

# Sorting and Fund Size in The Venture Capital Market\*

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March 15, 2019

## Abstract

Venture capitalists (VCs) add value to the projects they finance. I develop a matching model where VCs span their attention over more projects, and the entrepreneurs, who own the projects, direct their search to VCs based on their projects' quality. VCs attention is diluted as the fund grows. I derive conditions for positive sorting over VC attention and project quality to emerge. Anticipating positive sorting, VCs shrink their funds excessively. The entry of unskilled VCs feeds back into equilibrium sorting, increases returns at the top of the distribution - consistently with empirical evidence - and leads to a Pareto-improvement. In a dynamic extension, the model provides a new rationale for the prevalence of funds as finitely lived limited partnerships, which emerge in equilibrium even when they are socially wasteful: they allow VCs to attract the best entrepreneurs, who most value the exclusivity that such a fund structure guarantees.

**JEL codes: G24, G31, D82, D83**

## 1 Introduction

Venture capital has undoubtedly been a successful model of financing entrepreneurship.<sup>1</sup> The common view among practitioners and academics is that venture capitalists (VCs) add value to the companies they finance, on top of the capital they provide.<sup>2</sup> There is evidence that VCs

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\*I am indebted to Francesco Nava and Balázs Szentes for their continuous guidance, support and for many stimulating discussions. This paper has greatly benefited from comments and suggestions by Daniel Ferreira and Peter Kondor. I warmly thank Ulf Axelson, Michel Azulai, Matteo Benetton, Alberto Bennardo, Gianpaolo Caramellino, Daniele Condorelli, Amil Dasgupta, Alexia Delfino, Andrew Ellis, Juanita Gonzalez-Uribe, Wouter den Haan, Gilat Levy, Marco Pagano, Marco Pagnozzi, Nicola Persico, Ronny Razin, Markus Reisinger, Antonio Rosato, Emanuele Tarantino, Anjan Thakor and seminar participants at INSEAD (Finance), the Bank of Italy, Central European University, CSEF (Naples), Frankfurt School of Finance & Management, SSE (Finance), LSE Theory Work in Progress Seminar, LSE Finance PhD Seminar, the SAEe Meeting, and the Petralia Applied Economics Workshop for useful comments at various stages. A previous version of this paper circulated as: "A Theory of Venture Capital Fund Size with Directed Search". All errors are my own.

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<sup>1</sup>For a survey of research in private equity, see [Da Rin et al. \[2011\]](#).

<sup>2</sup>They do so through a number of activities such as monitoring, selecting top management, and experimenting innovative business strategies.

differ considerably in their ability to generate returns and to help companies reach the initial public offering stage; at the same time, it is largely recognized that access to a superior deal flow is an important driver of superior performance (Sørensen [2007], Korteweg and Sorensen [2017], Nanda et al. [2018]). The funds that VCs raise are often oversubscribed, and recent evidence suggests that they stay below the point where significant decreasing returns kick in (Rossi [2017]). In light of the role VCs play in boosting growth, it is important to understand how capital is allocated across and used by these scarce and differently skilled VCs. Is there an efficient amount of capital put to work in this industry? What drives sorting of VCs and entrepreneurs, and is it optimal?

This paper builds on the observation that in venture capital, companies receiving financing are also interested in matching with the best VCs. I argue that self-selection of different entrepreneurs seeking VC finance into different VC funds creates an additional source of decreasing returns to scale, is responsible for an inefficient choice of fund size by VCs, a consequent inefficient sorting of VCs and entrepreneurs, and can explain some of the regularities observed in this industry.

The success of the venture capital model has motivated many governments to try and stimulate the provision of such financing in various ways. This attempt has generated a debate, as well as some skepticism among academics, regarding the role of the public sector in improving private VC activity. In a thorough analysis of the subject, Lerner [2009] argues that public measures encouraging VC investment may favour the less efficient VCs, and even crowd out investment from the most knowledgeable ones.<sup>3</sup> In this paper I offer a reason why allowing less sophisticated VCs to participate in the economy can in fact be beneficial.

I tackle these issues by developing a matching model of fund management in venture capital. There are two sets of agents in the economy: VCs, and entrepreneurs. To capture scarcity in the quality and quantity of VCs' human capital and expertise, I assume that VCs value added - or *attention* - to each investment is diluted as the number of projects they finance increases. VCs differ in skill. Specifically, more skilled VCs can provide higher attention for any given portfolio size. The combination of VC skill and size ultimately determines the level of attention each VC can provide to each project they manage. On the other side of the market, each entrepreneur owns one project. Projects are heterogeneous in quality. A projects needs the input of a VC to become profitable. The return of a project is a deterministic function of its own quality and VC attention.

In the model, VCs move first and choose a fund size – or capacity – to which they commit. Entrepreneurs move after VCs. They decide whether to enter the market, and if they do so, they observe their project's quality. Then, they search for a suitable VC. Once a match is formed, returns are produced and shared exogenously between the VC and the entrepreneur.

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<sup>3</sup>See, in particular, the discussion of the Canadian Labor Fund Program in Chapter 6.

Once entrepreneurs have directed their search, as many entrepreneurs as vacancies available are matched at random in a given VC skill-size combination, which defines a submarket. Since the measure of VCs in the economy, and the capacity they commit to, are limited, entrepreneurs in a given submarket may get rationed. Hence, when choosing which VC to search for, entrepreneurs trade off matching with VCs that can devote more attention to their projects, against the lower search frictions in markets where VCs attention is lower. Complementarity between the two inputs of the returns function mean that for the best entrepreneurs, the first force - the value attached to higher attention - is relatively more salient. This generates positive sorting between VCs' attention and entrepreneurial quality.

This sorting, in turn, affects the initial stage of the game, since VCs anticipate that managing a fund of larger size attracts lower quality entrepreneurs. In equilibrium, some unskilled VCs shrink their funds below the welfare maximizing solution. This inefficiency arises because VCs do not internalize the effect of their choices on the equilibrium assignment: separation among entrepreneurs is driven by an increase in search frictions in markets where attention is higher compared to markets in which attention is lower. But if too many VCs offer high attention, this increase in frictions is too small, and entrepreneur separation is suboptimal. That is, some entrepreneurs of relatively low quality search for high-attention funds, lowering the average quality of those submarkets. In addition, multiple equilibria may emerge, with Pareto-dominated equilibria being characterized by smaller funds size.

In this environment, subsidizing the entry of low-skill VCs that are inactive – for example, because their ability to generate returns is insufficient to cover the fixed costs of starting operations – always results in net aggregate gains. The reason is that these agents will absorb low-quality entrepreneurs; in contrast, efficient VCs who choose to provide higher attention will attract even better projects, because only the worst entrepreneurs to which they were originally matched will find it worthwhile to switch in the larger market associated with low attention. In some cases, the total measure of projects funded by incumbent VCs will also increase. This possibility offers a new angle from which to think about public intervention in this market and a more optimistic view on policies that encourage fundraising devoted to venture investments. Interestingly, [Brander et al. \[2014\]](#) find evidence that the presence of government-sponsored VCs does not crowd out, but increases investments from private VCs at the aggregate level.<sup>4</sup>

The model provides novel implications of the entry of new VCs on the whole returns distribution. Specifically, when more unskilled VCs enter the market, a larger share of funded projects end up on the lower side of the returns distribution; however, those at the top of the distribution deliver higher returns. This effect is consistent with the findings in [Kaplan and Schoar \[2005\]](#) and [Nanda and Rhodes-Kropf \[2013\]](#): the former find that in times of more intense activity in the industry, capital flows disproportionately to worse funds; the latter doc-

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<sup>4</sup>An empirical assessment of the effects of subsidized funds activity on the profitability of investments made by incumbent, non-subsidized VCs is still missing in the literature.

uments that investments made during “hot” periods are more likely to fail and yield higher returns conditional on not failing.

A benchmark model with random matching or homogeneous entrepreneurs produces neither inefficiency in equilibrium fund size nor the beneficial effect of new VCs entering the market and their effect in changing the shape of the returns distribution described above.

The mechanism described above crucially relies on the assumed power of VCs to commit not to amend fund size ex post, that is, once entrepreneurs have self-selected into the fund. This assumption is precisely the focus of the second part of this paper, where I study one aspect of the typical VC fund structure: venture capital funds are limited partnerships with a specified finite horizon. As a consequence, capital does not flow into and out of a VC fund. The nature of the partnership into which VCs enter with investors makes it very difficult for VCs to agree with dispersed owners on amendments on the original amount of committed capital. In addition, VCs are often contractually restricted from starting a new fund before the current has been substantially invested, or until a given date.<sup>5</sup> Such arrangements are likely to generate distortions. For example, these limitations might force VCs to give up investment opportunities that are discovered too late in the fund’s life, when much of the agreed-upon capital has been already invested.<sup>6</sup> The common understanding is that such distortive fund configuration helps mitigate agency problems between limited partners – the investors – and VCs. Would a different arrangement emerge in the absence of such problems? In other words, is forming limited partnerships also in the best interest of VCs or merely an unavoidable cost?

To answer this question I accommodate the model to a dynamic setting where projects do not realize returns immediately, and in which VCs can match to one entrepreneur every period and follow the projects until they are ready to produce returns. I allow VCs to choose between a short-term contract and a long-lasting open credit relationship with investors. In the former case, VCs are forced to wait until the current project has realized its returns before they can manage a new fund and return to the market for entrepreneurs. In the latter, VCs have access to the investors’ money and can add a new project to the fund while the first investment is still ongoing. Projects under the management of a VC who has entered a short-term contract with investors will not overlap.<sup>7</sup> Thus, such contract allows the VC to commit its attention to the current project.

I show that there is no equilibrium where every VC is in a long-lasting credit relationship

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<sup>5</sup>See Sahlman [1990] and Gompers and Lerner [1996] for a detailed discussion of contractual arrangements.

<sup>6</sup>Kandel et al. [2011] find suggestive evidence that being closer to the end of the fund induces myopic behavior by VCs. Barrot [2016] finds that the length of the investment horizon is associated to selection of different startups, meaning that it has real effects on the VCs’ investment strategy.

<sup>7</sup>This is a somewhat extreme, stylized representation. In reality, fund managers can open new funds and manage them in parallel. However, fundraising is typically time consuming, which strongly limits the extent to which VCs can put projects on hold until enough money is raised; this practice is also limited by the contractual restrictions described above.

with investors, even when this is the most efficient arrangement. This happens because by choosing the short-term contract, a deviating VC will be able to skim the market and attract the very best entrepreneurs: those who are willing to endure the highest search friction in order to match with a committed VC. This finding provides a new rationale for the prevalence of closed, finite-horizon funds in venture capital, rather than the open funds we observe in contexts where fund managers invest in public securities and are not subject to a two-sided matching problem.

**Discussion of the Main Assumptions.** In the model, I make two main assumptions. The first is that VCs experience dilution in human capital when more firms are monitored in parallel. Evidence of diseconomies of scale at the (VC) fund level is found in [Kaplan and Schoar \[2005\]](#), [Harris et al. \[2014\]](#) and [Robinson and Sensoy \[2016\]](#).<sup>8</sup> This can be the result of several forces, but [Humphery-Jenner \[2011\]](#), [Cumming and Dai \[2011\]](#) and [Lopez-de Silanes et al. \[2015\]](#) provide evidence that scarcity in human capital is an important driver of diseconomies of scale in the industry. The second assumption is that entrepreneurs direct their search to different VCs. One major distinction between the activities of VCs and those of other fund managers (e.g. buyouts, mutual funds) is that the former invest in targets that are in turn interested in their ability to add value. After all, entrepreneurs remain owners of a significant fraction of the firm they develop with the VC. In a seminal contribution, [Sørensen \[2007\]](#), documents positive sorting in the industry between better VCs and better start-up firms. Importantly, it has been shown this is not necessarily due to VCs’ screening ability: [Hsu \[2004\]](#) finds that entrepreneurs are willing to accept worse terms in order to affiliate with VCs that can provide greater value added. More recently, [Nanda et al. \[2018\]](#) find that initial VC’s success – and a noisy signal of it – generates superior future performance through access to a better deal flow. This is consistent with the idea that the best entrepreneurs self select into the VC funds that they perceive can add more value.<sup>9</sup> Clearly, screening also happens in reality. But a model that allows some ex-post (costly) screening by VCs would not eliminate the effect of initial self-selection, and it would be considerably less tractable.

**Relation to the Literature.** The paper contributes to the literature focusing on delegated fund management, particularly the venture capital asset class. The main conceptual contribution is to offer a framework to study capital allocation in delegated fund management when the delegated managers are active in a search market characterized by two-sided heterogeneity.

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<sup>8</sup>In particular, it appears that 1) in the cross-section, there is a positive size- returns relationship at the fund level and 2) accounting for fund managers fixed effects, average returns to investors are decreasing in fund size. This is consistent with my modelling assumptions.

<sup>9</sup>Interestingly, [Nanda et al. \[2018\]](#) quote Chris Dixon, partner at Andreessen Horowitz: “Success in VC is probably 10% about picking, and 90% about sourcing the right deals and having entrepreneurs choose your firm as a partner”.

In a seminal paper, [Berk and Green \[2004\]](#) derive predictions concerning fund flows in the mutual fund industry. As in that paper, fund managers in my model possess scarce skills and, therefore, extract all the rents from investors. In [Berk and Green \[2004\]](#) this results in an efficient allocation of money across managers; however, in my model, adding entrepreneurs self selection produces: 1) a generically inefficient choice of fund size, 2) multiple equilibria that are not welfare equivalent and 3) a feedback effect of entry of unskilled managers on returns at the top of the distribution. [Fulghieri and Sevilir \[2009\]](#) model the optimal investment strategy of a VC who trades off the higher value added from a small portfolio, with the diversification gains from a large one. [Inderst et al. \[2006\]](#) hold constant the number of projects in the portfolio, and show that committing to limited capital at the refinancing stage allows the financier to induce stronger competition among entrepreneurs. This motivates the use of covenants and other arrangements typically employed in limited partnerships. As in [Inderst et al. \[2006\]](#), I find in a dynamic extension that VCs benefit from this commitment power; however, that benefit happens through the selection of different entrepreneurs. Rather than fearing the fierce competition that investors' "shallow pockets" induce, entrepreneurs in my model enjoy exclusivity, since a committed VC will not be able to excessively dilute his human capital. Another crucial difference is that I model the interaction of many VCs who anticipate entrepreneurs self-selection; consequently, the emergence of limited partnerships - the constrained finance solution - may well be inefficient.

In my model, there is a simple but essential asymmetry between VCs and entrepreneurs in how their payoff is affected by a particular match: while the entrepreneur is solely interested in the return from his project, the VC cares about the returns of the whole fund. Several works have modeled this conflict. In [Michelacci and Suarez \[2004\]](#), the focus is on identifying the institutional market characteristics that increase total welfare by alleviating this trade-off and allowing VCs to free up their human capital more quickly, without destroying too much of the monitored firm's value; [Jovanovic and Szentes \[2013\]](#) find conditions under which the optimal contractual arrangement in the presence of moral hazard on the entrepreneurs' part takes the form of an equity contract; they also explain the returns premia to VC-backed firms. [Silveira and Wright \[2015\]](#) study project selection on the VC's side and the optimal amount of capital to raise when start-up investment costs are random but committing funds entails forgone returns from alternative assets. Contrary to my model, none of the aforementioned models analyze the sorting of different entrepreneurs into different VC funds in the presence of these forces. More importantly, while I assume diseconomies of scale, I do not restrict intermediaries to run one project at a time. This more realistic assumption allows me to study 1) the equilibrium fund size and, therefore, 2) the allocation of investors' funds across fund managers. The VC-entrepreneur relationship has also been also the subject of a large strand of literature focusing on the inherent agency problems associated with venture capital financing, and the contractual

arrangements aimed at solving these problems.<sup>10</sup> I abstract from these issues and take a reduced-form approach to the determination of the returns to a project and assume an exogenous equity contract between the two parties. However, in my model, project's quality could be interpreted as a (negative) measure of the severity of the moral hazard problem on the entrepreneur's side, naturally affecting the total surplus from a match.

Similarly to this paper, [Marquez et al. \[2014\]](#) builds upon the observation that investments in venture capital are special in that they are subject to a two-sided matching problem. [Marquez et al. \[2014\]](#) develop a signal-jamming model where VCs with differential abilities to produce returns distort the fund size decision in order to affect entrepreneurs' learning.<sup>11</sup> On the contrary, in my model, VC ability is common knowledge. Moreover, while [Marquez et al. \[2014\]](#) take a reduced-form approach to the determination of a fund's portfolio quality, I study and characterize sorting explicitly; since the relative gains from committing higher attention are endogenous, I can derive conditions under which an equilibrium where every VC chooses a certain (efficient) fund structure might unravel. Modeling sorting allows me to study the efficiency of funds allocation across VCs, and to study the effects of entry of VCs on the entire allocation and returns distribution.

This paper also contributes to the literature that studies the contractual arrangements at the basis of investment funds, such as [Stein \[2005\]](#), who study the emergence of the open-ended fund structure in mutual funds, and [Axelson et al. \[2009\]](#) who explain why buyout funds exhibit a mix of outside debt and equity financing.

On a more abstract level, the way I introduce adverse selection to a matching problem is similar to [Guerrieri et al. \[2010\]](#): uninformed principals (the VCs) post contracts (the level of attention) and informed agents (the entrepreneurs) direct their search based on their type; in this model too the probability of matching serves as a separating device. In [Guerrieri et al. \[2010\]](#), the contract space – which defines the space of submarkets – is assumed to be rich enough to ensure full types separation. In my model instead VCs can only commit on fund size and consequently on their attention. Moreover, while [Guerrieri et al. \[2010\]](#) assume a fixed measure of homogeneous principals, in my model, VCs are heterogeneous ex ante and I study the effects of entry of new unskilled VCs. Finally, my paper provides conditions for sorting in a matching environment with non-transferable utilities and search frictions. [Eeckhout and](#)

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<sup>10</sup>Notable examples are [Cornelli and Yosha \[2003\]](#) and [Repullo and Suarez \[2004\]](#), both of which analyze the optimal security design when new information is produced about the investment at an intermediate stage, which is an essential characteristic of this environment; in [Schmidt \[2003\]](#) the double moral hazard problem between the two parties justifies the use of convertible preferred equity, while [Hellmann \[2006\]](#) extends this analysis to allow for a distinction between exit via IPO and via private acquisition and finds that automatic conversion is only triggered under exit via IPO in the optimal contract; finally, [Casamatta \[2003\]](#) studies the endogenous emergence of external financing from VCs who also provide human capital and shows that the optimality of common stocks versus preferred equity depends on the relative amount invested by the VC.

