The Effects of Capital Requirements on Good and Bad Risk-Taking

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Abstract

We study capital requirements in a model in which deposits are held by firms subject to idiosyncratic shocks and the liquidity value of deposits is microfounded. Higher capital requirements reduce excessive risk-taking by banks, but hinder good risk-taking by non-financial firms. The marginal private value of deposits, measured by the deposit liquidity premium in the data, is in general greater than the marginal social value, which is relevant for regulatory purposes. Thus, the optimal capital requirement is higher in comparison to the level derived in the literature, which typically uses deposits in the utility function and in which private and social value of deposits are equalized. Nonetheless, even if the marginal social value of deposits is zero, setting capital requirements too high is suboptimal.

Keywords: deposit insurance, capital requirements, idiosyncratic risk, safe assets
JEL Classifications: E21, G21, G32
1 Introduction

After the 2007–2008 financial crisis, the revision of the regulatory framework of financial intermediaries has become a central topic of discussion by regulators and academics. The Basel III accord has tightened bank regulation with the aim of reducing the likelihood and depth of financial crises. One of the key sets of rules at the center of this debate is capital requirements, namely, limits on the fraction of debt that banks can use to finance their investment. Appealing to the Modigliani and Miller (1958) theorem, Admati and Hellwig (2013) argue that capital requirements should be raised even further to eliminate the moral hazard induced by government guarantees and implicit too-big-to-fail subsidies, and some regulators have made a case for similar rules.\(^1\)

The typical argument against tighter capital requirements is that banks are “special” because their liabilities are valued for their safety and liquidity (DeAngelo and Stulz, 2015). This value is often modeled and measured in a simple and transparent way by including deposits in the utility function, without modeling explicitly what deposits are used for (van den Heuvel 2008, Begenau 2016, Davydiuk 2017). Under this view, raising capital requirements reduces excessive risky lending by banks, but at the cost of reducing the supply of valuable bank deposits. Optimal capital regulation trades off these costs and benefits.

In this paper, we provide a microfoundation of the usefulness of deposits that challenges the regulatory implications derived in models with deposits in the utility function. Deposits in our model are held by firms for precautionary reasons and facilitate firms’ acquisition of their production input. If financial regulation is relaxed and the availability of deposits increases, firms will try to hire more inputs to increase their production. Yet, in general equilibrium, prices of input will increase, partially offsetting the ability of firms to scale up their production. In this setting, the liquidity premium on deposits, which determines deposit holdings by firms, is informative only about the \textit{private} value of deposits, that is, the value of deposits for a firm that takes input prices as given. The welfare-relevant \textit{social} value of deposits depends not only on the liquidity premium but also on the way firms’ input prices react in general equilibrium to changes in regulation, and is lower than the private value of deposits.

Our first main result is that the optimal capital requirement is higher in comparison to models in which the value of deposits arises from a deposits-in-the-utility-function formulation. In these models, the social and private marginal value of deposits are equal to each other, and thus regulators that use these models may over-value deposits by ignoring the general equilibrium effects of raising capital requirements.\(^2\)

\(^{1}\)See for example the Minneapolis Plan discussed by Kashkari (2016).
The second result of our paper is that even if the marginal social value of deposits is zero, there is still a quantifiable trade-off in raising capital requirements, because higher capital requirements can induce greater bad risk-taking by banks. Raising capital requirements has two effects on banks’ assets: holding the return on deposits fixed, it forces banks to finance themselves with more-costly equity, causing them to reduce lending (leverage effect). This is the desired effect of raising capital requirements. However, in equilibrium, raising capital requirements also causes deposits to become more scarce, lowering the return on deposits and thus inducing greater lending by banks (funding effect). Optimal capital regulation can trade off these two forces even in an economy with a zero marginal social value of deposits. However, the optimal capital requirement in this scenario is higher than one would calculate using the deposit premium as a sufficient statistic for the benefits of capital requirements.

Building on a framework introduced by Quadrini (2017), deposits in our model are held by firms subject to uninsurable idiosyncratic shocks to the productivity of their employees. The wage bill must be paid independently of the realization of the shocks, and thus the productivity risk is borne by the firm. The effect of these shocks is to reduce the firm’s willingness to hire workers (i.e., to reduce the firm’s labor demand) in comparison to an economy without idiosyncratic shocks. Safe deposits held by firms reduce the volatility of firms’ assets, so that a high availability of and return on deposits increases firms’ willingness to hire more workers. We call this channel “good risk taking” because increasing labor demand is socially valuable but not fully exploited due to the lack of insurance against idiosyncratic risk.  

Our main contribution is to show that even if tighter capital requirements limit the availability of deposits, the effects on real economic activity and welfare through this channel may be modest. In our model, tighter capital requirements reduce labor demand, but the ultimate impact on welfare depends on what happens to the equilibrium level of employment, which is affected not only by labor demand but also by labor supply. To clarify this point, consider two extreme cases: one with infinitely elastic labor supply and another with fixed labor supply. With elastic labor supply, a reduction in labor demand produces a one-for-one reduction in the equilibrium level of employment and, thus, a large negative impact on consumption and welfare. With fixed labor supply, the only effect is that wages adjust in general equilibrium to equalize demand and supply in the labor market. In this second case, the reduction of deposits does not produce any impact on welfare through the labor market. This is not to say that capital requirements are irrelevant: in addition to affecting the labor market, tightening capital requirements alters the “bad risk taking” by banks. The

\[\text{The fact that non-financial firms in our model hold financial assets is consistent with the well-known large increase in firms’ cash holdings over the last few decades (Bates, Kahle and Stulz, 2009).}\]
key point here is that, in the case of fixed labor supply, capital requirements affect welfare only through banks’ lending decisions, and the marginal impact of deposits on welfare is zero even though the deposit premium is positive.

For moderate values of the Frisch elasticity of labor supply, changing capital requirements affects both wages and employment. Nonetheless, our main message is unaltered: the marginal private value of deposits, which can be measured using the liquidity premium in the data, does not equal the marginal social value of deposits, which is relevant for the welfare analysis of financial regulation. When deciding on the amount of deposits to hold, each firm equalizes the marginal private cost of deposits (i.e., the liquidity premium) to the marginal private benefits (i.e., the reduction in firms’ asset volatility). However, marginal private benefits take the wages paid to employees as given. In contrast, the marginal social benefit of deposits accounts for the fact that wages may adjust in general equilibrium. If wages drop in response to tighter capital requirements, the wage bill of firms decreases, and a bad idiosyncratic shock does not preclude a firm from financing future projects even if it has fewer deposits.\(^3\)

We confirm our intuition in a quantitative analysis. Preliminary results show that the optimal capital requirement is mainly determined by the objective of minimizing banks’ bad risk-taking, and considerations about the value of deposits play a small quantitative role.

The second result of the paper is to show that, even if the marginal social value of deposits is zero because labor supply is fixed, there is still a trade-off inherent in raising capital requirements. This consideration follows from two results. First, firms in our model require a minimum amount of safe deposits to operate. Thus, the total social value of deposits is positive, even though their marginal social value is zero with fixed labor supply, implying that a 100\% capital requirement is not optimal. Second, increasing capital requirements above a certain threshold actually increases banks’ lending. This point, which is also noted by Begenau (2016), follows from the fact that tighter capital requirements restrict the availability of deposits, which in turn reduces the return on deposits. If capital requirements are sufficiently large, the effect of lower funding costs from reducing the return on deposits is larger than the effect of forcing banks to rely more on costly deposits rather than cheaper equity, and bank lending increases. Thus, setting capital requirements too high is not optimal even if the marginal social value of deposits is zero, because such a policy stance exacerbates excessive lending by banks. This second result stands in contrast to the standard view of the literature, which typically emphasizes the reduction in deposits as the only cost of raising

\(^3\)Note that we are discussing the marginal values of deposits (either private or social), rather than their total value. This is because, even if the marginal social value of deposits can be as low as zero in our model, deposits might need to be at or above some minimum threshold to make sure that firms have enough safe assets to operate.
capital requirements. We derive our results using some simplifying assumptions that keep our model tractable and allow us to isolate our channel from other effects. In particular, even if idiosyncratic shocks create heterogeneity across firms’ wealth, the equilibrium in our model depends only on aggregate firm wealth, and other moments of the firm size distribution are irrelevant.

We focus on capital requirements due to their primary role in banking regulation, and we abstract from other policies. However, two remarks are in order. First, our results have implications not only for capital requirements but also for any regulation that might negatively impact the creation of “safe” and “liquid” assets by financial intermediaries, such as the monetary policy tools discussed by Stein (2012). Second, capital requirements are introduced to limit the negative impact of subsidized deposits insurance. We follow a common approach in the literature that imposes deposit insurance and motivates it with its role in preventing runs, as in Diamond and Dybvig (1983), but does not explicitly include runs nor analyze the optimality of such a policy. Nonetheless, we note that fully eliminating deposit insurance might not be optimal in our framework because it would make deposits unsafe, thereby adding rather than reducing the volatility of firms’ assets. We leave the joint determination of optimal deposit insurance and capital requirement regulation to future research. More generally, deposits insurance in our model can also be interpreted as any government guarantee on bank debt, such as the Temporary Liquidity Guarantee Program set up by the Federal Deposit Insurance Corporation (FDIC) in 2008.

We motivate firms’ aversion to idiosyncratic risk with an agency problem, following a literature that has highlighted the importance of these frictions (Panousi and Papanikolaou, 2012; Glover and Levine, 2015, 2017). Firms are owned by households that can fully diversify away their exposure to idiosyncratic risk by holding a well-diversified portfolio of equity. However, each firm is run by a manager who holds only an equity stake in the firm she manages and thus is exposed to the idiosyncratic risk of the firm. As a separate contribution of the paper, we introduce some modeling assumptions that make the model tractable and allow us to ignore managers’ consumption when performing welfare analysis, without altering the implications of the agency friction.

2 Literature Review

Thakor (2014) surveys the vast literature on the effects of bank capital and changes in capital requirements, and notes that much of the work on the effects of bank capital tends to be qualitative or abstracts from general equilibrium. For example, Berger and Bouwman (2013) show that, in the cross-section, large and medium banks with higher capital ratios gain
market share during financial crises. This cannot be a potential benefit of increasing capital requirements, since not everyone can gain market share in general equilibrium. Similarly, Mehran and Thakor (2011) show that better-capitalized banks are more valuable to acquirers, in contrast to theories that predict that they are equally valuable as less-well-capitalized banks (Modigliani and Miller 1958) or less valuable (Diamond and Rajan 2001). Likewise, their empirical results are identified in the cross-section of banks abstracting from general equilibrium effects.

A more recent and growing literature has embedded the analysis of capital requirements into quantitative general equilibrium models. This literature includes van den Heuvel (2008), Christiano and Ikeda (2013), Corbae and D’Erasmo (2014), Nguyen (2014), Gertler, Kiyotaki and Prestipino (2016), Begnau (2016), Begnau and Landvoigt (2017), Davydiuk (2017), and Dempsey (2017). Egan, Hortaçsu and Matvos (2017) develop a structural model of the US banking sector including runs and partial deposit insurance and argue that capital requirements should be at least 18%.

A related paper by Allen et al. (2018) studies the effect of government guarantees on banks’ choices of liquidity and investments in risky projects. Both our paper and that of Allen et al. (2018) point out that higher risk taking can be a good consequence of financial regulation. In their paper, more risk-taking by banks is associated with greater liquidity provision. In our paper, more risk-taking by firms has a positive impact, in general, on employment and output.

Our approach for modeling firms’ risk builds on Quadrini (2017), who also emphasizes the role of bank liabilities for insurance purposes. There are, however, two main differences. First, he focuses mainly on banks’ risk-taking and crises, whereas our focus is on the impact of financial regulation on firms’ risk-taking. Second, we extend his model so that we can reinterpret the agents subject to idiosyncratic risk as large firms run by managers who cannot diversify away idiosyncratic risk because of an agency friction. This extension facilitates the welfare analysis and the comparison of the model with the data, and it represents a separate contribution of our paper. The building block of our model that describes firms can be applied for other questions at the intersection of corporate finance and macroeconomics.

Firms in our model use deposits to self-insure against their idiosyncratic shocks; the availability of other safe assets not tied directly to banks’ balance sheets and capital regulation, such as Treasury bonds, would weaken our channel. However, our results go through so long as the other assets are imperfect substitutes for bank deposits, for example because they are less liquid. van den Heuvel (2018) analyzes a model of both liquidity and capital requirements regulation and finds that households do value the liquidity services provided

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by deposits more than those provided by government debt.

Our paper is also related to the literature that studies financial intermediaries as suppliers of safe assets, such as Stein (2012), Magill, Quinzii and Rochet (2016), and Diamond (2016). This literature builds on the ideas of Gorton and Pennacchi (1990) and Dang et al. (2017), in which the debt of banks is riskless to enhance its liquidity value or to overcome an informational friction. Bank debt is valuable in our model for a related but slightly different reason: there is a demand for securities that are uncorrelated with the idiosyncratic risk of firms.

