde{x}$ for each firm is public at noon, as are the identities of winners and losers. For type θ, by the law of large numbers, the mass of winners is $\theta f(\theta)$, and the mass of losers is $(1-\theta)f(\theta)$. In the aggregate (among legacy asset sellers), the mass of winners is $\int_{\underline{\theta}}^{\theta^*} \theta f(\theta)d\theta$, and the mass of losers

[9] One may interpret X_{np} as a hidden cash flow of the legacy asset that cannot be extracted by a buyer of the asset but can be accessed by the firm itself. As in Holmstrom and Tirole (1997), the pledgeable part of the payoff X_{p} can be viewed as the maximum repayment investors can demand before precipitating a switch by the firm to actions making the firm non-creditworthy.

is $\int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta$. A loser cannot finance its continuation investment δI from pledging the project's (non-pledgeable) return Y. In contrast, a winner can use its legacy asset's non-pledgeable payoff, X_{np}, to invest δI and continue its project;[10] this requires $X_{\text{np}} \geq \delta I$. We further assume that X_{np} is sufficiently high, but not too high, so a winner can acquire exactly *one* loser's project at noon and invest δI to continue the acquired project, after investing δI for its own project.[11] Parametric restrictions to make this precise are in Assumption 4.

Winners and losers interact in an over-the-counter market at noon to trade losers' projects. We model trading with a (one-shot) random search model. Let the buyer/seller ratio n denote the market tightness, which determines the probability that project buyers (winners) and project sellers (losers) are matched. Following earlier descriptions, $n = \frac{\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta}{\int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta}$. It is straightforward to show that $\frac{dn}{d\theta^*} > 0$,[12] since higher-θ firms are more likely to become winners. Note that firms that did not participate in legacy asset sales at dawn to raise financing for their initial project investments are not considered part of the trading and are excluded from the market. That is, to continue an acquired project at noon, the firm must have made the initial investment at dawn and gained some experience from its own project (i.e., learning by investing).[13]

For tractability, we follow the monetary economics literature (e.g., Kiyotaki and Wright (1993)) and use a common specification of the meeting technology

$$\mu(n) = \frac{\lambda n}{1+n} \tag{1}$$

to parameterize the probability a seller meets a buyer. The probability a buyer meets a seller is $\frac{\mu(n)}{n} = \frac{\lambda}{1+n}$. Trading efficiency is captured by $\lambda \in [0,1]$, with a larger λ corresponding to higher efficiency. After a buyer and a seller are matched, they negotiate the ownership $\alpha \in [0,1]$ of the seller's project acquired by the buyer in exchange for the buyer's investment of δI to continue the seller's project. This is determined according to Nash bargaining with β being the buyer's bargaining power, and $1-\beta$ the seller's bargaining power. Winners thus make a profit $\alpha Y - \delta I$ conditional on trade. We assume that the well-known Hosios condition (Hosios (1990)) is satisfied, i.e., an agent's bargaining power is commensurate with its contribution to matching. Specifically, a buyer's bargaining power equals the elasticity of $\mu(n)$ with respect to the market tightness n, i.e., $\beta = \frac{n\mu'(n)}{\mu(n)} = \frac{1}{1+n}$; a seller's bargaining power is $1-\beta = \frac{n}{1+n}$.

[10]The pledgeable payoff X_{p} is obtained by the legacy asset buyer.

[11]Our results do not qualitatively change if a winner is permitted to acquire two or more projects, as long as it is not so big that it can corner the market.

[12]Note that $\frac{dn}{d\theta^*} \propto \theta^* \int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta - (1-\theta^*)\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta = \int_{\underline{\theta}}^{\theta^*}(\theta^*-\theta)f(\theta)d\theta > 0$.

[13]Or, as in Botsch and Vanasco (2019), banks learn by lending.

Bailout: The frictional search market may result in some project buyers and sellers remaining unmatched, so some losers' continuation investment needs at noon are not met while some winners' spare investment capacities are not utilized. This leaves room for government intervention that bails out (some) losers. We model a bailout as the government injecting δI of equity capital in a loser's project in exchange for an ownership $\alpha_{\mathrm{g}} \in [0,1]$, similar to the Capital Purchase Program (CPP) employed by the U.S. Treasury during the 2007-09 crisis to bail out banks.[14] A bailed-out firm continues its project at noon, without having to search for a buyer for the project in the private market. Besides α_{g}, the government also determines the mass of losers to bail out, γ. The government must consider the shadow cost of a bailout: each unit of public funds used incurs a shadow cost of ω, so for the $\gamma\delta I$ units of equity injected into the γ mass of losers, the total social cost is $\omega\gamma\delta I$. The private market continues to operate alongside the bailout (as in Philippon and Skreta (2012) and Tirole (2012)), so each loser can either opt in the bailout or sell its project in the market. The government takes all these into account in setting α_{g} and γ to maximize social welfare at noon.

Summary and Parametric Restrictions: The timeline in Table 1 summarizes the model.

Table 1: Sequence of Events

Dawn	• Each firm decides whether to sell its legacy asset to raise the initial investment I.
Noon	• Realizations of legacy asset payoff $\tilde{x}$ are publicly observed, as are winners and losers. • Government chooses the bailout size (γ) and announces its offer (α_{g}). • Losers decide whether to opt in the bailout or remain in the private market. • Trade between winners and remaining losers in the private market takes place. • Continuation investment δI, if made available, is deployed.
Night	• Project return Y is realized if the continuation investment was made at noon.

Assumption 1. *$\mathbb{E}[\theta]X_{\mathrm{p}} > I > \underline{\theta}X_{\mathrm{p}}$, so there exists a unique interior cutoff, $\theta_{\mathrm{d}} \in (\underline{\theta}, \bar{\theta})$, such that*

$$m(\underline{\theta}, \theta_{\mathrm{d}})X_{\mathrm{p}} = I. \tag{2}$$

Moreover, $\theta_{\mathrm{d}} \geq \frac{1}{2}\big(1 + \frac{I}{X_{\mathrm{p}}}\big)$.

This assumption has two implications. First, to make the dawn investment feasible, the highest type θ^* engaging in asset sales must be no lower than θ_{d}, so the sale proceeds are enough to cover the investment need, i.e., $m(\underline{\theta}, \theta^*)X_{\mathrm{p}} \geq I$. Second, when the dawn investment is feasible ($\theta^* \geq \theta_{\mathrm{d}}$), we have $\theta^* \geq \frac{1}{2}\big(1+\frac{I}{X_{\mathrm{p}}}\big)$, so the highest type participating in asset sales θ^* is sufficiently more likely to become a winner than a loser.[15]

[14] See Mücke, Pelizzon, Pezone, and Thakor (2023).

[15] This is a bit restrictive – theoretically nothing precludes the type θ^* from being more likely to become a loser than a

Assumption 2. *In the benchmark without trading and bailout, the highest type θ^* raising financing at dawn lies in the interior of $(\underline{\theta}, \bar{\theta})$. A sufficient condition for this is*

$$B + \underline{\theta}(Y - \delta I) - I > 0 > B + \bar{\theta}(Y - \delta I) - I - \big(\bar{\theta} - \mathbb{E}[\theta]\big)X_{\text{p}}. \tag{3}$$

Assumption 3. *The expected trading profit of a winner at noon only partially compensates for its adverse selection loss in the legacy asset market at dawn, so the skimming property is sustained, i.e., a higher type is less inclined than a lower type to raise financing, even when trading is permitted. A sufficient condition for this is*

$$X_{\text{p}} \geq [1 + \lambda(1 - 2\underline{\theta})](Y - \delta I). \tag{4}$$

Assumption 4. $0 < \mathbb{E}[\theta]X_{\text{p}} - I < \delta I$ *and* $2\delta I \leq X_{\text{np}} < 3\delta I - \big(\mathbb{E}[\theta]X_{\text{p}} - I\big)$.

The first condition says that the maximum possible[16] cash left over after using the asset sale proceeds to finance the dawn investment, $m(\underline{\theta}, \bar{\theta})X_{\text{p}} - I = \mathbb{E}[\theta]X_{\text{p}} - I$, is less than δI, so a loser can never continue its project at noon by itself. The second condition implies (i) For a winner, its legacy asset's non-pledgeable payoff (X_{np}) alone is high enough to allow it to continue its own project and also acquire one loser's project ($2\delta I \leq X_{\text{np}}$); (ii) But X_{np} is not too high: even with the maximum possible remaining cash from the asset sale and dawn investment, $\mathbb{E}[\theta]X_{\text{p}} - I$, a winner still cannot acquire two losers' projects, i.e., $X_{\text{np}} + \big(\mathbb{E}[\theta]X_{\text{p}} - I\big) < 3\delta I$.

Assumption 5. *$F(\theta)$ is sufficiently log-concave such that*

$$\frac{\partial m(\underline{\theta}, z)}{\partial z} < 1 - \frac{[1 + \lambda(1 - 2\underline{\theta})](Y - \delta I)}{X_{\text{p}}} \quad \forall z \in (\underline{\theta}, \bar{\theta}]. \tag{5}$$

A standard assumption in the adverse selection literature (to obtain equilibrium uniqueness) is $\frac{\partial m(\theta, z)}{\partial z} < 1$, a sufficient condition for which is that $F(\theta)$ is log-concave, i.e., $\frac{f(\theta)}{F(\theta)}$ monotonically decreases with θ.[17] This log-concavity assumption is widely adopted (e.g., Tirole (2012)). Condition (5) is a bit stronger – it requires $F(\theta)$ to be more log-concave than what is typically assumed in the literature. This is due to the

winner (although this is unrealistic). However, this restriction is sufficient to ensure that the *expected* impact of a bailout on the highest type seeking financing (θ^*) is negative.

[16] This occurs when $\theta^* = \bar{\theta}$.

[17] Note $\frac{\partial m(\theta, z)}{\partial z} < 1$ is equivalent to $\frac{f(z)}{F(z)} < \frac{\int_{\underline{\theta}}^{z} f(\theta)d\theta}{\int_{\underline{\theta}}^{z} F(\theta)d\theta}$ $\forall z \in (\underline{\theta}, \bar{\theta}]$, which can be guaranteed by requiring $F(\theta)$ to be log-concave. See, for example, Bagnoli and Bergstrom (2006).

additional modeling of trading, which reduces a high type's adverse selection loss. Consequently, to preserve equilibrium uniqueness, we need $\frac{f(\theta)}{F(\theta)}$ to decrease faster at the margin as θ increases.[18] While equilibrium uniqueness keeps the analysis tractable, it is not critical for the main results, namely that trading can unfreeze a market that would be frozen by adverse selection absent trading, but bailout anticipation weakens this effect. With multiple equilibria, we would take the solutions corresponding to the socially efficient outcome with the maximal possible mass of firms raising financing; see footnote 22 for more details.

3 Results

This section presents our results. Section 3.1 analyzes a benchmark with no trading and no bailout at noon, and shows that there may be a market freeze at dawn. Section 3.2 introduces trading and shows that this may prevent the market freeze. Section 3.3 introduces a bailout. We derive the optimal size and offer of the bailout, and examine its implications for winners in the trading market at noon and consequently their incentives to participate in legacy asset sales at dawn. We show that bailout anticipation can have a first-order (negative) *net* welfare impact. Section 3.4 studies the impact of bailout anticipation on a market-unfreezing intervention at dawn. In all cases, we postulate that types $[\underline{\theta}, \theta^*]$ sell legacy assets (skimming property), verify the skimming property, and then solve for θ^*. We compare θ^* across cases to examine the effects of trading and the bailout.

3.1 Benchmark without Trading and Bailout

Suppose types $[\underline{\theta}, \theta^*]$ sell legacy assets at dawn. Assume $\theta^* \geq \theta_{\mathrm{d}}$ (θ_{d} is defined in (2)), so these firms can finance their dawn investments. Given this, a type θ's net gain from selling its asset instead of staying out is

$$\pi(\theta; \theta^*) = B + m(\underline{\theta}, \theta^*)X_{\mathrm{p}} - I + \theta(X_{\mathrm{np}} + Y - \delta I) - \theta(X_{\mathrm{p}} + X_{\mathrm{np}}). \tag{6}$$

In (6), B is the morning private benefit due to the dawn investment, and $m(\underline{\theta}, \theta^*)X_{\mathrm{p}}$ is the price reflecting the average quality of legacy assets being sold,[19] from which I is invested at dawn. At noon, with probability θ, the firm realizes a legacy asset payoff of $\tilde{x} = X$, uses the non-pledgeable part of this payoff

[18] Note $1 - \frac{[1+\lambda(1-2\underline{\theta})](Y-\delta I)}{X_{\mathrm{p}}} \in (0, 1)$, due to condition (4).

[19] Note that asset buyers only value the pledgeable part of the legacy asset's payoff, X_{p}.

$X_{\rm np}$ to make the continuation investment δI, and receives Y at night. With probability $1-\theta$, the firm has $\tilde{x}=0$, so it cannot continue its project on its own.[20] In contrast, if the firm does not sell its legacy asset, it cannot invest at dawn, in which case its expected payoff simply equals that from its legacy asset, $\theta(X_{\rm p}+X_{\rm np})$.

Condition (4) ensures that $\frac{\partial \pi(\theta;\theta^*)}{\partial \theta}<0$, so the conjectured skimming property holds. Thus, θ^*, denoted by $\theta^*_{\rm NT}$ in this no-trading benchmark, is determined by $\pi(\theta^*_{\rm NT};\theta^*_{\rm NT})=0$. The proof of Lemma 1 shows that the existence of $\theta^*_{\rm NT}\in(\underline{\theta},\bar{\theta})$ is ensured by condition (3), and its uniqueness is guaranteed by condition (5).

Lemma 1 (Benchmark)**.** *Without trading and a bailout, if $\theta^*_{\rm NT}$, which is uniquely determined by*

$$\pi(\theta^*_{\rm NT};\theta^*_{\rm NT})=\underbrace{B-I+\theta^*_{\rm NT}(Y-\delta I)}_{\textit{project investment gain}}-\underbrace{[\theta^*_{\rm NT}-m(\underline{\theta},\theta^*_{\rm NT})]X_{\rm p}}_{\textit{adverse selection loss}}=0, \tag{7}$$

is no smaller than $\theta_{\rm d}$ given in (2), then types $[\underline{\theta},\theta^*_{\rm NT}]$ sell their legacy assets to raise financing at dawn. If $\theta^*_{\rm NT}<\theta_{\rm d}$, then the legacy asset market is frozen and no firm can raise financing at dawn.

In (7), the marginal type $\theta^*_{\rm NT}$ incurs an adverse selection loss because its legacy asset is worth $\theta^*_{\rm NT}X_{\rm p}$, but is sold at a pooling price $m(\underline{\theta},\theta^*_{\rm NT})X_{\rm p}$. Incurring this loss allows the firm to invest I in its project at dawn, enjoying a morning private benefit B, and with probability $\theta^*_{\rm NT}$ it can continue the project at noon using its legacy asset's non-pledgeable payoff, thereby gaining $Y-\delta I$ at night.