<sup>11</sup>VCs' reputation building motive is also at the heart of [Piacentino \[2019\]](#): she shows that the conservatism of reputationally-motivated unskilled VCs can be beneficial because it generates a certification effect, making VC-backed firms more likely to raise capital in an IPO.

Kircher [2010] derive general results on the consequences of search frictions in an assignment problem where sellers commit to posted prices. Requirements on the match-value function for positive and negative sorting are found to depend on the elasticity of substitution in the matching technology. In my model, where utilities are non-transferable, the strongest form of supermodularity (and submodularity) is needed to guarantee sorting, under any specification of the matching function. Additional results related to my setting can be found in Eeckhout and Kircher [2018] who study the interaction between the choice of span of control and the sorting pattern in an assignment economy; they look at competitive equilibria where types are observable on both sides, and the allocation is not limited to one-to-one matching. An important difference is that in my model there is no direct type complementarity.

**Roadmap:** Section 2 introduces the setup, followed by the characterization of the equilibria; Equilibria are Pareto ranked and compared to a second-best solution in Section 3; Section 4 explores the effects of entry of new VCs in the economy; Section 5 is devoted to the analysis of the choice between short and long-term investors-VC relationships; Section 6 shows one generalization of the main model; Section 7 concludes. All proofs are relegated to Appendix A, further results are in Appendix B and C.

## 2 Model

**Agents.** The economy consists of heterogeneous VCs, identical investors and ex ante identical entrepreneurs. There is an arbitrarily large measure of investors. Each investor is endowed with money, which they can invest into funds, each managed by a single VC. VCs are exogenously endowed with ability, denoted  $x$ , according to the measure  $G$ , that admits a continuous density  $g$  with full support  $[\underline{x}, \bar{x}] \subset \mathbb{R}_+$ . The measure of VCs in the economy is fixed. Entrepreneurs are in large supply, and can enter the market upon paying startup cost  $c$ . If they do so, they draw a type  $\lambda$ , the quality of the project they own, from a continuous distribution  $f$  that is strictly positive on the entire support  $[\underline{\lambda}, \bar{\lambda}] \subset \mathbb{R}_+$ . A higher  $\lambda$  indicates a better project in the sense specified in the next paragraph. Entrepreneurs need money and VC input to turn their projects into profitable firms.<sup>12</sup>

**Projects.** All projects need only one unit of money to become a firm. Call  $m$  the measure of projects a given VC is matched to in equilibrium. Define  $a$  the attention the VC devotes to each project. Assume  $a \in \{a_0, a_1\}$ , with  $a_1 > a_0$ . VC's attention, or managerial input, is a function of his ability and the number of firms he is matched to,  $a := a(m, x)$ . In particular

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<sup>12</sup>A natural interpretation – which fits the common view of the role of VCs – is that young firms need to be constantly monitored, because entrepreneurs are unexperienced. Another one is that they lack the collaterals necessary to secure alternative sources of financing, such as loans from traditional banks.

$a(m, x)$  is the step function:

$$a(m, x) = \begin{cases} a_1 & \forall m \in [0, m_1^x] \\ a_0 & \forall m \in (m_1^x, m_0^x] \end{cases}$$

with  $m_0^x - m_1^x = \Delta > 0$  for all  $x$ ,  $\partial m_i^x / \partial x > 0$  for  $i = 0, 1$  and all  $x$ , and  $m_i^x$  is continuous in  $x$ . In words, 1) VC input is diluted by working on more projects in parallel and that 2) better managers can run more projects at a given level of attention. A manager with ability  $x$  can be matched to a maximum of  $m_0^x$  projects. Continuity of  $m_i^x$  simplifies the analysis.<sup>13,14</sup> Each project's return  $R$  is assumed to be a function of attention  $a$  and of the project's quality  $\lambda$ . Call this function  $R(a, \lambda)$ .<sup>15</sup> Assume (as natural) that  $R_a(a, \lambda)$  and  $R_\lambda(a, \lambda) > 0$  for all  $a, \lambda \in \mathbb{R}$ . I further assume that  $R(a, \lambda)$  is twice continuously differentiable in its arguments.

**Matching and Information.** While VCs' size and ability are common knowledge, the entrepreneur's type,  $\lambda$ , is his private information. Therefore, I study directed search from the long and informed side of the market, the entrepreneurs. Each VC's combination of size and ability,  $(w, x)$ , will therefore form a submarket in which entrepreneurs select, possibly depending on their type. Finally, assume that as many matches as possible are formed in each submarket, that is, the number of matches as a function of the measure of entrepreneurs searching,  $q_e$ , and the measure of money available (or "vacancies"),  $q_k$ , is given by  $M(q_k, q_e) = \min\{q_k, q_e\}$ .

**Payoffs, Strategies and Timing.** In the first stage of the game, each VC offers investors a contract  $(w, p)$ , which specifies the size of the fund  $w$  and fixed fee  $p$  that the VC receives from the investors for every unit of money invested.<sup>16</sup> As all projects require one unit of money, I will refer to fund size  $w$  as the fund's capacity, that is the maximum measure of entrepreneurs with which VC can be matched. Investors can accept the contract, and provide the VC with the amount  $w$ , or reject and invest in an alternative technology delivering constant returns  $R_0$ . When investing in a certain VC, they will get a fixed share  $\alpha \in (0, 1)$  of the VC's average

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<sup>13</sup>The assumption that attention jumps discontinuously with  $m$  is of no consequence to the qualitative results obtained in this Section, but it allows to guarantee existence of equilibria when size is the VC's choice. In Section 6 I show that the equilibrium characterization follows through in the more general case where  $a(m, x)$  is allowed to have  $N > 2$  steps.

<sup>14</sup>A more general setting allows for  $\Delta$  to be a function of  $x$ . In which case ensuring perfect separation of VCs in equilibrium requires to impose the single crossing condition  $\partial \left( \frac{m_0^x}{m_0^x - \Delta(x)} \right) / \partial x < 0$ , which is satisfied when  $\Delta$  is constant across  $xs$ .

<sup>15</sup>The direct implication is that attention is all that matters to a given type of entrepreneur. In other words, project's quality does not interact with VC's ability or fund size *per se*. This separability greatly simplifies the analysis.

<sup>16</sup>As it will be clear when studying size determination, the assumption that VCs receive no performance-based compensation is without loss of generality. This is due to: 1) the fact that there is no agency conflict between investors and VCs, nor uncertainty about the VC's ability, and 2) the presence of a large measure of investors, which implies that investors' participation constraint will bind in all equilibria.

returns from the fund. In the second stage, entrepreneurs observe the joint distribution of  $(w, x)$  induced by the first stage, and choose whether or not to pay the startup cost. Those who do, can direct their search towards different VCs. Conditional on being matched, they receive the residual  $-(1 - \alpha)$  share of the returns from their projects. All agents are risk neutral and maximize expected returns.

## 2.1 Entry and Sorting Subgames

**Market Tightness.** Let me first study the subgame where entrepreneurs make the entry decision and direct their search at different VCs. Assume that the allocation of investors' money generates a fund size between  $\underline{w}$  and  $\bar{w}$  with  $\bar{w} > \underline{w}$ .  $H(w, x)$  denotes the measure of VCs with fund size below  $w$  and ability below  $x$ .<sup>17</sup> Upon entry, the search strategy for an entrepreneur is described by a distribution over  $[\underline{w}, \bar{w}] \times [\underline{x}, \bar{x}]$ . Formally, the entrepreneur strategy is a mapping

$$s : [\underline{\lambda}, \bar{\lambda}] \rightarrow \Delta([\underline{w}, \bar{w}] \times [\underline{x}, \bar{x}]).$$

The strategy generates for every  $\lambda$  a cumulative density function  $S(w, x; \lambda)$ .  $E$  is the measure of entrepreneurs who decide to enter. Define  $\tilde{S}(w, x, E)$  the measure of entrepreneurs searching in markets with size below  $w$  and ability below  $x$ , given  $E$ . This is given by summing the search strategy over all entrepreneurs, so  $\tilde{S}(w, x; E) = \int_{\lambda} ES(w, x; \lambda) dF(\lambda)$ . On the other side of the market, as a VC managing a fund of size  $w$  can follow up to  $w$  projects in parallel, the amount of vacancies in submarkets below  $(w, x)$  is given by  $\int_{-\infty}^x \int_{-\infty}^w \hat{w} dH(\hat{w}, \hat{x})$ . To define expected payoffs properly, let  $\theta(w, x; E)$  be the expected ratio of vacancies to entrepreneurs in submarket  $(w, x)$ , when  $E$  entrepreneurs have entered. I will refer to  $\theta(w, x; E)$  as market tightness. The function will solve:

$$\int_{-\infty}^x \int_{-\infty}^w \hat{w} dH(\hat{w}, \hat{x}) = \int_{-\infty}^x \int_{-\infty}^w \theta(\hat{w}, \hat{x}; E) d\tilde{S}(\hat{w}, \hat{x}; E).$$

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<sup>17</sup>This is endogenous, as it is determined by the investors and VCs equilibrium choice. Hence no assumption on  $H$  is made at this stage, except that this measure will have to be zero in all submarkets  $(w, x)$  where no entrepreneurs is searching.

<sup>18</sup> Finally, define  $Q(w, x; E)$  the probability an entrepreneur finds a match when searching in market  $(w, x)$ . Given that the matching function is Leontief, this is:<sup>19</sup>

$$Q(w, x; E) := \min \{ \theta(w, x; E), 1 \}.$$

I can now write type- $\lambda$  entrepreneur's expected payoff from choosing to search in market  $(w, x)$  as:

$$(1 - \alpha) Q(w, x; E) R(a(m(w, x; E), x), \lambda)$$

where  $m(w, x; E)$  is the measure of projects per VC in market  $(w, x)$ . Note that  $m(w, x; E) \leq w$ , but the condition may, in principle, not bind. To save on notation, I will denote  $\pi_\lambda(E, s^*)$  the *equilibrium* value of a type- $\lambda$  entrepreneur's expected payoff. I can now describe what is an equilibrium of this subgame.

**Definition 1. (*Equilibrium in the Subgame*).** *An equilibrium in the entry and sorting subgame is characterized by a vector  $(E, s^*)$  such that:*

$$(i) \quad s^*(\lambda) = \arg \max_s \mathbb{E}_{w,x} [Q(w, x; E, s^*) (1 - \alpha) R(a(m(w, x; E, s^*), x), \lambda)]$$

$$(ii) \quad \int_\lambda \pi_\lambda(E, s^*) dF(\lambda) = c$$

Part (i) imposes optimality. Part (ii) follows from the assumption that entrepreneurs are in infinite supply: it states that, ex ante, entrepreneurs must be indifferent between entering the market and staying out.

An immediate observation is that, in this model, an entrepreneur's search strategy imposes an externality on the other entrepreneurs not only through the usual effect on search frictions, but also by affecting VCs attention. In principle, this can generate multiple equilibria where the value of a VC is ultimately determined by the measure of entrepreneurs searching in a given submarket. However, one additional assumption can be shown to substantially simplify the sorting game. The assumption requires that less VC attention is not *too* detrimental to the average type, as formalized below.

**Assumption A1.**  $(1 - \alpha) \mathbb{E}_\lambda R(a_0, \lambda) > c.$

<sup>18</sup>For a more formal definition,  $\tilde{H}(w, x)$  denotes the measure of available vacancies in submarkets below  $w$  and  $x$ , given by  $\int_{-\infty}^x \int_{-\infty}^w \hat{w} dH(\hat{w}, \hat{x})$ . As no VC will offer funds in submarkets where no entrepreneur is searching, the measure  $\tilde{H}$  is absolutely continuous with respect to  $\tilde{S}$ . Therefore, market tightness at every submarket is well defined and given by the Radon-Nikodym derivative  $d\tilde{H}/d\tilde{S}$ .

<sup>19</sup>The assumption that the matching function is Leontief does not affect the equilibrium characterization. However, it is relevant in the welfare analysis. By assuming that as many matches as possible are formed in every submarket, I can abstract from inefficiencies that might arise from matching frictions within the submarket, and focus on those coming from the directed search assumption alone.

A1 states that, ex ante, an entrepreneur would strictly benefit from paying the startup cost and matching to a VC in the absence of search frictions, even when the VC's attention is completely diluted (i.e., at its lowest level  $a_0$ ). When A1 holds, because entrepreneurs are in large supply, new ones will enter the market until search frictions kick in. This also implies that the situation where some VCs attract no entrepreneur cannot be an equilibrium of the subgame since those VCs would be able to provide the highest attention with no search friction, creating a strict incentive for entrepreneurs to deviate. This is formally stated below.

**Lemma 1.** *Under A1, in any equilibrium, in each submarket there are more entrepreneurs than vacancies. That is,  $Q(w, x, E) < 1$  and  $m(w, x, E) = w, \forall(w, x)$*

The implication of Lemma 1 is that all VCs operate at full capacity. The next result is a direct consequence of Lemma 1, and will help characterize the equilibrium strategies in the sorting subgame.

**Lemma 2.** *Given  $E$ , in any equilibrium,  $Q(w, x; E)$  is a function of  $a(w, x)$  only.*

Intuitively, because VCs must operate at full capacity in every equilibrium, attention in market  $(w, x)$  is given by  $a(w, x)$ . As returns are only a function of attention and project quality, an entrepreneur must be indifferent between searching in two markets where attention is the same. This suggests that, in essence, the entrepreneur's strategy is reduced to determining the attention level  $a$  with which to seek for a match.

**Lemma 3.** *For a given  $E$ , any equilibrium of the sorting subgame is mirrored by one from a game where entrepreneurs can only direct their search towards different attention levels, and they are then matched with VCs who can provide the chosen amount of attention, in proportion to the VC's size.*

In other words, because entrepreneurs must be indifferent to searching in any market where attention is the same, any equilibrium can be equivalently represented by one where the search strategy consists of choosing only among different levels of attention, which, in this reduced model, is a fixed predetermined characteristic of the VC. The distribution of vacancies will reflect total size summed across all VCs at a given iso-attention region in the original model.

Lemma 3 is useful because it allows to focus on a particular type of sorting equilibrium, where the sole characteristic of a VC, and hence what defines the sub-market in which to search, is attention. The interest is then determining the requirements the return function should obey so that in a general setting, independent of the distribution of types, sorting would emerge. If such conditions are identified, one can conclude that the same sorting pattern would emerge in the original model, once the mixed strategies are adjusted accordingly.

Let  $A^s(a)$  be the set of entrepreneurs searching in market  $a$  under strategy  $s$ ,  $A^s(a) := \{\lambda: s(a; \lambda) > 0\}$ .

**Definition 2.** An equilibrium exhibits positive (negative) assortative matching if  $\forall a, a'$  with  $a > a'$

$$\lambda \in \Lambda^s(a) \cap \lambda' \in \Lambda^s(a') \Rightarrow \lambda > (<) \lambda'.$$

Intuitively, under positive assortative matching (PAM), more attention cannot be associated with a lower-quality entrepreneur; however, the pooling of different entrepreneurs at a given attention level is allowed. I can now state the main result of this section, which establishes necessary and sufficient conditions for equilibria to exhibit PAM or negative assortative matching (NAM).

**Proposition 1. (Sorting).** All equilibria exhibit PAM (NAM) if and only if  $R(a, \lambda)$  is everywhere logsuper(sub)modular.

Note that logsuper(sub)modularity implies super(sub)modularity, while the opposite does not hold. To build intuition why a stronger form of supermodularity is necessary for *every equilibrium* to exhibit PAM, notice that, as emphasized by [Eeckhout and Kircher \[2010\]](#), when allowing for search frictions in matching models, two forces drive the sorting pattern, in opposite directions: the “trading security motive”, which motivates higher types to select into less crowded markets, and the “match value motive”, that is the value of being matched to better types, conditional on finding a match. In this setting, the latter motive corresponds to the value of the VC’s attention, which is a bigger concern when  $\lambda$  is high.<sup>20</sup> This trade-off becomes evident if one looks at the difference in expected payoffs from searching in two markets,  $a$  and  $a'$ , with  $a > a'$ , and differentiates it with respect to  $\lambda$ . This difference is increasing in  $\lambda$  when:

$$\underbrace{\frac{-(Q(a) - Q(a'))}{Q(a')} R_\lambda(a, \lambda)}_{\text{trading security motive}} < \underbrace{\frac{\partial (R(a, \lambda) - R(a', \lambda))}{\partial \lambda}}_{\text{match value motive}}.$$

In words, only when complementarities in the returns function between attention and quality are sufficiently strong does a higher- $\lambda$  entrepreneur prefer to search for more attention and face the stronger search frictions in this more crowded market.

To understand why logsupermodularity is sufficient, note that a function  $R(a, \lambda)$  is logsupermodular if and only if, for any  $(a, a')$  with  $a' > a$ , the ratio  $R(a', \lambda) / R(a, \lambda)$  is strictly increasing in  $\lambda$ . This means that, if for some type  $\tilde{\lambda}$ ,  $Q(a') R(a', \tilde{\lambda}) > Q(a) R(a, \tilde{\lambda})$ , the same would be true for all  $\lambda > \tilde{\lambda}$ . This ensures separation.

<sup>20</sup>It should be noted that the condition in Proposition 1 is particularly strong because utilities are non-transferable. In the framework proposed by [Eeckhout and Kircher \[2010\]](#), where sellers can commit to posted prices, it is shown that, although supermodularity per se is generally not sufficient, the requirements for PAM to emerge are milder. In particular, the degree of supermodularity depends on the elasticity of substitution in the matching function. Notably here, with directed search and non-transferable utilities, the result that  $R$  must be logsupermodular holds true under any specification of the matching function.

The rest of this analysis focuses on the case when  $R(a, \lambda)$  is logsupermodular.<sup>21</sup>

**Assumption A2.**  $R(a, \lambda)$  is everywhere logsupermodular.

## 2.2 Choice of Fund Size and Equilibrium in the Supergame

In this section I study the VC's choice at the initial stage, when contracting on size and fees with investors. Therefore, I endogenize the distribution  $H(w, x)$ , and, hence, characterize equilibria of the entire game. I will restrict my attention to equilibria where both VCs and entrepreneurs play symmetric, pure strategies.