Finally, our paper relates to a large literature on firms’ exposure to idiosyncratic risks. Smith and Stulz (1985) and Froot, Scharfstein and Stein (1993) derive theories in which firms do not completely hedge away idiosyncratic risks because external financing is costly. Minton and Schrand (1999) provide empirical evidence that firms that are more exposed to idiosyncratic risk invest less and have higher costs of external financing than other firms, consistent with an inability or unwillingness to completely insure against idiosyncratic shocks. In this setting, idiosyncratic shocks are costly to the firm. In addition, we build on the results of Panousi and Papanikolaou (2012) and Glover and Levine (2015, 2017) and we assume that firms are run by managers that are subject to the idiosyncratic risk of the firm they run. Thus, as pointed out by Panousi and Papanikolaou (2012), risk aversion of managers plays a role in shaping firm decisions. In our model, this implies that deposits provide insurance against the idiosyncratic risk, consistent with the finding of Bates, Kahle and Stulz (2009) of a rising trend in firms’ cash holdings since the 1980s that they attribute to an increasing precautionary motive.

3 Model

3.1 Environment

Time is discrete and infinite. There are four main players in the economy: large firms (which we will also refer to simply as “firms”) run by managers, banks, households, and the government. There is also a bank-financed production sector, representing the assets in which banks invest. Before presenting the details of the model, we briefly summarize the main structure.

Banks issue deposits and equity in order to provide loans to bank-financed firms. We keep the modeling of the bank-financed sector as simple as possible because our focus is on the role played by banks’ liabilities as opposed to their lending. Deposits are fully insured by the government. In equilibrium, firms value the safety of deposits, while risk-neutral
households supply all the equity and thus receive the profits from banks.

Firms are owned by households but are run by risk-averse managers. Because of an agency friction, managers act in their own interest rather than in those of shareholders. Crucially, the agency friction motivates managers’ exposures to firm-specific idiosyncratic risk, even though households are able to fully diversify away the idiosyncratic risk by holding a well-diversified portfolio of equity. These assumptions are motivated by the results of Panousi and Papanikolaou (2012), who document that managerial compensation affect firms’ investments in response to idiosyncratic risk, and Glover and Levine (2015, 2017), who calibrate structural models of firm investment and show that managers respond more to firm-specific shocks in comparison to what shareholders would choose in the absence of agency frictions. We model the agency frictions in a simpler way to keep the firms’ building-block of the model tractable and focus our analysis on financial regulation. The simpler approach, however, does not alter the main implication—namely, that idiosyncratic risk affects managers’ and, thus, firms’ behavior. Finally, we use some modeling assumptions so that the welfare of managers can be ignored when performing welfare calculations without limiting the impact of idiosyncratic risk on firms’ behavior.

3.1.1 Firms and managers

There is a continuum of firms that are owned by households. Each firm is run by a manager who behaves in her own interest because of an agency friction and owns an equity stake in the firm.\(^5\)

At time \(t\), firm \(i\) begins with wealth \(x_i^t\), which is deposited in the banking system. We denote the deposits as \(d_i^t\), and thus \(d_i^t = x_i^t\). Deposits pay a gross return \(R_d^t\) at \(t + 1\). The government provides full deposit insurance, so deposits are safe and \(R_d^t\) is known at time \(t\).

The role of the manager is to choose the amount of labor \(l_i^t\) that is hired every period by the firm. The total output produced by the firm is \(z_{t+1}^i l_i^t\), where \(z_{t+1}^i\) is the firm’s productivity, which is subject to an idiosyncratic shock realized at the beginning of \(t + 1\). Crucially, the productivity \(z_{t+1}^i\) is realized after after the manager has chosen the labor input \(l_i^t\) and has committed to pay the wage bill \(w_t l_t\), where \(w_t\) is the wage.

At the beginning of \(t + 1\), the total wealth available to the firm is given by

\[
\Psi_{t+1}^i = \left( z_{t+1}^i - w_t \right) l_i^t + R_d^t d_i^t,
\]

where the first term on the right-hand side denotes the profits obtained by hiring workers

\(^5\)We can microfound the optimal equity stake of the manager by determining the level that makes the manager unwilling to divert resources away from the firm for personal use. However, this would complicate the exposition and would not affect the results, and thus we simply impose the contract exogenously.
(which can be negative if the productivity shock is low) and the second term denotes the gross return on deposits. A fraction $1 - \alpha_{t+1}^i$ of this wealth is retained by the firm, whereas a fraction $\alpha_{t+1}^i$ is paid out as dividends. We allow $\alpha_{t+1}^i$ to be firm-specific and time-varying (and, thus, possibly correlated with the shock $z_{t+1}^i$); however, we will later impose restrictions on the total dividends paid at the economy-wide level. Note that $\alpha_{t+1}^i$ is not chosen by the manager, and in this sense, it can be understood as a part of the contract between the manager and the shareholders. We parametrize the equity stake of the manager by $\kappa$, so that manager’s dividends are $\alpha_{t+1}^i(1 - \kappa)\Psi_{t+1}^i$ and dividends paid to shareholders (i.e., to households) are $\alpha_{t+1}^i(1 - \kappa)\Psi_{t+1}^i$.

We can thus formalize the problem of the manager as follows:

$$V_t^m(x_t^i) = \max_{l_t^i} \beta^m E_t \left\{ \theta \log c_{t+1}^i + V_{t+1}^m(x_{t+1}^i) \right\}$$

(1)

where

$$d_t^i = x_t^i,$$

$$x_{t+1}^i = \left(1 - \alpha_{t+1}^i\right) \left[ (z_{t+1}^i - w_t) l_t^i + R_t^d d_t^i \right],$$

$$c_{t+1}^i = \alpha_{t+1}^i \kappa \left[ (z_{t+1}^i - w_t) l_t^i + R_t^d d_t^i \right],$$

$$\pi_{t+1}^i = \alpha_{t+1}^i (1 - \kappa) \left[ (z_{t+1}^i - w_t) l_t^i + R_t^d d_t^i \right],$$

(2) (3) (4) (5)

$\theta > 0$, and $\beta^m < 1$ is the discount factor of the manager. The term $x_{t+1}^i$ in Equation (3) is the wealth of the firm after paying dividends. The term $c_{t+1}^i$ in Equation (4) is the consumption of the manager, which produces a utility value $\theta \log c_{t+1}^i$. The term $\pi_{t+1}^i$ in Equation (5) describes the dividends paid by firm $i$ to shareholders (i.e., to households).

Since the manager hires workers $l_t^i$ before the realization of the productivity $z_{t+1}^i$ (and thus chooses the wage bill $w_t l_t^i$ before knowing $z_{t+1}^i$), the manager’s consumption fluctuates over time. For any given choice of $l_t^i$, a high value of $z_{t+1}^i$ implies that firms’ wealth will be high in $t + 1$, and dividends and manager’s consumption will be high too. The vice versa holds as well.

Our model is consistent with some of the evidence in Opler et al. (1999). First, riskier cash flow (i.e., higher variance of $z_{t+1}^i$) implies that the firm is willing to hold more cash. Second, firms that do well (i.e., firms hit by a sequence of good values of $z_{t+1}^i$) accumulates cash internally, and firms that experience losses (i.e., firms with bad realizations of $z_{t+1}^i$) experience decreases in cash. Third, the flexible process for $\alpha_{t+1}^i$ can be chosen so that
payout to shareholders are consistent with the data.

A note on timing and notation: all variables indexed with a $t$ subscript are known to agents at the beginning of time $t$ when they make decisions. Thus, for the manager, the key unknown is the productivity $z_{i,t+1}^i$. Firm $i$’s idiosyncratic output $z_{i,t+1}^i$ depends on future productivity $z_{i,t+1}$ and occurs at the beginning of period $t+1$, immediately after the shocks are realized.

After solving the problem of managers, we impose parametric restrictions on $\theta$ and $\kappa$ which imply that managers’ consumption is arbitrarily small and all dividends will be paid to shareholders, while at the same time preserving the implications of the agency friction on firms’ behavior.

### 3.1.2 Banks and bank-financed firms

Banks live for a single period: each bank is set up at time $t$ and is liquidated at the beginning of time $t+1$. This assumption is made without loss of generality because our model does not include adjustment costs on banks’ size nor costs to raise equity.

Each newly created bank receives an amount $n_t$ of net worth from its shareholders (i.e., households). Then, the bank collects deposits $d_t$ and uses the resources $n_t + d_t$ to purchase physical capital $k_t$. In the economy as a whole, capital accumulates endogenously, similar to a standard neoclassical growth model.

At the beginning of $t+1$, banks are hit by an idiosyncratic quality shock $\varepsilon$, with $E\{\varepsilon\} = 1$. That is, after the shock, the stock of capital held by a particular bank is $\varepsilon k_t$; the total stock of capital in the banking sector as a whole is unchanged, because the shock $\varepsilon$ is idiosyncratic. Banks lend the physical capital $\varepsilon k_t$ to bank-financed firms, which then return the undepreciated fraction $1 - \delta$ of the physical capital plus a return $r_{t+1}$ to the banks. Thus, profits are given by the cash flow $\varepsilon k_t (1 - \delta + r_{t+1})$ net of the repayment $R^d_t d_t$ to depositors, where $R^d_t$ is the gross return on deposits. Profits are bounded below by a limited liability constraint; that is, banks with negative profits pay zero to shareholders.

Banks face a capital requirement that limits their ability to raise deposits. Formally, their equity ratio $n_t/k_t$ must be weakly larger than the regulatory requirement $\zeta$.

We can now formalize the problem of banks. Given $n_t$, the bank’s problem is

$$\max_{k_t,d_t} E_t \int \left\{ \varepsilon k_t (1 - \delta + r_{t+1}) - R^d_t d_t \right\}^+ dF_{t+1}(\varepsilon)$$

s.t.

$$k_t = d_t + n_t$$

$$n_t \geq \zeta k_t$$

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where (8) reflects the capital requirement, \( \{ \cdot \}^+ = \max \{ \cdot, 0 \} \) is the positive part of bank’s profits, \( F_{t+1} (\cdot) \) is the CDF of banks’ idiosyncratic productivity shocks, which may vary over time, and the expectation is taken over the distribution of \( A_{t+1} \), the productivity of bank-financed firms (defined below). The deposits at banks that receive a low value of the productivity shock \( \varepsilon \), such that \( \varepsilon k_t (1 - \delta + r_{t+1}) < R_{t+1}^d d_t \), are fully repaid to depositors thanks to the deposit insurance intervention.

When shareholders invest in banks’ equity, they invest in a mutual fund that diversifies its holdings of equity over all banks in the economy. Investments in bank equity are exposed to the aggregate risk in \( A_{t+1} \). In addition, the distribution of \( \varepsilon \) affects the return on equity in the sense that only banks hit by a sufficiently large shock \( \varepsilon \) pay positive returns to shareholders. The realized return on equity is given by

\[
R_{t+1}^E = \frac{1}{n_t} \int \left\{ \varepsilon k_t (1 - \delta + r_{t+1}) - R_{t+1}^d d_t \right\}^+ dF_{t+1} (\varepsilon). \tag{9}
\]

Equation (9) implies that \( \varepsilon_{t+1} \), the highest value of \( \varepsilon \) at which banks default on their depositors, is implicitly defined as:

\[
R_{t+1}^d d_t = \varepsilon_{t+1} k_t (1 - \delta + r_{t+1}). \tag{10}
\]

From a mathematical perspective, the problem of banks can be solved as follows. Because of the assumption of constant returns to scale, and because in equilibrium the return on deposits will be less than the return on banks’ capital, the capital requirement (8) is always binding. Given \( n_t \), (8) can be used to compute the size \( k_t \) of the banks’ assets, and then the budget constraint (6) can be used to solve for deposits \( d_t \). Finally, the equilibrium value of \( n_t \) is determined such that (10) holds, given the return on equity demanded by banks’ shareholders (i.e., households) in equilibrium.

Subsidized deposit insurance creates an incentive for banks to lower their lending rate \( r_{t+1} \), thereby increasing the amount of physical capital in the economy. The logic of this result is the usual combination of limited liability and a lack of responsiveness of the deposit rate \( R_{t+1}^d \) because of deposit insurance. The overaccumulation of capital due to the distortion of subsidized deposit insurance entails a welfare loss. The objective of capital requirements is to reduce this overaccumulation.

In principle, we could allow banks to choose the riskiness of their balance sheet, as in van den Heuvel (2008) and Begenau (2016), but we conjecture that this would strengthen our results. If banks could choose the riskiness of their balance sheet, subsidized deposit insurance would make banks riskier, and capital requirements would have the objective of
reducing this effect as well. Thus, if the social value of deposits is small, capital requirements should be set even higher, not only to reduce the overaccumulation of physical capital but also to restrain the risk-taking of banks.

We close this section by describing the problem of bank-financed firms. These firms rent capital $k_t$ from banks and use it for production, according to the production function $A_{t+1}k_t^\gamma$, with $\gamma \in (0, 1)$. Thus, their profits $\pi_{t+1}^{\text{small}}$ are obtained by maximizing

$$\pi_{t+1}^{\text{small}} = \max_{k_t} A_{t+1}k_t^\gamma - r_{t+1}k_t,$$

which implies the first-order condition

$$A_{t+1}k_t^{\gamma-1} = r_{t+1}.$$ (12)

Since bank-financed firms produce using a decreasing return to scale technology, their profits $\pi_{t+1}^{\text{small}}$ are positive and are rebated lump-sum to households. Although $k_t$ is chosen at time $t$ before $A_{t+1}$ is known, $r_{t+1}$ depends on $A_{t+1}$ through equation (12).

