

Dividend Payouts and Rollover Crises

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Abstract

We study the interaction between bank dividend payouts and rollover crises. By increasing available liquidity, a reduction in dividends has a *resilience effect* – a direct positive effect on the bank’s ability to survive a rollover episode. However, it also has a *signaling effect*, since short-term lenders that decide on rolling over their debt use the dividend payment to infer the bank fundamentals. When the signaling effect is weak and the resilience effect dominates, banks exert a negative externality on other banks when paying dividends, which ends up amplifying financial instability. In contrast when the signaling effect is strong, banks use dividend payments to manage the rollover crisis. We use our framework to analyze the effects of different dividend regulation policies during periods of financial stress and also discuss the empirical relevance of our theory.

Key words: Rollover crises, global games, noisy signaling, dividend policies, financial stability, amplification.

JEL Codes: G01, G21, G35

1 Introduction

The dividend policies of banks received much attention in the wake of the 2007-2008 financial crisis. The U.S. banking sector maintained large dividend payouts throughout 2007 and 2008, even as losses were increasing rapidly (Acharya, Shin, and Gujral, 2009). Aggregate dividends paid by U.S. banks in 2008 exceeded their aggregate earnings by about 30 percent (Floyd, Li, and Skinner, 2015). Moreover, for the 19 largest U.S. banks, the dividends paid from the fall of 2007 to the fall of 2008 correspond to roughly 50 percent of the funds that were used in bailing out these banks (Rosengren et al., 2010).

One explanation for banks' dividend policies during the early stages of the financial crisis is that they reflected a form of moral hazard. Scharfstein and Stein (2008) argue that banks engaged in "risk shifting" and that their dividend policies were "... an attempt by shareholders to beat creditors out the door". Another explanation focuses on a potential signaling role of dividends. Acharya, Gujral, Kulkarni, and Shin (2011) suggest that U.S. banks were worried that cutting dividends could induce a run by their short-term creditors. Floyd, Li, and Skinner (2015), and Hirtle (2014) compare the evolution of dividend payouts and share repurchases by U.S. banks – two ways to return cash to shareholders – prior to and during the crisis. While dividends and share repurchases followed similar patterns prior to the crisis, banks cut their share repurchase programs substantially in 2007-2008 but maintained dividend payments.¹

Can these two views of dividend payments both play a role to explain bank behavior? Also, is there an informational role of dividends when banks face dispersed short-term lenders that try to coordinate their decisions to roll over maturing debt? Finally, what is the impact of dividend regulation policies on rollover risk when both "risk shifting" and "signaling" motives are present? In this paper, we address these questions by examining theoretically the role of dividends when banks are subject to coordination-based rollover crises or runs (Diamond and Dybvig, 1983).

In our framework, a bank (owner) can use dividend payouts to precipitate its failure and "beat creditors out the door" during a rollover episode. We call this direct effect on survival the *resilience effect* of dividends. Absent any other interactions, lowering dividends increases resilience and improves financial stability. However, dividends also convey information about

¹Such a "signaling" view goes beyond dividend payouts and concerns a number of bank actions that seem to worsen banks' proximate liquidity position in times of financial stress. For example, Duffie (2010) provides a description of a hypothetical dealer bank's actions in response to financial stress. He notes that the bank "... takes actions that worsen its liquidity position in a rational gamble to signal its strength and protect its franchise value. [The bank] wishes to reduce the flight of its clients, creditors, and counterparties." Such actions include compensating clients for losses on investments arranged by the bank or continuing with OTC derivative trades that reduce available liquidity. Although these actions are not our proximate motivation, our theoretical framework can be interpreted more broadly and used to analyze their signaling effects as well.

the bank's underlying assets and its ability to survive a rollover episode. Thus, a bank's dividend policy affects the incentives of short-term lenders to roll over their debt. We call this second indirect effect on survival the *signaling effect*. Whenever the signaling effect is sufficiently strong, some banks use their dividend policies to help lenders coordinate their rollover decisions. Therefore, even though a bank appears to be reducing its resilience by paying dividends, it may in fact be reducing the impact of the rollover episode.

We now provide further details for our analysis. We consider a bank that is financed by a continuum of short-term lenders (or lenders, for short).² Lenders simultaneously choose whether to roll over their maturing debt but face a coordination problem – if a sufficient number of lenders refuse to roll over (run, for short), then the bank does not have enough liquidity to repay all lenders and is forced to fail. In that case, an individual lender is better off running than rolling over. Lenders, however, have incomplete and dispersed information about the quality of the bank's portfolio, which also determines the total liquidity available to the bank.