As in Berk and Green [2004], VCs contract with competing investors over fund size and a fee that the VC will receive per-unit of money under management. Note that, for every unit of money invested in the fund, the investors' participation constraint gives:

$$\alpha \mathbb{E}[R(a(w, x), \lambda) | \lambda \in \Lambda^s(a(w, x))] - p \geq R_0. \quad (1)$$

Since VCs have all the bargaining power, it must be that the net return to investors equals their outside option,  $R_0$ . In other words (1) has to bind. It follows that VCs will choose  $w$  to maximize total excess returns and then set  $p$  such that the investors' participation constraint binds in order to extract the full surplus and maximize total fees.

**VC Strategy.** The VC's decision can be further simplified by noting that, as entrepreneur's selection is affected by fund size only through its effect on attention, a VC will never set a size strictly in one region where the function  $a(m, x)$  is constant. It follows that the relevant strategic choice of a VC is which level of attention  $a_i$  to offer. The VC will consequently propose to investors the maximum size conditional on  $a_i$ , that is  $m_i^x$ . VC's strategy is therefore fully described by a mapping  $\sigma : [\underline{x}, \bar{x}] \rightarrow \{a_0, a_1\}$ . I will sometimes refer to funds associated with more attention as to more focused funds, although it should be emphasized that a more focused fund could well be of larger size than a less focused one if it is managed by a more efficient VC. Define the set of VC types choosing to offer  $a_i$  given  $\sigma$ ,  $X_i^\sigma := \{x : \sigma(x) = a_i\}$ . Finally, define the set of attention levels offered in equilibrium  $I^* := \{a_i : X_i^\sigma \neq \emptyset\}$ . For any  $a_i \in I^*$ , and given  $s$ ,  $E$ , and  $\sigma$ , one can then compute the probability for an entrepreneur to find a match,

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<sup>21</sup>Focusing on the case when  $R$  leads to assortative matching is motivated by the fact that, as it will be clear in the next section, this will guarantee all equilibria exhibit positive sorting between firms and managers, which is consistent with the evidence started by Sørensen [2007]. Interestingly, the idea that better entrepreneurs gain more from VCs' advice appears to be at the core of the following quote by Fred Wilson, a managing partner at Union Square Ventures: "When it's clear the founder only wants your money and has no interest in your advice, it is hard to get excited about the investment. When it seems that all the founder wants is your advice and isn't worried about getting money, it makes you want to work with that founder" (see the full text at <https://avc.com/2015/12/advice-and-money/>).

or, equivalently, market tightness, as:

$$Q(a_i; \sigma, s, E) = \frac{\int_{x \in X_i^\sigma} m_i^x dG(x)}{E \int_{\lambda \in \Lambda^s(a)} dF(\lambda)}.$$

Before I state what is an equilibrium of the entire game, it is necessary to specify how VC beliefs about the composition of entrepreneurs in a given market are formed. The notion of Weak Perfect Bayesian Equilibrium only disciplines beliefs on the equilibrium path, by restricting these to be computed via Bayes rule.<sup>22</sup> Formally, the belief  $\beta$  is a mapping:

$$\beta : \{a_0, a_1\} \rightarrow \Delta([\underline{\lambda}, \bar{\lambda}]).$$

and, using Bayes rule, we have that, for  $a_i \in I^*$ ,

$$\beta_{a_i}(\lambda) = \frac{f(\lambda)}{\int_{\lambda \in \Lambda^s(a_i)} dF(\lambda)}.$$

where  $\beta_{a_i}(\lambda)$  is the pdf  $\beta_{a_i}$  evaluated at  $\lambda$ . What about beliefs for markets where no VC is positioned, that is for any  $a_j \notin I^*$ ? I am going to impose a restriction on these beliefs. The approach I follow is based on the argument adopted by Guerrieri et al. [2010] in a similar setting. Let me first state the restriction, and then explain the intuition behind it.

**Requirement 1.** *Take an off equilibrium  $a_j \notin I^*$ . Given a subgame equilibrium  $(s^*, E)$  and associated entrepreneur expected payoff  $\pi_\lambda^*$ , the belief  $\beta_{a_j}(\lambda)$  is strictly positive if and only if the set:*

$$Q(\lambda; a_j) := \{Q \in [0, 1] \mid Q(1 - \alpha)R(a_j, \lambda) \geq \pi_\lambda\}$$

*is maximal.*<sup>23</sup> *If  $Q(\lambda; a_j)$  is empty for all  $\lambda$ , the VC expects no entrepreneur to search in market  $a_j$ .*

Essentially, for every  $\lambda$ , one can construct the set of  $Q$ s such that the entrepreneur would (weakly) benefit from deviating and searching in market  $a_j$ . A VC that is contemplating offering such a level of attention must believe that this offer would attract the type(s) that are willing to face the highest search friction, that is, to deviate at the lowest level of  $Q$ .<sup>24</sup>

<sup>22</sup>For a formal definition of Weak Perfect Bayesian Equilibrium see definition 9.C.3 in Mas-Colell et al. [1995]

<sup>23</sup>For a given collection of sets  $Q(\lambda; a_j)$ ,  $\lambda \in [\underline{\lambda}, \bar{\lambda}]$ ,  $Q(\hat{\lambda}; a_j)$  is said to be maximal if it is not a subset of any other  $Q(\lambda; a_j)$ .

<sup>24</sup>Note that the value of  $Q(\lambda; a_j)$  can come from the VCs off-equilibrium behavior, the vacancies posted at attention  $a_j$ . Requirement 1 can be then interpreted as follows: “the type that is expected to search in  $a_j$  is the one for which there is a larger set of VCs actions that would make this deviation profitable”. In this sense, Requirement 1 is an adaptation of condition D1 introduced by Cho and Kreps [1987] for signaling games.

In comparing welfare in different equilibria, I will sometimes need to compute market tightness in empty markets. To do this, I will use the lowest  $Q$  such that the type(s) selected by Requirement 1 (weakly) benefits from deviating. Armed with the definitions above, I can now formally define an equilibrium of the game.

**Definition 3. (Equilibrium).** *An Equilibrium is a vector  $(E, s^*, \sigma^*, \beta)$  constituting a Weak Perfect Bayesian Equilibrium, with the restriction that  $\beta$  satisfies Requirement 1 off the equilibrium path. Off equilibrium, market tightness is computed as the infimum of the set  $Q(\lambda; a_j)$  for  $\lambda$  selected by Requirement 1.*

Before I proceed with the equilibrium characterization, I make one further restriction to avoid the emergence of a – trivial – degenerate equilibrium.

**Assumption A3.**  $R(a_1, \underline{\lambda}) / R(a_0, \underline{\lambda}) < (1 - \alpha) \mathbb{E}_\lambda R(a_1, \lambda) / c$ .

If A3 would not hold, there would always exist an equilibrium where every VC – no matter how inefficient – would choose to raise small funds because no entrepreneur would find it worthwhile to search in the alternative market, no matter how large the difference in search frictions would be across markets. Under A3 instead, such equilibrium could only emerge because of selection considerations by VCs.

In what follows, I characterize all equilibria of the game. The main message will be that, in any equilibrium, better VCs will necessarily offer more attention and match with higher quality entrepreneurs. This comes directly from the result in the previous section, together with the properties of the function  $a(m, x)$ , ensuring that the best VCs have to give up fewer projects in order to provide higher levels of attention. To simplify the notation,  $Q_i^*$  denotes the level of market tightness in market  $a_i$  in equilibrium.

**Proposition 2. (Partitional Equilibria).** *All equilibria are described by a partition of the set  $[\underline{x}, \bar{x}]$  given by  $X_0^{\sigma^*} = [\underline{x}, x_0]$ ,  $X_1^{\sigma^*} = [x_0, \bar{x}]$  and a partition of  $[\underline{\lambda}, \bar{\lambda}]$  given by  $\Lambda_0^{\sigma^*} = [\underline{\lambda}, \lambda_0]$ ,  $\Lambda_1^{\sigma^*} = [\lambda_0, \bar{\lambda}]$ . If  $X_0^{\sigma^*} = \emptyset$ , then  $\Lambda_0^{\sigma^*} = \emptyset$ ,  $x_0 = \underline{x}$  and  $\lambda_0 = \underline{\lambda}$ ; if  $X_1^{\sigma^*} = \emptyset$ , then  $\Lambda_1^{\sigma^*} = \emptyset$ ,  $x_0 = \bar{x}$  and  $\lambda_0 = \bar{\lambda}$ . The measure  $E$  is such that on the equilibrium path, the frictions  $Q_i^*$  satisfy free entry. The cutoffs  $x_0$  and  $\lambda_0$  satisfy:*

*If  $I^* = \{a_0, a_1\}$  (interior equilibrium)*

$$(i) \quad m_0^{x_0} (\alpha \mathbb{E} [R(a_0, \lambda) \mid \lambda \leq \lambda_0] - R_0) = m_1^{x_0} (\alpha \mathbb{E} [R(a_0, \lambda) \mid \lambda \geq \lambda_0] - R_0).$$

$$(ii) \quad Q_0^* (1 - \alpha) R(a_0, \lambda_0) = Q_1^* (1 - \alpha) R(a_1, \lambda_0).$$

*If  $i \in I^*$  and  $j \notin I^*$  (non-interior equilibrium),*

$$(iii) \quad m_i^x (\alpha \mathbb{E} [R(a_i, \lambda)] - R_0) \geq m_j^x (\alpha R(a_j, \lambda_0) - R_0) \quad \text{for all } x.$$

Let me describe in words the equilibria, starting from the interior ones. Entrepreneurs and VCs select into different attention levels according to their type. In particular, they separate in two groups, with low quality entrepreneurs and inefficient VCs matching in the market for low attention, and high quality and efficient VCs matching in the alternative market. Conditions (i) and (ii) impose that the cutoff types defining the equilibrium partitions are indifferent between the two attention levels. Condition (iii) is where the requirement on off-equilibrium beliefs kicks in. Note that, if  $a_0 \notin I^*$ ,  $\lambda_0 = \underline{\lambda}$ . Hence, condition (iii) requires that, in a candidate equilibrium where all VCs are offering high attention, no VC finds it profitable to deviate to low attention, given that this deviation would attract the lowest quality entrepreneur, the type that is most willing to sacrifice attention in exchange of matching with a higher probability. Viceversa, if  $a_1 \notin I^*$ ,  $\lambda_0 = \bar{\lambda}$ : in an equilibrium where all VCs coordinate on offering low attention funds, Requirement 1 implies that deviating to high attention will attract instead the highest quality entrepreneur. Condition (iii) again ensures that no VC would be better off deviating to  $a_1$  in such an equilibrium.

### 2.3 What if VCs Could Post $\alpha$ ?

The assumption that project returns are shared exogenously between the VC and the entrepreneur can now be examined in greater details. The assumption rules out, for example, the possibility that a VC could try to attract better entrepreneurs by offering them a larger share of the returns; in other words, it rules out transfers. In practice there is little evidence of transfers. This is – at least partially – due to the fact that financing happens in stages and new VCs may join in a partnership at later rounds, making it difficult to commit to transfers ex-ante (see the discussion in Sørensen [2007] who maintains the same assumption). Yet one might ask whether and how allowing VCs to commit both to size – hence attention – and the equity contract,  $\alpha$ , would affect the sorting equilibrium. In Appendix C it is shown that, in a model where VCs can commit to different  $\alpha$ s and entrepreneurs can direct their search based on this additional dimension too, the qualitative features of the model would remain the same. Allowing VCs to commit to a given share  $\alpha$  does not undo the equilibrium sorting that generates in the original model. To understand why, note that the share that goes to the entrepreneurs –  $(1 - \alpha)$  – enters linearly their payoff. The linear nature of the equity contract means that more favorable terms – a lower  $\alpha$  – can not attract entrepreneurs differentially; the equity share would play the same role as the search friction, and therefore not give VC any additional power to induce advantageous self-selection of entrepreneurs.

## 3 Efficiency and Excessively Small Funds

### 3.1 Pareto Ranking and Second-Best

Ex ante, total welfare in the economy amounts to the expected fees VCs receive from investors. This is because investors engage in perfect competition for VCs, and more entrepreneurs enter the market until they make zero expected payoff. In expectation, VCs are the only agents extracting rents. Here,  $W_i^*$  denotes total vacancies in a given market  $i$ , in a given equilibrium.  $W_i^*$  depends on the particular equilibrium strategy profile that is examined. Since the fees VCs get equal the total excess returns to investors, for a given equilibrium, aggregate welfare is then given by:

$$V(E, s^*, \sigma^*) = \sum_{i \in \{0,1\}} W_i^* \left( \alpha \mathbb{E} \left[ R(a_i, \lambda) \mid \lambda \in A_i^{\sigma^*} \right] - R_0 \right).$$

Generally, equilibria need not be unique. A first question one can ask is whether some equilibria are more desirable than others from an ex ante point of view. The next proposition states that some type of equilibria can be unambiguously ranked. Interestingly, undesirable equilibria are those in which markets for higher levels of attention are thicker, relatively to those for lower attention.

**Proposition 3. (*Ranking Equilibria*).**

(i) *An equilibrium of the game induces higher welfare than (and Pareto dominates) any another equilibrium where the market for high attention is thicker, that is  $Q_1^*/Q_0^*$  is larger.*

(ii) *An equilibrium of the game induces higher welfare than (and Pareto dominates) any another equilibrium where the ratio  $W_1^*/W_0^*$  is larger.*

(iii) *An equilibrium Pareto-dominates another equilibrium if and only if  $x_0$  is larger; hence, in a Pareto-dominant equilibrium the average fund size at every submarket and in the economy is larger, and the amount of money raised by VCs in the economy is larger.*

Equilibria where markets for high attention are thicker are Pareto inferior because, when the increase in search frictions in the two markets is small, the resulting assignment is characterized by a worse selection at the top, that is, the cutoff  $\lambda_0$  is lower, leading to lower average quality at each attention level. The second part of the proposition is a consequence of this feature and of the fact that whenever  $W_1/W_0$  is larger, entrepreneurs' search behavior adjusts so that the relative search friction between the two markets,  $Q_1/Q_0$ , is also larger. The emergence of Pareto-dominated equilibria is due to a typical coordination failure on the VC side: when many choose to raise a more focused fund, it is relatively easy for entrepreneurs to find a match in the associated market. As a result, only very low quality entrepreneurs are willing to give up this higher level of attention and select a (only slightly) less crowded market. In these equilibria, this tendency exacerbates the adverse selection associated with setting a larger fund capacity,

and the economy is stuck in a situation where even (relatively) inefficient VCs choose to raise focused funds.

I now study what is the welfare maximizing allocation of VCs into fund sizes when the induced aggregate effect on sorting is taken into account. Below, I define a Second-Best Allocation as a solution to this problem.

**Definition 4.** A *Second-Best Allocation* is a mapping  $\tilde{\sigma} : [\underline{x}, \bar{x}] \rightarrow \{a_0, a_1\}$  solving:

$$\max_{\sigma} \sum_{i \in \{0,1\}} W_i^{\sigma} \left( \alpha \mathbb{E} \left[ R(a_i, \lambda) \mid \lambda \in \Lambda_i^{s*} \right] - R_0 \right).$$

Observe that – by the continuity of the functions  $m_i^x$  and  $g(x)$  – a Second-Best allocation must also be characterized by a partition of  $[\underline{x}, \bar{x}]$ , with more skilled VCs being assigned to higher attention. Call  $x_0^{sb}$  the limit of this partition. The next result compares the equilibrium with the Second-Best solution.

**Proposition 4. (*Inefficiently small funds*).** *In equilibrium, too many VCs choose high attention compared to the second-best solution. That is,  $x_0^{sb} > x_0$ .*

There is a simple intuition behind this result. A solution to the Second-Best problem involves a tradeoff between allocating VCs to their optimal size, and the motive to increase relative search frictions so to induce a higher cutoff, and hence higher average quality in both markets. However, starting from any equilibrium – including the Pareto superior one – a marginal increase in  $x_0$  comes at a negligible (close to zero) cost in terms of the misallocation of VCs to a larger fund size, but has a strictly positive impact on the sorting outcome through the increase in  $\lambda_0$ .

## 3.2 Implications on the Size-Return Relationship

Before moving to the next section, it is worth emphasizing some implications of this theory on the empirical relationship between fund size and returns in venture capital. Equilibrium sorting of entrepreneurs exacerbates the effect of fund size on returns: not only is the VC who is managing a larger fund contributing less to projects returns, he is also attracting a worse pool of projects. However, the strength of this second effect depends on variables – such as the severity of asymmetric information, dispersion in project quality, and distribution of VC ability in the market – that are likely to change over time and across markets. VCs that are raising a new fund anticipate this. Therefore, observations of large fundraising episodes are concentrated in periods and markets where this sorting effect is less pronounced. Hence, an empirical study is naturally subject to a sample selection bias that leads to underestimating the importance of diseconomies of scale.

To visualize how the distribution of project quality may affect equilibrium fund size, define the function:

$$\phi(a, a', \tilde{\lambda}) := \frac{\alpha \mathbb{E} [R(a, \lambda) | \lambda \geq \tilde{\lambda}] - R_0}{\alpha \mathbb{E} [R(a', \lambda) | \lambda \leq \tilde{\lambda}] - R_0}.$$

This is the expected *per unit of money* excess return from choosing attention  $a$  and attracting entrepreneurs above some  $\tilde{\lambda}$ , *relative* to the excess return from choosing attention  $a'$  and attracting entrepreneurs below the same threshold. The function  $\phi(a, a', \tilde{\lambda})$  need not be monotone in  $\tilde{\lambda}$ . Below is an example where it is always decreasing.

**Example 1.** Quality  $\lambda$  is uniformly distributed over the support  $[0, 1]$ . Returns are given by  $R(a, \lambda) = a + (a - k) \rho(\lambda)$  with  $a > k > 0$ .<sup>25</sup> If  $\rho(\cdot)$  is any increasing linear function, it can be verified that the ratio  $\phi(a, a', \tilde{\lambda})$  is decreasing in  $\tilde{\lambda}$  for any  $a > a'$  and any  $k, R_0 > 0$ .

The properties of  $\phi(\cdot)$  depend entirely on the primitives of the model. When it is decreasing, it is particularly important for VCs not to attract entrepreneurs at the low end of the distribution. When many VCs coordinate to offer small funds, those are precisely the entrepreneurs that will self-select in the market for large funds, and, therefore, a bad equilibrium as characterized in Proposition 3 – equilibrium “ $I$ ” in Figure 1 – emerges.

In general, this theory offers an argument why not only the returns but also the derivative of returns with respect to fund size can well vary over time and cross-sectionally. An empirical study of the size-return relationship in venture capital should take this into account.