3.2 Trading without Bailout

As described in Section 2, a winner's (buyer's) surplus from trade is $\alpha Y-\delta I$. Without trade, a loser's project is abandoned at noon. So, the loser's (seller's) surplus from trade, relative to no trade, is $(1-\alpha)Y$. The total trade surplus is thus $(\alpha Y-\delta I)+(1-\alpha)Y=Y-\delta I$, which is exactly the surplus from continuing the loser's project – surplus that would have been lost in the absence of trade. The winner's ownership α solves a Nash bargaining problem that splits this $Y-\delta I$:

$$\max_{\alpha}\ (\alpha Y-\delta I)^{\beta}[(1-\alpha)Y]^{1-\beta}, \tag{8}$$

where $\beta=\frac{1}{1+n}$ (using (1)) is the winner's bargaining power. It can be shown that $\beta=1-m(\underline{\theta},\theta^*)$.[21]

[20] This is due to the first condition in Assumption 4.

[21] Using $n=\frac{\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta}{\int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta}$, we have $\beta=\frac{1}{1+n}=\frac{\int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta+\int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta}=\frac{\int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta}{F(\theta^*)}=1-m(\underline{\theta},\theta^*)$.

Solving (8), we see that the winner gets $\alpha Y - \delta I = [1 - m(\underline{\theta}, \theta^*)](Y - \delta I)$ and the loser obtains $(1-\alpha)Y = m(\underline{\theta}, \theta^*)(Y - \delta I)$. The winner gets $1 - m(\underline{\theta}, \theta^*)$ share of the surplus $Y - \delta I$, and the loser obtains the remaining share $m(\underline{\theta}, \theta^*)$. A higher θ^* increases the winner/loser ratio n ($\frac{dn}{d\theta^*} > 0$; footnote 12), weakening a winner's bargaining position. Thus, the winner's share decreases with θ^*, while the loser's share increases with θ^*.

With trading, the net gain for a type-θ firm from selling its legacy asset and hence making the investment at dawn instead of staying out is

$$\pi(\theta;\theta^*) + \underbrace{\theta \frac{\lambda}{1+n}\frac{1}{1+n}(Y - \delta I) + (1-\theta)\frac{\lambda n}{1+n}\frac{n}{1+n}(Y - \delta I)}_{\text{expected trading benefit (without bailout)}}. \tag{9}$$

In (9), $\pi(\theta;\theta^*)$, given by (6), is the net benefit as in the no-trading benchmark. The remaining terms capture the firm's expected trading benefit. As described earlier, the winner/loser ratio is $n = \frac{\int_{\underline{\theta}}^{\theta^*} \theta f(\theta) d\theta}{\int_{\underline{\theta}}^{\theta^*} (1-\theta) f(\theta) d\theta}$. With probability θ, the firm becomes a winner, in which case it finds a loser with probability $\frac{\mu(n)}{n} = \frac{\lambda}{1+n}$, making a profit $[1 - m(\underline{\theta}, \theta^*)](Y - \delta I) = \frac{1}{1+n}(Y - \delta I)$. With probability $1 - \theta$, the firm becomes a loser, in which case it locates a winner with probability $\mu(n) = \frac{\lambda n}{1+n}$, making a profit $m(\underline{\theta}, \theta^*)(Y - \delta I) = \frac{n}{1+n}(Y - \delta I)$.

We can show that the net profit in (9) monotonically decreases with θ (ensured by condition (4)), so the skimming property is preserved with trading. Therefore, the equilibrium can be characterized by setting the net profit in (9) to zero at $\theta = \theta^*$. Denote θ^* in this case with trading as θ^*_{T}.

Proposition 1 (Trading Equilibrium)**.** *When trading is permitted at noon, θ^*_{T} is uniquely determined by*

$$\pi(\theta^*_{\mathrm{T}};\theta^*_{\mathrm{T}}) + \lambda(Y - \delta I)\left\{\theta^*_{\mathrm{T}}[1 - m(\underline{\theta}, \theta^*_{\mathrm{T}})]^2 + (1 - \theta^*_{\mathrm{T}}) m(\underline{\theta}, \theta^*_{\mathrm{T}})^2\right\} = 0, \tag{10}$$

which exceeds θ^*_{NT} determined by (7) and is strictly increasing in λ. If θ^*_{T} is no smaller than θ_{d} given in (2), then types $[\underline{\theta}, \theta^*_{\mathrm{T}}]$ sell their legacy assets to raise financing at dawn.

Trading (partially) compensates for a high type's adverse selection loss in the legacy asset market, so the highest type selling its asset exceeds that in the no-trading benchmark ($\theta^*_{\mathrm{T}} > \theta^*_{\mathrm{NT}}$).[22] Higher trading efficiency increases a high type's expected trading profit, so this effect of trading gets stronger, i.e., $\frac{d\theta^*_{\mathrm{T}}}{d\lambda} > 0$. This means it is possible that the legacy asset market is frozen due to severe adverse selection

[22]The proof of Proposition 1 shows that the uniqueness of θ^*_{T} is guaranteed by condition (5). The uniqueness of the solution is, however, not critical for the main result that the availability of trading can unfreeze a market frozen by adverse selection absent trading. With multiple equilibria, we would take the largest solutions for θ^*_{T} and θ^*_{NT} (corresponding to the socially efficient outcome with the maximal possible mass of firms raising financing) and still have $\theta^*_{\mathrm{T}} > \theta^*_{\mathrm{NT}}$.

in the no-trading benchmark ($\theta^*_{\mathrm{NT}} < \theta_{\mathrm{d}}$), but when trading is permitted with a sufficiently high λ, the market thaws and a subset of firms engage in asset sales to raise financing ($\theta^*_{\mathrm{T}} > \theta_{\mathrm{d}}$). That is, there may be trading-induced market unfreezing, which is more likely to occur with higher trading efficiency.

3.3 Main Analysis: Trading with Bailout

The analysis has three steps. The first two steps are in Section 3.3.1 which examines the bailout at noon, taking types $[\underline{\theta}, \theta^*]$ as given. The first step in this section is the determination of the government's ownership α_{g} for every γ (mass of losers to be bailed out). The second step determines the optimal γ. Then, Section 3.3.2 characterizes θ^*, which is the third step. Lastly, Section 3.3.3 examines the bailout's net welfare impact.

3.3.1 Bailout at Noon

To examine the optimal bailout for given types $[\underline{\theta}, \theta^*]$, we use backward induction. First, we suppose the government wants to bail out γ mass of losers. We ask: how should it implement this by choosing its offer α_{g}, i.e., the ownership in the loser's project that it demands in exchange for its equity injection δI? Second, we solve for the optimal γ to maximize social surplus at noon.

Step 1 (Government sets α_{g} for a chosen γ): After γ mass of losers opt in for a bailout, the winner/loser ratio in the private market becomes $n_{\mathrm{g}} = \frac{\int_{\underline{\theta}}^{\theta^*} \theta f(\theta) d\theta}{\int_{\underline{\theta}}^{\theta^*} (1-\theta) f(\theta) d\theta - \gamma}$, which is higher than that absent the bailout, i.e., $n = \frac{\int_{\underline{\theta}}^{\theta^*} \theta f(\theta) d\theta}{\int_{\underline{\theta}}^{\theta^*} (1-\theta) f(\theta) d\theta}$. The government's choice of α_{g} takes this into account and makes a loser indifferent between joining the bailout and remaining in the private market:

$$(1 - \alpha_{\mathrm{g}})Y = \frac{\lambda n_{\mathrm{g}}}{1 + n_{\mathrm{g}}} \frac{n_{\mathrm{g}}}{1 + n_{\mathrm{g}}} (Y - \delta I). \tag{11}$$

A loser joining the bailout continues its project with certainty.[23] The project returns Y and the loser gets $1 - \alpha_{\mathrm{g}}$ fraction of it. This is the LHS of (11). For a loser opting out of the bailout, with fewer other losers remaining in the post-bailout market, both its probability of finding a winner to trade with and bargaining position vis-à-vis a winner improve – these are now $\mu(n_{\mathrm{g}}) = \frac{\lambda n_{\mathrm{g}}}{1+n_{\mathrm{g}}}$ and $1 - \beta_{\mathrm{g}} = \frac{n_{\mathrm{g}}}{1+n_{\mathrm{g}}}$, respectively, higher than the corresponding $\mu(n) = \frac{\lambda n}{1+n}$ and $1 - \beta = \frac{n}{1+n}$ absent the bailout. Conditional

[23]Our analysis is robust to the possibility that the government may ration some firms in the bailout (i.e., a loser opting in for the bailout may continue its project only probabilistically) as long as continuation investment is more likely for a loser joining the bailout than remaining in the private market, which motivates the bailout in the first place.

on trade, the loser's gain is $(1-\beta_{\rm g})(Y-\delta I) = \frac{n_{\rm g}}{1+n_{\rm g}}(Y-\delta I)$, as explained after (8). This explains the RHS of (11).

Substituting $n_{\rm g} = \frac{\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta}{\int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta-\gamma}$ into (11) leads to the following result:

Lemma 2 (Bailout Implementation)**.** *Suppose types $[\underline{\theta},\theta^*]$ sell legacy assets to raise financing at dawn. To bail out γ mass of losers at noon, where $\gamma \in \left[0, \int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta\right]$, the government injects δI equity into each of these losers in exchange for an ownership $\alpha_{\rm g}$, which is uniquely determined by*

$$(1-\alpha_{\rm g})Y = \lambda\left(\frac{\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta}{F(\theta^*)-\gamma}\right)^2(Y-\delta I). \tag{12}$$

This also pins down the ownership α that a private buyer acquires in the market, which is uniquely determined by

$$(1-\alpha)Y = \frac{\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta}{F(\theta^*)-\gamma}(Y-\delta I). \tag{13}$$

Corollary 1 (Comparative Statics on $\alpha_{\rm g}$ and α)**.** *(i) $\alpha_{\rm g} > \alpha$; (ii) $\frac{d\alpha_{\rm g}}{d\gamma} < 0$, $\frac{d\alpha}{d\gamma} < 0$; and (iii) $\frac{d\alpha_{\rm g}}{d\lambda} < 0$, $\frac{d\alpha}{d\lambda} = 0$.*

This corollary contains several interesting results. First, despite both contributing δI to a loser's project, the government demands a larger ownership for providing equity in the bailout than what private buyers demand in the market ($\alpha_{\rm g} > \alpha$). This is due to the loser's indifference between participating in the bailout and not participating that holds in equilibrium; see (11). In the former, the loser continues its project with certainty. In the latter, it finds a buyer for its project only probabilistically. This search friction in the market requires the loser to retain a larger ownership if it opts out of the bailout than if it opts in so as to remain indifferent, $1-\alpha > 1-\alpha_{\rm g}$, i.e., $\alpha_{\rm g} > \alpha$.

Second, with a larger bailout (larger γ), the *per capita* cost is higher both for the government ($\frac{d\alpha_{\rm g}}{d\gamma} < 0$) and private buyers ($\frac{d\alpha}{d\gamma} < 0$).[24] The reason is that a larger γ leaves fewer losers as sellers in the market, which increases the odds of a seller finding a buyer and improves the seller's bargaining power, thereby lowering α. Both effects make it more attractive for the seller to stay in the private market, so the government must reduce $\alpha_{\rm g}$ to make the seller indifferent between the bailout and the market.

Third, an increase in trading efficiency also makes the private market option more attractive to a loser,

[24] It can also be shown that $\frac{d^2\alpha_{\rm g}}{d\gamma^2} < 0$ and $\frac{d^2\alpha}{d\gamma^2} < 0$, i.e., $\alpha_{\rm g}$ and α decrease at a *faster rate* as γ increases. This means that the bailout also becomes more expensive per capita *at the margin* as the bailout size increases.

which, for the same reason as above, makes the bailout more expensive for the government ($\frac{d\alpha_{\mathrm{g}}}{d\lambda} < 0$). In contrast, a private buyer's ownership α is determined by bargaining *conditional on trade*, so trading efficiency (which affects the probability of trade) has no impact on α ($\frac{d\alpha}{d\lambda} = 0$).

This discussion shows that, because larger bailouts improve each loser's private market option by more, they force the government to be more generous to losers to match their improved private option. As in Philippon and Skreta (2012) and Tirole (2012), the government becomes its own worst enemy. But there is an added effect here – the government also competes with private buyers in bailouts, so larger bailouts hurt high-quality firms more,[25] inducing their *ex ante* anticipatory exit.

Step 2 (Government determines γ): How does the government choose the bailout size γ? The social benefits of a bailout are twofold: (i) *direct benefit* – a bailed-out loser continues its project with certainty (instead of probabilistically as with private market trading); and (ii) *indirect benefit* – due to the increased winner/loser ratio in the post-bailout market, each loser that is not bailed out has a higher probability of finding a buyer for its project in the market than is possible without the bailout.[26] The government must trade off these benefits against the shadow cost of using public funds for equity injection. Formally, the government's problem at noon is

$$\max_{\gamma} \quad \underbrace{\gamma\left(1-\frac{\lambda n}{1+n}\right)(Y-\delta I)}_{\text{direct benefit to bailed-out losers}} + \underbrace{\left[\int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta-\gamma\right]\lambda\left(\frac{n_{\mathrm{g}}}{1+n_{\mathrm{g}}}-\frac{n}{1+n}\right)(Y-\delta I)}_{\text{indirect benefit to losers not bailed out}} - \omega\gamma\delta I, \tag{14}$$

where $n = \frac{\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta}{\int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta}$ is the market buyer/seller ratio absent the bailout, and $n_{\mathrm{g}} = \frac{\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta}{\int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta-\gamma}$ is the market buyer/seller ratio with the bailout.

The total mass of losers is $\int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta$, from which mass γ is bailed out. Each bailed-out loser continues its project with certainty, which yields a net surplus of $Y-\delta I$. Absent the bailout, the loser finds a buyer for its project in the market only with probability $\mu(n) = \frac{\lambda n}{1+n}$. This direct benefit to bailed-out losers is captured by the first term in (14). The mass of losers remaining in the market is $\int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta-\gamma$. For each of them, the bailout-induced reduction of losers in the market increases its probability of finding a buyer for its project (with an associated surplus of $Y-\delta I$) from $\frac{\lambda n}{1+n}$ to $\frac{\lambda n_{\mathrm{g}}}{1+n_{\mathrm{g}}}$. This indirect benefit to losers outside the bailout is represented by the second term. The last term is the bailout cost – the amount

[25] In their models, private investors in the post-intervention market pay a higher price, but the elevated price correctly reflects the enhanced average quality of assets in the post-intervention market. So, the government creates enemy *only for itself*.

[26] The fact that the elevated winner/loser ratio also enhances losers' bargaining power vis-á-vis winners in the post-bailout market only has consequences for surplus distribution, but *not* surplus creation.

of public funds injected is $\gamma\delta I$, with a unit cost of ω.