At an initial stage, prior to the rollover episode, the bank (owner) chooses a dividend to maximize its payoff. It derives a positive payoff from consuming the dividend paid out but incurs a cost in terms of a reduction in the value of bank assets, *conditional* on surviving the rollover episode. Therefore, a bank which expects to fail the rollover episode (and does not care about its continuation value) has an incentive to pay as much in dividends as it feasibly can. Additionally, we assume that while liquidating assets is costly for any bank, conditional on survival, it is relatively more costly for banks with lower portfolio quality. Therefore, in the absence of a rollover episode, higher quality banks choose to pay higher dividends. In that case higher dividends constitute good news about a banks's portfolio quality.

For simplicity, we start our analysis by restricting the choice of dividends to one of two levels – either pay a fixed positive dividend or do not pay any dividend. In that setting we compare two cases. First, we switch off the signaling effect completely by assuming that lenders do not observe the dividend policy of the bank and instead observe an exogenous private signal about the bank's type. In that case only the resilience effect is present, and a bank that pays out a dividend ends up increasing the liquidity outflow it will eventually experience. Since a higher liquidity outflow can be met only by a bank of higher quality, the option to make a dividend payment increases the fraction of failing banks.

Moreover, in this case banks impose a negative externality on other banks when they choose to pay a dividend (and end up failing) due to the rollover decision of lenders. Specifically, strengthening the incentives to pay a dividend induces more banks to choose to pay

²Even though we motivate and frame our analysis in the context of banking and dividend payouts, the implications are applicable more generally to any firm that is subject to rollover risk and which can take an action that has a direct negative effect on its liquidity position but also conveys information to lenders.

a dividend and fail, increasing the fraction of failing banks. This in turn makes lenders less willing to roll over their debt for *all* banks, which induces more banks to choose to fail. The resulting equilibrium feedback, thus, ends up amplifying financial instability.

Next, we introduce the signaling effect by assuming that the lenders observe the bank's dividend and make inferences about the bank's type based on that information and their prior beliefs. In that case, the dividend choice of the bank acts as an endogenous signal about the bank's type. To ensure equilibrium uniqueness, we additionally assume that lenders observe the dividend with small idiosyncratic noise. Therefore, while we think of dividends as publicly-available information, we do not model it as public information in the game-theoretic sense.³

When lender signals are sufficiently precise, the signaling effect is strong, so paying a dividend can actually *decrease* the total liquidity outflow that the bank experiences. There are two reasons for this stark outcome. First, observing higher dividends constitutes good news about the bank's survival if there is a sufficiently large share of high-quality banks, which always survive the rollover episode and pay a dividend, and of low-quality banks, which never survive the rollover episode and cannot pay a dividend. In that case, paying a dividend allows a bank to pool with high-quality surviving banks and separate from very low-quality failing banks. Second, the high signal precision of lenders implies that the dividend choice of the bank influences the actions of a large group of these lenders. Thus, while the direct effect of paying a dividend is to reduce resilience and increase the liquidity outflow from the bank, the indirect effect through the rollover actions of lenders decreases the liquidity outflow from the bank. When lender signals are sufficiently precise and lenders are sufficiently coordinated, the indirect effect dominates the direct effect.

We then consider a richer model, in which the bank can pay any non-negative dividend that is feasible given its portfolio quality. In that case, a strong signaling effect lowers the sensitivity of dividends to the bank fundamentals relative to their dividend payout in the absence of rollover. Put differently, surviving banks with very different fundamentals may choose to pay similar dividends in equilibrium. Intuitively, banks with lower fundamentals have strong incentives to distort their dividends upward and pay a dividend similar to higher-quality banks to help lenders coordinate on rolling over. On the other hand, since lenders only care about whether the bank fails or survives the rollover episode (rather than the specific

³We are not the first to treat publicly-available information in this way (see, e.g. Woodford (2003), Myatt and Wallace (2014), Kolbin (2015), Angeletos and Lian (2016a), or Gaballo (2016)). If dividends are common knowledge, the economy trivially admits multiple equilibria. Introducing a small amount of private noise in the observation of dividends removes the common knowledge aspect from the dividend signal. Note, also, that adding a small amount of private noise to a signaling action in a global coordination game does not necessarily lead to a unique equilibrium (e.g. Angeletos, Hellwig, and Pavan, 2006). Nevertheless, under certain conditions, there will be a unique equilibrium in our framework.

