Social Progress and Corporate Culture

October 7, 2018

Abstract

Social progress through improved treatment of minority groups (the embrace of anti-racist and anti-sexist norms, for example) may or may not spread to corporate cultures through competition. Sometimes the market fails to adapt on its own and government must pass legislation to secure changes in the workplace. We show how corporate culture is determined, why a variety of corporate cultures exist, and whether progressive corporate cultures can oust regressive ones.

JEL classification: D23, G30, G38, J31, L13

Keywords: corporate culture, organizational behavior, sociology, industrial organization, entry and exit, wage gaps
1 Introduction

During the mid-1950s to late-1960s, firms in the U.S. had mixed reactions to the civil rights movement. Martin Luther King Jr. and other central figures of the movement had realized that pressuring businesses to racially integrate was an effective strategy to propel the plight of black Americans into the national conscience (Roberts and Klibanoff (2007)). The Christian Science Monitor wrote at the time, “there are many students of America’s racial problem who believe that only business and industry, acting on their own, can provide the massive improvement in the [black American’s] condition which will help heal the situation” ("Help and Self-help" (1967)).

Many businesses were slow to racially integrate. Newsweek reported at the time, “some white executives preferred talking among themselves. . . than to lower class black employees.” Other executives had “concerns about men and women of different races working together” (Russell and Lamme (2013)). With the passage of the Civil Rights Act of 1964, however, many business leaders began saying to one another: “comply with the law or you’ll suffer economically” ("Mississippi business plans for integration" (1965)).

Other firms were amenable to the burgeoning social movement before the legislation. “[Black employees] are as efficient and in some cases more productive than other employees,” said an International Harvester executive (Fournier (1956)). Charles Marshall of the Illinois Bell Telephone Company wrote, “I like to think we’re big enough to abolish racial discrimination because it’s an injustice that we can’t stand to live with in a free society” (Marshall (1968)).

The American industrial firm Cummins Inc. treated diversity as central to its identity. Decades later, the firm even championed and lobbied for LGBT rights in the 1990s despite local backlash. One of the company’s most prominent CEOs in its history, J. Irwin Miller, saw corporations as instruments of social reform and instilled that perspective into the firm’s corporate culture (Reed (2017)).

A defining characteristic of a firm, perhaps the defining characteristic, is its corporate culture. The management author Peter Drucker wrote, “[corporate] culture eats strategy for breakfast, technology for lunch, and products for dinner, and soon thereafter everything else too” (Campbell et al. (2011)). A variety of empirical work has also documented that corporate culture even affects firm performance (Denison (1984), Gordon and DiTomaso (1992), Kotter and Heskett (1992), Sørensen (2002), Guiso et al. (2015a), Guiso et al. (2015b), Martinez et al. (2015)).

But after spending a week at two different firms, one will likely come away identifying two different corporate cultures. If corporate culture truly is fundamental to an organization, driving its performance, why do we observe such variety across firms? And if social progress advances by changing corporate culture, as was the case during the civil rights movement, when does corporate culture reform on its own? Why might corporate cultures remain stagnant and require outside intervention, such as government legislation, to progress? In this paper, we answer these questions.
Social progress that targets corporate culture did not end after the civil rights movement. A social campaign that recently confronted firms is the “Me Too” movement. The campaign began in 2006 to help survivors of sexual violence and bring attention to the “breadth and impact of sexual violence worldwide” in order to end the horrendous behavior (me too. (2018)). In late 2017, public awareness of Me Too erupted and the activity of the movement focused on eradicating sexual harassment in the workplace.

In response, corporate boards began re-examining their corporate cultures, asking whether active harassment was common, how such cases were handled internally, whether perpetrators were tacitly supported, and whether women and men felt that jobs and promotions were limited by their genders (Temin (2018)). By May 2018, nearly three hundred senior executives across U.S. companies were let go or forced to resign after accusations of sexual misconduct, including those at the Weinstein Company, Nike, Intel, Uber, Fox News, Wynn Resorts, and Lululemon (Dishman (2017), Williams and Lebsock (2018), Nocera (2018), Creswell et al. (2018), Temin (2018), Ovide (2018)).

In our analysis, corporate culture is inseparable from the culture(s) of the people who make up a firm. Explaining what we mean by corporate culture, therefore, first requires us to make plain our notion of culture. We view a person’s culture through an anthropological lens. One of the earliest articulations of the term was made by the late 19th century English anthropologist Edward Burnett Tylor, who wrote, “culture...is that complex whole which includes knowledge, belief, art, morals, law, custom, and any other capabilities and habits acquired by [humans] as member[s] of society” (Tylor (1871)). We adopt this interpretation of culture and expand it to explicitly include symbols, language, values, norms, mores, and typical behavioral patterns people share, interact, and communicate. Because we focus on corporations, though, we limit the components of culture to those involving a workplace.

Making this idea analytical and open to modeling requires imposing a mathematical structure. We do so by treating culture as the importance people assign to various cultural elements. This importance is modeled as a function that maps cultural elements to different weights between zero and one. We remain agnostic about the precise content of the cultural elements that constitute the domain of the function—as in an exact cultural value like “fair employee rewards.”

The range of function matters more. The range is the weight, or importance, the culture places on an element of the domain. For example, if a culture places a weight close to one on “clearly articulating goals to everyone,” that element carries a great deal of importance to the culture. Conversely, if “secretly defrauding customers” maps to a weight close to zero, that element is unimportant or even abhorrent to the culture.

In the model, there are two types of people: the majority and the minority. The two types are endowed with distinct cultures, placing different weights on the possible cultural components. The majority and minority may differ along any observable characteristic. Age, gender, race, creed, political beliefs, and sexual orientation are examples, but not the only ones. Besides dif-
ferring from the minority in culture, the majority by definition makes up more of the employees at a firm and exclusively manages it.

In our framework, corporate culture is a decision of a firm. Indeed, the objective of a firm is to maximize profits by choosing its corporate culture. We define corporate culture as the optimal mixture (weighted average) of the majority and minority cultures. The two instruments the majority uses to choose the firm’s corporate culture are (1) the shares of majority and minority employees to hire and (2) the extent to which the hired minority is socialized into the majority’s culture.

Socialization is a concept in both sociology and organizational behavior: it is the process by which one group internalizes the culture of another (Bauer and Erdogan (2011), Macionis (2013)). We treat socialization as the minority conforming to the behavior implied by the majority’s culture. We model the cultural change as a transformation of the minority culture function into the majority’s. The transformation could be complete or incomplete, depending on the optimal choice of the majority. The residual “distance” between the minority’s culture after socialization and majority’s culture, weighted by the employee shares, is cultural conflict because it measures the disagreement between the two groups on what is culturally important at an organization.

Profits of a firm are increasing in employee diversity because it enhances innovation, but profits are decreasing in cultural conflict because it harms productivity. Employing all majority types, for instance, will be free of conflict but will also lack any diversity. By hiring more of the minority, a firm faces a trade-off between enhancing diversity and worsening cultural conflict. A firm can ease that trade-off using socialization.

To members of the minority, the socialization process may be positive or painful. It may better position them for understanding their responsibilities and expectations in the organization, or it may confront them with an environment wholly unpleasant and offensive, such as one that endorses harassment. Because one’s culture is so personally important, shifting a minority employee away from his or her culture will inevitably trigger some emotions (utility or disutility). How the minority reacts to the socialization is critical to whether a variety of corporate cultures emerge in equilibrium.

The hiring and socialization decisions of a firm turn out to be complements: the more minority employees a firm hires, the greater the benefit of socialization because doing so reduces cultural conflict. This complementarity can make choices of either more diversity (with more socialization) or less diversity (with less socialization) appealing. Because each choice generates a different mixture of the majority and minority cultures, each implies its own corporate culture. Whether the choices are equally optimal depends on how members of the minority emotionally react to the socialization, because their reaction will influence how

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1Østergaard et al. (2011) find that the likelihood of introducing a new product or service is higher when the employees of a firm are more diverse in education and gender.

2Bartel (1994) finds a positive relation between training programs—a form of socialization to reduce cultural conflict—and labor productivity.
much they must be paid to stay with the firm.

If retaining minority employees becomes too expensive for one corporate culture over another, for example, then a single corporate culture will be optimal. Otherwise, multiple corporate cultures may be equally profitable. And so, from firms that have no starting differences whatsoever, a variety of corporate cultures develop. The interaction between a firm’s desire for diversity while reducing cultural conflict and the minority’s reaction toward socialization generates the phenomenon.

When studying how social progress can spread to industry, we focus on progress that calls for the improved treatment of minority employees at work. We model that progress as a new, shared expectation and demand in society for less painful socialization. The progress occurs in the model exogenously, but one can imagine a long history of events and evolution of thought that lead to it.

Our economic environment features an incumbent firm that faces a threat of entry by another firm. The incumbent believes and practices in an old way of business that predates the advancement in views. The firm might be willfully ignorant of the change, stubborn to adapt, or even adversarial to the progress. Importantly, the incumbent sticks to its wage gap—the difference in pay between the majority and the minority—and is unwilling to adjust it in response to competition by a potential entrant.

Unlike the incumbent, the entrant embraces the progress, making the socialization process less painful for members of the minority for any extent of socialization. The entrant might be a wholly new firm that emerges in reaction to public outcry against the behavior of the incumbent. Or it may be an existing firm that reformed its corporate culture on its own and now competes for dominance with the incumbent. The potential entrant allows its wage gap to float and respond to competition with the incumbent.

Both majority and minority employees can leave the incumbent for the entrant should doing so be more favorable. While leaving for the progressive entrant is appealing for members of the minority, they bear a bias to stay at the incumbent despite the more painful socialization. This restraint from exiting may arise from any kind of general switching cost. Some examples are a perceived lack of full information about the new firm, a status quo bias or loss aversion (Kahneman et al. (1991)), a mere-exposure effect because the incumbent is more familiar (Zajonc (1968)), or the minority’s aversion to change because working at the incumbent culture has entered the minority’s sense of identity (Cote and Levine (2002), Weinreich (2003), Akerlof and Kranton (2005)).

In the competition between the incumbent and potential entrant, the incumbent’s wage gap determines whether it can successfully thwart entry. If the incumbent’s wage gap is severe and inequality between the two groups is large, whereby the incumbent pays its minority far less than its majority, the dominance of the incumbent corporate culture is lost. The progressive firm enters the market, ousts the incumbent, and a new corporate culture that reflects the social advancement permeates the market.
On the other hand, an incumbent with a narrower wage gap completely deters entry because the new firm is not profitable enough to enter. In this case, competition has no means to advance a corporate culture that embodies the social progress. Only an outside intervention, such as government legislation like the Civil Rights Act of 1964 can force change. A third possibility is that the incumbent and entrant are equally profitable, which leads both corporate cultures to coexist in the market.

The minority’s bias to remain with the incumbent controls the range of wage gaps for which the incumbent can shield itself from expulsion. One might think that a stronger bias gives room for the incumbent to survive over larger wage gaps because minority employees are less likely to leave despite bearing greater disutility from socialization. But it is the opposite. A greater bias shrinks the range of wage gaps that deters entry because the bias makes the entrant compete more aggressively. The entrant does so by easing the pain of socialization even more to overcome that bias and convince the minority to leave the incumbent. And so, the tighter the grip on the minority, the more fragile the dominance of the prevailing, but outdated, corporate culture.

The minority’s bias to stick with the incumbent also determines the amount that minority treatment at work improves should the progressive entrant successfully displace the regressive incumbent. A larger bias compels the entrant to compete more fiercely by lowering the pain of socialization. If profitable enough to enter, the entrant improves the treatment of minority employees by the size of the incumbency bias. The more the minority clings to the status quo, the larger is the improvement to their condition if the corporate cultural change is successful.

In summary, we provide a conceptual framework to analyze culture as a mapping between cultural elements and the weights that people place on them in terms of importance. We propose that corporate culture is a mixture of the individual cultures of the employees that make up a firm. We argue that firms optimally choose their corporate cultures by comparing an interest in diversity and an incentive to lower cultural conflict between employees. This trade-off, combined with the reaction of the minority to their socialization, can generate a multiplicity of corporate cultures. Finally, we show that firms can evolve positively in their corporate cultures when society advances in the treatment of a minority group. But firms do not always advance, as the power of incumbent attitudes is sometimes too great for the market to overcome them on its own.

We wait to discuss existing theories of corporate culture until the end. Next, we present the model, starting with the ingredients, turning to the emergence of corporate cultural variety, and ending with the competition between an incumbent and entrant.

2 Model

A firm consists of employees belonging to one of two groups: the majority and minority. The majority carries the decision-making authority over a firm. Let the initial share of the majority be $x_0 \geq \frac{1}{2}$ and the minority share be $1 - x_0$. 
2.1 Culture

The majority and minority employees are distinct only in their cultures. A culture of a group is the set of customs, behaviors, norms, traditions, values, symbols, and language that are shared among its members and which are deemed socially acceptable or unacceptable within a firm. An expectation for all to arrive to the office before 9 A.M. and leave after 8 P.M. is part of a culture. Celebrating birthdays, honoring retirements, chatting at the water cooler, talking sports at lunchtime, expecting overtime or encouraging personal time, punishing harassment or ignoring it, believing greed is good or bad, promoting extreme risk-taking or caution, obeying regulations or violating safety standards are parts as well.

Some elements of culture, such as the structure of compensation, are expressed using enforceable contracts, whereas others are not. The sheer act of writing contracts when possible rather than relying on informal agreements is also part of culture. So too is the language and symbols used by members of a group. One group may call each other employees, but the other insists on “team-members.” One may expect all to communicate by email, the other may never use email. One group may all wear suits whereas others wear shorts and t-shirts that bear the company logo.

The list can continue. What matters to our analysis is not the exact elements of what constitute a group’s culture. What matters is that whichever elements are chosen, the individuals of the group can weigh and order each element of that set in terms of importance.

Formally, let $\Omega$ be a non-empty set wherein the elements $o \in \Omega$ are the norms, beliefs, behaviors, traditions, values, language, symbols, etc. that could be a part of any corporate culture. Let $S$ be a $\sigma$-algebra of subsets of $\Omega$. We define culture as the following:

**Definition 1.** *Culture* is a real-valued function $c : S \to [0, 1]$ that assigns a number in $[0, 1]$ to every set $s_i \in S$, called the *weight* of $s_i$ in culture $c$. The function $c$ satisfies

1. *(Total Weight)* $c(\Omega) = 1$,
2. *(Countable Additivity)* $c\left(\bigcup_{i \in I} s_i\right) = \sum_{i \in I} c(s_i)$ for all countable collections $\{s_i\}$ of pairwise disjoint sets.

Moreover, the function $c$ induces an order via the binary relation $\preceq$ on $S$ such that for any sets $s_i, s_j \in S$,

1. *(Non-strict ordering)* $s_i \preceq s_j$ if and only if $c(s_i) \leq c(s_j)$,
2. *(Strict ordering)* $s_i \prec s_j$ if and only if $c(s_i) < c(s_j)$,
3. *(Equivalence)* $s_i \sim s_j$ if and only if $c(s_i) = c(s_j)$,

where $\prec$ is the strict relation induced by $\preceq$, and $\sim$ is the equivalence relation induced by $\preceq$.

If $s_i \succeq s_j$, we say that “$s_i$ carries at least as much weight as $s_j$ in culture $c.” In other words, members of a group adhering to culture $c$ deem the customs, norms, values, etc. of $s_i$ with at
least as much importance as those belonging to $s_j$. Figure 1 provides an illustration of the mapping and induced ordering.

**Figure 1: Culture**

![Diagram of culture mapping and induced ordering]

Our definition requires members of a group to make binary comparisons between elements (more precisely, sets of elements) that could make up a culture. One can see that the binary relation induced by the culture function $c$ generates a proper ordering that obeys both transitivity and completeness. Every potential element of a culture can be compared, and if $s_i$ carries at least as much weight as $s_j$, and $s_j$ carries at least as much as $s_k$, then $s_i$ carries at least as much weight as $s_k$.

We consider the culture of an individual to be a primitive, similar to how preferences, technology, information, markets, and property rights generally are primitives in economic models. A person does not choose a culture, but is endowed with one. That culture may have formed and evolved over many years before the person’s birth. Looking throughout history and countries, one can observe that cultures do indeed change over time, some at different speeds than others. For the period we consider in the model, however, a person’s culture is fixed.

