Financial Networks over the Business Cycle

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Abstract

This paper studies how the interconnectedness of financial networks evolves over the business cycle and shapes the economy’s exposure to occasional large-scale banking crises. Systemic risk emerges because of an endogenous correlation between intermediaries’ portfolios. Risk sharing reduces individual institutions’ default probabilities but increases the similarity of their exposures to shocks. Consistent with the empirical evidence, systemic crises burst at the end of credit booms when productive investment opportunities are depleted, intermediaries’ balance sheets are weak, and the financial system is densely interconnected. Under such circumstances, even a moderate negative shock can lead to widespread defaults due to magnified financial fragility.

JEL classification: D85, E32, E44, G01, G21

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1 Introduction

The events of the global financial crisis demonstrate that the financial system should be viewed as a complex network of interconnected nodes rather than a collection of isolated institutions. As recently argued by academics and policymakers, the network configuration shapes systemic risk, that is, the probability of simultaneous distress shared by many organizations.\(^1\) A growing body of research characterizes how the financial architecture affects the system’s capacity to absorb shocks. However, the evolution of financial interconnectedness over time and its contribution to a buildup of systemic risk in seemingly tranquil periods, crucial from the macroprudential regulation perspective, remains largely unexplored.

This paper constructs a tractable general equilibrium model of the dynamic interplay between real aggregates, intermediaries’ balance sheets, and financial interconnectedness to study the endogenous variation in systemic risk over time. In the model, banks raise funds from households and on the interbank market and extend credit to the real sector. Interbank debt is defaultable, and bankruptcy losses are priced rationally. To reduce their borrowing rates, banks diversify by investing in a finite number of risky primitive asset classes or projects. By engaging in risk sharing, banks become similarly exposed to underlying shocks and, thus, become interconnected. Because of common exposures, a negative project-specific surprise might result in widespread defaults, i.e. systemic crisis.\(^2\) Over the business cycle, time-varying credit supply and aggregate productivity govern the returns on the primitive assets, thereby affecting banks’ portfolio choices, interconnectedness and systemic risk.

Systemic crises do not burst at random in the model. Lending expansions, driven by positive but temporary aggregate shocks, saturate the economy with credit and gradually exhaust productive investment opportunities. When shocks fade away, marginal products of the underlying projects are depressed because of decreasing returns to scale. Intermediaries’ investments generate low payoffs, and their profit margins narrow down. To mitigate rising individual default probabilities, banks diversify more actively. Doing so drives up the correlation between their portfolios. The economy reaches the point of magnified financial fragility where a moderate negative shock can cause massive simultaneous defaults. As suggested by recent empirical evidence,

\(^1\)For example, Janet Yellen, in her speech at the American Economic Association in 2013, claims that “complex links among financial market participants and institutions are a hallmark of the modern global financial system” (Yellen, 2013). The importance of financial interconnectedness is also emphasized by Ben Bernanke (see, e.g., his testimony before the Financial Crisis Inquiry Commission, Bernanke, 2010).

systemic crises are typically preceded by credit booms and rising financial interconnectedness.\footnote{That large-scale financial crises follow credit booms is well established in the literature (e.g., Mendoza and Terrones, 2008 and Schularick and Taylor, 2012). Recent empirical evidence also points to a substantial increase in financial interconnectedness prior to the Great Recession (e.g., Billio, Getmansky, Lo, and Pelizzon, 2012, Hale, 2012, Cai, Eidam, Saunders, and Steffen, 2018). See Section 2 for a further discussion.}

More specifically, the financial system in this paper consists of two types of institutions, investing and noninvesting banks. Both types can invest in an inefficient riskless storage. On top of that, investing banks have access to a more productive risky technology. Such heterogeneity gives rise to the interbank market, where investing banks borrow from noninvesting ones. Each investing bank specializes in a particular asset class and has to bear a monitoring cost to invest in other projects. The optimal degree of portfolio diversification trades off monitoring expenses for a lower probability of individual bankruptcy and hence cheaper interbank debt.

In normal times underlying projects are sufficiently productive, so all funds within the financial sector are allocated to the risky technology. Expected payoffs to the investing banks’ portfolios are high, and interbank debt is cheap because of low default probabilities. The financial system is sparsely connected, because demand for diversification is limited. Strong intermediaries’ balance sheets and low interconnectedness make systemic crises unlikely. The network is therefore robust, i.e. a huge negative shock is required to trigger a large-scale banking crisis.

At the end of credit booms, risky projects generate below-average returns. Despite their profit margins being tight, investing banks are not willing to cut borrowing and slow the credit flow to the real sector. A more active diversification mitigates the counterparty risk and thus prevents the interbank borrowing rate from a sharp increase. Risk sharing redistributes financial intermediaries’ exposures from projects of their specialization to a broader pool of assets. While it does help investing banks to keep their individual default probabilities at modest levels, the financial system as a whole becomes fragile: affecting all entities in a similar way, even a moderate adverse project-specific shock can make many of them insolvent at once.

Large-scale banking crises, although painful, are a natural side effect of generally beneficial risk sharing. In the absence of diversification, the financial system is not connected. Investing banks do not simultaneously default but instead frequently go bankrupt in isolation. Facing high individual insolvency risks, they reduce their interbank borrowing and curb the supply of credit to the real sector. In fact, fully restraining banks from connecting and keeping the number of systemic crises at zero is suboptimal.

A natural question in this regard is whether the frequency of such events is efficient. In the model, an individual investing bank internalizes how its portfolio choice and default affect the rest of the economy.\footnote{For example, defaults of individual investing banks do not cause a domino effect (Acemoglu, Ozdaglar, and}
frequent systemic crises. The source of inefficiency is a pecuniary externality. Agents in the economy fail to internalize that credit expansions reduce the risky projects’ returns and, crucially, enhance intermediaries’ default probabilities. Therefore, the decentralized allocation is marked by overinvestment in the risky technology, inefficiently low returns, and an excessively connected, fragile financial system.

Recent financial innovations like securitization and credit derivatives (e.g., credit default swaps) have facilitated diversification for individual institutions, while intensifying the degree of their common exposures and contributed to lending boom (e.g., Shin, 2009, Stulz, 2010). The model interprets such innovations as a reduction in risk-sharing costs. This affects the economy in two ways. On the one hand, investing banks are more efficient in reducing their expected default losses. Their portfolios become better diversified and thus more correlated with each other. On the other hand, more accessible risk sharing expands investing banks’ borrowing capacities and improves the returns on households’ funds supplied to the financial sector. The credit supply grows, further depressing the productivities of the underlying projects and exacerbating the adverse effects of the pecuniary externality. Financial innovations in the model are destabilizing in the sense that they increase the number of systemic crises. However, the welfare implications are generally ambiguous.\textsuperscript{5}

The model is calibrated to match conventional macroeconomic moments as well as the frequency of systemic financial crises of about 1.7 per century (Romer and Romer, 2017). Besides the massive joint defaults of investing banks, the economy also experiences milder nonsystemic crises. They are marked by the bankruptcies of only those institutions whose portfolios are tilted toward projects hit by bad shocks. The two types of crises tend to happen under quite different conditions. Investing banks collapse together after prolonged credit booms during which financial interconnectedness and fragility are gradually increasing. Nonsystemic crises, on the contrary, are more likely in sparsely connected networks and are not normally preceded by unusual credit expansions. I find empirical support for these patterns in the data.

\textbf{Literature.} This paper builds on a rapidly growing literature focusing on contagion in financial networks. Early contributions by Kiyotaki and Moore (1997a), Allen and Gale (2000), and Freixas, Parigi, and Rochet (2000) present the first formal models of contagion in financial systems. More recently, the literature has identified several major channels through which granular idiosyncratic shocks might spread to the whole system, thereby causing aggregate problems (see Tahbaz-Salehi, 2015) or fire-sale spirals (Cifuentes, Ferrucci, and Shin, 2005). I also abstract from any herding behavior due to expected government interventions (Farhi and Tirole, 2012).

\textsuperscript{5}The view that financial innovations improve risk sharing is traditional in the literature (e.g., Allen and Gale, 1994). More recently, a potentially destabilizing role of financial innovations has been underscored by, among others, Allen and Carletti (2006), Gennaioli, Shleifer, and Vishny (2012), Brunnermeier and Sannikov (2014).
Cabrales, Gale, and Gottardi (2016) for a review). One is the domino effect: if institutions hold claims on each other, the failure of one organization might initiate a default cascade (Acemoglu et al., 2015, Glasserman and Young, 2015). Another channel is common exposures to the same underlying assets, as in Elliott et al. (2014) (their model also features the domino effect) and Cabrales et al. (2017).

This paper’s network modeling is most similar to that of Cabrales et al. (2017). However, whereas their study analyzes financial structures emerging under a wide set of shocks distributions, I focus on financial ties arising under sufficiently thin-tailed shocks and positive linking costs. Moreover, banks in my setting endogenously choose their liabilities.

More generally, in contrast to this strand of the literature, my paper presents a dynamic model of financial interconnectedness. While a static analysis is able to reveal that financial networks exhibit a robust-yet-fragile feature (Haldane, 2009), i.e. they are susceptible to occasional systemic crashes if hit by sufficiently negative shocks, only a dynamic model is able to shed light on how robust networks transform into fragile ones over time.

This paper is related to the literature introducing financial frictions into macroeconomic models, initiated by the classic works of Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997b), and Bernanke, Gertler, and Gilchrist (1999). These studies typically rely on log-linear approximations around the steady state to examine the role of financial accelerator for aggregate fluctuations. Nonlinear effects in economies with occasionally binding constraints are investigated by Mendoza (2010), He and Krishnamurthy (2012), and Brunnermeier et al. (2014). In my setting, financial crises are associated with painful recessions due to the simultaneous defaults of many institutions. This is reminiscent of Gertler and Kiyotaki (2015), who study runs on the whole banking sector.

Only a few recent papers attempt to explain boom-bust dynamics of credit around financial crises. In this respect, the works of Boissay, Collard, and Smets (2016) and Gorton and Ordoñez (2018) are most relevant. Like in my paper, in their settings credit expansions reduce a marginal product of the underlying production technology. When it becomes sufficiently low, a crisis bursts, either due to agency issues (Boissay et al., 2016) or due to production of information revealing the quality of the collateral (Gorton et al., 2018; see also their earlier work, Gorton and Ordoñez, 2014). In contrast, in my paper, crises are associated with insolvencies of

\footnote{Relatedly, Allen et al. (2012) and Babus and Farboodi (2018) analyze contagion due to information externalities in a setting with asset commonality. Informational contagion is also studied by Caballero and Sínsok (2013) and Alvarez and Barlevy (2014), among others. Additionally, contagion might arise because of fire sales (Cifuentes et al., 2005, Caccioli et al., 2014). A complementary strand of the literature studies contagion in large networks, usually relying on simulation techniques (e.g., Nier, Yang, Yorulmazer, and Alentorn, 2007, Gai, Haldane, and Kapadia, 2011, Gofman, 2017).}
interconnected financial institutions, which might simultaneously default even due to a shock to one relatively narrow sector. This is reminiscent of the worldwide Great Recession initiated by the U.S. subprime mortgage crisis (e.g., Brunnermeier, 2009).\footnote{The boom-bust dynamics also can arise due to learning about a new financial environment (Boz and Mendoza, 2014) or self-fulfilling expectations (Perri and Quadrini, 2018). Also related is the literature on bubbles in production economies (e.g., Martin and Ventura, 2012).}

The focus on banks’ borrowing decisions and financial interconnectedness relates this paper to Barattieri, Moretti, and Quadrini (2018). In their model, banks become interconnected by trading parts of their individual investments for a fully diversified interbank portfolio in order to expand their borrowing. My paper features two important differences. First, because of the granularity of the economy, systemic crises are triggered by nonaggregate surprises, and the degree of financial interconnectedness affects the probability of such events. Second, general equilibrium analysis underscores the role of endogenously changing interest rates for systemic risk.

The welfare analysis of pecuniary externalities shares similarities with Bianchi and Mendoza (2010), Jeanne and Korinek (2010), Bianchi (2011), and Boissay et al. (2016). My paper departs from the existing literature in that it studies the impacts of the externality on financial interconnectedness as well as its interactions with financial innovations.

The remainder of the paper is organized as follows. Section 2 provides some empirical evidence on the behavior of financial interconnectedness around the Great Recession. Section 3 lays out the model. Section 4 characterizes the model. Section 5 assesses the model numerically and illustrates its dynamic properties. Section 6 carries out the welfare analysis. Section 7 concludes.

2 Connectedness around the global financial crisis

Since the recent global crisis, a large empirical literature has attempted to construct measures of financial interconnectedness. This task is not straightforward. Institutions might become interrelated in various ways, from holding claims on each other’s payoffs to investing in similar assets. This section argues that various measures of interconnectedness exhibit similar time-series patterns. In particular, they tend to increase prior to the global financial crisis and recede afterward, suggesting a buildup of systemic risk during seemingly tranquil times.

The literature on systemic risk typically relies on comovements in asset prices to infer ties between financial institutions. Using principal component analysis and Granger-causality networks, Billio et al. (2012) document that U.S. banks, hedge funds, broker/dealers, and insurance companies became highly interrelated in the decade preceding the global financial crisis.
Figure 1: Four interconnectedness measures. All series are normalized to 1 in 2002 and smoothed using a 1-year moving average. The shaded area represents the Great Recession (NBER timing). Appendix C.1 gives full nonsmoothed, nonnormalized series. Panel (a) illustrates the industry/region-based overlap in the syndicated loan portfolios of the U.S. lead arrangers. Portfolios are considered overlapping if they have common exposures to a specific borrower industry and region. Source: Cai et al. (2018). Panel (b) illustrates the ratio of nonagency mortgage-backed securities and asset-backed securities over the total assets from the 100 largest U.S. bank holding companies. Source: FR Y-9C. Panel (c) illustrates the ratio of total liabilities net deposits (noncore liabilities) over the total assets from the 100 largest U.S. bank holding companies. Source: Barattieri et al. (2018) and FR Y-9C. Panel (d) illustrates the ratio of the total liabilities of all U.S. sectors over the total liabilities of the nonfinancial sector. Source: Greenwood et al. (2013) and U.S. Flow of Funds.

Diebold and Yilmaz, 2009, 2014). A few measures identify the tail dependency of individual institutions and the whole financial system (e.g., Huang, Zhou, and Zhu, 2009, Acharya, Pedersen, Philippon, and Richardson, 2017, Brownlees and Engle, 2017). These measures sharply rise in turbulent times, but do not capture a buildup of systemic risk prior to the recent financial crisis. As noted by Billio et al. (2012), this might be related to underrepresentation of large losses episodes in the precrisis data. Of particular interest here is forward-looking forward − ΔCoVaR by Adrian and Brunnermeier (2016), which has desired cyclical features.

Closely related to my paper are several studies attempting to measure the degree of asset commonality directly. Cai et al. (2018) document that interconnectedness due to the overlap in syndicated loan portfolios of the largest U.S. banks has been increasing between 2001 and 2007 but dropped during the crisis (panel (a) of Figure 1). Moreover, interconnectedness is

Ivashina and Scharfstein (2010) find that the share of syndicated loans retained by lead arrangers, a measure of loans’ ownership concentration, has been steadily decreasing since at least 1991 but spiked in 2008.
positively correlated with loan portfolio diversification and systemic risk measures in the cross-section. Coherent evidence is presented by Blei and Ergashev (2014), who consider correlations between the asset portfolios of major U.S. bank holding companies.

Securitization, created to facilitate credit risk sharing, is another contributor to portfolio similarity. During the run-up of the recent crisis, banks originated massive amounts of mortgage-backed and asset-backed securities. However, a large fraction of them were not transferred to outside investors. Instead, financial institutions traded them between each other and held them both on and off their balance sheets (e.g., Shin (2009) and panel (b) of Figure 1). As a result of multiple rounds of asset exchanges, financial market participants became exposed to similar underlying shocks. Relatedly, Nijskens and Wagner (2011) show that banks that start to use credit default swaps (CDS) and collateralized loan obligations (CLO), new credit risk transfer methods flourished prior to the Great Recession, experience increases in their market betas. Importantly, such increases come solely from higher correlation with the market, suggesting that risk sharing through holding CDS and CLO contribute to systemic risk. Similar effects of new credit risk transfer instruments are documented by Franke and Krahnen (2007) and Hänsel and Krahnen (2007).

A complementary strand of the literature considers the direct claims of financial institutions on each other. Panel (c) of Figure 1 uses the interconnectivity measure suggested by Barattieri et al. (2018). The measure is defined as banks’ liabilities held by other parties within the financial sector (noncore liabilities) over total assets (see also Hahm, Shin, and Shin, 2013 and Koijen and Yogo, 2016). A similar approach is used by Greenwood et al. (2013). Aiming to capture the number of steps involved in credit creation, the authors construct the credit intermediation index by dividing the total liabilities of all sectors by the liabilities of the nonfinancial sector only (panel (d) of Figure 1). A larger value of the index signals more lending between financial institutions. Similar time-series patterns (growth prior to the crisis and a reduction afterward) are observed for these types of interconnectedness measures.

Lastly, interconnectedness of global banking network is procyclical as well, both on the institutional (Hale, 2012) and on the country (Minoiu and Reyes, 2013) level.

### 3 Model

This section presents an infinite horizon model set in discrete time. By convention, time subscripts are omitted, and next-period variables are denoted by primes. Tildes mark random variables subject to intraperiod uncertainty.
The economy consists of a risk-averse, long-lived representative household and a risk-neutral, one-period banking sector. The household owns all assets but can only access production technologies through intermediaries. I abstract from any household-bank frictions. The household’s role is minimal: it funds banks, supplies labor to the real sector, and makes intertemporal consumption/savings decisions.

The banking sector includes a large number of ex ante identical institutions, with a small subset of them receiving risky investment projects ex post. Ex post heterogeneity gives rise to an intraperiod interbank market in which banks with investment opportunities borrow from the remaining ones. Interbank lending is subject to costly defaults. To mitigate expected default losses priced in the interbank rate, borrowers cross-invest in each other’s projects, forming risk-sharing linkages. These ties expose institutions to shocks to the investing projects of other banks. Common exposures make systemic crises associated with widespread defaults possible.

Sections 3.1–3.3 lay out the benchmark model and outline the key assumptions. Section 3.4 discusses the assumptions in more detail.

3.1 Banking sector

The banking sector consists of a finite number $N$ of islands. Each island is populated with $M$ banks. Banks are risk neutral and short lived. They are ex ante identical, and, therefore, at the beginning of each period, they raise the same amount of assets $a_o$ from the representative household. All banks have access to a riskless storage technology with a constant gross return $\rho_s$.

Banks are ex post heterogeneous. In particular, each period one institution per island becomes an investing bank (the remaining ones are noninvesting). The two types of institutions differ in their access to risky investment opportunities.\footnote{Two comments are worth making. First, the assumption of one-period banking sector is crucial to keep the number of state variables small. It would be identical to assume instead that banks are long-lived but their types are reshuffled every period, similar to Gertler and Karadi (2011). Second, the storage return $\rho_s$ is constant. As will be discussed at the end of Section 4.2, this is not crucial for the results.}

3.1.1 Production technology

Each island is associated with an investment project. A project of island $i \in \{1, \ldots, N\}$ (or simply project $i$) requires capital $k_i$ and labor $l_i$ to produce

$$\tilde{y}_i = zk_i^{\eta_1}l_i^{1-\eta} + (1 - \delta - \tilde{x}_i)k_i,$$
where $\eta$ is the capital share, $\delta$ is the depreciation rate, $\tilde{x}_i$ is a project-specific depreciation shock unknown at the beginning of the period, and $z$ is an aggregate productivity realized at the beginning of the period. \(^{10}\) \log z$ evolves according to a standard AR(1) process:

$$\log z' = \rho_z \log z + \sigma_z \epsilon', \quad \epsilon' \overset{iid}{\sim} N(0,1). \quad (1)$$

Project-specific shocks $\tilde{x}_i$, $i \in \{1, \ldots, N\}$ are treated as rare and large. Each period one and only one of $N$ projects receives a shock. \(^{11}\) The shock size follows a distribution with a sufficiently smooth cumulative distribution function $\Phi(\cdot)$. The support of $\Phi(\cdot)$ is $0 < \bar{x} < \bar{x} \leq \infty$. Because I am interested in contagion caused by adverse project-specific shocks, I focus on negative surprises and set the lower bound of the support to zero.