bank type), the dividend payments of banks with higher fundamentals are already interpreted by (most) lenders as evidence that the bank will survive the rollover episode, so any upward distortion in dividends for those bank types has only a small effect on the lenders' behavior.⁴

We also discuss some policy implications of our framework. We show that restricting dividend payouts during a rollover crisis has an *ambiguous* effect on bank failure. Intuitively, restricting dividends fully removes the bank's risk-shifting incentives. However, it also shuts down the signaling effect, impacting the banks' ability to manage the rollover episode. If the rollover crisis is sufficiently severe and the risk-shifting incentives are weak, restricting dividends ends up increasing the bank failure threshold (and vice versa). In contrast, we show that a (proportional) tax on dividends *unambiguously* reduces the bank failure threshold irrespective of the strength of the risk-shifting incentives or the severity of the rollover crisis. The reason is that a dividend tax both weakens the risk-shifting incentives but also maintains the signaling effects of dividends.

Finally, we discuss the empirical relevance of our theory. One implication is that rollover risk combined with a strong signaling effect reduce the sensitivity of dividends to fundamentals. Consistent with this, we show that banks which were more reliant on short-term funding prior to the crisis were less likely to cut dividends during the crisis. We also find cross-industry support for this link by showing that dividend payments are more stable in industries in which firms have greater reliance on short-term funding.

1.1 Related literature

Our paper is related to several strands of research. First, it is related to the growing literature on bank dividend payouts, particularly during a financial crisis, and the optimal policy response to those (Acharya, Le, and Shin (2013), Floyd, Li, and Skinner (2015), Hirtle (2014), Cziraki, Laux, and Loranth (2016)). Acharya, Le, and Shin (2013) study a model of bank dividend payouts, in which risk shifting by the bank equity holders because of a possible low future franchise value is an important motive for paying dividends. When banks are linked through an inter-bank market, there is an additional dividend externality that may lead to a systemic crisis, since a higher dividend payout by one bank makes it less likely to repay its inter-bank claims. This, in turn, reduces the franchise value of the bank's creditors and strengthens their incentives to risk shift. Our modeling approach complements this framework by studying the informational role of dividends when banks are exposed to a coordination-based run. When the signaling effect of dividends is weak or absent we also

⁴In the limit, as lenders get arbitrarily precise signals and are almost perfectly coordinated, the incentives to compress dividend payouts are so strong for surviving banks around the failure threshold so that, locally, banks pool on their dividend payouts.

uncover a negative dividend externality that banks impose on other banks, which reduces overall financial stability. However, rather than arising from direct spillovers via bank linkages, in our model, spillovers between banks arise through the inference of lenders and their rollover decisions.

The informational role of dividends in our model relates our paper to the seminal work of Bhattacharya (1979) and a large subsequent literature (Miller and Rock (1985), John and Williams (1985), Hausch and Seward (1993), Guttman, Kadan, and Kandel (2010), Baker, Mendel, and Wurgler (2016)). Bhattacharya (1979) argues that in the presence of asymmetric information about the prospects of a firm, dividends can serve as a signal to outside investors. In his paper, stronger firms have incentives to separate from weaker firms to ensure favorable stock valuations. In contrast, in our environment with coordination-based crises, surviving banks of intermediate strength have incentives to pay dividends similar to those of stronger banks and separate from very weak banks. Thus, our results are related to the partial pooling result of Guttman, Kadan, and Kandel (2010). However, while in their framework a partial pooling equilibrium is one of many possible equilibria (including a fully separating equilibrium), in our framework, the partial pooling equilibrium is unique (given some conditions). Furthermore, the pooling is only local (around the bank failure cutoff) in the limiting case where lenders have arbitrarily precise signals. Away from that limit and when dividend choices are unrestricted, surviving banks choose different (albeit similar) dividends.

Our paper is related to the large literature on global games of regime change (e.g Carlsson and Van Damme (1993) and Morris and Shin (1998)) and particularly to global game models of bank runs (Goldstein and Pauzner (2005), Rochet and Vives (2004)) and rollover crises (Morris and Shin (2004)).⁵ We contribute to this important literature by analyzing how banks use their dividend payouts to manage the rollover crisis. In addition, while most of these models assume an exogenous information structure for lenders or an exogenous resilience level for banks, both the information structure of lenders and the resilience level of banks are endogenous in our model.

