AGENCY IN INTANGIBLES

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Abstract

I argue that intangible assets promote agency conflicts between outside investors and inside specialists. I build a microfoundation for why highly intangible firms underinvest despite high profitability and valuations—a challenge for standard theories. This and several other model predictions are supported in the data. I also study trends in investment, profitability, valuations, compensation, idiosyncratic risk, and financing and conclude that the rise of intangibles have likely aggravated agency problems.

Understanding intangible capital is increasingly vital. Yet how much can we truly understand? Intangibles are valued indirectly and hard to quantify. And their development is at the hands of specialists who may possess their own private interest. In this paper I embed uncertainty into the dynamics of firms' intangible capital and show that this agency friction can account for puzzling features of the data.

To fix ideas, consider a firm's optimal choice for *physical* investment:

$$c'(i) = q. \tag{1}$$

Under an increasing and convex adjustment cost, $c(\cdot)$, the equation positively relates the investment rate $i = I/K$ to the level of marginal q. The right side is unobserved and influenced by many factors, including the firm's intangibility.

Complementarity of physical and intangible capital in production implies that average Q, which is observed, should still positively predict investment. The reasoning is straightforward and agrees with Tobin's original intuition: events that make firms valuable and raise Q tend to make them good places to invest. This is a tenet of investment theory, and an outcome of virtually all firm investment models in economics and finance.

I show empirically that this tenet is violated. Average Q's rise with intangibility, yet physical investment rates remain flat or even fall. This pattern holds in panel data, nonparametric portfolio sorts, and macroeconomic trends. It suggests that to better understand intangible capital, we need to go beyond traditional theory.

I begin in Section I by augmenting the standard firm investment model to distinguish physical capital, K, from intangible capital, N, by having the latter (i) not be directly observable to investors (principals) and (ii) be controlled by specialists (agents) who have their own private interest. Investors can only infer what has happened to intangibles through other observable channels, which seems realistic. They write a contract based on observables to enforce the appropriate action because specialists could shirk unknowingly to investors for private benefits.

Given the history of profits, the optimal contract prescribes agents' compensation, termination, and the firm's physical investment. Common to the dynamic contracting literature, agents are rewarded following good historical performance with higher compensation, effectively granting them more ownership of the firm and lessening the agency conflict, and are terminated following serial bad performance. Greater profitability in my model is de-

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livered, however, with higher intangibility, $n = N/K$, so in contrast to standard predictions the conflict does not necessarily diminish following a good history.

In the presence of agency conflicts, (1) still holds but does so from *investors' perspective*. In particular, marginal q now accounts for the marginal cost of compensating agents as physical investment shrinks their effective ownership. The incentive-compatible contract adjusts this marginal cost to minimize agency frictions and, more generally, dynamically smooths this cost, increasing agents' rents and aligning incentives in states in which the incentive problem is more costly. The desire for smoothing makes large rates of physical investment unappealing, especially when intangibility is high and agency conflicts severe.

As a result, profitability, net of a languid response in investment, must increase with intangibility. Thus, I provide a novel microfoundation for the great profitability, lofty valuations, yet puzzling small investment rates of highly intangible firms.

I describe how first-best investment theory will struggle to generate this empirical pattern in Section II. I then flesh out properties of the solution to the agency model and discuss extensions of it. To the best of my knowledge, my numerical solution method is a meaningful technical contribution that can be used to study many other economic problems—I elaborate in Appendix A. Finally, I characterize the stationary distribution of firms which allows for entry and exit that I use to analyze the data in the following sections.

In Section III I first compile data sources and quantify intangibility with a measure of organizational capital used in the accounting and finance literature. I then calibrate the model and in particular the flow of specialists' private benefits. In my model, the optimal contract endogenizes the sensitivity, β_t, of agents' compensation to realizations of intangibility. Specifically, β_t is proportional to the flow of private benefits over the quantity of intangible capital. New to the literature, I specify the flow as a function of intangibility and estimate its functional form in the data. I find the estimated form is consistent with the model's predictions for the form of the optimal contract.

I then analyze the model in Section IV and compare its predictions with those in the data. I confirm that the agency model can quantitatively capture the patterns of investment, valuations, and profitability in portfolio sorts, and provide evidence of intangibility as acting as a wedge between the interests of insiders and outsiders with panel regressions. I show the first-best model finds it challenging, if not impossible, even under various specifications of adjustment costs and allowing for the discounting for risk, to account for these patterns.

A model prediction is that agency conflicts grow with intangibility, which endogenously

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place increasingly tight financing constraints on these firms—in spite of their profitability. And I provide indirect evidence of this by showing that highly intangible firms naturally shift to sources of internal finance from external finance.

In Section V I examine recent macroeconomic trends. In the model I do this by comparing two stationary distributions that differ only by one parameter that influences the intensity of intangible capital in the production function. I find the model provides a microfoundation for trends in profitability, valuations, investment, compensation, idiosyncratic risk, and the shift from external to internal finance.[1] I reason that many aggregate trends and cross-sectional patterns are qualitatively consistent with a simple economic friction where an agency conflict is embedded into the intangibility of a firm.

Of course, there are other factors which matter to the development of intangible capital, like technological changes or market structure. For example, Lustig, Syverson and Van Nieuwerburgh (2011) document upward trends in managerial income inequality and pay-performance sensitivity and argue it reflects a shift towards general productivity growth from vintage-specific growth. In contrast, I argue that the incentive contracts of specialists are important in their own right. Many models of intangible capital, as in this paper, are predicated on special agents who possess some control over it, for example, Eisfeldt and Papanikolaou (2013) and Bolton, Wang and Yang (Forthcoming). New to these models, I focus on the *incentives of its development* and therefore the structure of specialists' compensation. The results here suggest that recent papers like Sun and Xiaolan (Forthcoming) and Kline, Petkova, Williams and Zidar (2019) that study the contracts and compensation of innovative firms are a fruitful direction for this line of research.

My paper contributes to topics in dynamic investment and agency.[2] Like DeMarzo, Fishman, He and Wang (2012), this paper develops an optimal contracting framework for investment, but I place the agency friction squarely on a second capital-type, intangibles, and study how agency affects both types. Great historical profitability, as I mentioned, does not necessarily imply a small agency friction. Bolton et al. (Forthcoming) study the impact of inalienability of human capital where a "key person's" potential departure is a risk to the firm. While my model nests this effect and can study its impact by changing a boundary

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[1] I thus provide a microfoundation of the empirical evidence of Campbell, Lettau, Malkiel and Xu (2001), Bates, Kahle and Stulz (2009), Frydman and Saks (2010), Gutiérrez and Philippon (2017), Doidge, Karolyi and Stulz (2017), Crouzet and Eberly (2018), Andrei, Mann and Moyen (2018), and Graham and Leary (2019) that links to the evidence presented in Hall (2001) and Corrado and Hulten (2010).

[2] A partial list is Albuquerque and Hopenhayn (2004), Quadrini (2004), Clementi and Hopenhayn (2006), DeMarzo and Fishman (2007), Biais, Mariotti, Plantin and Rochet (2007), Nikolov and Whited (2014), Morellec, Nikolov and Schürhoff (Forthcoming).

condition, it focuses instead on the incentives to develop intangible capital within the firm holding this departure risk fixed. In addition and new to literature, I embed the contracting environment into an stationary industry equilibrium (Hopenhayn (1992)) that permits a comparative statics analysis of the long-run distribution. Industry analysis is important because the implications of agency models are often directly influenced by the potential termination of these key employees and firm exit.

My paper also complements the large literature on financing and investment dynamics.[3] Rampini and Viswanathan (2013) show in a model of limited commitment that collateral is an important determinant of capital structure and provide a "user cost of (physical) capital" intuition where an additional term appears that captures the scarcity of internal funds. Thus, collateral (tangibility) affects the marginal cost of investment in contrast to my paper where intangibility affects the marginal benefit. Ai and Li (2015) develop a model that incorporates limited commitment of financial contracts into the neoclassical investment model. My paper also endogenizes financing constraints yet relates it to the level of intangibility. Different from these papers, highly intangible firms are typically smaller (in physical capital) and invest less although they are more profitable. Eisfeldt and Papanikolaou (2013) demonstrate that firms with a great deal of organizational capital are more likely to cite losing "key talent" as a systematic risk factor in their financial statements and possess higher discount rates. My paper highlights a feature of intangibility that is driven by idiosyncratic risk.

In Section I I introduce the model setup and describe the agency conflict, and then describe its solution in Section II. Next, in Section III I compile the data and calibrate the model. I analyze the model's predictions in Section IV and compare them with the data. In Section V, I conduct a comparative static analysis of the stationary equilibrium and contrast it with recent macroeconomic trends. I then conclude.

I. MODEL SETUP

I distinguish intangible capital from physical capital in three aspects. First, intangible capital is only indirectly observable to investors (principals). Second, only a small group of specialists (agents) have the ability to develop it. Third, in the event of termination of the firm and dismissal of agents, a greater fraction of it is lost relative to physical capital. The

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[3]A short list is Gomes (2001), Krueger and Uhlig (2006), Hennessey and Whited (2007), Rampini and Viswanathan (2010), Ai, Croce and Li (2013), Sun and Xiaolan (Forthcoming).

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model below is built around these ideas.

A. INTANGIBLE CAPITAL AND AGENCY FRICTION

The development of intangible capital is gradual and reflects the difficulty and effort in creating something new and verifying results. Agents' collective effort $e_t \in [0, 1]$ determines the growth of intangible capital, N, that evolves as the persistent process

$$dN_t = (ge_t - \delta_N)N_t dt + \sigma N_t dZ_t, \tag{2}$$

where $\delta_N \geq 0$ is the depreciation rate and $g > 0$ is an upper bound which can be interpreted as the difficulty of developing it.

The growth rate fluctuates with the increments of a Brownian motion, dZ_t, and its variability increases with $\sigma > 0$. Uncertainty here reflects the fundamental riskiness with developing a new technology or building a brand in the form of stochastic obsolescence or unproductive realized investment. Thus, the degree of uncertainty associated with the technology, and correspondingly the degree to which agents can hide their effort, scales with σ.

When agents exert effort e_t, they enjoy private benefits at rate $(1 - e_t)\Lambda(K_t, N_t)dt$ that I assume costs $(1 - e_t)gN_t$ of firm resources. Benefits being influenced by physical capital, $\Lambda(K, \cdot)$, is commonplace in the literature on empire building and intangible capital, $\Lambda(\cdot, N)$, as I argue, reflects an intellectual reward. Intangible capital is often developed by skilled labor who have typically chosen to attain advanced degrees. Cutting-edge research often goes hand-in-hand with traveling, attending conferences, and opining on technical issues that potentially influence policy. Related, Moskowitz and Vissing-Jørgensen (2002) calculate the nonpecuniary benefits of entrepreneurship to be effectively 143 percent of total annual income; a number they find plausible given the compensation financial economists give up to remain purportedly in academia. It therefore seems reasonable a priori that nonpecuniary private benefits could be a function of the intangibility of the firms in which specialists work. Furthermore, the empirical evidence I present in Section III is consistent with this assumption.

Because total effort spent across intangible development and enjoying private benefits sums to one, expenditure on development equals gN_t for all t. Effort only affects the drift of the process and a lack of it can be alternatively interpreted as shirking. In either interpretation the function $\Lambda(\cdot) > 0$ scales the severity of the agency friction.

B. PRODUCTION AND PHYSICAL INVESTMENT

The firm uses capital and employs (unskilled) labor, L, at wage rate w_L to generate (instantaneous) cash flows

$$\Pi_t = \max_{L_t \geq 0} \left\{ \overline{A} F(K_t, N_t)^\alpha L_t^{1-\alpha} - I_t - C(I_t, K_t) - g N_t - w_L L_t \right\}, \tag{3}$$

where $\overline{A} > 0$ is the level of productivity and α is the capital share of income. Capital comprises intangible capital, N, and physical capital, K, and is aggregated via the function $F(K, N)$. With fully and instantaneously adjustable labor, the optimal labor demand is proportional to capital, and substituting this optimum into (3) gives

$$\Pi_t = A F(K_t, N_t) - I_t - C(I_t, K_t) - g N_t, \tag{4}$$

where $A = \alpha \overline{A} \left(\frac{(1-\alpha)\overline{A}}{w_L} \right)^{\frac{1-\alpha}{\alpha}}$.

Gross physical investment is denoted by I and, following the vast literature on investment theory, it is subject to adjustment costs, $C(I, K)$. Physical capital, K, grows according to the standard accumulation equation,

$$dK_t = (I_t - \delta_K K_t) dt, \tag{5}$$

where $\delta_K \geq 0$ is its depreciation rate.