Labor is hired on a competitive market at the wage rate $w$. Solving the static labor optimization problem, the project’s operating profit is

$$\max_{l_i} \tilde{y}_i - w l_i = (R - \tilde{x}_i)k_i, \quad (2)$$

where

$$R = z\eta \left( \frac{(1 - \eta)z}{w} \right)^{\frac{1-\eta}{\delta}} + 1 - \delta. \quad (3)$$

$R - \tilde{x}_i$ is the return on capital invested in project $i$. Notice that $R$ does not depend on any island-specific variables.

### 3.1.2 Interbank market and risk-sharing linkages

While the storage technology is available to all banks, only investing ones have access to the risky underlying projects. Particularly, an investing bank from island $i$ has an expertise in project $i$ (i.e., its own project) and can invest in it at no cost. Moreover, it can allocate some of its assets to the remaining projects at an upfront cost of $f > 0$ per unit of investment, which includes any monitoring expenses. On the contrary, noninvesting banks cannot access the risky technology directly. This heterogeneity gives rise to the intraperiod interbank market in which investing banks borrow from noninvesting ones.

\(^{10}\)Assuming different timing of shocks keeps the analysis as simple as possible. The results are unchanged if the aggregate productivity $z$ is realized at the same time as project-specific shocks. See Appendix E.1 for more details.

\(^{11}\)At the expense of tractability but without changing the qualitative results of the paper, the analysis can be straightforwardly extended to handle a richer shock structure.
Assumption 1 *Interbank borrowing happens within the islands.*

Assumption 1 implies that the economy features a segmented interbank market, where each segment is represented by an island. This assumption captures the idea that establishing new interbank links might be costly; these costs prevent any cross-island borrowing. An island in the model includes banks with established relationships that can contact each other at no cost.\(^\text{12}\) Section 3.4.2 discusses the assumption in more detail.

The interbank market of island \(i\) is characterized by a large number \(M - 1\) of lenders (noninvesting banks) lending to one borrower (investing bank) at a rate of \(\rho_i\). I assume that the latter is a monopolist in this market. As a result, noninvesting banks in expectation earn the storage return \(\rho_s\).

Assumption 2 *Investing banks extract all the surplus from the interbank borrowing.*

Since returns on the investment projects are uncertain, interbank debt is subject to defaults that are associated with real resource losses. This is rationally priced in the interbank borrowing rate \(\rho_i\). Thus, even despite their risk neutrality and positive monitoring costs, investing banks are willing to invest in projects of other islands in order to diversify their individual risks and reduce costly default probabilities. As a result of cross-island investments, an investing bank \(i\) holds a portfolio \(\{\omega_{ij}\}_{j=1}^{N}\), \(\sum_{j=1}^{N} \omega_{ij} = 1\), where \(\omega_{ij} \geq 0\) represents the fraction of its assets under management \(a_i\) invested in project \(j\). Here, \(a_i\) consists of assets directly raised from the household and borrowed on the interbank market. The upfront cost of portfolio formation is \(fa_i\sum_{j \neq i} \omega_{ij}\).

The \(N \times N\) matrix of portfolio holdings \(B = \{\omega_{ij}\}_{i,j=1}^{N}\) describes the structure of linkages between investing banks. In the network terminology, \(B\) is an adjacency matrix of a weighted directed graph. The elements of \(B\) represent the financial institutions’ exposures to shocks to underlying projects. For example, a large value of \(\omega_{ij}\) implies that investing bank \(i\) is strongly affected by an adverse shock to project \(j\). As long as \(\omega_{ij} > 0\ \forall i \in \{1, \ldots, N\}\), a shock to project \(j\) affects all investing banks and can potentially lead to the simultaneous defaults of many institutions. Importantly, defaults themselves do not trigger cascading losses among investing banks, because they do not hold assets directly related to the payoffs of other intermediaries. Section 3.4.3 further comments on the contagion mechanism of the model.

\(^{12}\)Notice that the degree of the interbank market segmentation (i.e., the number of islands) coincides with the number of underlying risky projects \(N\). In principle, they do not have to be the same (like in, e.g., Elliott et al., 2014). The model can be extended to handle a more general case. I verify that such a generalization does not alter the main results in any substantial way (unreported).
3.1.3 Investing bank’s problem

The problem of investing bank $i$ can be now formulated. Because of model symmetry, island-specific indices are omitted where possible. Each investing bank starts a period with assets $a_o$, which are directly raised from the representative household. Additionally, it borrows $a_b$ at the rate $\rho$ on the interbank market and chooses an investment portfolio $\{\omega_{ij}\}_{j=1}^N$ in order to maximize its expected payoff. Assume that project $j$ is hit by an adverse shock of size $x$. At the end of the period, investing bank $i$ is insolvent when its portfolio payoff is below the face value of its debt,

$$a \left[ \sum_{s=1}^{N} R \omega_{is} - \omega_{ij} x \right] - \rho a_b < 0,$$

where $a = a_o + a_b$ is the total assets under the investing bank’s $i$ management. Because $\sum_{s=1}^{N} \omega_{is} = 1$, this expression can be rewritten as

$$x > \frac{R - \rho \mu}{\omega_{ij}},$$

(4)

where $\mu = \frac{a_b}{a}$ is the ratio of borrowed assets to total assets. Inequality (4) demonstrates that an adverse shock to project $j$ leads to default by investing bank $i$ if either the bank’s profit margin, captured by the spread $R - \rho \mu$, is small or its exposure $\omega_{ij}$ to such shock is large.

Investing bank $i$ maximizes an expected payoff to its portfolio under limited liability, net of risk-sharing costs.

$$\max_{\rho, \mu, \{\omega_{ij}\}_{j=1}^{N}} \frac{1}{1 - \mu} \mathbb{E}_x \left[ \frac{1}{N} \sum_{j=1}^{N} (R - \omega_{ij} x - \rho \mu) I \left\{ x \leq \frac{R - \rho \mu}{\omega_{ij}} \right\} - f \sum_{j \neq i} \omega_{ij} \right],$$

s.t. $\sum_{j=1}^{N} \omega_{ij} = 1, \quad \omega_{ij} \geq 0 \forall j \in \{1, \ldots, N\}$.

(5)

Here, $\mathbb{E}_x$ denotes the expectation with respect to the size of project-specific shock $x$. The first constraint requires portfolio weights to sum up to 1. The second set of constraints restricts short-selling.

11Notice that the diversification costs are paid by investing banks’ shareholders (the representative household) prior to any interbank relationships forming. Appendix E.2 elaborates on the version of the model in which $f$ is paid by investing banks ex post and thus directly affects their default cutoffs. The results are robust to this alternative modeling approach.
Investing bank $i$ is subject to the nondeviation constraint of its lenders:

$$
\rho_s \leq \mathbb{E}_x \left[ \rho \frac{1}{N} \sum_{j=1}^{N} \mathbb{I} \left\{ x \leq \frac{R - \rho \mu}{\omega_{ij}} \right\} + \frac{1}{\mu} \frac{1}{N} \sum_{j=1}^{N} (R - \omega_{ij} x - \theta) \mathbb{I} \left\{ x > \frac{R - \rho \mu}{\omega_{ij}} \right\} \right].
$$

(6)

In case of no default, each lender receives $\rho$. In case of default, lenders split the portfolio of their borrower equally. Default is subject to a loss of a fraction $\theta \geq 0$ of assets under the investing bank’s management.

Finally, each investing bank can operate at most all assets within its island. Therefore, it is subject to the following borrowing constraint:

$$
\mu \leq \bar{\mu} = \frac{M - 1}{M}.
$$

(7)

In equilibrium, the fraction $\frac{1 - \bar{\mu}}{1 + \bar{\mu}}$ of all assets in the economy is invested in the risky projects, and the remaining share $1 - \frac{1 - \bar{\mu}}{1 + \bar{\mu}}$ goes to the riskless storage technology.

To summarize the description of the banking sector, Figure 2 shows its structure and gives the timing of the intraperiod events.

### 3.2 Representative household

The representative household starts a period with assets $a_{hh}$ and rents them to the financial sector at the rate of $r$ (panel (a) of Figure 2), defined as an average return generated by all institutions within the banking sector (see expression (9) below). It also supplies labor $l$ for the wage $w$. At the end of a period, the household consumes and decides on the amount of assets in the next period $a'_{hh}$ (panel (f) of Figure 2). The household solves

$$
V(a_{hh}, \Omega) = \max_{a'_{hh}, c, l} \frac{1}{1 - \psi} \left( c - \frac{1}{1 + \nu} l^{1+\nu} \right)^{1-\psi} + \beta \mathbb{E} [V(a'_{hh}, \Omega')],
$$

s.t. $a'_{hh} = r(\Omega)a_{hh} + w(\Omega)l - c + \chi(\Omega), \Omega' = \Omega'(\Omega).
$$

(8)

The household is endowed with GHH (Greenwood, Hercowitz, and Huffman, 1988) preferences over consumption and labor with the inverse Frisch labor supply elasticity of $\nu$, the inverse elasticity of intertemporal substitution $\psi$, and the time-discounting factor $\beta$.\footnote{Notice that $\psi > 0$ implies that the household is risk averse, whereas bankers are assumed to be risk-neutral profit maximizers. While this assumption is not crucial for the results (see Appendix E.3), it substantially simplifies both theoretical analysis and numerical solution. Section 3.4.1 discusses the assumption in more detail.} $\chi = \chi(\Omega)$ represents the total monitoring costs paid by investing banks to the household in order to establish...
risk-sharing links.\textsuperscript{15} Finally, $\Omega$ is the set of the aggregate state variables whose evolution $\Omega'(\Omega)$ is taken by the household as given.

\subsection{3.3 Equilibrium}

The model has three state variables: the stock of assets $A$, aggregate productivity $z$, and the size of project-specific shock $x$, $\Omega = (A, z, x)$. By symmetry, the size of a project-specific shock, but not its location, matters for the aggregates. Moreover, investing banks’ portfolio weights are symmetric, $\omega_{ij}(\Omega_{-x}) = \omega_{ji}(\Omega_{-x})$, $\forall i, j \in \{1, \ldots, N\}$. Because project-specific shocks are realized after banks made their choices, a relevant set of the state variables for them is $\Omega_{-x} = (A, z)$.

\textsuperscript{15}For tractability, monitoring expenses are assumed to enter the household’s budget constraint in a lump-sum fashion. Alternatively, they can be modeled as a part of the household’s labor income. This would complicate the analysis, because the aggregate supply of labor then would be split between the real (risky projects) and banking (monitoring) sectors.
Moreover, in the absence of a wealth effect on the labor supply, the wage rate does not depend on \( x \) as well, \( w = w(\Omega - x) \). A decentralized recursive equilibrium consists of a set of quantities and prices described below.

(i) Investing banks’ decisions \( \mu(\Omega - x), \rho(\Omega - x), \{\omega_{ij}(\Omega - x)\}_{j=1}^{N} \) solve (5) subject to (6) and (7).

(ii) The value function \( V(a_{hh}, \Omega) \) and policy functions \( c(a_{hh}, \Omega), l(a_{hh}, \Omega - x), a'_{hh}(a_{hh}, \Omega) \) of the household solve (8) taking the prices \( r(\Omega), w(\Omega - x) \), the aggregate law of motion \( \Omega'(\Omega) \), and transfer \( \chi(\Omega - x) = A_{1} - \bar{\mu}_{1} \sum_{j \neq i} \omega_{ij}(\Omega - x) \) as given.

(iii) Capital invested in each project is \( k(\Omega - x) = a_{0}(A) \frac{a_{0}(A)}{1 - \mu(\Omega - x)} \), where \( a_{0}(A) = \frac{A}{N \cdot M} \).

(iv) The wage \( w(\Omega - x) \) clears the labor market, \( L(\Omega - x) = N \cdot l^{d}(\Omega - x) = l(A, \Omega - x) \), where \( l^{d}(\Omega - x) \) solves (2). The return on the household’s assets \( r(\Omega) \) is

\[
r(\Omega) = \left( 1 - \frac{1 - \bar{\mu}}{1 - \mu(\Omega - x)} \right) \rho_{s} + \frac{1 - \bar{\mu}}{1 - \mu(\Omega - x)} \left[ R(\Omega - x) - \frac{1}{N} x - \theta \frac{N^{d}(\Omega)}{N} - f(1 - \omega_{ii}(\Omega - x)) \right],
\]

where \( R(\Omega - x) \) is given by (3) and \( N^{d}(\Omega) \) is the number of investing banks in default, \( N^{d}(\Omega) = \sum_{i=1}^{N} \mathbb{1} \left\{ x > \frac{R(\Omega - x) - \mu(\Omega - x)}{\omega_{ij}(\Omega - x)} \right\} \).

(v) Individual decisions are consistent with the aggregate law of motion, \( a'_{hh}(A, \Omega) = A'(\Omega) \).

(vi) Aggregate goods market clear, \( A'(\Omega) = \left( 1 - \frac{1 - \bar{\mu}}{1 - \mu(\Omega - x)} \right) \rho_{s} A + z \left( \frac{1 - \bar{\mu}}{1 - \mu(\Omega - x)} A \right)^{\eta} L(\Omega - x)^{1 - \eta} + \frac{1 - \bar{\mu}}{1 - \mu(\Omega - x)} \left[ 1 - \delta - \frac{1}{N} x - \theta \frac{N^{d}(\Omega)}{N} \right] - C'(\Omega) \).

### 3.4 Discussions of assumptions

This section discusses several key assumptions of the model.

#### 3.4.1 Household’s and bankers’ attitudes to risk

In the model, the representative household has incentives to intertemporally smooth consumption, \( \psi^{-1} < \infty \). Because its utility function is from the constant relative risk aversion family, it automatically makes the household risk averse. At the same time, banks are assumed to be risk-neutral expected payoff maximizers.\(^{16}\) They do not weigh states of the world with the stochastic discount factor of their shareholders (the household). Although such an assumption

\(^{16}\)Risk neutrality of bankers/entrepreneurs is a typical assumption in the literature incorporating financial frictions into macroeconomic models (e.g., ?, Bernanke et al., 1999).
makes their problem tractable, it might raise a question about why bankers are not acting on behalf of the household.

A vast literature studies whether managers’ and stakeholders’ objectives are aligned. Managers of firms with mixed financial structures ignore the interests of outside equity and debt holders and take excessive risks from a social perspective (Jensen and Meckling, 1976). Imperfect contracts and compensation schemes, along with agency problems, might result in excessive risk-taking even from the shareholders’ point of view. Mutual funds deviate from maximizing risk-adjusted returns due to inflow-related benefits (e.g., Chevalier and Ellison, 1997). Option-like compensation schedules can induce excessive risk taking by managers (Lambert, 1986).

The main results are largely unaffected if objectives of the household and bankers are aligned (see Appendix E.3). Even though I am unable to characterize the solution analytically, the main features of the benchmark model are preserved.

3.4.2 Structure of the financial system

In the model, the financial system is two tiered. Noninvesting banks do not have access to efficient production technology and lend their funds to investing banks. The latter then invest in multiple projects on their behalf. Therefore, the interbank market exhibits a well-known core-periphery structure (e.g., Boss, Elsinger, Summer, and Thurner, 2004, in ’t Veld and van Lelyveld, 2014, Craig and von Peter, 2014, Craig and Ma, 2018). Only a few connections exist between the core and the periphery (each noninvesting bank lends to only one investing bank), and no ties exist within the periphery. The core is densely connected, because all investing banks hold diversified portfolios.

The model abstracts from explicitly modeling why the core-periphery structure arises endogenously. I instead assume that noninvesting banks within one island are restricted to lend to only one investing bank. The literature provides a number of microfoundations for this structure. A partial list includes a reduction in monitoring (Craig et al., 2018), intermediation (Farboodi, 2015) expenses, and inventory risk (Wang, 2016).

---

17Since the model is symmetric, the interbank rates are the same across different islands. Even if noninvesting banks could contact investing banks from other islands at no cost, they would not find it profitable to do so. Generally speaking, if cross-island borrowing was possible, the equilibrium interbank rate would depend on how the surplus is split between noninvesting and investing institutions. For example, the same interbank rate would arise if the surplus was fully extracted by the latter party, like in the benchmark analysis.
3.4.3 Contagion mechanism

In the framework of this paper, investing banks are interconnected through common portfolio holdings. Financial contagion happens when a large adverse shock to one underlying project affects all institutions and results in widespread bankruptcies (systemic crises), as in for example Cabrales et al. (2017). Importantly, investing banks do not hold any securities directly related to the payoffs of other intermediaries (e.g., their debt or their equity). Defaults of other institutions within the financial system thus do not cause direct problems to investing banks. Only within islands, where bankruptcy losses propagate from the core to periphery institutions, does the domino effect (e.g., Acemoglu et al., 2015) play a role.

As discussed in the review paper of Cabrales et al. (2016), the relation between interconnectedness and probability of systemic crisis is similar regardless of the underlying contagion mechanism (see, in particular, result 8 of their study): highly integrated systems are more susceptible to joint breakdowns. If individual players are strongly exposed to each other or to a common set of underlying shocks, they are more likely to crash together.\textsuperscript{18} Incorporating the domino effect into my setting, like in Elliott et al. (2014), is likely to make a densely connected interbank network even more susceptible to systemic collapses. Fragility due to commonality in asset holdings is also exacerbated by fire sales (e.g., Caccioli et al., 2014).

The evidence for whether the domino effect is an important contributor to systemic risk is mixed (see Upper (2011) for a survey). Comparing the relative strengths of the domino effect and the common exposures contagion mechanisms in the Austrian banking system, Elsinger, Lehar, and Summer (2006) find that the latter is a dominant source of systemic risk. The crucial role of common exposures for financial crises is also emphasized by Borio (2003).

4 Characterization

This section outlines how the interconnectedness of the financial system and systemic risk interact with the dynamic choices of the representative household. Section 4.1 characterizes the structure of investment banks’ portfolios. Section 4.2 analyzes the static interbank problem and establishes several comparative statics results regarding the dependence of connectedness and systemic risk on the aggregate state variables. Section 4.3 discusses the interplay between the representative household’s intertemporal decisions and the interbank outcomes. Appendix A

\textsuperscript{18}Here, the strength of exposures is understood as the intensity with which existing links are utilized, not the network structure (e.g., ring or complete). In line with the data, in this paper, the core of the financial system features a complete network (all elements of the adjacency matrix $B$ are positive). However, the connectedness (intensity of links) varies.
contains all proofs.

### 4.1 Network structure

I start by characterizing the structure of investing banks’ portfolios. In what follows, the distribution of project-specific shock size \( \Phi(x) \) is assumed to be sufficiently concave,

**Assumption 3** \( x \frac{\partial^2 \Phi}{\partial x^2} + 2 \frac{\partial \Phi}{\partial x} < 0 \).

As shown in Appendix B, Assumption 3 holds for several relevant loss distributions, including Pareto.