Our paper is particularly related to models of signaling in global games. Angeletos, Hellwig, and Pavan (2006) and Angeletos and Pavan (2013) consider a regime-change game in which the regime can undertake a costly policy action to influence the cost for agents of attacking. They show that the information conveyed by the policy action may restore multiplicity. Edmond (2013) studies a model of regime-change in which the regime can engage in costly manipulation of the private information of agents considering staging a revolution. In equilibrium, agents try to infer the true type of the regime given the signals

⁵More recently, Vives (2014) uses a global games model of bank runs to analyze liquidity regulation.

they observe. In his framework there is a unique equilibrium.

As in Edmond (2013), our economy may also admit a unique equilibrium despite the signaling effect of the bank's actions and the endogenous information structure that arises. Relative to Edmond (2013), we are motivated by a different question and consider a different environment. In particular, we study how a bank optimally chooses its dividend policy when faced with a coordination-based run, while he studies how a regime engages in costly manipulation of agents' private signals about its type (i.e. propaganda). In our framework, paying out a dividend has a direct positive payoff to the bank, while in Edmond (2013) the regime incurs a cost when manipulating the agents' information. Thus, in our model a bank that is certain it would fail the rollover episode pays out dividends, while in his framework, a regime that is certain that regime change will take place does not try to manipulate the agents' beliefs. Also, in our framework, the direct effect of paying out dividends is to weaken the ability of the bank to survive the rollover episode, so it is not clear *a priori* if paying out dividends increases or decreases the bank failure threshold. In Edmond (2013), the costly action of manipulating agents' information does not influence directly the ability of the regime to survive, so the regime's action cannot be destabilizing.

Goldstein and Huang (2016a) study how a regime can increase the probability of survival by committing to abandoning the status quo for some fundamentals. However, the information transmission that takes place in their model, and which ends up stabilizing the regime, is more in the spirit of the Bayesian persuasion literature (Kamenica and Gentzkow, 2011) rather than through sending a costly signal.⁶

Finally, in its treatment of how the endogenous information structure induced by banks' dividend policies affects financial stability, our paper is related to the literature on information disclosure and financial stability (for instance from stress-testing as in Bouvard, Chaigneau, and Motta (2015), Faria-e Castro, Martinez, and Philippon (2015), and Goldstein and Leitner (2016), or credit ratings as in Goldstein and Huang (2016b), and Holden, Natvik, and Vigier (2016)) and also to papers studying the effects of information quality and transparency on stability (Iachan and Nenov (2015), Moreno and Takalo (2016)). In contrast to many of these papers, we focus on information generated by one of the parties in the rollover game, which maximizes its own payoff, rather than a third party (i.e. a regulator) who has an explicit objective to improve financial stability. Thus, the private incentives to pay dividends in our model are not necessarily aligned with concerns for financial stability.

⁶Shapiro and Skeie (2015) also study a model of signaling and banking crises. However, in their paper the sender of the costly signal is a policy maker rather than the bank itself. Also, runs on banks are not due to a coordination failure as in our framework.

2 Model

Consider an economy with three periods, $t \in \{0, 1, 2\}$. There is a bank with an exogenously given asset and liability structure at the beginning of $t = 0$. The bank has a continuum of short-term creditors who make rollover decisions on their debt at $t = 1$.

2.1 The bank

We let $\theta \in \mathbb{R}$ parametrize the portfolio quality of the bank (its “fundamentals”). A higher θ means stronger fundamentals. At the beginning of $t = 0$, the bank holds a portfolio consisting of assets with different $t = 0$ liquidation values and $t = 2$ payoffs. At $t = 0$, the bank can make changes to its asset structure. Specifically, it can convert part of its asset portfolio into liquid assets (cash and cash-like instruments) of size l . l is obtained by selling part of the portfolio or borrowing against it as collateral. Out of l the bank chooses a dividend payment d to make at $t = 0$. The bank uses the residual, $g = l - d$, to meet redemptions by short-term lenders at $t = 1$ (see below). Therefore, g captures the (endogenous) resilience of the bank.