C. CONTRACTING ENVIRONMENT

I assume the firm's physical capital, K_t, and historical cash flow rate process, $\{\Pi_s : 0 \leq s \leq t\}$, are observable and contractible. Therefore, from (5), investment I_t can be contracted upon. Since physical capital and investment are contractible, the realization of the rate of cash flows then allows investors to write contracts on the development of intangible capital.[4]

Principals thus monitor other variables to assess intangibles' development. In this sense, the idea is related to Holmstrom and Milgrom's (1991) study on multitasking. They show that tasks, one of which is hard to measure, being either complements or substitutes

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[4]Here is an example of what I have in mind. When investors measure Google's development of intangible capital, basically the efficacy of its search algorithm, they measure the incremental amount of revenues obtained from ad-clicks or number of searches, controlling for other factors.

differentially affects an agent's effort and compensation. This model feature is new to the investment literature, is realistic, and is consistent with empiricists' approach to measuring intangible capital. For example, the Bureau of Economic Analysis (BEA) measures intangible investment in software in part with programmers' wages.

The contract with agents can be terminated at any time. When the contract is terminated, investors recover a fraction $0 < l_K < 1$ of physical capital yet only $0 < l_N < l_K$ of intangible capital, altogether recovering $l_K K_t + l_N N_t$ at termination. Because only fractions of capital are recovered, termination is inefficient ex post.

Investors have unlimited wealth and can therefore provide funds for the purchase of physical capital whenever they deem it suitable. They are risk-neutral and discount at rate r. Agents are also risk-neutral but discount at rate $\gamma > r$, a common assumption that reflects their assumed impatience or the presence of outside employment opportunities.[5] Agents have no initial wealth and have limited liability, so they cannot be paid negative wages. If the contract is terminated, agents receive the value of their outside option that I normalize to zero.

II. MODEL SOLUTION

I study the model with no agency friction that is the first-best case. I then introduce the environment with the agency friction before discussing the model solution.

To simplify the model, I assume that the capital function, $F(K, N)$, adjustment costs, $C(I, K)$, and private benefits $\Lambda(K, N)$ are all homogeneous of degree one in their arguments. Specifically, $F(K, N) = Kf(n)$ where $f(\cdot)$ is increasing and concave; the total cost of investment is $I + C(I, K) = Kc(i)$, where $i = I/K$ is the investment rate; and $\Lambda(K, N) = K\lambda(n)$. I refer to the state variable $n = N/K$ as the *intangibility* of the firm. By (2), (5), and Ito's lemma, it evolves as

$$dn_t = d\left(\frac{N_t}{K_t}\right) = ((ge_t - \delta_N) - (i_t - \delta_K))n_t dt + \sigma n_t dZ_t. \tag{6}$$

A. FIRST-BEST CASE

Without benefits to shirking ($\lambda(\cdot) \equiv 0$), agents choose $e_t = 1$ for all t. This is the first-best case widely used in the investment literature. Because of the homogeneity assumption, I

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[5]While $\gamma = r$ may be a more neutral assumption, DeMarzo and Sannikov (2006) argue a contract can be made more robust by having investors assume that γ is higher than agents' true γ.

can simplify the problem and write its solution as an ordinary differential equation

$$rp(n) = \max_i \pi(n) + p(n)(i - \delta_K) + p'(n)(g - \delta_N - (i - \delta_K))n + \frac{1}{2}p''(n)n^2\sigma^2. \quad (7)$$

This equation equates the required return $rp(n)$ to *profitability* (per instant), $\Pi/K = \pi(n) = Af(n) - c(i) - gn$, plus expected capital gains: respectively, the gain from an additional unit of physical capital, intangible capital, and an adjustment for variability.

The choice of physical investment follows from the first-order condition

$$\underbrace{c'(i(n))}_{\text{marginal cost}} = \underbrace{\underbrace{p(n)}_{\text{average } Q} - p'(n)n}_{\text{marginal } q}. \quad (8)$$

The right side is *physical* marginal q. The marginal cost of investment is equated to physical average Q, $p(n)$, net of the reduction in the firm's intangibility, $p'(n)n$, which is profitable in general. Optimal physical investment thus trades off the marginal values of physical and intangible capital.

Under complementarity, an abundance of intangible capital implies that the marginal value of physical capital is large and therefore a high rate of investment is desirable. Conversely, when intangibility is low, so are the marginal value of physical capital and investment. Thus, absent uncertainty ($\sigma = 0$), a firm's physical investment choice would gravitate towards an optimum interior point of intangibility. In the presence of uncertainty ($\sigma > 0$) and given the concavity of $p(n)$, the firm's intangibility would be on average below this steady state point, though these gravitational forces would remain.

In summary, first-best models imply that investment, Q, and intangibility are all positively related: sorts on intangibility are equivalent to sorts on Q in regards to their predictions for investment.[6] This equivalence is not present in the data as I show in Section IV and therefore poses a challenge to these models. I now explore how conflicting interests break this result.

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[6]Firm value in (7) is before the agent has been compensated. If investors have promised to pay $W > 0$ to agents, then it is optimal to pay out W in cash immediately, because agents are relatively more impatient. On a per physical capital basis, $w = W/K$, the value of the firm to investors is $p(n) - w$.

B. Agency Case

I introduce the agency problem to the model by setting $\lambda(\cdot) > 0$. To maximize firm value, investors offer a contract that specifies agents' cumulative cash payment history, U_t, an investment policy, I_t, and a termination time, τ. All variables depend on the history of agents' performance, which is given by the evolution of intangibility. Limited liability requires incremental cash, dU_t, be nondecreasing. I let $\mathcal{C} = (I, U, \tau)$ represent the contract.

Given the contract, agents choose their action process to maximize the present value of compensation and private benefits,

$$W(\mathcal{C}) = \max_{\{e_t \in [0,1]: 0 \le t < \tau\}} \mathbb{E}^e \left[\int_0^\tau e^{-\gamma t} (dU_t + (1 - e_t)\Lambda(K_t, N_t)dt) \right], \qquad (9)$$

where the expectation, $\mathbb{E}^e[\cdot]$, is taken under the probability measure conditional on agents' effort process.

When the contract is written, the firm has K_0 units of physical capital and N_0 units of intangible capital. The problem investors face is

$$P(K_0, N_0, W_0) = \max_{\mathcal{C}} \mathbb{E} \left[\int_0^\tau e^{-rt} (\Pi_t dt - dU_t) + e^{-r\tau} (l_K K_\tau + l_N N_\tau) \right]$$

$$\text{s.t. } \mathcal{C} \text{ is incentive compatible and } W(\mathcal{C}) = W_0. \qquad (10)$$

Agents' initial wealth, W_0, is determined by the bargaining power of agents and investors when the contract is initiated. If, for example, investors possess all bargaining power, then $W_0 = \text{argmax}_{W \ge 0} P(K_0, N_0, W)$; if agents possess all power, then $W_0 = \max\{W : P(K_0, N_0, W) \ge 0\}$; more generally, W_0 could be a convex combination of the two extremes.

The contract is *incentive compatible* when it implements the efficient action, $e_t = 1$, for all t. Given this contract and the history up until time t, agents' *continuation payoff* is given by

$$W_t(\mathcal{C}) = \mathbb{E}_t \left[\int_t^\tau e^{-\gamma(s-t)} dU_s \right]. \qquad (11)$$

As is standard in dynamic contracting models, agents' incremental compensation at time t is composed of incremental cash, dU_t, and the incremental change in continuation payoff, dW_t (Spear and Srivastava (1987), Sannikov (2008)). These two sources must compensate

agents for their time preference on average, therefore,

$$\mathbb{E}_t[dU_t + dW_t] = \gamma W_t dt. \tag{12}$$

Furthermore, to maintain incentive compatibility, agents' compensation must remain sufficiently sensitive to the firm's cash flow process. Under the environment where intangibles can be contracted upon, we can formulate this sensitivity directly and express it using the martingale representation theorem (see DeMarzo and Sannikov (2006) for details):

$$dW_t + dU_t = \gamma W_t dt + \beta_t N_t \left(\frac{dN_t}{N_t} - (g - \delta_N)dt \right)$$

$$= \gamma W_t dt + \beta_t N_t \sigma dZ_t. \tag{13}$$

Agents who deviate reduce their compensation by $(1 - e_t)g\beta_t N_t dt$ and receive private benefits $(1 - e_t)\Lambda(K_t, N_t)dt$. Incentive compatibility is thus implemented with $\beta_t \geq \Lambda(K_t, N_t)/(gN_t)$, for each t. Because liquidation is ex post inefficient and therefore costly to enforce, the optimal contract minimizes the likelihood of this event and sets

$$\beta_t = \frac{\Lambda(K_t, N_t)}{gN_t} = \frac{\lambda(n_t)}{gn_t}, \text{ for all } t. \tag{14}$$

This condition is natural and equalizes the contract's sensitivity to the ratio of private benefits per unit of intangibility growth. Equation (13) then suggests, intriguingly, that the functional form of private benefits of intangibility, $\lambda(n)$, can be recovered from an analysis of how shocks to compensation depend on intangibility.

This insight is new and more general than it first might appear. Optimal contracts in these Sannikov-type environments endogenize a contract sensitivity in the form of a ratio of private benefits to growth potential. While outside the scope of this paper, this observation should carry over to other potential agency frictions: for example, firm cash holdings or financial intermediary net worth. Thus, looking at private benefits as a function rather than a fixed parameter suggests potentially fruitful empirical work in the area that studies how shocks to compensation respond to measures of agency conflicts.

Another novelty to much of the contracting literature is that the contract sensitivity, β_t, is potentially time-varying. As I will discuss later, this makes it more costly to investors to maintain agents' efficient action choice for a given continuation payoff in firms with greater intangibility.

C. SOLUTION DETAILS

I now describe some properties of the solution to the problem in (10). First, whatever the history of the firm up until date t, the only relevant state variables are the firm's capital stocks, K_t and N_t, and agents' continuation payoff, W_t. Therefore, investors' value function at time t, $P(K_t, N_t, W_t)$, can be solved using dynamic programming techniques. Second, homogeneity allows me to reduce the problem to two endogenous state variables— agents' scaled continuation payoff, $w = W/K$, and intangibility, $n = N/K$—and write $p(n, w) = P(K, N, W)/K$.

To begin, investors require that

$$rp(n, w)dt = \max_i \pi(n, w)dt + \mathbb{E}_t \left[d(Kp(n, w))/K \right]. \tag{15}$$

This is a partial differential equation, written out fully and further discussed in Appendix A, and as before required returns are equated to expected returns. Investors choose investment to maximize this expectation via the first-order condition

$$c'(i(n, w)) = p(n, w) - p_n(n, w)n - p_w(n, w)w. \tag{16}$$

On top of the considerations in (8), from investors' perspective the marginal benefit of investing also accounts for the marginal decrease in agents' continuation payoff w. Investors now internalize the effect of a marginal unit of physical investment on agents' payoff and therefore the agency conflict.

Boundaries.—I now discuss the boundaries that determine the solution to (15). First, the agent will be terminated immediately once their promised future payments reach the value of their outside option (normalized to zero), because otherwise they would immediately consume private benefits. Therefore,

$$p(n, 0) = l_K + l_N \times n, \text{ for each } n. \tag{17}$$

In effect, consequent to a sequence of adverse shocks ($dZ_t < 0$), the firm's cash flow rate slows, leading principals to believe agents have not been sufficiently developing intangible capital and to terminate the contract and fire the agents.

Second, because investors can always compensate agents with cash, it will cost at most one dollar to increase w by one dollar, and therefore $p_w(n, w) \geq -1$, for each n. As w

grows, the probability of termination falls and with it the event of costly liquidation. It is thus optimal to grow w as quickly as possible at low levels of w by setting $du_t = dU_t/K_t$ to zero. Since agents are impatient relative to investors, however, cash compensation for immediate consumption will be required at some threshold of w to discourage agents from consuming private benefits. At this threshold, investors will be indifferent between reducing their value by one dollar to pay agents one dollar in cash:

$$p_w(n, \overline{w}(n)) = -1, \text{ for each } n. \tag{18}$$

And finally, to ensure that this threshold is optimal on behalf of investors, I require the super contact condition

$$p_{ww}(n, \overline{w}(n)) = 0, \text{ for each } n. \tag{19}$$

Note that these conditions hold for each n, so I highlight the dependence of the payment boundary on the firm's intangibility: $\overline{w}(n)$.

Altogether, under the optimal contract, $du_t = 0$ within the payment and termination boundaries, and $\beta_t = \lambda(n_t)/n_t$ for all t. The evolution of w then follows directly from (5) and (13):

$$dw_t = (\gamma - (i_t - \delta_K))w_t dt + \lambda(n_t)\frac{\sigma}{g}dZ_t. \tag{20}$$

Shocks to agents' scaled continuation payoff are increasing in the degree of private benefits, $\lambda(n)$, and the inverse signal-to-noise ratio σ/g, as a greater g makes it easier to detect deviations. The optimal contract thus makes shocks to intangibility and agents' scaled continuation payoff, dn_t and dw_t, perfectly correlated.

To summarize, the upper boundary $\overline{w}(n)$ is the point above which agents receive cash compensation for operating the firm; that is, when $du_t > 0$. When $0 < w <= \overline{w}(n)$ for each n, agents' wealth is solely derived from promised utility and receives no incremental cash payments $du_t = 0$. When $w = 0$ at time τ for any n, the contract is terminated, agents are fired, and investors receive $l_K + l_N n_\tau$ per unit of physical capital.

