Designing M&A Selling Mechanisms: Go-Shop Negotiations

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Abstract

In the past decade, a new selling procedure called "go-shop negotiation" has gained popularity in mergers and acquisitions. With a dynamic mechanism design approach, I fully characterize the target's revenue-maximizing mechanism, and find that it resembles a go-shop negotiation under certain parameter values; with other parameter values, it is similar to a standard auction or a traditional "no-shop negotiation". The relevant parameters include the correlation of bidders' valuations, due diligence cost, and expected gains from trade. The results are broadly consistent with empirical evidence, providing a potential explanation for the prevalence of go-shop negotiations in financial deals and distressed deals.

Keywords: M&A; Dynamic Mechanism Design; Costly Information Acquisition; Negotiations; Go-Shop Provisions

JEL Codes: G33, G34, D82, D86

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1 Introduction

An important yet understudied aspect of mergers and acquisitions (M&A) is the selling procedure, and it takes varying forms. Historically, standard auctions have been popular. Also, private negotiations with "no-shop" provisions that prevent the target from actively soliciting further bids are quite prevalent. However, in the past decade, a new selling mechanism called "go-shop negotiation" has gained popularity, especially in private equity deals and distressed M&A.

Go-shop negotiations emerged a few years prior to the 2006-2008 leveraged buyout boom. In this procedure, an *auction* is preceded by a *first round negotiation*, in which an initial offer is made, and a termination fee is promised to the initial bidder if the target switches to another buyer later. An example of this is the sale of CKE Restaurants to a private equity firm, Apollo Management. In September 2009, three private equity firms expressed interest in buying the target. The target's board then set up a special committee to privately negotiate with each of the private equity firms. At the completion of the negotiation, the target signed a tentative merger agreement with the highest bidder from amongst the three, Thomas H. Lee Partners. The agreement specified the following: first, a minimum bid $11.05 per share for that bidder; second, the target's right to solicit other bids in a go-shop period after announcing the agreement; and third, a termination fee to Thomas H. Lee in the event that the target was sold to another buyer during the go-shop period. Then, the target publicly announced the merger agreement in a press release. During the subsequent 40-day "go-shop period", the target hired the investment bank UBS to contact 24 private equity firms and four potential strategic buyers, and solicited their interest in making a higher proposal. The initial bidder was also included in the second-stage bidding game, and was able to match any new offers. Among the new bidders, a private equity firm, Apollo Management, topped the original offer with a bid of $12.55 per share; an offer with which Thomas H. Lee was not able to compete. As a result, Apollo Management won the deal, while the target paid the termination fee to the initial bidder. Practitioners also refer to such a mechanism as a "post-signing market check", because most market checks are conducted after signing a tentative merger agreement.

Go-shop negotiations are quite different from traditional mechanisms such as standard auctions or private negotiations with a no-shop provision, in terms of the timing of market checks and the level of competition after a merger agreement has been signed.[1] Having originated with deals involving private equity buyers, go-shop negotiations currently continue to be more prevalent in deals attracting *financial buyers*, the majority of which are private equity firms.[3] Empirical evidence suggests that the prevalence of go-shop negotiations is higher in deals attracting mostly financial buyers (15%) than in deals attracting mostly strategic buyers (3%).[4] The frequency of the use of a go-shop negotiation in the period 2002-2011 has also been higher in bankruptcy sales under Section 363 of Chapter 11 (84%) than in non-bankruptcy M&As (4%).[5]

The empirical evidence motivates two questions: *Why are go-shop negotiations observed in practice? And what drives the cross-sectional variation in the use of go-shop negotiations?*

Conventional wisdom emphasizes the revenue-boosting advantage of standard auctions, while attributing the use of sequential negotiations to agency conflicts.[6] This paper is the first theoretical work to show that a go-shop negotiation could arise as a part of an *optimal*

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[1] An auction conducts the market check on potential bidders *before* signing a merger agreement with the winning bidder, while a go-shop negotiation postpones most of the market check till *after* an initial tentative agreement has been signed. In a no-shop negotiation, the target cannot actively solicit bids after the merger agreement has been signed, and a termination fee much higher than that in a go-shop negotiation is imposed if the target accepts an unsolicited superior offer from another buyer. This results in a significantly lower level of bidder competition after the initial merger agreement has been announced compared to a go-shop negotiation, in the form of a lower chance of having a competing bids (Gogineni and Puthenpurackal [2017]). Moreover, no-shop negotiations are more prevalent in strategic deals as compared to financial deals (the frequency of use of no-shop negotiations is 61% in strategic deals and 27% in financial deals)[2], while go-shop negotiations are more popular in financial deals as opposed to in strategic deals. In addition, no-shop negotiations are in general more commonly used in practice than go-shop negotiations.

[3] The gains from trade between a strategic buyer and the target comes from combining the two businesses. The gains from trade between the target and a financial buyer stems from the financial buyer's ability to improve the target's value, by restructuring the target with acquisitions and sales, as well as by improving the target's corporate governance and capital structure. See Guo, Hotchkiss, and Song [2011].

[4] Source of data: MergerMetrics of FactSet, January 2003-June 2018.

[5] Source of data of bankruptcy sales: Gilson, Hotchkiss, and Osborn [2016]. Source of data for non-bankruptcy M&As: MergerMetrics 2002-2011.

[6] Bulow and Klemperer [2009] suggest that an auction generates a higher revenue for the seller than a sequential negotiation, because the former attracts more bidders. Denton [2008] asserts that a target management chooses a go-shop negotiation so as to favor a particular bidder, who may have promised the management a large compensation package. The author believes that the go-shop period in the go-shop negotiation mechanism is essentially "window-dressing" to reduce litigation risk (see also Antoniades, Calomiris, and Hitscherich [2014]). It has been heavily debated in courts as to whether or not a go-shop negotiation mechanism has fulfilled the *Revlon* duties that require the target management to maximize the shareholders' value. See Subramanian [2008] for comparison of the Delaware Chancery Court's decisions between *In re Topps Company Shareholders Litigation* and *In re Lear Corporation Shareholder Litigation*.

dynamic selling mechanism that maximizes the target's revenue. In particular, I build a model with two potential bidders and one seller, where the bidders' valuations for the target firm are positively correlated.[7] It is costly for the bidders to learn their valuations. In addition, without information acquisition, no bidder is willing to buy the target firm at a positive price. Solving for the optimal selling mechanism is complicated due to the large space of dynamic mechanisms. However, by deriving a specific version of dynamic revelation principle, I manage to simplify the space into a family of two-stage obedient truthful direct mechanisms.

I solve for the optimal mechanism, and find that the optimal mechanism resembles a go-shop negotiation under certain conditions of parameter values; with other parameter values, the optimal mechanisms are similar to common mechanisms such as standard auctions or private negotiations with a no-shop provision.

The optimal mechanism takes the following form when it resembles a go-shop negotiation. The target invites one bidder to *privately* and *non-verifiably* acquire information about its valuation on the target firm. After acquiring information (or not), the bidder then decides whether to accept some "floor price" for an auction later, in exchange for a termination fee. This acceptance decision, observable to the other bidder, reveals information about the initial bidder's valuations. Only if the first bidder accepts the floor price, the second bidder has the opportunity to acquire information. Finally, there is an auction between the two bidders.

A go-shop negotiation has two key features: interim information transmission and a termination fee. Why are they important ingredients of an optimal mechanism? Here is some intuition. First, in this dynamic mechanism, the acceptance or rejection of the floor price after information acquisition reveals information about the first bidder's valuation of the target firm. Due to the correlated values, the first bidder's acceptance of a floor price signals favorable information about the deal to the second bidder. Suppose the cost of information acquisition is too high, and the second bidder is originally too pessimistic about the gains from trade, so that the second bidder is not willing to acquire information at all. Then this good signal may encourage it to acquire information and improves bidder participation, while the rejection of a floor price cannot hurt participation further. This "option-like" structure

[7] The setting of correlated information is the main deviation from the literature that makes my results different from the conventional wisdom. See more discussions in the related literature.

makes interim information transmission profitable to the target. Second, a termination fee is a necessary compensation to the initial bidder. The termination fee is payable to the first bidder if the bidder loses to the second bidder in the auction later, conditional on the first bidder accepting the floor price. For sufficiently high due diligence costs, the initial bidder is willing to acquire information and leak it to a potential competitor only if the termination fee is lucrative enough. The termination fee introduces asymmetry into the subsequent auction: the second bidder has to bid significantly higher than the initial bidder in order to win. Such "unfairness" against the second bidder causes it to pay more, but this does not distort allocation efficiency; as long as the game proceeds to the auction stage, the target firm is always sold to the bidder with the higher valuation.

In order for the optimal mechanism to take the form of a go-shop negotiation, three conditions are needed.

First, the correlation across bidders' valuations needs to be sufficiently positive. A high valuation of the first bidder has two competing effects on the second bidder: an encouragement effect due to correlated values, and a deterrence effect due to the competition between the two. When the correlation is positive enough, the encouragement effect dominates, so that the second bidder is motivated to join the competition if the first bidder accepts the floor price.

Second, the information acquisition cost should be sufficiently high, without being prohibitive. Specifically, it must be high enough that it is ex ante unprofitable to acquire information about the target. Such a high cost creates an option-like feature, and it makes the information transmitted from the first bidder to the second beneficial to the target. However, it cannot be too large, otherwise information acquisition is too costly to implement even in a go-shop negotiation.[8]

Third, the prior belief about the target firm should be sufficiently pessimistic, but not being excessive either. The argument is similar to that of the information acquisition cost.

For parameter combinations such that a go-shop negotiation is no longer optimal, I also characterize the optimal mechanism as one of four possible forms. In particular, when the cost of information acquisition is sufficiently low, the optimal mechanism resembles a standard English auction. When the cost increases, the optimal mechanism is a sequential negotiation with only partial information revelation about the initial bidder's decision. When

[8]In this case, the seller approaches at most one bidder for information acquisition.

the cost increases and the bidders' valuations are less positively correlated, the optimal mechanism reduces to sequential posting prices, which resembles a "no-shop negotiation" (private negotiation with a *no-shop* provision). When the cost is even higher, only one bidder is given a posting price and the other bidder is ignored. Finally, the target does not approach any bidder if the cost is prohibitively high.

The model has several empirical implications. First, go-shop negotiations are used more often when bidders' valuations for the target firm are more positively correlated, and when the cost of information acquisition is higher. This prediction provides a potential explanation for the prevalence of go-shop negotiations among financial deals (15%) as opposed to strategic deals (3%), and in distressed M&As (84%) as compared to non-distressed M&As (4%). Indeed, since the business models of financial buyers are very similar, their potential gains from trade with the target are also likely to be highly correlated. This view is shared by Gorbenko and Malenko [2014], and Leslie and Oyer [2008]. Therefore, the average within-deal bidder value correlation should be higher for financial deals, because such deals typically attract buyers with very similar business models.[9] For the case of distressed deals, Nesvold, Anapolsky, and Lajoux [2010] claim that compared to non-distressed M&As, buyers face more opaque and confusing data provided by the target. In addition, they have to process such confusing information under immense time pressure. Therefore, the cost of information acquisition is likely to be higher in a distressed M&A, making it necessary to use a go-shop negotiation (or in their language, a "stalking-horse auction") to boost information acquisitions.

Second, this paper argues that the uses of go-shop provisions are potentially due to target revenue maximization, and this is broadly consistent with Gogineni and Puthenpurackal [2017]. In particular, they found that deals with go-shop provisions attract significantly more competing bids than deals with no-shop provisions. Also, the initial bid premiums are higher in go-shop negotiation deals, and the market reacts more favorably to such deals as well. Moreover, go-shop provisions are more likely in deals with greater institutional

[9] I estimated the within-deal correlation of bid premia, using the hand-collected data in Gorbenko and Malenko [2014] and a Random Effect ANOVA (Analysis of Variance) model. Indeed, I found that the estimated within-deal correlation of bid premia is higher for deals won by a financial buyer than that of deals won by a strategic buyer. In particular, the correlation for deals won by a financial buyer is 0.96, which is higher than that of strategic deals (0.81), even if taking into account the 90% confidence interval. Note that deals won by a financial (strategic) buyer typically attract mostly financial (strategic) buyers. The details of the estimation is available upon request.

ownership, where the agency problems of the target management are potentially less severe.

Finally, the model predicts that no-shop negotiations are less likely to appear when bidders' valuations for the target firm are more positively correlated. This is consistent with the fact that no-shop negotiations are used less often in financial deals (27%) than in strategic deals (61%).

The rest of the paper is organized as follows. Following a literature review, Section 2 describes the model setup. Section 3 derives a dynamic revelation principle in order to simplify the dynamic mechanism design problem. Section 4 solves for the optimal selling mechanism. Section 5 illustrates how to implement the optimal mechanism using common mechanisms observed in practice (e.g. go-shop negotiations). Section 6 studies the comparative statics regarding relevance of go-shop negotiations, and shows that the implications are broadly consistent with empirical evidence. Section 7 concludes.

Related Literature To the knowledge of the author, this is the first theoretical work to rationalize a go-shop negotiation as a part of an optimal mechanism.

Since a go-shop negotiation is a type of sequential negotiation, this paper is related to the literature that studies the preemptive bidding feature of sequential negotiations, including Fishman [1988], Bulow and Klemperer [2009], and Roberts and Sweeting [2013]. These papers show that a high first bid would preempt the second bidder from learning their values and participating in the game. As a major difference between my paper and their work, I allow for a positive correlation between bidders' information, while they restrict bidders' signals to be independent. In my paper, the preemptive bidding effect is indeed salient when the correlation is low; however, when the correlation is sufficiently high, a high initial bid would encourage the second bidder to acquire information.

This paper belongs to the broader literature of sequential selling mechanisms. Bulow and Klemperer [1996] compare an English auction to the optimal mechanism with one less bidder, and show that the seller's revenue in the former is almost always higher due to the extra bidder. Therefore they conclude that if a sequential mechanism attracts less bidders than an English auction, it is always dominated by the auction revenue-wise. However, their paper did not analyze how the choice of mechanism affects the number of bidders participating, which is the focus of my paper. Indeed, under my setting, a sequential mechanism could actually attract *more* bidders than a standard auction, therefore generating a higher revenue.

In an environment where bidders are *asymmetrically* informed and information acquisition is *costless*, Povel and Singh [2006] show that the optimal mechanism for the seller is a sequential bidding game *without* a termination fee. In my paper, however, costly information acquisition is essential; also, the bidders are ex ante symmetric, although the optimal mechanism may be asymmetric. Betton, Eckbo, and Thorburn [2009] consider a sequential negotiation model of hostile takeover with independent bidder values and toehold. Opp and Glode [2017] study the trading protocol for the sale of a financial asset. They compare the social welfare between a sequential trading game and a static auction, in a setting where the two buyers' and the seller's values for the security are interdependent. On the contrary, my paper aims to explain corporate transactions. Therefore, I assume that the seller's stand-alone value (market capitalization before merger) is common knowledge, where the bidders' values are potentially correlated. In addition, I focus on seller revenue optimization instead of social welfare. Finally, instead of comparing two particular mechanisms, I solved for the optimal mechanism. Vasu [2018] compares the seller's revenue in a first-price auction to that of a sequential negotiation, and shows that the comparison depends on the number of potential bidders.

This paper is related to the studies on the benefit of information revelation. Milgrom and Weber [1982], and Eső and Szentes [2007] investigate the information disclosure in auctions, and show that more information revelation increases the seller's revenue. However, their models do not include costly information acquisition; in my paper, sufficiently high information acquisition cost is a necessary condition for information revelation to be valuable to the seller.

Another related literature is mechanism design with costly information acquisition. Persico [2000] compares the amount of information acquisition in first-price and second-price sealed-bid auctions, while my paper studies on the optimal selling mechanism instead of the comparison between particular mechanisms. Bergemann and Välimäki [2002] show that with costly information acquisition, ex ante and ex post efficiency generally cannot be reconciled in a common value setting; my paper focuses on the optimal selling mechanism instead of the social optimum. Gershkov and Szentes [2009] consider the optimal voting mechanism with costly information acquisition. However, as a voting model, they do not allow for transfers; in addition, they also investigate the social optimum.

With correlated signals between bidders, the paper is linked to the theoretical work on correlated information such as Crémer and McLean [1988]. However, my model also considers endogenous costly information acquisition, which is absent in their model. This is the reason why in my model the bidders' rents may not be fully extracted.

The intuition of this paper is connected to the incomplete contract literature on exclusivity, such as McAfee and Schwartz [1994], and Segal and Whinston [2000]. In their work and this paper, a certain extent of exclusivity reduces the hold-up problem and encourages the agents to exert more effort. However, the non-contractible effort in the exclusivity literature usually refers to the investment to reduce production costs, while in my model, such effort is about information acquisition. The nature of such effort leads to implications regarding information revelation and the correlation of bidder values, which were not present in the literature of exclusivity.

Other related theoretical works include the IPO under-pricing literature such as Sherman and Titman [2002] and Sherman [2005]. They show that IPO under-pricing serves to compensate the primary dealers for information acquisition about the quality of the issued equity, and such information is revealed to the secondary market investors by the bids of primary dealers. However, IPO under-pricing models involve the resale between buyers in the primary market and the secondary market, while my model involves only the primary market and no resale among bidders. The paper is also linked to work on multi-stage auctions such as Ye [2007], and tender offer auctions such as Schwartz [1986], and Berkovitch, Bradley, and Khanna [1989].

Finally, the paper is connected to the following empirical studies about mergers and acquisitions. First, it is related to Gorbenko and Malenko [2014] who investigate the difference between financial bidders and strategic bidders. They find that the valuations of financial bidders are less dispersed than those of strategic bidders, consistent with the explanation that different financial bidders applying similar post-acquisition strategies and each strategic bidder having relatively unique synergies. Their finding is in line with my argument that since the valuations of financial bidders are more correlated, go-shop negotiations can boost the target's revenue in financial deals.

Second, it is connected to a (surprisingly) small empirical literature on go-shop provisions, which are a key feature of go-shop negotiations. Denton [2008] states that go-shop is chosen over standard auctions due to agency conflicts between the target management and the shareholders. Antoniades, Calomiris, and Hitscherich [2014] believe that the over-use of go-shops reflects excessive concerns about litigation risks, possibly resulting from lawyers' conflicts of interest in advising targets. Subramanian [2008] and Jeon and Lee [2014] claim that go-shop provisions may benefit the seller compared to deals with no-shop provisions. Gogineni and Puthenpurackal [2017] examine whether go-shop provisions in merger agreements are used to benefit target shareholders or for agency/entrenchment reasons. Their results indicate that go-shops are effective contractual devices used to further target shareholder interests. Gilson, Hotchkiss, and Osborn [2016] discuss the "stalking horse auctions" in the bankruptcy process, which are essentially go-shop negotiations.

Other related empirical literature includes Burch [2001], Officer [2003], and Boone and Mulherin [2007a] who examine termination fees and other deal protections, Boone and Mulherin [2007b] who compare multi-bidder takeover deals with single-bidder takeover deals, and Aktas, De Bodt, and Roll [2010] who find that the threat of an auction following a one-to-one negotiation improves the bid premium in the negotiation.

2 Model Setup

There is a target firm with two potential bidders. All of them are risk neutral. The bidders' valuations on the target firm depend on their expertise in creating values if merging with the target firm. As an example, for strategic buyers, such expertise may include the ability to utilize the economics of scale or scope. For financial buyers, their expertise mainly involve the skills to create a more efficient management and a better capital structure. Although each bidder could be competent in their own ways in creating values through the merger, their expertise might still share a common component. For instance, different strategic bidders could be from the same industry; various financial bidders might also adopt similar post-merger practices.

To capture such a valuation structure, I decompose Bidder i's valuation on the target firm, u_i, into a correlated part and an idiosyncratic part:[10]

$$u_i = \underbrace{v_i}_{\text{correlated}} + \underbrace{w_i}_{\text{idiosyncratic}} \quad \text{for } i = 1, 2.$$

The first part v_i represents the expertise *similar* across the two bidders, with marginal distribution

$$v_i \in \begin{cases} -Z & \text{with probability } 1-p, \\ 0 & \text{with probability } p, \end{cases}$$

where $Z > 0$, $p \in (0, 1)$. The terms v_1 and v_2 are positively correlated, with coefficient $Corr(v_i, v_j) = \rho \in [0, 1)$. The second part w_i represents the *idiosyncratic* expertise across the bidders, with conditional distribution

$$\begin{aligned} &\Pr(w_i = 0|v_i = -Z) = 1; \\ &\Pr(w_i = h|v_i = 0) = q, \ \Pr(w_i = l|v_i = 0) = 1 - q, \end{aligned}$$

where $0 < l < h$, $q \in [0, 1]$, and w_i is independent of w_{-i} for $i = 1, 2$. In addition, w_i is independent of v_{-i} conditional on v_i. As a result, w_i is independent of u_{-i} conditional on v_i.[11]

Therefore, the valuation u_i $(i = 1, 2)$ takes three possible values in the set $U \equiv \{-Z, l, h\}$. The joint distribution of u_1 and u_2 is illustrated in Table 1. The constant b is defined as $\Pr(v_i = 0, v_{-i} = -Z)$, which equals to $p(1-p)(1-\rho)$. Note that the correlation of bidders' valuations could reach as low as zero when ρ reaches zero, however large the value of Z takes.