We denote the $t = 2$ value of the remaining part of the bank’s asset portfolio *conditional on* surviving the rollover episode at $t = 1$ by $v(\theta, l)$. $v(\theta, l)$ is twice continuously differentiable, with $v_\theta > 0$, $v_l < 0$, and $v_{ll} < 0$. Therefore, we assume that the value of the remaining part of the bank’s portfolio is strictly increasing in θ . It is decreasing in l , since holding cash and cash-like instruments is assumed to yield a lower return than the long-term loans that the bank is initially endowed with. We assume it is concave in l , since the bank has to sell progressively more illiquid assets to get an additional dollar in cash. In addition, we assume that $v_{l\theta} > 0$. Therefore, while liquidating assets is costly for any bank, conditional on survival, it is relatively more costly for banks with lower θ . This is a single-crossing condition, and as we show below (Proposition 1), it implies that in the absence of a rollover episode banks with higher θ would choose to pay higher dividends. While we will work with this general form throughout the paper, below we present some possible (partial) microfoundations for this asset structure.

Given the properties of $v(\theta, l)$, there is a limit on the maximum available liquid assets that a bank can obtain at $t = 0$ denoted by $\bar{\ell}(\theta)$, which satisfies

$$v(\theta, \bar{\ell}(\theta)) = 0. \tag{1}$$

By the properties of v , $\bar{\ell}(\theta)$ is strictly increasing in θ (i.e. $\bar{\ell}' = -v_\theta/v_l > 0$).

The $t = 0$ liabilities of the bank consist of dispersed short-term debt that matures at $t = 1$ with total face value normalized to 1. The short-term debt is held by a unit-measure

continuum of lenders, who at $t = 1$ choose whether to redeem it or roll it over into $t = 2$. The bank may fail at $t = 1$, if it does not have enough liquid assets to meet redemptions by short-term lenders. Specifically, if A denotes the fraction of short-term lenders that refuse to roll over, then the bank survives iff

$$g \geq A. \tag{2}$$

The expected $t = 2$ payoff of the bank owner conditional on surviving the rollover episode is given by $W_2(v, A)$. That payoff may, in general, depend on the value of remaining assets and on the fraction of short-term lenders that have refused to roll over their debt. However, to simplify the analysis, we will be working with $W_2(v, A) = v$. Hence, conditional on surviving the rollover episode at $t = 1$, there is no conflict of interest between the remaining short-term lenders and the bank owner, so that the bank owner cares about the full residual value of the bank's assets.⁷ Therefore, we can write the bank owner's $t = 0$ payoff as

$$W(\theta, g, d, A) = \lambda d + \mathbf{1}_{\{g \geq A\}} v(\theta, d + g), \tag{3}$$

where $\mathbf{1}_{\{g \geq A\}}$ is an indicator for whether the bank survives the rollover episode at $t = 1$ and $\lambda > 0$ parametrizes the degree to which the bank owner cares about paying out a dividend at $t = 0$ versus waiting for assets to mature.

Since the bank owner is assumed to care about the full residual value of assets at $t = 2$, while the dividend payoff carries a direct private benefit to him, one can interpret a lower value of λ as a proxy for the strength of corporate governance and the alignment of interests between equity and debt holders within the bank. Conversely, the higher is λ the stronger the incentives of the bank owners to “beat creditors out the door”.

2.1.1 Microfoundations

We now provide some (partially) microfounded examples for $v(\theta, l)$. Consider the asset portfolio of the bank and suppose that assets are indexed according to their liquidation (or collateral) value. We denote this index by $a \in [0, 1]$ and assume, without loss of generality, that asset liquidity is decreasing in a . Therefore, cash and cash-like assets have a liquidity index of $a = 0$, while fully illiquid assets have an index of $a = 1$. In between are partially illiquid assets which the bank can sell or borrow against but at a discount. Specifically, let

⁷Assuming that the bank owner cares only about his expected $t = 2$ equity payoff net of promised payments to maturing short-term lenders would strengthen the bank owner's incentives to pay out dividends and induce the bank's failure even when the bank has the resources to survive a rollover episode. Since such incentives are already present when the bank owner is assumed to care about the full residual value of the bank's assets, allowing for a more general continuation payoff for the bank owner will not change the qualitative predictions of our model but will come at a substantial reduction in tractability.

$\rho(a, \theta) \in [0, 1]$ denote the $t = 0$ liquidation discount or the haircut which is applied to assets with index a if the bank borrows against them. Therefore, for cash and cash-like assets $\rho(0, \theta) = 0$ and for fully illiquid assets $\rho(1, \theta) = 1$. Also, let $X(a, \theta)$ denote the expected $t = 2$ payoffs of assets with liquidity index a if left until maturity. Therefore, the $t = 0$ liquidation value of assets with liquidity index a is $(1 - \rho(a, \theta)) X(a, \theta)$.