Proposition 2 (Bailout Size: Government's Solution for γ)**.** *Suppose types $[\underline{\theta}, \theta^*]$ sell legacy assets. The government's bailout size is*

$$\gamma = \begin{cases} 0 & \text{if } \omega \geq \omega_H \\ F(\theta^*) - \sqrt{\frac{\lambda(Y-\delta I)}{Y-(1+\omega)\delta I}} \int_{\underline{\theta}}^{\theta^*} \theta f(\theta) d\theta \in \left(0, \int_{\underline{\theta}}^{\theta^*} (1-\theta) f(\theta) d\theta\right) & \text{if } \omega \in (\omega_L, \omega_H) \\ \int_{\underline{\theta}}^{\theta^*} (1-\theta) f(\theta) d\theta & \text{if } \omega \leq \omega_L, \end{cases} \tag{15}$$

with

$$\omega_L = (1-\lambda)\left(\frac{Y}{\delta I} - 1\right), \tag{16}$$

$$\omega_H = \left[1 - \lambda m(\underline{\theta}, \theta^*)^2\right]\left(\frac{Y}{\delta I} - 1\right). \tag{17}$$

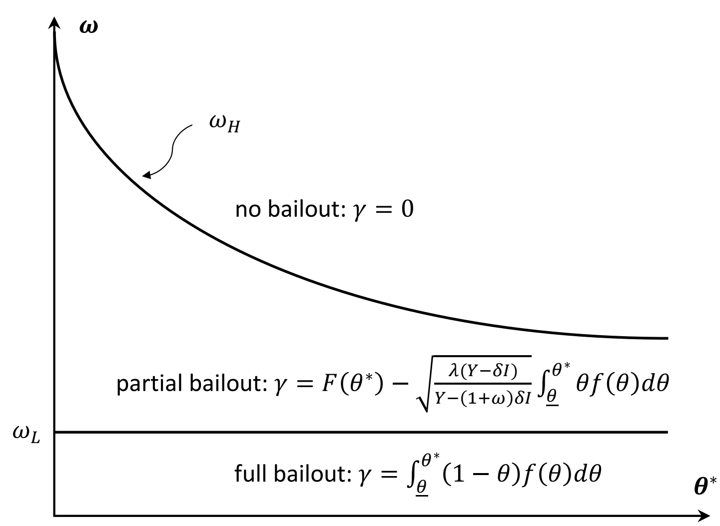

Figure 1: Bailout Size γ for Given ω and θ^*

This proposition formalizes the following idea. If the shadow cost of using public funds ω is sufficiently high ($\omega \geq \omega_H$), no loser is bailed out, i.e., $\gamma = 0$. If ω is sufficiently low ($\omega \leq \omega_L$), all losers are bailed out, i.e., $\gamma = \int_{\underline{\theta}}^{\theta^*} (1-\theta) f(\theta) d\theta$. For intermediate values of ω ($\omega \in (\omega_L, \omega_H)$), a fraction of losers are bailed out. The optimal bailout size γ, as a function of ω and θ^*, is depicted in Figure 1, which divides the (ω, θ^*) plane into three distinct regions: no bailout, partial bailout, and full bailout. Once γ is determined, the government implements the bailout by choosing its ownership α_g accordingly (Lemma 2).

Corollary 2 (Comparative Statics on Bailout Size)**.** *(i)* $\frac{d\omega_H}{d\theta^*} < 0$, $\frac{d\omega_L}{d\theta^*} = 0$*; (ii)* $\frac{d\omega_L}{d\lambda} < \frac{d\omega_H}{d\lambda} < 0$*; and (iii)* *for* $\omega \in (\omega_L, \omega_H)$, $\frac{d\gamma}{d\omega} < 0$, $\frac{d\gamma}{d\lambda} < 0$.

First, consider the effect of the quality of the highest participating type (θ^*). The result $\frac{d\omega_H}{d\theta^*} < 0$ says that as θ^* increases, the no-bailout region expands. The intuition is as follows. A higher θ^* increases the overall quality of firms seeking financing, so more firms are expected to become winners, leading to a higher buyer/seller ratio in trading.[27] As a result, each seller is more likely to find a buyer for its project, so the search-friction-induced inefficiency of private trading is lessoned, weakening the case for a bailout.

The result $\frac{d\omega_L}{d\theta^*} = 0$ (i.e., the full-bailout region is independent of θ^*) is a bit surprising. To understand it, suppose the government engages in a full bailout. If a single loser deviates and opts out of the bailout, then it becomes the *only* seller in the market. Consequently, it has a very high probability of finding a buyer and being in possession of full bargaining power while trading in the market. To make such a defector indifferent between the bailout and the market, the government would need to offer as much surplus to it with the bailout as it gets in the market. Since the market surplus such a loser gets is independent of θ^* – being the only seller in the market, it gets the same surplus regardless of the mass of buyers[28] – the full-bailout region is independent of θ^* as well.

Second, when trading efficiency (λ) improves, a loser is more likely to find a project buyer in the market. This lessens the inefficiency of trading that motivates a bailout. The overall bailout region (partial plus full) thus shrinks. But λ affects the full-bailout region more than the partial-bailout region, i.e., $\frac{d\omega_L}{d\lambda} < \frac{d\omega_H}{d\lambda} < 0$.[29] This is because a loser deviating from a full bailout, being the only seller in the market, has higher bargaining power vis-á-vis market buyers than a loser deviating from a partial bailout.[30] Thus, a loser deviating from a full bailout has more to gain from improved market trading efficiency, which implies a higher cost for the government to retain that loser in the full bailout. So the full-bailout region shrinks more with a higher λ.

Lastly, in the partial bailout region, $\omega \in (\omega_L, \omega_H)$, it is intuitive that the government bails out fewer losers when the cost of a bailout increases ($\frac{d\gamma}{d\omega} < 0$) or market trading efficiency improves ($\frac{d\gamma}{d\lambda} < 0$).[31]

Substituting the solution for γ in (15) into the government's objective function (14), we have

[27] As shown in footnote 12, the buyer/seller ratio in the market absent a bailout, $n = \frac{\int_{\underline{\theta}}^{\theta^*} \theta f(\theta) d\theta}{\int_{\underline{\theta}}^{\theta^*} (1-\theta) f(\theta) d\theta}$, increases with θ^*.

[28] The defector, being the only seller in the market (in which case the buyer/seller ratio $n \uparrow \infty$), meets a buyers with probability λ (the probability $\mu(n) = \frac{\lambda n}{1+n} \uparrow \lambda$ as $n \uparrow \infty$), and has full bargaining power (the seller's power $1 - \beta = \frac{n}{1+n} \uparrow 1$ as $n \uparrow \infty$). These are both independent of θ^*, so is the defector's surplus from the market.

[29] This implies that a higher λ, despite shrinking the overall bailout region, actually expands the partial-bailout region.

[30] A loser deviating from a partial bailout is *not* the only seller in the market.

[31] It can be shown that the fraction of losers bailed out, $\frac{\gamma}{\int_{\underline{\theta}}^{\theta^*} (1-\theta) f(\theta) d\theta}$, decreases with θ^* in this partial bailout region.

Corollary 3 (Surplus Increase from Bailout at Noon)**.** *Suppose types* $[\underline{\theta}, \theta^*]$ *sell legacy assets. At noon, the bailout increases social surplus by*

$$\Delta\mathcal{W}_{\text{noon}}(\theta^*) = \begin{cases} 0 & \text{if } \omega \geq \omega_H \\ \left\{\left[1-\sqrt{\frac{Y-(1+\omega)\delta I}{\lambda(Y-\delta I)}}\lambda m(\underline{\theta},\theta^*)\right](Y-\delta I)-\omega\delta I\right\}\left[F(\theta^*)-\sqrt{\frac{\lambda(Y-\delta I)}{Y-(1+\omega)\delta I}}\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta\right] & \text{if } \omega \in (\omega_L, \omega_H) \\ \{[1-\lambda m(\underline{\theta},\theta^*)](Y-\delta I)-\omega\delta I\}\int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta & \text{if } \omega \leq \omega_L. \end{cases} \tag{18}$$

In a full bailout ($\omega \leq \omega_L$), the mass of bailed-out losers is $\int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta$, with a cost of $\omega\delta I$ incurred for each loser. This full bailout increases each loser's probability of continuing its project from $\mu(n) = \frac{\lambda n}{1+n} = \lambda m(\underline{\theta}, \theta^*)$ to 1, where the surplus is $Y - \delta I$ from the continuation. This explains the expression for $\Delta\mathcal{W}_{\text{noon}}(\theta^*)$ when $\omega \leq \omega_L$. The expression for $\Delta\mathcal{W}_{\text{noon}}(\theta^*)$ when $\omega \in (\omega_L, \omega_H)$ refers to a partial bailout. In this region, the mass of bailed-out losers is $F(\theta^*) - \sqrt{\frac{\lambda(Y-\delta I)}{Y-(1+\omega)\delta I}}\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta$, and the increase in a loser's project continuation probability is $1 - \sqrt{\frac{Y-(1+\omega)\delta I}{\lambda(Y-\delta I)}}\lambda m(\underline{\theta}, \theta^*)$, which exceeds $1 - \lambda m(\underline{\theta}, \theta^*)$.[32] This increase in the continuation probability now accounts for both the direct benefit to a bailed-out loser and the indirect benefit to a loser outside the bailout. Finally, when no loser is bailed out ($\omega \geq \omega_H$), it is obvious $\Delta\mathcal{W}_{\text{noon}}(\theta^*) = 0$.

3.3.2 Financing at Dawn

Step 3 (Government's chosen γ now determines θ^*): We now characterize θ^*, the highest participating type at dawn. Given that types $[\underline{\theta}, \theta^*]$ sell legacy assets, the net gain for a type θ from participating in asset sales instead of staying out is

$$\pi(\theta;\theta^*) + \underbrace{\theta\frac{\lambda}{1+n_{\text{g}}}\frac{1}{1+n_{\text{g}}}(Y-\delta I) + (1-\theta)\frac{\lambda n_{\text{g}}}{1+n_{\text{g}}}\frac{n_{\text{g}}}{1+n_{\text{g}}}(Y-\delta I)}_{\text{expected profit from trading and bailout}}, \tag{19}$$

where $n_{\text{g}} = \frac{\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta}{\int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta - \gamma}$ is the post-bailout market buyer/seller ratio, with the bailout size γ in (15).

The expression (19) is similar to (9), the case without a bailout, except replacing n there with n_{g}. The first term $\pi(\theta;\theta^*)$ is the net benefit as in the no-trading and no-bailout benchmark, given by (6). The next

[32]With $\omega > \omega_L = (1-\lambda)\left(\frac{Y}{\delta I} - 1\right)$, it is clear that $\sqrt{\frac{Y-(1+\omega)\delta I}{\lambda(Y-\delta I)}} < 1$, so $1 - \sqrt{\frac{Y-(1+\omega)\delta I}{\lambda(Y-\delta I)}}\lambda m(\underline{\theta}, \theta^*) > 1 - \lambda m(\underline{\theta}, \theta^*)$.

two terms in (19) capture the expected benefit to the type θ from trading and bailout. If the firm becomes a winner (with probability θ), its probability of acquiring a loser's project in the post-bailout market is $\frac{\mu(n_{\mathrm{g}})}{n_{\mathrm{g}}} = \frac{\lambda}{1+n_{\mathrm{g}}}$. With bargaining power $\beta_{\mathrm{g}} = \frac{1}{1+n_{\mathrm{g}}}$, the winner obtains $\beta_{\mathrm{g}}(Y-\delta I) = \frac{1}{1+n_{\mathrm{g}}}(Y-\delta I)$ conditional on trade. If the firm ends up being a loser (with probability $1-\theta$), the government's bailout design makes it indifferent between joining the bailout and staying in the market; see (11). Thus, we compute the loser's expected payoff with a bailout as its expected payoff in the post-bailout market, which is computed as follows. The loser's probability of finding a project buyer in the post-bailout market is $\mu(n_{\mathrm{g}}) = \frac{\lambda n_{\mathrm{g}}}{1+n_{\mathrm{g}}}$, and with bargaining power $1-\beta_{\mathrm{g}} = \frac{n_{\mathrm{g}}}{1+n_{\mathrm{g}}}$, the loser gets $(1-\beta_{\mathrm{g}})(Y-\delta I) = \frac{n_{\mathrm{g}}}{1+n_{\mathrm{g}}}(Y-\delta I)$ conditional on trade.

We verify that the net profit in (19) monotonically decreases with θ,[33] as it does in (9), so the skimming property is preserved. The idea is that a bailout is more likely to adversely affect a higher type than a lower type – the former is more likely to become a buyer, and the bailout diminishes a buyer's ability to find a seller and also its bargaining power conditional on trade.[34] With the skimming property holding, we characterize θ^* by setting (19) to zero at $\theta = \theta^*$. Denote θ^* in this case with trading and bailout as θ^*_{TG}. Substituting $n_{\mathrm{g}} = \frac{\int_{\underline{\theta}}^{\theta^*_{\mathrm{TG}}} \theta f(\theta) d\theta}{\int_{\underline{\theta}}^{\theta^*_{\mathrm{TG}}} (1-\theta) f(\theta) d\theta - \gamma}$ into (19), we rewrite it as

$$\pi(\theta^*_{\mathrm{TG}}; \theta^*_{\mathrm{TG}}) + \lambda(Y-\delta I)\left[\theta^*_{\mathrm{TG}}\left(\frac{\int_{\underline{\theta}}^{\theta^*_{\mathrm{TG}}}(1-\theta)f(\theta)d\theta - \gamma}{F(\theta^*_{\mathrm{TG}}) - \gamma}\right)^2 + (1-\theta^*_{\mathrm{TG}})\left(\frac{\int_{\underline{\theta}}^{\theta^*_{\mathrm{TG}}}\theta f(\theta)d\theta}{F(\theta^*_{\mathrm{TG}}) - \gamma}\right)^2\right] = 0, \quad (20)$$

where the expression for $\pi(\theta^*_{\mathrm{TG}}; \theta^*_{\mathrm{TG}})$ is given by (7), replacing θ^*_{NT} there with θ^*_{TG}.

Condition (20) yields θ^*_{TG} for a given bailout size γ. But γ also depends on θ^*_{TG}; see (15), replacing θ^* there with θ^*_{TG}. So, θ^*_{TG} and γ are to be co-determined, leading to the potential for multiple equilibria, as shown below. Expressions for cutoffs $\omega_H(\theta^*_{\mathrm{T}})$ and ω_{trap} in the proposition below are given in the proof.[35]

Proposition 3 (Bailout Trap). *Equilibrium outcomes depending on the shadow cost ω:*

1. *When $\omega \leq \omega_L = (1-\lambda)\left(\frac{Y}{\delta I} - 1\right)$, the highest type θ^*_{TG} raising financing is uniquely determined by (A12), which is strictly lower than θ^*_{T}, and the government engages in a full bailout with $\gamma = \int_{\underline{\theta}}^{\theta^*_{\mathrm{TG}}}(1-\theta)f(\theta)d\theta$.*

2. *When $\omega_L < \omega < \omega_H(\theta^*_{\mathrm{T}})$, θ^*_{TG} is uniquely determined by (A15), which is strictly lower than θ^*_{T}, and the government engages in a partial bailout with $\gamma = F(\theta^*_{\mathrm{TG}}) - \sqrt{\frac{\lambda(Y-\delta I)}{Y-(1+\omega)\delta I}} \int_{\underline{\theta}}^{\theta^*_{\mathrm{TG}}} \theta f(\theta) d\theta$.*

[33] This is guaranteed by condition (4).