The target's stand-alone value is normalized to zero, so are the bidders' outside options if not acquiring the target firm. Such normalization is for simplicity only, and it does not affect the results.[12]

[10]Note that the valuation is assumed to be decomposed into a *correlated* and an idiosyncratic part instead of a *common* and an idiosyncratic part. This approach is completely due to the convenience to study the comparative statics regarding the correlation between the bidders' valuations. It does not affect the qualitative results.

[11]The assumption that w_i's distribution depends on v_i is not important. In an alternative setting where w_i is independent of v_i and follows marginal distribution $\Pr(w_i = h) = q$, $\Pr(w_i = l) = 1-q$, the qualitative results remain to be the same, but the illustration of the results is more complex.

[12]In particular, this setting is equivalent to the following. The target's stand-alone value is $m \geqslant 0$, and each bidder's stand-alone value is $n \geqslant 0$. That is, the target retains value m if no trade takes place, and Bidder i gets n if not merged with the target. If the merger eventually occurs between Bidder i and the

		u_2		
		$-Z$	l	h
u_1	$-Z$	$1-p-b$	$b(1-q)$	bq
	l	$b(1-q)$	$(p-b)(1-q)^2$	$(p-b)q(1-q)$
	h	bq	$(p-b)q(1-q)$	$(p-b)q^2$

Table 1: The joint distribution of u_1 and u_2. The constant b is defined as $\Pr(v_i = 0, v_{-i} = -Z)$, which equals to $p(1-p)(1-\rho)$.

While the outside options of the target and the bidders are commonly known to all, the target does not know the bidders' valuations. The bidders do not know their valuations ex ante; however, if invited by the target, a bidder has the option to learn its own valuation u_i after incurring an information acquisition cost $c > 0$. The action of information acquisition is *unobservable* and *unverifiable*. Formally, define $A = \{0, 1\}$ as the space of possible information acquisition choices. Bidder i can take a *hidden* action $a_i \in A$, with $a_i = 1$ representing the costly information acquisition conducted, and $a_i = 0$ implying otherwise.

The target commits to a dynamic selling mechanism which contains a game with potentially infinite stages. At each stage, the target can invite either 0, 1, or 2 bidders to acquire information, and then discloses certain information about the history observed by the target. After that, each invited bidder decides whether to acquire information, and takes an action (such as making a bid) based on the information it has. The bidders have the options to exit the mechanism, both before the decisions of acquiring information, and before taking actions. The target decides when it is the last stage to invite bidders. At the end of the last stage, the target determines the allocation of the target firm among the players, and the payments (or transfers) from the bidders to the target. If exiting the mechanism before the end of the last stage, a bidder is assigned with zero probability of winning the target firm and zero payment. This excludes entry fees from the model; in fact, entry fees are rarely seen in M&A.[13] The target designs the selling mechanism to maximize expected revenue,

target, then the merged firm has a value of $n + m + u_i$, where u_i follows the same distribution of the original setting. However, it now represents the gains from trade between Bidder i and the target, namely, the value created from the merger. Bidder i's value of the target firm is now represented by $m + u_i$. In the Appendix, we show that the problem in the new setting is equivalent to that of the original setting.

[13]In practice, this is likely due to the concern of a fake sale. In particular, a "target" firm can pretend to up for sale. After collecting the entry fees from the bidders, the firm can then announce that none of the bids are high enough and cancel the sale. If allowing for entry fees, the main qualitative results should not change. Negative entry fees to subsidize information acquisition are not necessary, since the action of information

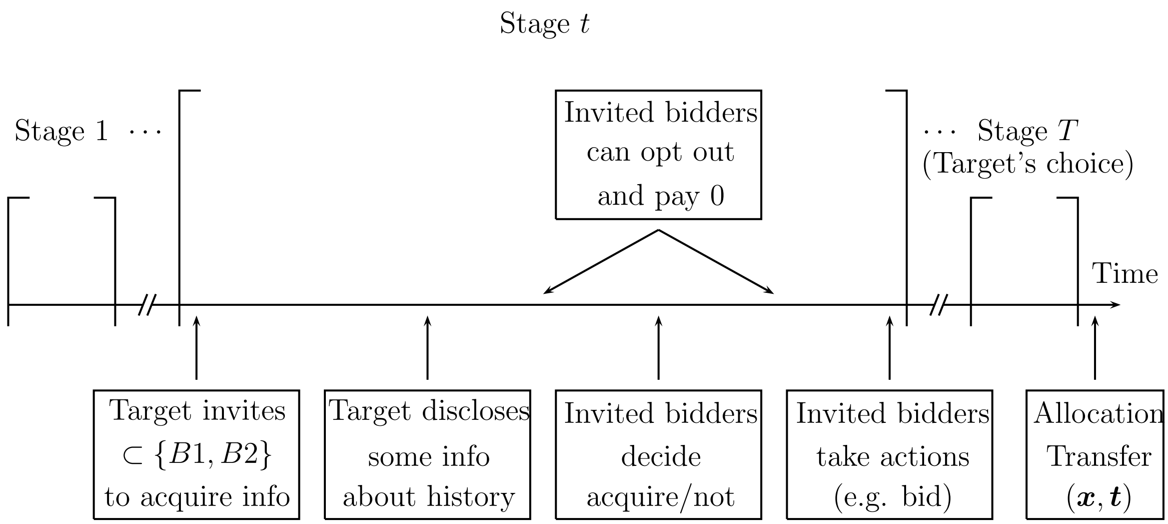

Figure 1: Timeline of a dynamic selling mechanism.

and the bidders make their decisions to maximize their expected payoffs. All players are patient. The timeline of a dynamic selling mechanism is illustrated in Figure 1.

As in the traditional mechanism design literature, the first step of optimization is to apply the revelation principle and optimize within a certain class of simpler mechanisms. However, since the mechanism design problem is dynamic, the standard argument from the static revelation principle does not apply. Therefore we need to derive a particular version of revelation principle for the problem, which is the task of the following section.

3 The Dynamic Revelation Principle

The space of the dynamic mechanisms is huge. Therefore, a *dynamic revelation principle* is needed to reduce the space. In particular, I introduce a family of simple dynamic mechanisms, called *canonical mechanisms*, to be defined below. I then show with a dynamic revelation principle that it is without loss of generality to optimize within this family, if imposing certain simplifying restrictions on the outcomes to be implemented.

acquisition is unobservable and unverifiable. Positive entry fees could potentially be used to extract bidders' ex ante rent. However, this will not affect the qualitative results about a go-shop negotiation. This is because in most of the parameter range where a go-shop negotiation arises as a part of the optimal mechanism, the bidders' ex ante rents are already fully extracted, even without entry fees.

Definition 3.1 (Canonical Mechanism) *A canonical mechanism has the following time line, where the target's actions are deterministic:*
(i) The target recommends Bidder 1 *whether to acquire information.*
(ii) Bidder 1 *decides whether to follow the recommendation; then it reports valuation* $\hat{u}_1 \in U$ *to the target;*
(iii) The target recommends Bidder 2 *whether to acquire information;*
(iv) Bidder 2 *decides whether to follow the recommendation; then it reports valuation* $\hat{u}_2 \in U$ *to the target;*
(v) The target determines that allocations and transfers.
The recommendation of information acquisition is of pure-strategy, and the target does not reveal any information other than the recommendations. In addition, the bidders obey the recommendations and report their valuations truthfully. Finally, the allocations and transfers for both bidders are measurable w.r.t. Bidder i*'s information under the target's recommendation. That is, they are constant whenever Bidder* i *is not recommended to acquire information, for* $i = 1, 2$.

Furthermore, for any Perfect-Bayesian-Nash-Equilibrium implemented by a dynamic mechanism, define a *type-acquisition correspondence* as follows:

Definition 3.2 *A type-acquisition correspondence* (U, A, R) *is a correspondence from Bidder* i*'s type space* $U = \{-Z, l, h\}$ *to Bidder* $-i$*'s information acquisition choice space* $A = \{0, 1\}$. *Here* R *is the relation between the element* $u_i \in U$ *and the element* $a_{-i} \in A$, *and such a relation exists if and only if the probability of Bidder* $-i$*'s information acquisition being* a_{-i} *conditional on Bidder* i*'s valuation being* u_i *is strictly positive.*

The following proposition states the dynamic revelation principle. According to the proposition, it is without loss of generality to optimize within the family of canonical mechanisms if implementing only certain types of outcomes.

Proposition 3.1 (Dynamic Revelation Principle) *Suppose a dynamic mechanism implements an equilibrium, of which the type-acquisition correspondence is a mapping, for* $i = 1, 2$. *Then the target's revenue in the equilibrium can also be achieved by a canonical mechanism.*

The basic idea of the proof of Proposition 3.1 is to show that it is without loss of generality to restrict attention to the mechanisms in which: (1) the target invites bidders sequentially;

(2) the target recommends bidders whether to acquire information, and the bidders follow the recommendation; (3) instead of taking actions, the bidders reports their valuations truthfully; (4) the target reveals no information other than the recommendations; (5) there are only two stages, where at each stage the target makes a recommendation to a bidder, and the bidder reports the valuation type; (6) the two-stage mechanism is deterministic, in that the target does not mix between recommending to acquire or not, nor does the target randomize the order of approaching the bidders; (7) Bidder 1 is the first bidder to receive a recommendation. Only the proof of (6) requires focusing on equilibrium outcomes with the type-acquisition correspondence being mapping.

The restriction is imposed only for the purpose of simplicity, and relaxing the restriction should not change the qualitative result. To see the intuition of the restriction, consider an example of an equilibrium where Bidder 1 may acquire information ahead of Bidder 2, and Bidder 1's action taken (such as a bid) is (partially) revealed to Bidder 2 by the target before Bidder 2 is invited to acquire information. Then Bidder 2's information acquisition choice a_2 may depend on Bidder 1's valuation u_1. The restriction allows only outcomes where a *single* type of Bidder 1's valuation does not correspond to *multiple* levels of Bidder 2's information acquisition; or intuitively speaking, equilibrium outcomes implemented by mechanisms with a "deterministic" nature. That is, in such a equilibrium, the bidders do not randomize when acquiring information or taking actions; the target invites the bidders with a deterministic order, and it does not randomize between different signals when revealing the history to the bidders. Without such a restriction, the target may achieve a higher revenue than that of the optimal mechanism in this paper, but the qualitative results should remain the same.[14]

[14]In particular, according to Step (1) to (5) of the proof of Proposition 3.1, for any outcome where the correspondence is not a mapping, there still exists a two-stage mechanism where the target recommends bidders for information acquisition, potentially based on the reported types in the past. However, the target may mix between recommending to acquire or not; the target might also randomize the order of approaching the bidders. In these cases, the probability of information acquisition increases continuously as the cost c increases; with pure-strategy recommendations and deterministic order, however, it increases in a discrete manner. As a result, the optimal mechanism also alters in a more gradual way as the parameters vary.

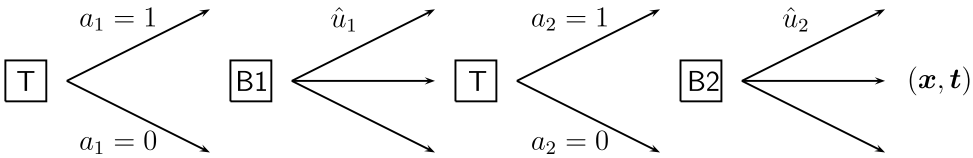

Figure 2: Timeline of a canonical mechanism

4 Solving for the Optimal Mechanism

Now we can formally define the key variables and solve for the optimal canonical mechanism. In a canonical mechanism, the message space is the type space

$$U = \{-Z, l, h\}.$$

The choice space of information acquisition is

$$A = \{0, 1\}.$$

A canonical mechanism is a quadruple $\big(\boldsymbol{x}(\cdot,\cdot), \boldsymbol{t}(\cdot,\cdot), a_1, a_2(\cdot)\big)$, where $\boldsymbol{x}(\hat{u}_1, \hat{u}_2)$ is the allocation vector if the reported valuations are $\hat{u}_1$ and $\hat{u}_2$, $\boldsymbol{t}(\hat{u}_1, \hat{u}_2)$ is the transfer (payment) vector, a_1 is the recommended acquisition decision for Bidder 1, and $a_2(\hat{u}_1)$ is the recommended acquisition decision for Bidder 2 given Bidder 1's report $\hat{u}_1$. In addition, we have $\hat{u}_i \in U$ and $a_i \in A$, for $i = 1, 2$. The time-line of a canonical mechanism is summarized by Figure 2.

The target's objective is to maximize the total payment collected from the two bidders, $\mathbb{E}_{\boldsymbol{u}}\big[t_1(\boldsymbol{u}) + t_2(\boldsymbol{u})\big]$, subject to the following constraints. First, $(IC1)$ and $(IC2)$ are the incentive compatibility (truth-telling) conditions for Bidders 1 and 2. That is, Bidder i with true type u_i weakly prefers to report truthfully than reporting any other type u_i', for $i = 1, 2$.

$$(IC1) \qquad \mathbb{E}_{u_2}\left[x_1(\boldsymbol{u})u_1 - t_1(\boldsymbol{u})\middle|u_1\right] \geqslant \mathbb{E}_{u_2}\left[x_1(u_1', u_2)u_1 - t_1(u_1', u_2)\middle|u_1\right], \forall u_1, u_1', \text{ if } a_1 = 1,$$

$$(IC2) \qquad \mathbb{E}_{u_1}\left[x_2(\boldsymbol{u})u_2 - t_2(\boldsymbol{u})\middle|u_2, a_2(u_1) = 1\right]$$
$$\geqslant \mathbb{E}_{u_1}\left[x_2(u_1, u_2')u_2 - t_2(u_1, u_2')\middle|u_2, a_2(u_1) = 1\right], \forall u_2, u_2'. \tag{1}$$

Second, $(IR1)$ and $(IR2)$ are the *interim* individual rationality conditions for Bidders 1 and 2, that is, both bidders have the option to exit the mechanism after learning their values. Since no entry fee is allowed, a bidder obtains its outside option (zero) if exiting the mechanism.

$$\begin{aligned}
(IR1) \qquad & \mathbb{E}_{u_2}\left[x_1(\boldsymbol{u})u_1 - t_1(\boldsymbol{u})\middle|u_1\right] \geqslant 0, \forall u_1, \text{ if } a_1 = 1,\\
& \mathbb{E}_{\boldsymbol{u}}\left[x_1(\boldsymbol{u})u_1 - t_1(\boldsymbol{u})\right] \geqslant 0, \text{ if } a_1 = 0,\\
(IR2) \qquad & \mathbb{E}_{u_1}\left[x_2(\boldsymbol{u})u_2 - t_2(\boldsymbol{u})\middle|u_2, a_2(u_1) = 1\right] \geqslant 0, \forall u_2,\\
& \mathbb{E}_{\boldsymbol{u}}\left[x_2(\boldsymbol{u})u_2 - t_2(\boldsymbol{u})\middle|a_2(u_1) = 0\right] \geqslant 0, \forall u_2.
\end{aligned} \tag{2}$$

Third, $(OB1)$ and $(OB2)$ are the obedience constraints to ensure that each bidder follows the recommendation whether to acquire information.

$$\begin{aligned}
(OB1) \qquad & \mathbb{E}_{\boldsymbol{u}}\left[x_1(\boldsymbol{u})u_1 - t_1(\boldsymbol{u})\right] - c\\
& \qquad \geqslant \max\left(0, \mathbb{E}_{\boldsymbol{u}}\left[x_1(u_1', u_2)u_1 - t_1(u_1', u_2)\right]\right), \forall u_1', \text{ if } a_1 = 1,\\
(OB2) \qquad & \mathbb{E}_{\boldsymbol{u}}\left[x_2(\boldsymbol{u})u_2 - t_2(\boldsymbol{u})\middle|a_2(u_1) = 1\right] - c\\
& \qquad \geqslant \max\left(0, \mathbb{E}_{\boldsymbol{u}}\left[x_2(u_1, u_2')u_2 - t_2(u_1, u_2')\middle|a_2(u_1) = 1\right]\right), \forall u_2'.
\end{aligned} \tag{3}$$

Finally, $(MS1)$ and $(MS2)$ correspond to the measurability properties stated in Definition 3.1, i.e. allocations and transfers should not respond to a bidder's report if that bidder is recommended not to acquire information. (FE) states the feasibility constraints on the allocations $\boldsymbol{x}$.

$$\begin{aligned}
(MS1) \qquad & \boldsymbol{x}(u_1, u_2) = \boldsymbol{x}(u_1', u_2), \boldsymbol{t}(u_1, u_2) = \boldsymbol{t}(u_1', u_2), \forall u_1, u_1' \text{ if } a_1 = 0,\\
(MS2) \qquad & \boldsymbol{x}(u_1, u_2) = \boldsymbol{x}(u_1, u_2'), \boldsymbol{t}(u_1, u_2) = \boldsymbol{t}(u_1, u_2'), \forall u_2, u_2' \text{ if } a_2(u_1) = 0\\
(FE) \qquad & x_1(\boldsymbol{u}) \in [0,1],\ x_2(\boldsymbol{u}) \in [0,1],\ x_1(\boldsymbol{u}) + x_2(\boldsymbol{u}) \leq 1, \forall \boldsymbol{u}.
\end{aligned} \tag{4}$$

To summarize, the target's problem is as follows:

$$\begin{aligned}
\max_{x(\cdot,\cdot), t(\cdot,\cdot), a_1, a_2(\cdot)} \ & \mathbb{E}_{\boldsymbol{u}}\left[t_1(\boldsymbol{u}) + t_2(\boldsymbol{u})\right]\\
\text{s.t. } & (1), (2), (3), \text{ and } (4).
\end{aligned} \tag{5}$$

Notice that when bidders are recommended not to acquire information, the incentive compatibility and obedience constraints are omitted. Incentive compatibility without information acquisition is ensured by $(MS1)$ and $(MS2)$, because their reports do not affect the outcome. Obedience constraints, when recommended not to acquire information, are also ensured by $(MS1)$ and $(MS2)$ because when the outcomes do not respond to reported types, the bidders will not incur the cost to learn their true types.[15]

4.1 Preliminary Analysis

I focus on the range of parameters specified in Assumption 4.1, where bidders' priors about gains from trade are pessimistic. That is, without information acquisition, a bidder's expected valuation is negative, even if knowing the other bidder has a positive valuation. It is stated formally in the following assumption.

Assumption 4.1 (Pessimistic prior)

$$\mathbb{E}[u_i|u_{-i} > 0] < 0.$$

Assumption 4.1 ensures that no bidder would submit a serious bid without information acquisition.[16] It is equivalent to[17]

$$Z > \left(\frac{1}{(1-p)(1-\rho)} - 1\right)(qh + (1-q)l).$$

It also implies that for the target, there are no gains from trade in expectation with an uninformed bidder. In addition, $-Z < 0$ implies that this is also true for an informed bidder with $u_i = -Z$. Hence, the target firm should not be sold inefficiently to such bidders, neither is any expected payment necessary. The following lemma formalizes this argument.

[15] Also, $(IC1)$, $(IR1)$ and $(OB1)$ are not written separately for regions where Bidder 2 is recommended to acquire information versus where Bidder 2 is not. When calculating Bidder 1's expected payoff where Bidder 2 is not recommended to acquire information, it is without loss of generality to assume that Bidder 2 nevertheless knows the true type of u_2 and reports truthfully. This is because in such a region, the outcomes are constant with respect to Bidder 2's report. Such a way of writing the constraints are completely for the purpose of a concise illustration, without affecting the results.

[16] Alternative assumptions such as bidders' risk aversion or ambiguity aversion should have a similar effect, but such assumptions would complicate the analysis.

[17] $\mathbb{E}[u_i|u_{-i} > 0] = \Pr(v_i = 0|u_{-i} > 0)\mathbb{E}[v_i + w_i|v_i = 0, u_{-i} > 0] + \Pr(v_i = -Z|u_{-i} > 0)\mathbb{E}[v_i + w_i|v_i = -Z, u_{-i} > 0] = (p-b)/p(qh + (1-q)l) + b/p(-Z)$, where $b = (1-p)p(1-\rho)$.