Because the upper boundary $\overline{w}(n)$ is reflecting, a given contract will be terminated with probability one. To assess the macroeconomic effects of intangibility and agency frictions, in the next section I allow entry and therefore a stationary distribution of firms to exist.

D. Aggregation

With the description of individual firm behavior complete I can now characterize the distribution of firms in the economy. Because each firm is described by its current state (n, w), the density of firms is defined over this state space. The non-stationary distribution at time t, $h(n, w, t)$, then satisfies the Kolmogorov forward equation

$$\frac{\partial h(n, w, t)}{\partial t} = \psi(n, w)m(t) + \mathcal{A}^* h(n, w, t),\tag{21}$$

where $\mathcal{A}^* h(n, w, t)$ is the adjoint of the infinitesimal generator of the bivariate diffusion process (dn, dw).[7] By construction, this generator contains the rates of exit that occur along the termination boundary $w = 0$. To ensure a stationary mass of firms, I add a product of an entry rate, $m(t)$, and an entry mass, $\psi(n, w)$, which integrates to one.

I pin down the entry rate with the requirement that the total mass of firms is constant, which I normalize to one: $\int_0^\infty \int_0^{\overline{w}(n)} h(n, w, t)dwdn = 1$ for all t. Twice integrating (21) then implies that the time derivative of this constant distribution on the left side is zero and therefore that the right side is time invariant. Thus, after integration the equation can be rearranged for the stationary entry rate

$$m = -\int_0^\infty \int_0^{\overline{w}(n)} \mathcal{A}^* h(n, w)dwdn.\tag{22}$$

By construction, the entry rate is equated with the exiting mass of firms at the termination boundary. I discuss the calculation of the stationary distribution in Appendix A.

When a firm's contract is terminated, a new, replacing firm's intangibility is drawn from a distribution with positive support. The entrant, however, is also starts with a new continuation payoff, w_0. The new payoff is determined by the bargaining power of agents and investors. It should be greater than zero $w_0 > 0$ because, as DeMarzo et al. (2012) note, it prevents the economy from exhibiting a "replacement frenzy" over a short period of time and mimics an economy where there is some cost to investors of starting a new firm.

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[7]Specifically, $\mathcal{A}^* h(n, w, t) = -\mathbb{E}_t[dn_t]h_n(n, w, t) - \mathbb{E}_t[dw_t]h_w(n, w, t) + \frac{1}{2}\mathbb{E}_t[(dn_t)^2]h_{nn}(n, w, t) + \frac{1}{2}\mathbb{E}_t[(dw_t)^2]h_{ww}(n, w, t) + \mathbb{E}_t[(dn_t dw_t)]h_{nw}(n, w, t)$.

E. POTENTIAL MODEL EXTENSIONS

The model could be extended in several ways, including additional shocks, endogenous recovery, private benefits of control, or the lack of commitment of human capital.

Physical Capital Shock.—The accumulation of physical capital could be extended to vary stochastically, similarly to Cox, Ingersoll and Ross (1985),

$$dK_t = (I_t - \delta_K K_t)dt + \sigma_K dZ_{Kt}, \tag{23}$$

where σ_K is the volatility of the capital depreciation shock. Intangibility would then evolve as

$$dn_t = ((ge_t - \delta_N) - (i_t - \delta_K) + (\sigma_K^2 - \sigma\sigma_K\rho))n_t dt + \sigma n_t dZ_t - \sigma_K n_t dZ_{Kt}, \tag{24}$$

where ρ is the correlation between Brownian shocks. Adding another shock effectively translates intangible capital's net growth rate, $g - \delta_N$, by $\sigma_K^2 - \sigma\sigma_K\rho$. One can also define a new normal shock, without loss of generality, with standard deviation $n_t\sqrt{\sigma^2 + \sigma_K^2 - 2\sigma\sigma_K\rho}$.

Endogenous Recovery.—DeMarzo et al. (2012) show the recovery fraction could be made endogenous. If the contract is terminated when the firm has intangibility n_t, then investors could be allowed to hire new specialists at payoff w_0

$$l_K + l_N n_t = \max_{w_0}(1 - \xi)p(n_t, w_0), \tag{25}$$

where $\xi \in (0, 1)$ is the cost replacing the specialists. The entry rate discussed above effectively assumes that the cost ξ becomes arbitrarily small.

Private Benefits of Control.—DeMarzo and Sannikov (2006) discuss the extension where agents receive private benefits of control from running the firm. If prior to termination agents earn additional utility at rate $\gamma\omega$, then with this private benefit agents' continuation payoff evolves as

$$dW_t = \gamma(W_t - \omega)dt - dU_t + \lambda(N_t)\frac{\sigma}{g}dZ_t. \tag{26}$$

Intuitively, the threat of liquidation and therefore the risk of losing private benefits improves agents' incentives.

Lack of Commitment of Human Capital.—Bolton et al. (Forthcoming) consider a model

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where an entrepreneur cannot commit their human capital to the firm and can leave to pursue their outside option and start a new firm. Within the context of this model, the change in commitment would redefine the default boundary in (17) to

$$p(n, \underline{w}) = p(\varphi n, w_0), \text{ for each } n. \tag{27}$$

where $\varphi \in (0, 1)$ measures agents' collective talent that determines the new firm's intangibility and $\underline{w}$ is their endogenous quitting boundary, which could depend on n.

III. DATA AND MODEL CALIBRATION

My data are from 1975 until 2015. Firm-level data are the widely-used annual accounting data from Compustat, annual compensation data from Execucomp, and monthly equity and return data from CRSP. I use Frydman and Saks's (2010) publically available dataset to extend the Execucomp sample that begins in 1992 to 1975. Publically traded firms provide a useful environment in which to study agency conflicts between agents and investors. Details are provided in Appendix B.

A. VARIABLE AND PORTFOLIO CONSTRUCTION

I quantify intangibles with a measure of organizational capital used in the accounting and finance literature (Lev and Radhakrishnan (2005)). It is constructed for every firm i for each year t using the perpetual inventory method on selling, general, and administrative expenses:

$$N_{it} = (1 - 0.15) \times N_{it-1} + 0.3 \times \frac{SG\&A_{it}}{cpi_t}, \tag{28}$$

where cpi_t is the consumer price index. Following previous work, I use a 15 percent depreciation rate and initialize a firm's N_{i0} at $SG\&A_{i0}/0.25$. Compustat's measurement of SG&A includes expenditure on research and development and a large part of it consists of expenses related to skilled labor (programmers) and information technology as well as marketing (brand capital). Eisfeldt and Papanikolaou (2013) validate this construction with evidence on managerial quality, investment in information technology, and productivity. I define a firm's intangibility as the ratio of organizational capital plus externally purchased intangible capital to the value of (real) property, plant and equipment.

I also compute every firm's physical investment rate and *physical* average Q (Peters and Taylor (2017)): the value of the firm over the replacement cost of *physical* capital. Profitability in the data is operating cash flows before depreciation minus capital expenditures, normalized by last year's book assets. Profitability is thus a scaled measure of free cash flow.

In my data, compensation is composed of salary, bonus, long-term incentive plans, and option and stock awards and is usually only available for upper management. I use the sum of total compensation (the sum of all components, consistent with Frydman and Saks's (2010) "comp3i" variable) across all managers within each firm-year pair and scale by physical capital. Unfortunately, only data on upper management are reported. The distinction of specialists evokes the idea of employees separate from management but for many firms this need not be the case. The model, moreover, interprets any form of labor not obtained in competitive markets (L_t) as specialists; that is, these could be executives and technicians.

After these key variables have been constructed, I create intangibility portfolios and look at their characteristics: every June, I rank firms by their intangibility and sort them into quintiles. Because accounting standards and production functions vary across industries, I rank firms' intangibility relative to their industry peers using two-digit NAICS codes, which better stratify modern services than do SIC codes. I then form five market-value-weighted portfolios based on these within-industry ranks and rebalance every year. Portfolio construction and rebalancing mitigates the effects of firm entry and exit, nonstationarity in firm-level data, and measurement error, which is important for analyses related to intangibility. Sorts also rank non-parametrically and do not impose a linear structure potentially inconsistent with (1).

B. CALIBRATION

I tabulate the calibration in Table I. I begin by setting the real interest rate, r, to 4 percent. For depreciation rates, I use the same rate for intangible capital that the BEA applies to research and development, $\delta_N = 0.15$, and I set $\delta_K = 0.125$ for physical capital (in line with DeMarzo et al. (2012) and Eberly, Rebelo and Vincent (2012)). I set $g = 0.17$ that implies a steady state growth rate of 2 percent ($i^{ss} - \delta_K = g - \delta_N$) and a steady state investment rate, i^{ss}, of 14.5 percent.

Production Function.—Krusell, Ohanian, Ríos-Rull and Violante (2000) and Eisfeldt, Falato

and Xiaolan (2018) argue for complementarity between physical capital and skilled labor and physical capital and human capital, respectively. While intangible capital is neither skilled labor nor human capital, it is plausible that they are all correlated. Following these two papers, therefore, I specify a constant elasticity of substitution (CES) production function between these two capital types:

$$F(K_t, N_t) = (\phi K_t^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)N_t^{\frac{\epsilon-1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}} = K_t \left((1-\phi) + \phi n_t^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} = K_t f(n_t) \quad (29)$$

where $\phi \in (0,1)$ is intangible capital's share of capital, $\epsilon \in (0,\infty)$ is the elasticity between physical and intangible capital. Krusell et al. (2000) estimate an elasticity of 0.67 and Eisfeldt et al. (2018) 0.87 in their benchmark models. I set $\epsilon = 0.8$.

I then pin down A and ϕ jointly by setting the two parameters to solve for the level of intangibility and profitability in the steady state (when $\sigma = 0$). In my data sample I estimate average intangibility and median profitability across all firm-years to be $n^{ss} = 2.05$ and 10 percent, respectively. These estimates require $A = 0.35$ and $\phi = 0.75$.

Stationary Distribution.—Because I focus on industry equilibrium, parameters that govern the shape of steady state distribution and entry and exit become important. First is the variability of the growth rate of intangibility, σ, which influences the severity of the agency friction and the likelihood of contract termination. To calibrate this, I examine within-firm volatility. Specifically, for every firm in my sample, I construct intangibility and then compute the volatility of its growth rate. This measure is fixed at the firm-level. I then take the median across all firms to get 20 percent.

For initial intangibility the entry mass, $\psi(n,w)$, draws from a log-normal distribution with mean n^{ss} and standard deviation σ. Given the intangibility draw, n, I assume agents possess all bargaining power to find the initial w: $w = \max\{w'|p(n,w') > 0\}$.

I specify a smooth adjustment cost technology as I am interested in the long-run properties of the model. It takes a quadratic form of the investment rate

$$c(i) = i + \frac{\kappa}{2}(i - \delta_K)^2, \quad (30)$$

where κ measures the magnitude of the adjustment cost. In the model this parameter captures the response of investment to marginal q: by equation (1), $i = (q-1)/\kappa + \delta_K$. I calibrate this response by setting $\kappa = 30$.

Specialists time rate of preference is $\gamma > r$. It primarily affects the average length of

the interval $[0, w(n)]$, as more impatient agents (higher γ) require relatively sooner current payments out of continuation utility which lowers $w(n)$. In the model, the agents' share of the firm is $(1 - \alpha) \times w/(w + p(n, w))$ where α reflects the share of the firm attributed to unskilled labor, leaving $(1 - \alpha)$ to the investors and agents. In order to convert this stock into a flow I multiply this share by γ. Thus, I narrow down γ by calibrating to average compensation per unit of physical capital. In the data, the average value is near one percent and I correspondingly choose $\gamma = 0.08$.

In equilibrium, the threat and potential implementation of termination in the optimal contract is used to ensure agents' proper incentives. I have contract termination impose a bankruptcy cost on physical capital on the order of 30 percent, close to the estimates reported in Glover (2016), putting $l_K = 0.7$. I assume half that fraction is obtained by investors in regards to intangible capital, $l_N = l_K/2 = 0.35$, which seems plausible.

Private Benefits.—Recall that (13) and its scaled analog (20) suggest that the function of private benefits of intangibility can be recovered from an analysis of how shocks to compensation depend on intangibility. I pursue this insight via three steps.

First, I run a panel regression of scaled compensation, w_{it}, on lagged scaled compensation, w_{it-1}, it and an interaction with the firm's investment rate, $i_{it} w_{it-1}$, and year, f_t, and firm, f_i, fixed effects. The regression has the coefficient estimates (with standard errors clustered at the firm-level in parentheses) of

$$w_{it} = f_i + f_t + \underset{(0.020)}{0.234} \times w_{it-1} + \underset{(0.0004)}{0.0062} \times i_{it} \times w_{it-1} + \epsilon_{it}, \tag{31}$$

with an overall R-squared of 63 percent. I estimate the residuals, which average to zero across all firm-years, and standardize them.

Next, I place the residuals into five portfolios that are annually rebalanced based on the quintiles of some underlying firm characteristic. Within a particular quantile-year, I calculate the mean and standard deviation of the residuals. I then take the time series average of these statistics for each portfolio. Table II reports the results for four portfolios formed on intangibility, profitability, book-to-market, and asset turnover.