## 4 Entry and Comparative Statics

### 4.1 Stable Equilibria and the Effects of Entry

The analysis so far has focused on an economy where the measure and distribution of VCs is fixed. As noted in Section 1, another object of interest is the effect of VC entry on the equilibrium allocation of investors money and projects to VCs. This analysis is mainly motivated by the debate around the effectiveness of policies that encourage VC investment, and by the recent finding that government-sponsored VCs do not crowd out investment by private VCs at the aggregate level. Moreover, there exists evidence that money committed in the venture capital industry is highly volatile and is subject to booms and busts and that the number of funds dedicated to this asset class vary over time, sometimes in response to the business cycle. Determining the reason why these cycles occur is beyond the scope of this paper. However, the

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<sup>25</sup>This function is logsupermodular whenever  $\rho' > 0$ .

model can predict how the distribution of returns is affected by the entry of new VCs into the economy.

**Two Submarkets.** Rewrite the two indifference conditions identifying the equilibrium vector  $(x_0, \lambda_0)$  as :

$$m_1^{x_0} (\alpha \mathbb{E} [R(a_1, \lambda) \mid \lambda \geq \lambda_0] - R_0) - m_0^{x_0} (\alpha \mathbb{E} [R(a_0, \lambda) \mid \lambda \leq \lambda_0] - R_0) = 0 \quad (2)$$

and

$$\frac{W_1(x_0)}{1 - F(\lambda_0)} (1 - \alpha) R(a_1, \lambda_0) - \frac{W_0(x_0)}{F(\lambda_0)} (1 - \alpha) R(a_0, \lambda_0) = 0. \quad (3)$$

Note from (2) that for a given equilibrium cutoff  $x_0$ , the induced equilibrium sorting is unique. In particular, it can be shown that the cutoff  $\lambda_0$  is a unique increasing function of  $x_0$ .

**Stable Equilibria.** Below I introduce one appealing property of a candidate equilibrium, that will help identify the comparative statics in this section. The property is based on a stability argument and will refine the set of equilibria. Call  $\eta(x_0, \lambda_0)$  the left hand side of (2) and  $\mu(\lambda_0, x_0)$  the left hand side of (3).

**Definition 5. (Stable Equilibria).** *An Equilibrium  $(\tilde{x}_0, \tilde{\lambda}_0)$  is stable if it is an asymptotically stable fixed point of the function  $\Theta: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  defined by:*

$$\Theta(x_0, \lambda_0) = \begin{bmatrix} \eta(x_0, \lambda_0) + x_0 \\ \mu(\lambda_0, x_0) + \lambda_0 \end{bmatrix}$$

In words, an equilibrium is stable if, after any small perturbation forces some agents' strategies away from it, behaviour will eventually converge back to the original equilibrium.<sup>26</sup>

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<sup>26</sup>In Appendix A, it is shown that this is equivalent to requiring stability of the constant solution  $(x_0, \lambda_0)$  to a system of differential equations where  $x_0$  is assumed to increase (decrease) proportionally to the marginal benefit (loss) to type  $x_0$  from choosing attention  $a_0$  rather than  $a_1$ , given  $\lambda_0$ , and the same is assumed for the differential equation governing the changes of  $\lambda_0$  for a given  $x_0$ .

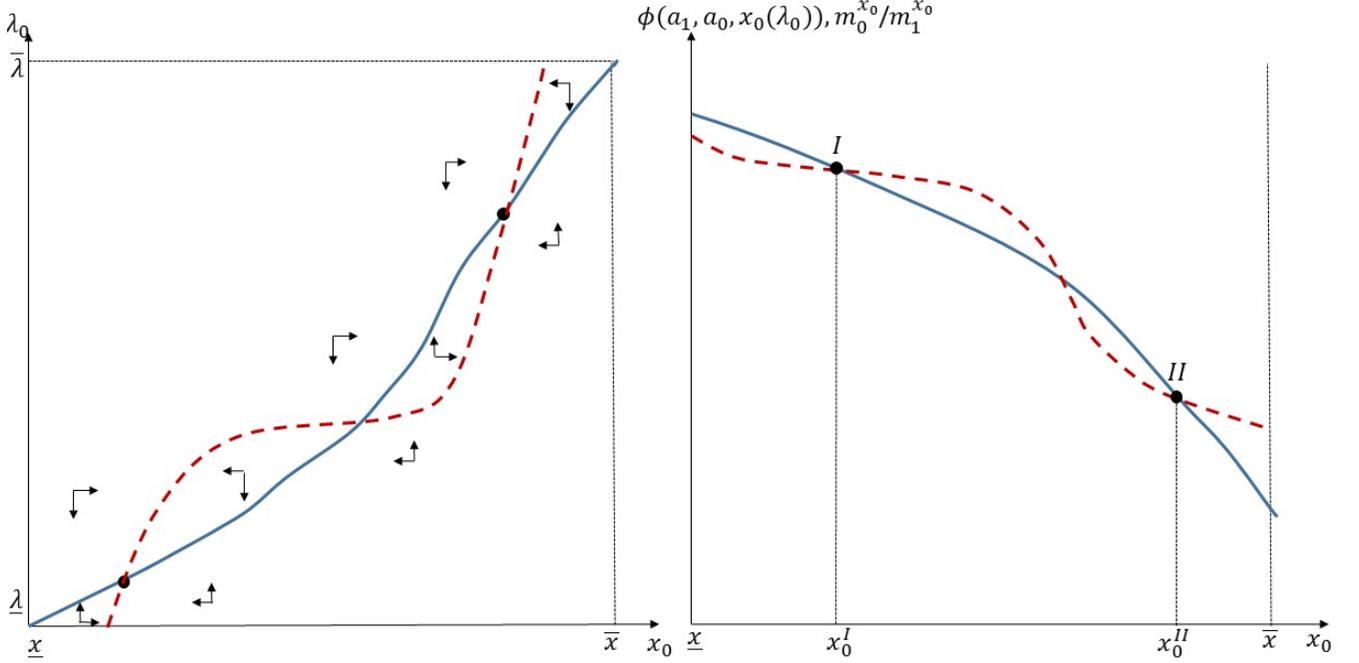


Figure 1: Left: The solid line is the solution to the entrepreneur's indifference condition for each level of  $x_0$ . Arrows above (below) this line point upwards (downwards) because if the population cutoff was type  $\lambda$ , he would strictly benefit (lose) from moving to market  $a_0$ . The dotted line connects all the indifferent VCs, for each  $\lambda_0$ . Arrows at the west (east) of the line point to the right (left) because if the population cutoff was type  $x$ , he would strictly benefit (lose) from moving to market  $a_0$ . The stable equilibria are the two intersection at the bottom-left and top-right of the picture. Right: The solid line is the function  $m_0^{x_0}/m_1^{x_0}$ , decreasing because the relative difference between  $m_0$  and  $m_1$  is smaller for better VCs. The dotted line is  $\phi(a_1, a_0, \lambda_0(x_0))$  which moves with  $x_0$  through its effect on  $\lambda_0$  and is decreasing because when  $x_0$  increases,  $\lambda_0$  increases, and  $\phi_{\lambda} < 0$  in this example. An equilibrium is an intersection of this two curves, and stable equilibria (denoted  $I$  and  $II$ ) are those where  $\phi$  is flatter than  $m_0^{x_0}/m_1^{x_0}$  at the intersection. In this example, there are three equilibria.  $I$  is the worse equilibrium, while  $II$  is the welfare maximizing equilibrium.

I will now conduct comparative statics around a stable equilibrium.

**Comparative Statics.** One interesting exercise is to study what happens when new, unskilled VCs enter the market. More precisely, imagine that the distribution of skills  $g$  is defined on a support larger that also includes types  $x < \underline{x}$ . Initially, only VCs in  $[\underline{x}, \bar{x}]$  operate. What will happen if some of the worst VCs previously excluded decide to enter? In other words, what are the consequences of a decrease in  $\underline{x}$ ? Note that the exclusion of some VCs from the market could result from the presence of barriers to entry. The expected payoff of VCs in equilibrium is strictly increasing in  $x$ ; thus, if being active in the market requires a fixed investment  $\kappa$ , the ex ante payoff to the marginal VC – denoted  $v(\underline{x})$  – would be given, in an interior equilibrium,

by:

$$v(\underline{x}) := m_0^{\underline{x}} (\alpha \mathbb{E}[R(a_0, \lambda) \mid \lambda \leq \lambda_0] - R_0) = \kappa.$$

In this environment, subsidizing the investment  $\kappa$  of the highest type outside the market would be equivalent to inducing a marginal decrease in  $\underline{x}$  in the venture capital market. Evidently, absent any effect on the rest of the economy, such a policy would have a neutral effect: the cost of it -  $\kappa$  - would be exactly offset by the excess returns produced by the marginal VC entering the economy. The next result states what is the effect of a marginal change in  $\underline{x}$  on all stable equilibria.

**Proposition 5. (*Entry of unskilled VCs*).** *For every stable equilibrium  $(x_0, \lambda_0)$ :*

- (i)  $\frac{\partial \lambda_0}{\partial \underline{x}} < 0$ .
- (ii) *As  $\underline{x}$  decreases, welfare increases by more than  $v(\underline{x})$*
- (iii)  $\frac{\partial x_0}{\partial \underline{x}} > 0$  *if and only if  $\phi_{\tilde{\lambda}}(a_1, a_0, \lambda_0) > 0$ .*

In words, the inflow of unskilled VCs leads some entrepreneurs to switch to the low-attention market: the indifferent entrepreneur's quality is higher. Total welfare increases by more than the rents collected by the marginal VC. When the function  $\phi(a, a', \tilde{\lambda})$  is decreasing in the cutoff  $\tilde{\lambda}$  - hence the relative gain from attracting entrepreneurs above versus below  $\tilde{\lambda}$  is lower the higher is  $\tilde{\lambda}$  - some VCs originally raising a small fund, will opt to form a large fund.

The intuition is simple. The relatively unskilled VCs who enter the market will select  $a_0$ . The larger number of vacancies in the market for low attention pushes the cutoff  $\lambda_0$  up. This implies that those VCs who will keep raising relatively smaller funds will select better projects. Essentially, the larger market for unfocused funds now absorbs some of the low-quality entrepreneurs from the economy. Because the increase in  $\lambda_0$  increases average quality in both submarkets, total welfare increases. When the function  $\phi(a, a', \tilde{\lambda})$  is decreasing in  $\tilde{\lambda}$ , the adverse selection problem associated with managing a larger fund is less severe, inducing more VCs to raise one. In this circumstance, the inflow of unsophisticated VCs increases the average fund size and aggregate investment in the market. This is consistent with the findings in [Brander et al. \[2014\]](#).

Consider now of the distribution of returns of funded projects in the industry. Returns will necessarily take values  $R \in [R(a_0, \underline{\lambda}), R(a_1, \bar{\lambda})]$ . The shape of the returns distribution will depend on that of the distribution of project quality -  $f$  - and on the equilibrium choices of VCs and entrepreneurs.

I show that the returns distribution is also affected by the entry of new, unskilled VCs. In the absence of sorting, the effect one should expect is mechanical: relatively more VC funds would now end up delivering low returns. When sorting is taken into account though, the positive externality for incumbents VCs who keep raising focused fund results in higher returns at the top of the distribution. The corollary below formalizes this observation.

**Corollary 1. (*Entry and returns distribution*).** *For each equilibrium, as  $\underline{x}$  decreases, there exists a point  $\tilde{R}$  in the new distribution of returns, such that  $\forall R > \tilde{R}$  expected returns are (weakly) higher conditional on being above  $R$  and are more likely to be below  $\tilde{R}$ . The first effect is strict for some  $R$  close enough to  $\tilde{R}$ .*

The effect is consistent with the findings in Kaplan and Schoar [2005] and Nanda and Rhodes-Kropf [2013]: the former find that, in times characterized by more intense activity in the industry, capital flows disproportionately to worse funds; the latter documents that investments made during “hot” periods are more likely to fail, but deliver greater returns conditional on not failing.

## 4.2 Negative NPV Projects

A natural objection to the result above is that in this model every project is assumed to deliver non-negative excess returns, and, therefore, allowing for more of them to be financed increases welfare by construction. In this subsection I emphasize that the key message of Proposition 5 is that entry of marginal VCs has a positive, indirect effect on the incumbent – inframarginal – VCs. To make this evident, allow projects to deliver negative excess returns. Assume that, for some  $a$  and some  $\lambda$ ,  $\alpha R(a, \lambda) < R_0$ . If, in equilibrium, the indifferent entrepreneur  $\lambda_0$  is such that  $\alpha \mathbb{E}[R(a_0, \lambda) \mid \lambda \leq \lambda_0] > R_0$ , the analysis above would apply and the fact that *some* projects deliver negative excess returns is of no consequences. If, instead, we have  $\alpha \mathbb{E}[R(a_0, \lambda) \mid \lambda \leq \lambda_0] < R_0$ , a VC would be better off not operating in the market, for any value of the fixed investment  $\kappa$ . To allow for such “bad funds” to exist, assume a VC receives private benefits  $-b > 0$  – anytime he manages a fund. Assume that this private benefits are wasteful and therefore would not enter the welfare calculation. It will still be the case in an interior equilibrium that the least efficient VC is indifferent between paying the fixed investment and not operating, so that

$$m_0^{\underline{x}}(\alpha \mathbb{E}[R(a_0, \lambda) \mid \lambda \leq \lambda_0] - R_0) + b = \kappa.$$

In this case, a policy that pays  $\kappa$  to the last VC  $\underline{x}$  would generate a direct *cost* given by  $m_0^{\underline{x}}(\alpha \mathbb{E}[R(a_0, \lambda) \mid \lambda \leq \lambda_0] - R_0)$  on top of the investment  $\kappa$ . On the other hand – just as in the original model – the resulting increase in  $\lambda_0$  would increase both  $\mathbb{E}[R(a_0, \lambda) \mid \lambda \leq \lambda_0]$  and  $\mathbb{E}[R(a_1, \lambda) \mid \lambda \geq \lambda_0]$  and, therefore, would benefit *all inframarginal* VCs. Such policy would then have an ambiguous effect overall. However, this analysis clarifies that the presence of negative NPV projects does not undermine the beneficial effect that entry of VCs from the bottom of the distribution has on all incumbent VCs.

## 5 VC Funds as Limited Partnerships

Contrary to standard firms and organizations, private equity funds are finitely lived limited partnerships. Capital committed in the vintage year imposes a cap on how much the fund manager will invest. Unlike the case of – for example – mutual funds, capital does not flow into and out of a venture capital fund. This can generate a number of inefficiencies. Perhaps the most obvious of these - which is not related to any asymmetric information or conflict of interest between fund managers and investors – is the fact that managers constrained by capital committed in the vintage year do not approach investors before returns from (a good share of) the first investments have materialized; hence, they might be forced to pass up investment opportunities that show up later in the fund’s life. While it is true that a manager can, in principle, open new funds in parallel, fundraising is time consuming, which strongly limits the extent to which VCs can put projects on hold until enough money is raised. This practice is also limited by contractual restrictions that are meant to protect current investors and prevent fund managers from forming successor funds before the existing one is substantially invested or has completed its investment period. Typically, this configuration is justified by agency problems between investors and fund managers. However, one could ask whether in the absence of such problems, a different arrangement would emerge. In other words, is forming such a limited partnership also in the VC’s best interest? The main point of this section is to show that this is the case. I will argue that this structure arises in equilibrium due to the incentive that endogenous entrepreneur sorting provides.

### 5.1 A Dynamic Setting

Time – denoted  $t$  – is discrete and the time horizon is infinite. To make the analysis more accessible, I assume in this section that all VCs are identical. VCs are long lived, and the original measure of VCs in the economy is normalized to one. At any time  $t$  a measure -  $\beta \in (0, 1)$  - of new VCs enters the market; at random, an equal measure of VCs dies. In each period, VCs maximise the sum of fees, with a common discount factor  $\delta \in (0, 1)$ . This is an effective discount factor, which already takes into account the probability of surviving the next period. Investors are in large supply and are endowed with one unit of money per period. In this section I normalize their outside option -  $R_0$  - to zero. Assume that VCs can only find up to one entrepreneur in each round of matching in which they participate. It takes two periods and the input of the VC in both for each project to develop and produce returns, independently of its quality. Returns are again a deterministic function of project’s quality and attention,  $R(a, \lambda)$ , which is assumed to be log-supermodular everywhere. Once a project has produced returns, the match expires. It follows that a VC searching for an entrepreneur can be in either of two states: he can be unmatched, or already engaged with a project in an intermediate stage.

**Diseconomies of Scale.** To capture the same quality-quantity trade-off as in the previous sections, I assume that VC attention to a particular project is determined by whether he has dealt with another one in any period of the project’s life span. The levels of  $a$  with and without overlapping projects are denoted  $a^h$  and  $a^l$  respectively, with  $a^h > a^l$ .

As before, the other side of the market consists of entrepreneurs. Assume that at each  $t$ , a new generation of entrepreneurs is born. They make an irreversible entry choice, and, conditional on entry, draw a type and direct their search. Those who are not matched die. Those who match receive a share  $(1 - \alpha)$  of the returns and leave the economy forever, as well as those who do not enter the market at all. Entrepreneurs discount the future at the same rate as VCs,  $\delta$ .<sup>27</sup>

## 5.2 Choice of Fund Structure: Open Credit Line vs Short-Term Contracts

At the beginning of each period, managers approaching investors can opt for either of two fund structures. They can choose an open fund that allows them to, at any time  $t$ , access the necessary cash to finance a new project. Alternatively, they can form a limited partnership with a finite two-period horizon. In the latter case, a fund consists essentially of a single investment, that matures and provides returns two periods from formation. In the model, I refer to this alternative as the short-term contract. Crucially, investors and VCs will not be able to renegotiate in the intermediate period. One can imagine that investors writing this type of contract will have their wealth invested in an alternative asset in the intermediate period. If approached by the VC in the intermediate period, investors would not have the liquidity to provide the VC with additional money to start a project. This is realistic: pension funds (representing a large share of investors in venture capital) usually meet capital calls by selling positions in liquid indexes.<sup>28</sup> Another interpretation is that in the typical private equity partnership, many dispersed investors own a share of the fund. This makes it harder for VCs to agree with the original investors on changes to the size of the fund. Regardless of the interpretation, all that matters in the model is that a short-term contract creates an endogenous commitment not to start new projects before the original one has produced returns.