### 3.1.3 Households

Households are infinitely-lived agents with linear utility of consumption $c_t$ and convex disutility of labor supply $l_t$. This quasi-linear specification allows us to easily characterize their choices. Households supply the labor used by firms and the equity to banks, and they earn the profits generated by large firms, banks, and bank-financed firms.

Households’ utility is given by

$$E \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \right\} = E \left\{ \sum_{t=0}^{\infty} \beta^t \left[ c_t - \beta \chi \frac{l_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right] \right\}$$

where $\chi > 0, \eta > 0$. Households choose labor at time $t$, but don’t provide that labor until the beginning of period $t+1$ when production actually occurs; thus we discount the disutility of labor chosen at $t$ with and additional $\beta$ in equation (13).

A household that starts with wealth $a_t$ solves the problem

$$V_t^h (a_t) = \max_{c_t, l_t, n_t} c_t - \beta \chi \frac{l_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} + \beta E_t \left\{ V_{t+1}^h (a_{t+1}) \right\}$$

subject to the budget constraint

$$c_t + n_t \leq a_t$$ (14)
and to an upper bound on the amount of hours worked, $l_t \leq \bar{l}$. Wealth at $t + 1$ is given by

$$a_{t+1} = w_t l_t + n_t R_{t+1}^E + \int \pi_{t+1}^i di + \pi_{t+1}^{small} - T_{t+1}. \quad (15)$$

At time $t$, the household chooses how to allocate its wealth $a_t$ between consumption $c_t$ and investment in bank equity $n_t$, and it chooses the labor supply $l_t$ taking as given the wage $w_t$. At $t + 1$, the wealth is the sum of its labor income $w_t l_t$, the gross return on banks' equity $n_t R_{t+1}^E$, the profits $\int \pi_{t+1}^i di$ and $\pi_{t+1}^{small}$ distributed by large and bank-financed firms, net of lump-sum taxes $T_{t+1}$.

Finally, note that households’ choice of $n_t$ affects the accumulation of physical capital $k_t$, given banks’ budget constraint (7) and firms’ deposits (2).

### 3.1.4 Government

The government taxes households in order to ensure that depositors at failed banks at the beginning of $t + 1$ are made whole. The government seizes output at failed banks (who return zero to their equity-holders) to partially defray the expenses of paying back depositors.

The total amount of tax to be collected is

$$T_{t+1} = \int_{-\infty}^{\xi_t} \left[ \frac{R_{t+1}^d}{\text{owed to depositors}} + \varepsilon k_t (1 - \delta + r_{t+1}) \right] dF_{t+1} (\varepsilon). \quad (16)$$

The amount of taxes to be collected depend on the realization of the aggregate shock $A_{t+1}$, which affects both the realized return on equity and the fraction of banks that default in equilibrium.

### 3.2 Equilibrium Definition

Given initial conditions $a_0$ and $x_i^0$ for all $i$, and exogenous stochastic processes for $A_t$ and $\{z_t^i\}$, equilibrium is a collection of firm policies, bank policies, households policies, and government taxes such that

1. Firms’ deposits $d_t^i$, managers’ consumption $c_t^i$, and managers’ choice for labor demand $l_t^i$ solve (1);
2. Banks’ choices for capital $k_t$ and deposits $d_t$ solve their problem (6);
3. Bank profits are returned to households holding bank equity through the return on equity given in equation (9);
4. The government taxes households lump-sum and uses the proceeds to pay depositors at failed banks according to equation (16).

5. Households’ choices for supply of labor $l_t$ and net worth $n_t$ maximize their utility (13).

6. The wage $w_t$ and the return on deposits $R^d_t$ clear the labor and deposit markets, respectively.

### 3.3 Agents’ choices and aggregation

This section discusses the optimality conditions that characterize the choices of firms, households, and banks. We show that firms’ choices can be easily aggregated despite the heterogeneity in their wealth. In addition, we impose a parametric restriction that implies that the importance of managers vanishes for welfare calculations but the implications of the agency frictions are not affected.

We begin the analysis by discussing the solution to the managers’ problem. The following proposition greatly simplifies the analysis by allowing us to aggregate easily across managers. All proofs are in Appendix A.

**Proposition 1.** Manager $i$’s optimal choice of labor is $l^i_t = \phi_t x^i_t$, where $\phi_t$ is independent of $x^i_t$ and satisfies the following first-order condition:

$$0 = E_t \left\{ \frac{z^i_{t+1} - w_t}{\Delta^i_{t+1}} \right\},$$

where

$$\Delta^i_{t+1} \equiv \left( z^i_{t+1} - w_t \right) \phi_t + R^d_t. \quad (18)$$

The key result of Proposition 1 is the first-order condition (17), which governs firms’ decisions to hire workers. The firm’s labor demand is lower in comparison to an economy in which the manager would be able to diversify away its exposure to the firms’ idiosyncratic risk. As a result, we will refer to the willingness of firms to hire workers as the “good” risk-taking decision because any force that create an incentive for firms to hire more workers brings the economy closer to the first best.

The result of Proposition 1 is independent of the equity stake $\kappa$ of the manager, provided that $\kappa > 0$. To simplify the welfare analysis, we impose that $\kappa$ is arbitrarily small so that consumption of managers is small as well. Under this assumption, we can perform welfare analysis of capital requirements simply by studying the welfare of households. We also impose some restrictions on $\theta$ (i.e., the parameter that affect the utility of managers). However, this
is done mostly for technical reasons, that is, to make sure that the managers’ problem is well-behaved even in the limit as $\kappa$ is small.

**Proposition 2.** Let $\theta = \eta_1 \kappa^{\eta_2}$, where $\eta_1$ and $\eta_2$ are any two constant satisfying $\eta_1, \eta_2 > 0$. If $\kappa \downarrow 0$, $\kappa > 0$, manager $i$’s optimal choice in Proposition 1 is not affected, manager $i$’s consumption converges to $c_{t+1}^i \downarrow 0$, and manager’s value function converges to $V_{t}^{m} \downarrow 0$. In addition, profits distributed to households become:

$$
\lim_{\kappa \downarrow 0, \kappa > 0} \pi_{t+1}^i = \alpha_{t+1}^i \left[ \left( z_{t+1}^i - w_t \right) \phi_t + R_{t}^{d} \right] x_t^i.
$$

The key result of Proposition 2 is that the importance of managers vanishes for welfare purposes (i.e., their consumption converges to zero) but, at the same time, the key first-order condition that governs their labor demand, (17), is unchanged. The implication of Proposition 2 is that we can just focus on the welfare of households when evaluating financial regulation, but at the same time the model exhibits a behavior driven by an agency friction. In addition, when performing a quantitative analysis, we can map the firms in the models to firms of any size in the data, including large firms, for which data are readily available.

Next, we characterize total firms’ labor demand. Since the wealth $x_t^i$ does not enter the first-order condition (17), labor demand does not depend on the distribution of $x_t^i$ across firms. Thus, total labor demand $l_t$ is

$$
l_t = \int l_t^i di
$$

$$
= \int \phi_t x_t^i di
$$

$$
= \phi_t X_t,
$$

(19)

where $X_t$ is the total wealth of firms, $X_t = \int x_t^i di$.

Similarly, we can solve for the law of motion of firms’ wealth. To do so, we first impose the restriction

$$
\int \alpha_t^i di = 1 - \beta \quad \text{for all } t
$$

(20)

and we also assume that $\alpha_{t+1}^i \perp x_t^i$. These assumptions allow us to obtain a benchmark result in which Modigliani-Miller holds if we shut down all shocks and deposit insurance (i.e., the return paid by banks on deposits and equity are the same; see Section 4.1). Then, aggregate wealth next period across all firms (after dividends are paid) is

$$
X_{t+1} = \beta \left[ (z_{t+1} - w_t) \phi_t + R_{t}^{d} \right] X_t,
$$

(21)
Next, we turn to the problem of households. The linear utility from consumption implies that their value function is linear as well, that is, \((V^h)'(a) = 1\). Thus, their labor supply curve is given by

\[
 w_t = \chi (L_t)^{\frac{1}{\gamma}}
\]

and the supply of banks’ net worth \(n_t\) is fully elastic (i.e., they are willing to supply any amount) provided that the expected return on equity \(E_t \{ R_{t+1}^E \}\) satisfies

\[
 E_t \{ R_{t+1}^E \} = \frac{1}{\beta}.
\]

Finally, the choices of banks are given by (7), (8) and

\[
 E_t \left\{ \int_{\xi_{t+1}}^{\infty} \varepsilon (1 - \delta + r_{t+1}) dF_{t+1} (\varepsilon) \right\} = \zeta E_t \{ R_{t+1}^E \} + (1 - \zeta) \Pr \{ \varepsilon \geq \xi_{t+1} \} R_t^d.
\]

which is obtained by combining (9) with (7), (8), and with the first-order condition of households in (23). Equation (24) equalizes the benefits and costs of purchasing an extra dollar of physical capital for a bank. The marginal benefit is the gross return \(\varepsilon (1 - \delta + r_{t+1})\) only in the states in which the bank is not in default, that is, for \(\varepsilon \geq \xi_{t+1}\). The marginal cost of financing the purchase corresponds to the cost of raising more deposits and more equity. A fraction \(\zeta\) of the purchase requires equity, for which households require a return \(E_t \{ R_{t+1}^E \}\). The remaining fraction \(1 - \zeta\) can be financed with deposits, which in turn requires a return \(R_t^d\). The bank internalizes the cost of depositors only in the states of the world in which it remains solvent, which happens with probability \(\Pr \{ \varepsilon \geq \xi_{t+1} \}\).

### 3.4 Welfare: definition

Before delving deeper into the analysis, we clarify the concept of welfare that we will employ to evaluate capital requirements. Under the assumption introduced in Proposition 2, managers’ consumption is negligible and thus we can ignore them for the purpose of performing welfare calculations.

We define the welfare-relevant criterion to be the expected present-discounted value of

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6Notice that households who choose labor \(l_t\) in period \(t\) receive their labor income at time \(t+1\), and that is also when they receive their disutility from labor.
households’ utility, given the initial conditions \( a_0 \) and \( x_i^0 \) for all \( i \):

\[
W = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ c_t - \beta \chi \frac{l_{t+1}}{1 + \frac{1}{\eta}} \right] \right\}. \tag{25}
\]

Note that welfare is equal to the value function of households, \( V_0^h(a_0) \).

## 4 Results

This section derives the main theoretical results of the paper. As a preliminary analysis, we characterize the results in the benchmark model in which we shut down all shocks (Section 4.1). In this case, we show that Modigliani-Miller holds, in the sense that the equilibrium is independent of how banks’ assets are financed. We then study the effects of capital requirements on firms’ good risk taking (Section 4.2) and banks’ bad risk taking (Section 4.3). Finally, in Section 5, we provide numerical examples to highlight how the good risk taking and bad risk taking are traded off in the determination of the optimal capital requirements.

### 4.1 Benchmark with no shocks: Modigliani-Miller

To begin with, we characterize the equilibrium in a version of the model in which we shut down all shocks—both the idiosyncratic shocks to entrepreneurs and banks, and the aggregate productivity shock. The objective is to establish a benchmark that can be used as a comparison for the analyses that follow.

An implication of shutting down the shocks to banks is that there is *de facto* no deposit insurance. This is because banks’ profits are fully deterministic and thus no bank fails in equilibrium, implying that deposit insurance disbursements are zero.

In this benchmark scenario, Modigliani-Miller holds in the sense that the equilibrium is independent of how banks’ assets are financed. To overcome the indeterminacy of Modigliani-Miller, we assume that capital requirements are imposed anyway by the regulator, even though there is no deposit insurance disbursement, and that they are satisfied with equality: \( n_t / k_t = \zeta \). We view this result as a “check” that our framework allows for the Modigliani-Miller theorem to hold once we shut down deposit insurance and the uninsurable idiosyncratic risk of firms.

**Proposition 3.** (*Benchmark equilibrium without any shock*) Suppose \( A_{t+1} = A \) is not random, \( z_{t+1}^i = 1 \), and \( \varepsilon = 1 \). Given \( \zeta \), the equilibrium is characterized by prices \( R^d = R^E = 1 / \beta \),
w = 1; banking variables \( k = \left[ \left( \frac{1}{\beta} - (1 - \delta) \right) \frac{1}{\lambda^2} \right]^{-\frac{1}{1 - \delta}}, d = k (1 - \zeta), \) and \( n = \zeta k; \) firms’ wealth \( X = k (1 - \zeta) \) and labor \( l = (1/\chi)^\eta; \) taxes \( T = 0; \) and consumption of households \( c = (1/\chi)^\eta + Ak^\gamma - \delta k. \)

### 4.2 Capital requirements and good risk-taking

This section presents the main result about the interaction between capital requirements and firms’ good risk-taking. We show that the effects of modifying capital requirements differ dramatically depending on how wages adjust in response to the policy change. The response of wages is in turn related to the Frisch elasticity of labor supply, which is equal to the parameter \( \eta \) in our model. To clarify this point, we consider two limiting cases: one in which the elasticity of labor supply converges to infinity (i.e., households have linear disutility from labor), and one in which it converges to zero (i.e., households supply a fixed amount of labor).