The ordering induced by a culture does not correspond to a preference relation. Nor does a cultural weight measure the strength of importance. That is, if $c(s_i) = 0.1$ and $c(s_j) = 0.5$, this does not imply that $s_i$ is five times as important than $s_j$ in culture $c$. It merely means that $s_j$ is more important than $s_i$. Finally, an individual’s culture will differ from corporate culture, which a firm chooses.

For mathematical convenience, we examine cultural weighting functions that are integrals of continuous probability densities defined over $\mathbb{R}$. So let $y$ be a random variable defined on $\Omega$ that maps to $\mathbb{R}$. Define the set $\{ y \in B \} \equiv \{ o \in \Omega; y(o) \in B \}$, where $B$ is a Borel subset of $\mathbb{R}$, and let the set be in $\mathcal{S}$. Let $c(\{ y \in B \})$ be the cultural weighting of that set. We define the cultural density function as the following:
Definition 2. The cultural density function $\rho(y)$ is a non-negative function defined for $y \in \mathbb{R}$ such that

$$c(\{a \leq y \leq b\}) = \int_a^b \rho(y) \, dy, \quad -\infty < a \leq b < \infty.$$ 

2.2 Corporate culture

The majority makes the decisions at a firm. They have two choices: (1) worker employment and (2) minority socialization. The employment decision determines the diversity of a firm. The socialization decision influences how closely the minority conforms with the culture of the majority. Both decisions set the corporate culture of the firm.

Diversity

A firm can hire from a pool of majority and minority types outside the firm, and the type of any potential hire is perfectly identifiable by the firm. A firm’s employment decision involves adding or deducting from the starting measure of the majority $x_0$.

Let $x$ denote the measure of new workers to add to or subtract from the majority. If $x \geq 0$, a firm increases the measure of the majority. To do so, a firm can either lay off some members of the minority or hire more of both types, but hire relatively more of the majority. If $x < 0$, a firm decreases the measure of the majority. Let $\tilde{x} = x_0 + x$ be the majority share that includes the employment changes. The corresponding minority measure is $1 - \tilde{x}$. Because choosing a new measure $x$ of workers is effectively the same as choosing the overall share $\tilde{x}$, we treat the choice variable of a firm to be $\tilde{x}$.

Define the diversity of a firm as

$$\Delta(\tilde{x}) = \tilde{x}(1 - \tilde{x}).$$

(1)

Diversity is maximized when $1 - \tilde{x} = \frac{1}{2}$, which lets the minority have the largest share possible. It is minimized when $\tilde{x} = 1$, meaning a firm is made up entirely of the majority types who share an identical culture. Because the majority cannot exceed 1 and the minority cannot exceed $\frac{1}{2}$, those are the upper and lower bounds on the choice variable.

Diversity at a firm is the degree to which the majority hires individuals from the minority group who share a different culture. Maintaining diversity will be an interest of a firm, which we discuss below when explaining a firm’s problem.

Socialization

The second decision of a firm is the socialization of the minority. Socialization is the process by which one group within an organization internalizes the culture of another group (Bauer and Erdogan (2011); Macionis (2013)). We treat socialization as the formal and informal ways the majority influences the minority to conform its behavior to the personal culture of the majority. Moreland and Levine (1984) argues that with socialization, “the group attempts to change the individual so as to maximize his or her contributions to the attainment of group goals.”
Socialization could take the form of training, onboarding programs, orientations, evaluations, recognition awards, codes of conduct, or group “summits.” It can also be less ceremonious, such as unspoken but observed dress codes, shared stories of legendary figures, tales of discharged deviants, open door policies, or superiors actively listening to subordinates. It can even be quite subtle, such as nodding to approve conforming actions, telling vile jokes, whispering uncomfortable comments about a person’s body, ignoring the lower-ranked, talking over others at meetings, or excluding groups from social events.

The sociologist Mindy Fried described a form of socialization at financial firms in the early 1980s in which woman “tried very hard to to play the part of, and even ‘look’ like, men as they struggled for respect and acceptance within a male-defined workplace culture. These women wore two-piece suits in solid colors with bow ties and medium-length skirts” (Fried (1998)). Similar behavior to sound and act masculine is seen among female entrepreneurs in Silicon Valley today (Brooks et al. (2014); Tariyal (2018); Robson (2018)).

So that the majority and minority cultures can be compared when we model socialization, it is helpful for the cultural weighting functions to map to the unit interval from the same $\sigma$-algebra. Therefore, let $\Psi \equiv \Omega \cup \Omega_m$ be the union of the sets for the majority ($\Omega$) and the minority ($\Omega_m$) that contain the elements of their respective cultures. And let $\mathcal{F}$ be a $\sigma$-algebra of subsets of $\Psi$. By uniting the elements of both cultures into a single set, we require members of the minority and the majority groups to be capable of ordering and weighing the values, traditions, language, customs, norms, etc. that could make up the cultures of either group. Let $z$ be a random variable defined on $\Psi$, and let $g(z)$ be the cultural density function of the majority and $f(z)$ the cultural density of the minority.

The cultural density of the socialized minority is
\[
\hat{f}(n,z) = \frac{g^n f^{1-n}}{a},
\]
where $a = \int g^n f^{1-n}dz$ is some integrating constant (as a function of $n$) that ensures $\hat{f}$ is a density, and $n \in [0, 1]$ is the extent of socialization. The majority has the means to transform the culture of the minority into that of the majority through the choice of $n$. In a moment, we shall get to the reason why the majority has an interest in doing this.

The culture of the socialized minority is a weighted average between the minority’s original density and the majority’s density. A perfectly socialized minority means $\hat{f} = g$, implying $n = 1$. It is important to emphasize that any extent of socialization $n$ chosen by a firm does not imply that the minority internally embraces the culture of the majority to that degree, permanently altering its original culture $f$. The minority simply act as if they do sufficiently well to conform to the behavior dictated by the majority’s culture. Later we shall discuss the effect that socialization has on the minority.

Socialization is costly to a firm. Formal training programs require resources. Extensive informal practices to instill the majority culture with the minority steals time from productive
work. The cost of socialization is \( \phi(n) \), and we assume the following properties to make it well-defined:

**Assumption 1. (Properties of \( \phi \))** The cost of socialization function \( \phi(n) \) is continuously differentiable, strictly increasing, strictly convex, satisfies \( \phi(0) = 0 \) and \( \lim_{n \uparrow 1} \phi(n) = \infty \).

**Corporate culture and conflict**

*Corporate culture* is a mixture of the majority’s personal cultural density \( g \) and the socialized minority’s cultural density \( \hat{f} \). The mixing weights is the measure of the two groups following the employment decision:

\[
\sigma(\tilde{x}, n, z) = \frac{g\tilde{x}\hat{f}^{1-\tilde{x}}}{a_\sigma},
\]

where \( a_\sigma = \int g\tilde{x}\hat{f}^{1-\tilde{x}}dz \) is an integrating constant. The corporate culture \( \sigma \) is a function of both choice variables of a firm. When \( \tilde{x} \) or \( n \) tend to 1, \( \sigma \to g \), making a firm’s corporate culture exactly match the majority culture.

The more the cultural density functions \( g \) and \( \hat{f} \) differ, the more the cultures of the majority and socialized minority conflict. Corporate cultural conflict means the two groups place contrasting importance on the elements that make up the set \( \Psi \). One group has beliefs, values, or norms that put it at odds with the other group, creating a kind of cultural “clash” (Turner (2005)). Perhaps the minority’s expectations which they consider common given their cultural backgrounds are not met when interacting with the majority at the workplace (Grewe (2005)). If spoken language is a major component of each group’s culture and they speak different languages, then cultural conflict would be significant disruptions in communication.

One way to measure cultural conflict is to calculate the “distance” between the cultural densities. A distance would capture the degree to which the two groups disagree on the elements of culture each considers important. But distance should account for the measures of the majority and minority groups. Significant discrepancies between \( g \) and \( \hat{f} \), for instance, would create more cultural conflict if the minority share \( 1 - \tilde{x} \) is larger. With this in mind, we measure cultural conflict as the distance between a firm’s corporate culture \( \sigma \) and the the corporate culture that has no conflict. A corporate culture with no conflict is the culture of the majority \( g \). Under full socialization \( (n = 1) \) or complete majority \( (\tilde{x} = 1) \), a firm’s corporate culture would equal \( g \) and the distance would be zero.

An innovation of our approach to modeling corporate culture is focusing on differences between cultures rather than choosing the components of any particular one. Many readers may differ on what they consider the correct elements that distinguish a culture. What likely is less controversial is that one culture is different from another. An apple may be difficult to define, but it is easy to point out that it is not a cucumber.

For the distance between densities, we use the Kullback-Leibler (KL) divergence. Roughly speaking, the KL divergence measures the information lost when one uses a different density to approximate a reference density rather than using the reference itself. The KL divergence is
not a true distance function because it is not symmetric: it needs that reference density.

The environment here does not suggest an obvious reference. If $g$ is used, the measure is the average difference in weights that a conflict-free firm would expect if it thought to change to culture $\sigma$, which bears some cultural conflict but also greater diversity. If $\sigma$ is used as the reference, the measure is the discrepancy a firm would expect if it completely socialized the minority or hired only majority types and sacrificed diversity. Neither density is a “true” one that is being approximated by the other.

Instead, we take the sum of the KL divergences when both $g$ and $\sigma$ are references. The sum is a true statistical metric that measures the distance between the two densities. When using the majority culture $g$ as a reference density, the KL divergence is

$$
D_{KL}(g||\sigma) = E_g [\log g - \log \sigma]
$$

Notice the expectation is taken with respect to the $g$ density because it is the reference density.

A KL divergence of 0 implies a firm can expect similar, if not the same, weighting under a firm’s culture $\sigma$ rather than a culture of all employees having culture $g$. Our distance measure is the sum of the two KL divergences:

$$
\delta(\tilde{x}, n) = D_{KL}(g||\sigma) + D_{KL}(\sigma||g).
$$

The distance function $\delta$ captures the corporate cultural conflict at a firm. Along with diversity, cultural conflict will enter a firm’s decision problem when it chooses the optimal employment $\tilde{x}$ and extent of socialization $n$. We turn to firm profits next.

2.3 Firm profits

A firm will make an employment decision $\tilde{x}$ and a socialization decision $n$ to maximize profits. The cost of hiring will be wages paid to the two types of workers. A wage $w$ is paid to the majority, whereas a wage $w_m$ is paid to minority, making the total wage bill $w\tilde{x} + w_m (1 - \tilde{x})$.

The profit function of a firm is

$$
\pi = A + \Delta(\tilde{x}) - \delta(\tilde{x}, n) - \phi(n) - w\tilde{x} - w_m (1 - \tilde{x}),
$$

where diversity $\Delta$ and cultural conflict $\delta$ are from (1) and (4), respectively, and $A$ is a constant that is large enough to ensure profits are non-negative even if diversity and conflict are zero.

Profits are increasing in diversity. There can be a number of reasons why. Variety of views, backgrounds, or beliefs among employees can enhance the creative process that generates ideas for improving firm productivity. Østergaard et al. (2011) find a positive relation between
diversity in education and gender and the likelihood of introducing a new product or service. Diversity through a variety of opinions might also produce higher quality decisions and in turn better financial performance. Richard (2000) finds that racial diversity increases return on equity and productivity, as measured by net income per employee.

Profits are decreasing in cultural conflict for a number of reasons. Conflict might create animosity between groups and give one group a feeling of moral license to engage in deviant behavior—such as shirking, free-riding, or theft (Kornblum (2011)). It might also make members of one group feel isolated or listless, harming their productivity. Finally, if cultural conflict is a sign of cultural weakness rather than strength, then firms with stronger corporate cultures (less conflict) have higher profits. Denison (1984) and Gordon and DiTomaso (1992) measure cultural strength as the consistency of survey responses within a company and find that greater strength is associated with better firm performance.

Socialization is a way to reduce the cultural conflict within the firm and increase profits. Among the socialization methods are formal training and orientation programs. Bartel (1994) shows that firms which implemented new employee training programs saw significant increases in labor productivity growth over a three year period. See also Klein and Weaver (2000) and references therein on how orientation programs significantly increase the levels of employee commitment, job involvement, and tenure at firms.

2.4 Employees

A unit measure of majority and minority types can enter employment. Each bears a single unit of labor available to supply inelastically. As an alternative to working, the majority and minority can pursue outside options. The majority’s outside option is worth \( b \) in utility, whereas the minority’s is worth \( b_m \). The utility of the majority and minority types, respectively, are

\[
U = w + \bar{x}, \quad U_m = w_m + (1 - \bar{x}) - v(n).
\]

Utility is increasing in consumption, which here is equal to the wage. Employees also have preference for working with others that are of the same type. The tendency to associate with others who are similar is called homophily. We take this preference to arise from the similarity in culture among those in the same group. That similarity raises utility by easing communication and encouraging working relationships (see McPherson et al. (2001) and the references therein).

The last component of preferences, \( v(n) \), comes from an employee’s feelings about the socialization process. We adopt insights from anthropological theory by envisioning culture as having a clear reason to exist: it “provid[es] principles for framing experience,” (Frake (1980)), sets instructions on how to “operate in a manner [that is] acceptable” (Goodenough (1957)), gives the “machinery individuals and groups employ to orient themselves in a world otherwise
opaque...and make sense of the events through which [they] live" (Geertz (1973)). And finally, culture not only informs a person about his or her own worldview, but also provides a “theory of what [his or her] fellows know, believe, and mean, a theory of the code being followed, the game being played” (Keesing (1974)).

Using these perspectives on the role of culture, we argue that the personal cultures of employees are important to them as individuals. So, when a minority employee enters a workplace environment that involves a socialization into a different culture (the majority’s), that employee will have an emotional reaction, feeling either utility or disutility from the process. We label \( v(n) \) the emotion function from the socialization. This component will have a powerful influence on whether a variety of corporate cultures can exist simultaneously in the market. For the moment, we only assume the following properties on the function \( v(n) \):

**Assumption 2.** *(Emotion function properties)* The emotion function is continuously differentiable, and the value of a non-socialized employee is zero, making \( v(0) = 0 \).

Under Assumption 2, the emotion function of those in the majority vanishes because members of that group are not socialized. For that reason we exclude the function in the majority utility in (5). Both types, however, have identical utility functions over consumption, homophily, and socialization. It is just the minority who are actually socialized.

We do not assign any further properties to the emotion function \( v(n) \) for now, but discuss some possibilities and their corresponding interpretations. If \( v(n) > 0 \) for all \( n \), for instance, minority employees have a strict distaste for socialization. They prefer not having to adjust their values, norms, language, etc. to conform with those of the majority. Such a distaste could grow at an increasing rate–making \( v(n) \) convex–which implies that minority employees find it increasingly painful to deviate from their personal cultures.

Alternatively, \( v(n) \) can be strictly negative for all \( n \). In this other extreme, a minority type actively wants to conform to the majority culture. To do so, he or she joins the firm for the socialization. An example would be an aspiring flight attendant who eagerly wishes to “live the Southwest way,” which emphasizes a “desire to excel...an ability to proactively serve customers...[and] a fun-loving attitude” (Weber (2015)).

Finally, \( v(n) \) may be highly non-linear and trace the emotional turbulence of enduring a workplace environment that pressures one to change personal values or beliefs. For instance, as \( n \) increases, a minority employee’s emotional path might follow something similar to the Kübler-Ross model in Kübler-Ross (1969): (1) shock (\( v(n) \) increases sharply); (2) denial (\( v(n) \) decreases); (3) anger (\( v(n) \) increases sharply); (4) bargaining (\( v(n) \) decreases); and (5) depression (\( v(n) \) increases sharply). Figure (2) illustrates a possible form of \( v(n) \) that follows this path, but also features an upward trend over \( n \) because of a general distaste for socialization.

### 3 A Special Category of Cultures

Having described the general problem of the firm, we solve it under a specific category of cultures. For both the majority and minority cultures, we use exponential distributions. The
Figure 2: An Example Emotion Function

Notes: The figure illustrates a possible form of the emotion function $v(n)$. The labels represent the emotions an employee is feeling at the corresponding extent of socialization $n$.

Cultural densities of the majority and minority, respectively, are

$$g(z) = \lambda e^{-\lambda z}, \quad (7)$$

$$f(z) = \lambda_m e^{-\lambda_m z}, \quad (8)$$

where $\lambda \neq \lambda_m$. The monotonically declining behavior of exponential densities permits a special interpretation of the numerical values of $z$. Under these densities, cultural weighting increases the fastest for lower values of $z$. We therefore can treat the weights associated lower values of $z$ as mappings to the values, traditions, language, norms, symbols, etc. a group considers the most important on the margin. For instance, if the group were asked to list a “small” set of elements (in the cardinality-sense) from $\Omega$ that carried the most weight in the culture, the lowest values of $z$ would map to this set.