Given this indexing, a bank that seeks to obtain l units of cash at $t = 0$ first liquidates/borrows against assets with liquidity index $a = 0$ and then moves on to assets with a higher liquidity index. Specifically, let $\tilde{a}(l, \theta)$ denote the index of the marginal assets that a type θ bank has to liquidate or borrow against to satisfy its demand for cash. Then \tilde{a} is implicitly defined by

$$l = \int_0^{\tilde{a}(l, \theta)} (1 - \rho(a, \theta)) X(a, \theta) da. \quad (4)$$

The remaining part of the bank's asset portfolio has an expected $t = 2$ value of

$$v(\theta, l) \equiv \int_{\tilde{a}(l, \theta)}^1 X(a, \theta) da. \quad (5)$$

Below we provide two specific structures for X and ρ , which result in an residual asset function, $v(\theta, l)$, with the properties assumed in Section 2.1.

Example 1: Higher $\theta \Rightarrow$ higher asset payoffs.

Suppose that ρ and X are continuously differentiable, and that $X_\theta > 0$. This can arise because a high θ bank has better quality assets with higher expected payoffs or a better monitoring technology than a low θ bank. Alternatively, it can be the case that a high θ bank has more assets (for the same amount of liabilities) compared to a low θ bank. Also, let $\rho(a, \theta) = \rho(a)$. By definition, $\rho_a > 0$. In the Appendix we show that these assumptions imply that $v_\theta > 0$, $v_l < 0$, $v_{ll} < 0$, and $v_{\theta l} > 0$.

Example 2: Higher $\theta \Rightarrow$ more liquid assets.

Suppose that ρ and X are continuously differentiable and let $X(a, \theta) = X(a)$. Also, suppose that $\rho_\theta < 0$. This can arise because a high θ bank has higher-quality assets, which are easier to liquidate or borrow against compared to a low θ bank. Alternatively, a high θ bank may have a superior asset liquidation technology, for example, because it is a market maker in some decentralized asset markets and can meet potential buyers with a higher probability. Finally, it may be the case that a high θ bank has better reputation than a low θ bank due to a history of repayment of liabilities, so it can borrow against the same asset with a lower

haircut compared to a lower θ bank. In the Appendix we show that these assumptions imply that $v_\theta > 0$, $v_l < 0$, $v_u < 0$, and $v_{\theta l} > 0$.⁸

2.2 The lenders

The lender side of the rollover game is standard (e.g. Morris and Shin, 2004). There is a unit-measure continuum of short-term creditors of the bank, which we refer to as the lenders. The lenders can either roll over their debt or refuse to roll over (run, for short). Lenders take their decisions at $t = 1$. If a lender runs at $t = 1$, she obtains

$$\pi_1(A, g) = 1,$$

which is the (normalized) face value of her short-term debt. If she rolls over at $t = 1$, she obtains

$$\pi_0(A, g) = \begin{cases} B & : g \geq A \\ k_0 & : g \leq A \end{cases}.$$

We assume that $k_0 < 1$, i.e. the payoff from rolling over is lower than the payoff from running if the bank fails. Conversely, we assume that $B > 1$, i.e. the payoff from rolling over is higher if the bank survives. Therefore, lenders' actions are strategic complements. We denote the net payoff from running versus rolling over by

$$\pi = \pi_1 - \pi_0,$$

so

$$\pi(A, g) = \begin{cases} 1 - B < 0 & , g \geq A \\ 1 - k_0 > 0 & , g < A \end{cases}.$$

With perfect information about the bank's type and the actions of the other agents, a lender runs iff $\pi \geq 0$.⁹

We assume that the lenders have some prior beliefs over θ distributed according to a distribution function F_p , which admits a density.¹⁰ Lenders observe additional information, which we detail in Section 3 below. Since the information will be heterogeneous across agents,

⁸Clearly, the conditions given in the two examples are only sufficient for obtaining a residual asset function with the desired properties, and one can obtain such an asset function in more general environments.

⁹We assume that a lender that is indifferent between running and rolling-over ends up running.