Tightening capital requirements reduces deposits, which in turn makes it harder for firms to self-insure against idiosyncratic risk.\(^7\) In response, firms reduce labor demand. The extent to which this change in labor demand produces movement in wages \( w_t \) or labor \( l_t \) depends on the Frisch elasticity of labor supply. If the Frisch elasticity is infinite, equation (22) implies that households’ labor supply is flat at the wage \( w_t = \chi. \) As a result, any change in labor demand is transmitted one-for-one into changes of the equilibrium value of labor, \( l_t. \) In contrast, if the Frisch elasticity of labor supply is zero, labor supply is completely inelastic, and thus changes in labor demand produces only changes in wages, but no changes in labor market quantities.

We summarize this result in the next proposition. To simplify the exposition, we shut down aggregate risk (i.e., \( A_t = A \) for all \( t \)) and we focus on a comparison across steady states, denoting the steady-state value of endogenous variables by dropping the time subscript.

**Proposition 4. (Capital requirements and good risk taking)** Assume that there exist an equilibrium with \( l > 0. \) Then:

- If the Frisch elasticity of labor supply is \( \eta \to \infty, \) then \( \partial w / \partial \zeta = 0 \) and \( \partial l / \partial \zeta < 0; \)
- If the Frisch elasticity of labor supply is \( \eta \to 0, \) then \( \partial w / \partial \zeta < 0 \) and \( \partial l / \partial \zeta = 0. \)

\(^7\)In the Appendix, we prove that \( \partial d / \partial \zeta < 0 \) for all \( \eta \) and in the limit as \( \eta \) goes to either zero or infinity.
The first implication of Proposition 4 is that the marginal social value of deposits differs from the marginal private value for all $\eta < \infty$. The marginal private value of deposits from the point of view of each firm is implicitly computed by taking the wage as given, but the social value for the whole firm-sector accounts for the fact that the wage may adjust in general equilibrium as the supply of deposits changes. To clarify this point, consider the limiting case in which the labor supply is fixed (i.e., $\eta \to 0$). An increase in the availability of deposits—due for instance to lower capital requirements—triggers an increase in the wage that exactly offsets the private benefits of the additional deposits. Thus, in this case, changing capital requirements does not affect firms’ good risk taking or the equilibrium value of output produced by firms. This is not to say that capital requirements do not produce any effect at all. As we discuss in the next section, even with fixed labor supply, capital requirements affect banks’ risk taking and thus the size of the banking sector. The key point here is just about the difference between private and social values of deposits for the firms that hold such deposits.

The second takeaway from Proposition 4 is that the existence of a liquidity premium on deposits in the data is not sufficient to conclude that deposits have a positive marginal social value. In our model, deposits display a positive liquidity premium (i.e., $1/\beta - R^d > 0$) even if they might have a zero marginal social value. This is because the return on deposits $R^d$ is determined by the firms’ first-order conditions, which account only for private marginal values. This result stands in contrast with models in which deposits enter the utility function. These theories imply that marginal and social values of deposits are the same, and thus a higher stock of deposits is always beneficial.

The last key message is that the existence of a wedge between marginal and social values of deposits has crucial implications for determining the optimal capital requirements. If deposits’ marginal social value is indeed less than private values, the availability of an additional dollar of deposits is not that important. As a result, capital requirements can be set at a somewhat high level to limit the negative effects of subsidized deposit insurance on banks’ bad risk taking (see next section). In contrast, theories based on deposits in the utility function may overemphasize the welfare importance of deposits and, thus, might suggest a level of capital requirements that is too low, in comparison to the optimal one.

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8This reasoning requires deposits to be above a certain threshold, which depends on parameters, so that firms hire workers in equilibrium; hence, we require $t > 0$ as an assumption of the proposition. If the quantity of deposits is too small, firms cannot insure against idiosyncratic risk and thus they might decide not to hire any workers.
4.3 Capital requirements and bad risk-taking

We now clarify the effects of deposit insurance and capital requirements on banks’ choices. The results in this section are similar to those obtained by other papers in the literature (e.g., Begenau (2016) and Davydiuk (2017)).

Tightening capital requirements produce two effects on the lending rate, which we label leverage effect and funding effect. The leverage effect is the standard increase of the lending rate that is often emphasized in policy debates. This effect forces banks to reduce leverage (i.e., to be financed proportionally more with equity). Since equity is more costly than deposits, banks react by charging a higher lending rate $r_{t+1}$, and the amount of physical capital in the economy drops. The funding effect is related to how the deposit rate changes in response to a modification of capital requirements, as noted by Begenau (2016). As tighter capital requirements make deposits become more scarce, depositors are willing to accept a lower return $R_d$. Such a lower return reduces the cost for banks to fund an additional dollar of loans, and thus reduces the lending rate $r_{t+1}$ and increases physical capital in the economy.

We emphasize that the funding effect exacerbates the overaccumulation of physical capital triggered by subsidized deposit insurance; therefore, this effect has a negative impact on welfare, while the leverage effects reduces physical capital and improves welfare. Which effect dominates depends (among other things) on the current level of capital requirements. We summarize the effects of increasing capital requirements in the next proposition.

Proposition 5. (Capital requirements and bad risk taking) Increasing the capital requirement $\zeta$ produces two effects:

- **Leverage effect.** Fixing $R^d$, banks’ debt-to-equity ratio $d/n$ decreases and banks’ lending rate $r$ increases; this effect reduces the equilibrium value of physical capital and thus the size of the banking sector;

- **Funding effect.** The deposit rate $R^d$ weakly decreases; this forces put a downward pressure on the banks’ lending rate $r$, generating a weak increase in the equilibrium value of physical capital and thus the size of the banking sector.

An implication of Proposition 5 is that the effect of capital requirement on banks’ lending is ambiguous, as noted by Begenau (2016). Nonetheless, there is one extreme case in which we can unambiguously sign the effect of changing capital requirements, and such a case is relevant in conjunction with the analysis of good risk-taking: if the demand for deposits

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9This is the case if the demand for deposits is downward sloping in the cost of deposits (i.e., in the inverse of the interest rate, $1/R^d$).
is infinitely elastic, then such a demand pins down a unique return on deposits $R^d$ that is independent of the amount of deposits available in equilibrium.\textsuperscript{10} In this extreme case, the funding effect described in Proposition 5 is zero, and thus tightening capital requirements unambiguously reduces banks’ lending.

In our model, the elasticity of banks’ deposits demand is related to the Frisch elasticity of labor supply, $\eta$. To clarify this statement, we consider again the two extreme cases of infinitely elastic and fixed labor supply (i.e., $\eta \to \infty$ and $\eta \to 0$, respectively). Fully elastic labor supply implies that $R^d$ does not vary with capital requirements, so that only the leverage effect of Proposition 5 obtains and raising capital requirements unambiguously reduces bad risk-taking. Of course, in this case raising capital requirements does reduce good risk-taking by firms, so there is still an optimal capital requirement that balances costs and benefits.

When labor supply is fully elastic (i.e., $\eta \to \infty$), the wage is fixed as we discussed in the previous section (see Proposition 4). A tightening of capital requirements that reduces deposits implies that firms hire fewer workers. Because wages are fixed, so is the benefit to the firm of insuring against the productivity shock by holding deposits. Thus, in steady-state equilibrium, the insurance benefit of deposits is constant as well, that is, it does not vary with capital requirements. Since the benefits of insurance is given by the liquidity premium on deposits $1/\beta - R^d$, and $1/\beta$ is fixed, then $R^d$ does not vary in steady-state with the capital requirements as well.\textsuperscript{11}

If instead the labor supply is fixed (i.e., $\eta \to 0$), a decrease in capital requirements increases the wage (see Proposition 4). This makes the marginal worker more expensive and thus firms’ insurance through deposits more valuable. As a result, firms are now willing to accept a lower return on deposits $R^d$, meaning that the demand for deposits is downward sloping. In the general case where $\eta \in (0, \infty)$ the demand for deposits is also downward sloping.

\section{Numerical examples}

This section discusses in details the trade-offs associated with determining the optimal capital requirements. The objective of capital requirements in our model is to reduce the bad risk-taking induced by deposit insurance without limiting too much the good risk-taking induced by deposits.

\textsuperscript{10} We establish formally this result in Lemma 8 in the Appendix.

\textsuperscript{11} On the date when capital requirements are increased, $R^d$ will fall in order to ensure that banks still return $1/\beta$ to their shareholders. Over time, however, as the level of of deposits falls, it will converge back to the same steady-state value that it had before capital requirements were changed.
We begin by summarizing the results of Sections 4.2 and 4.3. A marginal increase in capital requirements produces three main effects that are possibly relevant for welfare:

1. Deposits drop, which in turn reduces the demand for labor by firms, as discussed in Section 4.2. The ultimate result of this effect is a change of the equilibrium value of either labor $l$ or wages $w$, or both.

2. The leverage effect described in Proposition 5 implies that the stock of physical capital in the economy, $k$, decreases.

3. The funding effect described in Proposition 5 implies that the stock of physical capital in the economy, $k$, increases.

To clarify how these three effects impact the optimal capital requirement, we study two numerical examples: one for the case of infinitely elastic labor supply (i.e., $\eta \to \infty$) and one with fixed labor supply (i.e., $\eta \to 0$).

### 5.1 Optimal capital requirements with infinitely elastic labor supply

We now explore a numerical example to illustrate how capital requirements balance the good risk-taking of entrepreneurs against the bad risk taking of banks when labor supply is infinitely elastic (i.e., $\eta \to \infty$). We consider a benchmark scenario with a capital requirement $\zeta = 8\%$, and we ask whether this policy choice is optimal or whether $\zeta$ should be increased or decreased.

To keep the analysis simple, we assume that the idiosyncratic bank shock $\varepsilon$ is log-normally distributed with variance $\sigma^2$ and $E \{ \varepsilon \} = 1$ and that aggregate TFP $A$ is constant. We also assume that firms’ productivity shocks are i.i.d. and distributed as

$$z' \sim \begin{cases} 0 & \text{probability } 1 - p_z \\ \frac{1}{p_z} & \text{probability } p_z \end{cases}$$

so that the average productivity across all firms is normalized to 1. The process for $\{ \alpha_i \}$ does not need to be specified as long as it satisfies the restriction in (20). In this economy, both banks and firms face idiosyncratic risks, but because there is no aggregate uncertainty, aggregate variables such as output, consumption, and returns are all constant. Individual firms will grow and shrink as they receive a different sequence of shocks, but the firm-size

\[12\]In other words, $\log \varepsilon \sim N \{-1/2 \sigma^2, \sigma^2\}$. 

22
distribution does not affect aggregate variables. Because aggregate values in this economy are constant over time, in what follows we drop t subscripts where convenient. See Appendix B for details on the computation of the model solution.

Panels A and B of Table 1 report the parameter values we used for this exercise, as well as the numerical targets we chose. We set the discount rate to $\beta = 0.95$, the probability $p_z$ of successful firm output to 0.70, the depreciation rate to $\delta = 0.1$, and the decreasing-return parameter of the bank-financed sector to $\gamma = 0.95$.13 We then calibrate $A$, $\sigma$, and $\chi$ as follows. We set $A$ so that consumption is 1 in steady-state under the benchmark capital requirement of $\zeta = 8\%$. We choose $\sigma$ in order to match a bank default probability of 10% when the capital requirement is $\zeta = 8\%$. We set $\chi$ to match a deposit premium $\frac{1}{\beta} - R^d$ of 2%. The parameter $\chi$ primarily affects the deposit premium in equilibrium by affecting the level of wages; holding firm productivity fixed, a higher $\chi$ implies that the wage is higher (because $w = \chi$ when $\eta \to \infty$) and thus the risk associated with hiring the marginal worker is higher; as a result, firms are willing to accept a lower return on deposits. Panel B of Table 1 reports the parameter values that match these targets.

We perform the following policy experiment. Given the economy in steady-state with an 8% capital requirement, we vary the capital to a new level, ranging from 3% to 20%. For each new level of capital requirement in this range, we solve for the equilibrium transition path to the new steady-state. We then compute welfare over this path using equation (25).14

Figure 1 plots welfare for two extreme values of the elasticity of labor supply $\eta$, normalizing the welfare at the optimal capital requirement to one in each case. Details on these calculations are in Appendix B. With fully elastic labor supply, welfare is maximized at a capital requirement of 5%.

13 Even if this is just a numerical example, we note that $\gamma$ is the parameters that governs the decreasing return to scale of the bank-financed sector because such a sector does not use labor. With this in mind, our choice is in line with the value of 0.85 of Midrigan and Xu (2014).