With this in mind, we can also relate the densities of the majority and minority group. Figure 3 provides an illustration of majority cultural density $g(z)$ with parameter $\lambda$ and two cultural densities $f(z)$ for the minority. One minority culture has $\lambda_m < \lambda$, whereas the other has $\lambda_m > \lambda$. When $\lambda_m < \lambda$, the minority cultural density $f(z) < g(z)$ for lower values of $z$. In this case the minority treats as less important what matters most to the majority. For example, if the majority considers punishing harassment most important, the minority does
not. Alternatively, if \( \lambda < \lambda_m \), then \( f(z) > g(z) \) for lower values of \( z \). This relation implies the minority puts relatively more weight on the things the majority considers most important. In this case, the minority considers punishing harassment relatively more important than the majority does.

**Figure 3: Exponential Cultural Densities**

\[
\lambda_m > \lambda \\
\lambda_m < \lambda
\]

**Notes:** The cultural density of the majority \( g(z) \) is exponential with parameter \( \lambda \). Two minority cultural densities \( f(z) \) are displayed: one with parameter \( \lambda_m > \lambda \), and the other with \( \lambda_m < \lambda \).

The more interesting case is when \( \lambda_m < \lambda \), so we make that assumption:

**Assumption 3. (Relative majority-minority weighting)** The minority puts relatively less weight on what the majority considers most important in the majority’s culture, making \( \lambda_m < \lambda \).

By using the exponential form, we implicitly assume there exists a latent, common cultural factor that determines a common general ordering of cultural elements among people in the economy. For each group, the most important elements on the margin come “first” in the support. Therefore, the majority and minority cannot be too vastly different in their personal cultures. The two groups will only differ on the relative weights they put on each cultural element.

### 3.1 Corporate culture and conflict within the category

Applying the socialized minority function from (2) to exponential distributions gives

\[
\hat{f}(n, z) = \frac{1}{c} \left( \lambda e^{-\lambda z} \right)^n \left( \lambda_m e^{-\lambda_m z} \right)^{1-n}.
\]
For this to be a density, the integrating constant must be
\[ c = \frac{\lambda^n \lambda_m^{1-n}}{n \lambda + (1-n) \lambda_m}. \]

Substituting \( c \) and rearranging terms gives the socialized minority density:
\[ \hat{f}(n, z) = \hat{\lambda} e^{-\hat{\lambda} z}, \]
where \( \hat{\lambda} = n \lambda + (1 - n) \lambda_m \). Using (3) gives the corporate culture:
\[ \sigma(\tilde{x}, n, z) = \bar{\lambda} e^{-\bar{\lambda} z}, \]
where \( \bar{\lambda} = \tilde{x} \lambda + (1 - \tilde{x}) \hat{\lambda} \). Applying (4), gives the corporate cultural conflict:
\[ \delta(\tilde{x}, n) = \left( \log \frac{\lambda}{\bar{\lambda}} + \frac{\bar{\lambda}}{\lambda} - 1 \right) + \left( \log \frac{\lambda}{\hat{\lambda}} + \frac{\hat{\lambda}}{\lambda} - 1 \right) = \frac{\lambda}{\bar{\lambda}} + \frac{\bar{\lambda}}{\lambda} - 2. \]

Although \( \delta \) is a function of a firm’s choice of \( \tilde{x} \) and \( n \), we can analyze it as a function of \( \bar{\lambda} \), which is a weighted average of the majority and minority cultural parameters \( \lambda \) and \( \lambda_m \) that accounts for the two choice variables. We make the following remark about some properties of the function when treated this way:

**Remark. (Properties of corporate cultural conflict)** For a fixed majority parameter \( \lambda \), the corporate cultural conflict function has the following properties:

1. \( \delta \geq 0 \),
2. \( \delta \) is strictly convex in \( \bar{\lambda} \),
3. \( \min_{\lambda} \delta = 0 \),
4. \( \arg \min_{\lambda} \delta = \lambda \),
5. \( \lim_{\bar{\lambda} \downarrow 0} \delta = \infty \), and \( \lim_{\bar{\lambda} \uparrow \infty} \delta = \infty \).

Figure 4 illustrates the function \( \delta \) for fixed \( \lambda \) and different values of \( \bar{\lambda} \). As a comparison, the figure also overlays the quadratic function
\[ \hat{\delta}(\tilde{x}, n) = \frac{\theta^2}{2} (\lambda - \hat{\lambda})^2, \]
which is a 2nd-order Taylor approximation of \( \delta \) around \( \bar{\lambda} = \lambda \), and \( \theta^2 \) is a constant. The quadratic \( \hat{\delta} \) bears the same properties as \( \delta \) that are mentioned in Remark 3.1, except the last. For the quadratic, \( \lim_{\bar{\lambda} \downarrow 0} \hat{\delta} = \frac{\lambda^2}{2} \). A second key distinction is that the derivatives away from \( \bar{\lambda} = \lambda \) are symmetric for \( \hat{\delta} \) but asymmetric for \( \delta \). For \( \hat{\delta} \), the derivative depends only on the distance...
between $\bar{\lambda}$ and $\lambda$ and not on whether the derivative is taken from the left or the right of $\lambda$. This implies that the distance between the cultural density $g$ and corporate culture $\sigma$ increases by the same amount whether the minority puts more or less weight than the majority does on the norms, values, customs, traditions, etc. the majority considers most important.

Figure 4: Corporate Cultural Conflict $\delta$ and $\hat{\delta}$

Notes: The figure plots both the exact corporate cultural conflict $\delta = \frac{\bar{\lambda}}{\lambda} + \frac{\bar{\lambda}}{\lambda} - 2$ and its approximation $\hat{\delta} = \frac{1}{2} (\lambda - \bar{\lambda})^2$ as a function of the weighted average $\bar{\lambda} = \bar{x}\lambda + (1 - \bar{x})\hat{\lambda}$, where $\hat{\lambda} = n\lambda + (1 - n)\lambda_m$.

For $\delta$, the derivatives are asymmetric around $\lambda$. In fact, the derivative is larger for $\bar{\lambda} < \lambda$ than $\bar{\lambda} > \lambda$. Because $\bar{\lambda}$ is a weighted average between the majority parameter $\lambda$ and the minority’s $\lambda_m$, this asymmetry implies that that cultural conflict increases faster when the minority places less weight than the majority on the elements of culture that the majority considers most important ($\lambda_m < \lambda$). Conflict increases at a slower pace when the minority puts more weight on the things the majority considers most important ($\lambda_m > \lambda$).

Because of the similarity between the quadratic $\hat{\delta}$ and $\delta$, we use the simpler function $\hat{\delta}$ for the rest of our analysis. This way the intuition of the results can be presented more clearly without losing much. In Appendix 11, we solve the firm problem using the exact corporate cultural conflict $\delta$ and show that a key result—the complementarity between the minority share and the extent of socialization—remains. As we explain below, that relation can generate a variety of corporate cultures despite no starting differences between firms.

Under the approximate corporate cultural conflict $\hat{\delta}$, the firm problem in the exponential
category of cultures is

**Problem.** (*Firm problem, exponential cultures*)

\[
\max_{\{\tilde{x}, n\}} A + \tilde{x} (1 - \tilde{x}) - \frac{\theta^2}{2} (\lambda - \overline{\lambda})^2 - \phi(n) - w \tilde{x} - w_m (1 - \tilde{x}).
\] (10)

4 A Single Firm

Suppose potential employees can work only at a single firm. The outside options to working are fixed at \(b\) for majority types and \(b_m\) for minority types. We define an equilibrium when there is a single firm as the following:

**Definition 3.** (*Equilibrium, single firm*) When there is a single firm, an equilibrium is the tuple \(E_1 = \{\tilde{x}, n, w, w_m\}\), where \(\tilde{x} \in (\frac{1}{2}, 1]\) is a majority share and \(n \in [0, 1]\) is an extent of socialization that together maximize firm profits in (10), and \(\{w, w_m\}\) are the majority and minority wages, respectively, that clear the majority and minority labor markets.

Clearing of the labor markets requires both types of employees to be indifferent between working and their outside options. Under the employee utility functions from (5)-(6), the two indifference conditions are

\[
b = w + \tilde{x},
\]

\[
b_m = w_m + (1 - \tilde{x}) - v(n).
\]

Subtracting the two indifference conditions delivers the majority-minority wage gap:

\[
\omega = (b - b_m) - (2\tilde{x} - 1) - v(n).
\] (11)

We call the difference in outside options of the majority and minority \((b - b_m)\) the outside option gap. A larger outside option gap will increase the wage gap because the threat of exit is weaker for the minority. So too will a larger minority share. The firm takes advantage of the desire for homophily by offering a lower minority wage. Finally, if socialization is painful so that \(v(n) > 0\), the wage gap will decrease. The firm will have to offer monetary compensation to the minority for enduring the socialization.

So that the minority group still has an incentive to join a firm at all when there is a single firm, we place the following restriction on the emotion function:

**Assumption 4.** (*Emotion function bound, single firm*) When there is a single firm, the emotion function obeys the upper bound \(v(n) \leq b - b_m\) for all \(n\).

The higher is the outside option \(b_m\) of the minority group, the tighter the bound. In equilibrium, a fraction \(\tilde{x}\) of the majority will work while the remaining fraction will pursue the outside option. Similarly, a fraction \(1 - \tilde{x}\) of the minority will work and the remaining will take their outside options.
4.1 Optimal corporate culture

The first order conditions of the firm problem in (10) with respect to $\tilde{x}$ and $n$ are

$$\theta^2 (\lambda - \lambda_m)^2 (1 - n)^2 (1 - \tilde{x}) = \omega + (2\tilde{x} - 1), \quad (12)$$

$$\theta^2 (\lambda - \lambda_m)^2 (1 - n) (1 - \tilde{x})^2 = \phi' (n). \quad (13)$$

The left-hand-side of (12) is the marginal benefit of increasing the size of the majority. A larger majority reduces cultural conflict. The marginal benefit is declining in $\tilde{x}$, but increasing in the cultural distance between the majority and the minority $(\lambda - \lambda_m)^2$ and the marginal cost of cultural conflict $\theta^2$. The term $(\lambda - \lambda_m)^2$ can be considered the “quantity” of cultural conflict between employees, whereas $\theta^2$ is its “price” in terms of lost profits. The marginal benefit is also decreasing in the extent of socialization $n$. If the minority is already highly socialized to match the majority culture, then the additional benefit of hiring a majority type to reduce cultural conflict is diminished. The right-hand-side of (12) is the marginal cost of hiring more of the majority type. The larger the wage gap $\omega$, the more expensive the majority is relative to the minority. A greater majority also decreases the diversity of the firm at rate $(2\tilde{x} - 1)$.

The left-hand-side of (13) is the marginal benefit of socialization. This marginal benefit is declining in $n$, but increasing in the cultural distance and the share of the minority $1 - \tilde{x}$. The right-hand-side is the marginal cost of socialization $\phi'$, which we assume to be strictly increasing.

4.2 Corporate cultural variety

The fact that the marginal benefit of socialization increases with the minority share implies that the extent of socialization $n$ and the minority share $1 - \tilde{x}$ are complements. The marginal benefit of socialization is higher if the firm employs a larger minority share. This complementarity can lead to an upward-sloping marginal benefit of socialization. Indeed, substituting (12) into (13) and then inserting the wage gap from (11) gives

$$\frac{(b - b_m - v(n))^2}{\theta^2 (\lambda - \lambda_m)^2 (1 - n)^3} = \phi' (n). \quad (14)$$

The left-hand-side of (14) is the marginal benefit of socialization after taking into account the optimal adjustment to the minority share $1 - \tilde{x}$ and the movement in the wage gap $\omega$ that makes the labor markets clear. For a given extent of socialization $n$ obtained from (14), the corresponding optimal minority share $1 - \tilde{x}$ comes from rearranging (12):

$$1 - \tilde{x} = \frac{b - b_m - v(n)}{\theta^2 (\lambda - \lambda_m)^2 (1 - n)^2}. \quad (15)$$

The optimal minority share is positive by Assumption 2. Examining (14), one can observe that the marginal benefit of socialization is declining in the size of the cultural conflict $(\lambda - \lambda_m)^2$.
between the majority and minority. The reason is because a larger gap between the cultures induces the majority to hire less of the minority, as seen in (15). In turn, the complementarity between the minority share $1 - \tilde{x}$ and $n$ implies the marginal benefit of socialization decreases precisely because the size of the minority at the firm is smaller. Similar logic applies to explain why the marginal benefit is declining in the marginal cost of cultural conflict $\theta^2$. Finally, the marginal benefit of socialization is increasing in the outside option gap because the minority wage will be lower, which leads the firm to hire minority types, raising the marginal benefit of socialization.

The emotion function $v(n)$ also enters the left-hand-side of (14). The behavior of that function is crucial to the properties of the marginal benefit of socialization. The function $v(n)$ can reinforce the complementarity between the minority share and the extent of socialization by generating regions over $n \in [0, 1]$ such that the marginal benefit is increasing in $n$. This property permits the marginal benefit curve and the marginal cost curve to intersect more than once. Multiple crossings implies the possibility of multiple solutions to the firm problem.

Figures 5(a)-5(b) illustrate two possible shapes of the marginal benefit curve. The difference between the two panels is the emotion function $v(n)$. In Figure 5(a), the marginal benefit crosses the marginal cost curve only once ($n^*$), whereas in Figure 5(b), it crosses multiple times. In Figure 5(a), the $v(n)$ we use increases everywhere, but at a decreasing rate. For values of $n$ less than $n^*$, the marginal benefit of socialization exceeds the marginal cost, whereas it falls below the marginal cost for values greater than $n^*$. In this case, $n^*$ is the global optimal choice of socialization.

In Figure 5(b), the emotion function $v(n)$ oscillates in a similar way to the example given in Figure 2. That behavior generates the multiple crossings. At $n^*$, the marginal benefit curve crosses the marginal cost curve “from below,” making it a local minimum and suboptimal. On the other hand, both $n_1^*$ and $n_2^*$ are local maxima because at those points, the marginal benefit curve crosses the marginal cost curve “from above.” In the example, $n_2^*$ is the global optimum and the unique choice of socialization because the area $B$ exceeds the area $A$.

### 4.3 Corporate cultural variety equilibrium

Although we speak in this section of an equilibrium in which a single firm exists, we can interpret the environment as a broader economy in which there are many segmented firms that do not compete in the same labor markets. (For example, the firms may demand different types of labor skills.) In this context, a diverse collection of corporate cultures can coexist in the economy in equilibrium.

Corporate cultural variety can exist in equilibrium if and only if the multiple solutions to the firm’s problem are all global maxima. That way the firm will have no preference to choose one corporate culture over another. All firms could coordinate on a single solution, but they may not, leading to a variety of corporate cultures between firms despite no starting heterogeneity.

To construct such an equilibrium, we first need to choose a free parameter in (14) that can be adjusted to ensure all local maxima generate the same level of profits. The outside option
Figure 5: Two Examples of the Optimal Extent of Socialization

Notes: The figures illustrate the marginal benefit curve $\frac{(b-b_m-v(n))^2}{\theta^2(\lambda-\lambda_m)^2(1-n)^3}$ and the marginal cost curve $\phi'(n)$ for two different emotion functions $v(n)$. The first panel has $v(n) = n^v$ with $v \in (0, 1)$, whereas the second has $v(n) = \alpha_1 + \alpha_2 n + \alpha_3 \sin(2\pi fn + \psi)$ with the parameters arranged so $v(0) = 0$. Neither emotion function is shown. The first panel presents a case in which a single optimal solution $n^*$ exists, whereas the second presents one in which a single local minimal solution exists ($n^*$) and two locally maximal solutions exist ($n_1^*$ and $n_2^*$). Only one local maximal ($n_2^*$) is the unique global solution because the area $B$ exceeds the area $A$.

gap $b - b_m$ could do it. In that case, if the outside option gap was too small because the value of the minority’s outside option was too large, hiring more minority types beyond a certain share would be too expensive, rendering all but one corporate culture suboptimal. The same could be said for an outside option gap that was too large: only a single corporate culture would be optimal. The outside option gap would have to be at a certain value for multiplicity in corporate cultures to be sustained in equilibrium.