Lemma 4.1 *For any solution to Problem (5),*
(i) $x_i(\boldsymbol{u}) = 0$, $\forall \boldsymbol{u}$ *s.t.* $a_i = 0$; $\mathbb{E}_{\boldsymbol{u}}[t_i(\boldsymbol{u})\big|a_i = 0] = 0$.
(ii) $x_i(\boldsymbol{u}) = 0$, $\forall \boldsymbol{u}$ *s.t.* $u_i = -Z$; $\mathbb{E}_{\boldsymbol{u}}[t_i(\boldsymbol{u})\big|u_i = -Z] = 0$.

Now consider a new set of constraints, which is a subset of the constraints in Problem (5). The new (IC) constraints only require that a type $u_i = h$ does not mimic a type $u_i = l$, for $i = 1, 2$.

$$\begin{aligned}
(IC1) \qquad & \mathbb{E}_{u_2}\left[x_1(\boldsymbol{u})u_1 - t_1(\boldsymbol{u})\big|u_1\right] \\
& \geqslant \mathbb{E}_{u_2}\left[x_1(u_1', u_2)u_1 - t_1(u_1', u_2)\big|u_1\right], \text{for } u_1 = h, u_1' = l, \text{ if } a_1 = 1, \\
(IC2) \qquad & \mathbb{E}_{u_1}\left[x_2(\boldsymbol{u})u_2 - t_2(\boldsymbol{u})\big|u_2, a_2(u_1) = 1\right] \\
& \geqslant \mathbb{E}_{u_1}\left[x_2(u_1, u_2')u_2 - t_2(u_1, u_2')\big|u_2, a_2(u_1) = 1\right], \text{for } u_2 = h, u_2' = l.
\end{aligned} \tag{6}$$

For bidders that have acquired information, the new (IR) constraints only ask a type l to have a non-negative payoff.

$$\begin{aligned}
(IR1) \qquad & \mathbb{E}_{u_2}\left[x_1(\boldsymbol{u})u_1 - t_1(\boldsymbol{u})\big|u_1\right] \geqslant 0, \text{ for } u_1 = l, \text{ if } a_1 = 1, \\
& \mathbb{E}_{\boldsymbol{u}}\left[x_1(\boldsymbol{u})u_1 - t_1(\boldsymbol{u})\right] \geqslant 0, \text{ if } a_1 = 0, \\
(IR2) \qquad & \mathbb{E}_{u_1}\left[x_2(\boldsymbol{u})u_2 - t_2(\boldsymbol{u})\big|u_2, a_2(u_1) = 1\right] \geqslant 0, \text{ for } u_2 = l, \\
& \mathbb{E}_{\boldsymbol{u}}\left[x_2(\boldsymbol{u})u_2 - t_2(\boldsymbol{u})\big|a_2(u_1) = 0\right] \geqslant 0, \forall u_2.
\end{aligned} \tag{7}$$

The new (OB) constraints require the equilibrium payoff of a bidder recommended to acquire information to be no less than that of quitting the mechanism (zero).

$$\begin{aligned}
(OB1) \qquad & \mathbb{E}_{\boldsymbol{u}}\left[x_1(\boldsymbol{u})u_1 - t_1(\boldsymbol{u})\right] - c \geqslant 0, \text{ if } a_1 = 1, \\
(OB2) \qquad & \mathbb{E}_{\boldsymbol{u}}\left[x_2(\boldsymbol{u})u_2 - t_2(\boldsymbol{u})\big|a_2(u_1) = 1\right] - c \geqslant 0.
\end{aligned} \tag{8}$$

And the new (MS) and (FE) constraints are the same as (4).

Define a *relaxed* version of Problem (5) as below:

$$\max_{\substack{x(\cdot,\cdot),t(\cdot,\cdot)\\ a_1,a_2(\cdot)}} \mathbb{E}_{\boldsymbol{u}}\big[t_1(\boldsymbol{u}) + t_2(\boldsymbol{u})\big]$$

$$\text{s.t. (6), (7), (8), and (4).} \tag{9}$$

Compared with the original problem, several constraints have been removed or modified. First, (IRi) with $u_i = h$ are implied by other constraints, and hence are deleted. The justification is provided in the Appendix, by Lemma A.1. Second, certain constraints are ignored for now, and will be verified later. In particular, (ICi) of type $u_i = l$ to mimic type $u_i = h$ is ignored, as is typically true in a hidden-type problem. (ICi) of any other types mimicking $u_i = -Z$ are also ignored. Intuitively, no allocation or transfer is necessary if the reported type is $u_i = -Z$. Therefore, such incentive compatibility conditions could be implied by the individual rationality conditions of $u_i = -Z$. Moreover, we ignore (ICi) of type $u_i = -Z$ to mimic type $u_i = h$ and $u_i = l$, for $i = 1, 2$. Intuitively, when $-Z$ is low enough, the loss from buying an unwanted firm should be large enough to dwarf any monetary reward[18] to a type $u_i = l$ or $u_i = h$. Finally, we relax the (OBi) and replace the term *max* with 0. Intuitively, the expected valuation of an uninformed bidder is so low, that the loss from mimicking a winning higher type is too costly compared to any potential monetary reward from the target if there is any.

The following lemma claims that to find the solution for the original problem (Problem (5)), it is sufficient to first solve the relaxed problem (Problem (9)) with certain conditions imposed, then verify that the resulting solution satisfies all constraints in the original problem.

Lemma 4.2 *Suppose a mechanism is a solution to the problem that imposes the following additional constraints to the relaxed Problem (9):*
(i) $x_i(\boldsymbol{u}) = 0$ *and* $t_i(\boldsymbol{u}) = 0$, $\forall \boldsymbol{u}$ *s.t.* $u_i = -Z$ *or* $a_i = 0$*;*
(ii) $a_2(h) = 0$.
Moreover, suppose this mechanism satisfies all constraints in the original problem, Problem (5). Then it is also a solution for Problem (5).

Lemma 4.2 implies it is without loss of generality to consider the relaxed problem (9) only,

[18] Such monetary reward may be used by the target to incentivize bidders to acquire information.

and to restrict attention to mechanisms that have (i) $x_i(\boldsymbol{u}) = 0$ and $t_i(\boldsymbol{u}) = 0$ whenever $u_i = -Z$ or $a_i = 0$, and (ii) $a_2(h) = 0$. The intuition for (i) is similar to that of Lemma 4.1; that is, no allocation or transfer is necessary for a bidder that expects no gain from trade with the target. The intuition for (ii) is that if Bidder 1 is the highest type h already, then due to cost efficiency, there is no need to ask Bidder 2 to acquire information.

With the above simplifications, we can rule out some nuisance cases of the optimal contract. The possible shapes of optimal canonical mechanisms are significantly reduced, taking one of the following forms.

Lemma 4.3 *It is without loss of generality to restrict attention to the following candidates of the optimal canonical mechanisms:*
Candidate (I): Bidder 1 *acquires information; Bidder* 2 *acquires if* $u_1 = -Z$ *or* $u_1 = l$*.*
Candidate (II): Bidder 1 *acquires information; Bidder* 2 *acquires only if* $u_1 = l$*.*
Candidate (III): Bidder 1 *acquires information; Bidder* 2 *acquires only if* $u_1 = -Z$*.*
Candidate (IV): Only Bidder 1 *acquires information.*
Candidate (V): No bidder acquires information.

4.2 Optimal Mechanism

To solve for the optimal mechanism, I take three steps. First, for the relaxed problem (9) with the two conditions in Lemma 4.2 being imposed, I find the optimal mechanism *within* each of the five forms described in Lemma 4.3; second, I compare the revenues for the five optimal candidate mechanisms to find the globally optimal mechanism for the relaxed problem; third, I verify that the optimum obtained in the second step also satisfies all constraints in the original problem (5). By Lemma 4.2, such optimum is also the optimum for the original problem.

To illustrate the analysis, I focus on the second form of mechanism, Candidate (II), i.e. Bidder 1 acquires information and Bidder 2 is asked to acquire only if $u_1 = l$. The reason is that this mechanism is closely related to the “go-shop negotiations” and the “stalking-horse auction” we observed in practice, as we will see in Section 5. The analysis of the four other forms is similar and would be provided in the Appendix.

To further simplify, for the rest of the paper we restrict attention to the parameter range with

$$l - qh > 0, \ q \leq 1/2, \ \text{and} \ Z > \frac{3h}{2p(1-p)}. \tag{10}$$

Intuitively, the first condition implies that l is high enough, so that the target is better-off setting a low price and selling the firm for sure, rather than setting a high price and selling the firm only when the type turns out to be h (with a probability q). The second condition implies that the idiosyncratic part is less likely to take the higher value than the lower value. The third condition ensures that under the optimal solution obtained in this paper, a buyer with negative gains from trade with the target, e.g. a buyer with type $u_i = -Z$ or an uninformed buyer, does not mimic a type l or h. Intuitively, when $-Z$ is negative enough, the loss from buying the target is very high. Although a buyer with such negative expected value could benefit potentially from a termination fee by mimicking a positive-value type, the risk of winning is too detrimental compared to such benefit. Without the three simplifying assumptions, the qualitative results will not change, but the optimal mechanism will take a more complicated form.

In a Candidate (II) mechanism, by definition we have $a_1 = 1$, $a_2(l) = 1$ and $a_2(u_1) = 0, \ \forall u_1 \neq l$. Combining with condition (i) Lemma 4.2, we get

$$\begin{aligned} &x_1(-Z, u_2) = 0, \ t_1(-Z, u_2) = 0, \ \forall u_2 \\ &x_2(l, -Z) = 0, \ t_2(l, -Z) = 0 \\ &x_2(u_1, u_2) = 0, \ t_2(u_1, u_2) = 0, \ \forall u_1 \neq l, \ \forall u_2. \end{aligned} \tag{11}$$

Certain notations could be simplified. $(MS1)$ and $(MS2)$ imply that the outcomes should not respond to a bidder's reported type if that bidder is recommended not to acquire information. That is, $x_1(u_1, u_2)$ and $t_1(u_1, u_2)$ are constant in u_2 when $u_1 \neq l$. Therefore, I denote $x_1(u_1, u_2)$ as $x_1(u_1, 0)$ and $t_1(u_1, u_2)$ as $t_1(u_1, 0)$, $\forall u_1 \neq l$, $\forall u_2$, where "0" indicates that Bidder 2 is recommended not to acquire information ($a_2 = 0$).

As a result, the target's objective function in the optimization problem (9) becomes

$$\begin{aligned} &\Pr(u_1 = -Z) \cdot 0 + \Pr(u_1 = h) t_1(h, 0) + \Pr(u_1 = l, u_2 = -Z) t_1(l, -Z) \\ &\quad + \Pr(u_1 = l, u_2 = l)(t_1(l, l) + t_2(l, l)) + \Pr(u_1 = l, u_2 = h)(t_1(l, h) + t_2(l, h)). \end{aligned}$$

The constraints are as follows. For $i = 1, 2$, Bidder i with $u_i = h$ should not mimic $u_i = l$. The corresponding incentive compatibility constraints read

$$\begin{aligned} x_1(h,0)h - t_1(h,0) \geqslant\ & \Pr(u_2 = -Z|u_1 = h)(x_1(l,-Z)h - t_1(l,-Z)) \\ & + \Pr(u_2 = l|u_1 = h)(x_1(l,l)h - t_1(l,l)) \\ & + \Pr(u_2 = h|u_1 = h)(x_1(l,h)h - t_1(l,h)), \\ x_2(l,h)h - t_2(l,h) \geqslant\ & x_2(l,l)h - t_2(l,l). \end{aligned}$$

For $i = 1, 2$, individual rationality constraints of Bidder i with $u_i = l$ are

$$\begin{aligned} & \Pr(u_2 = -Z|u_1 = l)(x_1(l,-Z)l - t_1(l,-Z)) \\ & \quad + \Pr(u_2 = l|u_1 = l)(x_1(l,l)l - t_1(l,l)) \\ & \quad + \Pr(u_2 = h|u_1 = l)(x_1(l,h)l - t_1(l,h)) \geqslant 0, \\ & x_2(l,l)l - t_2(l,l) \geqslant 0. \end{aligned}$$

Note that the individual rationality constraint for Bidder 1 if $a_1 = 0$ is removed, since such case does not exist in a Candidate (II) mechanism. The individual rationality constraint for Bidder 2 when $a_2 = 0$ is also omitted, because it is implied by (11). The obedience constraints for the two bidders are

$$\begin{aligned} & \Pr(u_1 = -Z)0 + \Pr(u_1 = h)(x_1(h,0)h - t_1(h,0)) \\ & \quad + \Pr(u_1 = l, u_2 = -Z)(x_1(l,-Z)l - t_1(l,-Z)) \\ & \quad + \Pr(u_1 = l, u_2 = l)(x_1(l,l)l - t_1(l,l)) \\ & \quad + \Pr(u_1 = l, u_2 = h)(x_1(l,h)l - t_1(l,h)) \geqslant c, \\ & \Pr(u_2 = l|u_1 = l) \cdot (x_2(l,l)l - t_2(l,l)) \\ & \quad + \Pr(u_2 = h|u_1 = l)(x_2(l,h)h - t_2(l,h)) + \Pr(u_2 = -Z|u_1 = l)0 \geqslant c. \end{aligned}$$

Finally, we have the feasibility constraints:

$$x_i(u_1, u_2) \in [0, 1],\ x_1(u_1, u_2) + x_2(u_1, u_2) \leq 1, \forall u_1, u_2$$

The above is a linear programming problem, therefore it is straightforward to solve.

Similarly, we can also compute the optimal mechanisms within the other four candidate mechanisms. Comparing those optimal mechanisms, we can find the globally optimal mechanism for the relaxed problem (9) with the two conditions in Lemma 4.2 imposed. Such a mechanism can be verified to satisfy all constraints in the original problem (5), hence it is also a solution to the original problem according to Lemma 4.2. The form of the optimal mechanism for Problem (5) is illustrated in Proposition 4.1 below, while the exact solution is demonstrated in the proof of the proposition in the Appendix.

Recall c represents the cost of information acquisition; ρ is the correlation between the two bidders' similar expertise (v_1 and v_2), where a higher ρ indicates a higher correlation between the bidders' valuations. In addition, p is the probability that there are gains from trade between a bidder and the target. In the (c, ρ) space, define the following five regions:

$$\begin{aligned}
&\text{Region (I)} : c < \min(c_3(\rho, p), \max(c_1(\rho, p), c_2(\rho, p)));\\
&\text{Region (II)} : \rho > \frac{lp}{lp + (h-l)q},\\
&\qquad\qquad c \in [\max(c_1(\rho, p), c_2(\rho, p)), \min(c_3(\rho, p), c_4(\rho, p)));\\
&\text{Region (III)} : \rho \leq \frac{lp}{lp + (h-l)q},\\
&\qquad\qquad c \in [c_3(\rho, p), c_2(\rho, p));\\
&\text{Region (IV)} : \rho < \min\left(1, \frac{lp}{(h-l)q(1-p)}\right),\\
&\qquad\qquad c \in [\max(c_2(\rho, p), c_3(\rho, p)), c_5(p));\\
&\text{Region (V)} : c \geqslant \max(c_5(p), c_4(\rho, p)),
\end{aligned} \tag{12}$$

where $c_1(\rho, p) = \frac{(h-l)pq(1-q)+bhq(1+q)+bl(1-q-q^2)}{2-pq}$, $c_2(\rho, p) = \frac{b(l+(h-l)q)}{1-p}$, $c_3(\rho, p) = \frac{(h-l)(p-b)q}{p}$, $c_4(\rho, p) = \frac{hq(-b+p(2-q)+bq)+l(1-q)(p(1-q)+bq)}{1+p(1-q)}$, $c_5(p) = p(l + (h-l)q)$, and $b = \rho(1-p)p$.

Proposition 4.1 (Global Optimum) *The optimal mechanism for Problem (5) takes the form of a Candidate (i) mechanism in Region (i), for* $i = I, II, III, IV, V$.

Figure 3 illustrates each of the different regions in (12) and Proposition 4.1. There are two panels in Figure 3 since the figure takes slightly different forms when the parameters vary, but the intuitions for the two panels are essentially the same. When the information acquisition cost c is low, Bidder 1 is always asked to acquire information, and Bidder 2 is

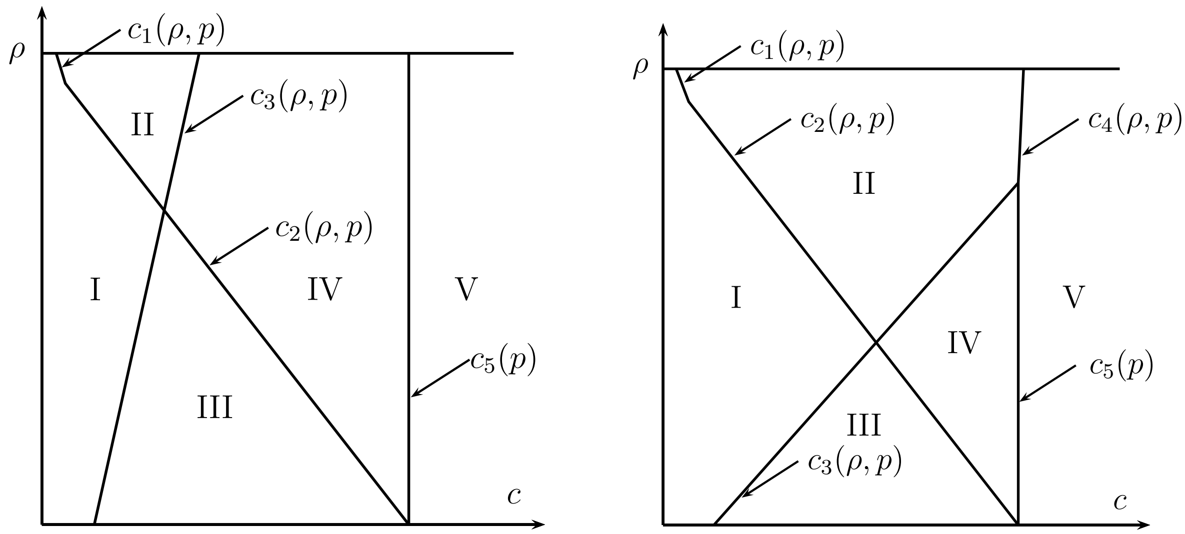

Figure 3: The globally optimal candidate mechanism for different combinations of c and ρ. Left panel: the case where $\frac{lp}{(h-l)(1-p)q} \leq 1$. Right panel: the case where $\frac{lp}{(h-l)(1-p)q} > 1$. The region labels represent the candidate mechanisms in Lemma 4.3.

asked to acquire information if Bidder 1's valuation is not the highest possible (h). Therefore, in region (I), candidate mechanism (I) is optimal.

As the cost c increases, it is no longer optimal to ask Bidder 2 to acquire information when both $u_1 = -Z$ and $u_1 = l$, because too much rents need be given to Bidder 2 to induce such frequent information acquisition. Alternatively, the target could ask Bidder 2 to acquire information only when Bidder 1's value is too low ($u_1 = -Z$). Indeed, when the Bidders' valuations are not highly correlated (low ρ, region (III)), it is optimal to do so. The reason is that knowing Bidder 1 is a weak competitor, Bidder 2 is encouraged to acquire information. However, this is no longer true when the bidders' valuations are highly positively correlated (high ρ, region (II)). It is very difficult to induce Bidder 2 to acquire information if Bidder 2 knows that there are no gains from trade between Bidder 1 and the target; Bidder 2 is discouraged from the information revelation. Therefore, in region (II), the optimal mechanism only asks Bidder 2 to acquire information when Bidder 1's value is as high as l. Encouraged by Bidder 1's high valuation, Bidder 2 then becomes more optimistic and is willing to pay the cost to acquire information.

When the cost of information acquisition is even higher, Bidder 2 will not be recommended to acquire information at all, as in region (IV). Finally, when the cost is extremely high, no one is recommended to acquire information, and there would be no sale, as in region (V).

5 Implementation

The optimal mechanisms in Proposition 4.1 are direct mechanisms where the bidders report their types. Their outcomes can be implemented with indirect mechanisms that are commonly observed in practice. As in the previous section, I focus on the implementation for Candidate (II). It is useful to illustrate a few terminologies before presenting the actual mechanisms.