[34] Even if the bargaining weights were to remain fixed (as in some standard search and matching models), *all* our results would be qualitatively sustained because the bailout still lowers a buyer's chance of finding a seller in the market. The impact of bailout anticipation on adverse selection at dawn would also remain, but would be weaker.

[35] $\omega_H(\theta^*_{\mathrm{T}})$ is given by (A13), and ω_{trap} is determined by (A17) and (A18) jointly.

3. *When $\omega_H(\theta^*_{\mathrm{T}}) \leq \omega < \omega_{\mathrm{trap}}$, there can be two equilibria. In one equilibrium, firms believe that $\gamma = 0$; consequently, $\theta^*_{\mathrm{TG}} = \theta^*_{\mathrm{T}}$, and there is no bailout. In another equilibrium, a bailout trap arises – firms expect that $\gamma > 0$; consequently, $\theta^*_{\mathrm{TG}} < \theta^*_{\mathrm{T}}$ is uniquely determined by (A15), and the government engages in a partial bailout with $\gamma = F(\theta^*_{\mathrm{TG}}) - \sqrt{\frac{\lambda(Y-\delta I)}{Y-(1+\omega)\delta I} \int_{\underline{\theta}}^{\theta^*_{\mathrm{TG}}} \theta f(\theta) d\theta}$.*

4. *When $\omega \geq \omega_{\mathrm{trap}}$, the equilibrium is unique with $\gamma = 0$ and $\theta^*_{\mathrm{TG}} = \theta^*_{\mathrm{T}}$.*

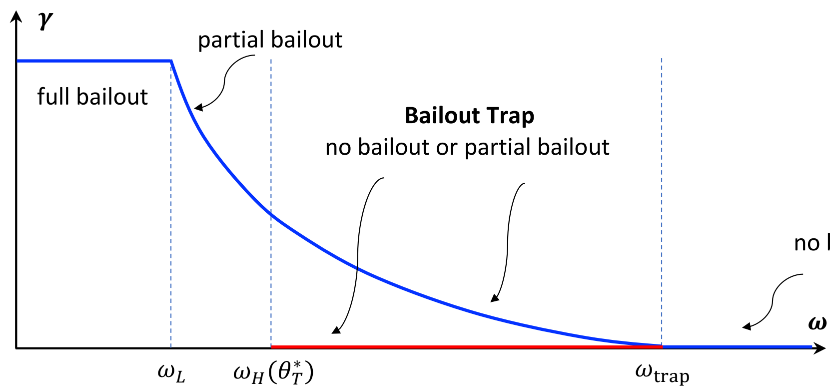

Figure 2: Equilibrium Outcomes: Bailout Size as a Function of the Shadow Cost of a Bailout

Figure 2 illustrates the equilibria. When $\omega \leq \omega_L$, the shadow cost of financing a bailout is so low that the government always bails out all losers. Corollary 2 shows that the full-bailout region is independent of the highest type seeking financing, so the government bails out all losers irrespective of θ^*_{TG}. The equilibrium θ^*_{TG} exactly balances this marginal type's adverse selection loss in the legacy asset market against its trading profit. The type must account for the adverse impact of the full bailout on its trading profit, so $\theta^*_{\mathrm{TG}} < \theta^*_{\mathrm{T}}$, where θ^*_{T} is the highest participating type absent a bailout (given by (10)).

When ω is of an intermediate value that is in the lower range of intermediate values, $\omega_L < \omega < \omega_H(\theta^*_{\mathrm{T}})$, the bailout cost is high enough to deter a full bailout. Thus, there is a partial bailout with θ^*_{TG} uniquely determined. Similarly, when $\omega \geq \omega_{\mathrm{trap}}$, the bailout cost is so high that the government effectively commits to not bail out any loser, so $\theta^*_{\mathrm{TG}} = \theta^*_{\mathrm{T}}$.

In the case in which ω is of an intermediate value that is in the higher range of intermediate values, $\omega_H(\theta^*_{\mathrm{T}}) \leq \omega < \omega_{\mathrm{trap}}$, there are two equilibria. Since the bailout cost is high enough, firms may assume at dawn that there will be no bailout at noon, in which case the highest type seeking financing is θ^*_{T}. Given this, and noting that the no-bailout region expands with the quality of the highest participating type (Corollary 2), at noon the government indeed finds it optimal to not bail out any loser. But if firms assume at dawn that there will be a partial bailout at noon, then the highest type seeking financing is lower than θ_{T}^{*}, in which case the no-bailout region shrinks and the government finds it optimal to engage in a partial bailout at noon.

3.3.3 Net Welfare Impact of Bailout

While the bailout increases social surplus at noon for a given set of firms selling legacy assets at dawn (Corollary 3), it lowers the highest participating type (Proposition 3). We now examine the bailout's *net* welfare impact, relative to when the government can credibly commit to no bailout (so the highest participating type is θ_{T}^{*} given by (10)).

If the market were frozen at dawn due to bailout anticipation (i.e., θ_{TG}^{*} as determined in Proposition 3 were to fall below θ_{d} in (2)), then there would be no investment at dawn by any firm and hence no trade at noon. In this case, the negative welfare impact of bailout anticipation clearly dominates, since at least some firms invest at dawn absent bailout anticipation. Therefore, the analysis below focuses on the more interesting case in which the market is not frozen at dawn ($\theta_{\mathrm{TG}}^{*} \geq \theta_{\mathrm{d}}$). Relative to the outcome with no bailout, bailout anticipation dissuades types $(\theta_{\mathrm{TG}}^{*}, \theta_{\mathrm{T}}^{*}]$ from participating in asset sale, resulting in a welfare loss of

$$\underbrace{\int_{\theta_{\mathrm{TG}}^{*}}^{\theta_{\mathrm{T}}^{*}} \{B - I + [\theta + (1-\theta)\lambda m(\underline{\theta}, \theta_{\mathrm{T}}^{*})](Y - \delta I)\} f(\theta) d\theta}_{\text{direct loss to types excluded from participation}}$$
$$+ \underbrace{\int_{\underline{\theta}}^{\theta_{\mathrm{TG}}^{*}} (1-\theta)\lambda[m(\underline{\theta}, \theta_{\mathrm{T}}^{*}) - m(\underline{\theta}, \theta_{\mathrm{TG}}^{*})](Y - \delta I) f(\theta) d\theta,}_{\text{indirect loss to types participating irrespective of the bailout}} \tag{21}$$

which could have been avoided absent the bailout. The first term in (21) is the direct loss to types $(\theta_{\mathrm{TG}}^{*}, \theta_{\mathrm{T}}^{*}]$. For each type $\theta \in (\theta_{\mathrm{TG}}^{*}, \theta_{\mathrm{T}}^{*}]$, participation would allow the firm to invest I in its project at dawn, enjoying a private benefit B. If the project is continued at noon, the firm would further gain $Y - \delta I$. Continuation occurs when the firm becomes a winner at noon (with probability θ) or becomes a loser but sells the project to a winner (with probability $(1-\theta)\lambda m(\underline{\theta}, \theta_{\mathrm{T}}^{*})$). The second term captures the indirect loss to types $[\underline{\theta}, \theta_{\mathrm{TG}}^{*}]$ that participate irrespective of the bailout. For each type $\theta \in [\underline{\theta}, \theta_{\mathrm{TG}}^{*}]$ ending up being a loser, participation by higher types $(\theta_{\mathrm{TG}}^{*}, \theta_{\mathrm{T}}^{*}]$ would improve its chance of finding a project buyer from $\lambda m(\underline{\theta}, \theta_{\mathrm{TG}}^{*})$ to $\lambda m(\underline{\theta}, \theta_{\mathrm{T}}^{*})$.[36]

[36]To see this, note that the buyer/seller ratio n increases with the highest participating type; see footnote 12. A higher n, in turn, increases a seller's chance of finding a buyer.

Subtracting the bailout-induced surplus increase at noon, $\Delta\mathcal{W}_{\text{noon}}(\theta^*_{\text{TG}})$ characterized in Corollary 3,[37] from the above welfare loss due to the bailout-induced exclusion of types $(\theta^*_{\text{TG}}, \theta^*_{\text{T}}]$, we obtain the bailout's *net* welfare impact $\Delta\mathcal{W}$, which is characterized in Appendix B. The bailout has a negative net welfare effect if $\Delta\mathcal{W} > 0$ and a positive net welfare effect if $\Delta\mathcal{W} < 0$. The complexity of the expressions for $\Delta\mathcal{W}$ in (B2) (for a full bailout) and (B4) (for a partial bailout) permits only a numerical analysis. Results are plotted in Figure 3. For a given bailout cost ω, the figure plots $\Delta\mathcal{W}$ for different levels of trading efficiency λ. The black and red curves correspond to a full bailout and a partial bailout, respectively.[38] It is evident that $\Delta\mathcal{W} > 0$ when λ is sufficiently high. The intuition is that when λ is higher, a high-quality firm obtains a larger gain from trading in the market without a bailout relative to trading with a bailout. Thus, bailout anticipation dissuades more high types from participation when λ is higher, so the bailout's negative impact on *ex ante* adverse selection outweighs the surplus it generates *ex post*, resulting in an overall negative welfare impact. This implies that *bailout anticipation will have a sizable adverse impact in an economy with high trading efficiency.* Not surprisingly, the bailout is more likely to be harmful with a higher ω (i.e., $\Delta\mathcal{W} > 0$ occurs for a wider range of λ values when $\omega = 0.5$ than when $\omega = 0.3$).

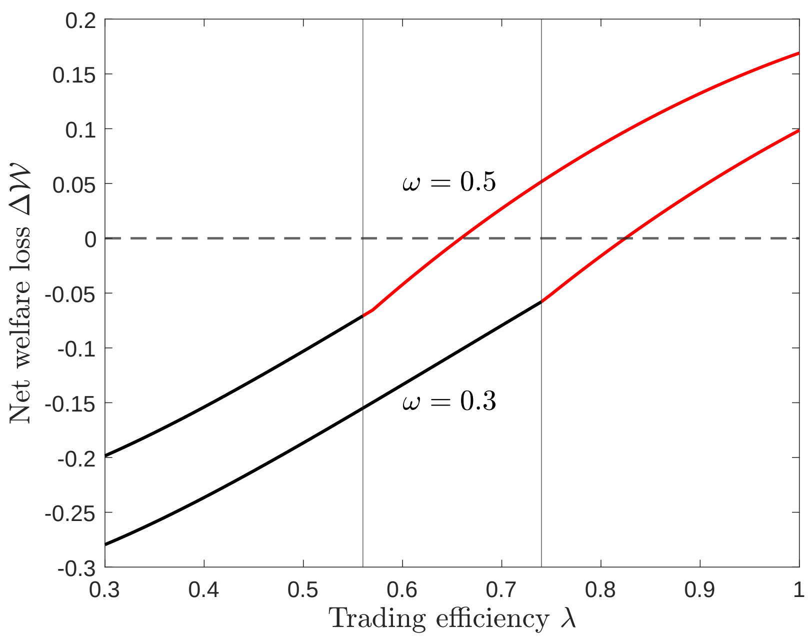

Figure 3: Net Welfare Impact of Bailout as a Function of Asset Market Trading Efficiency. Parameter values: $I = 1$, $B = 1.3$, $X_{\text{p}} = 2.95$, $Y = 1.73$, $\delta = 0.8$, $\theta \sim U[0, 0.9]$.

We believe that the potentially large welfare impact of a bailout makes the *ex ante* (anticipatory) effect

[37]The expression for $\Delta\mathcal{W}_{\text{noon}}(\theta^*_{\text{TG}})$ is given by (18), replacing θ^* there with θ^*_{TG}.

[38]For a given ω, a full bailout occurs for low λ values. This is because, as shown in Corollary 2 and illustrated in Figure 1, the full-bailout region expands (i.e., the boundary $\omega_L = (1-\lambda)(\frac{Y}{\delta I} - 1)$ increases) when λ decreases.

that we have identified a first-order effect of bailouts. It is an interesting empirical challenge to test this key implication of our analysis.

3.4 Extension: Asset Buyback at Dawn

Proposition 3 shows that the anticipation of a bailout worsens adverse selection ($\theta^*_{\mathrm{TG}} < \theta^*_{\mathrm{T}}$). This effect, if strong enough, may cause a market freeze at dawn, i.e., θ^*_{TG} determined by (A12) when $\omega \leq \omega_L$ or (A15) when $\omega \in (\omega_L, \omega_{\mathrm{trap}})$ falls below θ_{d} given in (2). The following analysis is predicated on such an outcome and shows that bailout anticipation also increases the cost of an intervention to unfreeze the dawn market.

The government announces a price at which to buy back legacy assets at dawn. Clearly, types $\theta \in [\underline{\theta}, \underline{\theta}_{\mathrm{buy}})$ sell assets to the government, while types $\theta \in [\underline{\theta}_{\mathrm{buy}}, \bar{\theta}_{\mathrm{buy}}]$ sell assets in the post-buyback market at a pooling price $m(\underline{\theta}_{\mathrm{buy}}, \bar{\theta}_{\mathrm{buy}})X_{\mathrm{p}}$, and types $\theta \in (\bar{\theta}_{\mathrm{buy}}, \bar{\theta}]$ hold on their assets and thus eschew projects. We characterize $\underline{\theta}_{\mathrm{buy}}$ and $\bar{\theta}_{\mathrm{buy}}$ below.

To jump-start the frozen market, we need $m(\underline{\theta}_{\mathrm{buy}}, \bar{\theta}_{\mathrm{buy}})X_{\mathrm{p}} = I$, i.e., the post-buyback market price of legacy assets exactly covers dawn financing. This means the government must pay a price exactly equal to I to buy back all legacy assets with $\theta \in [\underline{\theta}, \underline{\theta}_{\mathrm{buy}})$.[39] Each asset in this region is worth strictly less than I, $\theta X_{\mathrm{p}} < I \ \forall \theta \in [\underline{\theta}, \underline{\theta}_{\mathrm{buy}})$, so the buyback involves overpayment with public funds, which is socially costly.

If $\bar{\theta}_{\mathrm{buy}}$ is lower, then $\underline{\theta}_{\mathrm{buy}}$ must be higher to maintain $m(\underline{\theta}_{\mathrm{buy}}, \bar{\theta}_{\mathrm{buy}})X_{\mathrm{p}} = I$. A higher $\underline{\theta}_{\mathrm{buy}}$ means that the government must buy more bad assets at the inflated price I, i.e., the region $[\underline{\theta}, \underline{\theta}_{\mathrm{buy}})$ expands. In what follows, we show that the anticipation of a noon bailout, which causes the market freeze at dawn in the first place, also leads to a lower $\bar{\theta}_{\mathrm{buy}}$, making the asset buyback more expensive.