Alternatively, one could use the amount of cultural conflict $(\lambda - \lambda_m)^2$ between the majority and minority at the firm. Selecting that parameter poses an intriguing question: what amount of cultural conflict between employees is enough for two otherwise identical firms to adopt distinct corporate cultures? To simplify notation, let \( \Lambda \equiv (\lambda - \lambda_m)^2 \) and define

\[ m(n, \Lambda) \equiv \frac{(b-b_m-v(n))^2}{\theta^2 \Lambda (1-n)^3}. \]

Using the example depicted in Figure 5(b), we illustrate the construction of an equilibrium with corporate cultural variety in Figure 6. The panel displays $m(n, \Lambda)$ for three values $\Lambda_1 < \Lambda_2 < \Lambda_3$. The marginal benefit of socialization is clearly decreasing in $\Lambda$. Importantly, at the quantity of cultural conflict $\Lambda_2$ and extents of socialization $n_1^*$ and $n_2^*$, the areas $A$ and $B$ are equal, meaning the majority at the firm would be indifferent between choosing $n_1^*$ and $n_2^*$ because they generate the same level of profit. The optimal employment condition from (15)
then pins down the corresponding minority shares \(1 - \tilde{x}_1\) and \(1 - \tilde{x}_2\).³

Figure 6: Corporate Cultural Variety in Equilibrium

Notes: When the amount of cultural conflict between the majority and minority \(\Lambda \equiv (\lambda - \lambda_m)^2\) reaches \(\Lambda_2\), a firm becomes indifferent between choosing \(n_1^*\) and \(n_2^*\) as the extent of socialization. In the illustration, \(\Lambda_1 < \Lambda_2 < \Lambda_3\), and the areas \(A\) and \(B\) are equal. The emotion function of the minority is \(v(n) = \alpha_1 + \alpha_2 n + \alpha_3 \sin (2\pi fn + \psi)\) with the parameters arranged so \(v(0) = 0\).

The complementarity between the extent of socialization and the minority share implies that two firms could be equally profitable with a corporate culture having more socialization and a larger minority \((n_2^*, 1 - x_2^*)\) or one with less socialization and a smaller minority \((n_1^*, 1 - x_1^*)\). Proposition 1 formalizes the existence of corporate cultural variety in equilibrium. As part of the proposition, we also establish the relation in the minority shares between firms that make different optimal choices of \(n\).

**Proposition 1.** (Corporate cultural variety) If an equilibrium exists at cultural conflict \(\Lambda = \hat{\Lambda}\) and constant parameters \(\theta\) and \(b - b_m\), then such an equilibrium features corporate cultural variety when there exists at least one \(n \in (0, 1)\) satisfying

\[
2v'(n) (b - b_m - v(n)) = \theta^2 \hat{\Lambda} (1 - n)^2 (3\phi'(n) - (1 - n) \phi''(n)).
\] ¹(16)

In this kind of equilibrium, there exist at least two tuples \(\{n_i^*, 1 - x_i^*\}, i \geq 2\), between which a³Here is why the two areas must be equal for a firm to be indifferent: both \(n_1^*\) and \(n_2^*\) are local maxima, whereas the value of \(n\) between \(n_1^*\) and \(n_2^*\) at which \(m\) crosses \(\phi'\) from below is a local minimum. The profit function decreases as \(n\) moves away from \(n_1^*\) until \(n\) reaches the middle crossing point, after which profits begin increasing until \(n_2^*\). The two areas \(A\) and \(B\) must equal so that the “height” of the profit function at \(n_1^*\) and \(n_2^*\) is the same.
firm is indifferent. Among the tuples, if $n_i^* > n_j^*$, then $1 - \bar{x}_i^* > 1 - \bar{x}_j^*$ for $i \neq j$.

The proposition places a condition on the relation between the emotion function and the marginal cost of socialization (and their derivatives) that must be satisfied for a variety of corporate cultures to exist in equilibrium. In the example of Figure 6, corporate cultural variety exists when the cultural differences between the minority and majority groups are not too small or too large so that (16) is met. At extreme values of $\Lambda$, a unique corporate culture will be optimal. Too small a cultural gap and the majority at any firm would do best to hire a larger group of minority and engage in more socialization over all other choices. That case would be similar to $\Lambda = \Lambda_1$ and $n_2^*$ in the figure. On the other hand, too large a cultural gap (as in $\Lambda = \Lambda_3$ in the figure) would lead the majority everywhere to hire more of itself and forego much socialization: if the world were made up of only MBAs and Visigoths, the MBAs would rather hire themselves than trouble with socialization.

When the cultural gap is not extreme, a variety of corporate cultures arise because the complementarity between the choices of $n$ and $1 - \bar{x}$ makes the firm problem non-convex. That non-convexity, along with the shape of $v(n)$, opens the possibility for multiple optimal corporate cultures. Identical firms with the same technology, information, and profit functions would choose different corporate cultures featuring different levels of diversity and socialization in equilibrium. Importantly, the wage gaps at the firms would differ as well.\(^4\,5\)

4.4 Equilibrium wage gap

Substituting the optimal minority share from (15) into (11) expresses the wage gap solely as a function of the firm’s choice of $n$. For an optimal extent of socialization $n^*$, the equilibrium wage gap is

$$\omega(n^*) = (b - b_m - v(n^*)) \left(1 + \frac{2}{\theta^2 (\lambda - \lambda_m)^2 (1 - n^*)^2}\right) - 1.$$  \hspace{1cm} (17)

An interesting feature of (17) is that the wage gap could be negative, meaning the firm pays the members of the minority relatively more than the majority, which can be a sign of minority power. A minority group’s credible threat to exit from either a strong outside option $b_m$ or severe disutility from socialization could turn the wage gap negative in order to entice the minority to stay.

Figures 7(a)-7(b) display how the wage gap varies with a firm’s choice of $n$ when the emotion functions are the same as those used in Figures 5(a) and 6. The emotion function $v(n)$ used in the first panel is increasing and concave, whereas the wage gap is everywhere positive, but U-shaped, similar to the behavior of the marginal benefit curve of socialization. The emotion function for the second panel oscillates and reaches it peak near $n_2^*$. The wage gap curve in turn

\(^4\text{Introducing non-convexity into the cost of socialization function } \phi \text{ would create an additional source of multiple corporate cultures. But the interpretation would be distinct from the one we focus on, which arises from the complementarity between } n \text{ and } 1 - \bar{x}, \text{ as well as the shape of } v(n).\)

\(^5\text{Gârleanu et al. (2015) also feature a non-convex optimization problem, but in the portfolio problem of investors. There, a complementarity between participation across markets and leverage generates multiple optimal portfolio choices for otherwise identical investors.}\)
is U-shaped, but turns negative. At both optimal choices of socialization, the minority earns more than the majority. When the extent of socialization at the higher level \( n^*_2 \), the minority is paid significantly more. Members of the minority must be compensated sufficiently enough to retain them at the firm despite their distaste for that high degree of socialization.

**Figure 7: Two Examples of the Relation between the Wage Gap and the Extent of Socialization**

(a) Single solution  
(b) Multiple solutions

Notes: The figures illustrate the equilibrium wage gap from (17) as a function of a firm’s choice of \( n \). The first panel uses the emotion function \( v(n) = n^* \) with \( v \in (0, 1) \), whereas the second uses \( v(n) = \alpha_1 + \alpha_2 n + \alpha_3 \sin(2\pi fn + \psi) \) with the parameters arranged so \( v(0) = 0 \). The optimal choices of \( n \) under the two emotion functions are marked, just as in Figures 5(a) and 6.

An examination of (17) gives insight into the behavior of the wage gap under different extents of socialization and corporate cultures more generally. On the one hand, the complementarity between socialization and the minority share suggests that if a firm chooses to socialize members of the minority more, it would hire more of them. That in turn would imply the minority is paid relatively less (generating a larger wage gap) because the minority at that firm is compensated with greater homophily. But the exact relation between the wage gap and the choice of \( n \) will depend on how that complementarity compares to the minority member’s emotions from undergoing more socialization. Lemma 1 presents the conditions that determine the relation between the wage gap and corporate culture.

**Lemma 1. (Wage gaps across corporate cultures)** A firm that socializes its minority more in its corporate culture will also feature a higher wage gap if

\[
\frac{v(n^*_2) - v(n^*_1)}{v(n^*_1)} < \frac{v(n^*_1)}{\frac{\theta^2}{2} \Lambda (1 - n^*_1)^2} - \frac{v(n^*_2)}{\frac{\theta^2}{2} \Lambda (1 - n^*_2)^2},
\]

where \( n^*_1 < n^*_2 \) are two optimal choices of \( n \).

So far we have only studied the case of a single firm. Options for employment were limited and alternatives to working were fixed. We turn next to a setting in which a single incumbent
firm operates within an industry, but the incumbent faces the threat of entry from a newcomer. This richer environment allows us to study whether one corporate culture can drive out another in the marketplace without intervention from an external authority.

5 Firms and Social Progress

In many ways, corporate culture echoes society’s culture. The civil rights and “Me Too” movements discussed earlier are examples of corporate cultures adapting to social progress, whether voluntarily or forcefully through government legislation. Another example is the movement in the U.S. for protected family medical leave while employed.

Beginning in the mid-1980s, as an increasing number of women were entering the workplace, a variety of voices called for generous leave policies during pregnancy, after the birth of a child, or for personal or family illness. Some of the proponents were clinical experts, social science researchers (Fried (1998)), constituency groups such as the National Partnership for Women & Families, the Children’s Defense Fund, AARP, the American Academy of Pediatrics, the Catholic Conference, and even companies such as Ben & Jerry’s, Stride-Rite, and Fel-Pro (Lenhoff and Bell (2002)).

Ultimately, Congress passed the Family and Medical Leave Act of 1993 (FMLA), which entitled eligible employees for up to twelve weeks of unpaid, job-protected leave for qualified family and medical reasons (United States Department of Labor (2018)). Before that, only a quarter to one-third of leave policies at firms offered as many protections as the FMLA required, with many voluntary policies omitting authorized leave to care for newborns, or a gravely ill parent, child, or spouse (Commission on Family and Medical Leave (1996)). With the passage of the FMLA, the notion of a “work-life” balance for employees broadly entered corporate cultures (Fried (1998); Hoch (2013); Glynn (2013)).

5.1 Competition between corporate cultures

When society witnesses a cultural advancement, one might ask: can that progress spread to the corporate cultures in industry by competition, or must the government intervene? Indeed, companies often use their contrasting corporate cultures to compete.

For instance, the ride-hailing firm Lyft saw its competitor Uber’s myriad scandals, including accusations of widespread sexual harassment and gender discrimination, as an opportunity to position itself as a more ethical alternative. The head of brand strategy at Lyft, Gina Ma, said, “diversity and inclusivity are key to the Lyft mission.” That proclaimed corporate culture is a sharp contrast to Uber’s “always be hustlin’,” win-at-all-cost corporate culture, where “you can never get ahead unless someone else dies” (Solon (2017)).

Uber’s difficulties helped increase Lyft’s customer base (Wong (2017)). By July 2018, the U.S. Equal Employment Opportunity Commission had put Uber under investigation over the accusations against the company of gender discrimination, adding to the long list of federal investigations into the company’s practices (Kuchler and Bond (2018)).

Dating app Bumble is another example of two firms competing over corporate cultures.
Whitney Wolfe Herd had co-founded Tinder—another online dating app—but left and sued Tinder for sexual harassment. She started Bumble as a women-led company and the “first feminist dating app” where women make the initial move (Alter (2015), Yashari (2015)). The company was created in part to compete against the male-focused and overly sexualized Tinder. In mid-2017, Bumble rejected a buyout offer by Tinder’s parent company and was valued at well over $1 billion (O’Connor (2017), Wagner (2018)).

To study the propagation of social progress to the marketplace, we introduce an environment in which an incumbent firm operates in a sector or industry that another firm considers entering. While we speak of firms, they stand for competing corporate cultures. The incumbent represents the prevailing, regressive corporate culture prior to some progressive social change. The incumbent could be the corporate culture that is common among many firms within an industry or the corporate culture of a single firm that dominates an industry. A crucial distinction of this incumbent is that it defies adopting any aspect of the social progress.

A potential entrant, on the other hand, exemplifies the social progress. It incorporates into its operations the change that has come to society. It could be a start-up firm, but it need not be. The potential entrant may be as old and as large as the incumbent, but simply advances its corporate culture in a way the incumbent does not. While society can progress in a number of ways, the social change we focus on is the acceptable manner in which a minority is treated in a workplace.

5.2 An incumbent and entrant

Let $i$ denote the incumbent and $e$ the entrant. Both are managed by members of the majority who make employment and socialization decisions. Just as in the previous section, majority employees have personal cultural density $g$ from (7), whereas minority employees have $f$ from (8). The incumbent and entrant engage in Stackelberg competition (von Stackelberg (1952)). The incumbent selects its corporate culture (choosing $n_i$ and $\tilde{x}_i$) first, and the entrant follows sequentially with $\{n_e, \tilde{x}_e\}$ and its answer to the entry decision. The profit functions of the two firms are identical and the same as the profit function expressed in Problem 10 from the previous section.

Though sharing the same profit function, the incumbent and entrant will differ in their flexibility to adjust wages and their treatments of the minority. Whereas the entrant allows its wage gap $\omega_e$ to respond to competition in the majority and minority labor markets, the incumbent does not. The incumbent pegs its wage gap to $\omega_i$ and does not let it adjust in response to any strategy of the entrant. The entrant must allow its wage gap to be flexible in order to have a chance at entering.

We interpret this inflexible behavior of the incumbent as a stubbornness to change majority-minority pay differences despite outside pressure from the market in the form of a threat of a competitor’s entry. The incumbent is hard-nosed precisely because the market pressure arises from social change. In the face of progress, the incumbent clings to an antiquated view of what it believes to be the correct wage gap. Sticking to its wage gap $\omega_i$ is an endowed feature of the
incumbent and partly defines it.

The incumbent also differs from the entrant in how it treats members of the minority when socializing them. Both the incumbent and the entrant have an interest in reducing cultural conflict, which requires some degree of socialization. From here on we assume the socialization process for the minority is painful, captured by an emotion function \( v(n) > 0 \) for all \( n \). The social progress we consider is the entrant making the socialization process less painful for the minority. This change is in contrast to the incumbent’s attitude, who makes no advancement from its now obsolete view on minority treatment.

So let \( v_i(n) \) be the emotion function of an employee at the incumbent, and \( v_e(n) \) the emotion function at the entrant. By distinguishing the two functions, we presume that employees have state-dependent disutility from socialization. The “state” in this case is the firm an employee works. We model the entrant incorporating the social progress into its business with the following assumption:

**Assumption 5. (Emotion functions, Stackelberg game)** In the Stackelberg game, the emotion functions of employees at the incumbent and the entrant are continuous, strictly increasing, strictly convex, and for each \( n \in (0, 1] \), satisfy \( 0 < v_e(n) < v_i(n) \).

### 5.3 Employee utility

The outside option for employees is to leave the incumbent and join the entrant. They can freely do so because labor is perfectly mobile between the two firms. A single friction, however, dissuades the minority from leaving the incumbent. That friction is a utility cost \( \kappa > 0 \) to exiting that creates a mild preference of the minority to stay with the incumbent corporate culture. The indifference conditions for the majority and minority are

\[
\begin{align*}
  w_i + \bar{x}_i &= w_e + \bar{x}_e, \\
  w_{i,m} + 1 - \bar{x}_i - v_i(n_i) &= w_{e,m} + 1 - \bar{x}_e - v_e(n_e) - \kappa.
\end{align*}
\]

The left-hand-side of (18) is the majority’s utility from working at the incumbent, whereas the right-hand-side is the utility from joining the entrant. For the minority, the left-hand-side of (19) is the utility from working at the incumbent, whereas the right-hand-side is that from exiting for the entrant.

The utility cost \( \kappa \) stands broadly for any kind of switching cost for the minority to give up work at the incumbent and join the entrant. Up until the moment of social progress and the emergence of a potential entrant, the minority has felt its treatment at the workplace under the incumbent corporate culture. The entrant embodies a less painful socialization experience, but members of the minority might be uninformed of that assertion, distrust it, or perceive themselves undeserving of that better environment.