Under Assumption 3, diversification is useful for investing banks to reduce individual default probabilities. Proposition 1 follows.

**Proposition 1** As long as linking cost \( f \) is sufficiently small, the constraints \( \omega_{ij} \geq 0 \) do not bind \( \forall j \). Moreover, investing bank \( i \) invests a fraction \( \omega_i = \alpha \geq \frac{1}{N} \) of assets under its management in project \( i \) and invests equal shares of the remaining assets in other projects, \( \omega_{ij} = \frac{1-\alpha}{1-N} \leq \frac{1}{N}, \forall j \neq i \).

An investing bank fully internalizes how its expected bankruptcy losses are priced into the interbank rate \( \rho \), so it is willing to minimize them (subject to the diversification cost). The default probability of investing bank \( i \) is

\[
p^d_{\text{ind}} = \frac{1}{N} \mathbb{E}_x \left\{ \frac{R - \rho \mu}{\omega_{ij}} \right\} = \frac{1}{N} \sum_{j=1}^{N} h_1(\omega_{ij}), \text{ where } h_1(\omega_{ij}) = 1 - \Phi \left( \frac{R - \rho \mu}{\omega_{ij}} \right).
\]

Assumption 3 guarantees that \( h_1(\omega_{ij}) \) is convex. Since \( \sum_{j=1}^{N} \omega_{ij} = 1 \), \( p^d_{\text{ind}} \) is minimized when a portfolio is evenly exposed to the underlying shocks. In other words, the default probability can be reduced through diversification.

All projects yield the same diversification benefits. For investing bank \( i \), allocating resources toward all but its own project \( i \) is subject to equal nonnegative costs. Hence, the fraction of assets invested in project \( i \) is (weakly) larger than in all the remaining ones, \( \omega_i = \alpha \geq \omega_{ij} = \frac{1-\alpha}{1-N-1}, \forall j \neq i \). All off-diagonal elements of the adjacency matrix \( B \) equal to \( \frac{1-\alpha}{1-N-1} \) and a vector of \( \alpha \) is on its main diagonal:

\[
B = \begin{pmatrix}
\alpha & \frac{1-\alpha}{N-1} & \ldots & \frac{1-\alpha}{N-1} \\
\frac{1-\alpha}{N-1} & \alpha & \ldots & \frac{1-\alpha}{N-1} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{1-\alpha}{N-1} & \frac{1-\alpha}{N-1} & \ldots & \alpha
\end{pmatrix}.
\]
\( \alpha \) captures the strength of investing banks’ cross-exposures. If \( \alpha \) is close to one, portfolios are weakly diversified, and the network defined by \( B \) is sparsely connected. Investing banks go bankrupt predominantly because of shocks to their own projects. On the contrary, when \( \alpha \) approaches its minimum value of \( \frac{1}{N} \), ties between investing banks are strong. Shocks to individual projects affect all institutions in a similar way. Contagion and systemic crises are more likely. Below, \( IC(\alpha) = \frac{1-\alpha}{1-\frac{1}{N}} \in [0,1] \) is referred to as the interconnectedness of the financial system. Corollary 1 follows.

**Corollary 1** The number of investing banks in default \( N^d \) is

\[
N^d = \begin{cases} 
0, & x \leq \frac{R-\rho u}{\alpha}, \\
1, & x \in \left( \frac{R-\rho u}{\alpha}, \frac{R-\rho u}{\frac{1}{N}-1} \right), \\
N, & x > \frac{R-\rho u}{\frac{1}{N}-1}.
\end{cases}
\]

Hereafter, events with \( N^d = 1 \) and \( N^d = N \) are called nonsystemic and systemic financial crises, respectively.

### 4.2 Connectedness and systemic risk

This section analyzes the solution to the static investing banks’ problem. By Proposition 1, investing bank \( i \) optimally chooses \( \omega_{ii} = \alpha \geq \frac{1}{N} \) and \( \omega_{ij} = \frac{1-\alpha}{N-1} \leq \frac{1}{N} \forall j \neq i \). Therefore, the default probability of an investing bank is

\[
p_{d,\text{ind}} = \frac{1}{N} \left( 1 - \Phi \left( \frac{R-\rho u}{\alpha} \right) \right) + \frac{N-1}{N} \left( 1 - \Phi \left( \frac{R-\rho u}{\frac{1-\alpha}{N-1}} \right) \right),
\]

where \( p_{d,\text{own}} \) and \( p_{d,\text{oth}} \) are the probabilities of becoming insolvent because of shocks to its own and other projects, respectively. Clearly, \( \frac{\partial p_{d,\text{own}}}{\partial \alpha} > 0 \) and \( \frac{\partial p_{d,\text{oth}}}{\partial \alpha} < 0 \). Diversification makes defaults due to its own shocks less likely at the cost of elevated probability to go bankrupt because of shocks to other projects. Under Assumption 3, the former effect dominates, \( \frac{\partial p_{d,\text{ind}}}{\partial \alpha} > 0 \), and diversification reduces expected default losses.

While diversification helps individual institutions protect themselves against costly defaults, it also increases the chances of a systemic collapse. When the degree of portfolio overlap is sufficiently high, a shock to one project is more likely to trigger the simultaneous failure of all investing banks. Because all institutions allocate at least a fraction \( \frac{1-\alpha}{N-1} \) of their assets toward
each project, a joint default materializes when \( x > \frac{R - \rho \mu}{N - 1} \), which happens with probability

\[
p^d_{\text{syst}} = 1 - \Phi \left( \frac{R - \rho \mu}{\frac{1}{1 - \alpha} N - 1} \right).
\]

Clearly, \( \frac{\partial p^d_{\text{syst}}}{\partial \alpha} < 0 \). Contagion is more likely in a densely connected network ceteris paribus.

As diversification is costly, there is always a certain degree of heterogeneity across investing banks’ portfolios. Hence, there exists a range of project-specific shock sizes such that a shock within this range causes bankruptcy of only the investing bank whose project of specialization is hit (Corollary 1). The probability of such a nonsystemic crisis is

\[
p^d_{\text{nonsyst}} = \Phi \left( \frac{R - \rho \mu}{N - 1} \right) - \Phi \left( \frac{R - \rho \mu}{\alpha} \right).
\]

It is straightforward to see that \( \frac{\partial p^d_{\text{nonsyst}}}{\partial \alpha} > 0 \). Highly connected systems feature more homogeneous portfolios of individual institutions, resulting in fewer incidences of nonsystemic events.

In equilibrium, the variables affecting default probabilities, \( \alpha \) and the spread \( R - \rho \mu \), are set endogenously. Below, I investigate comparative statics of these variables with respect to changes in the aggregate state variables \( A \) and \( z \). I start by analyzing the case in which the underlying projects are sufficiently productive, so that investing banks are willing to borrow all funds within their islands, \( \mu = \bar{\mu} \). At the end of this section, I consider the region of the state space in which the underlying projects are unproductive and the inequality (7) is slack, \( \mu < \bar{\mu} \).

In what follows, I make an additional assumption about the shape of the project-specific shock size distribution,

**Assumption 3’**

\[
x^2 \frac{\partial^3 \Phi}{\partial x^3} + 4x \frac{\partial^2 \Phi}{\partial x^2} + 2 \frac{\partial \Phi}{\partial x} > 0.
\]

Assumption 3’ guarantees that diversification becomes more attractive when investing banks’ balance sheets weaken. In particular, \( \frac{\partial^2 p^d_{\text{ind}}}{\partial \xi \partial \alpha} < 0 \), where \( \xi = R - \rho \mu \); a reduction in the spread \( \xi \) makes the individual default probability \( p^d_{\text{ind}} \) more sensitive to changes in \( \alpha \), that is, diversification. Appendix B verifies that Assumption 3’ holds for several loss distributions, including Pareto.

Under Assumptions 3 and 3’, Proposition 2 establishes how investing banks’ choices are affected by changes in the aggregate state variables \( A \) and \( z \).

**Proposition 2** Assume that condition (7) holds as equality, \( \mu = \bar{\mu} \). Then \( \alpha(A, z) \) is decreasing in \( A \) and increasing in \( z \), and the spread \( \xi(A, z) = R(A, z) - \rho(A, z) \bar{\mu} \) is decreasing in \( A \) and increasing in \( z \).
The aggregate state variables affect the interbank allocation through the projects’ return $R$. Clearly, aggregate productivity $z$ is positively related to $R$. As long as $\mu = \bar{\mu}$, an increase in the amount of assets $A$ translates into higher investment in the risky projects. Since capital and labor are complementary in the production function, the labor demand goes up. Wages rise, and the return $R$ goes down (see Equation (3) and Appendix A.2 for formal proofs).

From the perspective of investing banks, $R$ is a nonrandom part of their returns on assets. Holding everything else equal, lower $R$ reduces the spread $R - \rho \bar{\mu}$ and increases the chance of costly defaults. Noninvesting banks, lenders on the interbank market, require compensation for the additional risk. The interbank rate $\rho$ goes up. To prevent their costs of funding from a sharp rise, investing banks (borrowers) diversify more.

Therefore, a change in the projects’ return $R$ affects the fragility of the financial system in two ways, which are illustrated by Figure 3. First, lower $R$ narrows the spread $R - \rho \bar{\mu}$ and makes both systemic and nonsystemic crises more likely (the latter is true by Assumption 3). Second, cross-exposures of investing banks become stronger. Importantly, the degree of portfolio overlap rises precisely at the time when insolvency risks are already large. More dispersed investments prevent the probability of individual default $p^d_{\text{ind}}$ from steeply increasing. On the other hand, contagion and joint collapses become more likely. Corollaries 2.A and 2.B follow.

**Corollary 2.A** As long as $\mu = \bar{\mu}$, the probability of systemic default $p^d_{\text{syst}}(A, z)$ is increasing in $A$ and decreasing in $z$. Individual and nonsystemic default probabilities, respectively $p^d_{\text{ind}}(A, z)$ and $p^d_{\text{nonsyst}}(A, z)$, and the interbank rate $\rho(A, z)$ are generally nonmonotone in $A$ and $z$.

**Corollary 2.B** Assume that $\mu = \bar{\mu}$. Define a threshold in the amount of assets $A^*(z, x)$ as

$$x = \frac{R(A^*(z, x), z) - \rho(A^*(z, x), z)\bar{\mu}}{\frac{1 - \alpha(A^*(z, x), z)}{N-1}}.$$

Simultaneous defaults of all investing banks happen if and only if $A > A^*(z, x)$. Moreover, $A^*(z, x)$ increases in $z$ and decreases in $x$.

Figure 4 summarizes the results of comparative statics with respect to the total amount of assets $A$ and aggregate productivity $z$. As long as condition (7) holds as an equality, the fraction of assets invested in the risky projects is $\frac{1 - \bar{\mu}}{1 - \mu} = 1$ (panel (a), the horizontal parts of the lines), the variables respond to changes in $A$ and $z$ in line with Proposition 2 and Corollary 2.A. When the return to the underlying projects $R$ is sufficiently low, the constraint $\mu \leq \bar{\mu}$ is slack. In this regime, the risky projects are unproductive, and investing banks optimally choose to reduce their borrowing. As a result, a nonzero amount of the economy’s total assets $A$ is allocated to
the storage technology. A further increase in $A$ barely affects the amount of assets invested in the risky projects and hence the return $R$. The default probabilities, spread, interbank rate, and interconnectedness all flatten.\footnote{Recall that, in the event of default, lenders equally split the recovered value of their borrower’s portfolio. Return to lenders in those states of the world is proportional to $\frac{a_s + a_o}{a_s} = \frac{1}{\mu}$ (see expression (6)). Therefore, a reduction in $\mu$ increases the expected return to noninvesting banks, who start to charge a lower rate $\rho$ on the interbank market. As a result, default probabilities $p_{\text{syst}}^d$ and $p_{\text{ind}}^d$ decrease somewhat.} Importantly, even in the states with $\mu < \bar{\mu}$ (high $A$, low $z$), the financial system stays densely connected, whereas systemic risk remains elevated.

Finally, it is worth commenting on the role of the constant storage return for Proposition 2 and Corollaries 2.A and 2.B. Because the productivity of the banks’ outside option does not depend on the state of the economy, a reduction in the projects’ return $R$ necessarily narrows the spread $R - \rho \bar{\mu}$. As long as the spread shrinks, investing banks become riskier and start to diversify more actively. Interconnectedness goes up, further increasing the probability of systemic crisis. While the assumption $\rho_s = \text{const}$ is sufficient for these results to hold, it is far from necessary. For example, Appendix A.4 verifies that even if the storage return varies one-to-one with $R$, the analog of Proposition 2 can be established.
4.3 Model dynamics

Even though the interbank problem is static, the financial institutions’ optimal choices, being functions of the state variables, are changing over time in a nontrivial way. This section discusses how the representative household’s intertemporal choices and the time-varying aggregate productivity \( z \) interact with the interbank outcomes.

Figure 5 presents a set of stylized asset accumulation policies \( A'(A, z, x) \) for two values of aggregate productivity \( z \), \( z_{\text{low}} < z_{\text{high}} \) and two values of project-specific shock size \( x \), \( x_{\text{low}} < x_{\text{high}} \). The discontinuities in policies at \( A^*(z_{\text{low}}, x_{\text{high}}) \) and \( A^*(z_{\text{high}}, x_{\text{high}}) \) are associated with joint failures of all investing banks, where the threshold \( A^*(z, x) \) is defined in Corollary 2.B.\(^{20}\)

Consider the economy starting at point \( O \), which is the steady state for \( z = z_{\text{low}} \) and \( x = x_{\text{low}} \). At this point, the total amount of assets is relatively low, and the risky projects are productive. Risk sharing is limited because of low default probabilities of individual investing banks. In states with highly productive projects and a sparsely connected financial network, systemic collapses are unlikely. An adverse realization of a project-specific shock, \( x = x_{\text{high}} \), brings the economy to \( B_O \). This is not exceptionally painful, because the financial system manages to avoid costly joint failures, \( A_O < A^*(z_{\text{low}}, x_{\text{high}}) \).

Consider now the economy at point \( C \). Here, the aggregate productivity is the same as at \( O \), \( z = z_{\text{low}} \). However, the amount of assets is significantly larger (\( A_C > A^*(z_{\text{low}}, x_{\text{high}}) > A_O \)),

\(^{20}\)For expositional purposes, the borrowing constraint (7) is assumed to bind, so that Corollary 2.B holds. Further, \( x_{\text{low}} \) is treated as sufficiently small, so systemic defaults do not occur for any combinations of \( A \) and \( z \), which are shown in Figure 5. Finally, policy discontinuities associated with nonsystemic crises are not shown, because they are small in comparison with those driven by systemic collapses.
which translates into a lower return $R$ and higher default probabilities for investing banks. They diversify actively, making their portfolios highly similar to each other. Chances of systemic collapse surge. The same project-specific shock that has only a modest negative impact on the economy at $O$ is more detrimental at $C$. It causes painful simultaneous bankruptcies of all investing banks, bringing the economy from $C$ to $B_C$.

How does the economy move from $O$ to $C$? A positive surprise to aggregate productivity $z$ (in Figure 5, $z$ increases from $z_{low}$ to $z_{high}$) drives up the return on risky projects, thus incentivizing the household to save. The economy starts to converge to point $O'$. Since $\log z$ is an AR(1) process, after a period of productivity boom $z$ eventually reverts to $z_{low}$, bringing the economy to point $C$, where assets are abundant, but not particularly productive. As discussed above, at $C$ the financial system is more fragile than at $O$. If a project-specific shock is bad at this state ($x = x_{high}$), all investing banks go bankrupt at once because of strong common exposures.

Why does the household keep accumulating assets even though expected default losses become high? One reason is consumption smoothing. At the same time, the agents in the economy do not internalize that investment in risky projects reduces $R$ and thus make the financial system more fragile. This gives rise to a pecuniary externality, which results in excessive amount of investment, inefficiently low return $R$, over-connected financial network and too frequent systemic crises in the decentralized equilibrium. The externality is formally analyzed in Section 6.

5 Quantitative analysis

This section presents a quantitative analysis of the model. The model is calibrated in Section 5.1. Section 5.2 presents the impulse responses to aggregate and project-specific shocks. Section 5.3 describes the economy’s behavior around systemic and nonsystemic financial crises. Section
5.4 emphasizes the importance of connectedness for financial fragility via several counterfactual experiments. Finally, Section 5.5 investigates whether the model matches untargeted moments related to the frequency and the severity of financial crises.

5.1 Parameterization

The period of the model is 1 year. Table 1 lists the parameters. Appendix C describes the moment construction in more detail. Appendix D presents the sensitivity analysis of several key variables.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
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<tbody>
<tr>
<td>Preferences</td>
<td></td>
<td>Banking sector</td>
<td></td>
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<tr>
<td>Inverse IES</td>
<td>ψ = 5</td>
<td>Number of islands</td>
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<td>Inverse Frisch elasticity</td>
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<td>Time discounting</td>
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<td>Diversification cost</td>
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<td>Production technology</td>
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<td>Project-specific shocks</td>
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<td>Capital share</td>
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<td>Storage return</td>
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<td>Depreciation rate</td>
<td>δ = 0.087</td>
<td>Default loss</td>
<td>θ = 0.1</td>
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<td>Aggregate shocks</td>
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<td>Tail index</td>
<td>γ = 3</td>
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<tr>
<td>Persistence</td>
<td>ρₙ = 0.83</td>
<td>Minimum value</td>
<td>xₘ = 0.088</td>
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<tr>
<td>St. dev. of innovations</td>
<td>σₙ = 0.019</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Parameterization.

Preferences. The time discounting factor is $\beta = 0.97$. The Frisch elasticity of labor supply is $\nu^{-1} = 1.67$. The intertemporal elasticity of substitution is $\psi^{-1} = 0.2$. These values are standard in the literature.

Production technology. The capital share is $\eta = 0.33$. The model treats project-specific shocks as capital quality shocks directly affecting the total amount of assets in the economy, as in for example Gertler et al. (2011). Therefore, the total depreciation rate consists of a constant part $\delta$ and a time-varying part associated with $\tilde{x}$ shocks. $\delta$ is set to 0.087, which implies that on average the household replaces 10% of assets every period.

Banking sector and project-specific shocks. In the model, large project-specific shocks trigger defaults of financial institutions. Many studies show that extreme realizations of losses have moderately heavy tails (e.g., Jansen and de Vries, 1991, Gabaix, 2009), well described by the Pareto distribution with the tail exponent between 2 and 5 (Ibragimov et al., 2011).
therefore assume that

$$
\Phi(x) = \begin{cases} 
1 - \left(\frac{x_m}{x}\right)^\gamma, & x \geq x_m, \\
0, & x < x_m,
\end{cases}
$$

where $x_m > 0$ is the lower bound of the support and $\gamma = 3$ is the tail index (Gabaix, 2009). Importantly, Assumptions 3 and 3' hold for this distribution.

In the baseline analysis $x_m$ is set to 0.088. The number of risky projects $N$ is 10. $N$ captures the level of the financial system’s granularity. Both $N$ and the size of project-specific shocks governed by $x_m$ directly affect the frequency of systemic defaults in the model. Higher $N$ implies that diversification is more effective in protecting banks against project-specific shocks, making financial crises rarer. Larger project-specific surprises work in the opposite direction. Given $N$, $x_m$ is set so that the frequency of systemic financial crises in the model is around 1.7 per century. This is in line with Romer et al. (2017) (more on this in Section 5.5). I verify that the results are unchanged for different values of $N$ as long as $x_m$ is simultaneously recalibrated to match the same target.