The mean and standard deviation of residuals rises across portfolios of increasing intangibility. Consistent with the model, the pattern reflects the history-dependence of the contract: a sequence of positive shocks to intangibility raises the growth of compensation and increases the variance. Thus the conditional distribution of residuals moves with intangibility in the right way and validates the model. Comparing the patterns across other char-

acteristics shows that something is indeed special about intangibility: profitability, book-to-market, and asset turnover all produce non-monotone relationships among characteristics and mean and variance.

In the third and final step, I regress the residuals on a polynomial expansion of intangibility to ascertain the functional form of the private benefits function $\lambda(n)$. The first-order expansion produces a point estimate of 0.17: a one standard deviation increase in intangibility growth raises compensation growth by $0.17/1.26 \times \sigma/g = 15.8$ percent, where 1.26 is the unconditional mean of scaled compensation. By comparison, Kline et al. (2019) study how patent-induced shocks causally affect worker compensation. They estimate that for every dollar of new firm surplus workers' earnings rise by nearly 30 cents, an elasticity of approximately 0.35. The elasticity they find is much larger than in previous studies and they attribute one possible reason is that failing to use an instrument obfuscates pure shocks to compensation with productivity shocks. Nevertheless, this difference suggests that the estimates here are biased towards zero and are therefore conservative.

In the end, I settle on the third-order approximation, as the fit still improves after the second iteration but by the fourth multicollinearity becomes problematic. Nearly 20 percent of the variation is explained by intangibility alone. Altogether, I specify the function of private benefits to be

$$\lambda(n) = 0.334n - 0.047n^2 + 0.003n^3. \tag{32}$$

IV. MODEL ANALYSIS

In this section I discuss the predictions of the calibrated model and compare them with the data. But I first describe the properties of the solution of the model. In what follows, I look at policy and value functions across four breakpoints based on the marginal distribution of intangibility: the 30th, 50th, 70th, and 90th percentiles.

A. INVESTORS' VALUE FUNCTION AND STATIONARY DISTRIBUTION

In Panel A of Figure 1 I plot the first derivative of investors' value function with respect to w. The first derivative measures the marginal cost of compensation: the marginal cost to investors of promising an additional dollar in continuation utility to agents. Optimality of the solution requires that all first derivatives equal -1 at the payment boundary. We can see that as intangibility rises, the marginal cost of compensation increases for every w in

general. This response is paramount in understanding the model.

Recall that in designing the optimal contract, investors adjust the marginal cost of compensation to minimize agency costs. There are several effects that determine the exact response of the marginal cost of compensation to an increase in intangibility:

1. Firm Value.—As intangibility rises, profitability and firm value do too, thus raising the benefit of avoiding termination. This decreases the marginal cost of improving agents' payoff, that is p_w rises, and therefore investors will optimally increase w to reduce agency costs.

2. Liquidation Value.—The firm's liquidation value rises with the quantity of intangible capital. This reduces the benefit of avoiding termination and counteracts, and could completely overturn, the first effect of a higher firm value.

3. Likelihood of Termination.—Equation (20) shows that this likelihood grows with intangibility for a given w because the upper payment boundary is reflecting and the lower termination boundary absorbing. Thus, for high n it is more likely an adverse shock will result in contract termination and liquidation. This amplifies the second effect.

The values of the second derivative at the payment boundary (along $\overline{w}(n)$) are plotted in Panel B. The super contact condition requires that these should all be zero to be optimal for investors. The norm (error) per grid point of intangibility (100 points) is less than 1 percent. The error is not uniformly zero, but numerical sensitivity analysis for different, local boundary curves suggest that the results which follow are not dramatically affected by this inaccuracy. The boundary curve I guessed is a two-part, piece-wise quadratic function that is evident in the shape of the value function depicted in Panel C.

Investors' scaled value function $p(n, w)$ is concave in w. Concavity arises from investors' aversion to fluctuations in agents' payoff and is generated by two opposing effects. First, agency conflicts are reduced as w grows away from the termination boundary as investors' and agents' incentives align. Agents' fear of termination dissipates and agrees with investors' desire to weaken this threat; $p(n, w)$ therefore grows at low w. Second, as w rises, agents extract a larger share of firm value, reducing investors' share and decreasing $p(n, w)$. The function in general declines with w. The value function increases in intangibility as profitability rises and because investors recover more in liquidation.

The stationary distribution of firms is depicted in Panel D. There are two parts to the distribution. The first part tracks out a path from $(n, w) = (2, 0)$ to the boundary curve

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near $(n^{ss}, \overline{w}(n^{ss}))$; this is the model's saddle path. Its particular gradient arises from the commonality in the drifts of states $(\mathbb{E}[dn], \mathbb{E}[dw])$. The path of investment is chosen so that both n and w grow together. But since the optimal contract minimizes agency costs that increase with n, investors optimally grow w relatively faster. The second part has to do with firms that have reached the payment boundary. Incentives are structured so that firms accumulate near the payment boundary as this is when agency frictions are minimized. Because of the model's stochastic property, the mass of firms are spread along the boundary.

B. INCENTIVES, INTANGIBILITY, AND INVESTMENT

In this section, I discuss how agents' incentives and intangibility interact to alter the predictions of first-best investment theory. Before doing so, I first define some variables in the model.

Total firm value including agents' claim is $P(K, N, W) + W$. Physical average Q, the ratio of total firm value to the physical capital stock, is given by

$$Q(n, w) = \frac{P(K, N, W) + W}{K} = p(n, w) + w. \tag{33}$$

Using this definition I rewrite (physical) marginal q of the Euler equation (16):

$$c'(i(n, w)) = q(n, w) = Q(n, w) - p_n(n, w)n - (1 + p_w(n, w))w. \tag{34}$$

Marginal q measures the incremental increase in firm value of a unit of physical capital whereas average Q is used in empirical studies due to the simplicity of its construction as a proxy for q. Moreover, whereas (16) reflects only investors' concerns, equation (34) reflects both investors' and agents' concerns.

Panel A of Figure 2 displays the value of physical Q as a function of w indexed by the four breakpoints. As intangibility rises, the firm becomes more profitable and total firm value split by both parties increases for any w.

Panel B depicts the marginal cost of intangibility. Recall that physical investment reduces intangibility, which is profitable, and so it subtracts from the benefit of investment. The effects of a reduction in profitability are quantitatively large, especially for firms of great intangibility. This profitability effect is crucial to understanding why investment rates are flat across portfolios of increasing intangibility.

Why is profitability so important? The reason relates to Panel C that depicts the net

benefit to both parties of shifting ownership in response to investment, $-(1 + p_w(n, w))w$. Physical investment reduces agents' effective claim on the firm and hence induces a more severe agency problem. Hence, this net benefit must be weakly negative (a cost). When w is near zero, shifts in ownership do not matter as contract termination is likely and liquidation becomes more certain. As w rises towards $\overline{w}(n)$, agents and investors incentives are aligned and again shifting ownership becomes insensitive to changes in intangibility.

Recall that the optimal contract adjusts the marginal cost of compensation to minimize agency costs. More generally, in this dynamic environment the optimal contract *smooths* the marginal cost of compensation, increasing agents' rents and aligning incentives in states in which the incentive problem is more costly. Notice that for greater levels of intangibility in the interior of $[0, \overline{w}(n)]$, the cost of shifting ownership is smaller. This is the manifestation of this optimal smoothing in response to a more costly incentive problem.

The desire for smoothing makes large rates of investment unappealing, especially when intangibility is large and agency conflicts severe. Even in response to a greater level of intangibility that in the first-best world would prescribe a high rate of investment, the smoothing motive ensures that investment is largely unresponsive. This can be seen in the patterns of investment rates in Panel D. Holding w fixed and raising intangibility results in a smaller and smaller increase in investment. Moreover, this smoothing motive can be so large that investment can actually decrease in intangibility at percentiles above the 90th. Common to these agency-investment models, investment increases in w holding n fixed.

We can now reconnect to profitability. As a result of this smoothness profitability, net of a languid response in investment, must increase with intangibility. But this mechanical interpretation masks the economics at hand. It masks the endogenous smoothing of the marginal cost of compensation to greater levels of intangibility. This reduction in variability is chosen to minimize the agency problem, both to the benefit of investors and agents. Alternatively put, great levels of profitability make the contract that commits to a high level of smooth compensation more credible, aligning incentives. Thus, profitability becomes extremely valuable and this explains the quantitatively large effects of reducing intangibility in Panel B.

This paper therefore provides a microfoundation for the high profitability, high valuation, and low investment of highly intangible firms through the stability of the marginal cost of compensation sought in designing the optimal contract. This is new and complements the existing literature. Michelacci and Quadrini (2009) and Guiso, Pistaferri and Schivardi (2013) show financially constrained firms defer wage payments as a source of internal fi-

nancing. Sun and Xiaolan (Forthcoming) demonstrate this deferment is endemic to highly intangible firms and further show this substitutes for debt capital.

In the next two sections, I compare the model's predictions with those in the data.

C. PORTFOLIO RESULTS

In Table III I tabulate statistics across the intangibility-sorted portfolios both in the data and in the model using the stationary distribution. I first discuss the direct predictions of the model before turning to secondary, indirect predictions.

Direct Predictions.—I document the primary empirical result by tabulating investment rates, average Q, and profitability for these portfolios of increasing intangibility. Increases in intangibility raise levels of average Q yet investment remains flat. These patterns are inconsistent with the first-best model that is based on investment theory's basic tenet that investment rates should rise with valuations of physical capital.

Related, profitability (net of investment) rises across these portfolios in the data. The first-best model has investment respond too much relative to the data. The agency model, as we discussed, endogenously generates higher profitability as an response to the optimal contract smoothing the marginal cost of compensation. Additionally, development rates are nearly constant at 19 percent across portfolios in the data and are consistent with the modeling assumption that g is constant. Endogenizing the development rate would likely therefore add little to the model.

Average returns, moreover, do not differ across portfolios. This discredits a simple story where firms of greater intangibility are systematically riskier, where higher discounting lowers marginal q and therefore investment. That average returns do not differ across portfolios is at odds with the findings in Eisfeldt and Papanikolaou (2013), so in Appendix A I extend my first-best model to include a risk premium on intangibility and development adjustment costs.[8] I find that even in the presence of these extensions, they cannot fully explain the flatness of investment with respect to intangibility.

I believe that intangible firms are, in some sense, riskier, but there could be factors other than discount rates at play, so I highlight my mechanism by conditionally sorting portfolios.

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[8]My ranking metric differs from theirs in several ways: they divide organizational capital by book assets, rank relative to SIC codes, accumulate organizational capital with a factor of 1 rather than 0.3, use a different sample period, only include firms with December-end fiscal years, among others. Intangibility of course is difficult to measure, so it is expected that various constructions could produce different interpretations of the data.

That is, I first sort firms on characteristics known to correlate with discount rates and then, within each discount rate sort, sort on intangibility. The idea is to hold discount rates fixed while varying intangibility.

In practice, I use the well-known value (book-to-market) characteristic of Fama and French (1992) which explains much of the cross-section of average stock returns.[9] More specifically, I sort firms by their book-to-market ratio into terciles and then within each book-to-market tercile I sort firms by their intangibility (within industry) into terciles. Terciles are grouped into Low (0-33), Medium (34-66), and High (67-100) and are used to ensure an adequate amount of firms within each portfolio.

I report the results of these conditional sorts in Table IV. Within a book-to-market sort we see the same pattern: average Q rises with intangibility yet investment rates across portfolios remain flat or even fall. Average returns increase in book-to-market ratios, reproducing the value premium. Yet average returns across intangibility portfolios do not differ within a book-to-market grouping, and so it is unlikely that differences in exposure to systematic risk are solely responsible for the results.

Therefore at first pass, an agency friction that is at least positively related to the relative quantity of intangible capital in firms' asset structure seems to be a reasonable description of the economics of these important firms. For completeness, in Appendix B I report sorts on average Q and show that the tenet of investment theory holds there, which suggests that there is something special about highly intangible firms. In conclusion, the results here suggest that first-best models are likely to be challenged in generating the correct patterns in investment and valuation across portfolios of intangible firms.

Indirect Predictions.—The agency friction microfounds and endogenizes an external cost of finance. Traditional theory distinguishes between internal and external financing. The model here makes no distinction between cash holdings, lines of credit, debt, and equity.[10] But one would naturally expect that firms with greater agency frictions to choose the option to finance externally a greater cost than doing so internally. Put differently, firms that have a severe agency friction would find it marginally less worthwhile to receive outside financing from investors. Therefore, the ability to finance internally, say by retaining earnings and

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[9]I relegate the results on the size (market equity) characteristic to Appendix B as its ability to explain variation in returns has been questioned in recent years (van Dijk (2011), Fama and French (2012)).

[10]While much of the dynamic contracting literature focuses on decentralizing the optimal contract with simple securities, my setup is not conducive to this analysis, for reasons similar to those discussed in He (2009). My setup, however, is more closely related to those in investment and macroeconomics literature and therefore provides a step towards bridging these literatures.

holding them in cash, would be consistent with the model but only an indirect prediction of it.