Note that, as clarified in Section 2, each manager’s choice maximizes the fund’s total excess returns, because they can set fees that hold investors to their participation constraint. Therefore, a fund manager’s objective is to maximize the discounted sum of expected excess

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<sup>27</sup>This is the effective discount factor used by VCs. There is an assumption behind this: in the event a funded project is interrupted due to the VCs death, the entrepreneur can not find a substitute for the VC. In such case, it is as if the entrepreneur did not find a match in the first place. It will become clear that the way entrepreneurs discount the future in infact of no consequence.

<sup>28</sup>See [Robinson and Sensoy \[2016\]](#).

returns.

**Summary of the Timing.** Let me now summarize the timing of the game:

- At each time  $t$ , newborn VCs and penniless ones approach investors, choose a fund structure, and contract over the fee  $p$  that investors pay upon realization of each project's returns. Then they search for entrepreneurs. Managers opting for a short-term contract at  $t - 1$  cannot approach investors before the project they are currently financing has produced returns and therefore are not actively searching at time  $t$ .
- Entrepreneurs observe the investors' strategy and make the entry choice. Those who enter the market privately observe their type  $\lambda$  and direct their search.
- At  $t + 1$ , managers who chose the open fund have access to money to search for a new project. Every match formed in  $t$  generates returns  $R(a, \lambda)$  at time  $t + 2$ . By then,  $t$ -generation entrepreneurs who matched leave the market forever.

**Assumption A4.**  $(1 + \delta) R(a^l, \lambda) > R(a^h, \lambda) \quad \forall \lambda.$

The assumption above limits the extent to which a manager's human capital is destroyed when working on parallel projects. Under A4, keeping quality fixed, it is always optimal to start a new project every period. In turn this means that a manager who is financing a project at its intermediate period, and expects to attract the same type of project, is always willing to search actively for new ones, provided he has the capital. If the inequality in A4 was reversed instead, the choice of which contract to enter into with investors would be inconsequential: managers would never start a new project before the current one was completed; the fact that they lack access to liquidity at any point in time would not constrain their choices. I will restrict my attention to equilibria where entrepreneur sorting is constant over time. When A4 holds, this means that entrepreneurs that are searching for a match will effectively face the choice between two distinct markets: one where uncommitted VCs will provide attention  $a^l$ , and the other where attention is at  $a^h$  because agents in a short-term contract with investors will not be able to search before the original investment has matured.

**The Sorting Subgame.** I first derive sorting behavior when a positive measure of vacancies is available in both markets. Call  $\gamma_t$  the share of managers who search for projects and are in an open contract at time  $t$ . It is immediate from the results in previous sections that the equilibrium search behavior at time  $t$  is characterized by a threshold  $\lambda^*$  such that entrepreneurs search in the high-attention market if and only if  $\lambda \geq \lambda^*$ . This is due to logsupermodularity of  $R(a, \lambda)$ . The threshold is implicitly defined by the equation:

$$\frac{1 - \gamma_t}{1 - F(\lambda^*)} R(a^h, \lambda^*) = \frac{\gamma_t}{F(\lambda^*)} R(a^l, \lambda^*). \quad (4)$$

**Lemma 4.** *The solution to (4) is unique: for any given share of managers in the open fund  $\gamma_t$ , there is a unique equilibrium of the sorting subgame. The function  $\lambda^*(\gamma_t)$  is continuous and strictly increasing in  $\gamma_t$ .*

To keep sorting constant over time, I restrict my attention to equilibria where, whenever VCs are indifferent between fund structures, the share of VCs opting for either stays constant. Such equilibria are always possible to construct, provided that at a certain time  $t$  agents are indifferent about which contract to choose, thanks to the assumption that new VCs enter the market at each  $t$ .

**Definition 6.** *A stationary equilibrium is an equilibrium in which the shares of actively searching VCs in either submarket is independent on  $t$ .*

**Lemma 5.** *There exists a measure of newborns -  $\hat{\beta}$  - such that, for all  $\beta \in (0, 1)$  with  $\beta < \hat{\beta}$ , if an equilibrium where  $\gamma_t = z \in (0, 1)$  for some  $t$  exists, then a stationary equilibrium where  $\gamma_t = z$  for all  $t$  exists.*

Let me focus on the case when selecting the best entrepreneur is appealing to the VC. Formally, this means imposing the following restriction.

**Assumption A5.**  $R(a^h, \bar{\lambda}) > (1 + \delta) \mathbb{E} [R(a^l, \lambda)]$ .

Under A5 the returns from following the best entrepreneur exclusively are higher than those from financing two average projects in two subsequent periods. The main results of this section can now be stated.

**Proposition 6. (Equilibrium Fund Structure).**

(i) *There is no stationary equilibrium where all VC choose the open fund.*

(ii) *The equilibrium's measure of VCs choosing the open fund,  $\gamma$ , is the solution to the equation:*

$$\mathbb{E} [R(a^h, \lambda) | \lambda \geq \lambda^*(\gamma)] = (1 + \delta) \mathbb{E} [R(a^l, \lambda) | \lambda \leq \lambda^*(\gamma)]$$

*whenever it exists.*

(iii) *When  $\mathbb{E} [R(a^h, \lambda)] \geq (1 + \delta) R(a^l, \underline{\lambda})$ , there is a stationary equilibrium where every VC chooses the short-term contract.*

Note that, because of A5, if the function  $\phi(a^h, a^l, \tilde{\lambda})$  was decreasing in  $\tilde{\lambda}$  – as it is in Example 1 – the condition in (ii) would have no solution, while the condition in (iii) would always hold. This means that a situation where every VC chooses the short-term contract would be the unique equilibrium. Unraveling of the equilibrium with open funds is due to the strong incentives generated by selection: in such candidate equilibrium, a deviating VC would attract the best entrepreneur, who is the most willing to face higher search frictions in order to enjoy exclusivity. This deviation is profitable under A5.

As similarly established in the general static model, the motive to attract better entrepreneurs generates an inefficiency, as VCs do not internalize the aggregate effect – due to equilibrium sorting – of their choices. In particular, one natural and policy-relevant question would be whether allowing these short-term contracts is desirable. It turns out that, under A4, banning short-term contracts is always beneficial.

**Proposition 7. (*Banning short-term contracts*).** *Every equilibrium of the game delivers lower welfare than the case where every VC chooses the open credit line. That is, banning short-term contracts improves welfare.*

It is easy to see why the corner equilibrium where every VC chooses the short-term contract is detrimental to welfare. In fact, note that under A4, the choice of which contract to sign involves a simple trade-off: on the one hand, starting a new project every period allows the VC to make the best use of his human capital, as the dilution of attention is assumed to be minimal; on the other hand, committing to an exclusive relationship helps the VC attract the best entrepreneurs. However, *in equilibrium*, this commitment confers no benefit at all, since every VC will look alike. In all interior equilibria, VCs that opt for long-term open funds attract a negatively selected subset of entrepreneurs. Since the expected returns are ultimately the same for all VCs – as they have to be indifferent to which contract they choose - it follows that the expected aggregate returns would be higher if all VCs chose the long-term contract and matched with the average entrepreneur.

## 6 Generalizing To $N > 2$ Submarkets

The analysis so far has focused, for tractability, on the case where VC attention can only take two values. In this section I introduce the more general setup when the attention function is allowed to have  $N > 2$  steps.

**General attention function.** Define the step function:

$$a(m, x) = \begin{cases} a_N & \forall m \in [0, m_N^x] \\ a_i & \forall m \in (m_{i+1}^x, m_i^x] \end{cases}$$

with  $m_i^x - m_{i+1}^x = \Delta > 0$  for all  $x$  and  $i$ , and  $\partial m_i^x / \partial x > 0$  for all  $i \in \{0, \dots, N\}$  and all  $x$ .<sup>29</sup> All the arguments in Section 2 are valid in this setup, and Requirement 1 is stated in such a way that it can be immediately applied to this more general environment. The next proposition characterizes equilibria.

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<sup>29</sup>As in the case with two attention levels, a general setting allows for  $\Delta$  to be a function of  $x$ . In which case ensuring perfect separation of VCs in equilibrium requires to impose the single crossing condition  $\partial \left( \frac{m_0^x}{m_0^x - \Delta(x)} \right) / \partial x < 0$ , which is satisfied when  $\Delta$  is constant across  $xs$ .

**Proposition 8. (Partitional Equilibria with  $N$  Submarkets).** *All equilibria are described by a partition of the set  $[\underline{x}, \bar{x}]$  defined by cutoffs  $\{\underline{x} = x_{-1}, \dots, x_i, \dots, x_N = \bar{x}\}$  and a partition of  $[\underline{\lambda}, \bar{\lambda}]$  defined by cutoffs  $\{\underline{\lambda} = \lambda_{-1}, \dots, \lambda_i, \dots, \lambda_N = \bar{\lambda}\}$  such that for any  $i \in I^*$ ,  $\Lambda_i^{s*} = [\lambda_{i-1}, \lambda_i]$  and  $X_i^{\sigma*} = [x_{i-1}, x_i]$ . If  $i \notin I^*$ ,  $\lambda_i = \lambda_{i-1}$  and  $x_i = x_{i-1}$ . The measure  $E$  is such that on the equilibrium path, the frictions  $Q_i^*$  satisfy free entry. For all adjacent  $i, j \in I^*$  with  $i > j$ :*

$$(i) \quad m_j^{x_j} \left( \alpha \mathbb{E} \left[ R(a_j, \lambda) \mid \lambda \in \Lambda_j^{s*} \right] - R_0 \right) = m_i^{x_j} \left( \alpha \mathbb{E} \left[ R(a_i, \lambda) \mid \lambda \in \Lambda_i^{s*} \right] - R_0 \right).$$

$$(ii) \quad Q_j^* (1 - \alpha) R(a_j, \lambda_j) = Q_i^* (1 - \alpha) R(a_i, \lambda_j).$$

(iii) For any  $a_j \notin I^*$ , and  $Q(\lambda; a_j) \neq \{\emptyset\}$  for at least one  $\lambda$ ,

$$m_i^x \left( \alpha \mathbb{E} \left[ R(a_i, \lambda) \mid \lambda \in \Lambda_i^{s*} \right] - R_0 \right) \geq m_j^x \left( \alpha R(a_j, \lambda_j) - R_0 \right) \quad \forall a_i \in I^* \text{ and } x \in X_i^{\sigma*}.$$

The only difference worth emphasizing is that here there could emerge partitional equilibria that are non-degenerate – in the sense that VCs and entrepreneur still separate to some extent – and yet some levels of attention are not chosen in equilibrium. In this case, condition (iii) imposes that – provided some entrepreneur would search in the off equilibrium market for some high enough matching probability – no VC finds it profitable to deviate to the off equilibrium  $a_j$ , given that this deviation would attract the highest quality entrepreneur in the set of those who select the closest lower  $a_i$  among  $a_i \in I^*$ .

**Welfare.** In this framework – without further knowledge on the distributions of VCs ability and project quality, and on the return function – it is not possible to rank all equilibria in welfare terms. This is because agents in the economy could coordinate, in principle, on different equilibria where the induced sorting implies higher average quality in some submarkets, and lower average quality in others. The next proposition therefore provides a partial welfare ranking.

**Proposition 9. (Ranking Equilibria with  $N$  Submarkets).**

(i) *An equilibrium of the game induces higher welfare than (and Pareto dominates) any another equilibrium where markets for higher attention are thicker, that is  $Q_i^*/Q_j^*$  is larger for all  $(i, j)$  and  $i > j$ .*

(ii) *An equilibrium of the game induces higher welfare than (and Pareto dominates) any another equilibrium where the ratio  $W_i^*/W_j^*$  is larger for all  $(i, j)$  and  $i > j$ .*

(iii) *In a Pareto-dominant equilibrium as in (i) and (ii), for all markets  $i$ ,  $x_i$  is larger than in the dominated equilibrium; hence, the average fund size in the economy and in every submarket is larger, and the amount of money raised by VCs in the economy is larger.*

The result mimics the one exposed in Proposition 3 and highlights that the same coordina-

tion failure on the VCs side can occur in a more general setting. This section confirms that the forces operating in the original model would play an important role in a more general environment, and would produce similar qualitative results. However, generalizing to  $N$  submarkets comes at a considerable cost in terms of tractability, since it adds more degrees of coordination in the agents' strategies.

## 7 Conclusion

I have introduced a matching model of fund management where the two key ingredients are scarcity in fund manager human capital and directed search from entrepreneurs who are heterogeneous in project quality. The main insight is that fund managers subject to an endogenous flow of investment opportunities have an additional incentive to manage smaller funds: on top of the motive to not dilute their human capital, they want to attract the best projects. This sheds new light onto recent empirical findings that suggest VC funds are operating at a size below their capacity. The result that – due to its effect on equilibrium sorting – the entry of unskilled VCs increases returns at the top of the distribution and leads to higher welfare, can inform the debate about public intervention in venture capital and the roles of alternatives to VC, such as angel investing and crowdfunding. Finally, in the last part of this paper, it is shown that the very same mechanism disciplining equilibrium fund size can rationalize limited-partnerships, the prevalent organizational form of VC funds. These results are a consequence of entrepreneur self-selection and would not result from a model with random matching or homogeneous entrepreneurs.

The model's main features – diseconomies of scale, directed search, and technological complementarities – are inspired by several stylized facts and empirical findings about the venture capital industry. However, it is tempting to interpret the model as describing other economic environments. A consulting firm may limit its size and be more focused in order to attract the best clients, those who attach greater value to the consultant's knowledge. Schools may offer smaller programs not only because they can make better use of their resources but also because the best students are more willing to sacrifice their time and energy to enter such programs. Likewise, a supervisor might want to limit advising to a small number of PhD students in order to attract those who will gain more from their mentoring. To the extent that in such environments, agents that want to attract good matches can commit to a certain capacity and that heterogeneity in quality is of first-order importance – as is the case in venture capital – it should be expected that forces similar to those in the model would prevail. The specific application would then guide future research towards the most appropriate and interesting extensions.

## Appendix A. Proofs

**Proof of Lemma 1.** Consider first those  $(w, x)$  for which  $\tilde{S}(w, x; E) > 0$ . Take one submarket  $(\tilde{w}, \tilde{x})$  where  $Q(\tilde{w}, \tilde{x}) = 1$ . By entering and searching in it, an entrepreneur that is outside the market gets in expectation:

$$(1 - \alpha)\mathbb{E}R(a(m(\tilde{x}, \tilde{w}, E), \tilde{x}), \lambda) - c \geq (1 - \alpha)\mathbb{E}R(a_0, \lambda) - c > 0.$$

Hence, when  $\tilde{S}(w, x; E) > 0$ , it must be that  $Q(\tilde{w}, \tilde{x}) < 1$ . To show that  $\tilde{S}(w, x; E) > 0$  for all  $(w, x)$ , assume not and denote  $(\hat{w}, \hat{x})$  the submarket where no entrepreneur searches. Then, any type  $\lambda$  who entered the market would like to deviate and search in  $(\hat{w}, \hat{x})$ , as this would give:

$$(1 - \alpha)R(a_1, \lambda) > Q(w, x; E)(1 - \alpha)R(a(m(w, x), x), \lambda) \quad \forall (w, x).$$

■

**Proof of Lemma 2.** By Lemma 1, returns to type  $\lambda$  in market  $(w, x)$  conditional on matching are given by  $R(a(w, x), \lambda)$ . Take two markets  $(w, x)$  and  $(w', x')$ , with associated attention levels  $a$  and  $a'$ , with  $a = a'$ . Assume that  $Q(w, x, E) > Q(w', x', E)$ . Then, any entrepreneur searching in  $(w', x')$  could deviate to  $(w, x)$  and get:

$$Q(w, x, E)R(a, \lambda) > Q(w', x', E)R(a', \lambda).$$

■

**Proof of Lemma 3.** In the original model entrepreneurs maximize  $Q(w, x, E)R(a(w, x), \lambda)$ , and, by Lemma 2,  $Q(w, x, E) = \theta(w, x, E)$ . Because  $R(a(w, x), \lambda)$  is constant across an iso-attention region, and since Lemma 2 must apply to the transformed model, all that remains to show is that market tightness is the same in submarket  $a$  as it is at any point in the iso-attention region in the original model. That is, formally,  $\theta(a) = \theta(w, x, E) \forall (w, x) : a(w, x) = a$ . Call  $\Gamma(a)$  the measure of vacancies across all VCs at a given iso-attention region -  $a$  - and  $S(a)$  the measure of entrepreneurs searching in the same region. We have:

$$S(a) := \int_{\{(w,x):a(w,x)=a\}} \int_{\lambda} E dS(w, x; \lambda) d\tilde{a}.$$

And

$$\Gamma(a) := \int_{\{(w,x):a(w,x)=a\}} wdH(w, x) d\tilde{a}.$$

Notice that

$$d\tilde{S}(w, x, E)\theta(w, x, E) = wdH(w, x).$$

Integrating on both sides over a given iso-attention region, and taking  $\theta(w, x, E)$  outside of the

integral by Lemma 2, it follows that:

$$\theta(w, x, E) \int_{\{(w,x):a(w,x)=a\}} d\tilde{S}(w, x, E) = \int_{\{(w,x):a(w,x)=a\}} wdH(w, x).$$

Therefore,

$$\theta(w, x, E) = \left( \int_{\{(w,x):a(w,x)=a\}} wdH(w, x) / \int_{\{(w,x):a(w,x)=a\}} d\tilde{S}(w, x, E) \right) = \theta(a).$$

■

**Proof of Proposition 1.** (Sufficiency). Assume  $R(a, \lambda)$  is logsupermodular everywhere. If there is an equilibrium that does not exhibit PAM everywhere, then there must exist at least two markets  $a_i, a_j$  with  $a_i > a_j$ , and two types  $\lambda', \lambda$  with  $\lambda' > \lambda$  such that  $\lambda \in \Lambda_i$  and  $\lambda' \in \Lambda_j$ .<sup>30</sup> Optimality of the search strategy requires that type  $\lambda$  is at least as well off searching in  $a_i$  rather than in  $a_j$  and similarly  $\lambda'$  (weakly) prefers  $a_j$  to  $a_i$ , that is:

$$Q(a_i) R(a_i, \lambda) \geq Q(a_j) R(a_j, \lambda) \tag{5}$$

$$Q(a_j) R(a_j, \lambda') \geq Q(a_i) R(a_i, \lambda') \tag{6}$$

The two inequalities imply

$$\frac{R(a_i, \lambda)}{R(a_j, \lambda)} \geq \frac{R(a_i, \lambda')}{R(a_j, \lambda')}$$

which contradicts the fact that  $R(a, \lambda)$  is logsupermodular.<sup>31</sup>

(Necessity). Assume  $R(a, \lambda)$  is not logsupermodular at some point  $(\hat{a}, \hat{\lambda})$ . The continuity properties of  $R(a, \lambda)$  (see Section 2) imply that there exists a number  $\varepsilon > 0$ , s.t. the function is not logsupermodular anywhere in  $[\hat{a} - \varepsilon, \hat{a} + \varepsilon] \times [\hat{\lambda} - \varepsilon, \hat{\lambda} + \varepsilon]$ . I construct an economy where NAM could be supported, hence a contradiction arises. Let  $F$  be defined on  $[\hat{\lambda} - \varepsilon, \hat{\lambda} + \varepsilon]$ , and  $a_i \in [\hat{a} - \varepsilon, \hat{a} + \varepsilon]$ , for all  $i$ . By construction, all matches will be in  $[\hat{a} - \varepsilon, \hat{a} + \varepsilon] \times [\hat{\lambda} - \varepsilon, \hat{\lambda} + \varepsilon]$ . In order for only PAM sorting patterns to emerge, a necessary condition is that for at least two

<sup>30</sup>This proof follows through when  $a$  can take any finite number of values. Infact, the result holds in the continuum too, but must be proven via different techniques. In other words, Proposition 1 is still valid when the domain of the attention function is assumed to be any subset of  $\mathbb{R}$ .