First, suppose trading is permitted, but the government can credibly commit to not bailing out at noon. The resulting highest type participating in asset sales is θ^*_{T}, given by (10). Suppose $\theta^*_{\mathrm{T}} < \theta_{\mathrm{d}}$, so adverse selection is so severe that the asset market is frozen at dawn, despite no bailout at noon. For type $\bar{\theta}_{\mathrm{buy}}$, its net gain from selling its legacy asset in the post-buyback market, instead of holding on the asset, is

$$\underbrace{B + \bar{\theta}_{\mathrm{buy}}(Y - \delta I) - \bar{\theta}_{\mathrm{buy}} X_{\mathrm{p}}}_{\text{net gain without trading}} + \underbrace{\lambda(Y - \delta I)\left\{\bar{\theta}_{\mathrm{buy}}[1 - m(\underline{\theta}, \bar{\theta}_{\mathrm{buy}})]^2 + (1 - \bar{\theta}_{\mathrm{buy}}) m(\underline{\theta}, \bar{\theta}_{\mathrm{buy}})^2\right\}}_{\text{expected trading profit (without bailout)}}. \tag{22}$$

For type $\bar{\theta}_{\mathrm{buy}}$, its legacy asset is worth $\bar{\theta}_{\mathrm{buy}} X_{\mathrm{p}}$, but is sold at a lower pooling price I at the post-buyback

[39] Paying a price higher than I will cost the government more than necessary to let the market rebound; see discussions after Proposition 4 for this price specification.

market. But incurring this adverse selection loss allows the firm to invest I in its project at dawn, enjoying a morning private benefit B, and with probability $\bar{\theta}_{\text{buy}}$ it can continue the project at noon using its legacy asset's non-pledgeable payoff and gain $Y - \delta I$ at night. This explains the first part in (22). The second part computes the firm's expected trading profit at noon, which can be understood similarly to (10), so we do not repeat the discussion here. Note that firms selling assets to the government, $\theta \in [\underline{\theta}, \underline{\theta}_{\text{buy}})$, also invest in their projects at dawn, and hence participate in trading at noon along with firms selling assets in the post-buyback market. We verify that the net profit in (22) monotonically decreases with $\bar{\theta}_{\text{buy}}$, so $\bar{\theta}_{\text{buy}}$ in this case is uniquely determined by setting (22) to zero.

Now, suppose the government cannot credibly commit to not bailing out at noon. Then, the type $\bar{\theta}_{\text{buy}}$'s net gain from selling its legacy asset in the post-buyback market, instead of holding on the asset, becomes

$$\underbrace{B + \bar{\theta}_{\text{buy}}(Y - \delta I) - \bar{\theta}_{\text{buy}} X_{\text{p}}}_{\text{net gain without trading}} \\ \underbrace{+ \lambda(Y - \delta I) \left[\bar{\theta}_{\text{buy}} \left(\frac{\int_{\underline{\theta}}^{\bar{\theta}_{\text{buy}}} (1-\theta) f(\theta) d\theta - \gamma}{F(\bar{\theta}_{\text{buy}}) - \gamma} \right)^2 + (1 - \bar{\theta}_{\text{buy}}) \left(\frac{\int_{\underline{\theta}}^{\bar{\theta}_{\text{buy}}} \theta f(\theta) d\theta}{F(\bar{\theta}_{\text{buy}}) - \gamma} \right)^2 \right]}_{\text{expected trading profit (with bailout)}} \tag{23}$$

with $\gamma = F(\bar{\theta}_{\text{buy}}) - \sqrt{\frac{\lambda(Y-\delta I)}{Y-(1+\omega)\delta I}} \int_{\underline{\theta}}^{\bar{\theta}_{\text{buy}}} \theta f(\theta) d\theta$.

Expressions in (22) and (23) differ only in their second parts. The second part of (23) follows from (20), replacing θ^*_{TG} there with $\bar{\theta}_{\text{buy}}$. As in (20), due to the anticipated bailout, the expected trading profit to type $\bar{\theta}_{\text{buy}}$ accounts for the consequent competition (from the government) to buyers in the private market at noon. It can be shown that $\bar{\theta}_{\text{buy}}$ determined by (23) (by setting (23) to zero) must be lower than that determined by (22).[40] The consequence of this, as explained earlier, is that the marginal type joining the government's asset buyback at dawn, $\underline{\theta}_{\text{buy}}$, must be higher with a noon bailout than without. This leads to the following result:

Proposition 4 (Asset Buyback)**.** *Firms' anticipation of a noon bailout causes the government to buy back more bad legacy assets at an inflated price to jump-start the frozen asset market at dawn.*

The analysis above relies on a fixed buyback price, call it p_{buy}, at I. As pointed out earlier (footnote 39), having $p_{\text{buy}} = I$ is just enough to jump-start the market. This does not mean that the optimal buyback price should be exactly I. Having $p_{\text{buy}} > I$ would induce even higher types to participate, i.e.,

[40]See the proof of Proposition 4.

$\bar{\theta}_{\text{buy}}$ would rise above that determined by (23). This benefit needs to be traded off against the increased total buyback cost. Tirole (2012) examines the optimal buyback price based on this tradeoff. However, we have a different goal, which is just to show that the jump-start becomes harder due to bailout anticipation. For that purpose, it suffices to establish the result at the lowest jump-start price, $p_{\text{buy}} = I$, but Proposition 4 holds for any $p_{\text{buy}} > I$ as well. Deriving the optimal p_{buy} here would be much more complex than in Tirole (2012) because we have a dynamic optimization problem – computing the social welfare at dawn with a specific p_{buy} must account for what would consequently occur in the bailout at noon.

This notwithstanding, some observations emerge. When p_{buy} increases, $\bar{\theta}_{\text{buy}}$ rises. Since the no-bailout region expands with the quality of the highest participating type (Corollary 2), the higher $\bar{\theta}_{\text{buy}}$ consequently makes no-bailout more likely at noon (i.e., the no-bailout cutoff ω_H falls). Thus, there exists a unique p^*_{buy}, such that for $p_{\text{buy}} \geq p^*_{\text{buy}}$, there would be no bailout at noon. This means that a large enough buyback today can prevent a bailout in the future. That is, if the government is willing to spend significantly more than what is needed to simply jump-start the market, then it would stop future bailouts.

4 Connections to Facts and Policy Implications

The literature on government intervention in frozen credit markets has focused on its role in cleaning up the industry by removing bad assets or bailing out troubled banks. Our analysis offers two novel insights. First, the anticipation of bailouts worsens *ex ante* adverse selection, which may actually lead to a market freeze. Second, the larger the anticipated *future* bailout, the more expensive it is for the government to unfreeze the market *ex ante*. That is, government intervention can become a *self-fulfilling prophecy.*

The costs of bailouts we identify are in addition to the usual argument that they increase moral hazard and lead to excessive risk taking, leverage and correlated asset choices (e.g., Farhi and Tirole (2012) and Greenbaum, Thakor, and Boot (2019)). Many regulatory strategies have evolved to limit such bailout-induced moral hazard. One is increasing capital and liquidity requirements, especially after the 2007-09 crisis. The other is tying these regulatory requirements to stress-test results. These measures are intended to not only alter the risk-taking and leverage incentives of financial institutions, but also track and monitor them, so as to intervene *ex ante* and obviate the need for a bailout *ex post*. Combined with normal regulatory supervisions, there appears to be an extensive prudential regulation machinery already in place to deal with the specter of moral-hazard-induced failures of institutions that anticipate being bailed out.

Our analysis highlights a different problem. We show that more markets may have been frozen due to anticipation of bailouts than if governments could have credibly precommitted to avoid bailouts. Moreover, even when there is trading in the private market, it is likely to be retarded by anticipation of bailouts, since potential asset sellers hold out for better deals from buyers in the post-bailout market. This makes private market acquisitions more costly for buyers and hence precipitates a market freeze, according to our analysis.[41] That is, because the anticipation of a bailout may lead to an *ex ante* market freeze that requires a bailout *ex ante*, the problem takes on the nature of an *addiction* to bailouts.

During the 2007-09 crisis, the bailouts undertaken by the U.S. government mirrored the way we model bailouts. Under the Capital Purchase Program (CPP) authorized by the TARP legislation, the U.S. Treasury injected equity into over 700 banks in exchange for ownership stakes in these banks. The program was designed to deal with bailout-related moral hazard (e.g., Mücke, Pelizzon, Pezone, and Thakor (2023)). However, according to our analysis, the extensive nature of the program – especially the need to bail out over 700 banks (large γ in our model) – may have been influenced by the *anticipation* of a bailout program of this nature. Just as importantly, the government's *per bank* bailout cost was likely higher because of the larger number of bailed-out banks than it would have been with a smaller program (Corollary 1).

One of the stated goals of the Dodd-Frank Act was to end bailouts. Most people recognize that such pronouncements are not time consistent. Faced with widespread bank failures, it is hard to imagine any government allowing mass bank failures. In other words, precommitting to avoid bailouts in all circumstances seems infeasible. Our analysis implies that this, in turn, makes it more likely that there will be an adverse-selection-induced market freeze. And given *ex post* social welfare considerations, the government will find it optimal to intervene in such a subgame. How does the government get out of this "bailout trap"? Addressing this question leads to some policy implications of our analysis that we discuss below.

Implication 1: The government should focus on improving the efficiency of trading in the secondary market for assets. In the case of banks, this is the market for bank loans. The idea that bank regulators should focus on improving interbank loan trading efficiency to reduce the need for both *current* interventions and *future* bailouts is novel. It highlights a policy channel that has previously not been examined.

Implication 2: The government can also directly target to reduce the likelihood of an *ex ante* adverse-selection-induced market freeze. One way to do this would be for the government to indicate that it would be willing to provide financial assistance to *asset buyers* in the private market, but *only* in case there is a bailout of asset sellers. This is similar to the kind of government assistance provided to financial institutions

[41] Anecdotal evidence of this is that Lehman rejected initial renegotiation offers from its creditors because it expected a government bailout.

to acquire failed savings and loan associations (S&Ls) during the S&L crisis in the 1980s. A more recent example is the assistance provided to JP Morgan Chase to acquire Bear Stearns during the 2007-09 crisis. While this kind of assistance is sometimes questioned – why should the government help shareholders of healthy banks that acquire failing banks – our analysis rationalizes this practice. Such actions serve to restore some trading profits that asset buyers expect to make at noon, pushing up the quality threshold below which firms participate in financing at dawn. That is, in addition to the usual approach of helping weak institutions during a crisis, the government should also assist strong institutions.

Implication 3: The policy suggestion in Implication 2 obviously raises the shadow cost of intervention to the government. How will it be paid for? One possibility is to establish a capital assistance fund (CAF) akin to formal deposit insurance wherein firms pay a premium *ex ante* – before they know whether they will be asset buyers or sellers in a subsequent crisis. This builds a fund that the government could tap to provide capital assistance for both future bailouts as well as asset purchases. This would not necessarily increase the bargaining power of asset buyers vis-à-vis sellers, but it would help boost the expected profits of buyers and thereby arrest the *ex ante* quality erosion that would lead to a market freeze.

Formal Analysis of the CAF: The analysis is predicated on the outcome that the market is frozen at dawn due to bailout anticipation (i.e., θ^*_{TG} as determined in Proposition 3 falls below θ_{d} in (2)). We show how the CAF may be designed to unfreeze the market, avoiding the bailout trap. With the CAF, suppose types $[\underline{\theta}, \theta^*]$ sell legacy assets, with $\theta^* \geq \theta_{\mathrm{d}}$, so a participating firm's asset sale proceeds suffice to cover its project investment, i.e., $m(\underline{\theta}, \theta^*)X_{\mathrm{p}} \geq I$, and the firm has extra cash $m(\underline{\theta}, \theta^*)X_{\mathrm{p}} - I$ after the investment. The analysis determines θ^*, which depends on the CAF design. Each firm participating in the asset sale (hence, project investment) is required to contribute $\zeta \in [0, m(\underline{\theta}, \theta^*)X_{\mathrm{p}} - I]$ to the CAF at dawn, so the fund size is $\int_{\underline{\theta}}^{\theta^*} \zeta f(\theta) d\theta = \zeta F(\theta^*)$. Firms are penniless at dawn, so their contributions must come from post-investment asset sale proceeds. Contributions are not required from non-participating types $\theta > \theta^*$.

At noon, since all contributions have been made, regulators find it *ex post* optimal to use the entire CAF to bail out as many losers as possible, so as to maximize *ex post* social surplus. Anticipation of this will make high-θ firms avoid contributing to the CAF because this outcome is worse than not having a CAF: high-θ firms now face competition from the government bailout at noon (as in the main analysis), and are *also* forced to finance the bailout by contributing to the CAF. Therefore, rules about how the CAF will be split between winners and losers at noon must be stipulated at dawn, and regulators must credibly commit to adhere to that rule. This differs from regulators not being able to commit to no bailout. We continue to assume that regulators cannot commit to not bail out losers. However, regulators must credibly commit to the fraction of the CAF, $\phi \in [0,1]$, that will be used to bail out losers, and the fraction $1-\phi$ that will go to buyers.

The CAF is designed to ensure that the bailout is self-funded. At noon, regulators will not tap public funds outside the CAF.[42] As a result, the social cost of using public funds (ω) in our main analysis is irrelevant here. Even if regulators cannot commit to not tap public funds outside the CAF, as long as winners receive assistance from the CAF ($\phi < 1$), high types will be encouraged to participate with the CAF than without it, although the effect of the CAF will be quantitatively weaker. If this quantitative weakening is large enough, the CAF may be unable to unfreeze the dawn market. So, the stricter the adherence to the bounds imposed by the CAF, the more effective it will be.