I examine these in Table III. Indirect predictions on the frequency of external finance suggest these should be lower for firms with greater agency frictions. I construct a indicator for external financing that takes the value one if the firm issues common or preferred equity greater than 3 percent of its market value of equity in a given year (McKeon (2015)) or if long-term debt issuance is positive. The frequency of issuance falls as the portfolio's intangibility rises which is consistent with intuition of a growing divergence of interests between insiders and outsiders. Moreover, issuance size, as a fraction of firm market value, is decreasing across portfolios with the exception of the last portfolio. Smaller issuance sizes are consistent with a higher marginal cost of external finance, all else equal.

Related, if external financing has become more costly, it is natural to assume that internal financing has on the margin become more appealing. This speaks directly to the story of profitability described above. Consistent with this intuition, patterns in cash holdings increase across portfolios.

After completing this analysis on the characteristics of intangibility-sorted portfolio, in the next section I go into finer detail to test the effects of intangibility on physical investment at the firm-level.

D. FIRM-LEVEL RESULTS

Table V looks at local average effects from panel regressions. All regressions use firm fixed effects to control for unobserved differences across firms and to let variation within a firm determine average responses. Because variation could come from aggregate or industry-level trends, I cluster standard errors at the firm level.

The first specification documents the well-known result that average Q (weakly) positively correlates with investment (Summers (1981)): an unit increase in Q predicts a 1.67 percentage point rise in the investment rate. The second column documents the non-parametric result from portfolio sorts: an increase in intangibility, holding Q fixed, lowers the physical investment rate of a firm.

I then study the interaction of these variables to better uncover the underlying economic story. In column (3), the interaction coefficient on Q and intangibility is negative and statistically significant. Thus, the prediction is that an additional unit of Q's effect on

physical investment *decreases* as the intangibility of the firm rises.

$$\frac{\partial \mathbb{E}[i|X]}{\partial Q} = 2.095 - 0.116 \times n, \tag{35}$$

where the equation denotes the conditional expectation of investment rate, $\mathbb{E}[i|X]$, given the set of regressors, X. In a sense, n measures the wedge between first-best theory's prediction of the positive relationship between Q and investment. More intriguingly, the model measures n as the wedge between investors' and agents' interests.

Using the stationary distribution of firms, I replicate these panel regressions in the model. I do this simulating the model at a monthly frequency and keeping track of entry and exit of firms according to the policy listed in the calibration. Monthly investment is summed over the year and, as in the data, regressed on last year's lagged average Q and intangibility and a firm fixed effect.

I tabulate these regression coefficients for various specifications across the first-best and agency models. In the first-best model, Q is nearly a sufficient statistic for investment. The model confounds the distinction between Q and intangibility: they effectively provide the same information. This can be seen by adding both rows in column (2) under the first-best model, where the sum basically recovers the same coefficient in column (1). Holding Q fixed while increasing intangibility raises investment in first-best, and this is counterfactual to the data.

The main difference of the agency model from first-best is that it can recreate the empirical patterns of the negative coefficients on the interaction of intangibility with Q. The agency model is able to replicate intangibility's role as a wedge between Q and investment.

Altogether, I find again that agency frictions that are in some sense embedded in intangibles seem to be an important device to model the behavior of highly intangible firms. In the next section, I use the stationary distribution to examine the impact of a shift in the production function on several documented trends in the macroeconomics literature.

V. MACROECONOMIC IMPLICATIONS

Over the past four decades, there have been several macroeconomic trends within the United States to which the model can directly or indirectly speak:

- Higher Intangibility

- Decreased Physical Investment in the Face of Greater Profitability and Raised Valuations

- Increased Compensation

- Elevated Idiosyncratic Risk

- A Shift from External to Internal Finance

Hall (2001), Corrado and Hulten (2010), and many others, document the rising trend in intangibility of firms. And while a single explanation is unlikely to completely explain all of these facts, I show that embedding an agency friction into intangibility is consistent with all of them while first-best theory is not. I split the data sample into two subsamples, comparing statistics computed over 1975 to 1995 with those over 1996 to 2015.

I analyze these subsamples by conducting a comparative statics analysis of the stationary equilibrium (Hopenhayn (1992)). Steady-state analysis has been used in economics to study the long-run properties of dynamic models. I use it here to understand how a rise in intangibility affects the structural characteristics of industries and the distributions of intangibility, profits, compensation, value, and investment. Specifically, I calibrate the models to match only the average values of intangibility across both subsamples by only changing the parameter ϕ and re-calibrating the boundary curve to achieve super contact.

Table VI summarizes the results of the exercise. Average intangibility rose and the width of the distribution, measured by the interquartile range, spread out, as do the model counterparts. The spread in first-best is a full point off, in contrast to the agency model that discourages high levels of intangibility where the agency conflict is formidable. In response to greater intangibility, investors in the agency model compensate agents more, and thus the model can capture the rise in compensation as documented in (Frydman and Saks (2010)). Moreover, the interquartile range of compensation, too, has widened in the model, consistent with the evidence in Lustig et al. (2011).

Average Q and profitability increase while investment rates fall in both data and the agency model. Gutiérrez and Philippon (2017) and Crouzet and Eberly (2018) document that investment is weak relative to measures of profitability and valuation, particularly Tobin's Q, and that this weakness starts even before the new millenium. First-best cannot achieve this, as a higher value is impeded by a decline in profitability, altogether leading to a flat investment response counterfactual to the data.

Campbell et al. (2001) document a secular increase in the idiosyncratic returns of public firms and Andrei et al. (2018) report an improved relationship between investment and

average Q that they attribute to a growing volatility in Q. In the model, the instantaneous variance is

$$\text{var}_t \left(\frac{dQ}{Q} \right) = \left(\frac{p_n(n, w)n\sigma + (1 + p_w(n, w))\lambda(n)\sigma}{p(n, w) + w} \right)^2 dt, \qquad (36)$$

which is increasing in intangibility. Thus, the agency model is able to generate this pattern, although the magnitudes are lower, partly from the empirical measures capturing both systematic and idiosyncratic risk. By contrast, first-best shows a large decline, as $p_n(n)$ becomes flatter with higher n from decreasing returns to scale. Moreover, the average return across intangibility portfolios has increased only modestly, further challenging first-best models that appeal to an increase in discount rates as reconciling the evidence.[11]

I now turn to indirect predictions of the model: the shift from external to internal finance. As empirically documented by Bates et al. (2009) and Graham and Leary (2019), corporate cash holdings have increased over the sample period.[12] Moreover, the frequency of issuance falls over time. Consistent with these shifts and a growth in agency conflicts, marginal q has fallen in the model. The magnitude of the friction has risen by $0.56/0.32 - 1 = 75$ percent.

Related is an extensive margin of external finance: initial public offerings. Doidge et al. (2017) examine US stock market listings and describe a "gap" between the post-1996 experience and what historical norms would have dictated. I follow their work by examining the decline in the rate of offerings and the increase in intangibility by NAICS industry. Figure 3 depicts the result that shows that in the industries which generated the greatest increases in intangibility correspond to those with the largest declines in offerings. The specific correlation across each of the 10 industry's changes is -0.40. While not a direct prediction of the model, the shift from external to internal finance is consistent with a growing separation of interests between insiders and outsiders.

In sum, many of the aggregate trends and cross-sectional patterns are qualitatively consistent with a simple economic mechanism where an agency friction is embedded in intangibles.

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[11]More broadly, the evidence in DeLong and Magin (2009) and Avdis and Wachter (2017) suggest the equity risk premium of the entire market has probably declined.

[12]Related, Falato, Kadyrzhanova, Sim and Steri (2018) explicitly model this phenomenon by appealing to a decline in tangibility that leads to a shrinking in the debt capacity of firms. If is worth commenting on the difference between approaches. There, another dollar of internal cash is less expensive than external cash, as it is assumed to be costly. Thus, greater holdings lower the cost of capital of these firms. In my model, it is not the firm's cost of capital that changes, but rather the return on physical investment.

Conclusion

I embed an agency friction into a firm's intangibility. The idea follows from intangible capital being only indirectly observed by outsiders. It provides a microfoundation for the high profitability, high valuation, and low investment of highly intangible firms through the stability of the marginal cost of compensation sought in designing the optimal contract. Many of the models predictions are confirmed in the data.

I also embed firms' dynamic investment environment into industry equilibrium, bridging this literature with modern macroeconomics. Trends in investment, profitability, valuations, compensation, idiosyncratic risk, and financing suggest that the rise of intangibles have likely aggravated agency problems.

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A. TECHNICAL APPENDIX

A. EXTENSIONS UNDER FIRST-BEST

In this appendix I show that including either adjustment costs on intangible development or a risk premium as a function of intangibility does not change the qualitative implication of first-best theory that investment is monotonically related to intangibility. Table A-I summarizes the results. I find that adding risk premia can explain approximately 40 percent and development costs 54 percent of the slope of investment rates over intangibility. Neither extension completely explains the flatness of the slope in the data.

Development Adjustment Costs.—I include adjustment costs on the development of intangible capital with the function $d(g) = g + \kappa/2(g - \delta_N)^2$, where I use the same κ used for $c(i)$. This changes profitability to $Af(n) - c(i) - d(g)n$ and results in an additional first-order condition to (8) in the problem (7): $d'(g) = p'(n)$. As before, at high levels of intangibility the investment rate and Q are high. New is that development is also chosen to be low, and, given the quadratic adjustment cost, the loss on profitability is so great that investment is reduced relative to first-best to help offset the loss. Thus, this loss lowers the investment rate at high intangibility and flattens the investment profile across portfolios. Development costs flatten the slope by over 50 percent relative to the benchmark model.

Risk Premium on Intangibility.—To study the effects of a discount rate correlated with intangibility I posit a exogenous process for a stochastic discount factor, M, as $dM_t = -rM_tdt - \Gamma M_t dZ_t$, where Γ is the price of risk. Girsanov's transformation ($dZ_t^{\mathbb{Q}} = dZ_t + \Gamma dt$) allows me to solve for the value of the firm under the risk-neutral distribution, dZ_t^Q, where the firm's intangible capital evolves as $dN = (g - \delta_N - \Gamma\sigma)N_t dt + \sigma N_t dZ_t^Q$. Given this transformation, the analysis in Section I's first-best case goes through. This results in a risk premium of

$$\mathbb{E}_t[dR_t] - rdt = -\text{cov}_t\left(\frac{dM_t}{M_t}, \frac{dp(n_t)}{p(n_t)}\right) = \Gamma p'(n_t)n_t\sigma^2 dt \tag{A1}$$

that captures the idea that investors will demand compensation for exposure to intangibility.

I run two calibrations under this extension. I set $\Gamma = 0.53$ across both following Eisfeldt and Papanikolaou (2013). The first calibration changes the production function to linear by changing two values $(\epsilon, \phi) = (\infty, 1)$ to match the slope in risk premia across portfolios sorted on intangibility. The table below summarizes the targets of this calibration.

The top two rows take the values of intangibility (their O/K ratio) and risk premia attributed to organizational capital (net of market risk—4.4 percent) from Eisfeldt and Papanikolaou (2013), denoted EP. The third row reports my model's risk premia under a linear production function at Eisfeldt and Papanikolaou's (2013) values of intangibility. Finally, the fourth row repeats the exercise in the third but uses my benchmark calibration with a CES production function.

	Portfolio				
	1	2	3	4	5
Intangibility (EP)	0.19	0.42	0.66	1	1.65
Risk Premia (%, EP, Net of Market Risk)	0.05	1.07	1.86	2.79	4.29
Risk Premia (%, Perfect Substitutes)	0.42	1.22	1.82	2.82	4.55
Risk Premia (%, Calibration)	0.53	0.97	1.16	1.39	1.54

Similarly to including adjustment costs on development, the investment rate at a high level of intangibility falls when accounting for differences in risk. Altogether, attaching discounting to intangibility reduces the slope by approximately 40 percent. I take this as evidence that discount rates cannot fully explain the cross-sectional investment patterns in these intangibility portfolios.

B. DETAILS OF SOLUTION METHOD

The partial differential equation is

$$
\begin{aligned}
rp(n,w) = \max_i & \, Af(n) - c(i) - gn + p(n,w)(i-\delta) \\
& + p_n(n,w)(g - \delta_N - (i-\delta))n + p_w(n,w)(\gamma - (i-\delta))w \\
& + \frac{1}{2}p_{nn}(n,w)(\sigma n)^2 + \frac{1}{2}p_{ww}(n,w)\left(\lambda(n)\frac{\sigma}{g}\right)^2 + p_{nw}(n,w)\sigma n \lambda(n)\frac{\sigma}{g}. \quad \text{(A2)}
\end{aligned}
$$

I solve it with a finite difference method that approximates the function $p(n,w)$ on a two-dimensional non-rectangular grid: $n \in \{n_i\}_{i=1}^I$ and $w \in \{w_j(n_i)\}_{j=1}^{J^i}$, where I define $\overline{w}(n_i) = w_{J^i}(n_i)$. Each set of grid points along j, $w_j(n_i)$, depend on the value of n_i, because of the boundary curve $\{\overline{w}(n_i)\}_{i=1}^I$. The total number of grid points is thus $\sum_{i=1}^I J^i$.