The cost might also be general monetary expenses of transitioning to a new place of employment. It may be a status quo bias (a preference for the current state of affairs), or loss
aversion (Kahneman et al. (1991); Samuelson and Zeckhauser (1998)). It could be a consequence of the mere-exposure effect that makes the incumbent more familiar and preferable (Zajonc (1968); Bornstein (1989)). It could also be a psychological cost to leaving because working at the incumbent firm and undergoing the socialization there has entered the minority’s identity (Cote and Levine (2002); Weinreich (2003); Akerlof and Kranton (2005)). The bias need only give preference to the minority to remain with the incumbent rather than depart for the entrant.

### 5.4 Equilibrium

Our notion of an equilibrium to the Stackelberg competition game between the incumbent and entrant is a subgame perfect Nash equilibrium. The decisions of the two firms are sequential: the incumbent chooses its corporate culture, which the entrant observes. Then the entrant chooses its corporate culture and decides whether to enter. Information is perfect and complete, and strategies are common knowledge. The definition of the equilibrium is the following:

**Definition 4.** *(Equilibrium, incumbent and entrant)* When an incumbent and entrant engage in Stackelberg competition, an equilibrium is an entry decision by the entrant and a tuple $\mathcal{E}_2 = \{\bar{x}_i, n_i, \bar{x}_e, n_e, \omega_e\}$, where $\{\bar{x}_i, n_i\}$ and $\{\bar{x}_e, n_e\}$ are Nash equilibrium strategies for the incumbent and entrant, respectively, in every subgame, and where $\omega_e \equiv w_e - w_{e,m}$ is the majority wage gap of the entrant that clears the majority and minority labor markets (i.e., satisfy (18) and (19)) when the incumbent has wage gap $\omega_i = w_i - w_{i,m}$. The entry decision is either to enter or not. In equilibrium, only the firm earning the highest profits operates in the market.

Should the potential entrant decide to enter, that entry is costless. However, as part of our equilibrium, entry is Nash if and only if maximal profits of the entrant, given the strategy of the incumbent, *meet or exceed* that of the incumbent. It is not enough for the entrant simply to earn maximal profits that are positive. The entrant will not enter the market in equilibrium if it earns less than the incumbent. To displace the incumbent and operate alone, the entrant must even earn more.

Our notion of entry, then, is stricter than another firm starting operations in the industry beside the incumbent simply because entry is profitable. Entry in equilibrium means the entrant earns at least as much as the incumbent, and if it earns more, the entrant *drives out* the incumbent from the market. Any exit by the incumbent normally would take time, so we consider exit, if it were to occur, to take place over the longer run.  

Because the two firms represent competing corporate cultures, the incumbent’s exit need not represent a business shutting down. What exit truly means here is corporate cultural progress: the supplanting of an antiquated corporate culture by a new one that reflects the

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6There are many reasons why the most profitable firm will succeed in the marketplace rather than all those making strictly positive profits. Financial capital may flow over time only to the most profitable. Or, customers may eventually flock to the most profitable firm because of its perceived stability, and over time, the firm can build enough capacity to accommodate all the demand.
social progress of the time. The incumbent’s exit is the same as changing its corporate culture to reflect the social progress. We presume that such an event can occur in the marketplace only if the new corporate culture is more profitable than the previous. If the entrant is unsuccessful, it means the more progressive corporate culture could not permeate the market independently. Some kind of external intervention, such as government legislation, would be necessary to impose widespread adoption among firms.

5.5 Firm strategies

We derive the equilibrium using backward induction. The entrant solves its problem taking as given an arbitrary choice of the incumbent \( \{\tilde{x}_i, n_i\} \). The incumbent then decides its best response after observing the decisions \( \{\tilde{x}_e, n_e\} \) of the entrant.

The optimality conditions for the two firms are virtually identical to (12) and (13) from the single-firm case in the previous section. The only change reflects the outside options of the employees now being an exit to another firm rather than some fixed value. Subtracting the minority indifference condition in (19) from the majority’s in (18) gives

\[
\omega_i + (2\tilde{x}_i - 1) + v_i (n_i) = \omega_e + (2\tilde{x}_e - 1) + v_e (n_e) + \kappa. \tag{20}
\]

From (20), the entrant’s best response as implicit functions of the incumbent’s strategy are

\[
\begin{align*}
\frac{(\omega_i + (2\tilde{x}_i - 1) + v_i (n_i) - v_e (n_e) - \kappa)^2}{\theta^2 (\lambda - \lambda_m)^2 (1 - n_e)^3} &= \phi' (n_e), \tag{21} \\
\frac{\omega_i + (2\tilde{x}_i - 1) + v_i (n_i) - v_e (n_e) - \kappa}{\theta^2 (\lambda - \lambda_m)^2 (1 - n_e)^2} &= 1 - \tilde{x}_e. \tag{22}
\end{align*}
\]

Examining the left-hand-side of (21), one can see that the marginal benefit to socialization for the entrant is increasing in the incumbent’s wage gap, majority share, and extent of socialization (because \( v_i (n) \) is increasing in \( n \)). The intuition here is that an increase to any of these three objects makes staying at the incumbent less attractive. A weaker outside option to joining the entrant raises the incentive of the entrant to socialize the minority more.

This relation implies that the socialization decisions of the two firms are strategic complements. When the incumbent socializes more, the entrant has an incentive to do the same. It is true that the entrant embodies the social progress by lowering the pain from socialization. But it also wants to maximize profits. If the entrant observes the incumbent socializing the minority to a greater extent, it too will want to socialize more, but it would do so in a way to create less disutility for the minority.

Equation (22) is the entrant’s minority share. Similar to its socialization decision, the entrant wants to hire more members of the minority when their outside options are weaker. More socialization at the incumbent, a lower sense of homophily there, and a weaker bias all encourage the entrant to hire more minority.

We now consider the incumbent’s best response \( (\tilde{x}_i, n_i) \), which will be a function of the
entrant’s strategy. The optimality conditions that determine the best response of the incumbent are
\[
\frac{(\omega_e + (2\widetilde{x}_e - 1) + v_e(n_e) - v_i(n_i) + \kappa)^2}{\theta^2 (\lambda - \lambda_m)^2 (1 - n_i)^3} = \phi'(n_i), \tag{23}
\]
\[
\frac{\omega_e + (2\widetilde{x}_e - 1) + v_e(n_e) - v_i(n_i) + \kappa}{\theta^2 (\lambda - \lambda_m)^2 (1 - n_i)^3} = 1 - \tilde{x}_i. \tag{24}
\]
Again, the marginal benefit of socialization increases when the outside option of the minority is weaker. It also increases when the bias is more powerful (higher \(\kappa\)) because a higher utility cost from leaving compels the minority to stay with the incumbent, which encourages the incumbent to socialize them more. Finally, a larger bias emboldens the incumbent to hire more minority because they are less likely to leave.

### 5.6 Discussion of the game

Classical games of competitive entry (e.g., von Stackelberg (1952); Spence (1977, 1979); Dixit (1979, 1980)) typically involve the first mover making an irreversible decision that raises a barrier to entry. For example, as in Dixit (1980), an incumbent may choose in the first period a capacity level of output that cannot be reduced. The incumbent can expand capacity in the second period, but the per-unit cost of production is cheaper when output stays within that initial capacity. If another firm threatens entry, the incumbent can gain by investing in more starting capacity than it otherwise would if it were a pure monopolist. That first period decision allows the incumbent to credibly commit to a high level of production, which deters another firm from entering the market because doing so is no longer profitable.

In the competition here between corporate cultures, neither firm’s choice of \(\tilde{x}\) or \(n\) directly enters the profit function of the other firm, so the strategic interaction between the incumbent and entrant is not direct.\(^7\) The incumbent influences the profit of the entrant only indirectly via the labor decisions of the majority and minority. This single avenue of interaction and the power of an employee to freely exit a firm undermine any commitment value of the incumbent’s employment decision. The incumbent cannot physically restrain its employees from leaving so as to make its choice of \(\tilde{x}\) irreversible. Indeed, with the presence of only the indirect interaction, one can show that the solution to this Stackelberg game will match that of a simultaneous-move game between the incumbent and entrant.

This is not to say that the incumbent has no “first-mover advantage.” Its association with the societal culture prior to the social progress makes the incumbent the prevailing corporate culture, which the minority has a bias to stay with. Most importantly, though, the incumbent pegs its wage gap \(\omega_i\), to which the potential entrant needs to respond in order to compete. The incumbent’s stubbornness to change its wage gap, despite the threat of entry, becomes its

\(^7\)Mathematically, \(\frac{\partial \pi_i}{\partial \tilde{x}_e} = 0\) and \(\frac{\partial \pi_i}{\partial n_e} = 0\). The decisions of the two firms are strategic complements, however, because \(\frac{\partial \pi_i}{\partial \tilde{x}_i, \partial \tilde{x}_e} > 0\) and \(\frac{\partial \pi_i}{\partial n_i, \partial n_e} > 0\) for the incumbent, and likewise for the entrant.
commitment. When we provide explicit solutions below, we will show that this inflexibility can either insulate the incumbent from exit or foreshadow its downfall.

5.7 Solving the game

To obtain explicit solutions to the game, we assume the following functions for the cost of socialization (common to the two firms), and the two emotion functions:

\[ \phi(n) = \frac{\phi^2}{2} \left( \frac{1}{(1-n)^2} - 1 \right), \]
\[ v_k(n) = v_k \left( \frac{1}{(1-n)^2} - 1 \right), \]

for \( k \in \{i, e\} \) and \( v_e < v_i \). Note that our choice for \( \phi(n) \) obeys Assumption 2 and \( v_k(n) \) obeys Assumption 5. These functions will guarantee a unique equilibrium to the game. Because the outside options of employees is to join the other firm rather than obtain amounts \( b \) and \( b_m \), as in the previous section, we impose one other assumption on the parameters in order to regulate the firm solutions to remain within the bounds:

**Assumption 6.** The interval \( I \equiv \left[ \frac{2\phi}{\theta(\lambda - \lambda_m)} \left( \frac{\kappa + v_i}{v_i} \right), \min \left\{ \left[ v_e \theta(\lambda - \lambda_m) (\frac{\kappa + v_i - v_e}{v_i}) \right] \left( \frac{2\phi}{\theta(\lambda - \lambda_m)} \left( \frac{\kappa + v_i - v_e}{v_i} \right) \right) \right\} \) has positive measure and \( 1 - \phi \theta(\lambda - \lambda_m) + \omega_i \in I \).

We solve the game in four steps. First, we determine the best response of the entrant while restricting the majority and minority labor markets from clearing (i.e., holding \( \omega_e \) fixed). Second, we solve for the incumbent’s best response, which will account for the entrant’s strategy. Third, we allow labor market clearing to express the firm strategies in terms of exogenous objects. Fourth, and finally, we give the conditions in which the entrant can push out the incumbent, giving the complete solution to the game.

5.7.1 Best responses prior to labor market clearing

Applying the specified functions for \( \phi \) and \( v_k \) to the entrant’s best response in (21) and (22) provides the entrant’s explicit strategy, prior to the labor markets clearing. We present that result in Lemma 2.

**Lemma 2.** *(Entrant best response, fixed wage gaps)* The entrant’s best response to the incumbent’s strategy \((\tilde{x}_i, n_i)\) before the majority and minority markets clear is

\[
    n_e = 1 - \sqrt{\frac{v_e}{\omega_i + (2\tilde{x}_i - 1) + v_i (n_i) + v_e - \kappa - \phi \theta (\lambda - \lambda_m)}},
\]
\[
    1 - \tilde{x}_e = \frac{\phi \left( \omega_i + (2\tilde{x}_i - 1) + v_i (n_i) + v_e - \kappa - \phi \theta (\lambda - \lambda_m) \right)}{v_e \theta (\lambda - \lambda_m)}.
\]

The entrant’s choice of socialization can be interpreted as a discount from full socialization. The entrant socializes the minority less when socialization pains the minority more (higher \( v_e \)) and when the minority has a larger bias to stay with the incumbent (higher \( \kappa \)). It also
socializes less when the cost of socialization is higher (higher \( \phi \)) and when the personal cultural conflict between the majority and minority \( (\lambda - \lambda_m) \) is greater. More conflict also encourages the entrant to hire fewer minority types. Finally, a lower incumbent wage gap \( \omega_i \), greater homophily at the incumbent \( 1 - \tilde{x}_i \), and less painful incumbent socialization \( v_i (n_i) \) all discourage the minority to leave the incumbent. All three factors discourage the entrant from both socialization and hiring more minority.

We derive the incumbent’s strategy by inserting the explicit functions for \( \phi \) and \( v_k \) into the incumbent’s best response in (23) and (24). That best response is given in Lemma 3.

**Lemma 3. (Incumbent best response, fixed wage gaps)** The incumbent’s best response to the entrant’s strategy \( (\tilde{x}_e, n_e) \) in Lemma 2 is

\[
\begin{align*}
n_i &= 1 - \frac{\sqrt{\phi}}{\sqrt{\theta (\lambda - \lambda_m)}} \sqrt{\frac{R}{2 (v_e \theta (\lambda - \lambda_m) - \phi) + (S + T)}}, \\
1 - \tilde{x}_i &= 1 - \frac{2 (v_i \theta (\lambda - \lambda_m) - \phi) - (S + T)}{R},
\end{align*}
\]

where

\[
\begin{align*}
R &\equiv 2 \theta (v_i + v_e) (\lambda - \lambda_m) - 4 \phi, \\
S &\equiv 2 \phi (v_i - v_e - \omega_i + \kappa), \\
T &\equiv \theta (\lambda - \lambda_m) \left(2 \phi^2 + v_e (\omega_i + \omega_e) - 2 \phi v_e \theta (\lambda - \lambda_m)\right).
\end{align*}
\]

The presence of a new, more progressive corporate culture compels the dominate, but bygone one to modify in order to defend from displacement. The incumbent adjusts its corporate culture to respond when it otherwise would change nothing if no threat of entry existed. In this sense, social progress can affect the prevailing corporate culture even if that progress cannot entirely replace it. The entrant pressures the incumbent through its less painful treatment of the minority \( v_e \) and its wage gap \( \omega_e \). The entrant’s wage gap, though, must also clear the labor markets, which we turn to next.

**5.7.2 Labor market clearing**

The incumbent responds to the entrant’s wage gap \( \omega_e \), which the incumbent knows will be determined in equilibrium by the indifference of both the majority and minority employees to work at either firm. The entrant’s wage gap will be determined explicitly by the market clearing condition (20).

Substituting (20) into the incumbent’s best response in Lemma 3, and then inserting those solutions into the entrant’s best response in Lemma 2 delivers the optimal strategies entirely in terms of exogenous objects. Proposition 2 has the results.

**Proposition 2. (Best responses, labor markets clearing)** The optimal strategies of the incum-
bent, allowing the labor markets to clear, are
\[
1 - \bar{x}_i^* = \frac{1}{2} \left( 1 - \phi \theta (\lambda - \lambda_m) + \omega_i \right),
\]
\[
n_i^* = 1 - \frac{\phi}{\theta (\lambda - \lambda_m) (1 - \bar{x}_i)},
\]
whereas the optimal strategies of the entrant are
\[
1 - \bar{x}_e^* = \frac{1}{2} \left( \frac{v_i}{v_e} (1 - \phi \theta (\lambda - \lambda_m) + \omega_i) - \frac{2\phi}{\theta (\lambda - \lambda_m)} \left( \frac{\kappa + v_i - v_e}{v_e} \right) \right),
\]
\[
n_e^* = 1 - \frac{\phi}{\theta (\lambda - \lambda_m) (1 - \bar{x}_e)}.
\]

The first thing to notice is that the equilibrium best responses of the incumbent and entrant are quite similar. What tends to increase minority hiring at the incumbent does so at the entrant as well. The same can be said when comparing the socialization strategies of the two firms. The relation reveals the strategic complementarity in the best responses.

For example, a higher wage gap \( \omega_i \) at the incumbent implies that minority labor is relatively less expensive, so the incumbent wants to hire more of that type. For the entrant, a higher \( \omega_i \) has the same effect in encouraging more minority hiring. We will see below that the wage gap for the entrant \( \omega_e \) that clears the labor markets will depend on the incumbent’s wage gap \( \omega_i \). If the incumbent pegs to a larger wage gap, the entrant must follow suit in order to attract majority employees. But minority pay then becomes relatively low, which encourages the entrant to hire more minority. At the same time, a larger wage gap compels the entrant to hire more minority in order to compensate them with greater sense of homophily.