The regulators' problem at dawn is

$$\max_{\zeta,\phi} \int_{\underline{\theta}}^{\theta^*} [B - I + \theta(Y-\delta I)] f(\theta) d\theta + \left[\gamma + \left(\int_{\underline{\theta}}^{\theta^*} (1-\theta) f(\theta) d\theta - \gamma\right) \frac{\lambda \int_{\underline{\theta}}^{\theta^*} \theta f(\theta) d\theta}{F(\theta^*) - \gamma}\right] (Y - \delta I) \tag{24}$$

$$\text{s.t. } 0 \le \zeta \le m(\underline{\theta}, \theta^*) X_{\mathrm{p}} - I, \tag{25}$$

$$\gamma \delta I = \phi \zeta F(\theta^*), \tag{26}$$

$$\begin{aligned} &\pi(\theta^*;\theta^*) + \lambda(Y-\delta I)\left[\theta^* \left(\frac{\int_{\underline{\theta}}^{\theta^*}(1-\theta) f(\theta) d\theta - \gamma}{F(\theta^*) - \gamma}\right)^2 + (1-\theta^*)\left(\frac{\int_{\underline{\theta}}^{\theta^*} \theta f(\theta) d\theta}{F(\theta^*) - \gamma}\right)^2\right] \\ &+ \theta^* \frac{(1-\phi)\zeta F(\theta^*)}{\int_{\underline{\theta}}^{\theta^*} \theta f(\theta) d\theta} - \zeta = 0. \end{aligned} \tag{27}$$

Regulators choose the CAF contribution ζ and the sharing rule $\phi \in [0,1]$ to maximize social surplus at dawn in (24). For each participating type $\theta \in [\underline{\theta}, \theta^*]$, the dawn investment generates a net morning surplus $B - I$. If the firm becomes a winner at noon, which occurs with probability θ, it continues the investment by itself with an additional surplus $Y - \delta I$. This explains the first term in (24). The mass of losers is $\int_{\underline{\theta}}^{\theta^*} (1-\theta) f(\theta) d\theta$ at noon, among which mass γ is bailed out. Each bailed-out loser continues its project with certainty, generating a net surplus $Y - \delta I$. The mass of losers remaining in the market is $\int_{\underline{\theta}}^{\theta^*} (1-\theta) f(\theta) d\theta - \gamma$. Each of them finds a project buyer (with an associated surplus $Y - \delta I$) in the post-bailout market with probability $\frac{\lambda \int_{\underline{\theta}}^{\theta^*} \theta f(\theta) d\theta}{F(\theta^*) - \gamma}$.[43] This explains the second term in (24).

[42] We do not take up the issue of whether such a commitment would be feasible in practice. As with deposit insurance, there may be instances in which the temptation to go beyond the formal commitment is irresistible. But we assume that the political cost of doing so would be prohibitive in most cases.

[43] This obtains by substituting the post-bailout market buyer/seller ratio $n_{\mathrm{g}} = \frac{\int_{\underline{\theta}}^{\theta^*} \theta f(\theta) d\theta}{\int_{\underline{\theta}}^{\theta^*} (1-\theta) f(\theta) d\theta - \gamma}$ into the probability for a seller to meet a buyer in the market, $\mu(n_{\mathrm{g}}) = \frac{\lambda n_{\mathrm{g}}}{1+n_{\mathrm{g}}}$.

Constraint (25) says that, as explained earlier, a firm's CAF contribution ζ cannot exceed its post-investment asset sale proceeds $m(\underline{\theta}, \theta^*)X_{\mathrm{p}} - I$. Condition (26) stipulates that the amount of the CAF allocated to the bailout, $\phi\zeta F(\theta^*)$, equals the amount of equity injected into the mass γ of bailed-out losers (bailing out each loser requires δI). Condition (27) says that, relative to staying out, the net gain for the highest participating type θ^* is zero. This condition follows from the corresponding condition (20) without the CAF, except for the additional term, $\theta^* \frac{(1-\phi)\zeta F(\theta^*)}{\int_{\underline{\theta}}^{\theta^*} \theta f(\theta) d\theta} - \zeta$, which computes the net gain from the CAF assistance for type θ^* – with probability θ^*, the type-θ^* firm becomes a winner at noon, in which case it shares with all winners (total mass of $\int_{\underline{\theta}}^{\theta^*} \theta f(\theta) d\theta$) the amount of the CAF allocated to winners (which is $(1-\phi)\zeta F(\theta^*)$); netting this is the firm's own CAF contribution ζ.

In order for the CAF to induce $\theta^* > \theta_{\mathrm{d}}$ and unfreeze the dawn market, type θ^* must benefit from the CAF, i.e., $\theta^* \frac{(1-\phi)\zeta F(\theta^*)}{\int_{\underline{\theta}}^{\theta^*} \theta f(\theta) d\theta} - \zeta > 0$. For that, conditional on $\zeta > 0$, only a fraction of the CAF should be committed to bailouts, i.e., $\phi < 1 - \frac{m(\underline{\theta}, \theta^*)}{\theta^*} < 1$.

Proposition 5 (Capital Assistance Fund)**.** *A self-funded optimal CAF has the following properties:*

1. *It mandates each participating firm to contribute all of its post-investment asset sale proceeds to the fund, i.e.,* $\zeta = m(\underline{\theta}, \theta^*)X_{\mathrm{p}} - I$.
2. *There cannot be a full bailout at noon, i.e.,* $0 \leq \gamma < \int_{\underline{\theta}}^{\theta^*} (1-\theta) f(\theta) d\theta$.
3. *Regulators need to commit to a sharing rule that* ϕ *fraction of the CAF is used for bailouts, and the remaining* $1-\phi$ *fraction is used to assist winners at noon, where* $\phi \in (0, 1)$ *is endogenously determined by* (A20).

The intuition is as follows. It is efficient to mandate that each participating firm contribute all of its post-investment asset sale proceeds to the CAF, since these funds can be used at noon to bail out losers and/or assist winners. Both these noon initiatives increase *ex ante* social surplus, while not investing some funds in the CAF at dawn just keeps them idle.

The reason why the bailout should be partial is as follows. A firm's maximum CAF contribution, $m(\underline{\theta}, \theta^*)X_{\mathrm{p}} - I$, is less than δI,[44] the amount needed to bail out a loser. Thus, bailing out *all* losers requires the total CAF allocated to bailouts to exceed the total contribution made by all losers, which means the assistance received by winners must be strictly less than their contributions, making them worse off relative to not having a CAF. This will not unfreeze the dawn market as it discourages participation of

[44] To see this, note $m(\underline{\theta}, \theta^*)X_{\mathrm{p}} - I \leq \mathbb{E}[\theta]X_{\mathrm{p}} - I < \delta I$, where the second inequality is the first condition in Assumption 4. Recall that condition ensures the maximum possible post-investment asset sale proceeds $\mathbb{E}[\theta]X_{\mathrm{p}} - I$ falls below the continuation investment needs δI, so a loser can never continue its project at noon by itself (which motivates trade).

high types.

Finally, we explain how ϕ is determined. For a given θ^*, a higher ϕ permits a larger bailout and hence a bigger bailout-induced surplus increase at noon (the third term in (A20)). But a higher ϕ also leads to less assistance to winners, which discourages participation of high types by reducing θ^*. A reduction in θ^* has two negative welfare consequences: (i) it directly reduces the surplus generated by those high types (the first term in (A20)); and (ii) it reduces the price at which firms can sell their legacy assets, which in turn reduces their CAF contributions, resulting in a smaller fund and hence less money to bail out losers (the second term in (A20)). This second effect highlights that the regulators' desire to bail out more losers (by choosing a higher ϕ) can perversely reduce the fund available for the bailout in the first place.

Implication 4: An intriguing policy implication of our analysis is that *after* acquiring some sellers' projects, the government can *re-sell* its equity stakes in those sellers to winners who failed to find sellers in the post-bailout private market. By doing so, the government frees itself of its initial equity injection and also allows some unmatched winners to purchase projects. Of course, the government's resale price cannot be so low that potential buyers are better off not participating in the private market in the first place. So these buyers will be worse off even with this scheme than they would be had the bailout not occurred, but they are better off than they would be if the government did not engage in resales. Although governments may not have engaged in such asset acquisitions and resales precisely for the reasons outlined here, the practice of the government injecting equity in a distressed financial institution and then selling it later to private investors or another institution is not unfamiliar. For example, the government did this with AIG during the 2007-09 crisis.

5 Conclusion

This paper has analyzed the idea that anticipation of future government bailouts can cause an *ex ante* market freeze. While theoretical analyses of optimal bailouts (e.g., Philippon and Skreta (2012) and Tirole (2012)) take the adverse selection that induces the market freeze as exogenous, our analysis *endogenizes* it based on future bailout anticipation. The anticipation of a bailout can cause high-quality firms to exclude themselves from raising financing at an earlier date, thereby worsening the average quality of firms raising financing and precipitating a market freeze. Even if there is no market freeze, the worsening of the quality of firms raising financing means that the expected number of firms failing in the future goes up, increasing both the need for and extent of a future bailout. This analysis leads to a number of new policy implications.

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Appendix A: Proofs of Results

Proof of Lemma 1. For $\pi(\theta;\theta^*)$ in (6), we have $\frac{\partial \pi(\theta;\theta^*)}{\partial \theta} = (Y-\delta I) - X_{\mathrm{p}} < 0$, due to condition (4). This verifies the skimming property. Thus, θ^*_{NT} is determined by (7). The existence of $\theta^*_{\mathrm{NT}} \in (\underline{\theta}, \bar{\theta})$ is ensured by boundary conditions

$$\pi(\underline{\theta},\underline{\theta}) = B - I + \underline{\theta}(Y-\delta I) > 0, \tag{A1}$$

$$\pi(\bar{\theta},\bar{\theta}) = B - I + \bar{\theta}(Y-\delta I) - (\bar{\theta} - \mathbb{E}[\theta])X_{\mathrm{p}} < 0, \tag{A2}$$

which follow from condition (3). To show the uniqueness of θ^*_{NT}, it is sufficient to have $\frac{\partial \pi(\theta^*_{\mathrm{NT}};\theta^*_{\mathrm{NT}})}{\partial \theta^*_{\mathrm{NT}}} < 0$, equivalent to

$$\frac{\partial m(\underline{\theta};\theta^*_{\mathrm{NT}})}{\partial \theta^*_{\mathrm{NT}}} < 1 - \frac{Y-\delta I}{X_{\mathrm{p}}}, \tag{A3}$$

which is ensured by condition (5). ■

Proof of Proposition 1. Using $\mu(n) = \frac{\lambda n}{1+n}$ (see (1)) and $\frac{1}{1+n} = 1 - m(\underline{\theta},\theta^*)$ (see footnote 21), we can rewrite (9) as

$$\pi_{\mathrm{T}}(\theta;\theta^*) \equiv \pi(\theta;\theta^*) + \theta\lambda[1-m(\underline{\theta},\theta^*)]^2(Y-\delta I) + (1-\theta)\lambda m(\underline{\theta},\theta^*)^2(Y-\delta I). \tag{A4}$$

Differentiating $\pi_{\mathrm{T}}(\theta;\theta^*)$ with respect to θ:

$$\begin{aligned}\frac{\partial \pi_{\mathrm{T}}(\theta;\theta^*)}{\partial\theta} &= \frac{\partial \pi(\theta;\theta^*)}{\partial\theta} + \lambda[1-2m(\underline{\theta},\theta^*)](Y-\delta I)\\ &= \{1+\lambda[1-2m(\underline{\theta},\theta^*)]\}(Y-\delta I) - X_{\mathrm{p}}\\ &< [1+\lambda(1-2\underline{\theta})](Y-\delta I) - X_{\mathrm{p}}\\ &\le 0, \end{aligned} \tag{A5}$$

where the last inequality follows from condition (4). This verifies the skimming property. Thus, θ^*_{T} is determined by (10). The existence of $\theta^*_{\mathrm{T}} > \underline{\theta}$ is ensured by the boundary condition

$$\pi(\underline{\theta},\underline{\theta}) + \lambda(Y-\delta I)[\underline{\theta}(1-\underline{\theta})^2 + (1-\underline{\theta})\underline{\theta}^2] = B - I + \underline{\theta}(Y-\delta I) + \lambda\underline{\theta}(1-\underline{\theta})(Y-\delta I) > 0. \tag{A6}$$

To show the uniqueness of θ^*_{T}, it is sufficient to have $\frac{\partial\pi_{\mathrm{T}}(\theta^*_{\mathrm{T}};\theta^*_{\mathrm{T}})}{\partial\theta^*_{\mathrm{T}}} < 0$, which is equivalent to

$$\frac{\partial m(\underline{\theta};\theta^*_{\mathrm{T}})}{\partial\theta^*_{\mathrm{T}}} < \frac{X_{\mathrm{p}} - \{1+\lambda[1-2m(\underline{\theta},\theta^*_{\mathrm{T}})]\}(Y-\delta I)}{X_{\mathrm{p}} + 2\lambda[m(\underline{\theta};\theta^*_{\mathrm{T}}) - \theta^*_{\mathrm{T}}](Y-\delta I)}. \tag{A7}$$

The RHS is larger than $\frac{X_{\mathrm{p}} - [1+\lambda(1-2\underline{\theta})](Y-\delta I)}{X_{\mathrm{p}}}$, since $m(\underline{\theta};\theta^*_{\mathrm{T}}) - \theta^*_{\mathrm{T}} < 0$ and $m(\underline{\theta};\theta^*_{\mathrm{T}}) > \underline{\theta}$. Thus, a sufficient condition is $\frac{\partial m(\underline{\theta};\theta^*_{\mathrm{T}})}{\partial\theta^*_{\mathrm{T}}} < \frac{X_{\mathrm{p}} - [1+\lambda(1-2\underline{\theta})](Y-\delta I)}{X_{\mathrm{p}}}$, which is exactly condition (5).

The results $\theta^*_{\mathrm{T}} > \theta^*_{\mathrm{NT}}$ and $\frac{\partial\theta^*_{\mathrm{T}}}{\partial\lambda} > 0$ follow from the facts that $\lambda(Y-\delta I)\left\{\theta^*_{\mathrm{T}}[1-m(\underline{\theta},\theta^*_{\mathrm{T}})]^2 + (1-\theta^*_{\mathrm{T}})m(\underline{\theta},\theta^*_{\mathrm{T}})^2\right\} > 0$ and is *ceteris paribus* increasing in λ. ■

Proof of Lemma 2. Substituting $n_{\mathrm{g}} = \frac{\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta}{\int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta-\gamma}$ into (11), we can rewrite it as

$$(1-\alpha_{\mathrm{g}})Y = \lambda\left(\frac{n_{\mathrm{g}}}{1+n_{\mathrm{g}}}\right)^2(Y-\delta I) = \lambda\left(\frac{\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta}{\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta + \int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta - \gamma}\right)^2(Y-\delta I), \tag{A8}$$

which is (12), noting that $\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta + \int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta = \int_{\underline{\theta}}^{\theta^*}f(\theta)d\theta = F(\theta^*)$. For a seller in the private market, its surplus is $(1-\alpha)Y = m(\underline{\theta},\theta^*)(Y-\delta I) = \frac{n_{\mathrm{g}}}{1+n_{\mathrm{g}}}(Y-\delta I) = \frac{\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta}{F(\theta^*)-\gamma}(Y-\delta I)$, where the first equality is explained in the text after (8), the second equality follows from footnote 21. This proves (13). ■

Proof of Corollary 1. The result $\alpha_{\mathrm{g}} > \alpha$ follows from the fact that the RHS of (13) is bigger than the RHS of (12), since $\lambda \in [0,1]$ and $\frac{\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta}{F(\theta^*)-\gamma} = \frac{n_{\mathrm{g}}}{1+n_{\mathrm{g}}} \in (0,1)$. Since the RHSs of (12) and (13) are increasing in γ, we have $\frac{d\alpha_{\mathrm{g}}}{d\gamma} < 0$ and $\frac{d\alpha}{d\gamma} < 0$. Since the RHS of (12) is increasing in λ while the RHS of (13) is independent of λ, we have $\frac{d\alpha_{\mathrm{g}}}{d\lambda} < 0$ and $\frac{d\alpha}{d\lambda} = 0$. ■

Proof of Proposition 2. Substituting $n = \frac{\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta}{\int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta}$ and $n_{\mathrm{g}} = \frac{\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta}{\int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta-\gamma}$ into (14), we can rewrite the government's objective function as

$$\gamma(Y-\delta I) + \lambda(Y-\delta I)\left[\frac{\int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta - \gamma}{F(\theta^*)-\gamma} - \frac{\int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta}{F(\theta^*)}\right]\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta - \omega\gamma\delta I. \tag{A9}$$

The first-order derivative of (A9) with respective to γ is

$$Y - \delta I - \lambda(Y - \delta I)\left(\frac{\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta}{F(\theta^*) - \gamma}\right)^2 - \omega\delta I. \tag{A10}$$

It is clear that (A10) is strictly decreasing in γ, so the objective function in (14) is concave in γ. The mass of losers is $\int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta$, so $0 \leq \gamma \leq \int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta$.