I set $f = 0.0050$ to match the ratio of monitoring expenses over the loan value reported in Craig et al. (2018). Using a revealed preferences approach, they estimate that monitoring costs account for 23% of expected banks’ gross loan value net of funding costs. More specifically, Craig et al. (2018) consider a setting in which lending banks (noninvesting banks in my model) lend funds to borrowing banks through core intermediary institutions. The authors find that borrowing banks lose 23% of their values generated on monitoring. In my framework, investing banks directly lend to the real sector, playing the role of a conglomerate of borrowing and intermediary organizations. The value of an interbank loan of size $a$ is $\left(R - \frac{1}{N}\mathbb{E}_x x\right) a$, the funding cost is $\rho a$, and the amount lost on monitoring is $f(1 - \alpha)a$. $f$ is calibrated so that

$$
\frac{f(1-\alpha)}{R-\frac{1}{N}\mathbb{E}_x x-\rho}\text{ is on average } 23%.
$$

The storage return $\rho_s$ and the number of banks per island $M$ are calibrated using the data from the largest U.S. bank holding companies’ balance sheets from the FR Y-9C filings. The storage return controls for the investing banks’ cost of borrowing $\rho$. The number of banks per island $M$ is related to the borrowing limit $\bar{\mu} = \frac{M-1}{M}$. $\rho_s - 1 = 0.92\%$ and $M = 670$ are

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21The banking sectors of many developed economies are dominated by a few large institutions (Laeven, Ratnovski, and Tong, 2014); for example, the 10 largest U.S. bank holding companies account for almost 70% of all assets (FR Y-9C filings). As to the granularity of the portfolios of financial institutions, Billio et al. (2012) document that the first 10 principal components of the returns of major financial institutions and insurers are responsible for up to 83% of all variation. Using FR Y-9C reports, Duarte and Eisenbach (2018) study the strength of fire-sale spillovers; they identify 18 distinct asset classes in the portfolios of U.S. bank holding companies, of which 6 are almost riskless (i.e., they have a risk weight below 20%).
parameterized to match the average net interest income over assets $R - \frac{1}{N} \mathbb{E}_x x - \rho \mu$ (2.6% in the data) and the average interest income over assets net of interest expenses over liabilities $R - \frac{1}{N} \mathbb{E}_x x - \rho$ (2.4% in the data).

Finally, the fraction of assets lost in default is $\theta = 0.1$, a value close to Bernanke et al. (1999).

**Aggregate shocks.** Using the postwar U.S. data on GDP, hours, and investment from FRED, I construct a series of Solow residuals and log-linearly detrend it. The same exercise is then repeated within the model. $\rho_z$ and $\sigma_z$ are picked to match the data and model-implied persistence and the standard deviation of innovations in the AR(1) process (1). I obtain $\rho_z = 0.83$ and $\sigma_z = 0.019$. Appendix C.2 describes the procedure in more detail. I also verify that the model performs reasonably well in terms of matching the second moments of the major macroeconomic series.\(^{22}\)

### 5.2 Impulse response functions

I start investigating quantitative features of the model by considering impulse response functions to aggregate and project-specific innovations. Figure 6 shows impulse response functions to a positive aggregate shock hitting the economy at its long-run mean. An expansionary aggregate surprise improves the returns on risky projects. Default risks recede, and investing banks cut risk-sharing expenses. Interconnectedness goes down. Naturally, the household increases its accumulation of assets. The stock of assets is slow moving and reaches its peak with a lag (around period 12). At this point, assets are abundant, and $z$ has almost returned to its long-run mean. The risky projects are on average not exceptionally productive, and investing banks spend more on diversification. Interconnectedness rises. As a result, systemic crises become more likely. Consistent with Section 4.3, fragility of the financial system is elevated after a boom in productivity.

Innovations to aggregate productivity affect financial fragility (i.e., the probability of systemic crisis), but defaults themselves are triggered by adverse realizations of project-specific shocks. Given the state of the economy $(A, z)$, a project-specific surprise can cause three distinct bankruptcy events depending on its size (Corollary 1). If the size $x$ is small, all investing banks are solvent; the financial network effectively protects institutions against project-specific shocks. For a sufficiently large $x$, all investing banks collapse at once because of common exposures. Finally, as long as $x$ is not too small or too large, only the bank that specializes in a

---

\(^{22}\)Under this calibration, the underlying projects are on average productive relative to the storage, so that in the economy around its steady state all resources are allocated to the risky technology, $\mu = \bar{\mu}$ (the constraint (7) binds). Substantial deviations from the steady state, associated with a reduction in the projects’ return $R$, might bring the economy to the states in which (7) is slack. In simulations $\mu < \bar{\mu}$ 13% of time.
directly hit project fails to repay its debt. Transitions between the three regimes are associated with policy discontinuities, similar to those shown in Figure 5.

Figure 7 illustrates the economy’s responses to two project-specific surprises of slightly different sizes. For $x = x_1$, a nonsystemic crisis bursts. Only one investing bank defaults; the remaining ones are on the edge of collapse but still solvent. This shock has quite limited impacts on the economy, because real default losses are not exceptionally large (in particular, fraction $\theta/N$ of assets is lost). A marginally worse shock $x = x_1 + \epsilon$ causes a systemic crisis: contagion takes place, and all investing banks collapse.\(^{23}\) Real losses are magnified by a factor of $N$. The economy experiences a deep recession. Because the stock of assets is directly affected by the crisis, the recovery is slow even though the downturn is caused by a nonpersistent shock.

5.3 Economy around financial crises

The previous section has illustrated the distinct impacts of the aggregate and project-specific shocks: the former ones affect the economy’s exposure to the latter ones, which are the immediate triggers of financial crises. The goal of this section is to investigate how the economy, hit by these two types of shocks simultaneously, behaves around the events of financial crises. To do so, the model is simulated for 1,000,000 periods. The incidences of defaults of investing banks are identified. I start by analyzing systemic crises, when all investing banks become insolvent at once. The solid lines in Figure 8 represent the average paths of both exogenous and endogenous

\(^{23}\) $x \geq x_1$ and $x \geq x_1 + \epsilon$ with a probability of 1.16% and 0.99%, respectively. $\epsilon = 0.021$, implying that the amount of assets lost directly due to the shocks (but not defaults) differs by 0.21% only.
Systemic financial crises do not hit the economy at random. They tend to happen after booms in productivity and credit. In the run-up of a typical boom, a prolonged series of positive aggregate surprises incentivizes the household to save. The amount of assets and investment in the risky projects grow. At this stage, risky projects are still sufficiently productive because of above-average $z$, and the spread $R - \rho \mu$ is only marginally below its long-run mean. Diversification incentives are modest, and connectedness stays close to its steady-state level.

The probability of a joint collapse sharply rises at the second stage of a boom. Because of its AR(1) structure, the aggregate productivity reverts to its mean and even falls below it prior to $t = 0$, the period of systemic crisis. Consumption smoothing and the pecuniary externality prevent the household from quickly responding to such changes in $z$. The stock of assets stays high, and the return on risky projects deteriorates. Aiming to mitigate the rise in the interbank rate, investing banks diversify more actively and increase their cross-exposures. If one of the projects is hit by a sufficiently bad shock at this moment, the whole financial system crashes.

Prior to systemic crises, below-average values of the aggregate productivity contribute to the financial fragility. At the same time, $z$ is the main driver of credit booms preceding joint collapses. In the run-up of a typical systemic crisis, aggregate productivity is normally above its average. Panel (a) of Figure 9 shows the distributions of log $z$ at the moment of crisis ($t = 0$) and for the 10 periods prior to it ($t = -10$). They are clearly different from a symmetric zero-centered unconditional distribution: log $z$ tends to be negative at $t = 0$, whereas the opposite is
Figure 8: Average paths of the economy around systemic and nonsystemic crises ($t = 0$). The model is simulated for 1,000,000 periods. Series in panels (a)–(g) and (h) are expressed as the percentage deviations from their long-run averages. Series in panels (h) and (i) are expressed as a percentage.

true at $t = -10$.

At the end of credit booms, the spread $R - \mu$ is narrow, whereas interconnectedness $IC$ is high. Under these circumstances, a project-specific shock that in normal times does not trigger contagion might result in the simultaneous bankruptcy of all investing banks. I find that $55\%$ of project-specific shocks initiating systemic crises in simulations would not cause such an event if the economy was at its steady state. Panel (b) of Figure 9 shows that a substantially smaller project-specific shock is required to cause a joint default in a densely connected network. In fact, $88\%$ of all systemic crises occur when interconnectedness is above its long-run mean.

Unlike systemic collapses, when all investing institutions become insolvent at once, nonsystemic financial crises are marked by the default of only one institution. The dashed lines in Figure 8 represent the average paths of the economy around these events. In contrast to systemic defaults, nonsystemic ones tend to happen when interconnectedness is below average and the spread is elevated. Generally, narrower spreads make both systemic and nonsystemic crises
more likely ceteris paribus. However, investing banks respond to lower profitability by spreading their resources among the underlying projects more evenly. The increased degree of portfolio homogeneity reduces the chance of a nonsystemic event. The second force dominates: 60% of all nonsystemic crises happen when interconnectedness is below its long-run average.

5.4 Interconnectedness and financial crises

In the model systemic and nonsystemic crises are more likely in a strongly and weakly connected financial system, respectively. The networks dynamic responses to changing macroeconomic conditions shape the economy’s exposure to financial distress. This section illustrates how the level of, and the time variation in, connectedness affect the frequencies of the two types of crises via a series of counterfactual experiments.

The level of interconnectedness is governed by the diversification cost parameter \( f \). A decrease in \( f \) eases risk sharing and thus can be interpreted as a financial innovation (Allen et al., 1994). A reduction in \( f \) affects the economy in two ways. First, at each state \((A, z)\), investing banks naturally become more interconnected.\(^{24}\) Because of cheaper diversification, in the states of the world in which the constraint (7) is slack they also borrow more. Second, lower \( f \) improves the return on household’s assets due to diminished intermediation costs (Equation 9). The household saves more, pushing down the productivities of the underlying projects. In response, investing banks further increase diversification (panel (a) of Figure 10, solid line). As a result, the economy features fewer nonsystemic crises, while being more susceptible to massive intermediaries defaults

\(^{24}\)Given the state \((A, z)\), better risk sharing makes individual defaults less likely. The impact on the systemic crisis probability is in general ambiguous. On the one hand, joint collapses are more probable in densely connected systems. On the other hand, diversification increases investing banks’ profit margin thanks to a lower borrowing rate. Numerically, I find that the former force dominates: \( p_{syst}^d \) is a decreasing function of \( f \) state by state.
Next, I investigate the relative importance of the two effects of a reduction in $f$ (i.e., cheaper risk sharing and lower intermediation costs) for probabilities of systemic and nonsystemic financial crises. To do so, I assume that $f$ consists of two parts, $f = f_1 + f_2$. Both affect investing banks’ portfolio decisions, whereas only $f_1$ affects the return on household’s assets. In the benchmark economy $f_2 = 0$. For the purpose of isolating the role of connectedness, I vary $f_1$ and simultaneously change $f_2$ so that the average connectedness remains at its initial level (panel (a) of Figure 10, dashed line). In this experiment, a reduction in the intermediation cost only leads to elevated stock of assets and narrower investing banks’ profit margins but leaves the mean network density intact. The sensitivity of systemic crises frequency to $f_1$ declines (panel (b) of Figure 10, dashed line). In the absence of adjustments in the level of interconnectedness, narrower spreads also make nonsystemic crises more likely (panel (c) of Figure 10, dashed line).

In the benchmark model, flexibility in portfolio adjustments helps investing banks keep their expected bankruptcy losses relatively low when the productivities of the underlying primitive assets deteriorate. In particular, they shift portfolio compositions from their own projects, thereby reducing their individual default probabilities and increasing systemic risk. When barred from changing connectedness beyond its long-run average, investing banks remain predominantly exposed to their own projects. They also choose to borrow less on the interbank market (the constraint (7) is slack more often). Without time variation in interconnectedness, the frequency of nonsystemic crises jumps up almost twice, whereas systemic defaults become 10% rarer (panels (b) and (c) of Figure 10, square markers).\footnote{A reduction in the weight of their own project $\alpha$ by $d\alpha$ is associated with a quite modest increment in the degree of common exposures, because $d\alpha$ is evenly spread among all other projects. Consequently, holding interconnectedness fixed affects the probability of nonsystemic events more significantly.}
5.5 Financial crises statistics

This section investigates whether the model matches some untargeted moments related to the frequency and the severity of financial crises. I first verify that the model-implied frequencies of systemic and nonsystemic events are in line with the data. I then examine whether the model can numerically account for well-documented credit boom-bust dynamics around financial crises (e.g., Schularick et al., 2012).

In a long sample compiled by Jorda, Schularick, and Taylor, 2016 (JST) and covering 17 advanced economies since 1870, systemic banking crises happen on average 4.0 times per 100 years. However, it is unclear whether the definition of “systemicness” used by JST immediately applies in my setting. For example, in many cases their classification relies on Reinhart and Rogoff (2009), who identify systemic banking crises as episodes associated with “closure, merging, or takeover by the public sector of one or more financial institutions.” In the model, a failure of one investing bank is defined as a nonsystemic event. Using OECD Economic Outlook reports, Romer et al. (2017) (RR) construct a more detailed measure of financial distress severity for 24 countries (including all 17 from JST’s dataset) but use a shorter sample starting from 1967. Their measure suggests that the frequencies of systemic and nonsystemic crises are, respectively, 1.7% and 2.4% annually.\(^{26}\) The subsample of JST’s data over the same years implies an annual frequency of banking crises of 3.1%. In comparison with JST, RR appear to identify more episodes of financial distress overall, although fewer of them are classified as systemic.\(^{27}\)

The minimum value of project-specific shocks \(x_m\) is calibrated to generate around 1.7 joint failures of investing banks per 100 years. The model-implied annual frequency of nonsystemic crises is 2.5%, close to RR’s number. Importantly, this moment is not a calibration target.

In the model, systemic defaults tend to happen at the end of credit booms, when the simultaneous crash of all investing organizations is likely because of strong common exposures and weak balance sheets. Table 2 compares the model and data-implied amplitudes of boom-bust cycles.\(^{28}\) In comparison with all financial crises, systemic ones are preceded by larger increases and followed by more significant reductions in credit and output, both in the model (columns 1 and 2) and in the data (columns 3 and 4, RR’s chronology). The busts sizes are more pronounced

\(^{26}\)Following RR, I categorize an event as systemic if its measure reaches the value of 7. All other episodes in which the measure is above 0 is considered nonsystemic. Appendix C.4 gives more details.

\(^{27}\)For example, JST’s chronology implies that Finland, Belgium, and the Netherlands experienced systemic crises in 2008, whereas RR consider them to be nonsystemic crises. At the same time, some episodes (e.g., France in 1995, Australia in 2008) are treated as nonsystemic by RR but are absent from JST. See RR for a more detailed comparison between the chronologies.

\(^{28}\)To use credit and output data from JST, I restrict the analysis to 17 countries. Because of this, the frequencies of the financial distress episodes in columns 3 and 4 are slightly different from those reported in the text (1.7% vs. 1.8% for systemic and 2.4% vs. 2.6% for nonsystemic crises).
in the model. One reason is that the paper abstracts from any government interventions aimed at mitigating real losses during default events, while in reality bailouts are widespread.

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>RR</th>
<th>JST</th>
</tr>
</thead>
<tbody>
<tr>
<td>Credit boom [%]</td>
<td>1.75***</td>
<td>3.04***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.36**</td>
<td>2.85***</td>
</tr>
<tr>
<td>Credit bust [%]</td>
<td>-3.27***</td>
<td>-5.95***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1.96***</td>
<td>-2.77***</td>
</tr>
<tr>
<td>Output boom [%]</td>
<td>1.00***</td>
<td>1.21***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.34***</td>
<td>1.35*</td>
</tr>
<tr>
<td>Output bust [%]</td>
<td>-1.94***</td>
<td>-3.12***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>-2.20***</td>
<td>-2.70***</td>
</tr>
<tr>
<td>Frequency [%]</td>
<td>4.2</td>
<td>1.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.4</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Table 2: Boom/bust is defined as an average 2 years growth of the HP-filtered variable of interest prior/subsequent to crises. The smoothing parameter is $\lambda = 6.25$. Model figures are based on 1,000,000 simulations; credit is defined as the amount of assets invested in risky projects, $A_{t} - \bar{A}$. “JST” and “RR” stand for crisis chronologies by Jorda et al. (2016) and Romer et al. (2017), respectively. Credit and output data are from Jorda et al. (2016). “RR sample” represents a subsample of a long sample used by Jorda et al. (2016) (“Full sample”), for which financial crises severity measure of Romer et al. (2017) is available. ***, **, * mark significance of the difference between the mean of the variable and zero at 1%, 5% and 10% levels, respectively.

The last two columns report the same statistics using JST’s crisis chronology. The amplitude of credit boom-bust cycles is larger around RR’s episodes of systemic financial distress (columns 4 and 5), confirming that RR’s systemic crises are on average more severe than that of JST. Finally, in the full sample (column 6), the amplitude is somewhat larger than in more recent data.

### 6 Welfare analysis and policy implications

In the model, painful systemic crises tend to happen when the amounts of assets and, hence, interbank borrowing are above their long-run means. Under such conditions, the returns on risky projects are on average low; the financial sector is strongly interconnected; and contagion is likely. A natural question in this regard is why the household optimally chooses to have the stock of assets at such a high level and does not actively dissave in spite of an elevated financial fragility. One reason is consumption smoothing. At the same time, taking the return on assets as given, the household does not internalize how its intertemporal consumption/savings decisions affect the financial fragility. Similar to Boissay et al. (2016), this gives rise to a pecuniary externality. In Section 6.1, I solve the social planner’s problem to analyze the impacts of the externality on the size of credit booms, interconnectedness, the frequency of financial crises, and the household’s welfare. Section 6.2 discusses policy implications. Appendix A.5 provides
additional details about the planner’s problem.

6.1 Planner’s problem

In the first-best case, financial frictions are completely absent from the economy. In particular, investing banks are financed only through equity, so there are no costly bankruptcies. Investing banks do not spend on risk-sharing connections. Such an allocation, however, requires unrestricted planning abilities and is therefore hardly feasible. As is standard in the literature (e.g., Bianchi, 2011), I instead consider a constrained planner. It makes intertemporal consumption/savings decisions for the household but allows the interbank and labor markets to clear competitively. Formally, the constrained planner solves

$$V_{SB}(A, z, x) = \max_{A', C} \frac{1}{1 - \psi} \left( C - \frac{1}{1 + \psi} L(A, z)^{1+\psi} \right)^{1-\psi} + \beta \mathbb{E} \left[ V(A', z', x') \right],$$

s.t. $$A' + C = \left( 1 - \frac{1 - \mu(A, z)}{1 - \bar{\mu}} \right) \rho_s A + \left( z \left( \frac{1 - \mu(A, z)}{1 - \bar{\mu}} \right)^{\eta} L(A, z)^{1-\eta} + \frac{1 - \mu(A, z)}{1 - \bar{\mu}} A \left[ 1 - \delta - \frac{1}{N} x - \theta N^d(A, z, x) \right] \right),$$

where \( L(A, z), \mu(A, z) \) and \( N^d(A, z, x) \) are set as in the decentralized equilibrium \( (DE) \) defined in Section 3.3. The constrained planner’s allocation is denoted by \( SB \) (second best).