Definition 5.1 (Termination fee) *If Bidder i promises to bid at least a "floor price", then the target in exchange promises to pay a termination fee for Bidder i, denoted as tf_i, payable to Bidder i if eventually the target firm is not sold to that bidder.*

Definition 5.2 (Clock auction with termination fees) *Suppose Bidder i is promised a termination fee tf_i, $i = 1, 2$.*[19] *A "clock auction" is held between the two bidders, with a different "clock" for each bidder if with different termination fees. The time-line of the auction is:*
(1) Bidder i is asked if is willing to bid $l - tf_i$, for $i = 1, 2$. If only Bidder i drops out, sold to Bidder $-i$ at the initial price $l - tf_{-i}$. If both bidders drop out, the firm is never sold. If both stay, then:
(2) The target raises the price for Bidder i by $\Delta_i \in (0, h - l)$, from $l - tf_i$ to $l + \Delta_i - tf_i$, $i = 1, 2$. If both bidders drop out, Bidder i wins with probability $x_i(l, l) \in [0, 1]$ at a price $l - tf_i$. If only Bidder i stays at this price, Bidder i wins and pays $l + \Delta_i - tf_i$. If both bidders stay, then:
(3) The target further raises the price for Bidder i to $h - tf_i$, $i = 1, 2$. If both drop out, Bidder i wins with probability $\frac{1}{2}$ at price $h - tf_i$. If only Bidder i stays, then Bidder i wins, paying $h - tf_i$. If both stay, Bidder i wins with probability $\frac{1}{2}$ at price $h - tf_i$.[20]

[19] This termination fee could be 0, either offered by the target, or because Bidder i rejects the floor price required by a strictly positive termination fee.
[20] $\frac{1}{2}$ is not the only probability that works. Actually, any probability in $[0, 1]$ could work.

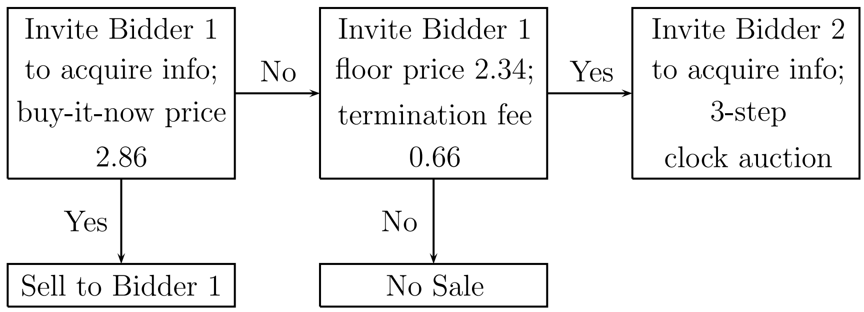

Figure 4: Time-line: an example of an implementation of the optimal candidate mechanism (II).

During the auction, termination fees would be paid to the losing bidder if the condition for payment is satisfied (remains in the auction at the floor price).

The following proposition gives an implementation of the optimal solution when it takes the form of a Candidate (II), that is, in region (II) of Figure 3. The solution could be implemented by an indirect mechanism similar to a go-shop negotiation (or a stalking-horse auction).

Proposition 5.1 (Optimal Candidate (II): Implementation) *The outcome of the optimal candidate mechanism (II) in Proposition 4.1 can be implemented by a mechanism with the following time-line:*
(1) The target invites Bidder 1 *to acquire information.*
(2) The target posts a buy-it-now price $t_1(h, 0)$ *to Bidder* 1*. If accepted, Bidder* 1 *wins at price* $t_1(h, 0)$*, the game ends. Otherwise:*
(3) The target asks if Bidder 1 *would promise to bid at least* $t_1(l, -Z) = l - tf_1$*, a "floor price" lower than the buy-it-now price. If rejected, the game ends. Otherwise, the target promises to pay Bidder* 1 *a termination fee* tf_1 *if sold to Bidder* 2 *later.*
(4) The target invites Bidder 2 *to acquire information.*
(5) The target holds a "clock auction" between the two bidders as in Definition 5.2, with a termination fee tf_1 *and floor price* $l - tf_1$ *for Bidder 1, and neither termination fee nor floor price for Bidder 2 (*$tf_2 = 0$*).*

To further illustrate the implementation, we use the following example.

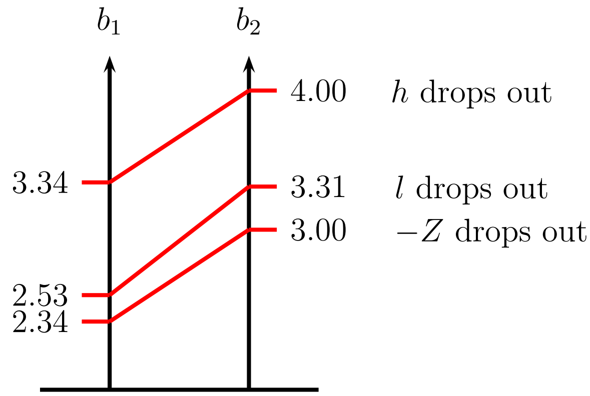

Figure 5: An illustration of a clock auction. It is essentially a discrete-type version of English auction with a termination fee.[22]

Example 5.1 *Suppose* $h = 4,\ l = 3,\ Z = 30,\ p = \frac{1}{3},\ q = \frac{1}{2},\ \rho = 0.8,\ c = 0.3$. *Figure 4 illustrates the time-line of the optimal candidate mechanism (II). The buy-it-now price for Bidder* 1 *is* $t_1(h, 0) = 2.86$. *The floor price that Bidder* 1 *promises is* $l - tf_1 = 2.34$, *with the termination fee for Bidder* 1 *being* $tf_1 = 0.66$. *In the second stage, the target holds a "clock auction" with a termination fee for Bidder 1 as demonstrated by Figure 5. This auction is essentially a discrete-type version of an English auction with a termination fee. The target raises the prices for Bidder* 1 *from 2.34, to 2.53, then to 3.34; the target raises the prices for Bidder* 2 *from 3, to 3.31, then to 4. If both bidders drop out at their second prices, Bidder* 2 *wins with probability* $x_2(l, l) = 0.19$. *The termination fee for Bidder* 2 *is* $tf_2 = 0$.

Note that in the clock auction, Bidder 2 has to bid significantly higher than Bidder 1 in order to win. In particular, the difference between the first price of Bidder 2 (3.00) and that of Bidder 1 (2.34, also equals to the floor price) is exactly Bidder 1's termination fee (0.66). It implies that the target will consider Bidder 2's bid only if it exceeds Bidder 1's initial floor price by the amount of the termination fee. Also, if Bidder 2 raises the bid from 3.00 to the second price 3.31, Bidder 1 is allowed to match Bidder 2's new bid by raising to the second price 2.53. Again, as a result of the Bidder 1's termination fee, Bidder 1's second price is

[22] Note that the differences between the first and the second prices for Bidder 1 and 2 are not the same. This is to make type $u_1 = h$ indifferent from dropping out at $l + \Delta_1 - tf_1$ and $h - tf_1$ if deviating to rejecting the buy-it-now price and accepting the floor price instead, making sure that type $u_1 = h$ accepts the buy-it-now price instead of entering the second stage auction and dropping out at $h - tf_1$. However, such complication is a result of discrete type, hence is not an important feature for the result. A continuous type version is much less tractable due to the correlation of information among bidders.

lower than that of Bidder 2's price.[23] If Bidder 2 further raises the bid to the third price, 4, Bidder 1 can also match the price by raising to 3.34. The difference between the two prices is again the amount of the termination fee.

This mechanism has two important features. First, it involves a *termination fee*, which is a necessary *compensation* to Bidder 1. The termination fee is payable to Bidder 1 if it loses to Bidder 2 in the auction later, conditional on Bidder 1 having accepted the floor price. For sufficiently high due diligence costs, Bidder 1 is willing to acquire information and leak it to a potential competitor only if the termination fee is lucrative enough. The termination fee also introduces *asymmetry* into the subsequent auction: Bidder 2 has to bid significantly higher than Bidder 1 in order to win. Such "unfairness" against Bidder 2 causes it to pay more, but this does not distort allocations; as long as the game proceeds to the auction stage, the target firm is always sold to the bidder with the higher valuation.

Second, it is a *dynamic* mechanism with *interim information revelation*. Bidder 1 is invited to acquire information first, and whether Bidder 1 is willing to accept the floor price reveals information about its willingness to pay, *before* Bidder 2 is given the chance to decide whether to acquire information. Bidder 2 has the opportunity to acquire information *only* when Bidder 1 accepts the floor price.

Compared with Candidate (I) where there is no information revelation to distinguish whether Bidder 1's valuation is $u_1 = l$ or $u_1 = -Z$, the information disclosure in Candidate (II) makes Bidder 2's information acquisition incentive to *bifurcate*. That is, Bidder 2 is more willing to acquire information under one circumstance, and less willing to under the other. The former may improve the target's revenue, while the latter may reduce the revenue. So when exactly does the information revelation *benefit the target?* The average effect depends on three parameters: the correlation of bidder valuations that increases with ρ, the information acquisition cost c, and the prior probability of the existence of gains from trade p. When the correlation and the cost are sufficiently high, and the expected gains from trade is sufficiently low, an *option-like* structure emerges, making such information revelation valuable to the target.

To see this, suppose the bidders' valuations are sufficiently positively correlated (ρ high enough). Then Bidder 1's acceptance of the floor price signals favorable information about the deal to the second bidder. Then Bidder 2 is *encouraged* to acquire information when

[23] As a result of the discreteness of types, the difference is not exactly 0.66.

Bidder 1's acceptance of the floor price reveals $u_1 = l$, while *discouraged* when Bidder 1's rejection of the floor price reveals that $u_1 = -Z$. When c is small (low information cost) or p is large (optimistic prior on the gains from trade), it is not excessively costly for the optimal Candidate (I) to induce Bidder 2 to acquire information on both the occasions of $u_1 = -Z$ and $u_1 = l$, even without revealing the actual state. That is, not too much rent is given to Bidder 2 to induce information acquisition. With the optimal Candidate (II), however, revealing $u_1 = -Z$ discourages Bidder 2's information acquisition under that occasion, while revealing $u_1 = l$ cannot improve upon Bidder 2's participation that is already good enough. Therefore, Candidate (II) reveals *too much information*, and it is dominated by Candidate (I).

When c is sufficiently large or when p is sufficiently low, however, it becomes too costly to induce Bidder 2 to acquire information on both when $u_1 = -Z$ and $u_1 = l$ using a Candidate (I). With a Candidate (II), revealing that Bidder 1's value is as high as l makes Bidder 2 more willing to acquire information on the occasion of $u_1 = l$, while revealing $u_1 = -Z$ cannot hurt Bidder 2's participation further. Such an option-like structure makes information revelation beneficial to the target.

The situation is different when the bidders' valuations are less correlated. When ρ is small, Bidder 1's valuation is not a good predictor of Bidder 2's valuation. Contrary to the case with a high ρ, Bidder 2 is *deterred* from information acquisition when Bidder 1 is revealed to be a strong competitor with a high value l, while it is *encouraged* when Bidder 1's rejection of the floor price reveals that $u_1 = -Z$. In this case, the target then uses a Candidate (III) and asks Bidder 2 to acquire information only when $u_1 = -Z$. Such mechanism is less costly to implement incentive-wise than a Candidate (II) when bidders' valuations have low correlation, precisely because the negative news about Bidder 1's valuation would encourage Bidder 2 to acquire information.

In line with the intuition above, the following corollary illustrates the comparative statics on when Candidate (II) is optimal, regarding the parameters ρ, c and p.

Corollary 5.1 (Comparative Statics: When is Candidate (II) Optimal)
(i) For a fix pair (c, p), Candidate (II) is optimal only if ρ is high enough;
(ii) For a fix pair (ρ, p), Candidate (II) is optimal only if c is neither too low nor too high;
(iii) For a fix pair (c, ρ), Candidate (II) is optimal only if p is neither too high nor too low.

Corollary 5.1 is a direct implication from Proposition 4.1. The first two results can be observed directly from Figure 3; the third result can be derived based on the additional fact that c_i is increasing in p, for $i = 1, 2, 3, 4, 5$. This corollary highlights the three conditions under which a Candidate (II) is optimal. The reason why c cannot be too high and p cannot be too low is that under those circumstances, information acquisition is too costly to implement even in a go-shop negotiation.[24]

Candidate mechanism (II) is very similar to a *go-shop negotiation*, which is a common mechanism used in M&A with mostly *financial* buyers. It is also the most common selling mechanism in *Bankruptcy* M&A under 363 Sales of Chapter 11, where the initial bidder is called the "*stalking-horse bidder*". In such mechanisms, typically an initial bidder is first invited to conduct information acquisition on the target firm during a period of exclusive negotiation with the target. At the end of the negotiation, the initial bidder agrees to pay at least a "floor price", in exchange for a termination fee payable to the bidder if the target firm is sold to another bidder later. Such "floor price" and the termination fee is then announced to the public, and the target actively solicit all potential bidders to investigate the firm. A bidding game on the target's firm is then held among all bidders, including the initial one. In order for future bidders to win over the initial bidder, their bids should exceed the initial bidder's bid by at least the termination fee.

Due to the similarity between a Candidate (II) and a go-shop negotiation, the comparative statics in Corollary 5.1 shed light on when go-shop negotiations are observed in practice. A formal analysis of the empirical implications regarding the frequency of use of go-shop negotiations is conducted in Section 6.

In addition to characterizing the implementation of the optimal canonical mechanism where it takes the form of a candidate mechanism (II), I also provide the implementations of the optimal mechanism elsewhere. The following proposition illustrates the implementation of the optimal mechanism in all regions.

Proposition 5.2 (Implementation) *In each region below, there exists an indirect mechanism that achieves the same target's revenue as that of the corresponding optimal canonical mechanism.*

Region (A): Symmetric clock auction without termination fees.

Region (I): Buy-it-now price for Bidder 1*/clock auction with termination fees for Bidder* 1

[24] In this case, the seller approaches at most one bidder for information acquisition.

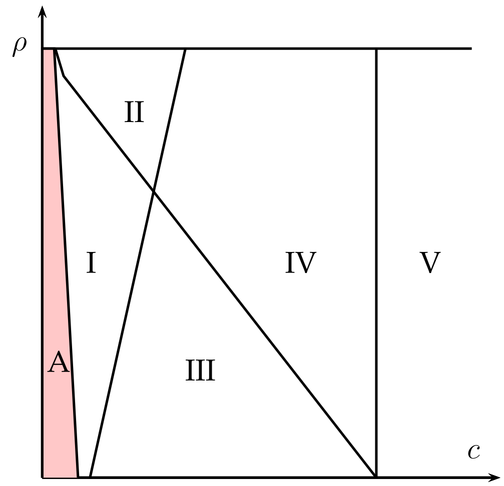

Figure 6: An Implementation of the global optimum. The numbers (I) to (V) represent the candidate mechanisms in Lemma 4.3. Here I only illustraste the case with $\frac{lp}{(h-l)(1-p)q} \leq 1$, which is the left panel of Figure 3. The case with $\frac{lp}{(h-l)(1-p)q} > 1$ (right panel of Figure 3) is similar and hence omitted.

and Bidder 2.

Region (II): Buy-it-now price for Bidder 1*/announce floor price, then clock auction with termination fee for Bidder* 1*.*

Region (III): Sequential posting prices.

Region (IV): Posting price for Bidder 1 *only.*

Region (V): No sale.

Here region (I) to (V) are as defined in (12). Region (A) is a subset of region (I), and it is defined as $c \in [0, c_6(\rho, p)]$*, where* $c_6(\rho, p) = \frac{1}{2}(h-l)pq(2 - p(1+q)(1-\rho) - (1+q)\rho)$*. The "clock auction" is defined in Definition 5.2.*

Figure 6 illustrates the various regions in Proposition 5.2. In region (I), the target first invites Bidder 1 for information acquisition and offers a buy-it-now price for Bidder 1. If Bidder 1 accepts, the target is sold to Bidder 1 at that price. If not, then the target invites Bidder 2 to acquire information, with Bidder 2 only knowing that Bidder 1 has rejected the buy-it-now price. Then the target asks Bidder i if willing to bid at least $l - tf_i$, in exchange for tf_i, for $i = 1, 2$. After that, the target holds a clock auction between the two bidders, with termination fees potentially for both bidders. Note that the main difference between the optimal mechanism in region (I) and (II) is that if Bidder 1 does not accept the buy-it-now price, in region (II) the target would ask whether Bidder 1 is willing to accept a floor price *before* Bidder 2 could acquire information. In region (I), however, there is no such information revelation before Bidder 2's decision of information acquisition.

In region (III), the target asks Bidder 1 for a posting price; if Bidder 1 rejects, then the target invites Bidder 2 for information acquisition and asks Bidder 2 for another posting price. In region (IV), the target only asks Bidder 1 for a posting price; Bidder 2 is never approached. In region (V), no bidder is invited to acquire information.

Notice that there is also an extra region (A) in Figure 6 as compared to Figure 3. Region (A) is a subset of region (I); in this region, the revenue of the optimal mechanism can be achieved by a symmetric auction without termination fees, where both bidders are approached simultaneously. Note that this auction is not as cost efficient as the optimal Candidate (I) and results in different allocations and transfers, but it can still achieve the same revenue when c is sufficiently small.

6 Empirical Implications: the Prevalence of go-shop/ no-shop negotiations

Sections 4 and 5 solve the optimal mechanism problem, and provide the conditions for each mechanism to be optimal. This section puts the theory into practice, and sheds lights on the cross-sectional differences in the prevalence of various mechanisms. Note that in the data, it is usually hard to observe the actual mechanism the target commits to; instead, only the outcome is observed. For example, if the target commits to an optimal candidate (II) mechanism, either it could be observed that only one bidder is approached and the buy-it-now price is accepted; or instead, a floor price is accepted by an initial bidder, with a "go-shop provision" attached to the tentative merger agreement specifying the floor price and a termination fee. The former appears when the initial bidder's valuation is very high, h; the latter occurs when the valuation is at an intermediate level, l. Therefore we define the following *outcomes* as possible *realizations* of the optimal mechanisms, and provide empirical predictions regarding the prevalence of these outcomes.

In particular, I study "go-shop negotiation" and "no-shop negotiation". While the former is the focus of the paper, the latter, which resembles a private negotiation without a "go-shop provision", is the most commonly observed outcome in practice. In fact, 55% of deals have been conducted this way.[25]

Definition 6.1 (Go-Shop Negotiations) *A go-shop negotiation is an outcome with the following time-line:*
(1) The target asks if Bidder 1 *is willing to accept a floor price in exchange for a termination fee. Bidder 1 accepts, and*
(2) The target holds a clock auction between the two bidders, with a termination fee promised to Bidder 1 *if sold to Bidder* 2*.*

Note that a go-shop negotiation could *only* arise as a possible realization of the optimal candidate mechanism (II); it occurs if Bidder 1 rejects the buy-it-now price but accepts the floor price. Therefore its occurrence indicates the use of the candidate (II).

Similarly, define a *no-shop negotiation* as follows.

Definition 6.2 (No-Shop Negotiations) *In a no-shop negotiation, either*
(i) The target only approaches one bidder with a posting price, and the bidder buys the target firm at that price;
(ii) Or the target approaches both bidders sequentially with their corresponding posting prices, with the first bidder rejecting its price, and the second bidder accepting its price.

According to Proposition 5.2, a no-shop negotiation could arise as a possible realization of the optimal candidate (I) and (II) when the first buy-it-now price is accepted; it could also occur as the optimal candidate (III) or (IV), since by definition (III) and (IV) lead to no-shop negotiations.

I then derive two empirical implications regarding the prevalence of go-shop negotiations and no-shop negotiations. In order to predict the likelihood of each outcome to occur, the distributions of the parameters ρ, c and p need to be involved, although the three parameters have been assumed to be fixed in the model. For simplicity, I focus on the case where ρ, c and p are independently distributed.[26]

[25] Source of data: MergerMetrics of FactSet, January 2003-June 2018.

[26] It is not clear whether deals with more correlated bidders are easier or harder to evaluate. It is possible that if the bidders' valuations are similar, it is a standard target firm, and hence information acquisition is

First, recall that Corollary 5.1 illustrates when a candidate (II) mechanism is optimal. Since go-shop negotiations only occur when candidate (II) mechanisms are used, Corollary 5.1 directly implies when go-shop negotiations are more likely to occur. The following corollary summarizes the comparative statics to when go-shop negotiations are more likely to happen.

Corollary 6.1 (Prevalence of go-shop negotiations) *Suppose the cost of information acquisition c and the prior of existence of gains from trade p are at a moderate level. Then, a go-shop negotiation could arise as an outcome of the optimal selling mechanism*
(i) for a wider range of values of parameter ρ and p when c is higher;
(ii) for a wider range of values of parameter c and p when ρ is higher;
(ii) for a wider range of values of parameter ρ and c when p is lower.
If ρ, c and p are independently distributed, then the probability of go-shop negotiations to occur is higher when c or ρ is higher, or when p is lower.