I approximate first derivatives of p using both backward and forward differences and second derivatives with central differences. All differences of n and w are calculated respectively over the fixed increments Δ_n and Δ_w. The boundary conditions imply that

$p(n, 0) = l_K + l_N n \Rightarrow p(n_i, w_0) \approx l_K + l_N n_i$ and $p_w(n, \overline{w}(n)) = -1 \Rightarrow p(n_i, w_{J^i+1}) \approx$ $p(n_i, w_{J^i}) - \Delta_w$ under a forward difference, where both conditions hold for all i. I impose the usual reflecting boundaries for n: $p(n_0, w_j) = p(n_1, w_j)$ and $p(n_{I+1}, w_j) = p(n_I, w_j)$ for all j, so that the non-termination-boundary rows of the transition matrix $\mathbf{Q}$ below sum to zero, although in practice these are never reached because the solution employs an upwind difference scheme that selects a drift of n towards its interior. The termination boundary rows do not sum to zero as they measure the (absorbing) exiting mass of firms.

I use an implicit method, following Candler (1999) and Achdou, Han, Lasry, Lions and Moll (2017) who discuss the "verify" part of the solution, which solves the vector $p^{k+1} = (p_{1,1}^{k+1}, \ldots, p_{1,J^1}^{k+1}, p_{2,1}^{k+1}, \ldots, p_{2,J^2}^{k+1}, \ldots, p_{I,J^I}^{k+1})'$ where I use the notation $p_{i,j} = p(n_i, w_j)$.[13] It provides an implicit definition that begins with a guess $b = 1$ and proceeds to recursively iterate until convergence ($< 10^{-7}$) on the value function

$$p^{b+1} \left[\left(\frac{1}{\Delta} + r - (i - \delta_K) \right) - \mathbf{Q} \right] = Af(n) - c(i) - gn + p^b/\Delta + B, \qquad \text{(A3)}$$

where i solves (16) on each iteration, $\Delta > 0$ is the step size of the iterative method, and the $\sum_{i=1}^I J^i \times \sum_{i=1}^I J^i$ matrix $\mathbf{Q}$ is the transition matrix defined by the diffusion processes of

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[13]Concisely put, Barles and Souganidis (1991) show that if the solution methods satisfies monotonicity, stability, and consistency, then as Δ_n and Δ_w get small its solution converges locally uniformly to the unique viscosity solution. Here, monotonicity is ensured by the upwind scheme; stability, by the implicit method (uniformly bounded and independent of Δ_n and Δ_w); and consistency, by the backwards time-step of the iterative method.

the states n and w and the boundaries described above

$$\mathbf{Q} = \begin{bmatrix}
q_{1,1}^{ss} & q_{1,1}^{su} & 0 & \cdots & 0 & q_{1,1}^{us} & q_{1,1}^{uu} & 0 & \cdots & 0 & \cdots & 0 \\
q_{1,2}^{sd} & q_{1,2}^{ss} & q_{1,2}^{su} & \ddots & \vdots & q_{1,2}^{ud} & q_{1,2}^{us} & q_{1,2}^{uu} & \ddots & \vdots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \ddots & \vdots \\
0 & \cdots & \cdots & q_{1,J^1}^{sd} & q_{1,J^1}^{ss} & 0 & \cdots & \cdots & q_{1,J^1}^{ud} & q_{1,J^1}^{us} & \ddots & \vdots \\
q_{2,1}^{ds} & q_{2,1}^{du} & 0 & \cdots & 0 & q_{2,1}^{ss} & q_{2,1}^{su} & 0 & \cdots & 0 & \ddots & \vdots \\
q_{2,2}^{dd} & q_{2,2}^{ds} & q_{2,2}^{du} & \ddots & \vdots & q_{2,2}^{sd} & q_{2,2}^{ss} & q_{2,2}^{su} & \ddots & \vdots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \ddots & \vdots \\
0 & \cdots & \cdots & q_{2,J^2}^{dd} & q_{2,J^2}^{ds} & 0 & \cdots & \cdots & q_{2,J^2}^{sd} & q_{2,J^2}^{ss} & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & q_{I,J^I}^{sd} & q_{I,J^I}^{ss}
\end{bmatrix}. \quad \text{(A4)}$$

The matrix $\mathbf{Q}$ is the discretized analogy of the infinitesimal generator of (dn, dw): $\mathcal{A}\varphi(n,w)$ for some arbitrary function $\varphi(\cdot)$. The elements of $\mathbf{Q}$ are based on an upwind scheme and defined as

- $q_{i,j}^{ss} = -\max(\mathbb{E}_t[dw], 0)/\Delta_w + \min(\mathbb{E}_t[dw], 0)/\Delta_w - \max(\mathbb{E}_t[dn], 0)/\Delta_n + \min(\mathbb{E}_t[dn], 0)/\Delta_n - \mathbb{E}_t[dw^2]/\Delta_w^2 - \mathbb{E}_t[dn^2]/\Delta_n^2$

- $q_{i,j}^{su} = \max(\mathbb{E}_t[dw], 0)/\Delta_w + \mathbb{E}_t[dw^2]/(2\Delta_w^2)$

- $q_{i,j}^{sd} = -\min(\mathbb{E}_t[dw], 0)/\Delta_w + \mathbb{E}_t[dw^2]/(2\Delta_w^2)$

- $q_{i,j}^{us} = \max(\mathbb{E}_t[dn], 0)/\Delta_n + \mathbb{E}_t[dn^2]/(2\Delta_n^2)$

- $q_{i,j}^{ds} = -\min(\mathbb{E}_t[dn], 0)/\Delta_n + \mathbb{E}_t[dn^2]/(2\Delta_n^2)$

- $q_{i,j}^{uu} = q_{ij}^{du} = q_{ij}^{ud} = q_{ij}^{dd} = \mathbb{E}_t[dwdn]/(4\Delta_n\Delta_w)$,

where the conditional moments of state variables are $\mathbb{E}_t[dw] = (\gamma - (i - \delta_K))w$, $\mathbb{E}_t[dn] = (g - \delta_N - (i - \delta_K))n$, $\mathbb{E}_t[dw^2] = (\lambda(n)\frac{\sigma}{g})^2$, $\mathbb{E}_t[dn^2] = (\sigma n)^2$, and $\mathbb{E}_t[dwdn] = \sigma n \lambda(n)\frac{\sigma}{g}$.

Lastly, the stacked $\sum_{i=1}^{I} J^i$ vector of constants B, required by the boundaries in (17)

and (18), takes the form

$$
B = \begin{bmatrix}
(q_{1,1}^{dd} + q_{1,1}^{ud} + q_{1,1}^{sd}) \times (l_K + l_N n_1) \\
\vdots \\
(q_{1,J^1}^{su}) \times (-\Delta_w) \\
(q_{2,1}^{dd} + q_{2,1}^{ud} + q_{2,1}^{sd}) \times (l_K + l_N n_2) \\
\vdots \\
(q_{2,J^2}^{su}) \times (-\Delta_w) \\
\vdots \\
\vdots \\
(q_{I,J^I}^{su}) \times (-\Delta_w)
\end{bmatrix}, \tag{A5}
$$

with two final adjustments made to $\mathbf{Q}$ for the non-rectangular grid as it is an approximation to the upper payment reflecting boundary ($\{J_i\}$): first, $q^{du} \times (-\Delta_w)$ is added to each (i, J_i) where $i > 1$ and $J^i = J^{i-1}$; and second, for all $i < I$ where $J^i > J^{i+1}$ I add q^{ud}, q^{us}, and q^{uu}, each times $(-\Delta_w)$, for each grid point $j \in J^{i+1}, \ldots, J^i$. These final adjustments ensure that the non-termination-boundary rows of the transition matrix $\mathbf{Q}$ sum to zero.

Stationary Distribution.—The stationary distribution vector of length $\sum_i J^i$, $h(n, w)$, is calculated by solving, $h(n, w) = -(\mathbf{Q}^T)^{-1}\psi$, where ψ is the entry vector of length $\sum_i J^i$. The rows of ψ that are non-zero are determined by the assumed shape of the entry distribution that isolates n and the assumption on how agents' initial continuation utility w is determined. The normalization of $h(n, w)$ to one implies that the entry rate equals $m = -\sum_i \mathbf{Q}^T h(n, w)\Delta_w\Delta_n$. In the first-best model, there is no exit and I compute the stationary distribution $h(n)$ by solving an eigenvalue problem of the adjoint of the $I \times I$-sized transition matrix $\mathbf{Q}$: $\mathbf{Q}^T h(n) = \mathbf{0}$.

C. DISCUSSION OF ADVANTAGE OF SOLUTION METHOD

Much previous work in the dynamic contracting literature is restricted to analyses involving problems of single-state (or nested) ordinary differential equation(s) (ODEs) across exogenous states. Early contributions which developed recursive formulations of the contracting problem include Green (1987), Spear and Srivastava (1987), Phelan and Townsend (1991), and Atkeson (1991). The exceptions, to my knowledge, are Piskorski and Tchistyi (2010) and DeMarzo et al. (2012), who solve two nested ODEs where the two states are exogenous

to the problem.

New to the literature, I solve a partial differential equation (PDE) with endogenous boundaries on a non-rectangular grid. The contribution is more than simply adding another endogenous state variable: it opens a door for economists to inquire more deeply into a broader set of questions where transparent continuous-time setups require endogenous boundaries.

In DeMarzo et al. (2012), building on the prior solution technique of Piskorski and Tchistyi (2010), solve a system of two ODEs, one for a low (L) and high (H) productivity state. The optimal compensation policy requires a state-dependent compensation adjustment ψ that is defined by $p'_L(w_L) = p'_H(w_L + \psi_{LH}(w_L)) = p'_H(w_H) = p'_L(w_H + \psi_{HL}(w_H))$, which measures how the agent's compensation changes in response to a change in state (from L to H and vice versa). This is an adjustment that needs to be added to the *domain* of the numerical derivative of the principal's value function $p'(w)$. This adjustment is solved for numerically in conjunction with an iterative procedure that alternates on solving the ODEs and determining the free boundaries $\{\overline{w}_L, \overline{w}_H\}$.

The main advantage of my approach replaces the compensation adjustment with a second-order partial derivative p_{wn}, which is analogous to limit of the compensation adjustment as the jump between productivity states goes to zero. This obviates the need to numerically solve for ψ within each step of the solution. The advantage is particularly obvious when considering more than two states, as the number of compensation adjustments requiring a solution equals $S(S-1)/2$, where S is the number of states. These $S(S-1)/2$ adjustments need to be solved in addition to the S ODEs and S free boundaries. I conjecture this becomes infeasible once S becomes large.

I overcome this dimensionality problem by simply solving the PDE on a non-rectangular grid. I guess a boundary *curve*, the collection $\{\overline{w}(n)\}$, and for each guess solve the PDE, which solves very quickly with the implicit method. It is then computationally cheap to guess boundary curves until the super contact condition ($p_{ww}(n, \overline{w}(n)) = 0$ for each n) numerically satisfies some criterion. I guess a piece-wise quadratic function, requiring two sets of three parameters and a contact point. The state space and boundary curve are depicted in Panel B of Figure 1. The cost of the approach is having to program the matrix $\mathbf{Q}$, which requires care. Overall, I think the most important advantage is that the solution is feasible.

B. DATA CONSTRUCTION

I use all industrial, standard format, consolidated accounts of firms in Compustat. I only include firms with common shares (shrcd = 10 and 11) that trade on the NYSE, AMEX, and NASDAQ (exchcd = 1, 2, and 3). I remove firms with book assets (at) or gross property, plant, and equipment (ppegt) of less than 5 million. I exclude firms without a NAICS code and in the utilities (22), financial (52-53), other (91), and public (92) industries. I group agriculture (11) and mining (21), education (61) and health (62), and arts (71) and accommodation (72), leaving 10 industries.