<sup>31</sup>Logsupermodularity of  $R(a, \lambda)$  implies that for any  $(a, a')$  with  $a' > a$ , the ratio  $\frac{R(a', \lambda)}{R(a, \lambda)}$  is strictly increasing in  $\lambda$ .

$(\lambda, \lambda')$  with  $\lambda > \lambda'$ , and two  $(a, a')$ , with  $a > a'$ ,

$$Q(a)R(a, \lambda) \geq Q(a')R(a', \lambda) \quad (7)$$

$$Q(a')R(a', \lambda') \geq Q(a)R(a, \lambda') \quad (8)$$

and, crucially, at least one of the two inequalities - for at least one such pair - is strict.<sup>32</sup> When either (7) or (8) or both are satisfied with strict inequality, it holds that

$$\frac{R(a, \lambda)}{R(a', \lambda)} > \frac{R(a, \lambda')}{R(a', \lambda')}$$

which means  $R(a, \lambda)$  is logsupermodular somewhere in  $[\hat{a} - \varepsilon, \hat{a} + \varepsilon] \times [\hat{\lambda} - \varepsilon, \hat{\lambda} + \varepsilon]$ , a contradiction. ■

**Proof Proposition 2.** Refer to the proof of Proposition 8 where a more general result is established. ■

**Proof Proposition 3.** Refer to the proof of Proposition 9 where a more general result is established. ■

**Proof Proposition 4.** First, observe that, for any allocation described by a cutoff  $x_0$ , so that VCs are assigned to the high attention market if and only if their ability is above  $x_0$ , the sorting outcome is described by a unique, increasing, and continuously differentiable cutoff  $\lambda_0(x_0)$ . To see why, rewrite the entrepreneur's indifference condition as:

$$\frac{R(a_1, \lambda_0)}{R(a_0, \lambda_0)} = \frac{W_0(x_0)}{W_1(x_0)} \frac{(1 - F(\lambda_0))}{F(\lambda_0)}. \quad (9)$$

The left hand side of (9) is continuous and strictly increasing in  $\lambda_0$  by assumption (as  $R$  is continuous in both arguments and assumed to be log-supermodular in this section). The right hand side is continuous and strictly decreasing in  $\lambda_0$ . Recall that  $W_0 = \int_{\underline{x}}^{x_0} m_0^x dG(x)$ , and  $W_1 = \int_{x_0}^{\bar{x}} m_1^x dG(x)$ . Therefore, the ratio  $W_0(x_0)/W_1(x_0)$  is continuous and decreasing in  $x_0$ , as  $\partial m_i^x / \partial x$  is positive,  $m_0^x$  is continuous, and the distribution  $g$  is continuous.

Denote  $x_0^*$  the largest equilibrium cutoff  $x_0$ , which corresponds to, by Proposition 3, the Pareto superior equilibrium. In the remainder of the proof I normalize the investors' outside option,  $R_0$ , to zero. This is only to better visualize each equation and the reader should note a strictly positive  $R_0$  would make no difference. The proof proceeds in two steps.

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<sup>32</sup>Otherwise, it would be possible to support an equilibrium with NAM, and the contradiction would immediately arise.

(Step 1). First, I show that any allocation  $\tilde{x}_0 < x_0^*$  delivers lower welfare than  $x_0^*$ . Notice that welfare induced by an allocation  $\tilde{x}_0$  is bounded above by what total returns would be if, given the sorting subgame, VCs could optimally select fund size. Formally:

$$V(\tilde{x}_0) \leq \int_x \max \{ m_0^x \alpha \mathbb{E}[R(a_0, \lambda) | \lambda \leq \lambda_0(\tilde{x}_0)], m_1^x \alpha \mathbb{E}[R(a_1, \lambda) | \lambda \geq \lambda_0(\tilde{x}_0)] \} dG(x).$$

Hence, for all  $\tilde{x}_0 < x_0^*$ :

$$V(x_0^*) = \int_x \max \{ m_0^x \alpha \mathbb{E}[R(a_0, \lambda) | \lambda \leq \lambda_0(x_0^*)], m_1^x \alpha \mathbb{E}[R(a_1, \lambda) | \lambda \geq \lambda_0(x_0^*)] \} dG(x) > V(\tilde{x}_0).$$

(Step 2). Second, I show that the Second Best Problem can be improved by a marginal increase in  $x_0$ , starting from  $x_0^*$ . To see why, write the objective function:

$$V(x_0) = W_0(x_0) \alpha \mathbb{E}[R(a_0, \lambda) | \lambda \leq \lambda_0(x_0)] + W_1(x_0) \alpha \mathbb{E}[R(a_1, \lambda) | \lambda \geq \lambda_0(x_0)].$$

So,

$$\begin{aligned} \frac{\partial V(x_0)}{\partial x_0} \Big|_{x_0=x_0^*} &= \underbrace{(m_0^{x_0^*} \alpha \mathbb{E}[R(a_0, \lambda) | \lambda \leq \lambda_0(x_0^*)] - m_1^{x_0^*} \alpha \mathbb{E}[R(a_1, \lambda) | \lambda \geq \lambda_0(x_0^*)])}_{=0} g(x_0) \\ &+ \alpha \left( W_0(x_0^*) \frac{\partial(\mathbb{E}[R(a_0, \lambda) | \lambda \leq \lambda_0(x_0^*)])}{\partial \lambda_0} + W_1(x_0^*) \frac{\partial(\mathbb{E}[R(a_1, \lambda) | \lambda \geq \lambda_0(x_0^*)])}{\partial \lambda_0} \right) \frac{\partial \lambda_0(x_0^*)}{\partial x_0} > 0, \end{aligned}$$

where the term in the first bracket is zero because  $x_0^*$  is indifferent in equilibrium. ■

**Stable Equilibria.** A fixed point of the vector function  $\Theta$  - defined in Section 4.1 - is asymptotically stable if and only if all eigenvalues of the the Jacobian of  $\Theta$  - denoted  $J(\Theta)$  - are smaller than one in absolute value. Therefore, in this context a necessary condition for  $(\tilde{x}_0, \tilde{\lambda}_0)$  to be asymptotically stable is that the determinant of the 2x2 matrix  $J(\Theta)$  is smaller than one in absolute value. Formally, the condition is:

$$\left| \det \begin{bmatrix} \eta_{x_0}(\tilde{x}_0, \tilde{\lambda}_0) + 1 & \eta_{\lambda_0}(\tilde{x}_0, \tilde{\lambda}_0) \\ \mu_{x_0}(\tilde{x}_0, \tilde{\lambda}_0) & \mu_{\lambda_0}(\tilde{x}_0, \tilde{\lambda}_0) + 1 \end{bmatrix} \right| < 1.$$

Since  $\eta_{x_0}$  and  $\mu_{\lambda_0}$  are always positive, for all stable equilibria it has to be the case that:

$$\eta_{\lambda_0}(\tilde{x}_0, \tilde{\lambda}_0) \mu_{x_0}(\tilde{x}_0, \tilde{\lambda}_0) < \eta_{x_0}(\tilde{x}_0, \tilde{\lambda}_0) \mu_{\lambda_0}(\tilde{x}_0, \tilde{\lambda}_0). \quad (10)$$

**An adjustment process.** Consider the following dynamic adjustment process. Take an initial  $(x_0, \lambda_0)$ . Impose that, starting from it, the cutoff  $x_0$  increases (decreases) proportionally to the benefit (loss) from selecting a large fund size against a small size, given the rest of

the agents are following the strategy described by the two cutoffs  $(x_0, \lambda_0)$ . Similarly, impose that the cutoff  $\lambda_0$  increases (decreases) proportionally to the benefit (loss) from searching in the low attention market versus searching for high attention. This process defines a system of autonomous differential equations as below:

$$\begin{cases} \dot{x}_0(t) = -b \left[ m_1^{x_0(t)} (\mathbb{E}[R(a_1, \lambda) \mid \lambda \geq \lambda_0(t)] - R_0) - m_0^{x_0(t)} (\mathbb{E}[R(a_0, \lambda) \mid \lambda \leq \lambda_0(t)] - R_0) \right] \\ \dot{\lambda}_0(t) = -b \left[ \frac{W_1(x_0(t))}{1-F(\lambda_0(t))} R(a_1, \lambda_0(t)) - \frac{W_0(x_0(t))}{F(\lambda_0(t))} R(a_0, \lambda_0(t)) \right] \end{cases} \quad (11)$$

for some  $b > 0$ . Notice the right hand side of the first part of (11) is  $-b\eta(x_0, \lambda_0)$  and the right hand side of the second part is  $-b\mu(x_0, \lambda_0)$ . One interpretation is that at each point in time a fraction of the population has the chance to readjust their strategies, starting from a state where all agents are following cutoff strategies and taking those strategies as given. An equilibrium of the game -  $(\tilde{x}_0, \tilde{\lambda}_0)$  - is clearly a constant solution to the system. One can then study local stability of such equilibria.

An equilibrium  $(\tilde{x}_0, \tilde{\lambda}_0)$  is locally asymptotically stable if all eigenvalues of the Jacobian:

$$-b \begin{bmatrix} \eta_{x_0}(\tilde{x}_0, \tilde{\lambda}_0) & \eta_{\lambda_0}(\tilde{x}_0, \tilde{\lambda}_0) \\ \mu_{x_0}(\tilde{x}_0, \tilde{\lambda}_0) & \mu_{\lambda_0}(\tilde{x}_0, \tilde{\lambda}_0) \end{bmatrix}$$

have negative real parts.<sup>33</sup> Since the trace of this matrix is:

$$-b \left( \eta_{x_0}(\tilde{x}_0, \tilde{\lambda}_0) + \mu_{\lambda_0}(\tilde{x}_0, \tilde{\lambda}_0) \right) < 0$$

local asymptotic stability of  $(\tilde{x}_0, \tilde{\lambda}_0)$  is implied by:

$$\eta_{\lambda_0}(\tilde{x}_0, \tilde{\lambda}_0) \mu_{x_0}(\tilde{x}_0, \tilde{\lambda}_0) < \eta_{x_0}(\tilde{x}_0, \tilde{\lambda}_0) \mu_{\lambda_0}(\tilde{x}_0, \tilde{\lambda}_0)$$

which is exactly equation (10).

**Proof of Proposition 5.** First, rewrite the system characterizing the vector of equilibrium cutoffs  $(x_0, \lambda_0)$  as:

$$m_1^{x_0} (\alpha \mathbb{E}[R(a_1, \lambda) \mid \lambda \geq \lambda_0] - R_0) - m_0^{x_0} (\alpha \mathbb{E}[R(a_0, \lambda) \mid \lambda \leq \lambda_0] - R_0) = 0 \quad (12)$$

$$\frac{W_1(x_0)}{1-F(\lambda_0)} R(a_1, \lambda_0) - \frac{W_0(x_0, \underline{x})}{F(\lambda_0)} R(a_0, \lambda_0) = 0 \quad (13)$$

---

<sup>33</sup>For details see for example Theorem 2.5 from [Acemoglu \[2008\]](#).

where I have made explicit in (13) the dependence of  $W_0$  on  $\underline{x}$ . In fact, note that  $W_0 = \int_{\underline{x}}^{x_0} m_0^x dG(x)$ , so  $\frac{\partial W_0}{\partial \underline{x}} = -m_0^{\underline{x}} g(\underline{x}) < 0$ . Call  $\Phi(x_0, \lambda_0, \underline{x})$  the left-hand side of (12) and  $\Psi(x_0, \lambda_0, \underline{x})$  the left-hand side of (13). Using the Implicit Function Theorem, one gets that:

$$\frac{\partial \lambda_0}{\partial \underline{x}} < 0 \iff \Phi_{\lambda_0}(x_0, \lambda_0, \underline{x}) \Psi_{x_0}(x_0, \lambda_0, \underline{x}) < \Phi_{x_0}(x_0, \lambda_0, \underline{x}) \Psi_{\lambda_0}(x_0, \lambda_0, \underline{x})$$

which is implied by condition (10). Similarly, for  $x_0$  one gets that:

$$\frac{\partial x_0}{\partial \underline{x}} > 0 \iff \frac{\Phi_{x_0}(x_0, \lambda_0, \underline{x})}{\Phi_{\lambda_0}(x_0, \lambda_0, \underline{x})} \Psi_{\lambda_0}(x_0, \lambda_0, \underline{x}) - \Psi_{\lambda_0}(x_0, \lambda_0, \underline{x}) > 0$$

and, using condition (10) because we are looking at stable equilibria, it follows that the sign of  $\partial x_0 / \partial \underline{x}$  is the same as that  $\Phi_{\lambda_0}(x_0, \lambda_0, \underline{x})$ , which is positive if and only if  $\phi_{\bar{\lambda}}(a_1, a_0, \lambda_0)$  is positive.

The effect on welfare directly follows from the fact that, as  $\lambda_0$  increases, average quality in both submarkets increases. Since VCs choose size to maximize returns, it must be that all of them are better off after the change in  $\lambda_0$ . Hence, for all equilibria,  $V(E, s^*, \sigma^*)$  increases by more than  $v(\underline{x})$ . ■

**Proof of Corollary 1.** The result above establishes the direction of changes in the equilibrium cutoffs  $\lambda_0$  and  $x_0$  due to a marginal change in  $\underline{x}$ . I therefore write the two cutoffs as  $\lambda_0(\underline{x})$  and  $x_0(\underline{x})$ . Notice that realized returns in any equilibrium - call them  $r$  - take values in the set:  $[R(a_0, \underline{\lambda}), R(a_0, \lambda_0(\underline{x}))] \cup [R(a_1, \lambda_0(\underline{x})), R(a_1, \bar{\lambda})]$ . Since, by assumption, the function  $R$  is monotonically increasing in  $\lambda$ , one can fix attention at  $a_0$  and  $a_1$  and respectively define the functions  $R(\lambda; a_0) := R_0(\lambda)$  and  $R(\lambda; a_1) := R_1(\lambda)$ , and their inverse  $R_0^{-1}(r)$  and  $R_1^{-1}(r)$ , mapping values of  $\lambda$  into a return -  $r$  - and viceversa. In the model, within a given submarket, say  $a_0$ , returns are heterogeneous because project's quality is heterogeneous. Their distribution is shaped by the underlying distribution of project's quality: this is the truncation of  $F$  for  $\lambda \in [\underline{\lambda}, \lambda_0(\underline{x})]$  in market  $a_0$  and the truncation of  $F$  for  $\lambda \in [\lambda_0(\underline{x}), \bar{\lambda}]$  in market for attention  $a_1$ . Fix an equilibrium, and consider the distribution of returns. The Cdf of the returns is a step function, denoted  $Y$ , taking the form:

$$Y(r; \underline{x}) = \begin{cases} \frac{W_0(x_0(\underline{x}), \underline{x})}{W_0(x_0(\underline{x}), \underline{x}) + W_1(x_0(\underline{x}))} \frac{F(R_0^{-1}(r))}{F(R_0^{-1}(R(a_0, \lambda_0(\underline{x}))))} & \text{if } r \leq R(a_0, \lambda_0(\underline{x})) \\ \frac{W_0(x_0(\underline{x}), \underline{x})}{W_0(x_0(\underline{x}), \underline{x}) + W_1(x_0(\underline{x}))} & \text{if } r \in (R(a_0, \lambda_0(\underline{x})), R(a_1, \lambda_0(\underline{x}))) \\ \frac{W_0(x_0(\underline{x}))}{W_0(x_0(\underline{x}), \underline{x}) + W_1(x_0(\underline{x}))} + \frac{W_1(x_0(\underline{x}))}{W_0(x_0(\underline{x}), \underline{x}) + W_1(x_0(\underline{x}))} \frac{F(R_1^{-1}(r)) - F(R_1^{-1}(R(a_1, \lambda_0(\underline{x}))))}{F(R_1^{-1}(R(a_1, \bar{\lambda}))) - F(R_1^{-1}(R(a_1, \lambda_0(\underline{x}))))} & \text{o.w.} \end{cases}$$

where  $W_0(x_0(\underline{x}), \underline{x}) = \int_{\underline{x}}^{x_0(\underline{x})} m_0^x dG(x)$ , and  $W_1(x_0(\underline{x})) = \int_{x_0(\underline{x})}^{\bar{x}} m_1^x dG(x)$ . The associated Pdf -  $y$  - is given by:

$$y(r; \underline{x}) = \begin{cases} \frac{W_0(x_0(\underline{x}), \underline{x})}{W_0(x_0(\underline{x}), \underline{x}) + W_1(x_0(\underline{x}))} \frac{f(R_0^{-1}(r))/R_0'(R_0^{-1}(r))}{F(R_0^{-1}(R(a_1, \lambda_0(\underline{x}))))} & \text{if } r \leq R(a_0, \lambda_0(\underline{x})) \\ 0 & \text{if } r \in (R(a_0, \lambda_0(\underline{x})), R(a_1, \lambda_0(\underline{x}))) \\ \frac{W_1(x_0(\underline{x}))}{W_0(x_0(\underline{x}), \underline{x}) + W_1(x_0(\underline{x}))} \frac{f(R_1^{-1}(r))/R_1'(R_1^{-1}(r))}{F(R_1^{-1}(R(a_1, \bar{\lambda}))) - F(R_1^{-1}(R(a_1, \lambda_0(\underline{x}))))} & \text{o.w.} \end{cases}$$

Since  $\partial \lambda_0 / \partial \underline{x} < 0$ , it has to be the case that  $W_0/W_1$  increases, so to satisfy the entrepreneur's indifference condition. Therefore,  $W_0/(W_0 + W_1)$  is now higher. Hence, at  $r = R(a_0, \lambda_0(\underline{x}))$ ,  $Y(r; \underline{x})$  has increased.