Unlike the representative household, the planner internalizes how its intertemporal savings decisions affect financial fragility. There are two main differences between the \( DE \) and \( SB \) asset accumulation policies. First, when facing increased systemic risk, the planner dissaves more aggressively. Figure 11 shows that, in comparison with the benchmark \( DE \) case, a typical systemic crisis in the \( SB \) economy is preceded by a smaller credit boom and triggered by a worse shock. Prior to nonsystemic crises, on the contrary, the planner accumulates more assets than does the representative household (not reported).

Second, the \( SB \) and \( DE \) economies feature different long-run steady states. On the one hand, the planner is willing to have fewer assets and a more sparsely connected financial system on average to avoid painful systemic crises. On the other hand, nonsystemic crises become more likely under such conditions. Moreover, the planner also internalizes that the diversification expenses are eventually rebated to the household, so, in the \( SB \) case, they do not dampen
Figure 11: Average paths of the benchmark (DE) and constrained efficient (SB) economies around systemic crises \((t=0)\). The models are simulated for 1,000,000 periods. All series are expressed as a percentage deviation from their long-run averages.

While it is generally unclear which effect dominates, the SB economy has, on average, 6\% fewer assets.

To quantify the differences between the SB and DE economies, I compute the compensating variation, that is, the percentage by which consumption in the DE allocation should be changed in order to achieve the same welfare as in the SB allocation. The compensating variation of moving from allocation \(i\) to allocation \(j\) is denoted by \(\kappa^{i\rightarrow j}\) and is defined as

\[
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u \left( (1 + \kappa^{i\rightarrow j})C_i^t, L_i^t \right) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u \left( C_j^t, L_j^t \right).
\]

Notice that \(\kappa^{i\rightarrow j}\) accounts for the transition from \(i\) to \(j\). Table 3 reports \(\kappa^{DE\rightarrow SB}\), along with the frequencies of systemic and nonsystemic crises, long-run means of assets, consumption, output, labor, and interconnectedness in the DE and SB economies (rows 1 and 2). The planner chooses to work and consume less. Output and asset stock are lower, and the financial system is more sparsely connected. The frequency of systemic events diminishes quite substantially, from 1.7\% to 1.1\%. At the same time, the number of nonsystemic crises increases marginally. The main macroeconomic series are less volatile under the constrained planner (Table A1 in Appendix C.2). The welfare gain from moving to the SB allocation is 0.05\% of permanent consumption, similar to Bianchi et al. (2010). It is worth noting that accounting for the transitional dynamics is important for the welfare calculation. During the initial stages of the \(DE \rightarrow SB\) transition, the household dissaves, contributing the lion’s share to the overall welfare gain.

### 6.2 Policy implications

I now discuss policies aimed at restoring constrained efficiency. The SB allocation can be decentralized by a state-contingent tax \(\tau(A, z, x)\) on savings \(a'_{hh}\), whereas tax proceeds are

\(^{29}\)Of course, this is true provided that the diversification costs are not deadweight losses.
Table 3: Columns report, respectively, the long-run averages of assets, consumption, labor, output, and interconnectedness; the annual frequencies of systemic and nonsystemic financial crises; and the compensating variation in $\kappa^{DE→i}$ for decentralized (DE), second-best (SB), decentralized with the optimal flat tax on savings (DE\text{optimal flat tax}), and decentralized with optimal diversification cost (DE\text{optimal f}) allocations. By definition, $\kappa^{DE→DE} = 0$. To compute $\kappa^{i→j}$, I simulate 10,000 paths of the economies $i$ and $j$ for 500 periods, starting from the ergodic distribution of $i$.

rebated to the household in a lump-sum fashion. This policy prevents a buildup of the interbank debt prior to systemic crises, avoiding the states of overconnected networks and inefficiently high financial fragility. The optimal tax is set so that the planner’s policies satisfy the representative household’s intertemporal Euler equation.

**Proposition 3** A state-contingent tax $\tau(A, z, x)$ on the household’s savings can restore constrained efficiency.

I find that the optimal tax is positive on average (0.38%), equalizing the long-run steady states of the DE and SB economies. It is positively correlated with $A$ and negatively correlated with $z$ and $x$. Prior to systemic events, the tax rate rises sharply (Figure 12), preventing larger credit booms and higher financial fragility. In the aftermath of systemic crises, $\tau$ drops below its long-run mean. In those states of the world the representative household, which does not internalize that the diversification expenses are eventually rebated to it, rebuilds the asset stock more slowly than would the planner (see also panel (c) of Figure 11).

![Tax rate](image)

Figure 12: An average path of the optimal tax rate $\tau(A, z, x)$ around systemic crises ($t = 0$). The model is simulated for 1,000,000 periods.

Since state-contingent policies might be difficult to implement in practice, I also study the
welfare implications of a simple flat tax on savings. I find that the optimal fixed tax rate is quite close to the long-run average of the state-contingent tax. The welfare gain, however, is about 60% of what is achieved by a time-varying policy (row 3 of Table 3). Unlike the state-contingent tax, the flat one only corrects the steady-state levels but, by definition, cannot prevent large fluctuations in credit and in interconnectedness.

Finally, I investigate the welfare implications of financial innovations associated with a reduction in the cost of establishing risk-sharing connections. As discussed in Section 5.4, a decrease in $f$ affects the economy in two ways. First, it incentivizes the household to accumulate more assets and thus exacerbates the oversaving problem. Second, it eases risk sharing and makes the financial system more interconnected. Holding everything else equal, more efficient diversification reduces expected default losses, which is generally welfare improving. At the same time, it makes systemic crises more frequent. This might be undesirable from the household’s point of view. Because bankers are risk neutral, whereas the household is risk averse, the latter experiences a larger utility loss in the states of joint defaults. Although how the household’s welfare is affected by financial innovations is unclear in principle, I find that an increase in $f$ by about 25% is optimal from the household’s perspective. The welfare gain, however, is only slightly above 0.01% of permanent consumption (row 4 of Table 3).

The policies considered in this section are mainly directed toward reducing the probability of systemic crises. Although the optimal state-contingent tax does help the economy recover faster after episodes of financial distress, it does not affect the immediate losses brought by breakdowns of the financial system. In this respect, interventions aimed at mitigating default losses, such as bailouts, might be desirable. Financial institutions, rationally expecting such policies, are likely to change their behavior ex ante (e.g., Acharya, 2009, Farhi et al., 2012). For example, within my framework, investing banks might choose to become overconnected in expectation of broad-based bailouts during episodes of systemic crises. However, as suggested by recent work (e.g., Bianchi, 2016, Allen, Carletti, Goldstein, and Leonello, 2017), the strength of a collective moral hazard problem might crucially depend on specific policy tools. A formal cost-benefit analysis of such interventions is beyond the scope of this paper.

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30 A decrease in $f$ improves the expected profitability of the investing banks’ portfolios. In the states of the world in which the constraint (7) is slack, investing organizations expand their borrowing and invest more in the risky projects. Because banks also do not internalize their impacts on the projects’ returns and hence financial fragility, they are subject to a welfare loss. Appendix A.5 provides further discussion.

31 Appendix E.3 considers the version of the model in which risk attitudes of the household and of bankers are aligned. In that economy, all agents weigh the same events equally, and, thus, at each state, banks’ portfolio choices are optimal from the household’s perspective. I still find that an increase in $f$ is welfare improving.
7 Concluding remarks

This paper has presented a general equilibrium model in which fragility arises because of dynamically evolving links connecting financial institutions. The overlap between banks’ portfolios shapes the economy’s exposure to financial crises associated with intermediaries defaults. During periods of credit expansion, banks evenly spread available funds across the underlying projects, thereby reducing their individual bankruptcy probabilities but magnifying systemic risk. Episodes of large-scale financial distress tend to happen after credit booms, when banks are densely connected and their balance sheets are weak.

To keep the analysis tractable, I have considered a setting in which banks internalize how their default decisions affect the rest of the economy and specifically other parties within the financial sector. Therefore, the model is likely to provide a lower bound on the importance of interconnectedness for fragility. It would be interesting to analyze how externalities (e.g., due to fire sales, a domino effect, anticipated government interventions, or noninternalized social costs of severe banking distress) affect the financial architecture and contribute to the evolution of systemic risk. These questions are left for future exploration.
References


Appendix

A Proofs

Appendices A.1–A.3 characterize the solution to the investing bank’s problem. As it is shown below, the equilibrium is symmetric, so the bank-specific indices are omitted where possible. Appendix A.4 considers the case when the storage return is time varying. Appendix A.5 proves Proposition 3.

A.1 Portfolio structure

Denote the investing bank’s objective function by

\[ \pi(\rho, \mu, \{\omega_{ij}\}_{j=1}^N) = \frac{a_o}{1 - \mu} \mathbb{E}_x \left[ \frac{1}{N} \sum_{j=1}^{N} (R - \rho \mu - \omega_{ij} x) \mathbb{I}\left\{ x \leq \frac{R - \rho \mu}{\omega_{ij}} \right\} - f \sum_{j \neq i} \omega_{ij} \right] . \]

The lenders’ break even constraint is

\[ \rho_s \leq g_0(\rho, \mu, \{\omega_{ij}\}_{j=1}^N), \quad (A1) \]

where

\[ g_0(\rho, \mu, \{\omega_{ij}\}_{j=1}^N) = \mathbb{E}_x \left[ \frac{1}{\rho} \frac{1}{N} \sum_{j=1}^{N} \mathbb{I}\left\{ x \leq \frac{R - \rho \mu}{\omega_{ij}} \right\} + \frac{1}{\mu} \frac{1}{N} \sum_{j=1}^{N} (R - \theta - \omega_{ij} x) \mathbb{I}\left\{ x > \frac{R - \rho \mu}{\omega_{ij}} \right\} \right] . \]

Denote the spread \( R - \rho \mu \) by \( \xi = \xi(\rho, \mu) > 0 \). Denote by \( \Phi(\cdot) \) and \( \phi(\cdot) \) cumulative and probability density functions of the shock size \( x \), respectively. Then \( \pi \) and \( g_0 \) can be written as\(^{32}\)

\[ \pi = \frac{a_o}{1 - \mu} \left[ \xi \frac{1}{N} \sum_{j=1}^{N} \Phi(\xi) \left( \frac{\xi}{\omega_{ij}} \right) - \frac{1}{\mu} \frac{1}{N} \sum_{j=1}^{N} \omega_{ij} \int_{x}^{\xi} x \phi(x) dx - f (1 - \omega_{ii}) \right] , \]

\[ g_0 = \rho + \frac{1}{\mu} \left[ (\xi - \theta) \left( 1 - \frac{1}{N} \sum_{j=1}^{N} \Phi(\xi) \left( \frac{\xi}{\omega_{ij}} \right) \right) - \frac{1}{N} \sum_{j=1}^{N} \omega_{ij} \int_{x}^{\xi} x \phi(x) dx \right] . \]

Lemma A1 In equilibrium, \( \frac{\partial g_0}{\partial \rho} > 0 \) and the break even constraint (A1) binds.

Proof: Differentiate \( \pi \) with respect to \( \rho \) to obtain

\[ \frac{\partial \pi}{\partial \rho} = -\frac{a_o}{1 - \mu} \frac{\mu}{1} \frac{1}{N} \sum_{j=1}^{N} \Phi(\xi) \left( \frac{\xi}{\omega_{ij}} \right) < 0 . \]

\(^{32}\) I implicitly assume that \( \frac{\xi}{\omega_{ij}} \in (\bar{x}, \bar{x}) \), so in the optimum the probability of investing bank’s \( i \) default due to shock to project \( j \) is strictly between zero and one.
Higher $\rho$ pushes $\pi$ down because it drives up investing bank’s liabilities and default chances.

Differentiation of $g_0$ with respect to $\rho$ yields
\[
\frac{\partial g_0}{\partial \rho} = \frac{1}{N} \sum_{j=1}^{N} \Phi \left( \frac{\xi}{\omega_{ij}} \right) - \theta \frac{1}{N} \sum_{j=1}^{N} \frac{1}{\omega_{ij}} \phi \left( \frac{\xi}{\omega_{ij}} \right).
\]

On the one hand, higher $\rho$ increases lenders’ payoff in the nondefault states, on the other hand, it increases chances of costly insolvency.

Assume that $\frac{\partial g_0}{\partial \rho} \leq 0$ in equilibrium. Then it is optimal for an investing bank to cut $\rho$ since it strictly increases its expected payoff $\pi$ and does not lead to violation of the break even constraint (A1). Hence this is not the case and $\frac{\partial g_0}{\partial \rho} > 0$. Then the constraint (A1) binds because otherwise a small reduction in $\rho$ both increases $\pi$ and does not violate the constraint.

Proposition 1 from the main text can be now established.

Proof: Consider a function $h_1(\omega_{ij}) = 1 - \Phi \left( \frac{\xi}{\omega_{ij}} \right)$.
\[
\frac{\partial^2 h_1}{\partial \omega_{ij}^2} = -\frac{\xi}{\omega_{ij}^3} \left[ 2\phi \left( \frac{\xi}{\omega_{ij}} \right) + \frac{\xi}{\omega_{ij}} \phi' \left( \frac{\xi}{\omega_{ij}} \right) \right] > 0,
\]
where the last inequality holds by Assumption 3. Further, consider a function $h_2(\omega_{ij}) = \omega_{ij} \int_{\frac{\xi}{\omega_{ij}}}^{\bar{x}} x \Phi(x) - \xi h_1(\omega_{ij})$.
\[
\frac{\partial h_2^2}{\partial \omega_{ij}^2} = \frac{\xi^2}{\omega_{ij}^2} \phi \left( \frac{\xi}{\omega_{ij}} \right) > 0.
\]

Write the function $g_0$ as
\[
g_0 = \rho - \frac{1}{\mu} \sum_{j=1}^{N} \left[ \theta h_1(\omega_{ij}) + h_2(\omega_{ij}) \right].
\]

Plug the constraint A1 into $\pi$ to obtain
\[
\pi = \alpha \frac{a_o}{1 - \mu} \left[ R - \rho_s \mu - \theta \frac{1}{N} \sum_{j=1}^{N} h_1(\omega_{ij}) - \frac{1}{N} \mathbb{E} x - f(1 - \omega_{ii}) \right].
\]

Let’s firstly show that $\omega_{ij}$ are the same $\forall j \neq i$. Assume not, $\exists k \neq i, l \neq i$ so that $\omega_{ik} \neq \omega_{il}$. Consider an alternative portfolio, with the same weights of all other projects and $\tilde{\omega}_{ik} = \tilde{\omega}_{il} = \frac{\omega_{ik} + \omega_{il}}{2}$. This portfolio is feasible and has the same formation cost since $\omega_{ii}$ stays unchanged. By convexity of $h_1(\cdot)$ and $h_2(\cdot)$, the constraint A1 is satisfied ($g_0$ increases), while expected payoff $\pi$ goes up. Hence, it must be the case that $\omega_{ij}$ are identical $\forall j \neq i$.

Let’s now show that $\alpha = \omega_{ii} \geq \omega_{ij} = \frac{1 - \alpha}{N-1}$. Assume not, $\exists k \neq i$ so that $\omega_{ik} > \omega_{ii}$. Then an alternative portfolio with the same weights of all other projects and $\tilde{\omega}_{ii} = \omega_{ik}$, $\tilde{\omega}_{ik} = \omega_{ii}$ is the same from the diversification perspective but reduces linking costs.
Finally, it is straightforward to see that $\alpha < 1$ as long as $f$ is sufficiently small.

A.2 Properties of $R(A, z)$

Before proving Proposition 2, I establish the following useful lemma.

**Lemma A2** Assume that the borrowing constraint (7) binds. Then

\[ R = \eta(1 - \eta) \frac{1-\eta}{\nu+\eta} z^{\frac{1+\nu}{\nu+\eta}} A^{\frac{\eta(1+\nu)}{\nu+\eta}-1}. \]

Consequently, $\frac{\partial R}{\partial A} < 0$ and $\frac{\partial R}{\partial z} > 0$.

**Proof:** If $\mu = \bar{\mu}$, all assets in the economy are invested in the risky technology. By Proposition 1, all projects receive the same amount of investment, $k = \frac{A}{N}$, and hence have the same labor input $l$. Solving the static labor problem (2), I obtain

\[ w = (1 - \eta)zk^{\eta}l^{-\eta}. \]

Optimal labor choice of the household implies that total labor supply $L = Nl$ is given by

\[ L^\nu = w. \]

Combining these two equations, I obtain

\[ L = [(1 - \eta)zA^\eta]^{\frac{1}{\nu+\eta}} \Rightarrow w = [(1 - \eta)zA^\eta]^{\frac{\nu}{\nu+\eta}}. \]

Recall that $R$ is given by (3). Plugging the wage equation to (3) yields

\[ R = \eta(1 - \eta) \frac{1-\eta}{\nu+\eta} z^{\frac{1+\nu}{\nu+\eta}} A^{\frac{\eta(1+\nu)}{\nu+\eta}-1}. \]

Clearly, $\frac{\partial R}{\partial z} > 0$. Since $\eta(1 + \nu) < \nu + \eta$, $\frac{\partial R}{\partial A} < 0$.