If $Y - \delta I - \lambda(Y - \delta I)\left(\frac{\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta}{F(\theta^*)}\right)^2 - \omega\delta I \leq 0$, i.e., $\omega \geq \left[1 - \lambda m(\underline{\theta}, \theta^*)^2\right]\left(\frac{Y}{\delta I} - 1\right)$, then (A10) is strictly negative for all $0 < \gamma \leq \int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta$, so the optimal solution is $\gamma = 0$.

If $Y - \delta I - \lambda(Y - \delta I)\left(\frac{\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta}{F(\theta^*) - \int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta}\right)^2 - \omega\delta I \geq 0$, i.e., $\omega \leq (1-\lambda)\left(\frac{Y}{\delta I} - 1\right)$, then (A10) is strictly positive for all $0 \leq \gamma < \int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta$, so the optimal solution is $\gamma = \int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta$.

If $(1-\lambda)\left(\frac{Y}{\delta I} - 1\right) < \omega < \left[1 - \lambda m(\underline{\theta}, \theta^*)^2\right]\left(\frac{Y}{\delta I} - 1\right)$, then the optimal γ is given by the first-order condition (FOC)

$$Y - \delta I - \lambda(Y - \delta I)\left(\frac{\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta}{F(\theta^*) - \gamma}\right)^2 - \omega\delta I = 0, \tag{A11}$$

i.e., $\gamma = F(\theta^*) - \sqrt{\frac{\lambda(Y-\delta I)}{Y-(1+\omega)\delta I}}\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta \in \left(0, \int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta\right)$. ■

Proof of Corollary 2. Results directly follow from (15), (16) and (17). ■

Proof of Corollary 3. Results directly follow from substituting (15) into (A9). ■

Proof of Proposition 3. First, we know from Proposition 2 that if $\omega \leq \omega_L = (1-\lambda)(\frac{Y}{\delta I} - 1)$, all losers are bailed out, irrespective of θ^*_{TG}. So, θ^*_{TG} is determined by letting $\gamma = \int_{\underline{\theta}}^{\theta^*_{\mathrm{TG}}}(1-\theta)f(\theta)d\theta$ in (20), which becomes

$$\pi(\theta^*_{\mathrm{TG}}; \theta^*_{\mathrm{TG}}) + \lambda(Y - \delta I)(1 - \theta^*_{\mathrm{TG}}) = 0. \tag{A12}$$

This pins down θ^*_{TG} for $\omega \leq \omega_L$. The solution is unique, since the LHS of (A12) monotonically decreases with θ^*_{TG}.[45] We prove $\theta^*_{\mathrm{TG}} < \theta^*_{\mathrm{T}}$, where the highest participating type without bailout, θ^*_{T}, is determined by (10). Suppose $\theta^*_{\mathrm{TG}} = \theta^*_{\mathrm{T}}$, then (A12) coincides with (10) when letting $m(\underline{\theta}, \theta^*_{\mathrm{T}}) = 1$ in (10). Since the LHS of (10) also monotonically decreases with θ^*_{T}, to show $\theta^*_{\mathrm{TG}} < \theta^*_{\mathrm{T}}$, it is sufficient to show that the LHS of (10) is decreasing in $m(\underline{\theta}, \theta^*_{\mathrm{T}})$, which is true: the partial derivative of the LHS of (10) with respect to $m(\underline{\theta}, \theta^*_{\mathrm{T}})$ is $2\lambda(Y - \delta I)[m(\underline{\theta}, \theta^*_{\mathrm{T}}) - \theta^*_{\mathrm{T}}] < 0$.

Second, denote

$$\omega_H(\theta^*_{\mathrm{T}}) \equiv \left[1 - \lambda m(\underline{\theta}, \theta^*_{\mathrm{T}})^2\right]\left(\frac{Y}{\delta I} - 1\right) \tag{A13}$$

as the no-bailout cutoff (see (17)). When $\omega_L < \omega < \omega_H(\theta^*_{\mathrm{T}})$, it cannot be an equilibrium in which $\gamma = 0$. Prove by contradiction. Suppose $\gamma = 0$. Then, (20) becomes identical to (10), the condition pinning down θ^*_{T}. So, $\theta^*_{\mathrm{TG}} = \theta^*_{\mathrm{T}}$, hence $\omega_H(\theta^*_{\mathrm{TG}}) = \omega_H(\theta^*_{\mathrm{T}})$. Given $\omega < \omega_H(\theta^*_{\mathrm{T}})$, we have $\omega < \omega_H(\theta^*_{\mathrm{TG}})$, i.e., ω is outside the no-bailout region, so

[45] We know from the proof of Lemma 1 that $\frac{\partial \pi(\theta^*_{\mathrm{TG}};\theta^*_{\mathrm{TG}})}{\partial \theta^*_{\mathrm{TG}}} < 0$, sufficient to ensure that the LHS decreases with θ^*_{TG}.

$\gamma > 0$: a contradiction. Given $\omega > \omega_L$, there cannot be a full bailout, either. The equilibrium thus has a partial bailout with

$$\gamma = F(\theta^*_{\text{TG}}) - \sqrt{\frac{\lambda(Y-\delta I)}{Y-(1+\omega)\delta I}} \int_{\underline{\theta}}^{\theta^*_{\text{TG}}} \theta f(\theta) d\theta. \tag{A14}$$

The equilibrium is then jointly determined by (20) and (A14). Substituting (A14) into (20), we can rewrite (20) as

$$\pi(\theta^*_{\text{TG}}; \theta^*_{\text{TG}}) + \lambda(Y-\delta I)\left[\theta^*_{\text{TG}} - 2\theta^*_{\text{TG}}\sqrt{\frac{Y-(1+\omega)\delta I}{\lambda(Y-\delta I)}} + \frac{Y-(1+\omega)\delta I}{\lambda(Y-\delta I)}\right] = 0. \tag{A15}$$

We show (A15) uniquely determines θ^*_{TG}. For this, it suffices to prove that the LHS of (A15) monotonically decreases with θ^*_{TG}, which is equivalent to showing

$$\frac{\partial m(\underline{\theta}; \theta^*_{\text{TG}})}{\partial \theta^*_{\text{TG}}} < 1 - \left\{1 + \lambda\left[1 - 2\sqrt{\frac{Y-(1+\omega)\delta I}{\lambda(Y-\delta I)}}\right]\right\} \frac{Y-\delta I}{X_{\text{p}}}. \tag{A16}$$

The RHS of (A16), decreasing in ω, obtains its minimum when $\omega = \omega_H(\theta^*_{\text{T}})$, given $\omega_L < \omega < \omega_H(\theta^*_{\text{T}})$. Given condition (5), a sufficient condition for (A16) is thus $\underline{\theta} < \sqrt{\frac{Y-(1+\omega_H(\theta^*_{\text{T}}))\delta I}{\lambda(Y-\delta I)}} = m(\underline{\theta}, \theta^*_{\text{T}})$ (the equality follows from (A13)), which is true. This proves that θ^*_{TG} is uniquely pinned down by (A15), with the corresponding γ uniquely given by (A14).

We now prove $\theta^*_{\text{TG}} < \theta^*_{\text{T}}$, where θ^*_{T} is determined by (10). Examine (20), which determines θ^*_{TG}. Let $\widetilde{m}(\underline{\theta}, \theta^*_{\text{TG}}) \equiv \frac{\int_{\underline{\theta}}^{\theta^*_{\text{TG}}} \theta f(\theta) d\theta}{F(\theta^*_{\text{TG}})-\gamma}$ in (20). Since $\gamma > 0$, we have $\widetilde{m}(\underline{\theta}, \theta^*_{\text{TG}}) > m(\underline{\theta}, \theta^*_{\text{TG}}) = \frac{\int_{\underline{\theta}}^{\theta^*_{\text{TG}}} \theta f(\theta) d\theta}{F(\theta^*_{\text{TG}})}$. Since the LHS of (10), which equals zero at θ^*_{T}, is decreasing in $m(\underline{\theta}, \theta^*_{\text{T}})$ (shown in the first step), the LHS of (20) must be strictly negative if $\theta^*_{\text{TG}} = \theta^*_{\text{T}}$, due to $\widetilde{m}(\underline{\theta}, \theta^*_{\text{T}}) > m(\underline{\theta}, \theta^*_{\text{T}})$. Since the LHS of (20) monotonically decreases with θ^*_{TG}, we thus must have $\theta^*_{\text{TG}} < \theta^*_{\text{T}}$.

Then, given that the no-bailout cutoff is decreasing in θ^* (see Corollary 2 which shows $\frac{d\omega_H}{d\theta^*} < 0$), the no-bailout cutoff in this case, $\omega_H(\theta^*_{\text{TG}})$, must be strictly higher than $\omega_H(\theta^*_{\text{T}})$. Since $\omega < \omega_H(\theta^*_{\text{T}})$, we have $\omega < \omega_H(\theta^*_{\text{TG}})$, consistent with the conjecture that the equilibrium involves a partial bailout in this case. These fully characterize the equilibrium when $\omega_L < \omega < \omega_H(\theta^*_{\text{T}})$.

Third, consider $\omega \geq \omega_H(\theta^*_{\text{T}})$. Multiple equilibria may now arise. If at dawn firms believe the government will not bail out any loser at noon (i.e., $\gamma = 0$), then $\theta^*_{\text{TG}} = \theta^*_{\text{T}}$ (as argued earlier; (20) becomes identical to (10) when $\gamma = 0$). Given $\theta^*_{\text{TG}} = \theta^*_{\text{T}}$, the no-bailout cutoff is $\omega_H(\theta^*_{\text{T}})$. Given $\omega \geq \omega_H(\theta^*_{\text{T}})$, it is indeed optimal for the government to not bail out any loser at noon. So, $\gamma = 0$ and $\theta^*_{\text{TG}} = \theta^*_{\text{T}}$ are mutually consistent, constituting an equilibrium.

There can be another equilibrium if ω, despite exceeding $\omega_H(\theta^*_{\text{T}})$, is not sufficiently high (cutoff to be determined below). Suppose firms expect a bailout (i.e., $\gamma > 0$). Then, $\theta^*_{\text{TG}} < \theta^*_{\text{T}}$, which can be proved as in the second step. Given $\theta^*_{\text{TG}} < \theta^*_{\text{T}}$ and the result that the no-bailout cutoff decreases with the highest type (see Corollary 2 which shows $\frac{d\omega_H}{d\theta^*} < 0$), the no-bailout cutoff corresponding to firms' beliefs of $\gamma > 0$, $\omega_H(\theta^*_{\text{TG}})$, exceeds $\omega_H(\theta^*_{\text{T}})$. Therefore, for ω in a region $[\omega_H(\theta^*_{\text{T}}), \omega_{\text{trap}})$, with ω_{trap} determined later, the firms' bailout anticipation lowers the highest type raising financing at dawn, which in turn induces a bailout at noon. Firms' beliefs and the government's action are again mutually consistent, constituting another equilibrium. In this bailout trap region $\omega \in [\omega_H(\theta^*_{\text{T}}), \omega_{\text{trap}})$, γ and θ^*_{TG} are again jointly determined by (20) and (A14). Since $\omega > \omega_L$, the bailout is partial. Same as in the second case wherein $\omega_L < \omega < \omega_H(\theta^*_{\text{T}})$, substituting (A14) into (20), θ^*_{TG} is determined by (A15), with the corresponding γ given by (A14). That is, in the bailout trap region $\omega \in [\omega_H(\theta^*_{\text{T}}), \omega_{\text{trap}})$, γ and θ^*_{TG} in the bailout equilibrium are characterized in the same way as in the second case wherein $\omega_L < \omega < \omega_H(\theta^*_{\mathrm{T}})$ (as shown there, the pair of γ and θ^*_{TG} is unique).

Lastly, we characterize ω_{trap}. When ω increases in the bailout trap region to ω_{trap}, the corresponding θ^*_{TG} in the bailout equilibrium is uniquely determined by (A15) with $\omega = \omega_{\mathrm{trap}}$, i.e.,

$$\pi(\theta^*_{\mathrm{TG}};\theta^*_{\mathrm{TG}}) + \lambda(Y-\delta I)\left[\theta^*_{\mathrm{TG}} - 2\theta^*_{\mathrm{TG}}\sqrt{\frac{Y-(1+\omega_{\mathrm{trap}})\delta I}{\lambda(Y-\delta I)}} + \frac{Y-(1+\omega_{\mathrm{trap}})\delta I}{\lambda(Y-\delta I)}\right] = 0. \tag{A17}$$

When the corresponding no-bailout cutoff $\omega_H(\theta^*_{\mathrm{TG}}) = \left[1-\lambda m(\underline{\theta},\theta^*_{\mathrm{TG}})^2\right]\left(\frac{Y}{\delta I}-1\right)$ exactly equals ω_{trap}, i.e.,

$$\omega_{\mathrm{trap}} = \left[1-\lambda m(\underline{\theta},\theta^*_{\mathrm{TG}})^2\right]\left(\frac{Y}{\delta I}-1\right), \tag{A18}$$

then, for any $\omega \geq \omega_{\mathrm{trap}}$, the equilibrium cannot involve a partial bailout any more (to be proved below), so there will be again only one equilibrium, involving no bailout: $\gamma = 0$ and $\theta^*_{\mathrm{TG}} = \theta^*_{\mathrm{T}}$. (A17) and (A18) jointly pin down ω_{trap}.