14 Although the economy only approaches the new steady-state asymptotically, we find that 1,000 time periods is sufficient for welfare to converge.
Figure 1. Both lines in the figure plot total welfare $W$ from equation (25) for values of the capital requirement $\zeta$ ranging from 3% to 20%, starting from an initial steady-state where $\zeta = 8\%$. Parameter values are reported in Table 1, with the following exception: the dotted line assumes a fully-elastic labor supply, i.e. $\eta \rightarrow \infty$. The solid line assumes a fixed labor supply, i.e. $\eta \rightarrow 0$, where labor supply is fixed at the steady-state value in the infinitely-elastic case when $\zeta = 8\%$. Welfare values are normalized by their value at the optimal capital requirement for each line individually. Details on these calculations are in Appendix B.
Since the elasticity of labor supply is $\eta \to \infty$, changing capital requirements affects welfare primarily through its effect on employment (good risk-taking) and through the leverage effect described in Proposition 5 (bad risk-taking). That is, as discussed in Section 4.3, the funding effect highlighted in Proposition 5 does not operate in the long run because the steady-state return on deposits $R^d$ does not change with capital requirements. In the short-run, $R^d$ does fall on impact and this does affect welfare along the transition path to the new steady-state. Let us stress again that in this case with fully-elastic labor supply, the equilibrium wage is $w = \chi$ for all levels of capital requirements, and thus the marginal and social value of deposits are equalized.

5.2 Optimal capital requirements with fixed labor supply

We now consider the other extreme case in which the labor supply is fixed (i.e., $\eta \to 0$). We perform the same exercise as in the previous section. That is, starting from a baseline capital requirement of $\zeta = 8\%$, we vary it to a new level ranging from 3\% to 20\%.

For the baseline case of $\zeta = 8\%$, we calibrate the model as in the previous section, that is, with the same parameters reported in Table 1 although we now use a Frisch elasticity of labor supply of $\eta \to 0$. This implies that the labor supply is fixed, and thus we must choose the amount of labor $l$ which is supplied by households. We set $l = 0.8382$, which is the steady-state value of $l$ in the elastic-labor economy when $\zeta = 8\%$; this implies that the equilibrium under the benchmark capital requirement of $\zeta = 8\%$ is identical to that obtained with fully elastic labor supply in the previous section, when $\zeta = 8\%$.

The result of changing capital requirements are plotted in the bottom panel of Figure 1. As in the previous section, we normalize the welfare at the optimal capital requirement to one.

Two results stand out when comparing the two lines plotted in Figure 1. First, the optimal capital requirements in the economy with fixed labor supply is 9\%, much higher than the 5\% optimal capital requirement of the economy with fully-elastic labor supply. Even though this is just a numerical example, we emphasize that the difference between the two (i.e., four percentage points) is approximately the same as the difference between the capital requirements under Basel II and III. Second, when moving away from the optimal capital requirements, the magnitude of the welfare losses are much smaller with fixed labor supply. For instance, if a policymaker wants to set capital requirement at 20\%, the welfare loss is about 5\% with fully elastic labor supplied; however, the loss is an order of magnitude smaller with fixed labor supply, that is, less than 1\%.

To clarify further the role of fixed labor supply, Figure 2 plots the steady-state level
Figure 2. Both lines in the figure plot the steady-state level of physical capital $k$ for values of the capital requirement $\zeta$ ranging from 3% to 20%. Parameter values are reported in Table 1, with the following exception: the dotted line assumes a fully-elastic labor supply, i.e. $\eta \to \infty$, while the solid line assumes a fixed labor supply, i.e. $\eta \to 0$, where labor supply is fixed at the steady-state value in the infinitely-elastic case when $\zeta = 8\%$. Details on these calculations are in Appendix B.
of physical capital, \( k \), as a function of the capital requirement \( \zeta \). With fully elastic labor supply (dotted line), increasing capital requirements unambiguously decreases the level of physical capital. This is not just the result of this numerical example, but it holds in general as discussed in Section 4.3. With fixed labor supply (solid line), the steady-state level of capital is non-monotonic in the level of capital requirement \( \zeta \). This is because the leverage effect described in Proposition 5 dominates for low \( \zeta \), whereas the funding effect described in Proposition 5 dominates for larger values of \( \zeta \).

With fixed labor supply, steady-state physical capital is minimized at \( \zeta = 12\% \). The difference between this value and the optimal capital requirement \( \zeta = 9\% \) is simply due to discounting: recall that we compute the welfare by accounting for the transition to the new steady state. As the discount factor tends to one, the optimal level of capital requirements approaches the level that minimizes steady-state capital. This is because the demand for deposits due to idiosyncratic risk and deposit insurance gives rise to overaccumulation of physical capital in comparison to an economy with no idiosyncratic shocks, but this overaccumulation is socially wasteful because labor supply is fixed. In other words, not only the marginal social value of deposits is zero, as we discussed in Section 4.2, but the marginal social value of the physical capital held by banks is negative. As a result, the optimal level of capital requirement minimizes the physical capital in the economy.

Figure 3 illustrates the dynamic properties of the model for a capital requirement change from 8\% to 12\%. Because deposits are riskless, bank liabilities cannot change immediately in response to the shock, and thus banks respond to the increased capital requirement by immediately issuing equity (bottom left panel) and increasing the size of their balance sheets (top left panel). This reduces the lending rate \( r_{t+1} \) (not plotted), and so to maintain a return on equity of \( 1/\beta \) the deposit rate \( R_{t}^{d} \) must fall on impact (top right panel).

Total deposits in the system \( X_{t} \) are pre-determined and so do not jump on impact (bottom right panel). However, the lower \( R_{t}^{d} \) reduces labor demand and through it, subsequent values of \( X_{t} \); thus \( X_{t} \) falls over time. As it does, \( R_{t}^{d} \) rises, and the total amount of equity needed to keep the new capital requirement falls. The fall in equity and deposits causes the amount of physical assets held by the banking sector to fall over time.

The dynamics for fully-elastic and fixed labor are quite different; Figure 3 plots both fully-elastic (\( \eta \to \infty \), dotted line) and fixed labor supply (\( \eta \to 0 \), solid line) as well as an intermediate case (\( \eta = 1 \), dash-dot line). Although \( R_{t}^{d} \) falls on impact in all three cases, in the flexible-labor case (dotted lines) \( R_{t}^{d} \) eventually returns to its previous level, and the adjustment to the new steady-state occurs solely in quantities: lower \( X_{t} \), lower \( k_{t} \) and \( n_{t} \), and lower labor (not plotted). When labor is fixed, \( R_{t}^{d} \) settles down at a lower level in the new steady-state, resulting in a smaller drop (after impact) in bank assets and deposits, and
Figure 3. All panels plot the dynamics of endogenous variables in the model to a capital requirement change from 8% to 12% at $t = 1$. The solid lines plot values for the fixed-labor case ($\eta \rightarrow 0$), the dashed lines plot the values for the elastic-labor case ($\eta \rightarrow \infty$), and the dash-dot lines plot values for the case of $\eta = 1$. Labor in the fixed-labor case is assumed to equal the steady-state value in the $\eta = 1$ economy before the policy change, $l = 0.9880$. The top-left panel plots total physical assets held by banks $k_t$; the top-right panel plots the return on deposits $R^d_t$; the bottom-left panel plots the total amount of equity $n_t$, and the bottom-right panel plots the total amount of deposits $X_t$. The graphs of $k_t$, $n_t$, and $X_t$ plot percent deviations from the pre-change steady-state; the graph of $R^d_t$ is in percent.
a higher total level of equity than before the change. When labor is fully flexible, the size of
the banking sector shrinks so dramatically that in the steady-state there is less equity and
fewer deposits than before the change. The economy also converges to the new steady-state
much faster when labor is fixed and adjustments occur through prices.

Figure 3 highlights the importance of accounting for dynamics and general equilibrium
effects when analyzing capital requirement regulation. Admati et al. (2014) note that there
are three ways a bank can respond to an increased capital requirement: by shrinking its assets
while maintaining the total amount of equity, by recapitalizing and keeping its assets constant,
or by expanding its assets and financing the expansion with equity. In our model, because
deposits at \( t+1 \) are pre-determined at \( t \) (they are riskless), banks respond on impact to a
capital requirement change from \( t \) to \( t+1 \) by expanding their balance sheets and financing
the expansion with new equity. However, in dynamic general equilibrium where labor is
flexible (\( \eta > 0 \)) this expansion does not last: over time, deposits and bank assets shrink,
and with them, total output and (potentially) welfare. Total bank equity always increases
on impact and declines thereafter, but whether it stays at a level above or below the original
steady-state depends on the Frisch elasticity \( \eta \): when labor is fully flexible and wages are
fixed, the reduction in labor demand is so large that long-run bank equity is actually lower
than before the increased capital requirement.

5.3 Private versus social value of deposits: measurement

In the numerical examples of Sections 5.1 and 5.2, we have analyzed two economies that
differ in the elasticity of labor supply, \( \eta \). However, the parameterization of such an elasticity
affects welfare not only through the value of deposits, as noted in Proposition 4, but also
through the effects of capital requirements on bad risk-taking, as discussed in Proposition 5
and Section 4.3.

We now take the analysis one step further by computing the wedge between the private
and social marginal value of deposits. By doing so, we can compute the capital requirement
that would be chosen by a regulator that mistakenly uses the private value of deposits for
welfare calculation, rather than the social value. This would be the case, for instance, if the
regulator interprets the results of our model economy through the lens of a framework with
deposit in the utility function. By doing so, we are able to concentrate only on the channel
highlighted in Proposition 4, abstracting from the effects of labor supply on bad risk-taking.

Proposition 6. Let \( tvd_{social}(\zeta) \) and \( tvd_{private}(\zeta) \) be the total social and private value of

\footnote{In the discussion in Admati et al. (2014), in which they address the worry that banks will curtail lending,
increased investment by banks in response to a higher capital requirement is a good thing, whereas in our
model (in which deposit insurance leads to moral hazard and banks over-invest), it is a bad thing.}
deposits, respectively. Then

\[ t_{vd}^{social}(\zeta) = \sum_{t=0}^{\infty} \beta^t \left[ zl_t - \beta^{t+1} \frac{l_t^{1+1/\eta}}{1+1/\eta} \right]. \]  

(26)

and

\[ t_{vd}^{private}(\zeta) = \sum_{t=0}^{\infty} \beta^t \left[ (\bar{z} - w_t)l_t \right]. \]  

(27)

The marginal private value of an additional $1 of deposits is given by

\[ \frac{\partial t_{vd}^{private}(\zeta)}{\partial d_t} = \frac{X_{t+1}}{X_t} \frac{1}{\beta} - R_t^d, \]  

(28)

Furthermore, the optimal capital requirement \( \zeta^* \) satisfies

\[ 0 = \frac{\partial W(\zeta^*)}{\partial \zeta}, \]  

(29)

whereas the capital requirement \( \hat{\zeta} \) chosen by a regulator that mistakenly uses the private rather than the social value of deposits to compute welfare satisfies

\[ 0 = \frac{\partial W(\hat{\zeta})}{\partial \zeta} + \left( \frac{\partial t_{vd}^{private}(\hat{\zeta})}{\partial \zeta} - \frac{\partial t_{vd}^{social}(\hat{\zeta})}{\partial \zeta} \right), \]  

(30)

and \( \hat{\zeta} < \zeta^* \) provided that \( \eta < \infty \).

Proposition 6 extracts from total welfare \( W \) the component that depends only on the good risk-taking side of the model, that is, the total value of deposits \( t_{vd} \). There are two ways to make this construction; the first considers the total value of deposits from the point of view of households, who receive both the profits from firms as well as the wages, and also incur a disutility from providing labor. As such, the total value of deposits is the present value of the aggregate output of firms minus the disutility of labor. This is equation (26), which we label the social value of deposits \( t_{vd}^{social} \).

Another, partial-equilibrium construction of the total value of deposits considers the value of deposits to firms. From the point of view of households thinking of themselves solely as owners of firms, the value of deposits is the present of aggregate firm profits, that is, output less the wage bill (and there is no disutility of labor term). This is equation (27), which we label the private value of deposits \( t_{vd}^{private} \). Equation (28) shows that in steady-state, the marginal private value of an additional $1 of deposits is exactly equal to the deposit
Figure 4. The figure plots two measures of the marginal benefit of changing capital requirements $\zeta$ for the fixed-labor economy ($\eta \to 0$), for values of $\zeta$ ranging from 3% to 20%, starting from an initial steady-state where $\zeta = 8\%$. Parameter values other than $\eta$ are reported in Table 1. The solid line plots the marginal effect on welfare of increasing $\zeta$, $\partial W(\zeta)/\partial \zeta$. The dashed line plots the marginal effect on welfare of increasing $\zeta$, augmented for the marginal effect on the private value of deposits in excess of the social value: $\partial \left[ W(\zeta) + tvd_{\text{private}}(\zeta) - tvd_{\text{private}}(\zeta) \right] /\partial \zeta$. Both lines are normalized by the level of welfare $W(\zeta)$.

premium. We conjecture that a similar construction for the marginal social value of an additional $1$ in deposits is weakly smaller than the private value.