¹⁰In Section 3 we will need to impose some conditions on the prior, since with two dividend levels, the priors remain important for the lenders' inference. In Section 4, we will assume that lenders have a uniform prior about θ over $[-K, K]$ and take K to be large.

we will denote the expectation (resp. probability) with respect to lender i 's information set by E_i (resp. $\Pr_i \{.\}$). Therefore, the expected net payoff from running versus rolling over is

$$E_i[\pi(A, g)] = (1 - B) \Pr_i \{g \geq A\} + (1 - k_0) \left(1 - \Pr_i \{g \geq A\}\right). \quad (6)$$

Dividing by $(B - 1) + (1 - k_0)$ and defining

$$p \equiv \frac{1 - k_0}{(1 - k_0) + (B - 1)} \in (0, 1),$$

a lender i runs iff

$$\Pr_i \{g \geq A\} \leq p. \quad (7)$$

Thus, as is standard in regime-change games, p parametrizes how aggressive lenders are when taking their actions. As p increases, the incentives of the lenders to run are strengthened. Finally, we define a normalized expected net payoff from running by

$$\hat{\pi}_i = p - \Pr_i \{g \geq A\}, \quad (8)$$

so that a lender i runs iff $\hat{\pi}_i \geq 0$. In our analysis below, we will work with this object when characterizing the lenders' actions.

2.3 Dominance regions

We assume that there exist lower and upper dominance regions.

- *Lower dominance region:* There exists a $\underline{\theta}$, such that for $\theta < \underline{\theta}$, $\bar{\ell}(\theta) < 0$.
- *Upper dominance region:* There exists a $\bar{\theta}$, such that for $\theta > \bar{\theta}$, $\bar{\ell}(\theta) > 1$, and $\lambda \bar{\ell}(\theta) < v(\theta, 1)$.
- *Multiplicity region:* For $\theta \in (\underline{\theta}, \bar{\theta})$, $\bar{\ell}(\theta) \in (0, 1)$.

Therefore, banks with very weak fundamentals are insolvent and fail with probability one for any $A \in [0, 1]$. Conversely, banks with very strong fundamentals can meet all demands for withdrawals. Furthermore, it is never optimal for such banks to liquidate all their assets at $t = 0$.¹¹ In between, whether a bank can survive or not, depends on whether lenders coordinate on running or rolling over. If all lenders run, then the bank cannot survive, and if

¹¹To see this, observe that $\bar{\ell}(\theta) > 1$ and the properties of v (i.e. $v_l \leq 0$), imply that $\lambda + v_l(\theta, \bar{\ell}(\theta)) \leq \lambda + v_l(\theta, 1)$. Furthermore, $\lambda \bar{\ell}(\theta) < v(\theta, 1) < \lambda + v(\theta, 1)$, and since $W_d = \lambda + v_l(\theta, d)$ is monotone decreasing in d (since $v_l \leq 0$), it follows that $\lambda + v_l(\theta, \bar{\ell}(\theta)) < 0$.

all lenders roll over, the bank can survive. The equilibrium concepts that we work with are standard and are included in the Appendix.

2.4 Dividend policy without rollover

To highlight the interaction between dividends and rollover risk, it is useful to start by characterizing the dividend payout of a bank that does not face a run. To this end we make the following assumption about v ,

Assumption B1. $\frac{v_\theta(\theta, l)}{v_l(\theta, l)}$ is strictly increasing in l .

The assumption ensures uniqueness of the bank failure threshold – the cutoff on the bank fundamentals below which banks pay out all available liquid assets as dividends (and choose not to survive a run if there is one), and above which banks pay out only part of their liquid assets as dividends (and choose to survive a run if there is one). We maintain this assumption throughout the paper. The failure threshold and bank dividend policy in the absence of runs are described in the following Proposition.

Proposition 1. *Consider a bank that does not face a run, i.e. $A = 0$. If $\lambda < -v_l(\underline{\theta}, 0)$, then banks with $\theta > \underline{\theta}$ choose $d_{nr}(\theta) = d^*$, where d^* solves the first-order condition*

$$(\lambda + v_l(\theta, d^*)) d^* = 0. \quad (9)$$

If $\lambda \geq -v_l(\underline{\theta}, 0)$, then there is a unique cutoff $\theta^ \in [\underline{\theta}, \bar{\theta})$, that solves*

$$\lambda = -v_l(\theta^*, \bar{\ell}(\theta^*)), \quad (10)$$

such that banks with $\theta \leq \theta^$ choose $d_{nr}(\theta) = \bar{\ell}(\theta)$, and banks with $\theta > \theta^*$ choose $d_{nr}(\theta) = d^*$, where d^* solves the first-order condition*

$$\lambda = -v_l(\theta, d^*). \quad (11)$$

In both cases, $d_{nr}(\theta)$ is increasing in θ .