The second corollary studies the comparative statics to when no-shop negotiations are more likely to happen. It is a direct implication of Proposition 5.2.

Corollary 6.2 (Prevalence of no-shop negotiations) *A no-shop negotiation can arise as an outcome of the optimal selling mechanism for a wider range of values of parameter c when the correlation between bidders' valuations is lower (ρ lower).*
If ρ and c are independently distributed, then the probability of no-shop negotiations to occur is higher when ρ is lower.

As can be seen in Figure 3, when the correlation of bidders' valuations are lower (ρ lower), for a fixed ρ, the range of c when (III) and (IV) are optimal enlarges, while the range of c when (I) and (II) are optimal shrinks.Therefore the region of (II) contracts and (III) expands when ρ decreases. A no-shop negotiation could take place in a candidate (I) and (II) if the first bidder's value is the highest possible, h. However, it always occurs whenever a candidate (III) or (IV) is optimal, disregarding the realization of bidders' valuations. Therefore, if ρ and c are independently distributed, a no-shop negotiation is more likely to occur in practice when ρ is lower.

Corollary 6.1 and 6.2 are broadly consistent with the empirical evidence.

less costly; it is also possible that such target firms are troubled mature businesses, and therefore the quality of information is low. In addition, It is not clear whether the expected gains from trade with the target is correlated with the cost of due diligence or the correlation between bidders' valuations.

Financial Deals v.s. Strategic Deals. Corollary 6.1 implies that if bidders' valuations are more correlated, then go-shop negotiations are more likely to occur. This is consistent with the empirical evidence that such mechanism is much more common in deals with mostly financial buyers as compared to strategic deals.

In particular, the business models of financial buyers are very similar, hence their potential gains from trade with the target are also likely to be highly correlated. This view is shared by Gorbenko and Malenko [2014], Leslie and Oyer [2008]. Therefore, the average within-deal bidder value correlation should be higher for financial deals,[27] because such deals typically attract buyers with very similar business models.

Indeed, according to the data ranging from 01/01/2003 to 06/30/2018 provided by MergerMetrics of FactSet, the frequency of the use of go-shop negotiations is 15% (94 out of 613 cases) for deals with mostly financial buyers, and 3% (82 out of 3016 cases) for deals attracting mostly strategic buyers.[28]

In addition, Corollary 6.2 indicates that no-shop negotiations are less likely to appear when bidders' valuations are more correlated. Indeed, the frequency of use of no-shop negotiations is lower for financial deals (27%, 165 out of 613 deals) than strategic deals (61%, 1845 out of 3016 deals), consistent with the prediction.

Distressed Deals v.s. Non-Distressed Deals. Corollary 6.1 also shows that when the cost of information acquisition is higher (but not extremely high), a go-shop negotiation is more likely to be used. This is probably a reason why a go-shop negotiation, or equivalently, a "stalking-horse auction", is the major mechanism in bankruptcy sales (84% according to Gilson, Hotchkiss, and Osborn [2016]),[29] while it is only 4% in non-bankruptcy mergers and

[27] I also estimated the within-deal correlation of bid premia, using the hand-collected data in Gorbenko and Malenko [2014] and a Random Effect ANOVA (Analysis of Variance) model. Indeed, I found that the estimated within-deal correlation of bid premia is higher for deals won by a financial buyer than that of deals won by a strategic buyer. In particular, the correlation for deals won by a financial buyer is 0.96, which is higher than that of strategic deals (0.81), even if taking into account the 90% confidence interval. The details of the estimation is available upon request.

[28] I consider completed deals with public targets only. Deals with mostly financial (strategic) buyers in the data are deals won by a financial (strategic) buyer. According to Gorbenko and Malenko [2014], in the deals won by financial (strategic) buyers, the majority of bidders are financial (strategic). In particular, Table I of Gorbenko and Malenko [2014] shows that for a deal won by a financial bidder, there are seven financial bidders and two strategic bidders on average; for a deal won by a strategic bidder, there is one financial bidders and three strategic bidders on average.

[29] The bankruptcy sales refer to the sales of all assets under Section 363 of Chapter 11.

acquisitions.[30]

In particular, Nesvold, Anapolsky, and Lajoux [2010] claim that compared to non-distressed M&A, in a typical distressed M&A, buyers face more opaque and confusing data provided by the target. In addition, they have to process such confusing information under immense time pressure. Therefore, the cost of information acquisition is likely to be higher in a distressed M&A, making it necessary to use a go-shop negotiation to boost information acquisitions.

Go-Shop Provisions: Target Revenue Maximizing or Agency Problem? Finally, this paper argues that the use of go-shop provisions is potentially due to target revenue maximization. However, an alternative explanation is that go-shop provisions are simply a window-dressing practice that cannot increase bidder participation. The target management uses it to favor the initial bidder, while hurting the shareholders' value.

The findings in Gogineni and Puthenpurackal [2017] are more consistent with the former argument. In particular, they found that deals with go-shop provisions attract significantly more competing bids than deal with no-shop provisions. Also, the initial bid premiums are higher in go-shop negotiation deals, and the market reacts more favorably to such deals as well. Moreover, go-shop provisions are more likely in deals with greater institutional ownership, where the agency problems of the target management are potentially less severe.

7 Conclusion

There are a variety of selling mechanisms used in mergers and acquisitions, yet the literature on this topic is limited. In particular, go-shop negotiations, or equivalently, "stalking-horse auctions", are commonly used in financial deals and distressed deals. With a theoretical framework of dynamic mechanism design, this paper fully characterizes the revenue-maximizing mechanism for the target, and is the first to show that a go-shop negotiation could arise as an outcome of the optimal selling mechanism. In addition, the optimality obtains when first, the correlation of the bidders' valuations on the seller's firm is sufficiently large; second, the cost of information acquisition is sufficiently, but not prohibitively high; third, the expected gains from trade are sufficiently, but not excessively low.

[30] Data range: 2002-2011.

The results are broadly consistent with the empirical evidence in M&A, which suggests that the use of a go-shop negotiation could potentially be driven by seller's revenue maximization. It provides a possible explanation for the prevalence of go-shop negotiations in private equity deals and in distressed M&A.

In addition, the paper sheds lights on why common mechanisms like no-shop negotiations are used in practice, and derives predictions regarding when such mechanism are more likely to occur in practice. The predictions are also broadly consistent with the empirical evidence.

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A Appendix

Proof of Proposition 3.1. I prove the proposition with the following steps.

(1) For the equilibrium allocations and transfers of a mechanism where the bidders are invited simultaneously at a certain stage, there exists another mechanism that implements the same allocations and transfers, where the bidders are invited *sequentially* in two consecutive stages. Since all players are patient, postponing acquiring information and taking actions has no impact on the players' payoffs. Besides, as long as the target discloses exactly the same information between the two stages (i.e., no disclosure about the previous actions), the information set and the belief of the second bidder remains the same.

(2) For the equilibirum allocations and transfers of a mechanism where the target invites the bidders and the invited bidders choose the information acquisition levels, there exists another mechanism that implements the same allocations and transfers, where the target *recommends* the bidders whether to acquire information, and the bidders follow the recommendations. In particular, in an equilibrium under the original mechanism, suppose the target invites Bidder i to acquire information, and then discloses a certain signal about the history. Then Bidder i chooses the information acquisition level at the resulting information set. Suppose at this information set, Bidder i's information acquisition is of pure-strategy; then let the target recommend the equilibrium information acquisition to the bidder instead. If at this information set, Bidder i mixes between acquiring or not with probability $k \in (0,1)$;

then with probability k, let the target recommend the bidder to acquire information, and with probability $1 - k$, let the target recommend the bidder not to acquire information. In the resulting mechanism, the target recommends the bidders whether to acquire information, and the recommended acquisition actions are of pure-strategy. Also, the bidders follow the recommendations.

In addition, in any history of a game under such a new mechanism, the target will not recommend a bidder to acquire information for more than once. This is because when recommended for a second time, no new information could be obtained in exchange for the costly information acquisition; a bidder will not follow such a recommendation.

(3) Next, I show that instead of the bidders taking actions at each stage, we can construct another mechanism with the same allocations and transfers, where the bidders *report their types* instead. Suppose at a certain stage, with probability $k \in [0, 1]$, a bidder is recommended to acquire information and takes its own action; with $1 - k$ the bidder is recommended not to acquire information. In the former case, let the bidder report the type after the information acquisition, and let the target take the corresponding actions on behalf of the bidder. In the latter case, still let the bidder report a type in U after receiving the recommendation, although such a bidder is uninformed at the time of reporting. Then the target takes the corresponding actions under the original mechanism on behalf of the uninformed bidder. The resulting new mechanism implements the same allocations and transfers, and both bidders report their types truthfully.

Note that if a bidder is recommended not to acquire information and submits an uninformed report, the allocations and transfers for both bidders are *unresponsive* to that report. This is because in the new mechanism, both bidders' allocations and transfers equal to the corresponding levels of the original mechanism; hence they do not vary with the uninformed reports in the new mechanism.

(4) Now further modify the mechanism, so that the target *reveals no information other than the recommendations*. The resulting new mechanism achieves the same allocations and transfers, and the bidders still follow the recommendations and report types truthfully. In particular, let the target redesign the signal structures, so that all events under which the target recommends the same information acquisition action to a bidder leads to a single information set for that bidder. Then under such a mechanism, the bidder does not know any information other than the recommendations. Note that this creates a possibly *coarser*

information structure for that bidder. However, in the resulting new mechanism, the bidder still follows the recommendations (obedience); in addition, the bidder reports the type truthfully (incentive compatibility), and stays in the game (individual rationality). This is because under the original finer information structure, the obedience, incentive compatibility and the individual rationality constraints are all satisfied at each information set. Under the coarser information structure, the corresponding new constraints are the weighted average of those of the original structure; therefore, they are also satisfied. The allocations and transfers also remain the same as a result.

(5) With the transformations in (1) to (4), I show that any equilibrium allocations and transfers in the original mechanism M can be implemented by a new mechanism M', where the target approaches the bidders in a potentially random sequence. At each stage, the target approaches one bidder; with probability $k \in [0, 1]$, the target recommends the bidder to acquire information, and with probability $1 - k$, the target recommends the bidder not to acquire information. The target reveals no more information other than the recommendations, and recommends a bidder to acquire information at most once. The bidders follow the recommendations. In addition, a bidder who is recommended to acquire information truthfully reports the type; a bidder recommended not to acquire information reports a type in U, but the allocation and transfers do not depend on the uninformed reports.

Next, I show that we can restrict attention to a new mechanism M'' with only *two stages*. Such a mechanism is a mixing of the three two-stage sub-mechanisms below:
Mechanism 1: at stage 1, Bidder 1 is recommend to acquire information and asked to report its type. At stage 2, with probability $k_2 \in [0, 1]$, Bidder 2 is recommended to acquire information and asked to report its type; with probability $1 - k_2$, Bidder 2 is recommended not to acquire information, but still has to report a type.
Mechanism 2: the same as Mechanism 1, except that the roles of the two bidders are switched.
Mechanism 3: at stage 1, Bidder 1 is recommend not to acquire information and asked to report its type; at stage 2, Bidder 2 is recommended not to acquire information and asked to report its type.
The mixing among the three mechanisms is designed such that (i) Mechanism i takes place with the same probability that Bidder i is the first bidder to be recommended to acquire information in mechanism M', for $i = 1, 2$; and that (ii) Mechanism 3 takes place with the

same probability that no bidder acquires information under mechanism M'. In addition, in the second stage of Mechanism i, conditional on Bidder i's true type, the probability that Bidder $-i$ is recommended to acquire information is the same as that under M'.

The new two-stage mechanism M'' implements the same allocations and transfers as that of M'. To see this, consider Bidder 1 as an example, and the case with Bidder 2 is omitted due to symmetry. Suppose at an information set under mechanism M', Bidder 1 is recommended to acquire information. Then either (i) Bidder 1 is recommended to acquire information before Bidder 2 has acquired information, or (ii) Bidder 2 is recommended to acquire information first, and Bidder 1's recommendation is based on Bidder 2's reported type. In the new mechanism M'', by construction, Bidder 1 is recommended to acquire information whenever Bidder 1 is recommended to do so in the old mechanism M'; this is true whether Bidder 1 is the first to acquire information (Mechanism 1 takes place) or not (Mechanism 2 occurs). However, Bidder 1 now may have less information as compared to the case under mechanism M'. That is, Bidder 1 does not know which of the two cases occur, while it may know under M'; it may also know less information about Bidder 2's reported type. Similar to the argument in Step (4), such coarser information structure does not invalidate the constraints of obedience, incentive compatibility and individual rationality, because the new constraints are the weighted averaged of those in M'. Therefore, M'' recommends Bidder 1 to acquire information exactly when M' recommends Bidder 1 to do so; at the information set where Bidder 1 is recommended to acquire under M'', Bidder 1 follows the recommendation and reports the type truthfully. Similarly, it is also the case for the information set where Bidder 1 is recommended not to acquire information. In addition, a similar argument applies for Bidder 2. Therefore, M'' implements the same allocations and transfers as M'.

(6) Finally, I show that if implementing only the outcomes of equilibria with the type-acquisition correspondences being mappings, we have that (i) for each of the three two-stage sub-mechanisms, the target does not randomize when making recommendations, and that (ii) the target does not mix among the three sub-mechanisms. To see why (i) is true, consider Mechanism 1. If under a type u_1, the probabilities that Bidder 2 is recommended to acquire and not to acquire are both non-zero. Then the restriction is violated directly. The same argument applies for Mechanism 2. There is no mixing of recommendation with Mechanism 3 by definition.

To see why (ii) is true, suppose the target mixes Mechanism 1 with Mechanism 2. If in Mechanism 1, there exists a type of u_1 such that Bidder 2 is recommended not to acquire information, then according to the restriction, in the mixed mechanism Bidder 2 should always be recommended not to acquire information under u_1. However, in Mechanism 2, Bidder 2 is always recommended to acquire information for any u_1. Therefore, for the restriction on type-acquisition correspondences to hold, the only possible case with Mechanism 1 mixed with Mechanism 2 is that for all u_1, Bidder 2 is recommended to acquire information in Mechanism 1. Similarly, it also requires that for all u_2, Bidder 1 is recommended to acquire information in Mechanism 2. The mixing of such two mechanisms generates the same allocations and transfers as that of non-mixing mechanism where both bidders are always recommended to acquire information, hence it is without loss of generality to restrict attention to mechanisms that do not mix Mechanism 1 with 2.

Now suppose the target mixes Mechanism 1 with Mechanism 3. Then for any u_2, Bidder 1 is recommended to acquire information whenever Mechanism 1 occurs, and it is recommended not to acquire information whenever Mechanism 3 occurs. This violates the restriction directly. The argument for the case of mixing Mechanism 2 with 3 is similar.

(7) Applying (6) to (5), it is clear that the only types of mechanism M'' that satisfies the type-acquisition correspondence restriction are Mechanism 1, 2, or 3. Due to the symmetry between Bidder 1 and 2, Mechanism 1 and 2 are revenue-equivalent. Therefore, we could restrict attention to Mechanism 1 and 3, that is, canonical mechanisms. Note that in a canonical mechanism, if Bidder 1 is recommended not to acquire information, it does not fall into Mechanism 1 or 3. To see this is not a problem, suppose Bidder 2 is recommended to acquire information; then this is a redundant case that revenue-equivalent to a case in Mechanism 1. If Bidder 2 is recommended not to acquire information, it is revenue-equivalent to Mechanism 3. ■

Comment on Footnote 12. Given the notations at the beginning of Section 4, we can prove the equivalence of the new setting in Footnote 12 and the original setting. In the new setting, let $W_i = n + m + u_i$ represents the merged firm value if Bidder i's merges with the target. Also, define $P_i(\mathbf{u})$ as the transfer from Bidder i to the target. Then given any information set A_i, Bidder i's expected payoff given information set A_i is $\mathbb{E}_{u_1,u_2}[W_i x_i + n(1-x_i) - P_i|A_i]$. Let $t_i = P_i - mx_i$. Intuitively, with the new setting where the target's outside option is m, the price Bidder i pays should be higher by m than those in the original setting with target's outside option being 0. Therefore after removing m from the price Bidder i pays in the new setting when Bidder i wins, t_i should be the transfer in the setting with target's outside option being 0. Plug in $W_i = n + m + u_i$ and $t_i = P_i - mx_i$, we have $\mathbb{E}_{u_1,u_2}[W_i x_i + n(1-x_i) - P_i | A_i] = n + \mathbb{E}_{u_1,u_2}[u_i x_i - t_i | A_i]$. Write down the constraints for the maximization problem for the new setting in a similar fashion as in problem (5), then it is straightforward to verify that the new (IC), (IR), (OB), (MS) and (FE) constraints are the same as in the original problem (with the term n canceled out). In addition, the target's revenue in the new setting is $\mathbb{E}_{u_1,u_2}[P_1 + P_2 + (1 - x_1 - x_2)m]$, where the first two terms are the transfer from each bidder, and the third term is due to the fact that when the target is not sold with probability $1 - x_1 - x_2$, the target still enjoys the outside option m. Plug in $t_i = P_i - mx_i$ into the revenue, we have $\mathbb{E}_{u_1,u_2}[P_1 + P_2 + (1 - x_1 - x_2)m] = \mathbb{E}_{u_1,u_2}[t_1 + t_2]$, which is the same as in the original problem. Therefore the new setting is equivalent to the original setting in the model. ■

Proof of Lemma 4.1. (i) Suppose at the optimum, $a_i = 0$, but $x_i > 0$. Assumption 4.1 implies that Bidder i gets negative utility from owning the target firm with probability x_i. Due to the individual rationality condition when Bidder i is not recommended to acquire information, the expected payment from Bidder i to the target is strictly negative (target paying the bidder). Therefore target could modify both $x_i(\mathbf{u})$ and the payment $t_i(\mathbf{u})$ when $a_i = 0$ to be 0, increasing the revenue strictly. Note that reducing x_i does not affect x_{-i}, hence Bidder $-i$'s constraints are intact. In addition, this modification does not violate any Bidder i's constraints in target's optimization problem (it only involves (OB) and (IR) of $a_i = 0$, which are trivially satisfied). Hence $x_i > 0$ is not optimal when $a_i = 0$. That is, in any optimal mechanism, $x_i = 0$ whenever $a_i = 0$. In addition, with a similar argument, it can be shown that $\mathbb{E}_{\mathbf{u}}[t_i(\mathbf{u}) \big| a_i = 0] = 0$ at the optimum (otherwise, pushing the transfer to 0 without changing the allocation increases the revenue strictly). That is, (IRi) when $a_i = 0$ are binding.

(ii) Now suppose with the previous modification in the proof of part (i), the resulting new optimal mechanism has $x_i > 0$ when $a_i = 1$ and $u_i = -Z$. The assumption that $-Z < 0$ implies that Bidder i gets negative utility from owning the target firm with probability x_i. Due to the individual rationality condition when Bidder i is not recommended to acquire information, the expected payment from Bidder i to the target, $\mathbb{E}_{u_{-i}}[t_i(\mathbf{u})\big|a_i = 1, u_i = -Z]$, is strictly negative (the target paying the bidder). Therefore, the target could modify both $x_i(\boldsymbol{u})$ and the payment $t_i(\boldsymbol{u})$ where $a_i = 1, u_i = -Z$ to be 0, increasing the revenue strictly. Such modification does not interfere with the previous adjustment for the case with $a_i = 0$, since the two cases occur under different realizations of $\boldsymbol{u}$. Similar to the case with $a_i = 0$, reducing x_i does not affect x_{-i}, hence Bidder $-i$'s constraints are intact. But this could affect the (ICi) conditions between types $u_i = -Z$ and types $u_i = l, h$. Therefore we increase $t_i(\mathbf{u})$ where $u_i = l$ and $u_i = h$ by the positive amount, $-\mathbb{E}_{u_{-i}}[t_i(\mathbf{u})\big|a_i = 1, u_i = -Z]$. Consider $u_i = -Z$'s motive to mimic l. The difference of payoffs between telling the truth and misreporting a type l is now even higher. This is because if telling the truth, $u_i = -Z$ no longer suffers from owning the firm, while the utility from owning the firm if reporting l remains the same due to the unchanged allocation when $\hat{u}_i = l$. In addition, the transfers with truth-telling and with lying increase by the same amount. Hence $u_i = -Z$'s motive to mimic l is actually weaker. Similarly, $u_i = -Z$ has less motive to mimic h as well. The incentive of type l (or h) to mimic type $u_i = -Z$ is also weaker, since mimicking results in lower allocation and higher transfer.