The social value of deposits $tvd_{\text{social}}$ is a component of total welfare $W$, so that the optimal capital requirement $\zeta^*$ satisfies equation (29). On the other hand, were a planner to mistakenly use the private rather than the social value of deposits to set the optimal capital requirement, that capital requirement $\hat{\zeta}$ would instead satisfy equation (30). This would occur, for example, if a social planner in our economy, using a model with deposits in the utility function, inferred the marginal value of deposits using the deposit premium. Proposition 6 shows that such a mistaken planner will always set the capital requirement too low (unless $\eta \to \infty$ and wages are fixed).

We quantify the size of this effect in the extreme case where labor is fixed, using the same parameters from Table 1. To do so, Figure 4 plots true marginal welfare $\partial W(\zeta)/\partial \zeta$

\footnote{In the numerical example with fully elastic labor supply of Section 5.1, private and social value of deposits are equalized, and thus the capital requirement chosen by the planner that uses the private value of deposits for welfare purposes, $\zeta$, is equal to the socially-optimal capital requirement, $\zeta^*$. On the other hand, focusing on the case where labor is fixed simplifies the calculation because the marginal social value of deposits is zero in this case, as shown in Section 5.2.}
as a solid line, and “mistaken” marginal welfare \( \partial \left[ W(\zeta) + tvd^{\text{private}}(\zeta) - tvd^{\text{social}}(\zeta) \right] / \partial \zeta \) as a dashed line for values of the capital requirement \( \zeta \) ranging from 3% to 20%. Both marginal effects are expressed as a fraction of \( W(\zeta) \), for each \( \zeta \). The solid line intersects the horizontal axis at 9%; thus, the true optimal capital requirement is \( \zeta^* = 9\% \). In contrast, the dashed line intersects the horizontal axis at 7%, and thus \( \hat{\zeta} = 7\% \) is the capital requirement chosen by a planner that mistakenly uses the private rather than the social value of deposits for welfare calculations. The difference between \( \zeta^* \) and \( \hat{\zeta} \) is two percentage points; this is a large number because it corresponds to about half of the change in capital requirement implemented by Basel III in comparison to Basel II, to 80% of the room of maneuver allowed by regulators for the countercyclical capital buffer (CCyB), and is equal to maximum variation in the CCyB recommended by Davydiuk (2017).

### 6 Quantitative Model

In this section we extend the model by introducing an aggregate productivity shock and allowing agents to have constant relative risk aversion utility over consumption. The main takeaway of our quantitative results is that considerations about the value of deposits are not first-order in setting the capital requirements, similar to the baseline model with fixed labor supply.

We use the same parameters as in Section 5. In future work, we plan to calibrate the extended model to match similar moments to other quantitative dynamic general equilibrium models of optimal capital requirements, such as Begenau (2016) and Davydiuk (2017).

We assume that \( A_{t+1} \) follows an AR(1) process in logs and that households have constant relative risk aversion rather than quasi-linear utility. The law of motion for bank asset productivity \( A_{t+1} \) is

\[
\log A_{t+1} = (1 - \rho) \log \bar{A} + \rho \log A_t + \sigma_A \varepsilon_{t+1}^A
\]

where \( \varepsilon_{t+1}^A \sim N(0,1) \). The value function of households becomes

\[
V^h_t (a_t) = \max_{c_t, l_t, n_t} \left( \frac{c_t^{1-\gamma_c}}{1 - \gamma_c} - \beta \chi \frac{l_t^{1+\frac{\gamma}{\eta}}}{1 + \frac{\gamma}{\eta}} + \beta E_t \left\{ V^h_{t+1} (a_{t+1}) \right\} \right)
\]

s.t.

\[
c_t + n_t = a_t
\]

\[
a_{t+1} = \phi_t X_t + A_{t+1} k_t^\gamma + (1 - \delta) k_t - (1 - \zeta_{t+1}) k_{t+1}
\]

(31)
where the equation for $a_{t+1}$ includes equations (9), (16), (21), and the result of Proposition 2, and $\gamma_c$ is the household’s coefficient of relative risk aversion.

In this quantitative version of the model, we also relax the dividend-policy rule (20) that pins down the fraction $\alpha^i_t$ of firms’ wealth that is paid out as dividends. In particular, $\alpha^i_t$ is now chosen so that the stream of dividends maximizes the value of the firm from the households’ point of view (i.e., using the household discount factor), as shown by the next proposition. The inability of the manager to differentiate away firms’ idiosyncratic risk is unchanged, and so is the first-order condition (17).

**Proposition 7.** The optimal dividend policy from the point of view of households implies that $\alpha^i_t$ is chosen so that

$$1 = E_t \left\{ \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma_c} \left[ (\bar{z} - w_t) \phi_t + R^d_t \right] \right\}.$$  

(32)

Note that equation (32) evaluated in steady-state implies

$$1 = \beta \left[ (\bar{z} - w) \phi + R^d \right],$$

which is identical to equation (21), in steady-state, for the model of Section 3.3 under the exogenous dividend policy $\alpha = 1 - \beta$.

Table 2 reports the parameters we use for this extended model. Panel A reports the fixed parameters for which we do not have a calibration target; they are identical to Table 1, with the exception of $\eta$ which we now set to 0.75 in line with the results of Chetty et al. (2011). We assume that $\gamma_c = 1$ (log utility) and that $\rho = 0.95$ in line with Davydiuk (2017).

Panel B of Table 2 reports parameters that we plan to calibrate towards the indicated data moments. Because the model is non-linear, each parameter affects all moments; however in Panel B of Table 2 we have indicated target moments that are particularly affected by each parameter in equilibrium. Thus for example a higher value of average bank-financed firm productivity $\bar{A}$ will increase the share of GDP coming from banks; Begenau (2016) estimates this value at 5%, and we set $\bar{A}$ to try and match this value. However, decreasing $\bar{A}$ also increases the deposit premium; thus as we lower $\bar{A}$ to reduce the bank share of GDP, we need to increase $\chi$ to keep the deposit premium near its target of 57 bps, which we take from Davydiuk (2017).

We calibrate the firm-success probability $p_z$ to match the average employment growth
Figure 5. The top panel plots household welfare for different values of $\zeta$, starting from a steady-state in which $\zeta = 8\%$, for values of $\zeta$ ranging from 3\% to 20\%. Welfare is defined in consumption equivalent units; that is, the percentage decrease in consumption one would have to give households in order to make them indifferent between staying in the $\zeta = 8\%$ economy, and transitioning to the economy with $\zeta$ indicated on the x-axis. We compute welfare by averaging across 1,000 simulated economies for 900 periods each, discarding the first 200 time periods and making the capital requirement change at $t = 400$, which is when we begin calculating welfare. The bottom panel plots the average across simulations of $k_t$ at $t = 900$ for each value of $\zeta$. 
Panel A: Set Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$</td>
<td>0.75</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.95</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.05</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.85</td>
</tr>
<tr>
<td>$\gamma_c$</td>
<td>1</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Panel B: Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Target</th>
<th>Value (data)</th>
<th>Value (model)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{A}$</td>
<td>0.09</td>
<td>Bank Share of GDP</td>
<td>5%</td>
<td>10.8%</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>0.01</td>
<td>Volatility log GDP</td>
<td>1.8%</td>
<td>1.6%</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.0345</td>
<td>Bank Default Probability</td>
<td>0.76%</td>
<td>0.78%</td>
</tr>
<tr>
<td>$\chi$</td>
<td>130</td>
<td>Deposit Premium $R^f - R^d$</td>
<td>0.57%</td>
<td>1.01%</td>
</tr>
<tr>
<td>$p_z$</td>
<td>0.93</td>
<td>Continuers Employment Growth</td>
<td>2.5%</td>
<td>2.2%</td>
</tr>
</tbody>
</table>

Table 2. Calibrated Parameter Values

rate of firms who receive the high value of $z_{t+1}^i$, defined as

$$g_{t+1}^i \equiv \frac{L_{t+1}^i - L_t^i}{\frac{1}{2} (L_{t+1}^i + L_t^i)}$$

$$L_t^i = \phi_t x_t^i$$

$$L_{t+1}^i = \phi_{t+1} x_{t+1}^i$$

$$= \left[ \left( \frac{1}{p_z} - w_t \right) \phi_t + R_t^d \right] (1 - \alpha_{t+1}) x_t^i$$

Haltiwanger, Jarmin and Miranda (2013) show that for all but the youngest firms this growth rate is around 2.5%, though with $p_z = 0.93$ it is slightly lower at 2.2%.

The top panel of Figure 5 reports welfare in consumption-equivalent percent units for capital requirements ranging from 3% to 20%, assuming that the economy is in steady-state with an 8% capital requirement. The optimal policy in this economy is actually to lower the capital requirement slightly, to 7.2%; however, the welfare gain is relatively small. What the figure does reveal is that very low or very high capital requirements lead to significant declines in welfare, as much as 4–5% of annual consumption.

The bottom panel of Figure 5 plots average $k_t$ for each simulation after the transition
to the new ergodic distribution is complete. As in the discussion of Figure 2, when labor supply is only partially elastic ($\eta < \infty$) the effect of capital requirements on physical capital is non-monotonic. This is due to the leverage effect of Proposition 5 dominating at low levels of $\zeta$, while the funding effect is stronger at higher values of $\zeta$. In particular, for values of $\zeta$ greater than 7.6%, raising $\zeta$ decreases the banks’ lending rate and thus increases lending by banks. As we mentioned before, higher lending by banks reduces welfare because lending is inefficiently high due to subsidized deposit insurance.

The final takeaway is that the optimal capital requirement (i.e., 7.2%), is close to the level of the capital requirement that minimizes bank lending (i.e., 7.6%). The difference between the two capital requirements is a consequence of the transition between steady states, similar to the baseline model, and to a modest gain by reducing capital requirement below 7.6% due to the value of deposits. Nonetheless, the value of deposits affect the optimal capital requirement by less than one percentage point.

7 Conclusion

We have proposed a new channel which generates an endogenous liquidity premium on safe and liquid deposits held by firms subject to idiosyncratic productivity risk. Our microfoundation of the deposit premium gives rise to a wedge between the marginal private and social values of deposits, with important implications for determining optimal capital regulation. In contrast to other general equilibrium models of optimal capital requirements, the deposit premium itself is not a sufficient statistic for the welfare costs of tighter capital requirements, but instead recovers the private marginal value of deposits. Regulators using this private value when the true social marginal value of deposits is zero will set capital requirements lower than the true optimum.

This paper can open up several directions for future research. We have followed the literature in assuming complete deposit insurance, but Egan, Hortaçsu and Matvos (2017) show that only about half of all deposits in the US are in fact FDIC insured. Our model can be extended to study the optimal degree of deposit insurance. In our model, eliminating deposit insurance altogether increases firms’ asset value volatility, and thus the optimal outcome is likely to be at least partial deposit insurance. This is another important novelty of our model—the literature that studies financial regulation quantitatively uses models in which zero deposit insurance would be optimal, and thus studying partial insurance in those models might require complicated extensions. In addition, depending on how firms’ idiosyncratic productivities $z_{it+1}$ evolve over time, the value of safe deposits might be time-varying and the optimal regulation might vary over the cycle as well, as in Davydiuk (2017).
References


Kashkari, Neel. 2016. “The Minneapolis Plan to End Too Big to Fail.” Inaugural Address given at the Economic Club of New York, November 16, 2016. 2


A Proofs

Proof of Proposition 1. We conjecture and later verify that the value function has the form

\[ V_t^m(d_t) = \theta \frac{\beta}{1-\beta} \log(d_t) + \Xi_t, \]  

(33)

where \( \Xi_t \) is independent of \( d_t \). Then, the problem of the manager implies the first-order condition in (17). Since \( l_t^i \) is independent of \( d_t^i \), the conjecture about the value function can be verified, obtaining

\[ \Xi_t = \theta \frac{\beta}{1-\beta} \left\{ \log(\kappa \alpha) + \frac{\beta}{1-\beta} \log(1 - \alpha) + \frac{1}{1-\beta} E_t \log \left[ (z_{t+1}^i - w_t) \phi_t + R_t^d \right] \right\}. \]  

(34)

\( \square \)

Proof of Proposition 2. The first-order condition in (17) does not depend on \( \kappa \) and thus holds for any \( \kappa > 0 \). Manager \( i \)'s consumption converges to zero, using (4). To show that the manager’s value function converges to zero, we use the specification of the value function stated in the proof of Proposition 1, equation (33); under the assumption that \( \theta = \eta_1 \kappa^{\eta_2} \) with \( \eta_1, \eta_2 > 0 \), we have

\[ \theta \frac{\beta}{1-\beta} \log(d_t) \downarrow 0 \]

and \( \Xi_t \downarrow 0 \) as \( \kappa \downarrow 0 \), where \( \Xi_t \) is defined in equation (34). The result about dividends follows using equation (5).

\( \square \)

Proof of Proposition 3. Since there are no idiosyncratic shocks to firms, the first-order condition (17) implies \( w = 1 \), which in turn implies the equilibrium value of labor stated in the proposition, using (22). The law of motion of deposits, (3), evaluated in steady state (i.e., at \( x_t^i = x_{t+1}^i \)) and using the restriction on \( \alpha \) in (20), implies \( R^d = 1/\beta \). Given this result, and since no bank fail in equilibrium because shocks are shut down, equation (24) implies \( 1 - \delta + r = 1/\beta \) which, together with the first-order condition of bank-finance firms, (12), implies the value of capital \( k \) stated in the proposition. The value of deposits and equity follows from the bank’s budget and capital-requirement constraints, (7) and (8), and firms’ wealth \( X \) follows from (2). Taxes \( T = 0 \) follow from the fact that no bank fails in equilibrium, and consumption follows from Walras’ Law.