A.3 Comparative statics results

Investing banks choose $\rho$, $\mu$ and $\alpha$, taking $R$ as given. Assuming that the linking cost $f$ is positive but sufficiently small, so that $\alpha > \frac{1-\alpha}{N-1} > 0$ in the optimum, the problem can be written as

\[
\max_{\rho, \mu, \alpha} \pi(\rho, \mu, \alpha) = \frac{\alpha}{1-\mu} \left[ R - \rho s \mu - \theta g_1(\alpha, \xi) - \frac{1}{N} \mathbb{E}_x x - f(1 - \alpha) \right],
\]

s.t. $\rho_s = g_0(\rho, \mu, \alpha) = \rho - \frac{1}{\mu} \left( \theta g_1(\alpha, \xi) + g_2(\alpha, \xi) \right)$, $\mu \leq \bar{\mu}$,

where

\[ \xi = R - \rho \mu, \]

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\[ g_1(\alpha, \xi) = \frac{1}{N} \left( 1 - \Phi \left( \frac{\xi}{\alpha} \right) \right) + \frac{N - 1}{N} \left( 1 - \Phi \left( \frac{\xi}{\frac{1}{N-1}} \right) \right), \]
\[ g_2(\alpha, \xi) = -\xi g_1(\alpha, \xi) + \frac{1}{N} \left( \alpha \int_{\frac{\xi}{N}}^{\xi} x d\Phi(x) + (1 - \alpha) \int_{\frac{\xi}{N-1}}^{\xi} x d\Phi(x) \right). \]

**Lemma A3** Derivatives of \( g_i(\alpha, \xi), i \in \{1, 2\} \) have the following signs,
\[ \frac{\partial g_i}{\partial \alpha} > 0, \frac{\partial^2 g_i}{\partial \alpha^2} > 0, \frac{\partial g_i}{\partial \xi} < 0, \frac{\partial^2 g_i}{\partial \xi^2} > 0, \frac{\partial^2 g_i}{\partial \xi \partial \alpha} < 0. \]

Moreover, \( \frac{\partial g_2}{\partial \xi} = -g_1 \) and \( \frac{\partial g_1}{\partial \xi} + \left( \frac{\partial g_1}{\partial \alpha} \right)^2 < 0. \)

**Proof:** Recall that by Assumption 3, \( x^2 \Phi'(x) \) is a decreasing function of \( x \) and hence \( x \Phi''(x) + 2 \Phi'(x) < 0 \) (the lower bound of the support \( x > 0 \)). By Assumption 3', \( x^3 \Phi''(x) + x^2 \Phi'(x) \) is an increasing function of \( x \). Moreover, \( \alpha > \frac{1}{N} > \frac{1 - \alpha}{N-1} \) by Proposition 1. Then

\[ \frac{\partial g_1}{\partial \alpha} = \frac{1}{N} \left( \frac{\xi}{\alpha} \right)^2 \Phi' \left( \frac{\xi}{\alpha} \right) - \left( \frac{\xi}{\frac{1}{N-1}} \right)^2 \Phi' \left( \frac{\xi}{\frac{1}{N-1}} \right) > 0, \]
\[ \frac{\partial^2 g_1}{\partial \alpha^2} = -\frac{1}{N} \left( \frac{\xi}{\alpha} \right)^4 \Phi'' \left( \frac{\xi}{\alpha} \right) + 2 \left( \frac{\xi}{\alpha} \right)^3 \Phi' \left( \frac{\xi}{\alpha} \right) + \frac{1}{N - 1} \left( \frac{\xi}{\frac{1}{N-1}} \right)^4 \Phi'' \left( \frac{\xi}{\frac{1}{N-1}} \right) > 0, \]
\[ \frac{\partial g_1}{\partial \xi} = -\frac{1}{N} \left( \frac{\xi}{\frac{1}{N-1}} \right)^2 \Phi' \left( \frac{\xi}{\frac{1}{N-1}} \right) + (N - 1) \frac{\xi}{\frac{1}{N-1}} \Phi' \left( \frac{\xi}{\frac{1}{N-1}} \right) < 0, \]
\[ \frac{\partial^2 g_1}{\partial \xi^2} = -\frac{1}{N} \left( \frac{\xi}{\frac{1}{N-1}} \right)^2 \Phi'' \left( \frac{\xi}{\frac{1}{N-1}} \right) + (N - 1) \left( \frac{\xi}{\frac{1}{N-1}} \right)^2 \Phi'' \left( \frac{\xi}{\frac{1}{N-1}} \right) > 0, \]
\[ \frac{\partial^2 g_1}{\partial \xi \partial \alpha} = \frac{1}{N} \left( \frac{\xi}{\frac{1}{N-1}} \right)^3 \Phi'' \left( \frac{\xi}{\frac{1}{N-1}} \right) - \left( \frac{\xi}{\frac{1}{N-1}} \right)^2 \Phi' \left( \frac{\xi}{\frac{1}{N-1}} \right) < 0, \]
\[ \frac{\partial g_2}{\partial \alpha} = \frac{1}{N} \left[ \int_{\frac{\xi}{N}}^{\xi} x d\Phi(x) - \int_{\frac{\xi}{N-1}}^{\xi} x d\Phi(x) \right] > 0, \]
\[ \frac{\partial^2 g_2}{\partial \alpha^2} = \frac{1}{N} \left( \frac{\xi}{\alpha} \right)^3 \Phi' \left( \frac{\xi}{\alpha} \right) + \frac{1}{N - 1} \left( \frac{\xi}{\frac{1}{N-1}} \right)^3 \Phi' \left( \frac{\xi}{\frac{1}{N-1}} \right) > 0, \]
\[ \frac{\partial g_2}{\partial \xi} = -g_1 < 0, \frac{\partial^2 g_2}{\partial \xi^2} = -\frac{\partial g_1}{\partial \xi} > 0, \frac{\partial^2 g_2}{\partial \xi \partial \alpha} = -\frac{\partial g_1}{\partial \alpha} < 0. \]
Finally, observe that

\[
\frac{\partial g_1}{\partial \xi} \frac{\partial^2 g_2}{\partial \alpha^2} + \left( \frac{\partial g_1}{\partial \alpha} \right)^2 = -\frac{1}{N} \frac{1}{\xi^2} \left[ \frac{1}{N - 1} \frac{\xi}{\alpha} \left( \frac{1 - \alpha}{N - 1} \right)^3 \Phi' \left( \frac{\xi}{\alpha} \right) \Phi' \left( \frac{\xi}{\frac{1 - \alpha}{N - 1}} \right) + \right.
\]

\[
(N - 1) \left( \frac{\xi}{\alpha} \right)^3 \frac{1 - \alpha}{N - 1} \Phi' \left( \frac{\xi}{\alpha} \right) \Phi' \left( \frac{\xi}{\frac{1 - \alpha}{N - 1}} \right) + 2 \left( \frac{\xi}{\alpha} \frac{1 - \alpha}{N - 1} \Phi' \left( \frac{\xi}{\alpha} \right) \Phi' \left( \frac{\xi}{\frac{1 - \alpha}{N - 1}} \right) \right)^2 < 0.
\]

The Proposition 2 can be now proved.

**Proof:** The proof proceeds in two steps. In the first step, the first order conditions for the investing bank’s problem are derived. They implicitly define the functions of interest, \( \alpha(A, z) \) and \( \xi(A, z) = R(A, z) - \rho(A, z)\bar{\mu} \). In the second step, the properties of interest are established by means of the implicit function theorem.

**Step 1.** I start by deriving the first order condition of (A2). First, the borrowing constrain (7) binds if

\[
\left[ \frac{\partial \pi}{\partial \mu} + \frac{\partial \pi}{\partial \rho} \frac{\partial \rho}{\partial \mu} \right]_{\mu = \bar{\mu}} \geq 0.
\]

Second,

\[
\frac{\partial \pi}{\partial \alpha} + \frac{\partial \pi}{\partial \rho} \frac{\partial \rho}{\partial \alpha} = 0,
\]

which can be written as

\[
\frac{\partial \pi_1}{\partial \alpha} + \frac{\partial \pi_2}{\partial \alpha} \frac{\partial \pi_2}{\partial \xi} - \frac{\partial \pi_2}{\partial \xi} \frac{\partial \pi_2}{\partial \alpha} - \frac{f}{\theta} = 0.
\]

(A3)

Here the implicit function theorem is used to calculate \( \frac{\partial \rho}{\partial \alpha} = -\frac{\partial \pi_1}{\partial \alpha} \).

**Step 2.** If \( \mu = \bar{\mu} \), the optimal choice effectively consists of two variables, \( \alpha \) and \( \rho \), while \( \mu = \bar{\mu} = \text{const} \). In equilibrium, the variables are functions of the aggregate states \( A \) and \( z \). The break-even constraint of the problem (A2) and the first order condition (A3) implicitly define \( \alpha = \alpha(A, z) \) and \( \rho = \rho(A, z) \),

\[
Q_1(A, z, \alpha, \rho) = \rho - \frac{1}{\bar{\mu}} \left( \theta g_1(\alpha, \xi) + g_2(\alpha, \xi) \right) - \rho_s = 0,
\]

\[
Q_2(A, z, \alpha, \rho) = \frac{\text{NUM}(\alpha, \xi)}{\text{DEN}(\alpha, \xi)} - \frac{f}{\theta} = \frac{\frac{\partial \pi_1}{\partial \alpha} + \frac{\partial \pi_2}{\partial \alpha} \frac{\partial \pi_2}{\partial \xi} - \frac{\partial \pi_2}{\partial \xi} \frac{\partial \pi_2}{\partial \alpha}}{1 + \theta \frac{\partial \pi_1}{\partial \xi} + \frac{\partial \pi_2}{\partial \xi}} - \frac{f}{\theta} = 0,
\]

where \( \xi = \xi(A, z, \rho) = R(A, z) - \rho \bar{\mu} \). By Lemma A2, \( \frac{\partial \xi}{\partial A} = \frac{\partial R}{\partial A} < 0, \frac{\partial \xi}{\partial z} = \frac{\partial R}{\partial z} > 0 \) and
\( \frac{\partial \xi}{\partial \rho} = -\bar{\mu} < 0 \). Applying the implicit function theorem, I get

\[
d\alpha \over dv = -\frac{\partial Q_1 \partial Q_2}{\partial \alpha \partial \rho} - \frac{\partial Q_1 \partial Q_2}{\partial \rho \partial \alpha},
\]

where \( v \in \{A, z\} \).

Observe that by Lemma A3,

\[
\frac{\partial Q_1}{\partial \alpha} = -\frac{1}{\mu} \left( \theta \frac{\partial g_1}{\partial \alpha} + \frac{\partial g_2}{\partial \xi} \right) < 0,
\]

\[
\frac{\partial Q_1}{\partial A} = -\frac{1}{\mu} \left( \theta \frac{\partial g_1}{\partial \xi} + \frac{\partial g_2}{\partial \alpha} \right) \frac{\partial R}{\partial A} < 0,
\]

\[
\frac{\partial Q_1}{\partial z} = -\frac{1}{\mu} \left( \theta \frac{\partial g_1}{\partial \xi} + \frac{\partial g_2}{\partial \alpha} \right) \frac{\partial R}{\partial z} > 0.
\]

By Lemma A1,

\[
\frac{\partial Q_1}{\partial \rho} = 1 + \theta \frac{\partial g_1}{\partial \xi} + \frac{\partial g_2}{\partial \alpha} > 0.
\]

Next, differentiate \( NUM(\alpha, \xi) \) and \( DEN(\alpha, \xi) \) to obtain

\[
\frac{\partial NUM}{\partial \alpha} = \frac{\partial^2 g_1}{\partial \alpha^2} + \frac{\partial^2 g_1}{\partial \alpha^2} \frac{\partial g_2}{\partial \xi} + \frac{\partial g_1}{\partial \alpha} \frac{\partial^2 g_2}{\partial \xi \partial \alpha} - \frac{\partial^2 g_1}{\partial \xi \partial \alpha} \frac{\partial g_2}{\partial \alpha} - \frac{\partial g_1}{\partial \xi} \frac{\partial^2 g_2}{\partial \alpha^2} \frac{\partial g_1}{\partial \alpha} < 0,
\]

\[
\frac{\partial^2 g_2}{\partial \alpha^2} \frac{\partial g_1}{\partial \alpha} (1 - g_1) - \frac{\partial^2 g_1}{\partial \xi \partial \alpha} \frac{\partial g_2}{\partial \alpha} < 0,
\]

and

\[
\frac{\partial NUM}{\partial \xi} = \frac{\partial^2 g_1}{\partial \xi \partial \alpha} \frac{\partial g_2}{\partial \alpha} + \frac{\partial g_1}{\partial \alpha} \frac{\partial^2 g_2}{\partial \xi^2} - \frac{\partial^2 g_1}{\partial \xi \partial \alpha} \frac{\partial g_2}{\partial \alpha} - \frac{\partial g_1}{\partial \xi} \frac{\partial^2 g_2}{\partial \alpha \partial \xi} \frac{\partial g_1}{\partial \alpha} < 0.
\]

Finally,

\[
\frac{\partial DEN}{\partial \alpha} = \theta \frac{\partial^2 g_1}{\partial \alpha \partial \xi} + \frac{\partial^2 g_2}{\partial \xi^2} < 0,
\]

\[
\frac{\partial DEN}{\partial \xi} = \theta \frac{\partial^2 g_1}{\partial \xi^2} + \frac{\partial^2 g_2}{\partial \xi^2} > 0.
\]

All inequalities follow from Lemma A3.

Then it follows that

\[
\frac{\partial Q_2}{\partial \alpha} = \frac{\partial NUM}{\partial \alpha} \frac{DEN}{DEN^2} - \frac{\partial NUM}{\partial \alpha} \frac{DEN}{DEN^2} NUM > 0,
\]

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Using Equation (A4), I obtain

\[
\frac{d\alpha}{dA} = -\frac{\partial Q_1}{\partial A} < 0, \quad \frac{d\alpha}{dz} = -\frac{\partial Q_1}{\partial z} > 0.
\] (A5)

Finally, the full derivatives of the spread \(\xi\) with respect to the aggregate states \(A\) and \(z\) are

\[
\frac{d\xi}{dv} = \frac{\partial R}{\partial v} - \frac{d\rho}{dv} \bar{\mu} = \frac{\partial Q_1}{\partial v} + \frac{\partial Q_1}{\partial \alpha} \frac{d\alpha}{dv} < 0, \quad \frac{d\xi}{dv} = -\frac{\partial Q_2}{\partial \rho} < 0.
\]

From (A5),

\[
\frac{\partial Q_1}{\partial A} + \frac{\partial Q_2}{\partial \alpha} \frac{d\alpha}{dA} < 0, \quad \frac{\partial Q_1}{\partial z} + \frac{\partial Q_2}{\partial \alpha} \frac{d\alpha}{dz} > 0.
\]

\section*{A.4 Comparative statics results: Time-varying storage return}

The goal of this Appendix is to show that Proposition 2 holds even if the return on storage is not constant. Since the goal of this exercise is purely expositional, I simply assume that the storage return is proportional to the projects’ return \(R = R(A, z)\). The break-even constraint for lenders on the interbank market becomes

\[
Q_1(A, z, \alpha, \rho) = \rho - \frac{1}{\bar{\mu}} \left( \theta g_1(\alpha, \xi) + g_2(\alpha, \xi) \right) - \rho_s R(A, z) = 0.
\]

The constant \(\rho_s\) is strictly below 1 to guarantee that the storage technology is less productive than risky projects. The second condition implicitly defining \(\alpha(A, z)\) and \(\rho = \rho(A, z)\), \(Q_2(A, z, \alpha, \rho) = 0\) does not change.

Notice that now

\[
\frac{\partial Q_1}{\partial A} = \left[ -\rho_s - \frac{1}{\bar{\mu}} \left( \theta \frac{\partial g_1}{\partial \xi} + \frac{\partial g_2}{\partial \xi} \right) \right] \frac{\partial R}{\partial A}.
\]
\[
\frac{\partial Q_1}{\partial z} = \left[ -\rho_s - \frac{1}{\bar{\mu}} \left( \theta \frac{\partial g_1}{\partial \xi} + \frac{\partial g_2}{\partial \xi} \right) \right] \frac{\partial R}{\partial z}
\]

have in principle uncertain signs. On the one hand, a reduction in \( R(A, z) \) (either due increase in the amount of assets \( A \) or decline in the aggregate productivity \( z \)) narrows the spread \( \xi(A, z) \) and makes defaults more likely. On the other hand, the outside option of lenders also becomes less attractive.

The full derivative of \( \alpha \) with respect to \( A \) is

\[
\frac{d\alpha}{dA} = \left( \frac{\partial Q_1}{\partial A} \frac{\partial Q_2}{\partial \rho} - \frac{\partial Q_1}{\partial \rho} \frac{\partial Q_2}{\partial A} \right) \propto \left( \frac{\partial Q_1}{\partial \rho} + \frac{\partial Q_2}{\partial A} \right) + \frac{1}{\bar{\mu}} \frac{\partial R}{\partial A} \propto \left( \frac{\partial Q_1}{\partial A} \frac{\partial Q_2}{\partial \rho} - \frac{\partial Q_1}{\partial \rho} \frac{\partial Q_2}{\partial A} \right) + \frac{1}{\bar{\mu}} \frac{\partial R}{\partial A} < 0,
\]

where \( D_1 \propto D_2 \) means that \( \frac{D_1}{D_2} = a > 0 \). The proof that \( \frac{d\alpha}{dA} < 0 \) does not change. Inequalities \( \frac{d\alpha}{dz} > 0 \) and \( \frac{d\xi}{dz} > 0 \) can be established analogously.

\[\Box\]

### A.5 Planner’s problem and optimal policy

In this appendix, the constrained planner’s problem and the pecuniary externality are discussed in more details. Proposition 3 is then established.

Consider the planner who decides for the representative households but allows the labor and interbank markets to clear as in the decentralized case. Its problem is

\[
V_{SB}^{SB}(A, z, x) = \max_{A', C} \frac{1}{1 - \psi} \left( C - \frac{1}{1 + \nu} L(A, z)^{1+\nu} \right)^{1-\psi} + \beta \mathbb{E} \left[ V(A', z', x') \right],
\]

s.t. \( A' + C = \left( 1 - \frac{1 - \mu(A, z)}{1 - \bar{\mu}} \right) \rho_s A + \rho_s \rho_s + z \left( \frac{1 - \mu(A, z)}{1 - \bar{\mu}} A \right)^\eta L(A, z)^{1-\eta} + \frac{1 - \mu(A, z)}{1 - \bar{\mu}} A \left[ 1 - \delta - \frac{1}{N} x - \theta N^d(A, z, x) \right].
\]

The number of investing banks in default is

\[
N^d(A, z, x) = \begin{cases} 0, & x \leq \frac{\xi(A, z)}{\alpha(A, z)}, \\ 1, & x \in \left[ \frac{\xi(A, z)}{\alpha(A, z)}, \frac{\xi(A, z)}{1 - \alpha(A, z)} \right], \\ N, & x > \frac{\xi(A, z)}{1 - \alpha(A, z)}, \end{cases}
\]

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where the spread $\xi(A,z) = R(A,z) - \rho(A,z)\mu(A,z)$ and $R(A,z) = \eta z\left(\frac{1-\mu}{1-\mu(A,z)}\right)^{\eta-1} L(A,z)^{1-\eta}$.

In contrast to the household, the planner’s savings are not dampened by the diversification costs because it internalizes that they are not deadweight losses and are eventually rebated to the household. Further, the planner takes into account how its decisions impact $\mu(A,z)$, $\alpha(A,z)$, $L(A,z)$ and hence $R(A,z)$, $\xi(A,z)$ and eventually $N^d(A,z,x)$. The pecuniary externality in the decentralized equilibrium arises due to three reasons.

First, in the states of the world where $\mu = \bar{\mu}$ the representative household accumulates too much assets intertemporally. It fails to understand that by doing so it reduces $R(A,z)$ and increases expected $N^d(A,z,x)$, i.e. financial fragility.

Second, the return on household’s assets (9) is an average return on the aggregate portfolio of the financial sector. In particular, the representative household does not understand how its assets are split between the risky projects and storage on the margin. Around the steady state, when $\mu = \bar{\mu}$ and all assets in the economy are invested in the risky projects, the average and marginal returns are equal. However, when $\mu < \bar{\mu}$ it is no longer the case. In those states of the world the financial sector allocates almost all additional resources to the riskless storage (see Figure 4). The household therefore accumulates more aggressively then the constrained planner would do.

Third, investing banks do not internalize that the return on the risky projects $R(A,z)$ goes down with $\mu$. Effectively, they do not fully understand how their borrowing decisions affect their default probabilities. As a result, the constraint (7) binds too often. When it is slack, investing banks pick too high $\mu$.

By definition, the constrained planner can only address the first two problems and the rebate issue because it takes the interbank outcomes as given. An increase in the diversification cost $f$ mitigates the strength of the pecuniary externality on the interbank market. Facing larger costs, investing banks borrow (weakly) less state by state. Of course, a change in $f$ also impacts the intertemporal savings decisions of the household as well as the efficiency of risk sharing.

A state-contingent tax on savings $\tau(\Omega)$ is able to restore the second best allocation. The problem of the representative household under $\tau(\Omega)$ is

$$V^\tau(a_{hh},\Omega) = \max_{a_{hh}',c,l} \frac{1}{1-\psi} \left( c - \frac{1+\nu}{1+\nu} \right)^{1-\psi} + \beta \mathbb{E}[V^\tau(a_{hh}',\Omega')]$$

s.t. $(1 + \tau(\Omega))a_{hh}' = r(\Omega)a_{hh} + w(\Omega-x)l - c + \chi(\Omega-x) + T(\Omega)$,

where the prices $r(\Omega)$ and $w(\Omega-x)$ as well as the transfer $\chi(\Omega-x)$ are set as in Section 3.3. Tax proceeds are rebated to the household in a lump sum fashion, $T(\Omega) = \tau(\Omega)A'_{SB}(\Omega)$, where $A'(\Omega)$

---

As in Bianchi et al. (2010), I consider the planner who is unable to pick the supply of labor for the household. If it was allowed to do so, another policy tool (labor tax) would be required to restore the constrained efficiency. Due to discontinuities associated with financial crises, it is not possible to derive the first order conditions of the planner’s problem. I am therefore unable to directly compare them with their analogues in the decentralized case.
is the aggregate asset accumulation policy. Observe that the intertemporal Euler equation is

$$1 + \tau(\Omega) = \beta E \left[ \frac{(c' - \frac{1}{1+\nu} l'-\nu)^{-\psi}}{(c - \frac{1}{1+\nu} l^1+\nu)^{-\psi}} \times r(\Omega) \right].$$

Set $\tau(\Omega)$ so that the Euler equation holds for the constrained planner’s policies. Recall that the constrained planner lets the labor market clear competitively. Then the representative household optimally makes the same decisions as the constrained planner. Proposition 3 holds.