The central distinction between the incumbent and entrant strategies is that the entrant accounts for the bias \( \kappa \) and its relative pain from socialization \( \frac{\omega_i}{v_e} \), whereas the incumbent does not. The incumbent does not even account for its own socialization pain \( v_i \) when choosing its socialization policy. The entrant, on the other hand, will reduce its extent of socialization the more painful the process is for its minority and the larger is the bias.

Why does the incumbent effectively ignore these two components? To answer this question, it is useful to examine the incumbent’s optimal policy if it faced no threat of entry. Similar to the single-firm environment of the previous section, suppose majority and minority employees could not leave for another firm, but had fixed outside options \( b \) and \( b_m \). The incumbent’s employment and socialization decisions would be
\[
1 - \bar{x}_i = \frac{\phi (b - b_m + v_i + \kappa - \phi \theta (\lambda - \lambda_m))}{v_i \theta (\lambda - \lambda_m)},
\]
\[
n_i = 1 - \frac{v_i}{\sqrt{b - b_m + v_i + \kappa - \phi \theta (\lambda - \lambda_m)}}.
\]
Here, the incumbent *does* consider both $\kappa$ and $v_i$ in its choices: a larger bias to work at the firm lets the incumbent socialize the minority more without risk of losing those employees, but a more painful socialization compels the incumbent to relax the extent of it.

When facing no other firm’s competition, the incumbent accounts for both the bias and the pain of socialization, but once the threat of entry arises, the incumbent stops. The reason is the change in the employee outside options after a new corporate culture emerges. When alone, the incumbent only competes against the alternatives to working, whose values are fixed. Those outside options effectively anchor the incumbent’s strategies, forcing the incumbent to respond to them and internalize both $\kappa$ and $v_i$.

However, when another firm attempts to enter the market, the outside options of the employees no longer stay fixed, but adjust to the strategy of the new firm. As the first-mover, the incumbent knows that any entrant’s strategy must respond to the incumbent’s choices. So the *incumbent* now becomes the anchor of competition to which the outside options must respond, rather than the other way around. In its best response, the incumbent does not need to internalize the bias or the pain it inflicts on the minority through socialization because it knows the entrant will instead.

Having the best responses, we can also examine how the minority’s pain from socialization compares while working at each firm. Substituting each firm’s socialization strategy into the emotion function gives

$$v_i (n_i^*) = v_i \left( \frac{\theta (\lambda - \lambda_m)}{2} (1 - \phi \theta (\lambda - \lambda_m) + \omega_i) - 1 \right),$$

$$v_e (n_e^*) = v_i (n_i^*) - \kappa.$$

Unequivocally, the minority will bear less pain from socialization at the entrant than at the incumbent. In this sense does the entrant embody the social progress of the time: it makes the socialization process less painful for the minority, in contrast to the incumbent. The amount by which the entrant reduces the pain from socialization depends entirely on the bias $\kappa$. The greater bias to stick with the prevailing environment, the more the entrant must reduce the pain from socialization to encourage the minority to leave that environment behind.

The entrant is the means by which progress in society can penetrate corporate culture. Although the economy is static, we discussed in the definition of the equilibrium that successfully expelling a prevailing corporate culture in reality takes time. Corporate cultural advancement in this economy, therefore, can be thought of as occurring over a period in which the entrant may eventually push out the incumbent.

In equilibrium, the entrant anchors its minority’s emotion function at the value of the incumbent’s. From that position, the entrant reduces the socialization pain by the amount of the bias in an effort to displace the dominate corporate culture. If one envisions such a displacement as a process, where $v_i (n_i^*)$ is the past “state” and $v_e (n_e^*)$ is today’s “state,” the relation between $v_i (n_i^*)$ and $v_e (n_e^*)$ above reveals the process to be Markovian. Corporate
cultures advance to reflect society’s progress in a way that depends only on the current state. Improvement is based on how good or bad things are currently rather than over history. And if the improvement comes, the size of that change ($\kappa$) depends solely on how strong the resistance was to it.

5.7.3 Wage gaps within and between firms

We can express the entrant’s equilibrium wage gap by substituting the best responses in Proposition 2 into the market clearing condition (20):

$$\omega_e = \left( \frac{v_i}{v_e} \right) \omega_i + (1 - \phi \theta (\lambda - \lambda_m)) \left( \frac{v_i - v_e}{v_e} \right) - \frac{2\phi}{\theta (\lambda - \lambda_m)} \left( \frac{\kappa + v_i - v_e}{v_e} \right).$$  \hspace{1cm} (25)

The market clearing wage gap for the entrant adjusts around the incumbent’s fixed wage gap $\omega_i$. If the incumbent sets a larger wage gap, the entrant’s must also increase in order to appeal to majority employees and clear the labor markets. If the pain from socialization $v_i$ is larger at the incumbent, the entrant can get away with a larger wage gap because the minority’s outside option is weaker. However, if the bias to remain with the incumbent $\kappa$ is greater, the entrant’s wage gap must fall in equilibrium in order to entice the minority to exit and join the new corporate culture.

So far we have examined the pay differences between majority and minority employees at the same firm. We can label $\omega_i$ and $\omega_e$ the “within-firm” wage gaps. Alternatively, we can also study the wage gaps of the same type of employee between firms. The “between-firm” wage gaps for the majority and minority, respectively, are determined using the majority and minority indifference conditions in (18) and (19). Those wage gaps are

$$w_i - w_e = \tilde{x}_e - \tilde{x}_i,$$

$$w_{i,m} - w_{e,m} = \tilde{x}_i - \tilde{x}_e + v_i (n_i) - v_e (n_e) - \kappa.$$

Having the best responses of both firms completely specified, we can determine the between-firm wage gaps, which we present in Lemma 4.

**Lemma 4.** *(Between-firm wage gaps)* The difference in majority pay at the incumbent and entrant is

$$w_i - w_e = \left( \frac{\kappa + v_i - v_e}{v_e} \right) \left( \frac{\phi}{\theta (\lambda - \lambda_m)} \right) - \frac{1}{2} \left( \frac{v_i - v_e}{v_e} \right) (1 - \phi \theta (\lambda - \lambda_m) + \omega_i),$$

whereas the difference in minority pay between the two firms is

$$w_{i,m} - w_{e,m} = -(w_i - w_e).$$

The sign of the minority wage gap is always opposite that of the majority. So if the incumbent pays its *majority* more than the entrant pays its majority, the entrant will pay its *minority*
more than the incumbent will pay its minority.

5.7.4 The market entry of social progress

The final step to complete the solution of the game is determining the entry decision. Again, our definition of entry in equilibrium is the entrant coexisting with or displacing the incumbent, which only occurs if \( \pi_i \leq \pi_e \). Substituting the best responses of the two firms from Proposition 2 into the difference \( \pi_i - \pi_e \) generates a function \( P(\omega_i) \) that is quadratic in the incumbent’s (within-firm) wage gap.

When the wage gap \( \omega_i \) is at a value for which \( P(\omega_i) < 0 \), the entrant can earn more profits than the incumbent. In this case, the new corporate culture enters the market and forces the dominant one to exit. Conversely, at values of \( \omega_i \) for which \( P(\omega_i) > 0 \), the entrant’s corporate culture is not profitable enough to enter the market, so it is deterred from displacing the incumbent. Finally, where \( P(\omega_i) = 0 \), profits of the two firms match, so a firm cannot expel or bar the other. The incumbent accommodates the entrant without being forced out, meaning the old and progressive corporate cultures coexist in the market.

Each case represents a distinct kind of equilibrium. We label the three equilibria forced exit \( (P(\omega_i) < 0) \), deterred entry \( (P(\omega_i) > 0) \), and accommodation \( (P(\omega_i) = 0) \). In the next proposition, we discuss the regions of \( \omega_i \) that determine the equilibrium.

Proposition 3. (Forced exit, deterred entry, accommodation equilibrium) A subgame perfect Nash equilibrium to the Stackelberg competition exists and is unique. Moreover, if

\[
1 - \phi \theta (\lambda - \lambda_m) > \frac{2\phi}{\theta (\lambda - \lambda_m)} \left( \frac{\kappa + v_i - v_e}{v_i - v_e} \right),
\]

the quadratic \( P(\omega_i) \) has two roots \( \omega_{i,-} \) and \( \omega_{i,+} \) that are arranged \( \omega_{i,-} < 0 < \omega_{i,+} \). The roots are

\[
\omega_{i,-} = \frac{2\phi}{\theta (\lambda - \lambda_m)} \left( \frac{\kappa + v_i - v_e}{v_i - v_e} \right) - (1 - \phi \theta (\lambda - \lambda_m)),
\]

\[
\omega_{i,+} = \frac{2\phi}{\theta (\lambda - \lambda_m)} \left( \frac{\kappa + v_i - v_e}{v_i + v_e} \right) - (1 - \phi \theta (\lambda - \lambda_m)) \left( \frac{v_i}{v_i + v_e} \right) + (1 + \phi \theta (\lambda - \lambda_m)) \left( \frac{v_e}{v_i + v_e} \right).
\]

The roots separate the three possible equilibria. The equilibrium is forced exit when \( \omega_i < \omega_{i,-} \) or \( \omega_i > \omega_{i,+} \); deterred entry when \( \omega_i \in (\omega_{i,-}, \omega_{i,+}) \); and accommodation when \( \omega_i = \omega_{i,-} \) or \( \omega_i = \omega_{i,+} \).

Figure 8 illustrates the function \( P \) as well as the three regions that separate the equilibria. Two values of the incumbent wage gap, \( \omega_{i,-} \) and \( \omega_{i,+} \), constitute the accommodation equilibrium. At the first value, the incumbent pays its minority more than its majority, whereas at the second, it pays its minority less. At both values, the profit difference between the incumbent and entrant vanishes, so both firms can operate in the market simultaneously.

The proposition reveals that when the magnitude of the incumbent wage gap exceeds either of those two values, forced exit takes place. In a situation of an extreme wage gap at the incumbent—which can be considered as severe inequality in pay between the two groups—a new corporate culture has room to push out the old one. What the proposition also reveals, however,
is that forced exit is not possible and entry is deterred when the incumbent’s wage gap is within a specific range. If there are not large differences in pay between the two groups at the firm, there is no way for the market to adopt the social progress on its own. The prevailing, outdated corporate culture would persist unless some external authority mandated change.

Figure 8: Forced Exit, Deterred Entry, Accommodation Equilibrium

Notes: The figure plots the quadratic function $P(\omega_i) = \pi_i - \pi_e$. Where $P(\omega_i) > 0$ is deterred entry because the potential entrant’s profits cannot eclipse the incumbent’s, so there is no entry. Where $P(\omega_i) < 0$ is forced exit because the new corporate culture is more profitable than the old, so the entrant displaces the incumbent. Finally, the points at which $P(\omega_i) = 0$ are accommodation because the incumbent and entrant are equally profitable, so both coexist in the market.

The intuition for why forced exit occurs at extreme wage gaps is the following. Generally speaking, there are two strategies a firm can choose. The first strategy is to employ more minority employees to increase diversity and choose more socialization to reduce conflict. The second is to employ more majority employees to reduce conflict and choose less socialization while sacrificing diversity. When the wage gap is large and positive, minority employees are much less expensive to hire, so the first strategy is more profitable. In contrast, when the wage gap is large and negative, majority employees are less costly to hire, so the second strategy is more profitable. From Proposition 2, one can observe that the entrant’s employment decision is more responsive to the incumbent’s wage gap, making it better at executing the more profitable strategy when the wage gap is extreme. When the wage gap is narrower, however, the entrant’s advantage is weaker, the two strategies delivery roughly the same profits, and the incumbent
cannot be displaced.

The region $\omega_{i+} - \omega_{i-}$ is the range where the dominant corporate culture can shield itself from displacement. Taking the difference between the two expressions in Proposition 3 gives the size of the range:

$$\omega_{i+} - \omega_{i-} = \frac{2v_e (\theta (\lambda - \lambda_m) (v_i - v_e) - 2\phi (\kappa + v_i - v_e))}{\theta (\lambda - \lambda_m) (v_i^2 - v_e^2)}.$$

We put attention on the effect of the bias $\kappa$ on the range. A greater bias to resist leaving for the entrant surprisingly shrinks the incumbent’s protected range. One might think that a stronger bias of the minority to stick with the incumbent awards the incumbent a wider range to protect itself from being driven out. It is the opposite: the tighter the incumbent’s grip on the minority, the more fragile it is. When the entrant observes a large bias, it competes more aggressively by reducing the minority’s pain from socialization by a greater amount. That competitive pressure weakens the incumbent’s power to sustain a large wage gap. The bias for the incumbent, therefore, regulates both the size of the market’s progress in corporate culture should the change occur, as well as how much of a chance the entrant has at effecting the change.

5.8 A flexible incumbent

So far we have taken the incumbent’s wage gap $\omega_i$ as an object endowed to the firm that is a conspicuous feature of the prevailing corporate culture. And that corporate culture itself was an offspring of an outworn societal culture that has since progressed. The incumbent was firmly committed to maintaining $\omega_i$ at the endowed value irrespective of any threat of entry. We then studied the conditions in which that commitment to $\omega_i$ can either shield the incumbent from being displaced or force its exit.

If the incumbent were more flexible and willing to adjust $\omega_i$ in response to the risk of being pushed out by a potential entrant, however, a moment’s reflection would make one realize that entry would always be deterred. If $P(\omega_i) < 0$ at the incumbent’s current wage gap $\omega_i$, the incumbent would shrink the magnitude of the wage gap just enough to the value $\omega_i'$ so that $P(\omega_i') > 0$. The entrant would be kept out, and there would be no way in this economy for the market by itself to adopt social progress into widespread corporate culture. In this setting the first-mover advantage of the incumbent is most apparent. An outside intervention would be the only means to impose corporate cultural change and better the workplace conditions of the minority.

6 Empirical predictions

The model lends itself to a number of empirical predictions with respect to the shares of minority employees and the wage gaps at firms. All of those objects are observable. Examining the minority shares in Proposition 2 and the wage gaps in (25) and Lemma 4, one can notice that all are affine in the incumbent’s wage gap $\omega_i$. Therefore, an econometrician could fix an
industry and run a univariate linear time-series regression of an entrant’s minority share and wage gaps onto the wage gap of an incumbent and compare the coefficients to the relations predicted by the model. He or she could alternatively run a panel or cross-sectional regression across industries provided that each industry is segregated enough from every other so that firms across industries do not meaningfully compete.

6.1 Choice of groups and firms

The econometrician would inevitably have some freedom in the choice of the minority group as well as the incumbent and the entrant. The model is silent on the exact characteristics of the members of the minority, except that they collectively make up a share of all employees that does not exceed one-half. But that restriction is quite loose. In every firm, any employee could be a minority along enough dimensions. There is only one of each of us after all. In the selection of the majority and minority groups, the econometrician must choose an observable dimension that is broad enough to obtain meaningful shares—such as race, ethnicity, or gender—and the members of each group must reasonably be perceived to have similar personal cultures. Considerable discretion remains. The chosen groups must also match as closely as possible along every other dimension except that which distinguishes their minority or majority membership. For instance, the wage gap between a male and female manager should be compared rather than that of a male custodian and a female manager.

The most important characteristics of the incumbent when trying to identify such a firm within an industry is (1) its existence prior to the emergence of a progressive firm and (2) its treatment of the minority that is considered regressive upon the development of the social progress. The entrant must embrace the social progress insofar as its improved treatment of the chosen minority. It need not be a start-up firm. The entrant could be as old and as large as the incumbent, but simply distinguish itself as adhering to a more progressive corporate culture. The characteristics defining the two kinds of firms rightfully limit the econometrician, but they unfortunately also admit imprecision.

6.2 The predictions

After the selection of an incumbent and entrant and minority and majority groups, the most apparent prediction of the model is the relation between the majority and minority wage gaps between firms in Lemma 4. The two should be opposite in sign and most precisely, equal in magnitude, though the latter condition is less likely to hold empirically given its strictness. No regression needs to be run here, but only a comparison in averages through time or across industries.