\( \square \)

To prove Propositions 4 and 5, we first state and prove the following intermediate lemma.
Lemma 8. If there exists an equilibrium with \( l > 0 \), we have

\[
\frac{\partial d}{\partial \zeta} < 0, \quad \frac{\partial R^d}{\partial \zeta} < 0, \quad \frac{\partial w}{\partial \zeta} < 0, \quad \frac{\partial \phi}{\partial \zeta} > 0, \quad \frac{\partial l}{\partial \zeta} < 0,
\]

for all \( \eta \in (0, 1) \). Moreover the sign of \( \frac{\partial d}{\partial \zeta} \) is also preserved in the limit as \( \eta \to 0 \) and \( \eta \to \infty \), whereas \( \frac{\partial R^d}{\partial \zeta} < 0 \) as \( \eta \to 0 \) but \( \frac{\partial R^d}{\partial \zeta} \to 0 \) as \( \eta \to \infty \).

Proof of Lemma 8. We totally differentiate (17), the law of motion of deposits (3) evaluated in steady-state and integrated over \( i \), the labor demand equation \( l = \phi X \) evaluated at \( X = d \), and the first-order condition of banks (24). Thus, we obtain a system of four equations in four unknowns, where the unknowns are \( \frac{\partial w}{\partial \zeta}, \frac{\partial \phi}{\partial \zeta}, \frac{\partial R^d}{\partial \zeta}, \) and \( \frac{\partial d}{\partial \zeta} \). To derive the results, it is useful to define the following variables

\[
A \equiv E \left\{ \frac{R^d}{[\phi (z' - w) + R^d]^2} \right\} > 0
\]
\[
B \equiv E \left\{ \frac{(z' - w)^2}{[\phi (z' - w) + R^d]^2} \right\} > 0
\]
\[
C \equiv E \left\{ -\frac{(z' - w)}{[\phi (z' - w) + R^d]^2} \right\} > 0
\]
\[
D \equiv E \left\{ \frac{1}{\phi(z' - w) + R^d} \right\} > 0.
\]

The inequality for \( C \) can be established by noting that \( C \) is cross-partial derivative of the manager’s objective function with respect to \( R^d \) and \( \phi \), that is,

\[
\frac{\partial}{\partial R^d} \frac{\partial (\text{manager’s objective function})}{\partial \phi} = C.
\]

The signs of \( A, B, \) and \( D \) hold because the argument of the expectation is positive for all states. The sign of \( C \) follows from two remarks. First, since the objective function of the manager is concave in \( \phi \) (i.e., the first-order condition with respect to \( \phi \) pins down a maximum), then \( \partial / \partial \phi \) is decreasing in \( \phi \). Second, (17) implies a constant ratio \( \phi/R^d \), and thus a marginal increase in \( R^d \) implies an increase in \( \phi \). Thus, \( C \) must be positive when evaluated at equilibrium values.

We can then solve the system of four equations in four unknowns described before. For
deposits, we obtain

\[
\frac{\partial d}{\partial \zeta} = - \left( \frac{1}{\beta} - R^d \int_x^\infty dF (\varepsilon) + \frac{A(1-\gamma) \int_x^\infty dF (\varepsilon)}{(d^\gamma)(1-\zeta)^{\gamma-1}} \right) \left( d\Pi + \eta \frac{B+C(\bar{z}-w)}{w^{1-\eta} \chi^\eta} \right) + (1-\zeta) \int_x^\infty dF (\varepsilon) \phi \left[ A (\bar{z} - w) + B \phi \right] < 0,
\]

where the inequality follows from the fact that both the numerator and the denominator on the right-hand side are positive, using \( R^d < 1/\beta \) and \( w < \bar{z} \) (which must both hold otherwise firms would make negative profits) and the signs of \( A, B, C, \) and \( \Pi \) established before. The sign of the derivative is preserved in the limit as \( \eta \to 0 \) and \( \eta \to \infty \), using the labor supply equation (22).

Similar, for the return on deposits, we obtain

\[
\frac{\partial R^d}{\partial \zeta} = - \phi \left( A (\bar{z} - w) + B \phi \right) \left( \frac{1}{\beta} - R^d \int_x^\infty dF (\varepsilon) + \frac{A(1-\gamma) \int_x^\infty dF (\varepsilon)}{(d^\gamma)(1-\zeta)^{\gamma-1}} \right) \left( d\Pi + \eta \frac{B+C(\bar{z}-w)}{w^{1-\eta} \chi^\eta} \right) + (1-\zeta) \int_x^\infty dF (\varepsilon) \phi \left[ A (\bar{z} - w) + B \phi \right] < 0,
\]

and taking the limit as \( \eta \) goes to zero or \( \infty \) we can established the respective result.

For the wage, we obtain

\[
\frac{\partial w}{\partial \zeta} = - \phi \left( B + C (\bar{z} - w) \right) \left( \frac{1}{\beta} - R^d \int_x^\infty dF (\varepsilon) + \frac{A(1-\gamma) \int_x^\infty dF (\varepsilon)}{(d^\gamma)(1-\zeta)^{\gamma-1}} \right) \left( d\Pi + \eta \frac{B+C(\bar{z}-w)}{w^{1-\eta} \chi^\eta} \right) + (1-\zeta) \int_x^\infty dF (\varepsilon) \phi \left[ A (\bar{z} - w) + B \phi \right] < 0,
\]

and for the risk-taking parameter \( \phi \) we obtain

\[
\frac{\partial \phi}{\partial \zeta} = \frac{\phi d \Pi \left( \frac{1}{\beta} - R^d \int_x^\infty dF (\varepsilon) + \frac{A(1-\gamma) \int_x^\infty dF (\varepsilon)}{(d^\gamma)(1-\zeta)^{\gamma-1}} \right)}{\left( d\Pi + \eta \frac{B+C(\bar{z}-w)}{w^{1-\eta} \chi^\eta} \right) + (1-\zeta) \int_x^\infty dF (\varepsilon) \phi \left[ A (\bar{z} - w) + B \phi \right]} > 0.
\]

Finally, using \( l = \phi X \) and \( X = d \), totally differentiating with respect to \( \zeta \), and using the previous results, we obtain

\[
\frac{\partial l}{\partial \zeta} = - \left( \frac{1}{\beta} - R^d \int_x^\infty dF (\varepsilon) + \frac{A(1-\gamma) \int_x^\infty dF (\varepsilon)}{(d^\gamma)(1-\zeta)^{\gamma-1}} \right) \left( d\Pi + \eta \frac{B+C(\bar{z}-w)}{w^{1-\eta} \chi^\eta} \right) + (1-\zeta) \int_x^\infty dF (\varepsilon) \phi \left[ A (\bar{z} - w) + B \phi \right] < 0.
\]
Proof of Proposition 4. The results follow as corollaries of the proof of Lemma 8 by using the labor supply (22) and taking the appropriate limits with respect to \( \eta \) (for the case \( \eta \to \infty \)).

Proof of Proposition 5. Combining (12), (23), and (24) evaluated at the non-stochastic steady state, we obtain:

\[
\int_{\xi}^{\infty} \varepsilon (1 - \delta + A \gamma k^{\gamma-1}) \, dF(\varepsilon) = \xi \frac{1}{\beta} + (1 - \xi) R^d \int_{\xi}^{\infty} dF(\varepsilon)
\]

Totally differentiating with respect to \( \zeta \), we have:

\[
\int_{\xi}^{\infty} \varepsilon \left( A \gamma (\gamma - 1) k^{\gamma-2} \frac{\partial k}{\partial \zeta} \right) dF(\varepsilon) = \left[ \frac{1}{\beta} - R^d \int_{\xi}^{\infty} dF(\varepsilon) \right] + (1 - \xi) \frac{\partial R^d}{\partial \zeta} \int_{\xi}^{\infty} dF(\varepsilon)
\]

Fixing \( R^d \), and since the term in square brackets on the right-hand side is positive (because \( R^d < 1/\beta \) in equilibrium), then capital \( k \) drops when \( \zeta \) marginally increases (leverage effect).

The fundind effect follows from the fact that \( R^d \) weakly decreases, as established in Lemma 8.

Proof of Proposition 6. In our economy, deposits are valuable because they allow firms to operate despite the idiosyncratic risk. Thus, the total social value of deposits that arises when the capital requirement is set at \( \zeta \), which Proposition 6 denotes as \( tvd^{social}(\zeta) \), is given by equation (26). That is, \( tvd^{social}(\zeta) \) is the output produced by firms minus the disutility of labor. We can differentiate (26) with respect to \( \zeta \) to obtain the marginal social value of deposits arising from a change in the capital requirement:

\[
\frac{\partial tvd^{social}(\zeta)}{\partial \zeta} = \sum_{t=0}^{\infty} \beta^t \left[ (\bar{\varepsilon} - w_t) \frac{\partial l_t}{\partial \zeta} \right]
\]

where we have used the fact that differentiating the disutility of labor \( \chi^\frac{I + \eta}{1 + \eta} \) with respect to \( l \) gives the marginal disutility of labor, which is equal to the wage \( w_t \) in equilibrium. Finally, we can use the fact that \( l_t = \phi_t x_t \) from Proposition 1, \( x_t = d_t \) from (2), and \( d_t = (1 - \zeta) k_t \)
from (7) and (8) to obtain $l_t = \phi_t(1 - \zeta)k_t$. Thus,\footnote{We do not differentiate $k_t$ with respect to $\zeta$ because the welfare effects of changing $k_t$ through $\zeta$ are zero. We can show this formally by stating the constrained planner problem that replicates the decentralized equilibrium, and then using the envelope theorem, as in van den Heuvel (2008).} 

$$
\frac{\partial tvd^{social}(\zeta)}{\partial \zeta} = \sum_{t=0}^{\infty} \beta^t \left[ (\bar{z} - w_t) \frac{\partial (\phi_t(1 - \zeta)k_t)}{\partial \zeta} \right] \\
= \sum_{t=0}^{\infty} \beta^t \left[ (\bar{z} - w_t) \frac{\partial \phi_t}{\partial w_t} \frac{\partial w_t}{\partial \zeta} (1 - \zeta)k_t - \phi_t k_t \right].
$$

\begin{equation}
(36)
\end{equation}

Crucially, the marginal social value (36) accounts for the fact that the wage $w_t$ may adjust in response to a change in the capital requirement, as shown by the dependence of (36) on $\partial w_t/\partial \zeta$.

Next, we define the total private value of deposits, $tvd^{private}(\zeta)$. The total value of deposits equal the profits that are earned by firms each period by hiring workers, $(\bar{z} - w_t)l_t$:

$$
tvd^{private}(\zeta) = \sum_{t=0}^{\infty} \beta^t \left[ (\bar{z} - w_t)l_t \right] = \sum_{t=0}^{\infty} \beta^t \left[ (\bar{z} - w_t)\phi_t d_t \right],
$$

where the second line uses $l_t = \phi_t x_t$ from Proposition 1 and $x_t = d_t$ from (2). We emphasize that the marginal private value of deposits obtained by having an extra dollar of $d_t$ in the firms (e.g., by cutting dividends by $1$ at all firms) is equal, in equilibrium, to the liquidity premium on deposits:

$$
\frac{\partial tvd^{private}(\zeta)}{\partial d_t} = (\bar{z} - w_t)\phi_t = \frac{X_{t+1}}{X_t} \frac{1}{\beta} - R_d^t,
$$

where the second line uses (3) and the restriction in (20). The liquidity premium is given by the return $1/\beta$, adjusted to account for the growth rate of firms’ wealth $X_{t+1}/X_t$, net of the return on deposits $R_d^t$. Note that, in steady-state, $X_{t+1} = X_t$, and thus the liquidity premium is simply $1/\beta - R_d$.