**B Distribution of project-specific shock**

This appendix describes the conditions under which Assumptions 3 and 3’ hold. I explore several distributions used to model losses (e.g., Hogg and Klugman, 1984). For all distributions but Pareto Assumption 3 turns out to be a weaker version of Assumption 3’. In the Pareto case the two assumptions are equivalent.

**Pareto distribution.** Pareto distribution is described by the two parameters, the lower bound of support $x_m > 0$ and tail index $\gamma > 0$. As long as $x \geq x_m$, the cumulative distribution function is

$$\Phi(x) = 1 - \left( \frac{x_m}{x} \right)^\gamma.$$ 

Therefore,

$$x \Phi''(x) + 2 \Phi'(x) \propto 1 - \gamma,$$

$$x^2 \Phi'''(x) + 4x \Phi''(x) + 2 \Phi'(x) \propto -\gamma(1 - \gamma),$$

where the $\propto$ sign between two expressions means that they are proportional up to a positive variable.

Assumptions 3 and 3’ are satisfied as long as $\gamma > 1$, so that Pareto distribution has a finite first moment.

**Weibull distribution.** Weibull distribution is described by the two parameters, shape $k > 0$ and scale $\lambda > 0$. For $k = 1$ it reduces to exponential distribution. For $x \geq 0$ the cumulative distribution function is

$$\Phi(x) = 1 - \exp \left( \frac{x^k}{\lambda^k} \right),$$

Therefore,

$$x \Phi''(x) + 2 \Phi'(x) \propto k + 1 - x^k \frac{k}{\lambda^k},$$

$$x^2 \Phi'''(x) + 4x \Phi''(x) + 2 \Phi'(x) \propto x^{2k} \frac{k^2}{\lambda^{2k}} - x^k \frac{k}{\lambda^k} (3k + 1) + k(k + 1).$$
Assumptions 3 and 3’ are satisfied for sufficiently large \( x \geq x^* \), \( \frac{1}{x^*} = \frac{2k + 1 + \sqrt{5k^2 + 2k + 1}}{2k} \). If the distribution is truncated from below by \( x^* \), the assumptions of interest hold over all the support.

**Lognormal distribution.** Lognormal distribution is described by the two parameters, \( \mu \) and \( \sigma > 0 \) (mean and standard deviation of the corresponding normal distribution). For \( x > 0 \) the cumulative distribution function is

\[
\Phi(x) = \frac{1}{2} + \frac{1}{2} \text{erf}\left( \frac{\log x - \mu}{\sqrt{2}\sigma} \right),
\]

where \( \text{erf}(\cdot) \) is error function.

Therefore,

\[
x\Phi''(x) + 2\Phi'(x) \propto 1 - z(x)\sqrt{\frac{2}{\sigma^2}},
\]
\[
x^2\Phi'''(x) + 4x\Phi''(x) + 2\Phi'(x) \propto 2\frac{z(x)^2}{\sigma^2} - \sqrt{\frac{2}{\sigma^2}z(x)} - \frac{1}{\sigma^2},
\]

where \( z(x) = \frac{\log x - \mu}{\sqrt{2}\sigma} \).

Assumptions 3 and 3’ are satisfied for sufficiently large \( x \geq x^* \), \( z(x^*) = \frac{\sqrt{2} + \sqrt{2 + \frac{1}{\sigma^2}}}{4} \). If the distribution is truncated from below by \( x^* \), the assumptions of interest hold over all the support.

**Gamma distribution.** Gamma distribution is described by the two parameters, shape \( \alpha > 0 \) and rate \( \beta > 0 \). For \( x > 0 \) the cumulative distribution function is

\[
\Phi(x) = \frac{1}{\Gamma(\alpha)} \gamma(\alpha, \beta x),
\]

where \( \gamma(\cdot, \cdot) \) is the lower incomplete gamma function and \( \Gamma(\cdot) \) is gamma function. Therefore,

\[
x\Phi''(x) + 2\Phi'(x) \propto \alpha + 1 - \beta x,
\]
\[
x^2\Phi'''(x) + 4x\Phi''(x) + 2\Phi'(x) \propto x^2\beta^2 - 2x\beta(1 + \alpha) + \alpha(\alpha + 1).
\]

Assumptions 3 and 3’ are satisfied for sufficiently large \( x \geq x^* \), \( x^* = \frac{1 + \alpha + \sqrt{1 + \alpha}}{\beta} \). If the distribution is truncated from below by \( x^* \), the assumptions of interest hold over all the support.

**C Data sources and moment construction**

**C.1 Financial interconnectedness**

Figure A1 presents full nonsmoothed nonnormalized time series of the four interconnectedness measures discussed in Section 2 of the main text. Notice a sharp rise in the fraction of the noncore liabilities over assets and the credit intermediation index in the 1990s. A rapid credit expansion during this period outpaced the growth in pool of retail deposits (Hahm et al., 2013). Another important driving force is securitization (Greenwood et al., 2013).
Figure A1: Four interconnectedness measures. The shaded areas represent NBER recessions. Panel (a) illustrates the industry/region-based overlap in the syndicated loan portfolios of the U.S. lead arrangers. Portfolios are considered overlapping if they have common exposures to a specific borrower industry and region. Source: Cai et al. (2018). Panel (b) illustrates the ratio of nonagency mortgage-backed securities and asset-backed securities over the total assets from the 100 largest U.S. bank holding companies. Source: FR Y-9C. Panel (c) illustrates the ratio of total liabilities net deposits (noncore liabilities) over the total assets from the 100 largest U.S. bank holding companies. Source: Barattieri et al. (2018) and FR Y-9C. Panel (d) illustrates the ratio of the total liabilities of all U.S. sectors over the total liabilities of the nonfinancial sector. Source: Greenwood et al. (2013) and U.S. Flow of Funds.

C.2 Macroeconomic moments

The source of all macroeconomic data is FRED. In particular, consumption is the sum of ‘Personal Consumption Expenditures: Services’ and ‘Personal Consumption Expenditures: Nondurable Goods’. Investment is ‘Personal Consumption Expenditures: Durable Goods’ plus ‘Gross Private Domestic Investment’. Labor input is ‘Nonfarm Business Sector: Hours of All Persons’. GDP is the sum of consumption and investment. All series are deflated by ‘Gross Domestic Product: Implicit Price Deflator’ and transformed into per capita terms using ‘Population’. Capital series is constructed using perpetual inventory method, where the initial stock is picked to match the average capital-output ratio. The data spans from 1950 to 2017.

In order to calibrate the persistence $\rho_z$ and standard deviation of innovations $\sigma_z$ of the aggregate productivity process, I construct a series of Solow residuals, $\log S = \log Y - \eta \log K - (1 - \eta) \log L$, linearly detrend it, and fit an AR(1) specification. The same exercise is repeated.

I run the following iterative procedure in order to compute the average capital-output ratio: form an initial guess about the value of the ratio; construct the capital series; update the guess; iterate until the change in the capital series becomes trivial.

---

35 The data spans from 1950 to 2017.

35 I run the following iterative procedure in order to compute the average capital-output ratio: form an initial guess about the value of the ratio; construct the capital series; update the guess; iterate until the change in the capital series becomes trivial.
within the model.

Table A1 reports the second moments of the HP-filtered macroeconomic series. In comparison with the model with no financial frictions, the benchmark model implies somewhat higher volatilities of macro variables due to infrequent systemic crises. In the constrained efficient equilibrium the volatilities are in between the first best and benchmark cases.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Benchmark</th>
<th>Second best</th>
<th>No financial frictions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>1.98</td>
<td>2.29</td>
<td>2.14</td>
<td>2.02</td>
</tr>
<tr>
<td>Hours</td>
<td>1.70</td>
<td>1.60</td>
<td>1.39</td>
<td>1.27</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.74</td>
<td>1.71</td>
<td>1.55</td>
<td>1.43</td>
</tr>
<tr>
<td>Investment</td>
<td>5.06</td>
<td>4.16</td>
<td>4.11</td>
<td>3.76</td>
</tr>
</tbody>
</table>

Table A1: Standard deviations of macro variables in the model and in the postwar US data (1950-2017). All series are HP-filtered with the smoothing parameter of $\lambda = 6.25$.

C.3 Bank-related parameters

I use the FR Y-9C reports to parameterize the return on storage $\rho_s$ and the number of banks per island $M$. The sample includes $n = 10$ largest bank holding companies (the results are robust to various cutoffs) by assets and spans from 1990 and 2017. Goldman Sachs and Morgan Stanley are excluded since they only became bank holding companies in 2008. Results are unchanged if they are not excluded. The variables are defined following Kovner, Vickery, and Zhou (2014).

In order to calibrate $\rho_s$ and $M$, I construct two spreads, $\overline{\text{Margin}}_{1,t}$ and $\overline{\text{Margin}}_{2,t}$,

\[
\overline{\text{Margin}}_{j,t} = \frac{\sum_{i=1}^{n} \text{Assets}_{i,t} \times \text{Margin}_{j,i,t}}{\sum_{i=1}^{N} \text{Assets}_{i,t}}, \quad j \in \{1, 2\}, \text{ where}
\]

\[
\text{Margin}_{1,i,t} = \frac{\text{Interest Income}_{i,t} - \text{Interest Expense}_{i,t}}{\text{Assets}_{i,t}} - \frac{\text{Interest Expense}_{i,t}}{\text{Liabilities}_{i,t}}
\]

\[
\text{Margin}_{2,i,t} = \frac{\text{Interest Income}_{i,t} - \text{Interest Expense}_{i,t}}{\text{Assets}_{i,t}}
\]

The model analogues of $\overline{\text{Margin}}_{1,t}$ and $\overline{\text{Margin}}_{2,t}$ are long-run averages of respectively $R - \frac{1}{\lambda} x - \rho$ and $R - \frac{1}{\lambda} x - \rho \mu$.

C.4 Banking crises statistics

Romer et al. (2017) present a semiannual series on financial distress in 24 OECD countries starting from 1967. Their measure takes values from 0 to 15, where 0 corresponds to no financial distress and 15 to extremely severe disruption. A crisis episode in my analysis starts and ends with a period of no distress; all periods between the start and finish are marked by nonzero distress. As suggested by the authors, a crisis is called systemic if during a crisis episode distress reaches value of 7 or higher. All other episodes are treated as nonsystemic. The start of a crisis
is defined as the year when the measure passes corresponding threshold for the first time (7 for systemic events and 1 for nonsystemic ones).

### D Sensitivity analysis

This appendix contains sensitivity analysis with respect to several key variables. Table A2 summarizes the long-run statistics of the economies under different parametrizations. Table of figures A3 compares the average paths of the economies around systemic crises.

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>A</th>
<th>C</th>
<th>L</th>
<th>Y</th>
<th>IC</th>
<th>(p^d_{syst})</th>
<th>(p^d_{nonsyst})</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.26</td>
<td>1.27</td>
<td>1.08</td>
<td>1.71</td>
<td>0.941</td>
<td>1.7%</td>
<td>2.5%</td>
<td></td>
</tr>
<tr>
<td>(\psi = 2.5)</td>
<td>4.14</td>
<td>1.26</td>
<td>1.07</td>
<td>1.68</td>
<td>0.938</td>
<td>1.4%</td>
<td>2.7%</td>
</tr>
<tr>
<td>(\nu = 0.3)</td>
<td>4.75</td>
<td>1.39</td>
<td>1.78</td>
<td>1.87</td>
<td>0.943</td>
<td>1.9%</td>
<td>2.4%</td>
</tr>
<tr>
<td>(\theta = 0.05)</td>
<td>4.30</td>
<td>1.29</td>
<td>1.09</td>
<td>1.72</td>
<td>0.898</td>
<td>1.6%</td>
<td>6.5%</td>
</tr>
<tr>
<td>(\gamma = 3)</td>
<td>4.32</td>
<td>1.29</td>
<td>1.09</td>
<td>1.72</td>
<td>0.918</td>
<td>0.6%</td>
<td>2.1%</td>
</tr>
<tr>
<td>(N = 20)</td>
<td>4.31</td>
<td>1.29</td>
<td>1.09</td>
<td>1.72</td>
<td>0.904</td>
<td>0.4%</td>
<td>4.8%</td>
</tr>
</tbody>
</table>

Table A2: Columns report, respectively, the long run averages of assets, consumption, labor, output, interconnectedness; the annual frequencies of systemic and nonsystemic financial crises for the benchmark and alternative (different parameterizations) models. The models are simulated for 1,000,000 periods.

**Intertemporal elasticity of substitution.** The household with relatively more elastic preferences towards intertemporal substitution (\(\psi\) is decreased from 5 to 2.5) dissaves more aggressively in the run-up of systemic crises. Events of severe financial distress are therefore preceded by smaller credit booms. Moreover, the household accumulates less assets on average. As a result, the financial system becomes more sparsely interconnected. Annual frequency of systemic crises drops from 1.7% to 1.4%. Non-systemic events happen more frequently.

**Frisch elasticity of labor supply.** If labor supply is more elastic (I consider a decrease from \(\nu = 0.6\) to \(\nu = 0.3\)), labor becomes more responsive to aggregate shocks. The household works more in response to a sequence of smaller (relative to the benchmark case) positive surprises. Credit booms become larger. Systemic crises occur somewhat more frequently.

**Default loss.** In the model, investing banks become interconnected because their interbank debt is subject to real default losses. If default losses get smaller (\(\theta\) is reduced from 0.1 to 0.05), noninvesting banks require smaller compensation for costs they incur in case of bankruptcies. The financial system’s interconnectedness declines substantially. Systemic crises happen less frequently; recessions following them are naturally milder due to lower \(\theta\). Nonsystemic crises, on the other hand, hit the economy much more often.

**Tail index.** An increase in the tail index of the Pareto distribution \(\gamma\) from 3 to 4 makes large adverse realizations of project-specific surprises less likely. Under smaller shocks investing banks do not default that frequently. They spend less on diversification, and the financial interconnectedness drops. The frequency of systemic events reduces substantially. The number of nonsystemic crises also goes down. However, due to weaker commonality between investing
banks’ portfolios, this decline is much less pronounced.

<table>
<thead>
<tr>
<th></th>
<th>Aggregate productivity, $z$ [% deviation from mean]</th>
<th>Project-specific shock, $x$ [% deviation from mean]</th>
<th>Assets, $A$ [% deviation from mean]</th>
</tr>
</thead>
<tbody>
<tr>
<td>IES</td>
<td><img src="image" alt="Chart" /></td>
<td><img src="image" alt="Chart" /></td>
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<tr>
<td>Frisch elasticity</td>
<td><img src="image" alt="Chart" /></td>
<td><img src="image" alt="Chart" /></td>
<td><img src="image" alt="Chart" /></td>
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<tr>
<td>Default loss</td>
<td><img src="image" alt="Chart" /></td>
<td><img src="image" alt="Chart" /></td>
<td><img src="image" alt="Chart" /></td>
</tr>
<tr>
<td>Tail index</td>
<td><img src="image" alt="Chart" /></td>
<td><img src="image" alt="Chart" /></td>
<td><img src="image" alt="Chart" /></td>
</tr>
<tr>
<td># of islands</td>
<td><img src="image" alt="Chart" /></td>
<td><img src="image" alt="Chart" /></td>
<td><img src="image" alt="Chart" /></td>
</tr>
</tbody>
</table>

Table A3: Average paths of the benchmark and alternative (different parameterizations) models around systemic crises ($t = 0$). The models are simulated for 1,000,000 periods. All series are expressed in percentage deviations from their long-run averages.

**Number of islands.** Increasing the number of islands $N$ from 10 to 20 improves the efficiency of diversification. A shock hitting one project becomes less detrimental since the losses it brings are shared by more institutions. At the same time, individual investing banks become less risky. The probability that projects of their specializations are directly hit goes down from 0.1 to 0.05. Demand for diversification weakens, and the interconnectedness declines. Under these circumstances a much larger shock is required to trigger a systemic crisis. The frequency of such events declines sharply. Not surprisingly, nonsystemic crises, associated with collapse of only one institution, are more likely in the system with more investing banks.

It is worth mentioning that considering a change in $N$ in isolation may be misleading. One
of the model’s interpretations of $N$ is the number of distinct asset classes. A larger $N$ then corresponds to a finer definition of those asset classes (e.g., ‘Commercial and industrial loans’ might be further divided into industry-specific subcategories). Such a refinement is however likely to impact the correlation structure of the project-specific surprises. The model assumes that each period only one project (one asset class) is shocked. It should be taken into account that narrower defined projects within one asset class are likely to be subject to positively correlated losses.

E Model extensions

This appendix considers several generalizations of the benchmark model. Appendix E.1 analyzes a version of the model where both aggregate and project-specific uncertainties are realized after interbank lending takes place. Appendix E.2 considers a model where the diversification costs affect investing banks’ default cutoffs directly. Appendix E.3 presents a model where the household’s and bankers’ attitudes to risk are aligned.

E.1 Model with two types of intraperiod uncertainty

In the benchmark economy the aggregate productivity $z$ is realized at the beginning of each period, before the interbank market takes place. The only source of intraperiod uncertainty is therefore island-specific shocks. In this appendix the timing assumption is relaxed. To keep the analysis transparent I consider the case when the size of project-specific shock follows the Pareto distribution, $\Phi(x) = 1 - \left(\frac{x}{m}\right)^\gamma$, $\gamma > 1$.

Theoretical analysis: Interbank problem. I start by analyzing the investing banks’ problem. I show that the analogues of Propositions 1 and 2 as well as of Corollaries 2.A and 2.B hold in this version of the model.