Asset Turnover	= sales (sale(t)) / assets (at(t-1))
Average Q	= (market equity + long-term debt (dltt) + current debt (dlc)
	− inventory (invt)) / physical capital (ppegt)
Book Equity	= stockholders' equity (seq) - preferred stock
	+ deferred taxes and investment tax credits (txditc)
Cash to Assets	= cash (cash) / assets (at)
Compensation	= (salary (salary(t)) + bonus (bonus(t)) + LTIP (ltip(t)) + equity rewards(t))
	/ physical capital (ppegt(t-1))
Debt Issuance	= 1 if long-term debt issuance (dltis) > 0; 0 otherwise
Development Rate	= 0.3 × real SG&A (xsga(t)/cpi(t)) / Organizational Capital (N(t-1), see (28))
Equity Issuance	= 1 if sale of common and preferred stock (sstk)/market equity > 0.03;
	0 otherwise
Equity Rewards	= stock awards (stock_awards) + option awards (option_awards_blk_value)
Intangibility	= organizational capital (N, see (28))
	plus purchased intangible capital (intan) / real physical capital (ppegt/cpi)
Investment Rate	= physical investment (capx(t)) / physical capital (ppegt(t-1))
Issuance	= 1 if either equity issuance or debt issuance equal 1
Issuance Size	= (sstk×$\mathbf{1}$\{Equity Issuance\} + dltis×$\mathbf{1}$\{Debt Issuance\})
	/ (market equity + long-term debt (dltt) + current debt (dlc)
	− inventory (invt))
Market Equity	= price per share × shares outstanding
	(December values of abs(prc) x shrout from CRSP)
Preferred Stock	= Use the redemption value (pstkrv) , liquidation value (pstkl),
	book value (pstk), or zero, in decreasing order of preference
Profitability	= (EBITDA (ebitda(t)) - physical investment (capx(t))) / assets (at(t-1))
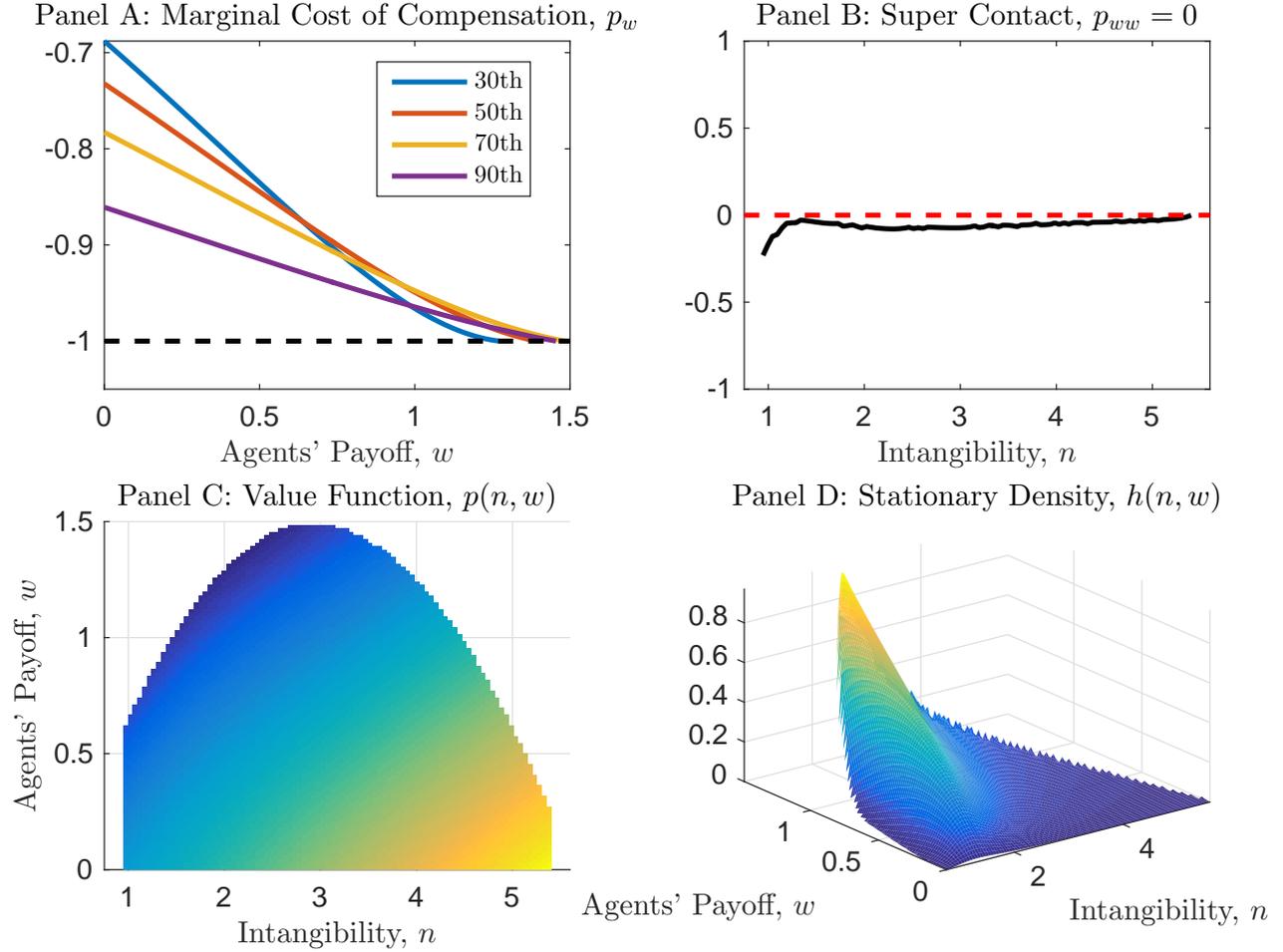
FIGURE 1: PROPERTIES OF MODEL SOLUTION

Panel A plots the first derivatives of investors' scaled value function with respect to w, agents' scaled continuation payoff. I denote the 30th, 50th, 70th, and 90th percentiles of the marginal distribution of intangibility. At the payout boundaries, smooth pasting holds: $p_w(n, \overline{w}(n)) = -1$. Panel B plots the investor's scaled value function, $P(K, N, W)/K$, as a function of intangibility $n = N/K$ and the agent's scaled continuation payoff $w = W/K$. The domain of the solution is non-rectangular. Panel C plots the second derivative of $p(n, w)$ with respect to w for each value of intangibility. This is the super contact condition on the payout boundary, $p_{ww}(n, \overline{w}(n))$. Panel D plots the stationary density of the model. In Panels C and D a brighter color represents a higher value.
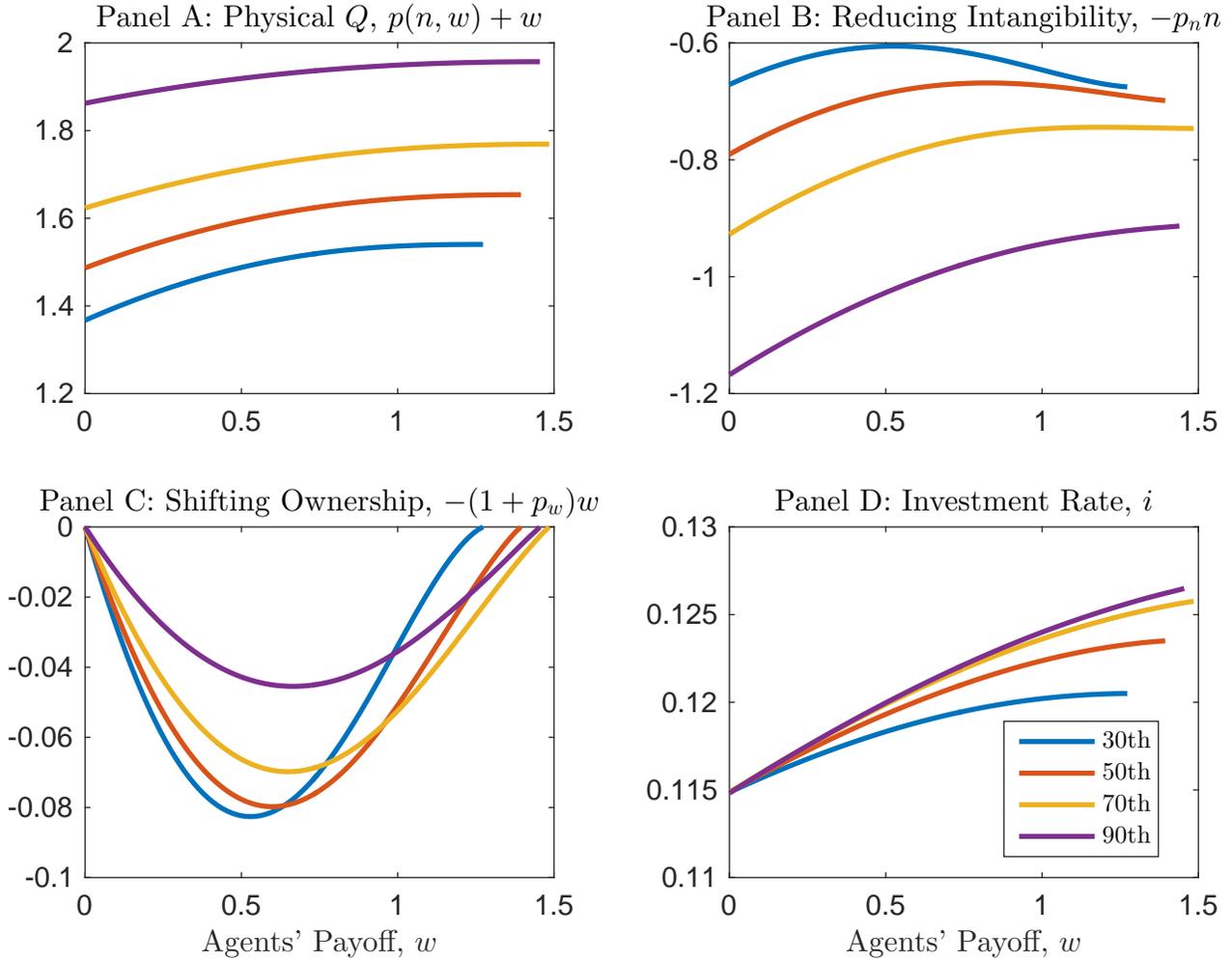
FIGURE 2: THE INVESTMENT DECISION

This figure decomposes the investment policy function: $c'(i) = p(n,w) + w - p_n(n,w)n - (1 + p_w(n,w))w$. I denote the 30th, 50th, 70th, and 90th percentiles of the marginal distribution of intangibility. Panel A plots average Q, $p(n,w) + w$. Panels B and C plot the cost of reducing intangibility, $-p_n n$, and shifting ownership, $-(1 + p_w(n,w))w$, respectively. Panel D plots the investment rate which, since $c(i)$ is quadratic, is linear in the sum of panels A + B + C.
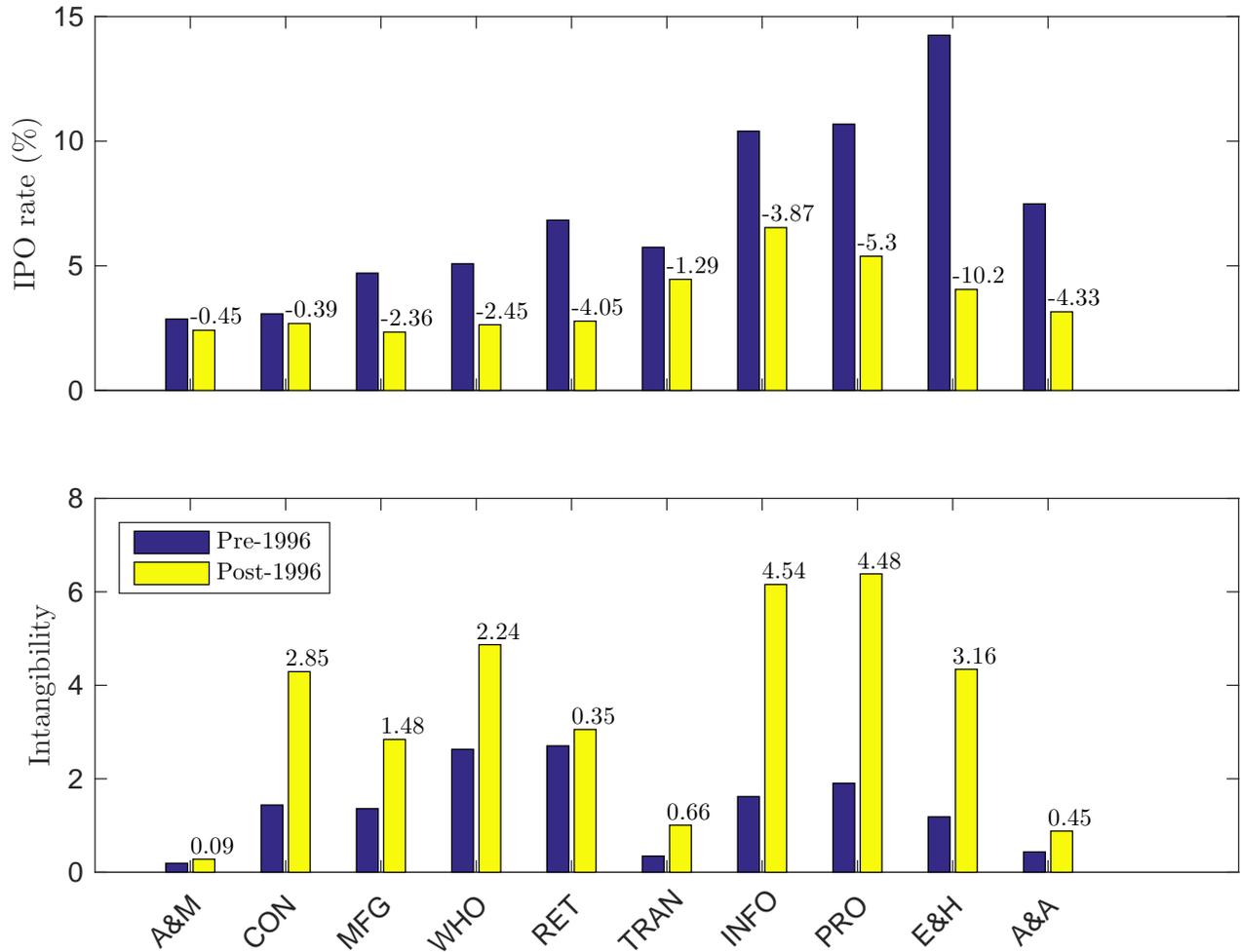
FIGURE 3: DECLINE IN OFFERINGS AND RISE IN INTANGIBILITY

This figure shows the decline in the rate of initial public offerings (IPOs) and the increase in the average intangibility of firms by industry. IPO data are from Jay Ritter's website. The IPO rate is the number of IPOs divided by the number of last year's listed firms. Construction of the intangibility (n) is the ratio of organization capital, constructed in 28, to property plants and equipment (Compustat's PPEGT). Differences between the pre-1996 and post-1996 values are listed above each industry's bar plots. Industry groups are based on 2-digit NAICS codes: 11–21, Agriculture and Mining (A&M); 23, Construction (CON); 31–33, Manufacturing (MFG); 42, Wholesale Trade (WHO); 44-45, Retail Trade (RET); 48–49, Transportation (TRAN); 51, Information (INFO); 54–56, Professional (PRO); 61–62, Education & Health (E&H); 71–72, Arts & Accommodation (A&A).