In addition, (OBi) is also relaxed after the change. In the original solution with $x_i > 0$ when $u_i = -Z$, (OBi) requires that the bidder is worse-off disobeying the information acquisition recommendation and mimicking $u_i = -Z$, $u_i = l$ or $u_i = h$. Since in the new (OBi), $\mathbb{E}_{u_{-i}}[t_i(\mathbf{u})|a_i = 1, u_i]$ for all u_i has been increased by the same positive amount, $-\mathbb{E}_{u_{-i}}[t_i(\mathbf{u})\big|a_i = 1, u_i = -Z]$, the difference between the transfer parts of acquiring information versus not acquiring but mimicking a certain type remains unchanged (changes in transfers canceled out), whichever type the uninformed Bidder i chooses to mimic. Allocation-wise, reducing x_i to 0 in the scenario of $u_i = -Z$ would increase the left-hand-side (payoff if acquiring information) of (OBi) by making type $u_i = -Z$ suffer less. This is the only change on the left-hand-side due to the changes in allocation. Suppose with the old allocation and transfer, the uninformed Bidder i chooses to mimic type $u_i = -Z$, then the right-hand-side of (OBi) would increase by the same amount in the scenario of $u_i = -Z$, canceling out the left-hand-side change due to the modified allocation. In the scenarios of $u_i = l$ and h, the right-hand-side would be reduced due to a lower allocation. Hence overall, acquiring information is more attractive if the uninformed chooses to mimic $u_i = -Z$. Suppose the uninformed mimics $u_i = l$ or h instead, then there would be no change on the right-hand-side due to the change in allocation. Acquiring information is more attractive as well due to the increase in the payoff if acquiring information. Therefore in the new (OBi), acquiring information is still better than not acquiring and mimicking $u_i = -Z$, $u_i = l$ or $u_i = h$. Finally, since after the change, x_i and t_i are both 0 if the uninformed bidder i reports $u_i = -Z$, the payoff if mimicking $-Z$ is exactly 0. Therefore acquiring information is better than quitting the game. Hence with the modified allocations and transfers, (OBi) remains to be true.

Moreover, we check that (IRi)s are not violated either. (IR) for type $u_i = -Z$ holds after the change, and (IR) for type $u_i = l$ is ensured by the (IC) of l mimicking $-Z$. To see this, take the (IC) of l mimicking $-Z$ as an example, and consider $i = 1$. We know that after the change, x_1 and t_1 are both 0, and the aforementioned (IC) holds. The expected payoff given $u_1 = l$ after the change is

$$\begin{aligned}
&\mathbb{E}_{u_2}\left[x_1(l, u_2)l - t_1(l, u_2)\middle|u_1 = l\right] \\
\geqslant\ &\mathbb{E}_{u_2}\left[x_1(-Z, u_2)l - t_1(-Z, u_2)\middle|u_1 = l\right] \\
=\ &0
\end{aligned}$$

where the first inequality is due to (IC1) of $u_1 = l$, the second due to x_1 and t_1 being 0 when $u_1 = -Z$. The case with $u_1 = -Z$ and Bidder 2 is similar.

(IRi) of type h is ensured by (IR) of $u_i = l$ and (IC) of $u_i = h$ mimicking l. In particular, take Bidder 1 as an example. The expected payoff given u_1 is

$$\begin{aligned}
&\mathbb{E}_{u_2}\left[x_1(h, u_2)h - t_1(h, u_2)\middle|u_1 = h\right] \\
\geqslant\ &\mathbb{E}_{u_2}\left[x_1(l, u_2)h - t_1(l, u_2)\middle|u_1 = h\right] \\
\geqslant\ &\mathbb{E}_{u_2}\left[x_1(l, u_2)l - t_1(l, u_2)\middle|u_1 = h\right] \\
=\ &\mathbb{E}_{u_2}\left[x_1(l, u_2)l - t_1(l, u_2)\middle|u_1 = l\right] \\
\geqslant\ &0
\end{aligned}$$

where the first inequality is due to (IC1) of $u_1 = h$, the second due to $x_1(l, u_2) \geqslant 0$ and $h > l$, the third line due to the fact that $u_1 = h$ and $u_1 = l$ imply the same information on u_2, and the last line as a result of (IR1) of $u_1 = l$. All other constraints remained the same. Therefore, the resulting mechanism strictly dominates the original one with $x_i > 0$ when $u_i = -Z$, indicating that an optimal mechanism must have $x_i = 0$ if $u_i = -Z$.

Finally, with a similar argument, it can be shown that $\mathbb{E}_{\mathbf{u}}[t_i(\mathbf{u})\big|u_i = -Z] = 0$ at the optimum (otherwise, changing the transfers in the same way but without changing the allocations increases the revenue strictly). That is, (IRi) when $u_i = -Z$ are binding.

■

Lemma A.1 *Removing (IRi) of Bidder i with $u_i = h$ from Problem (5) does not change the solution.*

Proof. For (IR1) of type $u_1 = h$, the expected payoff given u_1 is

$$\begin{aligned}
&\mathbb{E}_{u_2}\left[x_1(h,u_2)h - t_1(h,u_2)\big|u_1 = h\right] \\
\geqslant\ &\mathbb{E}_{u_2}\left[x_1(l,u_2)h - t_1(l,u_2)\big|u_1 = h\right] \\
\geqslant\ &\mathbb{E}_{u_2}\left[x_1(l,u_2)l - t_1(l,u_2)\big|u_1 = h\right] \\
=\ &\mathbb{E}_{u_2}\left[x_1(l,u_2)l - t_1(l,u_2)\big|u_1 = l\right] \\
\geqslant\ &0
\end{aligned}$$

where the first inequality is due to (IC1) of $u_1 = h$, the second due to $x_1(l,u_2) \geqslant 0$ and $h > l$, the third line due to the fact that $u_1 = h$ and $u_1 = l$ imply the same information on u_2, and the last line as a result of (IR1) of $u_1 = l$.

The proofs for Bidder 2 are similar.

■

Proof of Lemma 4.2. I first show that when solving the relaxed problem (9), it is without loss of generality to restrict attention to mechanisms with zero allocations and transfers for a bidder without gains from trade with the target, and for an uninformed bidder. The lemma below formalizes this argument.

Lemma A.2 *There exists a solution to Problem (9), where*
$x_i(\boldsymbol{u}) = 0$ and $t_i(\boldsymbol{u}) = 0$, $\forall \boldsymbol{u}$ s.t. $u_i = -Z$ or $a_i = 0$.

Proof. To see this, one can show first that the allocations and the *expected* transfers are zero when $u_i = -Z$ or $a_i = 0$. The proof is very similar to that of Lemma 4.1; in fact, the only part changed is that with the modified mechanism, fewer constraints are to be verified. Second, modify the optimum to have $t_i(\boldsymbol{u}) = 0$, *for all* $\boldsymbol{u}$ s.t. $a_i = 0$ or $u_i = -Z$. This would keep the expected payment where $a_i = 0$ or $u_i = -Z$ unchanged without affecting the target's revenue. In addition, such modification won't violate any other constraints. Therefore the modified mechanism is also an optimal mechanism. ■

Next, I show that if the problem with $a_2(h) = 0$ imposed to Problem (9) has a solution that satisfies all the constraints in Problem (5), then such solution is also an optimum for Problem (5). Combining with Lemma A.2, Lemma 4.2 will be proved.

The rest of the proof of Lemma 4.2 consists of two additional lemmas. For any mechanism M, define $R(M)$ as the target's revenue from such mechanism. The first lemma is as follows.

Lemma A.3 *Given any optimal mechanism M of Problem (5), there exists a possibly different mechanism M' such that:*
(i) In M', $a_2(h) = 0$,
(ii) M' satisfies the relaxed set of constraints in Problem (9), and
(iii) $R(M) \leq R(M')$.

Proof. Define $A \equiv \{u_1 : a_2(u_1) = 1\}$. Define B as the set of mechanisms satisfying the original constraints in Problem (5), B_R as the set of mechanisms satisfying the relaxed constraints in Problem (9). Let $B' \equiv \{M \in B_R : a_2(h) = 0\}$. The lemma says that $\max_{M \in B} R(M) \leqslant \max_{M \in B'} R(M)$.

To see why this is true, fix any given optimal mechanism $M \in B$. With M if $a_1 = 0$ or $a_2(h) = 0$ then we are done. If $a_1 \neq 0$ and $a_2(h) = 1$, then construct a new mechanism M' in the following way:
(i) For $\forall u_2$, $x_2'(h, u_2) = 0$, $t_2'(h, u_2) = 0$,
(ii) For $\forall u_2$, $x_1'(h, u_2) = 1$, $t_1'(h, u_2) = (1 - x_1(h, u_2))h + t_1(h, u_2)$,
(iii) For $\forall u_1 \in A - \{h\}$, $t_2'(u_1, h) = t_2(u_1, h) - \frac{\Pr(u_1=h|u_1\in A, u_2=h)(x_2(h,h)h - t_2(h,h))}{\Pr(u_1\neq h|u_1\in A, u_2=h)}$, $t_2'(u_1, l) = t_2(u_1, l) - \frac{\Pr(u_1=h|u_1\in A, u_2=l)(x_2(h,l)l - t_2(h,l))}{\Pr(u_1\neq h|u_1\in A, u_2=l)}$.
The first step is to exclude Bidder 2 if Bidder 1 has the highest type. The second step serves to increase allocation to Bidder 1 as much as possible if having the highest type, while still keeping the surplus of Bidder 1 unchanged. The final step is to keep Bidder 2's surplus conditional on $u_2 = h$ and $u_2 = l$ constant (the case with $u_2 = -z$ will not be affected), so that the IC's, IR's, and obedience constraint of Bidder 2 remain satisfied. Other allocations and payments remain unchanged.

With such a mechanism, we have:

$$
\begin{aligned}
& R(M') - R(M) \\
= \; & \Pr(u_1 = h)\mathbb{E}_{u_2}[(1 - x_1(h, u_2))h - t_2(h, u_2)|u_1 = h] \\
& - \Pr(u_1 \in A - \{h\}, u_2 = h)\frac{\Pr(u_1 = h|u_1 \in A, u_2 = h)\,(x_2(h, h)h - t_2(h, h))}{\Pr(u_1 \neq h|u_1 \in A, u_2 = h)} \\
& - \Pr(u_1 \in A - \{h\}, u_2 = l)\frac{\Pr(u_1 = h|u_1 \in A, u_2 = l)\,(x_2(h, l)l - t_2(h, l))}{\Pr(u_1 \neq h|u_1 \in A, u_2 = l)} \\
= \; & \Pr(u_1 = h, u_2 = h)((1 - x_1(h, h))h - t_2(h, h)) \\
& + \Pr(u_1 = h, u_2 = l)((1 - x_1(h, l))h - t_2(h, l)) \\
& - \Pr(u_1 = h, u_2 = h)\,(x_2(h, h)h - t_2(h, h)) \\
& - \Pr(u_1 = h, u_2 = l)\,(x_2(h, l)l - t_2(h, l)) \\
\geqslant \; & \Pr(u_1 = h, u_2 = h)\,(x_2(h, h)h - t_2(h, h) - x_2(h, h)h + t_2(h, h)) \\
& + \Pr(u_1 = h, u_2 = l)(x_2(h, l)h - t_2(h, l) - x_2(h, l)l + t_2(h, l)) \\
= \; & \Pr(u_1 = h, u_2 = l)x_2(h, l)(h - l) \\
\geqslant \; & 0. \qquad (13)
\end{aligned}
$$

It remains to verify that the new mechanism satisfies the relaxed set of constraints. The left-hand-side of the new (IC2) of type h mimicking l reads:

$$\begin{aligned}
& \mathbb{E}_{u_1}\left[x_2'(u_1,h)h - t_2'(u_1,h)|u_1 \in A-\{h\}, u_2=h\right] \\
= \; & \mathbb{E}_{u_1}\left[x_2(u_1,h)h - t_2(u_1,h)|u_1 \in A-\{h\}, u_2=h\right] \\
& + \frac{\Pr(u_1=h|u_1\in A, u_2=h)\left(x_2(h,h)h-t_2(h,h)\right)}{\Pr(u_1\neq h|u_1\in A, u_2=h)} \\
= \; & (\Pr(u_1\neq h|u_1\in A,u_2=h)\mathbb{E}_{u_1}\left[x_2(u_1,h)h - t_2(u_1,h)|u_1 \in A-\{h\}, u_2=h\right] \\
& + \Pr(u_1=h|u_1\in A, u_2=h)\left(x_2(h,h)h-t_2(h,h)\right))\frac{1}{\Pr(u_1\neq h|u_1\in A, u_2=h)} \\
= \; & \mathbb{E}_{u_1}\left[x_2(u_1,h)h - t_2(u_1,h)|u_1 \in A, u_2=h\right]\frac{1}{\Pr(u_1\neq h|u_1\in A, u_2=h)} \\
\geqslant \; & \mathbb{E}_{u_1}\left[x_2(u_1,l)h - t_2(u_1,l)|u_1 \in A, u_2=h\right]\frac{1}{\Pr(u_1\neq h|u_1\in A, u_2=h)} \\
= \; & \mathbb{E}_{u_1}\left[x_2(u_1,l)h - t_2(u_1,l)|u_1 \in A-\{h\}, u_2=h\right] \\
& + \frac{\Pr(u_1=h|u_1\in A, u_2=h)\left(x_2(h,l)h-t_2(h,l)\right)}{\Pr(u_1\neq h|u_1\in A, u_2=h)} \\
\geqslant \; & \mathbb{E}_{u_1}\left[x_2(u_1,l)h - t_2(u_1,l)|u_1 \in A-\{h\}, u_2=h\right] \\
& + \frac{\Pr(u_1=h|u_1\in A, u_2=l)\left(x_2(h,l)l-t_2(h,l)\right)}{\Pr(u_1\neq h|u_1\in A, u_2=l)} \\
= \; & \mathbb{E}_{u_1}\left[x_2'(u_1,l)h - t_2'(u_1,l)|u_1 \in A-\{h\}, u_2=h\right],
\end{aligned}$$

where the first inequality follows from the initial (IC2) of type h mimicking l , and the second inequality utilizes the fact that whether $u_2 = h$ or $u_2 = l$ is independent of u_1 conditional on v_2, and $x_2(h,l) \geqslant 0$, $h > l$.

The new (IR2) of type l reads:

$$\begin{aligned}
& \mathbb{E}_{u_1}\left[x_2'(u_1,l)l - t_2'(u_1,l)|u_1 \in A-\{h\}, u_2=l\right] \\
= \; & \mathbb{E}_{u_1}\left[x_2(u_1,l)l - t_2(u_1,l)|u_1 \in A-\{h\}, u_2=l\right] \\
& + \frac{\Pr(u_1=h|u_1\in A, u_2=l)\left(x_2(h,l)l-t_2(h,l)\right)}{\Pr(u_1\neq h|u_1\in A, u_2=l)} \\
= \; & \mathbb{E}_{u_1}\left[x_2(u_1,l)l - t_2(u_1,l)|u_1 \in A, u_2=l\right]\frac{1}{\Pr(u_1\neq h|u_1\in A, u_2=l)} \\
\geqslant \; & 0,
\end{aligned}$$

where the inequality follows from the initial (IR2) of type l.

The new (OB2) in the relaxed problem reads:

$$\begin{aligned}
& \mathbb{E}_{u_1 u_2}\left[x_2'(u_1,u_2)u_2 - t_2'(u_1,u_2)|u_1 \in A-\{h\}\right] \\
= & \mathbb{E}_{u_1,u_2}\left[x_2(u_1,u_2)u_2 - t_2(u_1,u_2)|u_1 \in A-\{h\}\right] \\
& +\Pr(u_2=h|u_1 \in A-\{h\})\frac{\Pr(u_1=h|u_1\in A, u_2=h)\left(x_2(h,h)h-t_2(h,h)\right)}{\Pr(u_1\neq h|u_1\in A,u_2=h)} \\
& +\Pr(u_2=l|u_1 \in A-\{h\})\frac{\Pr(u_1=h|u_1\in A, u_2=l)\left(x_2(h,l)l-t_2(h,l)\right)}{\Pr(u_1\neq h|u_1\in A,u_2=l)} \\
= & \mathbb{E}_{u_1,u_2}\left[x_2(u_1,u_2)u_2 - t_2(u_1,u_2)|u_1 \in A-\{h\}\right] \\
& +\frac{\Pr(u_1=h)}{\Pr(u_1\in A-\{h\})}\mathbb{E}\left[x_2(h,u_2)u_2-t_2(h,u_2)|u_1=h\right] \\
= & (\Pr(u_1\in A-\{h\}|u_1\in A)\mathbb{E}_{u_1,u_2}\left[x_2(u_1,u_2)u_2 - t_2(u_1,u_2)|u_1 \in A-\{h\}\right] \\
& +\Pr(u_1=h|u_1\in A)\mathbb{E}\left[x_2(h,u_2)u_2-t_2(h,u_2)|u_1=h\right])\frac{\Pr(u_1\in A)}{\Pr(u_1\in A-\{h\})} \\
= & \mathbb{E}_{u_1,u_2}\left[x_2(u_1,u_2)u_2 - t_2(u_1,u_2)|u_1 \in A\right]\frac{\Pr(u_1\in A)}{\Pr(u_1\in A-\{h\})} \\
\geqslant & \frac{c}{\Pr(u_1\in A-\{h\}|u_1\in A)} \\
> & c,
\end{aligned}$$

where the inequality follows from the initial (OB2) in the relaxed problem reads.

Other constraints in the relaxed problem (9) remain to be the same or can be trivially verified. ■

With Lemma A.3, we further derive the following lemma, so that when looking for the optimal mechanism for the original problem, we could restrict attention to the relaxed Problem (9) with $a_2(h)=0$ imposed.

Lemma A.4 *Suppose a mechanism is a solution to the problem that imposes $a_2(h)=0$ to the relaxed Problem (9). Moreover, this mechanism satisfies all constraints in the original problem, Problem (5). Then it is also a solution for Problem (5).*

Proof. Recall in the proof of Lemma A.3 that B is the set of mechanisms satisfying the original constraints in Problem (5), B_R is the set of mechanisms satisfying the relaxed constraints in Problem (9), and $B' \equiv \{M \in B_R : a_2(h)=0\}$.

Since M' is a solution to the relaxed Problem (9) with $a_2(h)=0$ being imposed, then by definition M' is the optimal mechanism within B'. In addition, M' satisfies all constraints in the original Problem (5), so $M' \in B$ as well. Suppose M^* is the optimal mechanism within B. Then according to Lemma A.3, there exists an $M'' \in B'$ such that $R(M^*) \leq R(M'')$. Since $R(M'') \leq R(M')$, we have $R(M^*) \leq R(M')$. But $M' \in B$, therefore $R(M') = R(M^*) = \max_{M \in B} R(M)$. That is, M' is also a solution to Problem (5). ■

Combining Lemma A.4 with Lemma A.2, the conclusion of Lemma 4.2 is reached. ■

Proof of Proposition 4.1. In order to prove Proposition 4.1, there are three steps. First, I find the optimal mechanism for the relaxed problem (9), within each candidate mechanism; second, I compare the revenues among the candidate mechanisms and find the global optimum for the relaxed problem (9); third, I show that such solution satisfy all constraints in the original problem (5), hence by Lemma 4.2, it is also the optimum for the original problem. Here I only show the solution for the optimal candidate mechanism (II); the solutions for other candidate mechanisms and the revenue comparison are in the internet appendix of the paper.

Solution for Candidate (II): $a_2 = 1$ iff $u_1 = l$. The relaxed maximization problem is

$$\begin{aligned}
\max_{x(\cdot,\cdot),t(\cdot,\cdot)} \quad & pqt_1(h,0) + b(1-q)t_1(l,-Z) \\
& +(p-b)(1-q)^2(t_1(l,l)+t_2(l,l)) + (p-b)q(1-q)(t_1(l,h)+t_2(l,h) \quad \text{s.t.} \\
(ICh1) \quad & x_1(h,0)h - t_1(h,0) \geqslant b/p(x_1(l,-Z)h - t_1(l,-Z)) \\
& \quad +(p-b)(1-q)/p(x_1(l,l)h - t_1(l,l)) + (p-b)q/p(x_1(l,h)h - t_1(l,h)) \\
(ICh2) \quad & x_2(l,h)h - t_2(l,h) \geqslant x_2(l,l)h - t_2(l,l) \\
(IRl1) \quad & b/p(x_1(l,-Z)l - t_1(l,-Z)) + (p-b)(1-q)/p(x_1(l,l)l - t_1(l,l)) \\
& \quad +(p-b)q/p(x_1(l,h)l - t_1(l,h)) \geqslant 0 \\
(IRl2) \quad & x_2(l,l)l - t_2(l,l) \geqslant 0 \\
(OB1) \quad & pq(x_1(h,0)h - t_1(h,0)) + b(1-q)(x_1(l,-Z)l - t_1(l,-Z)) \\
& \quad +(p-b)(1-q)^2(x_1(l,l)l - t_1(l,l)) + (p-b)q(1-q)(x_1(l,h)l - t_1(l,h)) \geqslant c \\
(OB2) \quad & \frac{(p-b)(1-q)}{p}(x_2(l,l)l - t_2(l,l)) + \frac{(p-b)q}{p}(x_2(l,h)h - t_2(l,h)) \geqslant c \\
(FE) \quad & x_1(u_1,u_2) \in [0,1],\ x_2(u_1,u_2) \in [0,1],\ x_1(u_1,u_2) + x_2(u_1,u_2) \leq 1, \forall u_1, u_2
\end{aligned}$$

This is also a linear programming problem. Let

$$b = (1-p)p(1-\rho), \tag{14}$$

then the solution is as follows.