Consider now the expectation of  $r$  conditional on being larger than  $R(a_0, \lambda_0(\underline{x}))$ . This is

$$\int_{R(a_0, \lambda_0(\underline{x}))}^{R(a_1, \bar{\lambda})} r dY(r; \underline{x}) / (1 - Y(R(a_0, \lambda_0(\underline{x})); \underline{x})),$$

which is higher the higher is  $\lambda_0(\underline{x})$ . The same is true when conditioning on  $r$  being above any number in the set  $(R(a_0, \lambda_0(\underline{x})), R(a_1, \lambda_0(\underline{x}))]$ . The conditional expectation given that  $r$  is above some  $\hat{r} > R(a_1, \lambda_0(\underline{x}))$  is instead given by:

$$\int_{\hat{r}}^{R(a_1, \bar{\lambda})} r dY(r; \underline{x}) / (1 - Y(\hat{r}; \underline{x})),$$

which is unaffected by the change in  $\underline{x}$  because  $(1 - Y(\hat{r}; \underline{x}))$  is equivalent to:

$$\int_{\hat{r}}^{R(a_1, \bar{\lambda})} dY(r; \underline{x}) = \frac{W_1(x_0(\underline{x}))}{W_0(x_0(\underline{x}), \underline{x}) + W_1(x_0(\underline{x}))} \int_{\hat{r}}^{R(a_1, \bar{\lambda})} \frac{f(R_1^{-1}(r))/R_1'(R_1^{-1}(r))}{F(R_1^{-1}(R(a_1, \bar{\lambda}))) - F(R_1^{-1}(R(a_1, \lambda_0(\underline{x}))))} dr$$

which gives:

$$\frac{\int_{\hat{r}}^{R(a_1, \bar{\lambda})} r dY(r; \underline{x})}{(1 - Y(\hat{r}; \underline{x}))} = \frac{\int_{\hat{r}}^{R(a_1, \bar{\lambda})} r f(R_1^{-1}(r))/R_1'(R_1^{-1}(r)) dr}{F(R_1^{-1}(R(a_1, \bar{\lambda}))) - F(R_1^{-1}(\hat{r}))}$$

Hence, returns conditional on being above any number in  $[(R(a_0, \lambda_0(\underline{x})), R(a_1, \lambda_0(\underline{x})))]$  are strictly higher when  $\underline{x}$  decreases, whereas those conditional on being higher than some  $\hat{r} > R(a_1, \lambda_0(\underline{x}))$  stay constant after the same marginal change.  $\blacksquare$

**Proof of Lemma 4.** It is convenient to rewrite (4) as:

$$\frac{R(a^h, \lambda^*)}{R(a^l, \lambda^*)} = \frac{\gamma_t (1 - F(\lambda^*))}{1 - \gamma_t F(\lambda^*)} \quad (14)$$

To prove existence and uniqueness, note the left-hand side of (14) is continuous and strictly increasing in  $\lambda^*$  by assumption (as  $R$  is continuous in both arguments and assumed to be log-supermodular in this section). The right hand side is continuous and strictly decreasing in  $\lambda^*$ . In particular, notice the left-hand side is positive and finite for all  $\lambda^* \in [\underline{\lambda}, \bar{\lambda}]$ . The right hand side is zero as  $\lambda^* \rightarrow \bar{\lambda}$  and tends to infinity as  $\lambda^* \rightarrow \underline{\lambda}$ .

Continuity of  $\lambda^*$  with respect to  $\gamma_t$  is ensured by continuity of  $F$  and  $R$ . Note finally that the right-hand side of (14) is strictly increasing in  $\gamma_t$ , hence  $\lambda^*$  is strictly increasing in  $\gamma_t$ . ■

**Proof of Lemma 5.** The proof proceeds in two steps.

(Step 1). First, I show that the equilibrium share of VCs in the open contract,  $\gamma$ , is continuous in  $\beta$ . To see why, denote  $\tilde{\delta}$  the discount rate, so that the effective discount rate is

$$\delta = \tilde{\delta}(1 - \beta).$$

Write the VC's indifference condition:

$$\alpha \mathbb{E} \left[ R(a^h, \lambda) \mid \lambda \geq \lambda^*(\gamma) \right] = \left( 1 + \tilde{\delta}(1 - \beta) \right) \alpha \mathbb{E} \left[ R(a^l, \lambda) \mid \lambda \leq \lambda^*(\gamma) \right]$$

and note that continuity is ensured by the continuity properties of  $R$ ,  $g$ , and the endogenous function  $\lambda^*(\gamma)$  which is continuous and monotone in  $\gamma$  by Lemma 4.

(Step 2). I now construct a stationary equilibrium. The fact that there exists a time  $t$  where a share  $-\gamma_t$  of active VCs is in the open contract and therefore a share of active VCs  $-1 - \gamma_t$  is in the short-term contract means that VCs are indifferent and hence there exists a solution to the equation:

$$\alpha \mathbb{E} \left[ R(a^h, \lambda) \mid \lambda \geq \lambda^*(\gamma_t) \right] = \left( 1 + \tilde{\delta}(1 - \beta) \right) \alpha \mathbb{E} \left[ R(a^l, \lambda) \mid \lambda \leq \lambda^*(\gamma_t) \right].$$

A strategy profile induces, in every period  $t$ , two measures: the measure of VCs that are actively searching for projects, denoted  $M_t$ ; and the measure of non-active VCs, denoted  $N_t$ . The former are composed of active VCs survived from period  $t - 1$  and in the open contract, VCs survived from period  $t - 2$  and in the short-term contract, and newborns. The latter is composed of VCs survived at  $t - 1$  and in a short-term contract, therefore still matched to one entrepreneur. Since in every period an equal measure of VCs dies and is born, it has to be the case that:

$$M_t + N_t = 1 \quad \forall t. \quad (15)$$

Construct now a stationary equilibrium where, at every  $t$ , these measure are constant:  $M_t = M$ ,  $N_t = N$ , and  $\gamma_t = \gamma$  by stationarity. Note that in such equilibrium, entrepreneurs sorting will be the same in every period and hence VCs will always be indifferent between the two contracts. If at a given period there are  $M$  active VCs, in the next period there are  $(1 - \beta) M\gamma + (1 - \beta) N + \beta$  active VCs. To make the share of active VCs offering low attention constant, assign  $\xi \in (0, 1)$  newborns to the open contract, so to satisfy:

$$\frac{(1 - \beta) M\gamma + \beta\xi}{(1 - \beta) M\gamma + (1 - \beta) N + \beta} = \gamma. \quad (16)$$

Since  $M$  is assumed constant, it has to be that

$$N = (1 - \beta) M (1 - \gamma). \quad (17)$$

Solving (16) for  $\xi$  and using (17) gives that, provided  $\beta \neq 0$ :

$$\xi = \gamma(1 - N)$$

and indeed  $\xi \in (0, 1)$ .

It remains to show there is a solution to the constant  $M$  induced by the strategy profile. Plug (15) into (17) to get:

$$M = \frac{1}{2 - \gamma - (1 - \gamma)\beta}. \quad (18)$$

Evaluate the solution in (18) at  $\beta = 0$ , in which case  $M = (2 - \gamma)^{-1} \in (0, 1)$ . The solution (18) depends on  $\beta$  also through the endogenous share  $\gamma$ . However, by Step 1,  $M$  is continuous in  $\beta$ . Therefore, there must exist a  $\hat{\beta} > 0$  close enough to zero such that  $M \in (0, 1)$ . ■

**Proof of Proposition 6.** (i). I first show that there is no equilibrium where every manager has the open credit line. Take an equilibrium where  $\gamma = 1$  and consider a manager who deviates to a finite-horizon structure. If there is a  $Q(a^h)$  for which some  $\lambda$  benefits from deviating to  $a^h$ , all types above  $\lambda$  would strictly deviate. This means the manager must expect to attract type  $\bar{\lambda}$ . This is profitable as long as:

$$R(a^h, \bar{\lambda}) > (1 + \delta) \mathbb{E} [R(a^l, \lambda)]$$

which is assumption A5.

(ii). The second part of the Proposition follows from the fact that all VCs are identical, therefore they must be indifferent in an interior equilibrium.

(iii). Finally, by an argument similar to that for part (i), when all managers are in a short-term contract with investors, a deviation to the open credit line must necessarily attract

$\underline{\lambda}$ . This is not profitable as long as:

$$\mathbb{E} \left[ R \left( a^h, \lambda \right) \right] \geq (1 + \delta) R \left( a^l, \underline{\lambda} \right)$$

■

**Proof of Proposition 7.** Recall that the mass of VCs is normalized to one in this section. Since all VCs are alike, ex-ante welfare,  $V$ , in the economy is the expected equilibrium payoff to the VC. Consider first an equilibrium where every VC opts for the finite-horizon fund. Welfare is:

$$V = \alpha \mathbb{E} \left[ R \left( a^h, \lambda \right) \right] < (1 + \delta) \alpha \mathbb{E} \left[ R \left( a^l, \lambda \right) \right]$$

where the second inequality is a consequence of A4 once expectations are taken on both sides. Hence this equilibrium is dominated by a situation where every VC is forced to choose the open credit line. Second, consider welfare from any interior equilibrium. This is given by:

$$V = \alpha \mathbb{E} \left[ R \left( a^h, \lambda \right) \mid \lambda \geq \lambda^* (\gamma) \right] = (1 + \delta) \alpha \mathbb{E} \left[ R \left( a^l, \lambda \right) \mid \lambda \leq \lambda^* (\gamma) \right] < (1 + \delta) \alpha \mathbb{E} \left[ R \left( a^l, \lambda \right) \right]$$

where the second inequality trivially holds since  $\lambda^* (\gamma) < \bar{\lambda}$ , for any  $\gamma \in (0, 1)$ . ■

**Proof Proposition 8.** By Proposition 1, sorting in the subgame must exhibit PAM. Each point in the sequence that forms the equilibrium partition of the set  $[\underline{\lambda}, \bar{\lambda}]$  is a type that must be indifferent between searching in the two markets where types just above and just below are assigned.

Consider a VC with ability  $x$  that is choosing between  $a_i$  (and associated size,  $m_i^x$ ) and  $a_j$  (with associated size  $m_j^x$ ), with  $i > j$  and  $i, j \in I^*$ . This VC will prefer the first option if and only if:

$$m_i^x \left( \alpha \mathbb{E} \left[ R \left( a_i, \lambda \right) \mid \lambda \in A_i^{s*} \right] - R_0 \right) > m_j^x \left( \alpha \mathbb{E} \left[ R \left( a_j, \lambda \right) \mid \lambda \in A_j^{s*} \right] - R_0 \right).$$

Which can be rewritten as:

$$\frac{m_i^x}{m_j^x} > \frac{\alpha \mathbb{E} \left[ R \left( a_j, \lambda \right) \mid \lambda \in A_j^{s*} \right] - R_0}{\alpha \mathbb{E} \left[ R \left( a_i, \lambda \right) \mid \lambda \in A_i^{s*} \right] - R_0}. \quad (19)$$

The right hand side of (19) is independent on  $x$ . The left hand side is continuous and *increasing* in  $x$ . Therefore, if (19) holds for some  $x$ , it will hold for any VC with ability above  $x$ . Moreover, if the same inequality is reverted for some  $x' < x$ , then, by the Intermediate Value Theorem, there exist a level of ability  $\tilde{x} \in [x', x]$  such that the payoff from  $a_i$  and  $a_j$  is the same.

Finally, consider a deviation to some  $a_j \notin I^*$ . Take the closest smaller market in  $I^*$  to  $a_j$ , call it  $a_i$ .

$$a_i := \arg \max_{a_h \in I^* \setminus \{a_h \geq a_j\}} a_h$$

It can be argued that the belief  $\beta(a_j)$  has to place all support on – if any – type  $\lambda_i$ , which, since all markets in between  $a_i$  and  $a_j$  are empty, is also equal to  $\lambda_j$ . Assume not, and first, say that  $\beta(a_j)$  is supported on another type  $\lambda \in \Lambda_h$  with  $a_h \in I^*$  and  $h > j$ . From the partitioned equilibrium,  $\lambda > \lambda_j$ . If this type finds a deviation to  $a_j$  weakly beneficial for at least some  $Q$ , then, the set of  $Q \in [0, 1]$  such that this is true is an interval  $[Q_\lambda, 1]$ , with  $Q_\lambda$

$$Q(a_h) R(a_h, \lambda) = Q_\lambda R(a_j, \lambda)$$

However, the fact that  $R(a_h, \lambda) / R(a_j, \lambda)$  is increasing in  $\lambda$  – by logsupermodularity – implies that,  $\forall \lambda' \in \Lambda_h$  and  $\lambda' < \lambda$ ,

$$Q(a_h) R(a_h, \lambda') < Q_\lambda R(a_j, \lambda').$$

That is, all types in  $\Lambda_h$  lower than  $\lambda$  would strictly benefit from deviating at  $Q_\lambda$ , and, similarly for some  $Q > Q_\lambda$ . Hence the set of  $Q$ s such that  $\lambda$  would deviate – defined as  $Q(\lambda; a_j)$  in Requirement 1 – is a subset of the set of  $Q$ s at which these types would deviate. Because this set is not maximal,  $\beta$  should place no density at  $\lambda$  according to Requirement 1. A contradiction.

Assume instead  $\beta(a_j)$  is supported on some  $\lambda \in \Lambda_h$  with  $a_h \in I^*$  and  $h < j$ . A similar argument applies. In this case,  $\forall \lambda' \in \Lambda_h$  and  $\lambda' > \lambda$ ,

$$Q(a_h) R(a_h, \lambda') < Q_\lambda R(a_j, \lambda').$$

Given what the off-equilibrium deviation attracts, condition (iii) from the Proposition guarantees that no VC offers  $a_j$ .

Naturally, condition (iii) does not have to hold in case the set  $Q(\lambda; a_j)$  is empty for every  $\lambda$ . In that case VCs will not deviate since they will expect the market to be empty of entrepreneurs.

It remains to show that, in case attention can only be in  $\{a_0, a_1\}$ , as in Proposition 3, the sets  $Q(\lambda; a_1)$  and  $Q(\lambda; a_0)$  are not empty in every equilibrium, and therefore the condition is stringent. First, take the case where  $I^* = a_0$ , that is, every VC is offering low attention. In this case, the free entry condition – condition (ii) in Definition 1 – gives:

$$Q^*(a_0) (1 - \alpha) \mathbb{E}_\lambda R(a_0, \lambda) = c$$

and rearranging:

$$Q^*(a_0) = \frac{c}{(1 - \alpha) \mathbb{E}_\lambda R(a_0, \lambda)}.$$

For  $Q(\lambda; a_1)$  to be non-empty, it must be that:

$$Q(1 - \alpha) R(a_1, \lambda) > Q^*(a_0) (1 - \alpha) R(a_0, \lambda).$$

for some  $Q \in [0, 1]$  and some  $\lambda$ . Clearly, this is true for any  $Q > Q^*(a_0)$  and any  $\lambda$ .

Consider now the case where  $I^* = a_1$ , that is, every VC is offering high attention. The free entry condition gives:

$$Q^*(a_1) = \frac{c}{(1 - \alpha) \mathbb{E}_\lambda R(a_1, \lambda)}.$$

To make sure there exists a  $Q$  and a  $\lambda$  that would search in  $a_0$  if the probability to find a match is  $Q$ , set  $Q = 1$  and consider type  $\underline{\lambda}$ . Then, searching in the off equilibrium market is worthwhile if:

$$(1 - \alpha) R(a_0, \underline{\lambda}) > \frac{c}{(1 - \alpha) \mathbb{E}_\lambda R(a_1, \lambda)} (1 - \alpha) R(a_1, \underline{\lambda}).$$

Rearranged, this gives:

$$\frac{(1 - \alpha) \mathbb{E}_\lambda R(a_1, \lambda)}{c} > \frac{R(a_1, \underline{\lambda})}{R(a_0, \underline{\lambda})}$$

which is exactly assumption A3. ■

**Proof Proposition 9.** (i). The proof of part (i) proceeds in two steps.

(Step 1). First, call  $I$  the equilibrium exhibiting lower ratio  $Q_i/Q_j$  for any two  $(i, j)$  with  $i > j$ , with  $II$  being the other equilibrium. Use the superscripts  $I$  and  $II$  to denote the limits of the equilibrium partitions  $X_i$  and  $A_i$  under equilibrium  $I$  and  $II$ . It can be shown that, for any  $i$ ,  $\lambda_i^I > \lambda_i^{II}$ . This is easily seen by looking at the entrepreneur's  $\lambda_i$  indifference condition. Rewrite it as:

$$\frac{R(a_{i+1}, \lambda_i)}{R(a_i, \lambda_i)} = \frac{Q_i}{Q_{i+1}} \quad (20)$$

Condition (20) has to hold under both equilibrium values  $\lambda_i^I$  and  $\lambda_i^{II}$ . Note that, when  $a_{i+1} \notin I^*$ ,  $\lambda_{i+1} = \lambda_i$  by Proposition 8, and (20) still holds because for the empty market, the measure  $Q_{i+1}$  is defined in such a way that the type selected by Requirement 1 – in this case,  $\lambda_i$  – is indifferent between searching in the off-equilibrium market and not deviating from the equilibrium strategy. The left hand side is increasing in  $\lambda_i$  by logsupermodularity. The right hand side is assumed to be larger under equilibrium  $I$ . Therefore,  $\lambda_i^I > \lambda_i^{II}$  for all  $i$ .

(Step 2). Given  $\lambda_i^I > \lambda_i^{II}$ , it can be proven that welfare is higher under equilibrium  $I$ . Notice that, for any  $i$ , average quality is higher under  $I$  at any market, including those that are not offered in equilibrium. That is:

$$\mathbb{E} \left[ R(a_i, \lambda) \mid \lambda_{i-1}^I \leq \lambda \leq \lambda_{i+1}^I \right] > \mathbb{E} \left[ R(a_i, \lambda) \mid \lambda_{i-1}^{II} \leq \lambda \leq \lambda_{i+1}^{II} \right].$$

Since equilibrium requires that VCs select  $a_i$  to maximise total returns, it must be that each one is strictly better off under  $I$  compared to  $II$ . Part (i) is therefore proven.