Denote $\xi = R - \rho \mu$. $R$ is now unknown at the beginning of a period because $z$ is realized ex-post. The condition of footnote 32 is assumed to hold, so for any realization of $z$ there always exist such a realization of project-specific shock $x < x_1 < \bar{x}$ hitting project $j$ that bank $i$ defaults when $x > x_1$ and does not default when $x \leq x_1$. The problem of investing bank $i$ can be then written as

$$\max_{\rho, \mu, (\omega_{ij})_{j=1}^N} \frac{a_\omega}{1 - \mu} \left[ \mathbb{E}_z \xi - \frac{\gamma}{\gamma - 1} \frac{x_m}{N} + \frac{1}{\gamma - 1} \mathbb{E}_z \xi^{1-\gamma} \frac{x_m}{N} \sum_{j=1}^N \omega_{ij}^\gamma - f(1 - \omega_{ii}) \right],$$

s.t. $\rho_s \leq \rho - \frac{1}{\mu} \left( \theta \mathbb{E}_z \xi - \gamma + \frac{1}{\gamma - 1} \mathbb{E}_z \xi^{1-\gamma} \right) \frac{x_m}{N} \sum_{j=1}^N \omega_{ij}^\gamma$,

$$\sum_{j=1}^N \omega_{ij} = 1, \, \omega_{ij} \geq 0 \forall j \in \{1, ..., N\}, \, \mu \leq \bar{\mu}.$$
It is straightforward to see that Lemma A1 and Proposition 1 hold. In particular, each investing bank holds a portfolio \( \omega_i = \alpha \geq \frac{1}{N} \) and \( \omega_j = \frac{1-\alpha}{N-1} \leq \frac{1}{N}, \forall j \neq i \). Observe that

\[
g_1(\alpha, \xi) = \xi^{-\gamma} \frac{x_0^\gamma}{N} \left( \alpha^\gamma + (N-1) \left( \frac{1-\alpha}{N-1} \right)^\gamma \right),
\]

\[
g_2(\alpha, \xi) = \frac{1}{\gamma-1} \xi^{1-\gamma} \frac{x_0^\gamma}{N} \left( \alpha^\gamma + (N-1) \left( \frac{1-\alpha}{N-1} \right)^\gamma \right).
\]

Denote \( g_p(\alpha) = \frac{x_0^\gamma}{N} \left( \alpha^\gamma + (N-1) \left( \frac{1-\alpha}{N-1} \right)^\gamma \right), \frac{\partial g_p}{\partial \alpha} > 0, \frac{\partial^2 g_p}{\partial \alpha^2} > 0 \). If \( \mu = \bar{\mu}, \alpha \) and \( \rho \) as functions of \( A \) and \( z_{-1} \) are given implicitly by

\[
Q_1(A, z_{-1}, \alpha, \rho) = \rho - \frac{1}{\mu} \left( \theta E_z \xi^{-\gamma} + \frac{1}{\gamma-1} E_z \xi^{1-\gamma} \right) g_p(\alpha),
\]

\[
Q_2(A, z_{-1}, \alpha, \rho) = \frac{\partial g_p}{\partial \alpha} \left[ E_z \xi^{-\gamma} + \frac{g_p(\alpha)}{1 - g_p(\alpha)} \left( \gamma \theta E_z \xi^{-\gamma} + \frac{\gamma}{\gamma-1} E_z \xi^{-\gamma-1} E_z \xi^{1-\gamma} \right) \right] - \frac{\bar{\mu}}{\bar{\theta}}.
\]

By the implicit function theorem,

\[
\frac{d\alpha}{dv} = -\frac{\partial Q_1}{\partial v} \frac{\partial Q_2}{\partial \alpha} - \frac{\partial Q_1}{\partial \alpha} \frac{\partial Q_2}{\partial v},
\]

where \( v \in \{A, z_{-1}\} \). It is easy to see that \( \frac{\partial Q_1}{\partial \alpha} < 0, \frac{\partial Q_2}{\partial \alpha} > 0, \frac{\partial Q_1}{\partial \rho} > 0 \) and \( \frac{\partial Q_2}{\partial \rho} > 0 \). Moreover, \( \frac{\partial Q_1}{\partial A} < 0 \) and \( \frac{\partial Q_2}{\partial A} > 0 \), while \( \frac{\partial Q_2}{\partial z_{-1}} > 0 \) and \( \frac{\partial Q_2}{\partial z_{-1}} < 0 \). Hence,

\[
\frac{d\alpha}{dA} < 0, \quad \frac{d\alpha}{dz_{-1}} > 0.
\]

Thus, the interconnectedness \( IC(\alpha) = \frac{1-\alpha}{1-1/N} \) increases in \( A \) but decreases in \( z_{-1} \), in line with the benchmark case. Observe that (see proof of Proposition 2)

\[
\frac{d\xi}{dA} < \frac{\partial R}{\partial A} + \frac{\partial g_p}{\partial \alpha} \bar{\mu}.
\]

Moreover, \( E_z \xi^{-a} \frac{\partial R}{\partial A} > E_z \xi^{-a} E_z \xi \frac{\partial g_p}{\partial \alpha} \), \forall a > 0 \), since both \( \xi^{-a} \) and \( \frac{\partial g_p}{\partial \alpha} \) are decreasing in \( z \). Therefore,

\[
\frac{d\xi}{dA} < \frac{\partial R}{\partial A} + \frac{\partial g_p}{\partial \alpha} \bar{\mu} < \frac{\partial R}{\partial A} - E_z \frac{\partial R}{\partial \alpha}.
\]

Expected spread \( E_z \xi \) is decreasing in \( A \). Similarly, \( E_z \xi \) is increasing in \( z_{-1} \). The analogue of Proposition 2 is therefore established.

Probability of systemic crisis is given by \( p^{d}_{syst} = \left( \frac{1-\alpha}{N-1} \right)^\gamma E_z \xi^{-\gamma} \). Corollaries 2.A and 2.B do not follow straightforwardly from Proposition 2 due to expectation terms. However, if the variance

\[36\]
of the conditional distribution \( z \mid z_{-1} \) is small, so that the second-order Taylor approximations for the moments of functions of the random variable \( z \) are sufficiently precise, it is possible to establish that \( \frac{d^2 \xi \gamma}{d \xi} > 0 \) and \( \frac{d^2 \xi \gamma}{dz_{-1}} < 0 \). Corresponding derivations are omitted for brevity and are available upon request. Importantly, numerically those inequalities are always satisfied.

Theoretical analysis: Household problem. The household’s problem is almost unchanged. The only difference is that now the state space includes four variables: the total amount of assets \( A \), the aggregate productivities of previous and current periods, \( z_{-1} \) and \( z \), respectively, and the project-specific shock size \( x \).

Numerical results. To investigate the role of uncertainty in the aggregate productivity, I solve the model under the benchmark calibration and compare the results of the benchmark and alternative models. Table A4 shows the long-run averages of the main variables as well as the frequencies of systemic and non-systemic financial crises for the two economies. A new source of uncertainty provides an additional incentive for investing banks to diversify. They spend more on risk sharing, which reduces the return on household’s assets. On average, the economy with two types of uncertainty features a smaller stock of assets, higher interconnectedness, more systemic and fewer non-systemic crises. However, the differences are quantitatively small.

<table>
<thead>
<tr>
<th></th>
<th>( A )</th>
<th>( C )</th>
<th>( L )</th>
<th>( Y )</th>
<th>( IC )</th>
<th>( p^d_{\text{syst}} )</th>
<th>( p^d_{\text{nonsyst}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>4.26</td>
<td>1.27</td>
<td>1.08</td>
<td>1.71</td>
<td>0.941</td>
<td>1.7%</td>
<td>2.5%</td>
</tr>
<tr>
<td>Uncertain ( z )</td>
<td>4.25</td>
<td>1.27</td>
<td>1.08</td>
<td>1.70</td>
<td>0.946</td>
<td>1.8%</td>
<td>2.5%</td>
</tr>
</tbody>
</table>

Table A4: Columns report, respectively, the long run averages of assets, consumption, labor, output, interconnectedness; the annual frequencies of systemic and non-systemic financial crises for the benchmark model and the model with 2 types of uncertainty (Uncertain \( z \)). The models are simulated for 1,000,000 periods.

Figure A2 compares the average paths of the two economies around systemic defaults. Not surprisingly, they are similar as well. In comparison with the benchmark case, in the model with two sources of uncertainty the aggregate productivity \( z \) plays a slightly more important role in the run-up of systemic crises. Since only \( z_{-1} \) is observable at the moment of portfolios formation, investing banks cannot timely respond to a lower than expected realization of \( z \). An adverse surprise to \( z \) increases default probabilities. Slightly smaller project-specific shocks are required to initiate a crisis.

E.2 Model with linking costs affecting default cutoffs

In the benchmark model the costs of portfolio formation are paid by investing banks’ shareholders (the representative household) prior to any interbank relationships are formed. This appendix considers the case when those costs are paid ex-post and thus directly affect investing banks’ default decisions. As in the previous section, I focus on the case when the size of project-specific shock follows the Pareto distribution, \( \Phi(x) = 1 - \left( \frac{x_m}{x} \right)^\gamma \), \( \gamma > 1 \).

Theoretical analysis. I assume that the diversification expenses are repaid before the interbank debt. Denote \( \xi = R - \rho \mu - f(1 - \omega_{ii}) \). The problem of investing bank \( i \) can be then
(a) Aggregate productivity, \( z \) [% deviation from mean]

(b) Project-specific shocks, \( x \) [% deviation from mean]

(c) Assets, \( A \) [% deviation from mean]

Figure A2: Average paths of the benchmark and extended (2 types of uncertainty) models around systemic crises \((t = 0)\). The models are simulated for 1,000,000 periods. All series are expressed in percentage deviations from their long-run averages.

written as

\[
\max_{\rho, \mu, (\omega_{ij})_{j=1}^N} \frac{a_o}{1 - \mu} \left[ \xi - \frac{\gamma}{\gamma - 1} \frac{x_m}{N} + \frac{1}{\gamma - 1} \xi^{1-\gamma} \frac{x_m^\gamma}{N} \sum_{j=1}^N \omega_{ij}^\gamma \right] ,
\]

s.t. \( \rho_s \leq \rho - \frac{1}{\mu} \left( \theta \xi^{-\gamma} + \frac{1}{\gamma - 1} \xi^{1-\gamma} \right) \frac{x_m^\gamma}{N} \sum_{j=1}^N \omega_{ij}^\gamma \),

\[
\sum_{j=1}^N \omega_{ij} = 1, \quad \omega_{ij} \geq 0 \quad \forall j \in \{1, \ldots, N\}, \quad \mu \leq \bar{\mu}.
\]

Proofs of Lemma A1 and Proposition 1 closely follow the ones for the benchmark case.

Similar to Appendix E.1, denote \( g_P(\alpha) = \frac{x_m^\gamma}{N} \left( \alpha^{\gamma} + (N - 1) \left( \frac{1}{N-1} \right)^\gamma \right) \), \( \frac{\partial g_P}{\partial \alpha} > 0, \frac{\partial^2 g_P}{\partial \alpha^2} > 0 \). Investing bank defaults with probability \( p_{ind}^d = g_P(\alpha) \xi^{-\gamma} \), where \( \xi = R - \rho \bar{\mu} - f(1 - \alpha) \) as long as the constraint (7) binds. Diversification now has a dual impact on \( p_{ind}^d \). On the one hand, low \( \alpha \) correspond to low \( g_P(\alpha) \), as in the benchmark case. Unlike the main scenario, on the other hand, now higher diversification expense also reduces the spread \( \xi \) which drives \( p_{ind}^d \) up. Investing banks therefore trade benefits of more dispersed portfolios for thinner profit margins as well as higher direct diversification costs.

It is generally unclear whether diversification is effective in keeping default probability low, so I can not establish an analogue of Proposition 2 in this case. I therefore rely on numerical analysis. As it is shown below, comparative statics results of Proposition 2 and Corollary 2.A still hold.

**Numerical results.** Solving the model under the benchmark calibration reveals how alternative modeling of the diversification costs impacts the allocation. First of all, the spread \( \xi = R - \rho \bar{\mu} - f(1 - \alpha) \) narrows ceteris paribus, making interbank debt riskier. Investing banks respond by increasing risk sharing. Despite now linking expenses directly enter \( \xi \) with a negative sign, the diversification benefits dominate. Portfolio overlap strengthens. Both forces contribute to increased probability of systemic crises: their frequency rises by 40%. The number of non-systemic events goes up to a smaller extent because the effect of reduction in \( \xi \) is compensated by a higher correlation between investing banks’ portfolios.

Figure A3 compares the average paths around systemic crises in the benchmark and alternative models. Boom-bust dynamics around such events is preserved. At the same time, the
amplitude of boom-bust cycles becomes somewhat smaller. The alternative economy is more susceptible to severe financial distress even at its steady state. A rise in the stock of assets and interconnectedness above their long-run averages has a smaller marginal impact on the probability of systemic crisis.

E.3 Model with no misalignment in risk attitudes

In the benchmark model the household has motives to intertemporally smooth consumption. Its preferences exhibit a finite elasticity of intertemporal substitution, $\psi > 0$. Since the utility is of CRRA class, risk aversion and elasticity of intertemporal substitution are one-to-one related. The household is therefore risk-averse, while banks are risk-neutral. Risk-neutrality of banks makes their optimization problem tractable. In particular, it allows for analytical characterization of the comparative statics. Moreover, it simplifies the numerical analysis substantially. Under banks’ risk-neutrality expectations with respect to the size of project-specific shock can be computed analytically which turns out to be important to obtain precise solution to the interbank problem.

The misalignment of the bankers’ and household’s attitudes to risk might be important for the welfare analysis of policies, especially of changes in the diversification cost $f$. In comparison with risk-neutral banks, the risk-averse representative household suffers more from large-scale systemic crises. Risk-neutral banks therefore might do too much risk sharing from the household’s perspective. If preferences are aligned, all agents weigh the same events equally. This appendix lays out the interbank problem where the household’s stochastic discount factor enters the banks’ objective. I then show that the results, including welfare implications of policies, are
largely unchanged in this version of the model.

**Interbank problem.** Banks weigh states of the world by the household’s marginal utilities at those states, $M(A, z, x) = (C(A, z, x) - \frac{1}{1+\nu}L(A, z)^{1+\nu})^{-\psi}$. Investing banks solve

$$\max_{\rho,\mu, \{\omega_{ij}\}} \frac{a_o}{1 - \mu} \mathbb{E}_x \left\{ M(A, z, x) \left[ \frac{1}{N} \sum_{j=1}^{N} (R - \rho \mu - \omega_{ij} x) \mathbb{I}\{x \leq \frac{R - \rho \mu}{\omega_{ij}}\} - f \sum_{j \neq i} \omega_{ij} \right] \right\},$$

s.t. $0 \leq \mathbb{E}_x \left\{ M(A, z, x) \left[ \frac{1}{N} \sum_{j=1}^{N} \left( R - \rho \mu \mathbb{I}\{x \leq \frac{R - \rho \mu}{\omega_{ij}}\} - \rho_s \right) \right] \right\}$,

$$\sum_{j=1}^{N} \omega_{ij} = 1, \quad \omega_{ij} \geq 0 \forall j \in \{1, \ldots, N\}, \quad \mu \leq \bar{\mu}.$$

Since stochastic discount factor $M(A, z, x)$ enters the objective function, I now cannot characterize the solution analytically and rely on numerical analysis instead.

**Numerical results.** In the benchmark analysis banks are willing to diversify purely because defaults are subject to real losses. Now banks take into account that the household is risk-averse and thus dislikes larger realizations of $x$, even if they do not lead to costly bankruptcies. However, numerically it turns out that default-related considerations play a dominant role. In particular, the variation in the household’s stochastic discount factor due to changes in $x$ within the three default regions (normal times, $N^d = 0$; nonsystemic crisis, $N^d = 1$; systemic crisis, $N^d = N$) is small in comparison with the variation due to changes across the regions. I therefore approximate the $M(A, z, x)$ function as

$$M(A, z, x) \approx \tilde{M}(A, z, x) = \begin{cases} M_0(A, z) = \mathbb{E}_x [M(A, z, x)|N^d = 0], & \text{if } N^d(A, z, x) = 0, \\ M_1(A, z) = \mathbb{E}_x [M(A, z, x)|N^d = 1], & \text{if } N^d(A, z, x) = 1, \\ M_N(A, z) = \mathbb{E}_x [M(A, z, x)|N^d = N], & \text{if } N^d(A, z, x) = N. \end{cases}$$

$\tilde{M}(A, z, x)$ takes three values given $(A, z)$. It is possible to compute expectations in the problem (A6) analytically, which considerably simplifies the solution.

The model is solved iteratively. Starting from the benchmark solution, I compute $\tilde{M}(A, z, x)$, solve (A6) and then the intertemporal household’s problem. This procedure is repeated until the change in the interbank optimal choices becomes trivial.

To make the models with and without preferences misalignment comparable, I recalibrate some of the parameters to match the same targets as in the main text. The change in parameterization is modest. First, noninvesting banks start to require higher compensation for defaults, pushing the interbank rate $\rho$ up. $\rho_s$ is reduced from 1.0092 to 1.0087 so that the average profit margin $R - \frac{1}{N} \mathbb{E}_x x - \rho$ is 2.4%. Second, the number of costly systemic crises goes down; $x_m$ is increased from 0.088 to 0.089 in order to set their frequency to 1.7 per 100 years. Investing
banks’ expenses on diversification are barely affected. The cost parameter $f$ is therefore held at the same level.

One noticeable difference between the benchmark and current models is the number of non-systemic crises. In absence of preferences misalignment, banks are more averse to large-scale defaults. They are willing to decrease their frequency at a cost of more milder non-systemic crises. Since I target the frequency of systemic events of 1.7 per 100 years in both versions of the model, it is not surprising that non-systemic crises are more numerous in the economy without preferences misalignment (3.0% versus 2.5% in the benchmark case).

The economy’s dynamics around systemic and non-systemic crises does not change in any substantial way. Corresponding results are omitted for brevity. In what follows, I discuss the welfare analysis in the alternative model.

A transition from the decentralized to constrained efficient equilibrium results in 0.05% of permanent consumption gain (Table A6). The amount of assets, consumption, working hours, output and interconnectedness all decrease. The number of systemic crises reduces from 1.7 to 1.1 per century. Non-systemic crises become slightly more frequent. All these results, both qualitatively and quantitatively, are very similar to those in the benchmark case. Welfare gains from a flat tax on savings are also almost identical. Different risk attitudes of the household and bankers therefore do not have any important impacts on the over-saving problem arising in my model.

Interestingly, in this version of the model I find also a similar impact of financial innovations on the household’s welfare. In particular, a 25% higher $f$ brings a modest 0.02% increase in the permanent consumption.

<table>
<thead>
<tr>
<th></th>
<th>$A$</th>
<th>$C$</th>
<th>$L$</th>
<th>$Y$</th>
<th>$IC$</th>
<th>$p^d_{syst}$</th>
<th>$p^d_{nonsyst}$</th>
<th>$\kappa_{DE\rightarrow i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DE$</td>
<td>4.27</td>
<td>1.27</td>
<td>1.08</td>
<td>1.71</td>
<td>0.938</td>
<td>1.7%</td>
<td>3.0%</td>
<td></td>
</tr>
<tr>
<td>$SB$</td>
<td>3.99</td>
<td>1.24</td>
<td>1.06</td>
<td>1.65</td>
<td>0.924</td>
<td>1.1%</td>
<td>3.3%</td>
<td>0.05%</td>
</tr>
<tr>
<td>$DE_{optimal flat tax}$</td>
<td>4.00</td>
<td>1.24</td>
<td>1.06</td>
<td>1.65</td>
<td>0.922</td>
<td>1.2%</td>
<td>3.3%</td>
<td>0.03%</td>
</tr>
<tr>
<td>$DE_{optimal f}$</td>
<td>4.20</td>
<td>1.26</td>
<td>1.07</td>
<td>1.69</td>
<td>0.923</td>
<td>1.5%</td>
<td>3.8%</td>
<td>0.02%</td>
</tr>
</tbody>
</table>

Table A6: Impacts of policies on the allocations in the economy with no preference misalignment between banks and the representative household. Columns report, respectively, the long-run averages of assets, consumption, labor, output, and interconnectedness; the annual frequencies of systemic and non-systemic financial crises; and the compensating variation in $\kappa_{DE\rightarrow i}$ for decentralized ($DE$), second-best ($SB$), decentralized with the optimal flat tax on savings ($DE_{optimal flat tax}$), and decentralized with optimal diversification cost ($DE_{optimal f}$) allocations. By definition, $\kappa_{DE\rightarrow DE} = 0$. To compute $\kappa_{i\rightarrow j}$, I simulate 10,000 paths of the economies $i$ and $j$ for 500 periods, starting from the ergodic distribution of $i$.  
References: Appendix