Case 1, $0 \le c < \frac{(h-l)q(p(1-q)+bq))}{1+p(1-q)}$. ICh1, ICh2, IRl1, IRl2 are all binding, but OB1 and OB2 are not.The allocations are

$$\begin{aligned} &x_1(h,0) = 1,\ x_1(l,-Z) = 1,\ x_1(l,h) = 0,\ x_2(l,h) = 1 \\ &x_1(l,l) = \frac{p - b(1+p)}{(p-b)(1-p(1-q))},\ x_2(l,l) = 1 - x_1(l,l) \end{aligned} \tag{15}$$

The transfers are

$$\begin{aligned} &t_1(h,0) = \frac{-p(l+hp) + b(h-l)q + (l-h(1-p))pq}{p(-1+p(-1+q))},\ t_1(l,-Z) = l, \\ &t_2(l,h) = h - (h-l)\delta,\ t_2(l,l) = l \cdot x_2(l,l),\ t_1(l,l) = l \cdot x_1(l,l),\ t_1(l,h) = 0, \end{aligned} \tag{16}$$

where $\delta = x_2(l,l)$. The optimal revenue is $lp + (h-l)(p-b)q$.

Case 2, $\frac{(h-l)q(p(1-q)+bq))}{1+p(1-q)} \le c < \frac{(h-l)q(p-b)}{p}$. OB1, OB2, ICh1, IRl2 are binding. ICh2, IRl1 are not. The allocations are the same as Case 1. The transfers are

$$\begin{aligned} &t_1(h,0) = \frac{l(1-q)(p(1-q)+bq) - c(1+p-pq) + h(p^2(1-q) - p(2-q)q - b(1-q)q)}{p(1+p(1-q))}, \\ &t_1(l,-Z) = l - tf_1,\ t_2(l,h) = h - (h-l)\delta - tf_2,\ t_2(l,l) = lx_2(l,l) - tf_2, \\ &t_1(l,l) = lx_1(l,l) - tf_1,\ t_1(l,h) = -tf_1. \end{aligned} \tag{17}$$

where

$$tf_1 = \frac{c - pq(h - t_1(h,0))}{p - pq},\ \delta = \frac{cp}{(h-l)(p-b)q},\ tf_2 = 0. \tag{18}$$

The optimal revenue is $lp - c(1-p(1-q)) + (h-l)(2p-b)q - (h-l)(p-b)q^2$.

Case 3, $c \geqslant \frac{(h-l)q(p-b)}{p}$. OB1, OB2, ICh1 are binding. IRl2, IRl1, ICh2 are not. The allocations are the same as Case 1 and 2. The transfers are the same except that

$$\delta = 1,\ tf_2 = \frac{cp}{(p-b)} - (h-l)q.$$

The optimal revenue is the same as Case 2.

Solving for the optimal mechanism for each of the candidates and compare revenues, we can identify the optimal mechanism within each region of the (c, ρ) space.

For $\rho \in [0,1)$, $c \geqslant 0$, we define the space into the following five regions:

$$\begin{aligned}
&\text{Region (I)} : c < \min(c_3(\rho,p), \max(c_1(\rho,p), c_2(\rho,p)));\\
&\text{Region (II)} : \rho > \frac{lp}{lp+(h-l)q}, c \in [\max(c_1(\rho,p), c_2(\rho,p)), \min(c_3(\rho,p), c_4(\rho,p)));\\
&\text{Region (III)} : \rho \leq \frac{lp}{lp+(h-l)q}, c \in [c_3(\rho,p), c_2(\rho,p));\\
&\text{Region (IV)} : \rho < \min(1, \frac{lp}{(h-l)q(1-p)}), c \in [\max(c_2(\rho,p), c_3(\rho,p)), c_5(p));\\
&\text{Region (V)} : c \geqslant \max(c_5(p), c_4(\rho,p)).
\end{aligned} \tag{19}$$

■

Proof of Proposition 5.1. From the proof of Proposition 4.1, when Candidate (II) is the optimal mechanism, it is either in Case 1 or Case 2. Therefore in the optimal solution of the direct mechanism, $tf_2 = 0$, $tf_1 \geqslant 0$. It remains to be shown that the indirect mechanism in Proposition 5.1 implements the same outcome as the optimal direct mechanism.

I show that under such indirect mechanism, Bidder 1 acquires information. Type $u_1 = h$ accepts the buy-it-now price $t_1(h, 0)$, and type $u_1 = -Z$ rejects both the buy-it-now price and the floor price $l - tf_1$. Type $u_1 = l$ rejects the buy-it-now price but accepts the floor price in exchange for the termination fee tf_1, and drops out at the price $l + \Delta_1 - tf_1$ at the clock auction, where $\Delta_1 = (h-l)x_2(l,l)$. Bidder 2 is recommended to acquire information if and only if Bidder 1 rejects the buy-it-now price but accepts the floor price, and Bidder 2 follows the recommendation. Type $u_2 = -Z$, $u_2 = l$, $u_2 = h$ drops out at prices of l, $l + \Delta_2$, h at the clock auction accordingly, where $\Delta_2 = (h-l)(1-\delta)$, and $\delta \geqslant x_2(l,l)$. Then I verify that under such equilibrium, the outcome is the same as that in the direct mechanism solved in the proof of Proposition 4.1.

Consider Bidder 2 first. If not approached, Bidder 2 is excluded, so both the allocation and transfer are 0, consistent with the direct mechanism. If approached to be recommended to acquire information, Bidder 2 knows $u_1 = l$ and would drop out at $l + \Delta_1 - tf_1$. Consider three cases. If $u_2 = -Z$, dropping out at l leads to 0 payoff. If dropping out at $l+\Delta_2$, Bidder 2 wins with probability $x_2(l, l)$ at price l, getting $x_2(l, l)(-Z - l) < 0$. If dropping out at h, the payoff is $-Z - (l + \Delta_2) < 0$. Therefore $u_2 = -Z$ drops out at l, and both the allocation and transfer are 0, consistent with the direct mechanism. Similarly, if $u_2 = l$, dropping out at l, $l+\Delta_2$ and h results in payoff 0, $x_2(l, l)(l-l) = 0$, and $l-(l+\Delta_2) < 0$, so would drop out at $l+\Delta_2$, with the allocation and transfer being $x_2(l, l)$ and $x_2(l, l) \cdot l$, consistent with the direct mechanism. If $u_2 = h$, dropping out at l, $l + \Delta_2$ and h results in payoff 0, $x_2(l, l)(h - l) > 0$, and $h - (l + \Delta_2) = h - l - (h - l)(1 - \Delta) \geqslant (h - l)x_2(l, l)$, so would drop out at h, with the allocation and transfer being 1 and $l + \Delta_2 = l + (h - l)(1 - \Delta)$, consistent with the direct mechanism. Finally, since Bidder 2's payoff is the same as the direct mechanism after information acquisition, Bidder 2 would follow the recommendation to acquire information as in the direct mechanism. Note that essentially the incentive constraints in this indirect mechanism coincide with the corresponding ICs in the direct mechanism.

Now consider Bidder 1. If has acquired information and $u_1 = -Z$, accepting the buy-it-now price results in payoff $-Z - t_1(h, 0) < 0$. Rejecting both the buy-it-now price and the floor price $l - tf_1$ gives 0. If accepting the floor price and dropping out at $l + \Delta_1 - tf_1$, the payoff is

$$
\begin{aligned}
&P(u_2 = -Z|u_1 = -Z)(-Z - l + tf_1) \\
&+P(u_2 = l|u_1 = -Z)((-Z - l + tf_1)x_1(l, l) + tf_1 x_2(l, l)) \\
&+P(u_2 = h|u_1 = -Z)tf_1.
\end{aligned}
$$

It is the same payoff if type $u_1 = -Z$ mimics type $u_1 = l$ in the direct mechanism, hence it is no higher than 0, the equilibrium payoff for type $u_1 = -Z$ in the direct mechanism. If dropping out at $h - tf_1$, the payoff is

$$
\begin{aligned}
&P(u_2 = -Z|u_1 = -Z)(-Z - l + tf_1) \\
&+P(u_2 = l|u_1 = -Z)(-Z - l - \Delta_1 + tf_1) \\
&+P(u_2 = h|u_1 = -Z)((-Z - h + tf_1) \cdot 1/2 + tf_1 \cdot 1/2),
\end{aligned}
$$

which is smaller than the previous payoff when plugging in $\Delta_1 = (h-l)x_2(l,l)$. So $u_1 = -Z$ rejects both prices and gets 0 as allocation, transfer and payoff, consistent with the direct mechanism.

We can also show in a similar fashion that type $u_1 = l$ would reject the buy-it-now price and drops out at $l + \Delta_1 - tf_1$. It is straightforward with the case of mimicking h or $-Z$ in the indirect mechanism, since the conditions are the same as the corresponding ICs in the direct mechanism. If deviating to dropping out at $h - tf_1$ instead (off-equilibrium path), the payoff is

$$
\begin{aligned}
&P(u_2 = -Z|u_1 = l)(l - l + tf_1) \\
&+P(u_2 = l|u_1 = l)(l - l - \Delta_1 + tf_1) \\
&+P(u_2 = h|u_1 = l)((l - h + tf_1) \cdot 1/2 + tf_1 \cdot 1/2) \\
&\quad < P(u_2 = -Z|u_1 = l)(l - l + tf_1) \\
&+P(u_2 = l|u_1 = l)(x_2(l,l)tf_1 + x_1(l,l)(l - l + tf_1)) \\
&+P(u_2 = h|u_1 = l)tf_1,
\end{aligned}
$$

where the right-hand-side equals to the equilibrium payoff of Bidder 1 if $u_1 = l$.

If $u_1 = h$ deviates to accepting the floor price and dropping out at $h - tf_1$ instead of accepting the buy-it-now price, the payoff is

$$
\begin{aligned}
& P(u_2 = -Z|u_1 = h)(h - l + tf_1) \\
& + P(u_2 = l|u_1 = h)(h - l - \Delta_1 + tf_1) \\
& + P(u_2 = h|u_1 = h)tf_1 \\
& \quad = P(u_2 = -Z|u_1 = h)(h - l + tf_1) \\
& + P(u_2 = l|u_1 = h)(x_2(l,l)tf_1 + x_1(l,l)(h - l + tf_1)) \\
& + P(u_2 = h|u_1 = h)((h - h + tf_1) \cdot 1/2 + tf_1 \cdot 1/2),
\end{aligned}
$$

where the equality is due to $\Delta_1 = (h - l)x_2(l,l)$.[31] The expression on the right-hand-side equals to the payoff of Bidder 1 if a type $u_1 = h$ mimics a type l in both the direct mechanism and the indirect mechanism. Then the IC condition in the direct mechanism ensures that a type h prefers to accept the buy-it-now price. The cases with deviating to mimic a type l or $-Z$ are trivial.

Finally, Bidder 1 would acquire information, because such condition is exactly the same as the (OB1) in the direct mechanism. ■

[31] Note that Δ_1 is chosen to be $(h-l)x_2(l,l)$ (potentially different from Δ_2) to make type $u_1 = h$ indifferent from dropping out at $l + \Delta_1 - tf_1$ and $h - tf_1$ if deviating to rejecting the buy-it-now price and accepting the floor price instead. However, such complication is a result of discrete type, hence is not an important feature for the result.

B Internet Appendix

Proof of Proposition 4.1. In order to prove Proposition 4.1, there are three steps. First, I find the optimal mechanism for the relaxed problem (9), within each candidate mechanism; second, I compare the revenues among the candidate mechanisms and find the global optimum for the relaxed problem (9); third, I show that such solution satisfy all constraints in the original problem (5), hence by Lemma 4.2, it is also the optimum for the original problem.

Step 1, finding the solution for each candidate mechanism.

Solution for Candidate (I): $a_2 = 1$ **iff** $u_1 = l$ *or* $u_1 = -Z$**.** The relaxed maximization problem is

$$\begin{array}{rl}
\max\limits_{x(\cdot,\cdot),t(\cdot,\cdot)} & pqt_1(h,0) + b(1-q)t_1(l,-Z) + (p-b)(1-q)^2(t_1(l,l)+t_2(l,l)) \\
& +(p-b)q(1-q)(t_1(l,h)+t_2(l,h)) + b(1-q)t_2(-Z,l) + bqt_2(-Z,h) \ \text{ s.t.} \\
(ICh1) & x_1(h,0)h - t_1(h,0) \geqslant b/p(x_1(l,-Z)h - t_1(l,-Z)) \\
& \quad +(p-b)(1-q)/p(x_1(l,l)h - t_1(l,l)) + (p-b)q/p(x_1(l,h)h - t_1(l,h) \\
(ICh2) & b/(b+(p-b)(1-q))(x_2(-Z,h)h - t_2(-Z,h)) \\
& \quad +(p-b)(1-q)/(b+(p-b)(1-q))(x_2(l,h)h - t_2(l,h)) \\
& \geqslant b/(b+(p-b)(1-q))(x_2(-Z,l)h - t_2(-Z,l)) \\
& \quad +(p-b)(1-q)/(b+(p-b)(1-q))(x_2(l,l)h - t_2(l,l)) \\
(IRl1) & b/p(x_1(l,-Z)l - t_1(l,-Z)) + (p-b)(1-q)/p(x_1(l,l)l - t_1(l,l)) \\
& \quad +(p-b)q/p(x_1(l,h)l - t_1(l,h) \geqslant 0 \\
(IRl2) & b/(b+(p-b)(1-q))(x_2(-Z,l)l - t_2(-Z,l)) \\
& \quad +(p-b)(1-q)/(b+(p-b)(1-q))(x_2(l,l)l - t_2(l,l)) \geqslant 0 \\
(OB1) & pq(x_1(h,0)h - t_1(h,0)) + b(1-q)(x_1(l,-Z)l - t_1(l,-Z)) \\
& \quad +(p-b)(1-q)^2(x_1(l,l)l - t_1(l,l)) + (p-b)q(1-q)(x_1(l,h)l - t_1(l,h)) \geqslant c \\
(OB2) & \dfrac{b(1-q)}{1-pq}(x_2(-Z,l)l - t_2(-Z,l)) + \dfrac{bq}{1-pq}(x_2(-Z,h)h - t_2(-Z,h)) \\
& \quad +\dfrac{(p-b)(1-q)^2}{1-pq}(x_2(l,l)l - t_2(l,l)) + \dfrac{(p-b)q(1-q)}{1-pq}(x_2(l,h)h - t_2(l,h)) \geqslant c \\
(FE) & x_1(u_1,u_2) \in [0,1],\ x_2(u_1,u_2) \in [0,1],\ x_1(u_1,u_2) + x_2(u_1,u_2) \leq 1, \forall u_1, u_2
\end{array}$$

This is a linear programming problem, therefore it is easy to solve. Let

$$b = (1-p)p(1-\rho), \tag{20}$$

that is, b is higher when ρ is smaller. Then the solution is as follows.

Case 1, $0 \leq c < \frac{(h-l)q(p(1-q)+bq+b))}{2-pq}$. ICh1, ICh2, IRl1, IRl2 are binding. OB1 and OB2 are not. The allocations are

$$x_1(h,0)=1,\ x_1(l,-Z)=1,\ x_1(l,h)=0,\ x_2(-Z,h)=1,\ x_2(-Z,l)=1,\ x_2(l,h)=1,$$
$$x_1(l,l)=\frac{-b+p(1-q)+bq(1+p)}{(p-b)(1-q)(2-pq)},\ x_2(l,l)=1-x_1(l,l)$$

The transfers are

$$t_1(h,0)=\frac{p(l+h)-b(h-l)(1+q)-(l-h(1-p))pq}{p(2-pq)},\ t_1(l,-Z)=l,\ t_1(l,l)=lx_1(l,l),$$
$$t_1(l,h)=0,\ t_2(l,h)=h-(h-l)\Delta,\ t_2(l,l)=lx_2(l,l),\ t_2(-Z,l)=l,\ t_2(-Z,h)=l,$$

where $\delta=x_2(l,l)$. The optimal revenue is $l(b+p)+(h-l)(p-b)q$.

Case 2, $\frac{(h-l)q(p(1-q)+bq+b))}{2-pq}\leq c<\frac{(h-l)q(p-(p-b)(\varepsilon(1-q)+q))}{1-pq}$, with ε being an arbitrary number $\in(0,1-x_2(l,l))$. OB1, OB2, ICh1, IRl2 are binding. ICh2, IRl1 are not. The allocations are the same as Case 1. The transfers are

$$t_1(h,0)=\frac{-lp(1-q)^2+c(2-pq)+b(h-l)(1-q^2)+hp(-1+q(-2+p+q))}{p(-2+pq)},$$
$$t_1(l,-Z)=l-tf_1,\ t_2(l,h)=h-(h-l)\delta-tf_2,\ t_2(l,l)=l\cdot x_2(l,l)-tf_2,$$
$$t_1(l,l)=l\cdot x_1(l,l)-tf_1,\ t_1(l,h)=-tf_1,\ t_2(-Z,l)=l-tf_2,\ t_2(-Z,h)=l-tf_2.$$

where

$$tf_1=c/p+\frac{(h-l)q(b+p+bq-pq)}{(-2+pq)p},\ \delta=\frac{c-b(h-l)q-cpq}{(h-l)(p-b)(1-q)q},\ tf_2=0.$$

The optimal revenue is $l(b+p)-c(2-pq)+(h-l)(p(2-q)+bq)q$.

Case 3, $c\geqslant\frac{(h-l)q(p-(p-b)(\varepsilon(1-q)+q))}{1-pq}$. OB1, OB2, ICh1 are binding. ICh2, IRl1, IRl2 are not. The allocations are the same as Case 1 and 2. The transfers are the same except that

$$\delta=1-\varepsilon,\ tf_2=\frac{c-cpq-(h-l)q(p-(p-b)(\varepsilon(1-q)+q))}{p+bq-pq},$$

where ε is an arbitrary number $\in(0,1-x_2(l,l))$. The optimal revenue is the same as Case 2.

Solution for Candidate (II): $a_2 = 1$ **iff** $u_1 = l$. The relaxed maximization problem is

$$\begin{aligned}
\max_{x(\cdot,\cdot),t(\cdot,\cdot)} \quad & pqt_1(h,0) + b(1-q)t_1(l,-Z) \\
& +(p-b)(1-q)^2(t_1(l,l)+t_2(l,l)) + (p-b)q(1-q)(t_1(l,h)+t_2(l,h) \;\text{ s.t.} \\
(ICh1) \quad & x_1(h,0)h - t_1(h,0) \geqslant b/p(x_1(l,-Z)h - t_1(l,-Z)) \\
& \quad +(p-b)(1-q)/p(x_1(l,l)h - t_1(l,l)) + (p-b)q/p(x_1(l,h)h - t_1(l,h)) \\
(ICh2) \quad & x_2(l,h)h - t_2(l,h) \geqslant x_2(l,l)h - t_2(l,l) \\
(IRl1) \quad & b/p(x_1(l,-Z)l - t_1(l,-Z)) + (p-b)(1-q)/p(x_1(l,l)l - t_1(l,l)) \\
& \quad +(p-b)q/p(x_1(l,h)l - t_1(l,h)) \geqslant 0 \\
(IRl2) \quad & x_2(l,l)l - t_2(l,l) \geqslant 0 \\
(OB1) \quad & pq(x_1(h,0)h - t_1(h,0)) + b(1-q)(x_1(l,-Z)l - t_1(l,-Z)) \\
& \quad +(p-b)(1-q)^2(x_1(l,l)l - t_1(l,l)) + (p-b)q(1-q)(x_1(l,h)l - t_1(l,h)) \geqslant c \\
(OB2) \quad & \frac{(p-b)(1-q)}{p}(x_2(l,l)l - t_2(l,l)) + \frac{(p-b)q}{p}(x_2(l,h)h - t_2(l,h)) \geqslant c \\
(FE) \quad & x_1(u_1,u_2) \in [0,1],\; x_2(u_1,u_2) \in [0,1],\; x_1(u_1,u_2) + x_2(u_1,u_2) \leq 1, \forall u_1, u_2
\end{aligned}$$

This is also a linear programming problem. With (20) the solution is as follows.