This table summarizes the calibrated parameters discussed in Section B.

Parameter	Value	Description/Target
Preferences		
(r, γ)	$(0.04, 0.08)$	Interest rate and agent compensation
$\lambda(n)$	$0.334n - 0.047n^2 + 0.003n^3$	See Section B and Table II
Technology		
(A, ϕ)	$(0.35, 0.75)$	Steady state intangibility (n^{ss}) and profitability
ϵ	0.8	Capital-skill complementarity
(δ_K, δ_N)	$(0.125, 0.15)$	Depreciation rates
g	0.17	Economy growth rate: $(g - \delta_N)$
κ	30	Adjustment cost parameter/Response of investment to q
σ	0.2	Variability of intangibility growth
(l_K, l_N)	$(0.7, 0.35)$	Recovery rates

TABLE II: ESTIMATION OF PRIVATE BENEFITS FUNCTION

Panel A reports the time series average values of the mean and standard deviation (in parentheses) of the standardized residuals obtained from regression (31). The residuals are grouped into portfolio defined by quintiles of a characteristic and are rebalanced every year. The four characteristics are intangibility, profitability, book-to-market, and asset turnover. Variable definitions are in Appendix B. Panel B reports the results of a regression of the standardized residuals on a polynomial expansion of intangibility across four orders. Robust standard errors are in parentheses.

Panel A: Conditional Distribution of Residuals				
Quintile	Intangibility	Profitability	BE/ME	Turnover
1	-0.43	-0.08	0.21	-0.07
	(0.37)	(0.85)	(1.16)	(0.91)
2	-0.25	0.2	0.17	0.19
	(0.51)	(1.18)	(1.06)	(1.10)
3	-0.04	0.11	0.15	0.17
	(0.77)	(0.98)	(0.95)	(1.10)
4	0.23	0.11	0.07	0.14
	(1.01)	(0.95)	(0.90)	(0.99)
5	0.88	0.21	-0.07	0.14
	(1.38)	(1.09)	(1.00)	(0.97)

Panel B: Functional Form Fitting				
	(1)	(2)	(3)	(4)
n	0.170	0.207	0.334	0.236
	(0.006)	(0.015)	(0.030)	(0.049)
n^2		-0.004	-0.047	0.014
		(0.002)	(0.009)	(0.027)
n^3			0.003	-0.008
			(0.001)	(0.005)
n^4				0.0010
				(0.0003)
Observations	28,840	28,840	28,840	28,840
$\bar{R}^2$	0.192	0.193	0.195	0.196

This table lists characteristics of portfolios sorted on intangibility. Firms are sorted into quintiles by intangibility within their two-digit NAICS code and are rebalanced every June. Portfolios are value-weighted by a firm's market capitalization based on these within-industry ranks. I report time series averages of median portfolio characteristics for all variables except for returns and issuance frequency which are averages and annualized. Variable definitions are in Appendix B. The sample period is from 1975 until 2015. In the model, intangibility is n, average Q is $p(n)$ or $p(n, w) + w$, investment rate is $i = I/K$, profitability is $Af(n) - c(i) - gn$, agents' flow share of the firm is $(1 - \alpha)\gamma w/(p(n, w) + w)$, and the agency friction, $\lambda(n)$, takes the form in (32).

		Portfolio				
		1	2	3	4	5
Data	Intangibility	0.2	0.8	1.4	2.3	4.2
	Physical Average Q	1.0	2.8	4.2	5.6	6.2
	Investment Rate (%)	10.5	13.2	14.0	13.7	13.1
	Profitability (%)	8.2	10.7	13.2	15.2	15.5
	Average Return (%)	11.8	11.1	12.8	13.2	11.6
	Development Rate (%)	18.3	20.0	20.6	20.0	19.6
	Compensation (%)	0.07	0.19	0.41	0.53	0.62
	Cash-to-Assets (%)	4.4	7.5	10.9	11.5	10.8
	Issuance Frequency (%)	87.6	79.3	71.5	73.1	71.3
	Issuance Size (%)	3.9	4.0	3.1	2.7	3.8
First-Best	Intangibility	0.5	1.0	1.5	2.1	3.4
	Average Q	0.9	1.8	2.4	3.0	3.7
	Investment Rate (%)	8.2	10.3	11.9	13.6	16.9
	Profitability (%)	1.0	7.2	9.8	10.7	5.9
Agency	Intangibility	1.5	1.9	2.3	2.7	3.5
	Average Q	1.4	1.5	1.6	1.8	2.0
	Investment Rate (%)	11.6	11.9	12.2	12.4	12.3
	Profitability (%)	10.1	12.0	13.0	13.8	14.3
	Compensation (%)	1.10	1.50	1.60	1.60	1.30
	Agency, $\lambda(n)$	0.41	0.49	0.55	0.62	0.72

This table lists characteristics of portfolios sorted on book-to-market ratios and intangibility. Firms are first sorted into portfolios on book-to-market ratios and then within each book-to-market portfolio are sorted by intangibility within their two-digit NAICS code. Portfolios are rebalanced every June and are value-weighted by a firm's market capitalization based on these book-to-market/within-industry ranks. Terciles are grouped into Low (0-33), Medium (34-66), and High (67-100). I report time series averages of median portfolio characteristics for all variables except returns and report average monthly returns, which I annualize. The sample period is from 1975 until 2015.

BE/ME	Intangibility			Intangibility		
	L	M	H	L	M	H
	Intangibility			Average Return (%)		
L	0.6	1.7	3.2	10.5	12.6	11.8
M	0.3	1.3	3.5	12.3	12.9	15.3
H	0.2	0.9	3.2	12.8	14.6	14.5
	Average Q			Investment Rate (%)		
L	2.9	6.3	7.8	15.0	15.1	14.5
M	1.0	1.8	3.6	9.9	12.0	11.9
H	0.6	0.8	1.7	9.7	10.3	11.6

TABLE V: INVESTMENT PANEL REGRESSIONS

This table reports empirical estimates of coefficients of the panel regression $I_{it+1}/K_{it} = f_i + \beta_i' X_{it} + \epsilon_{it}$, where i and t respectively index firm and year and f_i is a firm fixed-effect. The Compustat sample is from 1975 until 2015. Variable definitions are in Appendix B. All continuous variables are winsorized at the 5-95% level across all firm-year observations. Standard errors in parentheses are clustered at the firm level. I run annual panel regressions in the model using simulated monthly data and including firm fixed effects. Monthly investment is summed over the year and, as in the data, is regressed on last year's lagged average Q $(p(n,w) + w)$ and intangibility (n).

| | Investment Rate (%) | | | | | | | | |
| | Data | | | First-Best | | | Agency | | |
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Average Q	1.677	1.702	2.095	3.528	1.488	2.108	0.875	4.614	2.020
	(0.036)	(0.035)	(0.050)						
Intangibility		-0.195	0.604		1.659	-2.588		-1.244	0.724
		(0.070)	(0.091)						
$Q \times$ Intangibility			-0.116			0.846			-0.426
			(0.009)						
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	95,265	95,265	95,265						
Within R^2	0.129	0.129	0.136						

TABLE VI: MACROECONOMIC TRENDS

This table reports statistics computed for two subsamples: from 1975 to 1995 and from 1996 to 2015. In the data, within-firm Q variation is calculated by computing the volatility of the growth rate of average Q for every firm in my sample. This firm-level constant is then applied to every year the firm exists. For each year in each subsample, firms are grouped into one portfolio and weighted by market capitalization. I report time series averages of median or interquartile portfolio characteristics for all variables except for returns and issuance frequency which are averages and annualized. In the model, intangibility is n, average Q is $p(n)$ or $p(n, w) + w$, investment rate is $i = I/K$, profitability is $Af(n) - c(i) - gn$, compensation equals the agents' flow share of the firm, $(1 - \alpha)\gamma w / (p(n, w) + w)$, within-firm variation of Q is $\sqrt{\mathrm{var}_t(dQ/Q)}$, marginal q is $c'(i)$, and private benefits, $\lambda(n)$, take the form in (32).

	1975-1995			1996-2015		
	Data	First-Best	Agency	Data	First-Best	Agency
Direct						
Intangibility	1.1	1.1	1.1	2.4	2.4	2.4
Interquartile Range	0.5	1.8	1.1	2.1	3.9	2.1
Average Q	2.4	2.4	1.4	5.4	2.4	1.7
Investment Rate (%)	14.2	12.3	12.8	11.7	12.3	12.1
Profitability (%)	12.2	7.1	11.3	12.9	6.9	12.6
Compensation (%)	0.15		1.32	0.56		1.52
Interquartile Range	0.50		1.33	2.00		1.46
Within-Firm Q Variation (%)	26.9	52.8	9.3	34.9	33.4	11.5
Indirect						
Average Excess Return (%)	7.1			7.5		
Marginal q		0.94	1.09		0.93	0.89
Agency, $\lambda(n)$			0.32			0.56
Cash-to-Assets (%)	8.6			9.4		
Issuance Frequency (%)	79.8			73.5		

This table lists investment rates, in percent, of portfolios sorted on intangibility. Firms are sorted into portfolios by intangibility and rebalanced every June. I report the time series average of the portfolio's median investment rate. The sample period is from 1975 until 2015. The column (5-1) reports the slope: the average time-series difference between a portfolio 1 and portfolio 5. The last column reports the relative difference to the first-best model. Development Adjustment Costs includes the function $d(g)$ and endogenizes g. Risk Premia includes a price of risk of $\Gamma = 0.53$, following Eisfeldt and Papanikolaou (2013); Calibration refers to the benchmark calibration in Table I; Perfect Substitutes changes two parameters to $(\epsilon, \phi) = (\infty, 1)$.

	Portfolio						
	1	2	3	4	5	(5-1)	Relative
Data	10.5	13.2	14.0	13.7	13.1	2.6	
First-Best	5.8	9.4	11.8	14.5	20.0	14.2	0.0%
Development Adjustment Costs	2.2	3.7	4.8	5.8	8.6	6.4	-54.9%
Risk Premia (Perfect Substitutes)	-3.4	-1.9	-0.1	2.7	10.8	14.2	-0.1%
Risk Premia (Calibration)	-2.1	-0.4	1.2	3.0	6.5	8.6	-39.3%

TABLE A-II: DOUBLE SORTS ON MARKET EQUITY AND INTANGIBILITY

This table lists characteristics of portfolios sorted on market equity and intangibility. Firms are first sorted into portfolios on market equity and then within each market equity portfolio are sorted by intangibility within their two-digit NAICS code. Portfolios are rebalanced every June and are value-weighted by a firm's market capitalization based on these market equity/within-industry ranks. I report time series averages of median portfolio characteristics for all variables except returns and report average monthly returns, which I annualize. The sample period is from 1975 until 2015.

	Intangibility			Intangibility		
	L	M	H	L	M	H
ME						
	Intangibility			Average Return (%)		
L	0.4	1.4	3.7	15.2	17.1	16.2
M	0.4	1.4	4.1	14.3	14.8	14.5
H	0.3	1.4	3.0	11.3	11.7	12.7
	Average Q			Investment Rate (%)		
L	0.8	1.5	2.7	9.5	11.3	12.2
M	1.1	2.0	4.4	11.9	12.2	15.4
H	1.2	4.0	5.8	10.6	13.7	13.8

TABLE A-III: PORTFOLIOS SORTS ON AVERAGE Q

This table lists characteristics of portfolios sorted on physical average Q. Firms are sorted into portfolios by Q and rebalanced every June. Portfolios are value-weighted by a firm's market capitalization. I report time series averages of median portfolio characteristics for all variables except returns and report average monthly returns, which I annualize. The sample period is from 1975 until 2015.

	Portfolio				
	1	2	3	4	5
Intangibility	0.2	0.3	0.7	1.7	2.4
Average Q	0.6	1.0	1.8	3.5	9.9
Investment Rate (%)	9.2	9.4	11.5	13.2	17.4
Average Return (%)	13.3	13.0	11.6	12.3	11.8