Moving onto the minority shares, one can express the equations in Proposition 2 as

\[
1 - \bar{x}_i^* = \beta_{1-\bar{z}_i^*,0} + \beta_{1-\bar{z}_i^*,1}\omega_i,
\]

\[
1 - \bar{x}_e^* = \beta_{1-\bar{z}_e^*,0} + \beta_{1-\bar{z}_e^*,1}\omega_i.
\]
With this relation, the following sets of parameters can be identified from linear regressions of the minority shares on the incumbent wage gap:

\[ 1 - \phi \theta (\lambda - \lambda_m) = \frac{\beta_{1,-x^*_e,0}}{\beta_{1,-x^*_e,1}}, \]
\[ \frac{v_i}{v_e} = \frac{\beta_{1,-x^*_e,1}}{\beta_{1,-x^*_i,1}}. \]

Because of the sign restrictions on those sets of parameters, two empirical predictions of the model are

\[ \frac{\beta_{1,-x^*_e,0}}{\beta_{1,-x^*_e,1}} \in (0, 1), \quad (26) \]
\[ \frac{\beta_{1,-x^*_e,1}}{\beta_{1,-x^*_i,1}} > 1. \quad (27) \]

The first ratio (26) is a measure of the marginal cost of cultural conflict. A larger cultural gap between the majority and minority would imply a ratio closer to zero. The second ratio (27) of the two loadings from the regressions is a measure of the relative mistreatment of the minority by the incumbent. A greater ratio would imply a larger relative mistreatment.

The remaining wage gaps to examine are the entrant wage gap \( \omega_e \) in (25) and the majority wage gap between the two firms \( w_i - w_e \) in Lemma 4. The two linear relations here are

\[ \omega_e = \beta_{\omega_e,0} + \beta_{\omega_e,1} \omega_i, \]
\[ w_i - w_e = \beta_{w_i-w_e,0} + \beta_{w_i-w_e,1} \omega_i. \]

From (25), \( \beta_{\omega_e,1} = \frac{w}{v_e} \), making this coefficient another measure, besides (26), of the relative minority mistreatment between the incumbent and entrant. The wage gap equations then imply the three predictions

\[ \beta_{\omega_e,1} = \frac{\beta_{1,-x^*_e,1}}{\beta_{1,-x^*_i,1}}, \quad (28) \]
\[ \beta_{\omega_e,1} > 1, \quad (29) \]
\[ \beta_{w_i-w_e,1} < 0, \quad (30) \]

where the last prediction arises because \( \beta_{w_i-w_e,1} = -\frac{1}{2} \left( \frac{v_i-v_e}{v_e} \right) \). The relation in (26) is rather strict, so a more realistic one is that the loading \( \beta_{\omega_e,1} \) from the wage gap regression matches signs with the ratio of the minority share loadings \( \frac{\beta_{1,-x^*_e,1}}{\beta_{1,-x^*_i,1}} \).

### 7 Perfect Competition

The incumbent and entrant have so far differed in four ways: (1) the order of their decisions, (2) the incumbent’s commitment to its wage gap \( \omega_i \) but the entrant’s willingness to adjust its
wage gap to competition, (3) their different treatments of the minority that were represented by different emotion functions \(v_k(n)\) at each firm, and (4) the incumbent’s advantage of the minority’s bias \(\kappa\) to stay.

In this section, we examine the case in which there are no differences at all between the firms. No longer is one an incumbent or entrant. Both are actively competing in the market simultaneously. Both firms need to adjust their wages in response to competition in the majority and minority labor markets. The emotion function is also the same at both firms, with \(v_k = v\). Finally, any employee may leave one firm for another, and none has a bias to remain with either firm.

We consider this type of environment one of perfect competition because the firms are identical, there could be many of them, labor is freely mobile, and both firms and labor will be price-takers in the labor markets. The next proposition describes the outcome.

**Proposition 4.** (Equilibrium, perfect competition) When the two firms are identical and engage in perfect competition, the optimal employment and socialization decisions are the same between them and are

\[
1 - \tilde{x} = \frac{1}{2} (1 - \phi \theta (\lambda - \lambda_m) + \omega),
\]

\[
n = 1 - \sqrt{\frac{2\phi}{\theta (\lambda - \lambda_m) (1 - \phi \theta (\lambda - \lambda_m) + \omega)}},
\]

where \(\omega\) is the within-firm wage gap common to both firms. Profits of the two firms are identical, both majority and minority between-firm wage gaps are zero, and \(\omega\) must satisfy \(1 - \phi \theta (\lambda - \lambda_m) + \omega \in [0, 1)\).

Similar factors that influence the corporate culture decisions in the Stackelberg game do so in perfect competition as well. More costly socialization \(\phi\) reduces \(n\) and also the share of the minority. Greater personal cultural conflict \((\lambda - \lambda_m)\) between the majority and minority also shrinks the minority share. With fewer members of the minority, there is less incentive for a firm to engage in socialization, so that reduces as well.

Whereas earlier, the majority and minority between firms earned different wages, here that is not the case. Both types of labor expect the same homophily and socialization at either firm, so both firms must match wages for each type in order to retain employees. The majority must earn at one firm what it would earn at the other firm, and likewise for the minority. Free labor mobility and perfect competition are powerful enough forces to zero out those two wage gaps.

As for the within-firm wage gap \(\omega\), perfect competition guarantees that the difference in pay between a majority and minority type is the same across firms, but does not guarantee the difference is zero. In fact, multiple within-firm wage gaps may constitute an equilibrium with perfect competition so long as each one meets the condition provided in the proposition. Interestingly, no gap in pay between the majority and minority within a firm is an equilibrium, despite the two types contributing differently to marginal revenue. But so too are equilibria
with positive or negative wage gaps.

8 Existing theory on corporate culture

The theoretical literature on corporate culture in economics is reviewed in a thorough and enjoyable survey by Benjamin Hermalin (Hermalin (2001)). We define corporate culture mathematically in a different way than the previous literature and answer questions that have not yet been addressed: why does corporate cultural variety exist and when does corporate culture adapt to social progress?

Some of the existing work uses a setting with repeated games and treats corporate culture as one of multiple equilibria. Kreps (1990) considers corporate culture as principles a firm has a reputation for applying and communicating when unforeseen contingencies occur—events that are difficult or impossible to anticipate and contract on ex ante.

Crémer (1993) takes a different approach. He thinks in terms of teams in which there are no agency problems. But employees have limited capacity to process and transmit information. A growing stock of that information is common and lives on beyond the lives of individual employees. This common stock of knowledge is what Crémer calls corporate culture.

Van den Steen (2010b) considers corporate culture as shared beliefs (priors). Like us, he studies the costs and benefits of employee homogeneity. On the one hand, strong homogeneity makes a firm efficient in carrying out its tasks, featuring more delegation and less monitoring. However, the cost of that homogeneity is less experimentation and less information collection. He focuses on mergers and acquisitions and studies the change in firm behavior when two firms having incongruent beliefs combine. Van den Steen (2010a) shows how shared beliefs arise endogenously within a firm through screening, self-sorting, and joint learning.

Song and Thakor (Forthcoming (2018) study bank culture. In their setting, bank culture offers a way to improve upon explicit incentive contracting. A bank’s culture is the behavior it prefers loan officers to follow when extending credit: issuing loans indiscriminately to increase growth or exerting effort judiciously to discern creditworthy borrowers. Banks reinforce their cultures by penalizing deviant behavior. Common knowledge about a bank’s culture helps match a bank with loan officers who share aligned beliefs about default risk, which moves the loan officer’s behavior closer to the first-best.

Management and organizational behavior theories of corporate culture are nicely summarized in Gordon and DiTomaso (1992). An early example is Schein (1983), who considers corporate culture as “provid[ing] group members with a way of giving meaning to their daily lives, setting guidelines and rules for how to behave, and, most important, reducing and containing the anxiety of dealing with an unpredictable environment.” Martin (1992) categorizes an extensive part of the organizational behavior literature as interpreting corporate culture as shared beliefs and values.
9 Conclusion

With this paper we hope to provide fresh insight on why firms can differ vastly in their corporate cultures and whether the market can discipline itself to reform the treatment of minority groups at the workplace. To do so we introduce a new mathematical construction of culture as a mapping between values, norms, behaviors, symbols, etc. and weights that members of a group put on those elements in terms of importance.

We consider corporate culture as a deliberate choice of a firm, one that optimally combines the different cultures of its employees to improve diversity but also avoid cultural conflict. We show that such a trade-off can lead otherwise identical firms to choose distinct corporate cultures. Some firms elect greater diversity while inculcating the minority in the majority's culture. Other firms decide on less diversity without disturbing as much the culture of the minority.

Whether firms on their own can adapt to progressive development in society depends on the power of the incumbent corporate culture, as measured by the difference in pay between the majority and minority. Extreme differences in wages give room for an emergent, progressive corporate culture to displace a regressive, outdated one. In contrast, a narrower wage gap insulates the incumbent, thereby straining corporate culture to advance without forced intervention. The more entrenched an antiquated corporate culture, the more vulnerable it is to removal, as progressive firms compete more aggressively to change the minds of the minority to leave. And finally, this fiercer competition from a progressive outsider implies that a stronger bias of minority employees to remain with the status quo would lead to a larger improvement in how they are treated should the outsider succeed.

Surely, the way we model social progress is simple and incomplete. In history and the world, the process is slow and imperfect. At times it may seem as if society has advanced in its treatment of certain groups, only to revert to sad, sick behavior not long after. Just the same with progress in corporate culture. Deeply rooted tendencies of a firm that may have grown out of the beliefs of a founder and persist do not change rapidly. If corporate culture changes after pressure from the market, it does so in fits and starts.

Our model treats social progress as happening in advance of firm improvement, though at times society may lag firms. When North Carolina required transgender people to use the bathroom corresponding with their genders at birth, many corporations responded quickly by aborting plans to start or expand in the state, canceling conventions, concerts, and sporting events. North Carolina then modified its law (Bissell (2017)). How a firm positions itself to influence the society in which it attempts to optimize some objective is an issue worth studying.

We aim to draw more attention to corporate culture within the research area broadly interpreted as “theory of the firm.” That subject has a long and rich history dating as early as Coase (1937). Since then economists have advanced substantially, with many significant contributions sprouting from the field of organizational economics. See Gibbons and Roberts (2012) for a handbook of surveys. If corporate culture is part of the nature of a firm, then
it seems right to study it formally and deeply to better understand how it can shape firm
decisions, whether operational or financial. Here we have put attention on how firms respond
to changes in the society in which they do business. We hope we have provided a framework
handy enough to think about that continuous process and many others related to the firm.
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48


10 Appendix A: Proofs (For Online Publication)

10.1 Proof of Proposition 1

To demonstrate the existence of a corporate cultural variety equilibrium, we let \( n^* (\Lambda) \) be the set of optimal extents of socialization \( n \) that solve the firm problem in (10) when the cultural conflict \((\lambda - \lambda_m)^2 = \Lambda\) and the other parameters \( b - b_m \) and \( \theta \) are constants. For variety in equilibrium, it must be that \( n^* (\Lambda) \) is not single-valued. The theorem of the maximum implies that \( n^* (\Lambda) \) is an upper hemicontinuous correspondence. If \( n^* (\Lambda) \) is single-valued, it must then be a continuous function. To prove variety, we need only provide a condition under which \( n^* \) is not a continuous function.

Every element \( n \in n^* (\Lambda) \) must satisfy the necessary first-order condition (14), repeated here in rearranged form:

\[
(b - b_m - v (n))^2 - \theta^2 \Lambda (1 - n)^3 \phi' (n) = 0.
\]

Let \( f (\Lambda, n) \) be the continuous function on the left-hand-side of the above equation defined over the elements of \( n^* \). To prove that \( n^* \) is not a continuous function, we rely on a symmetric implicit function theorem proven in Dontchev and Rockafellar (2014) (Theorem 1B.8). Under Assumptions 1 and 2, \( f \) is continuously differentiable. Provided \( \frac{\partial f}{\partial \Lambda} \neq 0 \) at a point \((\bar{\Lambda}, \bar{n})\) for \( \bar{n} \in n^* \), there exists a single-valued function around \( \bar{\Lambda} \) for \( \bar{n} \) if and only if \( \frac{\partial f}{\partial n} (\bar{\Lambda}, \bar{n}) \neq 0 \). Therefore, if \( \frac{\partial f}{\partial n} (\bar{\Lambda}, \bar{n}) = 0 \), there must exist at least one \( \Lambda \) where there does not exist a single-valued function. This in turn implies that \( n^* \) must not be a continuous function and hence not single-valued.

The partial derivative of \( f \) with respect to the parameter \( \Lambda \) is

\[
\frac{\partial f}{\partial \Lambda} = -\theta^2 (1 - n)^3 \phi' (n).
\]

By Assumption 1, \( \phi' > 0 \), making \( \frac{\partial f}{\partial \Lambda} (\bar{\Lambda}, \bar{n}) \neq 0 \). The partial derivative \( \frac{\partial f}{\partial n} \) is

\[
\frac{\partial f}{\partial n} = -\left[ 2v' (n) (b - b_m - v (n)) + \theta^2 \Lambda \left( (1 - n)^3 \phi'' (n) - 3 (1 - n)^2 \phi' (n) \right) \right].
\]

Setting \( \frac{\partial f}{\partial n} \) equal to zero and rearranging gives the condition

\[
2v' (n) (b - b_m - v (n)) = \theta^2 \Lambda (1 - n)^2 (3\phi' (n) - (1 - n) \phi'' (n)).
\]

If the above relation holds for \( \bar{n} \in n^* \) when \( \Lambda = \bar{\Lambda} \), we have what we need to show that \( n^* \) is not single-valued. In the proposition we impose the stronger condition that the relation holds for any \( n \in (0, 1) \) when \( \Lambda = \bar{\Lambda} \). To complete the construction of a corporate cultural variety equilibrium, we note that market clearing in the labor markets is confirmed by construction given (14).
To prove the relation between multiple optimal firm decisions, suppose without loss of generality that \( \{n_1, n_2\} \in n^* (\Lambda) \). Let \( n_1 < n_2 \). Because \( n^* (\Lambda) \) is multivalued, an implication of the necessary first order condition (15) is that \( \tilde{x}^* (\Lambda) \) is also multi-valued. To complete the proof of the proposition, we need only show that \( 1 - \tilde{x}_1 < 1 - \tilde{x}_2 \). Because \( v(n) \leq b - b_m \), (14) can be expressed as

\[
b - b_m - v(n) = \sqrt{\theta^2 (\lambda - \lambda_m)^2 \phi'(n) (1 - n)^3}.
\]

Substituting this expression into (15) gives

\[
1 - \tilde{x} = \sqrt{\frac{\phi'(n)}{\theta^2 (\lambda - \lambda_m)^2 (1 - n)}}.
\]

Because \( \phi'(n) \) is strictly increasing, \( n_1 < n_2 \) implies \( 1 - \tilde{x}_1 < 1 - \tilde{x}_2 \).

### 10.2 Proof of Lemma 1

Substitute the optimality condition for \( 1 - \tilde{x} \) in (15) into the market clearing condition (11) to express the wage gap as

\[
\omega(n) = (b - b_m - v(n)) \left( 1 + \frac{2}{\theta^2 \Lambda (1 - n)^2} \right) - 1,
\]

where \( \Lambda \equiv (\lambda - \lambda_m)^2 \). Suppose \( n_1^* < n_2^* \) are two optimal extents of socialization. A firm that chooses \( n_2^* \) instead of \( n_1^* \) will feature a larger wage gap if

\[
\omega(n_2^*) - \omega(n_1^*) > 0.
\]

Substituting the wage gap into the inequality gives

\[
\frac{\theta^2}{2} \Lambda (v(n_1^*) - v(n_2^*)) + \frac{v(n_1^*)}{(1 - n_1^*)^2} - \frac{v(n_2^*)}{(1 - n_2^*)^2} > \frac{b - b_m - v(n_1^*)}{(1 - n_1^*)^2} - \frac{b - b_m}{(1 - n_2^*)^2}.
\]

Rearranging gives

\[
\frac{\theta^2}{2} \Lambda (v(n_1^*) - v(n_2^*)) + \frac{v(n_1^*)}{(1 - n_1^*)^2} - \frac{v(n_2^*)}{(1 - n_2^*)^2} > \frac{b - b_m - v(n_1^*)}{(1 - n_1^*)^2} - \frac{b - b_m}{(1 - n_2^*)^2}.
\]

Because \( n_1^* < n_2^* \), the right-hand-side of the above inequality is negative. Therefore, the inequality is satisfied if

\[
\frac{\theta^2}{2} \Lambda (v(n_1^*) - v(n_2^*)) + \frac{v(n_1^*)}{(1 - n_1^*)^2} - \frac{v(n_2^*)}{(1 - n_2^*)^2} > 0.
\]

Some further minor manipulation delivers the condition in the lemma.
10.3 Proof of Lemma 2

Substitute the specific function for $\phi$ into the entrant’s best response (implicit) function (21) to get

$$
(\omega_i + (2\tilde{x}_i - 1) + v_i (n_i) - v_e (n_e) - \kappa)^2 = \phi^2 \theta^2 (\lambda - \lambda_m)^2,
$$

which implies

$$
\omega_i + (2\tilde{x}_i - 1) + v_i (n_i) - v_e (n_e) - \kappa = \pm \phi \theta (\lambda - \lambda_m).
$$

(31)

Substitute the left-hand-side of this last equation into the entrant’s other best response function (22) to get

$$
\frac{\pm \phi}{\theta (\lambda - \lambda_m) (1 - n_i)^2} = 1 - \tilde{x}_e.
$$

(32)

By Assumption 3, $\phi (n) > 0$, making $\phi > 0$. Hence, the entrant’s minority share is positive. Next, rearrange (31) to get

$$
v_e (n_e) = \omega_i + (2\tilde{x}_i - 1) + v_i (n_i) - \kappa - \phi \theta (\lambda - \lambda_m).
$$

Now substitute the specific function for $v_e$ into this last equation and rearrange to get

$$
n_e = 1 - \sqrt{\frac{v_e}{\omega_i + (2\tilde{x}_i - 1) + v_i (n_i) + v_e - \kappa - \phi \theta (\lambda - \lambda_m)}},
$$

which matches the expression for $n_e$ in Lemma 2. Finally, substitute $n_e$ into (32) to get

$$
1 - \tilde{x}_e = \frac{\phi (\omega_i + (2\tilde{x}_i - 1) + v_i (n_i) + v_e - \kappa - \phi \theta (\lambda - \lambda_m))}{v_e \theta (\lambda - \lambda_m)},
$$

which also matches the Lemma.