Case 1, $0 \leq c < \frac{(h-l)q(p(1-q)+bq))}{1+p(1-q)}$. ICh1, ICh2, IRl1, IRl2 are all binding, but OB1 and OB2 are not.The allocations are

$$\begin{aligned}
& x_1(h,0) = 1,\; x_1(l,-Z) = 1,\; x_1(l,h) = 0,\; x_2(l,h) = 1 \\
& x_1(l,l) = \frac{p - b(1+p)}{(p-b)(1-p(1-q))},\; x_2(l,l) = 1 - x_1(l,l)
\end{aligned} \tag{21}$$

The transfers are

$$\begin{aligned}
& t_1(h,0) = \frac{-p(l+hp) + b(h-l)q + (l - h(1-p))pq}{p(-1+p(-1+q))},\; t_1(l,-Z) = l, \\
& t_2(l,h) = h - (h-l)\delta,\; t_2(l,l) = l \cdot x_2(l,l),\; t_1(l,l) = l \cdot x_1(l,l),\; t_1(l,h) = 0,
\end{aligned} \tag{22}$$

where $\delta = x_2(l,l)$. The optimal revenue is $lp + (h-l)(p-b)q$.

Case 2, $\frac{(h-l)q(p(1-q)+bq))}{1+p(1-q)} \leq c < \frac{(h-l)q(p-b)}{p}$. OB1, OB2, ICh1, IRl2 are binding. ICh2, IRl1 are not. The allocations are the same as Case 1. The transfers are

$$
\begin{aligned}
&t_1(h,0)=\frac{l(1-q)(p(1-q)+bq)-c(1+p-pq)+h(p^2(1-q)-p(2-q)q-b(1-q)q)}{p(1+p(1-q))},\\
&t_1(l,-Z)=l-tf_1,\ t_2(l,h)=h-(h-l)\delta-tf_2,\ t_2(l,l)=lx_2(l,l)-tf_2,\\
&t_1(l,l)=lx_1(l,l)-tf_1,\ t_1(l,h)=-tf_1. \qquad (23)
\end{aligned}
$$

where

$$tf_1=\frac{c-pq(h-t_1(h,0))}{p-pq},\ \delta=\frac{cp}{(h-l)(p-b)q},\ tf_2=0. \qquad (24)$$

The optimal revenue is $lp-c(1-p(1-q))+(h-l)(2p-b)q-(h-l)(p-b)q^2$.

Case 3, $c\geqslant\frac{(h-l)q(p-b)}{p}$. OB1, OB2, ICh1 are binding. IRl2, IRl1, ICh2 are not. The allocations are the same as Case 1 and 2. The transfers are the same except that

$$\delta=1,\ tf_2=\frac{cp}{(p-b)}-(h-l)q.$$

The optimal revenue is the same as Case 2.

Solution for Candidate (III): $a_2=1$ iff $u_1=-Z$. The relaxed maximization problem is

$$
\begin{aligned}
&\max_{x(\cdot,\cdot),t(\cdot,\cdot)} && p(qt_1(h,0)+(1-q)t_1(l,0))+b(qt_2(-Z,h)+(1-q)t_2(-Z,l))\ \text{ s.t.}\\
&(ICh1) && x_1(h,0)h-t_1(h,0)\geqslant x_1(l,0)h-t_1(l,0)\\
&(ICh2) && x_2(-Z,h)h-t_2(-Z,h)\geqslant x_2(-Z,l)h-t_2(-Z,l)\\
&(IRl1) && x_1(l,0)l-t_1(l,0)\geqslant 0\\
&(IRl2) && x_2(-Z,l)l-t_2(-Z,l)\geqslant 0\\
&(OB1) && p(q(x_1(h,0)h-t_1(h,0))+(1-q)(x_1(l,0)l-t_1(l,0)))\geqslant c\\
&(OB2) && \frac{b}{1-p}(q(x_2(-Z,h)h-t_2(-Z,h))+(1-q)(x_2(-Z,l)l-t_2(-Z,l))\geqslant c\\
&(FE) && x_1(u_1,u_2)\in[0,1],\ x_2(u_1,u_2)\in[0,1],\ x_1(u_1,u_2)+x_2(u_1,u_2)\leq 1,\forall u_1,u_2
\end{aligned}
$$

This is a linear programming problem. With (20) the solution is as follows.

Case 1, $0 \leq c < \frac{bq(h-l)}{1-p}$. ICh1, ICh2, IRl1, IRl2 are binding. OB1 and OB2 are not.The allocations are

$$x_1(h,0) = 1,\ x_1(l,0) = 1,\ x_2(-Z,h) = 1,\ x_2(-Z,l) = 1.$$

The transfers are

$$t_1(h,0) = l,\ t_1(l,0) = l,\ t_2(-Z,h) = l,\ t_2(-Z,l) = l.$$

The optimal revenue is $l(b+p)$.

Case 2, $\frac{bq(h-l)}{1-p} \leq c < (h-l)pq$. ICh1, ICh2, IRl1, OB2 are binding. OB1 and IRl2 are not. The allocations are the same as Case 1. The transfers are

$$t_1(h,0) = l,\ t_1(l,0) = l,\ t_2(-Z,h) = l - c(1-p)/b + (h-l)q,\ t_2(-Z,l) = t_2(-Z,h).$$

The optimal revenue is $l(b+p) - c(1-p) + (h-l)bq$.

Case 3, $c \geqslant (h-l)pq$. ICh1, ICh2, OB1, OB2 are binding. IRl1 and IRl2 are not. The allocations are the same as Case 1 and 2. The transfers are:

$$\begin{aligned} &t_1(h,0) = l - c/p + (h-l)q,\ t_1(l,0) = t_1(h,0) \\ &t_2(-Z,h) = l - c(1-p)/b + (h-l)q,\ t_2(-Z,l) = t_2(-Z,h). \end{aligned}$$

The optimal revenue is $l(b+p) - c(2-p) - (h-l)q(b+q)$.

Solution for Candidate (IV): $a_2 = 0$ for all u_1. The relaxed maximization problem is

$$\begin{aligned} \max_{x(\cdot,\cdot),t(\cdot,\cdot)} \quad & p(qt_1(h,0) + (1-q)t_1(l,0)) \ \text{ s.t.} \\ (ICh1) \quad & x_1(h,0)h - t_1(h,0) \geqslant x_1(l,0)h - t_1(l,0) \\ (IRl1) \quad & x_1(l,0)l - t_1(l,0) \geqslant 0 \\ (OB1) \quad & p(q(x_1(h,0)h - t_1(h,0)) + (1-q)(x_1(l,0)l - t_1(l,0))) \geqslant c \\ (FE) \quad & x_1(u_1,u_2) \in [0,1],\ x_2(u_1,u_2) \in [0,1],\ x_1(u_1,u_2) + x_2(u_1,u_2) \leq 1, \forall u_1, u_2 \end{aligned}$$

This is a linear programming problem. With (20) the solution is as follows.

Case 1, $0 \leq c < (h-l)pq$. ICh1, IRl1 are binding. OB1 is not. The allocations are

$$x_1(h,0) = 1,\ x_1(l,0) = 1.$$

The transfers are

$$t_1(h,0) = l,\ t_1(l,0) = l.$$

The optimal revenue is lp.

Case 2, $c > (h-l)pq$. ICh1, ICl1, OB1 are binding. IRl1 is not. The allocations are the same as Case 1. The transfers are

$$t_1(h,0) = l - c/p + (h-l)q,\ t_1(l,0) = t_1(h,0).$$

The optimal revenue is $lp - c + (h-l)pq$.

Solution for Candidate (V): $a_1 = a_2 = 0$ for all u_1, u_2. This case is trivial. All allocations and transfers are 0, and the optimal revenue is 0.

Step 2, revenue comparison. We compare the revenues of the different candidate mechanisms on a (c,ρ) space (where c is on the X-axis, and ρ is on the Y-axis), and identify the mechanism with the highest revenue in each region of the (c,ρ) space.

Compare Candidate (I) with (II). Case 2 and 3 of Candidate (II) could also be combined since they share the same revenue. It is straightforward to verify that when the revenue of Candidate (I) equals to that of (II), (I) is in Case 2 or 3. (II) is either in Case 1, or in Case 2 or 3.

When Case 2 or 3 of (I) intersects with Case 1 of (II), by equating the two corresponding revenues and plug in (20), the intersection is the line:

$$\begin{aligned} & c = c_1(\rho,p), \\ & \text{where } c_1(\rho,p) = \frac{(h-l)pq(1-q) + bhq(1+q) + bl(1-q-q^2)}{2-pq}. \end{aligned} \tag{25}$$

Candidate (I) is higher to the left of the line, while (II) is higher to the right of the line.

When Case 2 or 3 of (I) intersects with Case 2 or 3 of (II), by equating the two corresponding revenues and plug in (20), the intersection is the line:

$$\begin{aligned} & c = c_2(\rho, p), \\ & \text{where } c_2(\rho, p) = \frac{b(l + (h-l)q)}{1-p}. \end{aligned} \tag{26}$$

Candidate (I) is higher to the left of the line, while (II) is higher to the right of the line.
Compare Candidate (I) with (III). Case 2 and 3 of Candidate (I) could be combined since they share the same revenue. It is straightforward to verify that when the revenue of Candidate (I) equals to that of (III), (I) is in Case 2 or 3, and (III) is in Case 3. Therefore their intersection is solved by equating the two corresponding revenues. Recall (20), the intersection is the line:

$$\begin{aligned} & c = c_3(\rho, p), \\ & \text{where } c_3(\rho, p) = \frac{(h-l)(p-b)q}{p}. \end{aligned} \tag{27}$$

Candidate (I) is higher to the left of the line (when ρ is higher, or when c is smaller), while (III) is higher to the right of the line.
Compare Candidate (II) with (IV). It is straightforward to verify that when the revenue of Candidate (II) equals to that of (IV), (II) is in Case 2 or 3, and (IV) is in Case 2. By equating the two corresponding revenues and plug in (20), the intersection is the line:

$$c = c_3(\rho, p).$$

Candidate (II) is higher to the left of the line, while (IV) is higher to the right of the line. Note that this is the same line as the intersection of (I) and (III). Also, this line coincides with the line dividing Case 2 and Case 3 of (II), therefore when (II) dominates (IV), (II) is in Case 2 (with $tf_2 = 0$).
Compare Candidate (III) with (IV). It is straightforward to verify that when the revenue of Candidate (III) equals to that of (IV), (III) is in Case 3, and (IV) is in Case 2. By equating

the two corresponding revenues and plug in (20), the intersection is the line:

$$c = c_2(\rho, p).$$

Candidate (III) is higher to the left of the line, while (IV) is higher to the right of the line. Note that this is the same line as the intersection of (I) and (II) Case 2 or 3.

Compare Candidate (II) with (V). It is straightforward to verify that whenever the revenue of Candidate (II) equals to that of (V), (II) is in Case 2 or 3. By equating the two corresponding revenues and plug in (20), the intersection is the line:

$$\begin{aligned} & c = c_4(\rho, p), \\ & \text{where } c_4(\rho, p) = \frac{hq(-b + p(2-q) + bq) + l(1-q)(p(1-q) + bq)}{1 + p(1-q)}. \end{aligned} \tag{28}$$

Candidate (II) is higher to the left of the line, while (V) is higher to the right of the line.

Compare Candidate (IV) with (V). It is straightforward to verify that whenever the revenue of Candidate (IV) equals to that of (V), (IV) is in Case 2. By equating the two corresponding revenues and plug in (20), the intersection is the line:

$$\begin{aligned} & c = c_5, \\ & \text{where } c_5 = p(l + (h-l)q). \end{aligned} \tag{29}$$

Candidate (IV) is higher to the left of the line, while (V) is higher to the right of the line.

With the comparisons above, we can identify the optimal mechanism within each region of the (c, ρ) space.

For $\rho \in [0,1)$, $c \geqslant 0$, we define the space into the following five regions:

$$\begin{aligned}
&\text{Region (I)} : c < \min(c_3(\rho,p), \max(c_1(\rho,p), c_2(\rho,p)));\\
&\text{Region (II)} : \rho > \frac{lp}{lp+(h-l)q}, c \in [\max(c_1(\rho,p), c_2(\rho,p)), \min(c_3(\rho,p), c_4(\rho,p)));\\
&\text{Region (III)} : \rho \leq \frac{lp}{lp+(h-l)q}, c \in [c_3(\rho,p), c_2(\rho,p));\\
&\text{Region (IV)} : \rho < \min(1, \frac{lp}{(h-l)q(1-p)}), c \in [\max(c_2(\rho,p), c_3(\rho,p)), c_5(p));\\
&\text{Region (V)} : c \geqslant \max(c_5(p), c_4(\rho,p)).
\end{aligned} \tag{30}$$

Optimal mechanism in each region. We now show that in region (i) in (30), the optimal mechanism for the relaxed problem (9) is Candidate (i), for $i = I, II, III, IV, V$.

It is straightforward to verify that the regions defined in (30) is illustrated in the left panel of Figure 3 when $\frac{lp}{(h-l)(1-p)q} \leq 1$, and in the right panel when $\frac{lp}{(h-l)(1-p)q} > 1$. Also, the boundaries of each regions are illustrated in the figures as well. In both cases, for region (I), Candidate (I) dominates (II) and (III), while Candidate (II) dominates (IV), and Candidate (IV) dominates (V), hence (I) is the optimum. In region (II), Candidate (II) dominates (I), (IV) and (V), while (I) dominates (III), so (II) is the optimum. In region (III), Candidate (III) dominates (I) and (IV), while (I) dominates (II), and (IV) dominates (V), so (III) is optimal. In region (IV), Candidate (IV) dominates (II), (III) and (V), while (III) dominates (I), so (IV) is optimal. Finally, in region (V), Candidate (V) dominates (II) and (IV), while (II) dominates (I), (IV) dominates (III), so (V) is the optimal.

Step 3, verify optimality for the original problem (5). The solution we have obtained is the optimum for the relaxed problem (9). To show that it is also the optimum for the original problem (5), it is sufficient to verified that the constraints relaxed are satisfied according to Lemma 4.2. In particular, for Bidder i, $i = 1, 2$, we need to verify that (i) type l does not mimic h, (ii) type $-Z$ does not mimic l or h, (iii) if recommended to acquire information and chooses not to, mimicking $u_i = -Z$ is the best choice, and (iv) type l and h do not mimic a type $u_i = -Z$. The last condition, (iv), is trivially satisfied, since in the solution, $x_i = 0$ and $t_i = 0$ whenever $u_i = -Z$. Therefore such IC conditions are implied by the IR conditions of type l and h. For the rest of the constraints, we need to verify them for Case 1, 2, 3 of Candidate (I), Case 1 and 2 of Candidate (II), Case 3 of Candidate (III), and Case 2 of Candidate (IV), since only those cases occur at the optimum. The case with Candidate (V) is trivial.

Simple calculations show that if type $-Z$ or the uninformed one does not mimic l, then h will not be mimicked either. This is because for each of the cases above,

$$x_1(l, u_2) \leq x_1(h, u_2),\ t_1(l, u_2) \leq t_1(h, u_2),\ \forall u_2,$$
$$x_2(u_1, l) \leq x_1(u_1, h),\ t_1(u_1, l) \leq t_1(u_1, h),\ \forall u_1.$$

That is, mimicking h leads to more payment and higher likelihood to get the target firm, even if the valuation is negative. Therefore we could skip the constraints that type $-Z$ does not mimic h, and if recommended to acquire information and chooses not to, mimicking $u_i = -Z$ is better than mimicking h.

These conditions that have been relaxed essentially require that Z is high enough, so that a buyer with negative gains from trade with the target would not mimic a buyer with positive gains from trade at the optimum. The sufficient conditions for them to hold are Assumption 4.1 and Inequality (10). ■

Proof of Proposition 5.2, Region (I). The proof is similar to that of Candidate (II). Take the solution for the optimal Candidate (I) from the proof of Proposition 4.1. It remains to be shown that the indirect mechanism in Proposition 5.2 implements the same outcome as the optimal direct mechanism.

I show that under such indirect mechanism, Bidder 1 acquires information. Type $u_1 = h$ accepts the buy-it-now price $t_1(h, 0)$, and type $u_1 = -Z$ rejects both the buy-it-now price and the floor price $l - tf_1$. Type $u_1 = l$ rejects the buy-it-now price but accepts the floor price in exchange for the termination fee tf_1, and drops out at the price $l + \Delta_1 - tf_1$ at the clock auction, where $\Delta_1 = (h - l)x_2(l, l)$. Bidder 2 is recommended to acquire information if and only if Bidder 1 rejects the buy-it-now price, but Bidder 1's decision about the floor price is not revealed to Bidder 2 before being approached. Bidder 2 follows the recommendation. Type $u_2 = -Z$ rejects the floor price $l - tf_2$, while $u_2 = l$ and $u_2 = h$ accept the floor price in exchange for the termination fee tf_2. In the second stage auction, $u_2 = l$ and $u_2 = h$ drop out at the second price $l + \Delta_2 - tf_2$ and third price $h - tf_2$ accordingly, where $\Delta_2 = (h - l)(1 - \Delta)$, and $\Delta \geqslant x_2(l, l)$. Then I verify that under such equilibrium, the outcome is the same as that in the direct mechanism solved in the proof of Proposition 4.1.

Consider Bidder 2 first. If not approached, Bidder 2 is excluded, so both the allocation and transfer are 0, consistent with the direct mechanism. If approached to be recommended to acquire information, Bidder 2 knows $u_1 = l$, or $u_1 = -Z$. It is straightforward to show that the condition for a type $u_2 = -Z$ Bidder 2 prefers to drop out at the first price $l - tf_2$ rather than the other two prices is the same as the IC condition of $u_2 = -Z$ not to mimic a type l or h. Therefore the condition is satisfied. Similarly, the same situation holds for the conditions to make sure a type $u_2 = l$ ($u_2 = h$) drops out at the second price $l + \Delta_2 - tf_2$ (third price $h - tf_2$). As a result, Bidder 2 is also willing to acquire information when knowing that Bidder 1 rejects the buy-it-now price, as suggested by the (OB2) condition in optimal direct mechanism. Also, it can be verified that the outcome in this equilibrium implemented by such indirect mechanism is consistent with that of the optimal direct mechanism.

Now consider Bidder 1. Suppose Bidder 1 has acquired information. Similar to the case with Bidder 2, we only need to verify (1) $u_1 = h$ prefers accepting the buy-it-now price to rejecting it, accepting the floor price and dropping out at the third price $h - tf_1$; (2) $u_1 = l$ prefers dropping out at the second price $l + \Delta_1 - tf_1$ to dropping out at the third price $h - tf_1$; (3) $u_1 = -Z$ prefers dropping out at the floor price $l - tf_1$ to dropping out at the third price $h - tf_1$. All other incentive constraints are equivalent to the corresponding (IC) constraints in the direct mechanism. The proof is exactly the same as the case with Candidate (II).

Finally, Bidder 1 would acquire information, because such condition is exactly the same as the (OB1) in the direct mechanism. ■

Proof of Proposition 5.2, Region (III), (IV) and (V). The proof is similar to that of Candidate (II) and (III), but simpler. Since there is no off-equilibrium path in the indirect mechanism, the incentive constraints in the indirect mechanism coincide with those in the direct mechanism. The information acquisition recommendation is also followed. ■

Proof of Proposition 5.2, Region (A). A symmetric auction cannot implement the exact outcome of the optimal candidate (I); however, it achieves the same revenue when the information acquisition cost is small enough. Consider the region where[32]

$$c \leq c_6(\rho, p),$$

where $c_6(\rho, p) = \frac{1}{2}(h-l)pq(2 - p(1+q)(1-\rho) - (1+q)\rho)$. In such an auction, the target approaches both bidders simultaneously for information acquisition. Both bidders would acquire information. Then the target holds a symmetric clock auction *without* termination fees, with the three prices being l, $\frac{h+l}{2}$, and h. Type $u_i = -Z$ drops out at l, $u_i = l$ at $\frac{h+l}{2}$, and $u_i = h$ at h. The target's revenue is $l(b+p) - (h-l)(p-b)q$ with $b = (1-p)p(1-\rho)$, which is exactly the same as that of Case 1 of the optimal Candidate (I). ■

[32]This is the upperbound for c so that the OB constraints in the auction is not binding. It is also the upperbound for the revenue in the auction to equal that of the optimal candidate (I).