To complete the proof, we show the equilibrium cannot involve a partial bailout for $\omega \geq \omega_{\mathrm{trap}}$. We show that θ^*_{TG} monotonically increases with ω when $\omega \geq \omega_{\mathrm{trap}}$. This is because the LHS of (A15) monotonically increases with ω when $\omega \geq \omega_{\mathrm{trap}}$: the partial derivative of the LHS with respect to ω is $\frac{\delta I}{\lambda(Y-\delta I)}\left\{\theta^*_{\mathrm{TG}}\left[\frac{Y-(1+\omega)\delta I}{\lambda(Y-\delta I)}\right]^{-\frac{1}{2}} - 1\right\}$, which is positive if $\theta^*_{\mathrm{TG}} > \sqrt{\frac{Y-(1+\omega)\delta I}{\lambda(Y-\delta I)}}$. The RHS of this (wanted) inequality decreases with ω, so it obtains its maximum in the region $\omega \geq \omega_{\mathrm{trap}}$ when $\omega = \omega_{\mathrm{trap}} = \left[1-\lambda m(\underline{\theta},\theta^*_{\mathrm{TG}})^2\right]\left(\frac{Y}{\delta I}-1\right)$; in this case, the RHS is $m(\underline{\theta},\theta^*_{\mathrm{TG}})$, so $\theta^*_{\mathrm{TG}} > m(\underline{\theta},\theta^*_{\mathrm{TG}})$ clearly holds. Thus, if the equilibrium involves a partial bailout for $\omega > \omega_{\mathrm{trap}}$, then the corresponding θ^*_{TG} must be higher than that given by (A17) (when $\omega = \omega_{\mathrm{trap}}$), so the corresponding no-bailout cutoff must be lower than ω_{trap}. But given $\omega \geq \omega_{\mathrm{trap}}$, the government is then in the no-bailout region, contradicting the conjectured partial bailout. ■

Proof of Proposition 4. We only need to show that $\bar{\theta}_{\mathrm{buy}}$ determined by (23) is lower than that determined by (22). The proof is similar as that for $\theta^*_{\mathrm{TG}} < \theta^*_{\mathrm{T}}$ in the proof of Proposition 3. Specifically, let $\widetilde{m}(\underline{\theta},\bar{\theta}_{\mathrm{buy}}) \equiv \frac{\int_{\underline{\theta}}^{\bar{\theta}_{\mathrm{buy}}}\theta f(\theta)d\theta}{F(\bar{\theta}_{\mathrm{buy}})-\gamma}$ in (23). Since $\gamma > 0$, we have $\widetilde{m}(\underline{\theta},\bar{\theta}_{\mathrm{buy}}) > m(\underline{\theta},\bar{\theta}_{\mathrm{buy}}) = \frac{\int_{\underline{\theta}}^{\bar{\theta}_{\mathrm{buy}}}\theta f(\theta)d\theta}{F(\bar{\theta}_{\mathrm{buy}})}$. Since the LHS of (22) is decreasing in $m(\underline{\theta},\bar{\theta}_{\mathrm{buy}})$ (shown in the proof of Proposition 3), the LHS of (23) is strictly smaller than the LHS of (22) for the same $\bar{\theta}_{\mathrm{buy}}$ due to $\widetilde{m}(\underline{\theta},\bar{\theta}_{\mathrm{buy}}) > m(\underline{\theta},\bar{\theta}_{\mathrm{buy}})$. Since the LHSs of (23) and (22) are both decreasing in $\bar{\theta}_{\mathrm{buy}}$, $\bar{\theta}_{\mathrm{buy}}$ determined by (23) must be lower than that determined by (22). ■

Proof of Proposition 5. First, we show $\zeta = m(\underline{\theta},\theta^*)X_{\mathrm{p}} - I$. If $\zeta < m(\underline{\theta},\theta^*)X_{\mathrm{p}} - I$, then ζ can be increased slightly without violating (25) for a given θ^* and ϕ, which increases γ; see (26). Since (24) is increasing in γ,[46] this increase in ζ enhances the objective function, so $\zeta < m(\underline{\theta},\theta^*)X_{\mathrm{p}} - I$ cannot be optimal.

Second, we prove $\gamma < \int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta$. If $\gamma = \int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta$ (i.e., full bailout), then $\phi = \frac{\int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta}{F(\theta^*)}\frac{\delta I}{\zeta} = [1-m(\underline{\theta},\theta^*)]\frac{\delta I}{m(\underline{\theta},\theta^*)X_{\mathrm{p}}-I}$, where the first equality follows from (26) and the second equality uses $\zeta = m(\underline{\theta},\theta^*)X_{\mathrm{p}} - I$. Note that $m(\underline{\theta},\theta^*)X_{\mathrm{p}} - I \leq \mathbb{E}[\theta]X_{\mathrm{p}} - I < \delta I$, where the second inequality is due to the first condition in Assumption

[46] The first-order derivative of the objective function in (24) with respect to γ is $\left[1-\lambda\left(\frac{\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta}{F(\theta^*)-\gamma}\right)^2\right](Y-\delta I)$, which is positive because $\lambda \in [0,1]$ and $\frac{\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta}{F(\theta^*)-\gamma} < \frac{\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta}{F(\theta^*)-\int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta} = 1$.

4. Thus, $\phi > 1 - m(\underline{\theta}, \theta^*)$. But, as explained in the text, we need $\phi < 1 - \frac{m(\underline{\theta},\theta^*)}{\theta^*}$ for the CAF to benefit type θ^* to unfreeze the dawn market. Since $1 - m(\underline{\theta}, \theta^*) \geq 1 - \frac{m(\underline{\theta},\theta^*)}{\theta^*}$ (as $\theta^* \leq 1$), these two conditions, $\phi > 1 - m(\underline{\theta}, \theta^*)$ and $\phi < 1 - \frac{m(\underline{\theta},\theta^*)}{\theta^*}$, contradict. So, we cannot have $\gamma = \int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta$.

Third, we determine ϕ. Substituting $\zeta = m(\underline{\theta}, \theta^*)X_{\mathrm{p}} - I$ into (26), we have $\gamma = \frac{\phi[m(\underline{\theta},\theta^*)X_{\mathrm{p}}-I]F(\theta^*)}{\delta I}$. The regulators' problem can be rewritten as

$$\max_{\phi} \int_{\underline{\theta}}^{\theta^*}[B - I + \theta(Y - \delta I)]f(\theta)d\theta + \left[\gamma + \left(\int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta - \gamma\right)\frac{\lambda\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta}{F(\theta^*)-\gamma}\right](Y - \delta I) \tag{A19}$$

subject to $\gamma = \frac{\phi[m(\underline{\theta},\theta^*)X_{\mathrm{p}}-I]F(\theta^*)}{\delta I}$ and (27). Totally differentiating (A19) with respect to ϕ leads to

$$\begin{aligned}
&\underbrace{\left\{\begin{array}{l} B - I + \theta^*(Y-\delta I) \\ +\lambda(Y-\delta I)\left[\theta^*\left(\frac{\int_{\underline{\theta}}^{\theta^*}(1-\theta)f(\theta)d\theta-\gamma}{F(\theta^*)-\gamma}\right)^2 + (1-\theta^*)\left(\frac{\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta}{F(\theta^*)-\gamma}\right)^2\right]\end{array}\right\} f(\theta^*)\frac{d\theta^*}{d\phi}}_{\text{marginal cost of a higher } \phi \text{ in reducing } \theta^*} \\
&+ \underbrace{\left[1 - \lambda\left(\frac{\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta}{F(\theta^*)-\gamma}\right)^2\right](Y-\delta I)\frac{\phi}{\delta I}\frac{\partial\big([m(\underline{\theta},\theta^*)X_{\mathrm{p}} - I]F(\theta^*)\big)}{\partial\theta^*}\frac{d\theta^*}{d\phi}}_{\text{marginal cost of a higher } \phi \text{ in reducing the CAF size due to the decrease in } \theta^*} \\
&+ \underbrace{\left[1 - \lambda\left(\frac{\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta}{F(\theta^*)-\gamma}\right)^2\right](Y-\delta I)\frac{d\gamma}{d\phi}}_{\text{marginal benefit of a higher } \phi \text{ in increasing } \gamma} = 0,
\end{aligned} \tag{A20}$$

where $\gamma = \frac{\phi[m(\underline{\theta},\theta^*)X_{\mathrm{p}}-I]F(\theta^*)}{\delta I}$, $\frac{d\gamma}{d\phi} = \frac{[m(\underline{\theta},\theta^*)X_{\mathrm{p}}-I]F(\theta^*)}{\delta I} > 0$, and

$$\frac{d\theta^*}{d\phi} = -\frac{-\frac{\theta^*[m(\underline{\theta},\theta^*)X_{\mathrm{p}}-I]}{m(\underline{\theta},\theta^*)} - 2\lambda(Y-\delta I)\left(\theta^* - \frac{\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta}{F(\theta^*)-\gamma}\right)\frac{\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta}{[F(\theta^*)-\gamma]^2}\frac{[m(\underline{\theta},\theta^*)X_{\mathrm{p}}-I]F(\theta^*)}{\delta I}}{\left\{\begin{array}{l}(Y-\delta I) - \left[1 - \frac{\partial m(\underline{\theta},\theta^*)}{\partial\theta^*}\right]X_{\mathrm{p}} + 1 - \frac{2\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta}{F(\theta^*)-\gamma} - \frac{2f(\theta^*)}{F(\theta^*)-\gamma}\left(\frac{\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta}{F(\theta^*)-\gamma} - \theta^*\right)^2 \\ +\frac{\partial m(\underline{\theta},\theta^*)}{\partial\theta^*}\left[\frac{\theta^*(1-\phi)}{m(\underline{\theta},\theta^*)} - 1\right]X_{\mathrm{p}} + (1-\phi)[m(\underline{\theta},\theta^*)X_{\mathrm{p}} - I]\frac{\partial\left(\frac{\theta^* F(\theta^*)}{\int_{\underline{\theta}}^{\theta^*}\theta f(\theta)d\theta}\right)}{\partial\theta^*}\end{array}\right\}} < 0 \tag{A21}$$

by applying the Implicit Function Theorem to (27). ■

Appendix B: Characterization of the Net Welfare Loss $\Delta\mathcal{W}$ in Section 3.3.3

Rewrite (21) as

$$\begin{aligned}\int_{\theta^*_{\mathrm{TG}}}^{\theta^*_{\mathrm{T}}}[B - I + \theta(Y-\delta I)]f(\theta)d\theta &+ \int_{\underline{\theta}}^{\theta^*_{\mathrm{T}}}(1-\theta)\lambda m(\underline{\theta}, \theta^*_{\mathrm{T}})(Y-\delta I)f(\theta)d\theta \\ &- \int_{\underline{\theta}}^{\theta^*_{\mathrm{TG}}}(1-\theta)\lambda m(\underline{\theta}, \theta^*_{\mathrm{TG}})(Y-\delta I)f(\theta)d\theta.\end{aligned} \tag{B1}$$

If $\omega \le \omega_L = (1-\lambda)\left(\frac{Y}{\delta I}-1\right)$, a full bailout is anticipated. So, $\Delta\mathcal{W}_{\text{noon}}(\theta^*_{\text{TG}}) = \int_{\underline{\theta}}^{\theta^*_{\text{TG}}}(1-\theta)\{[1-\lambda m(\underline{\theta},\theta^*_{\text{TG}})](Y-\delta I)-\omega\delta I\}f(\theta)d\theta$, and

$$\begin{aligned}
\Delta\mathcal{W} = &\int_{\theta^*_{\text{TG}}}^{\theta^*_{\text{T}}}[B-I+\theta(Y-\delta I)]f(\theta)d\theta + \int_{\underline{\theta}}^{\theta^*_{\text{T}}}(1-\theta)\lambda m(\underline{\theta},\theta^*_{\text{T}})(Y-\delta I)f(\theta)d\theta \\
&-\int_{\underline{\theta}}^{\theta^*_{\text{TG}}}(1-\theta)\lambda m(\underline{\theta},\theta^*_{\text{TG}})(Y-\delta I)f(\theta)d\theta - \int_{\underline{\theta}}^{\theta^*_{\text{TG}}}(1-\theta)\{[1-\lambda m(\underline{\theta},\theta^*_{\text{TG}})](Y-\delta I)-\omega\delta I\}f(\theta)d\theta \\
= &\int_{\theta^*_{\text{TG}}}^{\theta^*_{\text{T}}}[B-I+\theta(Y-\delta I)]f(\theta)d\theta + \int_{\underline{\theta}}^{\theta^*_{\text{T}}}(1-\theta)\lambda m(\underline{\theta},\theta^*_{\text{T}})(Y-\delta I)f(\theta)d\theta - \int_{\underline{\theta}}^{\theta^*_{\text{TG}}}(1-\theta)(Y-\delta I-\omega\delta I)f(\theta)d\theta.
\end{aligned} \tag{B2}$$

If $\omega \in (\omega_L, \omega_{\text{trap}})$, where ω_{trap} is determined by (A17) and (A18) jointly, a partial bailout is anticipated, so

$$\begin{aligned}
\Delta\mathcal{W}_{\text{noon}}(\theta^*_{\text{TG}}) &= \left\{\left[1-z^{-1}\lambda m(\underline{\theta},\theta^*_{\text{TG}})\right](Y-\delta I)-\omega\delta I\right\}\left[F(\theta^*_{\text{TG}}) - z\int_{\underline{\theta}}^{\theta^*_{\text{TG}}}\theta f(\theta)d\theta\right] \\
&= \int_{\underline{\theta}}^{\theta^*_{\text{TG}}}(1-z\theta)\left\{\left[1-z^{-1}\lambda m(\underline{\theta},\theta^*_{\text{TG}})\right](Y-\delta I)-\omega\delta I\right\}f(\theta)d\theta,
\end{aligned} \tag{B3}$$

where $z \equiv \sqrt{\frac{\lambda(Y-\delta I)}{Y-(1+\omega)\delta I}} > 1$. Thus,

$$\begin{aligned}
\Delta\mathcal{W} = &\int_{\theta^*_{\text{TG}}}^{\theta^*_{\text{T}}}[B-I+\theta(Y-\delta I)]f(\theta)d\theta + \int_{\underline{\theta}}^{\theta^*_{\text{T}}}(1-\theta)\lambda m(\underline{\theta},\theta^*_{\text{T}})(Y-\delta I)f(\theta)d\theta \\
&-\int_{\underline{\theta}}^{\theta^*_{\text{TG}}}(1-\theta)\lambda m(\underline{\theta},\theta^*_{\text{TG}})(Y-\delta I)f(\theta)d\theta - \int_{\underline{\theta}}^{\theta^*_{\text{TG}}}(1-z\theta)\left\{\left[1-z^{-1}\lambda m(\underline{\theta},\theta^*_{\text{TG}})\right](Y-\delta I)-\omega\delta I\right\}f(\theta)d\theta \\
= &\int_{\theta^*_{\text{TG}}}^{\theta^*_{\text{T}}}[B-I+\theta(Y-\delta I)]f(\theta)d\theta + \int_{\underline{\theta}}^{\theta^*_{\text{T}}}(1-\theta)\lambda m(\underline{\theta},\theta^*_{\text{T}})(Y-\delta I)f(\theta)d\theta \\
&-\int_{\underline{\theta}}^{\theta^*_{\text{TG}}}\left\{(1-z\theta)(Y-\delta I-\omega\delta I)+(1-z^{-1})\lambda m(\underline{\theta},\theta^*_{\text{TG}})(Y-\delta I)\right\}f(\theta)d\theta.
\end{aligned} \tag{B4}$$

If $\omega \ge \omega_{\text{trap}}$, no bailout is anticipated, so $\Delta\mathcal{W} = 0$. ■