(ii). For part (ii), similarly call  $I$  the equilibrium exhibiting lower ratio  $W_i/W_j$  for any two  $(i, j)$  with  $i > j$ , with  $II$  being the other equilibrium. Use the superscripts  $I$  and  $II$  to denote

the limits of the equilibrium partitions  $X_i$  and  $A_i$  under equilibrium  $I$  and  $II$ . It can be shown that, for any  $i$ ,  $\lambda_i^I > \lambda_i^{II}$ . To prove this, assume this is not the case. That is, assume that, for at least some  $i$ ,  $\lambda_i^I \leq \lambda_i^{II}$ . First focus on the case where the inequality is strict for some  $i$ . Take the *largest*  $i$  such that this holds. Rewrite the indifference condition for the indifferent type,  $\lambda_i$ , as:

$$\frac{R(a_{i+1}, \lambda_i)}{R(a_i, \lambda_i)} = \frac{W_i}{W_{i+1}} \frac{F(\lambda_{i+1}) - F(\lambda_i)}{F(\lambda_i) - F(\lambda_{i-1})}. \quad (21)$$

When  $\lambda_i$  is lower, the left hand side of (21) decreases (due to logsupermodularity).  $W_i/W_{i+1}$  is higher in equilibrium  $I$  by assumption. Hence,  $(F(\lambda_{i+1}) - F(\lambda_i)) / (F(\lambda_i) - F(\lambda_{i-1}))$  is lower under  $I$ . The numerator is higher under  $I$ , since  $i$  is the largest submarket for which  $\lambda_i^I \leq \lambda_i^{II}$  (this is also true in case  $i = N - 1$  and hence  $\lambda_{i+1} = \bar{\lambda}$ ). Therefore, it must be that  $\lambda_{i-1}^I < \lambda_{i-1}^{II}$ , giving a contradiction. It remains to show that it is impossible that  $\lambda_i^I = \lambda_i^{II}$  for all  $i$ . Assume this is the case. This would imply that under the two equilibria, the left hand side of (19) stays constant, as well as the ratio  $(F(\lambda_{i+1}) - F(\lambda_i)) / (F(\lambda_i) - F(\lambda_{i-1}))$ . Because  $W_i/W_{i+1}$  is not the same under the two equilibria, (21) can not hold and the desired contradiction arises.

Given  $\lambda_i^I > \lambda_i^{II}$ , welfare is higher under equilibrium  $I$ , as proven for part (i). This completes the proof of part (ii).

(iii). Finally, to prove part (iii), assume, in anticipation of a contradiction, that there is at least one  $i$  such that in the dominant equilibrium ( $I$ ), the indifferent VC is smaller than in the dominated equilibrium ( $II$ ); that is, assume for at least one  $i$ ,  $x_i^I < x_i^{II}$ . Take the smallest such  $i$ . This means that the number of vacancies in market  $i$ ,  $W_i$ , is smaller in equilibrium  $I$ , since we have that  $x_i^I < x_i^{II}$  but  $x_{i-1}^I > x_{i-1}^{II}$ . If the amount of vacancies in market  $i + 1$  is larger in equilibrium  $I$ , we immediately get the contradiction since for no adjacent markets it is possible that the ratio  $W_{i+1}/W_i$  is larger in the dominant equilibrium. Therefore, assume  $W_{i+1}$  is smaller under equilibrium  $I$ , meaning that  $x_{i+1}^I < x_{i+1}^{II}$ . Continue with the same logic until either one contradiction arises or it leads to  $x_N^I < x_N^{II}$ . In this case, it has to be that  $W_N$  is larger in the dominant equilibrium, leading to the desired contradiction.

From the result above, one can conclude that, for all  $i$ ,  $x_i^I > x_i^{II}$ . This implies that, at every market, average fund size, which is given by

$$\frac{\int_{x_{i-1}}^{x_i} m_i^x dG(x)}{G(x_i) - G(x_{i-1})}$$

is larger in equilibrium  $I$ , since  $\partial m_i^x / \partial x > 0$ . The fact that total fundraising is larger in equilibrium  $I$  simply comes from the fact that, comparing the two economies, there is a subset of VCs that don't change their strategies; those who do are all VCs in  $x \in [x_i^{II}, x_i^I]$  who switch from a smaller to a larger fund size. ■

## Appendix B. Existence

**Proposition B.1.** *An equilibrium of the game always exists.*

**Proof of Proposition B.1.** The proof proceeds in four steps.

(Step 1). I first show that, given any equilibrium strategy profile, a measure  $E$  such that equilibrium  $Q_i^*$ s satisfy free entry exists. Note that free entry can be rewritten as:

$$\mathbb{E}_\lambda \max \{ Q_0^* (1 - \alpha) R(a_0, \lambda), Q_1^* (1 - \alpha) R(a_1, \lambda) \} = c.$$

Substituting the definitions of  $Q_0^*$  and  $Q_1^*$ ,

$$\mathbb{E}_\lambda \max \left\{ \frac{W_0^*}{E(F(\lambda_0))} (1 - \alpha) R(a_0, \lambda), \frac{W_1^*}{E(1 - F(\lambda_0))} (1 - \alpha) R(a_1, \lambda) \right\} = c.$$

Thus:

$$\mathbb{E}_\lambda \frac{1}{E} \max \left\{ \frac{W_0^*}{F(\lambda_0)} (1 - \alpha) R(a_0, \lambda), \frac{W_1^*}{(1 - F(\lambda_0))} (1 - \alpha) R(a_1, \lambda) \right\} = c. \quad (22)$$

Since by A1  $(1 - \alpha)\mathbb{E}_\lambda R(a_0, \lambda) > c$ , there must exist an  $E \in \mathbb{R}$  small enough such that the left hand side of (22) is larger than the right hand side. As  $E$  approaches infinity, the opposite inequality holds. Therefore, by the Intermediate Value Theorem, an  $E$  that satisfies (22) exists.

(Step 2). I show that any equilibrium VC strategy profile – hence any cutoff  $x_0$  – induces one and only one cutoff in the entrepreneurs' search strategy. For any interior cutoff on the VCs side,  $x_0$ , the indifferent entrepreneur,  $\lambda_0$ , solves equation (3). It is useful to rewrite it as:

$$\frac{R(a_1, \lambda_0)}{R(a_0, \lambda_0)} = \frac{W_0(x_0)(1 - F(\lambda_0))}{W_1(x_0)F(\lambda_0)} \quad (23)$$

The left-hand side of (23) is continuous and strictly increasing in  $\lambda_0$  by assumption (as  $R$  is continuous in both arguments and assumed to be log-supermodular in this section). The right hand side is continuous and strictly decreasing in  $\lambda_0$ . In particular, notice the left-hand side is positive and finite for all  $\lambda_0 \in [\underline{\lambda}, \bar{\lambda}]$ . The right hand side is zero as  $\lambda_0$  tends to  $\bar{\lambda}$  and tends to infinity as  $\lambda_0$  tends to  $\underline{\lambda}$ . Therefore, by the Intermediate Value Theorem, for any given  $x_0$ , there exists a cutoff  $\lambda_0$ , and this cutoff is unique due to monotonicity. Moreover, recall that the ratio  $W_0(x_0)/W_1(x_0)$  is continuous and increasing in  $x_0$ . Therefore, the (unique) function  $\lambda_0(x_0)$  is continuous and increasing. When  $x_0$  tends to  $\underline{x}$ ,  $W_0(x_0)/W_1(x_0)$  tends to zero and the solution  $\lambda_0$  approaches  $\underline{\lambda}$ . If  $x_0$  tends to  $\bar{x}$ ,  $W_0(x_0)/W_1(x_0)$  tends to infinity and  $\lambda_0$  tends to  $\bar{\lambda}$ .

(Step 3). Substitute  $\lambda_0 = \lambda_0(x_0)$  into the VCs indifference condition and observe that, if a solution to the equation below exists, an interior equilibrium  $(\lambda_0, x_0)$  exists.

$$m_1^{x_0} (\alpha \mathbb{E} [R(a_1, \lambda) \mid \lambda \geq \lambda_0(x_0)] - R_0) = m_0^{x_0} (\alpha \mathbb{E} [R(a_0, \lambda) \mid \lambda \leq \lambda_0(x_0)] - R_0).$$

(Step 4). Let me now cover the case where an interior equilibrium does not exist, and therefore, the two functions:

$$m_0^{x_0}/m_1^{x_0}$$

and

$$\phi(a_1, a_0, \lambda_0(x_0)) := \frac{\alpha \mathbb{E}[R(a, \lambda) | \lambda \geq \lambda_0(x_0)] - R_0}{\alpha \mathbb{E}[R(a', \lambda) | \lambda \leq \lambda_0(x_0)] - R_0}$$

do not intersect at any  $x_0 \in [\underline{x}, \bar{x}]$ . Start from the case where  $m_0^{x_0}/m_1^{x_0} > \phi(a_1, a_0, \lambda_0(x_0))$  for all  $x_0 \in [\underline{x}, \bar{x}]$ . Note that the inequality also applies at  $\bar{x}$ ; that is,  $m_0^{\bar{x}}/m_1^{\bar{x}} > \phi(a_1, a_0, \lambda_0(\bar{x})) = \phi(a_1, a_0, \bar{\lambda})$  and note further that  $\phi(a_1, a_0, \bar{\lambda}) = (\alpha R(a_1, \bar{\lambda}) - R_0) / (\alpha \mathbb{E}[R(a_0, \lambda)] - R_0)$ . Consider a candidate equilibrium where every VC is offering low attention. Then, from Proposition 2, by deviating to high attention a VC will attract the highest quality entrepreneur,  $\bar{\lambda}$ . This is not profitable as long as:

$$m_0^x(\alpha \mathbb{E}[R(a_0, \lambda)] - R_0) > m_1^x(\alpha R(a_1, \bar{\lambda}) - R_0).$$

The condition above is satisfied at  $x = \bar{x}$  because of the case we are restricting attention to, and, since  $m_0^x/m_1^x$  is decreasing in  $x$ , will apply to every VC. Therefore, an interior equilibrium where every VC offers  $a_0$  exists.

Assume instead that  $m_0^{x_0}/m_1^{x_0} > \phi(a_1, a_0, \lambda_0(x_0))$  for all  $x_0 \in [\underline{x}, \bar{x}]$ . Note that the inequality also applies at  $\underline{x}$ ; that is,  $m_0^{\underline{x}}/m_1^{\underline{x}} < \phi(a_1, a_0, \lambda_0(\underline{x})) = \phi(a_1, a_0, \underline{\lambda})$  and note further that  $\phi(a_1, a_0, \underline{\lambda}) = (\alpha \mathbb{E}[R(a_1, \lambda)] - R_0) / (\alpha R(a_0, \underline{\lambda}) - R_0)$ . Consider a candidate equilibrium where every VC is offering high attention. Then, from Proposition 2, by deviating to low attention a VC will attract the lowest quality entrepreneur,  $\underline{\lambda}$ . This is not profitable as long as:

$$m_0^x(\alpha R(a_0, \underline{\lambda}) - R_0) < m_1^x(\alpha \mathbb{E}[R(a_1, \lambda)] - R_0).$$

The condition above is satisfied at  $x = \underline{x}$  because of the case we are restricting attention to, and, since  $m_0^x/m_1^x$  is decreasing in  $x$ , will apply to every VC. Therefore, an interior equilibrium where every VC offers  $a_1$  exists. ■

## Appendix C. Posting Equity Share

Assume at the fundraising stage, a VC can also choose the equity share  $\alpha$ . At the sorting subgame, entrepreneurs would then direct their search to submarkets defined by a combination  $(w, x, \alpha)$ . The same considerations that led to reduce the  $(w, x)$  dimension into one – given by  $a(w, x)$  – are valid in this extended model. Consider now the sorting subgame when entrepreneurs can choose a market to search into, based on attention –  $a$  – and on the equity share –  $(1 - \alpha)$  – that funds in that market will guarantee. I start with one intermediate result.

**Lemma C.1.** *In any two equilibrium submarkets  $(a, \alpha)$  and  $(a', \alpha')$  and associated search frictions  $Q$  and  $Q'$ , if  $a = a'$ , then  $Q(1 - \alpha) = Q'(1 - \alpha')$ .*

It is no longer true that search frictions must be the same in two markets where VCs offer the same level of attention; the reason is that a market might be more crowded than another one, but at the same be characterized by a lower  $\alpha$ . What is necessary though, is that, for any two equilibrium submarkets with the same attention the product  $Q(1 - \alpha)$  is the same. If not, entrepreneurs would only search in the submarket where this product is largest. The characterization of the sorting subgame equilibrium then follows from one observation: solving sorting with different shares  $(1 - \alpha)$  offered at different attention levels is equivalent to solving for a modified model where  $\alpha$  is constant but the measure of vacancies at each submarket is readjusted to replicate the differences in  $(1 - \alpha)$ . This is due to the linear nature of the equity contract, which makes the equity share play the same economic role as the probability of matching. It follows from the reasoning above that sorting across attention levels will be positive assortative (under logsupermodularity). Crucially, the same types  $\lambda$  have to search in markets where attention and  $Q(1 - \alpha)$  are the same. Formally:

**Lemma C.2.** *Every equilibrium exhibits PAM between project's quality and attention. The set of types searching in any two equilibrium submarkets  $(a, \alpha)$  and  $(a', \alpha')$  such that  $a = a'$ , and  $Q(1 - \alpha) = Q'(1 - \alpha')$ , coincide.*

I now study the VC's optimal choice in the supergame. From what established above, any equilibrium contract  $(a, \alpha)$  is dominated by a contract  $(a, \alpha')$  where  $\alpha' > \alpha$ , since with the second contract VCs attract the same set of entrepreneurs, but also receive a larger share of the returns. Hence, the following is true:

**Lemma C.3.** *In every equilibrium, VCs that offer the same level of attention will also choose the same equity share.*

The last observation to be made is that the fact that VCs must separate across attention level with the least efficient VCs choosing large unfocused funds, and the efficient ones offering high attention, is robust to allowing for different  $\alpha$ s at different attention levels (see the Proof of Proposition 8 in Appendix A). Therefore the following holds:

**Lemma C.4.** *In every interior equilibrium, more efficient VCs select the  $(a_1, \alpha_1)$  contract, and less efficient VCs select the alternative  $(a_0, \alpha_0)$ .*

It is now possible to characterize equilibria. In this model no bargaining protocol is specified. Thus, given the abundance of entrepreneurs and the search frictions it generates, a VC could always deviate from an equilibrium contract –  $(a_i, \alpha_i)$  – by offering a slightly worse contract –  $(a_i, \alpha_i + \varepsilon)$  with  $\varepsilon$  taken arbitrarily small – and, due to lower search frictions, potentially attract *some* entrepreneur. To circumvent this problem I assume that given the contracts offered by other VCs, a VC can not offer a strictly dominated contract.

**Assumption C.1.** *Given the profile of  $\{(a_0, \alpha_0), (a_1, \alpha_1)\}$  offered in equilibrium, a VC can not offer another contract with  $\alpha > \min\{\alpha_0, \alpha_1\}$ .*

It is easy to see that the conditions that are necessary and sufficient for a vector  $(E, s^*, \sigma^*, \beta)$  to be an equilibrium of the game with constant exogenous  $\alpha$  are necessary conditions of an equilibrium in the extended game too. On top of those, one must ensure that VCs don't deviate to off equilibrium  $(a, \alpha)$ s, which, due to the additional dimension, generates more potential deviations. However, it can be shown that the possibility of posting more favorable financial terms – a lower  $\alpha$  – does not help VCs affect entrepreneurs' selection, and, therefore, no additional constraints need to be imposed for an equilibrium to be fully characterized.

**Proposition C.1.** *The constraint on off equilibrium  $\alpha$ s is never binding. Given the profile of  $(a, \alpha)$  offered in equilibrium, the conditions in Proposition 2 characterize the equilibrium of the extended game.*

**Proof of Proposition C.1.** Consider a candidate equilibrium strategy profile  $(x_0, \lambda_0)$ , and associated submarkets  $\{(a_0, \alpha_0), (a_1, \alpha_1)\}$ . Assume that the conditions in Proposition 2 hold.

Case I. Deviation to  $(a_0, \tilde{\alpha})$ . Entrepreneurs in  $\Lambda_0$  would profitably switch to searching in the new submarket if and only if:

$$Q(1 - \tilde{\alpha})R(a_0, \lambda) > Q_0(1 - \alpha_0)R(a_0, \lambda)$$

which gives:

$$Q > Q_0 \frac{(1 - \alpha_0)}{(1 - \tilde{\alpha})} \equiv \tilde{Q} \quad (24)$$

Entrepreneurs in  $\Lambda_1$  would profitably switch to searching in the new submarket if and only if:

$$Q(1 - \tilde{\alpha})R(a_0, \lambda) > Q_1(1 - \alpha_1)R(a_1, \lambda)$$

which gives:

$$Q > Q_1 \frac{(1 - \alpha_1)R(a_1, \lambda)}{(1 - \tilde{\alpha})R(a_0, \lambda)} \equiv \hat{Q}_\lambda \quad (25)$$

Observe that the threshold defined in (25) is increasing in  $\lambda$ . Therefore, the lowest type in  $\Lambda_1$ , that is type  $\lambda_0$ , has the largest incentive to deviate and is selected by Requirement 1 over the other types in  $\Lambda_1$ . Using the equilibrium indifference condition defining  $\lambda_0$ , one can see that  $\hat{Q}_{\lambda_0} = \tilde{Q}$ . Hence, the off-equilibrium submarket attracts all types in  $\Lambda_0$ . Since by Assumption C.1  $\tilde{\alpha} < \min\{\alpha_0, \alpha_1\}$ , it follows that no VC has a strict incentive to deviate to  $(a_0, \tilde{\alpha})$ .

Case II. Deviation to  $(a_1, \tilde{\alpha})$ . The same argument used for Case I can lead to conclude that a deviation to such contract attracts all types in  $\Lambda_1$ . Again, since the conditions in Proposition 2 hold in this candidate equilibrium, such deviation is not profitable to any VC. ■

Clearly the analysis does not help identifying the equilibrium  $\alpha$ s. In this model, with no additional structure, VCs could coordinate on many different combinations  $\{(a_0, \alpha_0), (a_1, \alpha_1)\}$ . The analysis however clarifies that - due to the linear nature of the equity contract - allowing VCs to commit to a given share  $\alpha$  does not undo the equilibrium sorting that generates in the original model.

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