10.4 Proof of Lemma 3

The optimality conditions of the incumbent that determine its best response are

$$
\frac{(\omega_e + (2\tilde{x}_e - 1) + v_e (n_e) + \kappa - v_i (n_i))^2}{\theta^2 (\lambda - \lambda_m)^2 (1 - n_i)^3} = \phi' (n_i),
$$

$$
\frac{\omega_e + (2\tilde{x}_e - 1) + v_e (n_e) + \kappa - v_i (n_i)}{\theta^2 (\lambda - \lambda_m)^2 (1 - n_i)^2} = 1 - \tilde{x}_i.
$$

Substituting the functional forms of $\phi$ and $v_k$ and rearranging delivers the two conditions:

$$
v_i (n_i) = \omega_e + (2\tilde{x}_e - 1) + v_e (n_e) + \kappa - \phi \theta (\lambda - \lambda_m),
$$

(33)

$$
1 - \tilde{x}_i = \frac{\phi}{\theta (\lambda - \lambda_m) (1 - n_i)^2}.
$$

(34)
From Lemma 2, the optimal conditions for the entrant are

\[ v_e(n_e) = \omega_i + (2\bar{x}_i - 1) + v_i(n_i) - \kappa - \phi \theta (\lambda - \lambda_m), \tag{35} \]

\[ 1 - \bar{x}_e = \frac{\phi (\omega_i + (2\bar{x}_i - 1) + v_i(n_i) + v_e - \kappa - \phi \theta (\lambda - \lambda_m))}{v_e \theta (\lambda - \lambda_m)}. \tag{36} \]

Substituting (35) into (33) and solving for the incumbent majority share \(1 - \bar{x}_i\) gives

\[ 1 - \bar{x}_i = \bar{x}_e + \frac{1}{2} (\omega_i + \omega_e) - \phi \theta (\lambda - \lambda_m). \]

Next, substitute (36) to obtain

\[ 1 - \bar{x}_i = L(v_i(n_i)), \]

where \(L\) is a function of the emotion function at the incumbent \(v_i(n_i)\). We set this equation equal to (34), but first we must express (34) in terms of \(v_i(n_i)\). Some manipulation gives

\[ 1 - \bar{x}_i = \frac{\phi (v_i(n_i) + v_i)}{v_i \theta (\lambda - \lambda_m)}. \tag{37} \]

Equating the right-and-side of (37) to \(L\) and solving for \(n_i\) gives

\[ n_i = 1 - \frac{\sqrt{\phi}}{\sqrt{\theta (\lambda - \lambda_m)}} \sqrt{\frac{R}{2 (v_e \theta (\lambda - \lambda_m) - \phi) + (S + T)}}, \]

where

\[ R \equiv 2 \theta (v_i + v_e) (\lambda - \lambda_m) - 4 \phi, \]

\[ S \equiv 2 \phi (v_i - v_e - \omega_i + \kappa), \]

\[ T \equiv \theta (\lambda - \lambda_m) (2 \phi^2 + v_e (\omega_i + \omega_e) - 2 \phi v_e \theta (\lambda - \lambda_m)). \]

The incumbent’s optimal minority share is found by substituting \(n_i\) into (34). Doing so yields the expression in the lemma.

**10.5 Proof of Proposition 2**

The equilibrium best responses of both the incumbent and entrant are determined by substituting the equilibrium entrant wage gap in (25) into the incumbent’s best responses in Lemma 3, then substituting those strategies into the entrant’s best response in Lemma 2. Extensive algebra delivers the expressions in the proposition. By Assumption 6, \(n_i, n_e \in [0, 1]\) and \(1 - \bar{x}_i, 1 - \bar{x}_e \in \left[0, \frac{1}{2}\right]\).

**10.6 Proof of Lemma 4**

The expressions in the lemma are derived by substituting the equilibrium best responses of the firms into the definitions of the between-firm wage gaps and performing some extensive
algebra. Adding the two wage gaps reveals

\[(w_i - w_e) + (w_{i,m} - w_{e,m}) = 0,\]

which is how we express the minority wage gap in the lemma.

10.7 Proof of Proposition 3

The entry decision of the subgame perfect Nash equilibrium is determined by the sign of the difference in profit functions of the two firms. The profit function of each firm is

\[\pi_k = A + \tilde{x}_k (1 - \tilde{x}_k) - \frac{\theta^2}{2} \left( \lambda - \tilde{\lambda}_k \right)^2 - \phi \left( n_k \right) - w_k \tilde{x}_k - w_{k,m} (1 - \tilde{x}_k),\]

for \( k \in \{i, e\} \). Some algebra shows

\[\left( \lambda - \tilde{\lambda}_k \right) = \left( \lambda - \lambda_m \right) (1 - \tilde{n}_k) (1 - \tilde{x}_k).\]

Substituting \( \phi \left( n_k \right) = \frac{\phi}{2} \left( \frac{1}{(1 - n_k)^2} - 1 \right) \) and taking the difference \( \pi_i - \pi_e \) gives

\[\pi_i - \pi_e = (\tilde{x}_i (1 - \tilde{x}_i) - \tilde{x}_e (1 - \tilde{x}_e)) - \frac{\theta^2}{2} \left( \lambda - \lambda_m \right)^2 \left( (1 - n_i)^2 (1 - \tilde{x}_i)^2 - (1 - n_e)^2 (1 - \tilde{x}_e)^2 \right) - \phi \left( n_e \right) - (w_{i,m} - w_{e,m}).\]

Next, substitute the minority wage gap \( w_{i,m} - w_{e,m} = \tilde{x}_i - \tilde{x}_e + v_i \left( n_i \right) - v_e \left( n_e \right) - \kappa \) and use the emotion function \( v_k \left( n_k \right) = v_k \left( \frac{1}{(1 - n_k)^2} - 1 \right) \) to get

\[\pi_i - \pi_e = \tilde{x}_e^2 - \tilde{x}_i^2 - \frac{\theta^2}{2} (\lambda - \lambda_m)^2 \left( (1 - n_i)^2 (1 - \tilde{x}_i)^2 - (1 - n_e)^2 (1 - \tilde{x}_e)^2 \right) - \phi \left( n_e \right) - (w_{i,m} - w_{e,m}) + (v_i - v_e) - \kappa.

After substituting the best responses from Proposition 2 and the entrant wage gap in (25) and doing some algebra, the difference in profit functions becomes

\[\pi_i - \pi_e = P \left( \omega_i \right).\]

The function \( P \left( \omega_i \right) \) is quadratic in the incumbent’s wage gap and is

\[P \left( \omega_i \right) = a\omega_i^2 + b\omega_i + c,\]
where the coefficients are
\[
a = -\frac{v_1^2 - v_e^2}{4v_e^2},
\]
\[
b = \frac{\phi\theta^2 (\lambda - \lambda_m)^2 (v_1^2 - v_e^2) - \theta (\lambda - \lambda_m) \cdot v_i (v_i - v_e) + 2\phi v_i (\kappa + v_i - v_e)}{2\theta (\lambda - \lambda_m) v_e^2},
\]
\[
c = -\frac{c_1 c_2 (v_1^2 - v_e^2)}{4\theta^2 (\lambda - \lambda_m)^2 v_e^2 (v_i + v_e) (v_i - v_e)},
\]
with
\[
c_1 = 2\phi (\kappa + v_i - v_e) - \theta (\lambda - \lambda_m) (v_i - v_e) + \phi\theta^2 (\lambda - \lambda_m)^2 (v_i - v_e),
\]
\[
c_2 = 2\phi (\kappa + v_i - v_e) - \theta (\lambda - \lambda_m) (v_i - v_e) + \phi\theta^2 (\lambda - \lambda_m)^2 (v_i - v_e).
\]

Because the function \(P(\omega_i)\) is continuous and the value \(P(\omega_i)\) is unique for each \(\omega_i\), the equilibrium of the Stackelberg game exists and is unique. The leading coefficient \(a\) is negative, so the parabola is concave down. The discriminant of the quadratic is
\[
\Delta = \frac{(2\phi - \lambda \theta)(v_i - v_e) + 2\phi^2}{4\lambda^2 \theta^2 v_e^2} > 0,
\]
so the root(s) of \(P(\omega_i)\) are real. From the quadratic formula, the roots of \(P(\omega_i)\) are
\[
\omega_{i,-} = \frac{2\phi}{\theta (\lambda - \lambda_m)} \left( \frac{\kappa + v_i - v_e}{v_i - v_e} \right) - (1 - \phi \theta (\lambda - \lambda_m)),
\]
\[
\omega_{i,+} = \frac{2\phi}{\theta (\lambda - \lambda_m)} \left( \frac{\kappa + v_i - v_e}{v_i + v_e} \right) - (1 - \phi \theta (\lambda - \lambda_m)) \left( \frac{v_i}{v_i + v_e} \right) + (1 + \phi \theta (\lambda - \lambda_m)) \left( \frac{v_e}{v_i + v_e} \right).
\]
The root \(\omega_{i,-}\) is negative if
\[
1 - \phi \theta (\lambda - \lambda_m) > \frac{2\phi}{\theta (\lambda - \lambda_m)} \left( \frac{\kappa + v_i - v_e}{v_i - v_e} \right).
\]
The root \(\omega_{i,+}\) is positive if
\[
1 - \phi \theta (\lambda - \lambda_m) < \frac{v_e}{v_i} \cdot \frac{2\phi}{\theta (\lambda - \lambda_m)} \left( \frac{\kappa + v_i - v_e}{v_i} \right) + \phi \theta (\lambda - \lambda_m) \left( \frac{v_e}{v_i} \right).
\]
The left-hand-side of (39) is strictly less than one, and the right-hand-side strictly exceeds
\[
\frac{\kappa + v_i - v_e}{v_i} \cdot \frac{2\phi}{\theta (\lambda - \lambda_m)} \left( \frac{\kappa + v_i - v_e}{v_i} \right).
\]
Under Assumption 6, \(\omega_{i,+} > 0\). Using this reasoning, one can also show that \(c_1 > 0\) and \(c_2 < 0\), making \(c > 0\). Therefore, if (38) holds, the roots of \(P(\omega_i)\) are arranged \(\omega_{i,-} < 0 < \omega_{i,+}\).
10.8 Proof of Proposition 4

The perfect competition equilibrium can be solved using the analysis that was done for the Stackelberg game. Just let one firm be called the “incumbent” and the other the “entrant” and let the decisions be made simultaneously. The within- and between-firm wage gaps are kept distinct for the moment then solved for. From the proof of Proposition 3, the difference in profit functions \( \pi_i - \pi_e \) is

\[
\pi_i - \pi_e \equiv \bar{x}_e^2 - \bar{x}_i^2 - \frac{\theta^2}{2} \left( \lambda - \lambda_m \right)^2 \left( (1 - n_i)^2 (1 - \bar{x}_i)^2 - (1 - n_e)^2 (1 - \bar{x}_e)^2 \right)
- \frac{\phi^2}{2} \left( \frac{1}{(1 - n_i)^2} - \frac{1}{(1 - n_e)^2} \right) - (\omega_i \bar{x}_i - \omega_e \bar{x}_e) + v \left( \frac{1}{(1 - n_i)^2} - \frac{1}{(1 - n_e)^2} \right).
\]

Each firm is solving the same problem, so will have identical optimal solutions. Applying this fact to the between-firm wage gaps in Lemma 4 and the “entrant’s” wage gap in (25), while also using \( v_i = v_e = v \) and \( \kappa = 0 \), one can see that the majority and minority between-firm wage gaps are zero and the two within-firm wage gaps \( \omega_i = \omega_e = \omega \). With these results, one can observe from the difference in profit functions that \( \pi_i - \pi_e \equiv 0 \) for any \( \omega \). Therefore, the equilibrium is not unique. Nevertheless, any \( \omega \) that constitutes an equilibrium must satisfy the condition in the proposition. That condition is determined by setting \( v_i = v_e = v \) and \( \kappa = 0 \) in Assumption 6.

11 Appendix B: Exact Corporate Cultural Conflict (For Online Publication)

The exact corporate cultural conflict function is

\[
\delta (\bar{x}, n) = \bar{\lambda} + \lambda - 2
\]

where \( \bar{\lambda} = \bar{x} \lambda + (1 - \bar{x}) \left( n \lambda + (1 - n) \lambda_m \right) \). The firm problem using \( \delta \) is

\[
\max_{\{\bar{x}, n\}} A + \bar{x} (1 - \bar{x}) - \theta^2 \delta (\bar{x}, n) - \phi (n) - w \bar{x} - w_m (1 - \bar{x})
\]

The two first order conditions are

\[
[\bar{x}] : \theta^2 \left( \lambda - \lambda_m \right) (1 - n) \left( \frac{\lambda}{\lambda^2} - \frac{1}{\lambda} \right) = \omega + (2 \bar{x} - 1), \quad (40)
\]

\[
[n] : \theta^2 \left( \lambda - \lambda_m \right) (1 - \bar{x}) \left( \frac{\lambda}{\lambda^2} - \frac{1}{\lambda} \right) = \phi' (n).
\]

When \( \lambda > \lambda_m \), the weighted average \( \bar{\lambda} \) of the exponential cultural parameters \( \lambda \) and \( \lambda_m \) is decreasing in the minority share \( 1 - \bar{x} \). Therefore, the marginal benefit to socialization (the left-hand-side of (41)) is increasing in the minority share. This relation means the extent of socialization and the minority share are complements, just as they are in the main text. And
that complementarity has the capacity to generate a variety of corporate cultures, depending on the behavior of the emotion function $v(n)$.

The wage gap $\omega$ that clears both the majority and minority labor markets again is

$$\omega = (b - b_m) - (2\bar{x} - 1) - v(n).$$

Substituting $\omega$ into (40) and then solving for the minority share gives:

$$1 - \bar{x} = \frac{\lambda(\lambda - \lambda_m)(1 - n) - \tau_1}{(\lambda - \lambda_m)^2(1 - n)^2},$$

where

$$\tau_1 = \sqrt{\frac{\theta^2 (\lambda - \lambda_m)^3 (1 - n)^3}{\lambda ((b - b_m) - v(n)) + \theta^2 (\lambda - \lambda_m)(1 - n)}}.$$  

Substituting the expression for $1 - \bar{x}$ into (41) delivers an equation exclusively in terms of $n$ that pins down the optimal extent of socialization. That equation is

$$\frac{\lambda((b - b_m) - v(n))(\lambda - \lambda_m)(1 - n) - \tau_1}{(\lambda - \lambda_m)^2(1 - n)^3} = \phi'(n).$$

The marginal benefit of socialization is on the left-hand-side. Just as in the main text, the properties of $v(n)$ will influence the behavior of the marginal benefit curve and determine whether corporate cultural variety can emerge in equilibrium.