We can now compute the marginal private value of deposits that arises from a marginal
change in the capital requirement $\zeta$:

$$\frac{\partial \text{tvd}_{private}(\zeta)}{\partial \zeta} = \sum_{t=0}^{\infty} \beta^t \left[ (\bar{z} - w_t) \frac{\partial l_t}{\partial \zeta} \right]_{\text{w} \text{fixed}},$$

$$= \sum_{t=0}^{\infty} \beta^t \left[ (\bar{z} - w_t) (-\phi_t k_t) \right], \tag{37}$$

where the second line uses $l_t = \phi_t d_t$ and $d_t = (1 - \zeta) k_t$, as in the derivation of (36). The key difference between (37) and (36) is that the wage is fixed in (37) because it is taken as given by firms, whereas the general equilibrium effect of $\zeta$ on $w_t$ are accounted for in (36). Combining (36) and (37), we have that

$$0 = \frac{\partial V_f(\hat{\zeta})}{\partial \zeta} + \left( \frac{\partial \text{tvd}_{private}(\zeta)}{\partial \zeta} - \frac{\partial \text{tvd}_{social}(\zeta)}{\partial \zeta} \right)$$

$$= \frac{\partial V_f(\hat{\zeta})}{\partial \zeta} - \left( \sum_{t=0}^{\infty} \beta^t (\bar{z} - w_t) \frac{\partial w_t}{\partial \zeta} \frac{\partial \phi_t}{\partial \zeta} \frac{\partial d_t}{\partial w_t} \right)$$

and the result of Proposition 6 follows. □

Proof of Proposition 7. Define the value of the firm before-dividends as $V_t^f(\Psi_t^i)$, from the point of view of shareholders. This value corresponds to the present-discounted stream of dividends, discounted using the stochastic discount factor of households, and where the choice of dividends (i.e., of $\alpha_t$ for all $t$) is made optimally to maximize households’ utility. That is,

$$V_t^f[\Psi_t^i] = \max_{\alpha_t} \alpha_t \Psi_t^i + \beta E_t \left\{ \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma c} V_{t+1}^f[\Psi_{t+1}^i] \right\},$$

where

$$\Psi_{t+1}^i = \left( z_{t+1}^i - w_t \right) l_t + R_t^d d_t$$

$$= \left[ \left( z_{t+1}^i - w_t \right) \phi_t + R_t^d \right] d_t$$

$$= \left[ \left( z_{t+1}^i - w_t \right) \phi_t + R_t^d \right] (1 - \alpha_t) \Psi_t^i$$

where the last line uses $d_t = (1 - \alpha_t) \Psi_t^i$. Note that $\phi_t$ is taken as given because it is chosen by the manager.
The first-order condition with respect to $t$ implies
\[ 1 = \beta E_t \left\{ \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma_c} \left( z_{t+1}^i - w_t \right) \phi_t + R_t^d \right\} \left( V_{t+1}^f \right)^{\prime} \left[ \Psi_{t+1}^i \right] \]
where the marginal value of the firm can be computed recursively using the envelope condition
\[ \left( V_t^f \right)^{\prime} \left[ \Psi_t^i \right] = \alpha_t + \beta E_t \left\{ \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma_c} (1 - \alpha_t) \left[ \left( z_{t+1}^i - w_t \right) \phi_t + R_t^d \right] \left( V_{t+1}^f \right)^{\prime} \left[ \Psi_{t+1}^i \right] \right\}. \]

Since $z_{t+1}^i$ is i.i.d. and independent of all other endogenous variables at $t + 1$, we can rewrite the previous two equations as
\[ 1 = \beta E_t \left\{ \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma_c} (\bar{z} - w_t) \phi_t + R_t^d \right\} \left( V_{t+1}^f \right)^{\prime} \left[ \Psi_{t+1}^i \right] \]
where the marginal value of the firm can be computed recursively using the envelope condition
\[ \left( V_t^f \right)^{\prime} \left[ \Psi_t^i \right] = \alpha_t + \beta E_t \left\{ \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma_c} (1 - \alpha_t) \left[ (\bar{z} - w_t) \phi_t + R_t^d \right] \left( V_{t+1}^f \right)^{\prime} \left[ \Psi_{t+1}^i \right] \right\}. \]

Combining (38) and (39), we obtain \[ \left( V_t^f \right)^{\prime} \left[ \Psi_t^i \right] = 1 \] for all $t$, and thus equation (32).

\section*{B Solution Method}

In this section we describe the numerical method for solving the model in section 5. We take all parameters as given and constant. In sections B.1 and B.2 we set $A_{t+1} = A$ constant, while in section B.3 we allow for $A_{t+1}$ to be stochastic.

\subsection*{B.1 Steady-State}

Here we assume that $\zeta_t = \zeta \ \forall t$ and compute steady-state endogenous variables. In this section we drop all time subscripts.

First, because the capital constraint always binds, equations (7) and (8) together imply
that

\[ n = \zeta k \]
\[ d = (1 - \zeta) k \]
\[ = X \]  \hspace{1cm} (40)

where the last line follows from equation (2). Equation (12) also implies that

\[ k = \left[ \frac{\gamma A}{r} \right]^{\frac{1}{1-\tau}}. \]  \hspace{1cm} (41)

Equation (21) in steady-state implies that

\[ R^d = \frac{1}{\beta} - (1 - w) \phi \]  \hspace{1cm} (42)

using the fact that \( E\{z^i\} = 1 \). Plugging this equation into equation (17) yields

\[ 0 = E \left\{ \frac{z' - w}{(z' - 1) \phi + \frac{1}{\beta}} \right\}, \]  \hspace{1cm} (43)

To solve for steady-state, we guess a value for \( w \). Given this value for \( w \), we solve equation (43) numerically for \( \phi \), noting that \( \phi \in (0, \frac{1}{\beta}) \), since there is positive probability that \( z' = 0 \). We then have \( R^d \) from equation (42). We then plug equation (10) into equation (24) and solve numerically for \( r \), noting that since \( \varepsilon \) is log-normal with mean 1,

\[ \int_{-\infty}^{\infty} \varepsilon dF(\varepsilon) = 1 - \Phi \left\{ \frac{1}{\sigma \varepsilon} - \frac{1}{2} \right\} \]
\[ \Pr\{\varepsilon \geq \varepsilon\} = 1 - \Phi \left\{ \frac{1}{\sigma \varepsilon} + \frac{1}{2} \right\} \]

where \( \Phi \{\cdot\} \) denotes the standard normal CDF. Given \( r \), we can find \( k \) and \( X \) from equations (40) and (41). Finally, we search over values of \( w \in (0, 1) \) to satisfy equation (22) using the implied values of \( X \) and \( \phi \).
B.2 Deterministic Path for $\zeta_t$

The following method works for any deterministic path for $\zeta_t$, although in the text we assume that

$$\zeta_t = \begin{cases} 
\zeta & t \geq 1 \\
0.08 & t < 1 
\end{cases}$$

for values of $\zeta$ ranging from .03 to 0.20.

Assume the economy is in steady-state up to $t = 0$. At the start of $t = 1$, a new time path for $\zeta_t$ is announced. However, $X_{t+1}$ is pre-determined at time $t$ from equation (21), since $\bar{z}_{t+1} = 1$. Thus $X_1 = X_0$.

We solve the model at each $t$ recursively. Suppose we know $X_t$. Then $k_t = \frac{X_t}{1 - \zeta}$, and $r_{t+1}$ comes directly from equation (12). We then plug equation (10) into equation (24) and solve numerically for $R^d_t$. Given $R^d_t$, we plug equation (22) into equation (17) and solve numerically for $\phi_t$ (and then recover $w_t$). Then, given $\phi_t$, $w_t$, and $R^d_t$, $X_{t+1}$ is given by equation (21) and we move to the next $t$.

Consumption at time $t + 1$ is given by

$$c_{t+1} = a_{t+1} - n_{t+1}$$

$$= w_tl_t + \int \pi_t^1 di + \pi_t^{small} + n_tR^E_{t+1} - T_{t+1} - \zeta_{t+1}k_{t+1}$$

$$= w_t\phi_tX_t + (1 - \beta) \left( (1 - w_t) + R^d_t \right) X_t + A_{t+1}k_t - r_{t+1}k_t + k_t (1 + \delta - r_{t+1}) - R^d_t d_t - \zeta_{t+1}k_{t+1}$$

$$= \phi_tX_t + A_{t+1}k_t + (1 - \delta) k_t - X_{t+1} - \zeta_{t+1}k_{t+1}$$

$$= \phi_tX_t + A_{t+1}k_t - \delta k_t - (k_{t+1} - k_t)$$

where the second line plugs in equations (15) and the binding (8), the third line plugs in equations (9), (11) and (16) as well as the main result of Proposition 2, the fourth line plugs in equations (2) and (21) and cancels terms, and the last line uses equation (2) at $t + 1$. 

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**B.3 Stochastic $A_{t+1}$ and CRRA utility**

The first-order conditions for $l_t$ and $n_t$ for problem (31), after plugging in the budget constraint, are

$$w_t = \frac{\chi}{E_t \left\{ c_t^{1-\gamma_c} \right\}^\frac{1}{\gamma_c}} (\phi_t X_t)^\frac{1}{\gamma_c}$$  

(44)

$$1 = E_t \left\{ \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma_c} R_{t+1}^E \right\}$$  

(45)

The labor-supply curve (44) includes an expectation of future marginal utility of consumption because, although labor is chosen at $t$, wages are earned at $t + 1$. The expected-return condition for bank equity (45) is standard.

In this economy there is a single exogenous state variable, $A_t$, and two endogenous state variables: $Y_t$, which is total output available for consumption and future investment, and $\Psi_t$, which is total wealth held by firms before they make dividend payments:

$$Y_t = \phi_{t-1} X_{t-1} + A_t k_{t-1}^\gamma + (1 - \delta) k_{t-1}$$

$$\Psi_t = (1 - w_{t-1}) \phi_{t-1} + R_{t-1}^d X_{t-1}$$

Notice that $\Psi_t$ is pre-determined at $t - 1$, much like $X_t$ was pre-determined at $t - 1$ in the model with quasi-linear utility. However, in this model $\alpha_t$ is a choice variable of firms, so that $X_t$ satisfies:

$$X_t = (1 - \alpha_t) \Psi_t$$

where the dependence of $X_t$ on the current state comes from its dependence on $\alpha_t$.

We solve the model globally on a grid of values for the state variables. Instead of using $Y_t$ and $\Psi_t$, however, we solve the model over $\log Y_t$ and $\omega_t$, where

$$\omega_t = \log \Psi_t - \psi_0 + \psi_1 \log Y_t$$

where $\psi_0$ and $\psi_1$ are constants. Solving the model on a grid for $\omega_t$ rather than $\log \Psi_t$ is more accurate because $\log Y_t$ and $\log \Psi_t$ are highly correlated in equilibrium, so that solving on a grid of $\log Y$ and $\log \Psi$ would include node points that with extremely low probabilities in the ergodic distribution.

We choose five points for (de-meaned) $\log A$ using the method of Rouwenhorst (1995), and for each $\log A$ point we approximate $\alpha_t, R_t^d, w$, and $\phi$ as 5-degree Chebyshev polynomials.
on a grid of 36 points for log $Y$ and $\omega$. In particular, at each of the $5 \times 36 = 180$ node points, we solve equations (44) and (17) for $w_t$ and $\phi_t$, given $X_t$ and $R^d_t$, and equations (45) and (32) for $R^d_t$ and $\alpha_t$, given $\phi_t$ and $w_t$. We solve equations (45) and (32) using the household’s budget constraint

$$c_t = Y_t - k_t$$
$$= Y_t - \frac{1 - \alpha_t}{1 - \zeta_t} \Psi_t,$$

iterating it forward for $c_{t+1}$ using the law of motion for the endogenous state

$$Y_{t+1} = \phi_t X_t + A_{t+1} k_t^\gamma + (1 - \delta) k_t$$
$$\Psi_{t+1} \equiv \left[ (1 - w_t) \phi_t + R^d_t \right] (1 - \alpha_t) \Psi_t.$$

This procedure requires a guess for $(\phi_t, w_t)$ to solve for $(\alpha_t, R^d_t)$, and vice versa, at each node-point. In addition, computing expectations of $c_{t+1}$ at time $t$ for each node point requires an interpolation for $\alpha_{t+1}$. We start with a guess for each variable from the quasi-linear utility model, and iterate until the maximum percent change across the four endogenous variables from iteration to iteration is zero to 3 decimal places in absolute value.

After solving the model equations, we simulate 1,000 economies for 700 periods each in order to verify that the model remains within the assumed bounds for the endogenous state variables. Each simulation starts at the steady-state of the quasi-linear utility model.

To estimate the implied moments of the model, we compute averages over time and over the 1,000 simulated economies, throwing away the first 200 observations of each simulation to reduce dependence on the initial state.

To compute welfare for a change to $\zeta$, we again simulate 1,000 economies, but this time for 900 periods, throwing away the first 200 periods to reduce dependence on the initial state, and assuming $\zeta$ jumps immediately to a new value at $t = 400$. This implies no change to either state variable, but does change the endogenous policy functions $\alpha$, $\phi$, $w$, and $R^d$ as a function of the state. We compute the 1,000 simulations for 250 values of $\zeta$ between 0.03 and 0.20, and find maximum and minimum values of log $Y$ and $\omega$ that are common to all 100 values of $\zeta$ (plus the initial $\zeta_0$ economy). We then solve the model for each value of $\zeta$ on this common, enlarged grid; we use the same values of $\psi_0$ and $\psi_1$ in each economy. Were we to allow a different Chebyshev grid for the endogenous state variables for each value of $\zeta$, changes in the grid would result in spurious differences to welfare and would be evident in Figure 5 as jagged discontinuities.

For the model parameters reported in Table 2, we use $\psi_0 = -0.05$ and $\psi_1 = 1.28$, which
we obtain by regressing log\(Y\) on log\(\Psi\) after solving the model. For these values of \(\psi_0\) and \(\psi_1\), we find that for values of \(\zeta\) ranging from 3% to 20% we can solve the model on a grid of \((-1.2, 0)\) for log\(Y\) and \((-0.16, 0.1)\) for \(\omega\).