Proof. See Appendix. □

Proposition 1 shows that, in the absence of runs, banks with higher portfolio quality pay higher dividends. This outcome is a direct implication of the single crossing condition,

$v_{l\theta} > 0$. Put differently, absent runs, higher dividends constitute good news about a bank’s type. This monotonicity in dividend payouts need not be preserved in the presence of a rollover episode, as we show below.

When the private benefits to the bank owner from paying dividends are sufficiently high ($\lambda \geq -v_l(\underline{\theta}, 0)$), then even banks with low values of θ find it optimal to pay dividends. Since those banks have limited liquid assets, they prefer using all available liquidity to pay out dividends. The “failure” threshold in that case – the cutoff at which a bank switches from a corner to an interior solution is given by θ^* . We view this particular case as the empirically-relevant one in light of our discussion of the risk-shifting incentives by banks in the Introduction and our interest in incorporating both risk-shifting and signaling incentives into a common framework. Therefore, for the rest of the paper we will assume that the second case in Proposition 1 applies.

Assumption B2. $\lambda \geq -v_l(\underline{\theta}, 0)$.

3 Equilibrium with two dividend levels

We start by restricting the set of dividends that a bank can choose to two levels, denoted by $d^0 = 0$ and $d^1 = m > 0$. Therefore, the bank faces a binary dividend choice – either pay a fixed positive dividend or do not pay any dividend. Later on (Section 4), we relax this assumption and analyze the equilibrium outcome when the bank can choose any non-negative dividend that is feasible given the bank’s type.

In the two-dividend setting, we compare equilibrium outcomes under two information structures. First, we consider only the resilience effect of dividends by assuming that lenders do not observe the dividend choice of the bank and instead observe an exogenous private signal about the bank’s type. We call this case the exogenous information case. Formally, lenders observe a private signal about θ , $\theta_i = \theta + \eta_i^\theta$, with $\eta_i^\theta \sim_{i.i.d.} N(0, \alpha_\theta^{-1})$, where α_θ denotes the signal precision. We analyze this environment in Section 3.1.

Next, we introduce the signaling effect by assuming that lenders observe the bank’s dividend and make inferences about the bank’s type based on that information and their prior beliefs. Hence, the dividend choice of the bank acts as an endogenous signal about the bank’s type. We additionally assume that dividends are observed by lenders with small idiosyncratic noise to abstract away from the possibility of multiple equilibria due to common certainty resulting from the observation of a public signal (as in Woodford (2003), Myatt and Wallace (2014), Kolbin (2015), Angeletos and Lian (2016a), or Gaballo (2016)).¹² Formally, lenders

¹²See Angeletos and Lian (2016b) for a survey of the implications of relaxing the assumption of common

observe a private signal about dividends, $d_i = d(\theta) + \eta_i^d$, with $\eta_i^d \sim_{i.i.d.} N(0, \alpha_d^{-1})$, where $d(\theta)$ is the dividend choice of a bank with type θ , and α_d denotes the signal precision. We analyze this environment in Section 3.2.

3.1 Exogenous information and the resilience effect

First, we consider the exogenous information case where dividends are not observed by lenders and serve no signaling role. In this case a dividend payout has a direct negative effect on bank survival. We consider equilibria in monotone strategies by lenders, i.e. lenders attack iff their signal $\theta_i \leq \hat{\theta}$. In addition, the bank's problem is characterized by a failure cutoff θ_f , such that a bank with quality $\theta < \theta_f$ fails the rollover episode and a banks with quality $\theta > \theta_f$ survive.

Given the distribution of signals and a monotone strategy summarized by the (strategic) cutoff $\hat{\theta}$, the fraction of lenders running is given by

$$A(\theta, \hat{\theta}) = \Phi\left(\sqrt{\alpha_\theta}(\hat{\theta} - \theta)\right). \quad (12)$$

Clearly, $A_\theta < 0$, so bank with higher fundamentals face a run of a smaller size. We make the following additional technical assumption.

Assumption A1: $m < p$.

Assumption A1 ensures that the failure cutoff θ_f is unique given a binary dividend choice for the bank. We show below that a bank at the failure cutoff will face a run of size p . Assumption A1, thus, restricts the dividend payout that a bank can make to be smaller than the smallest liquidity outflow that a failing bank suffers.¹³

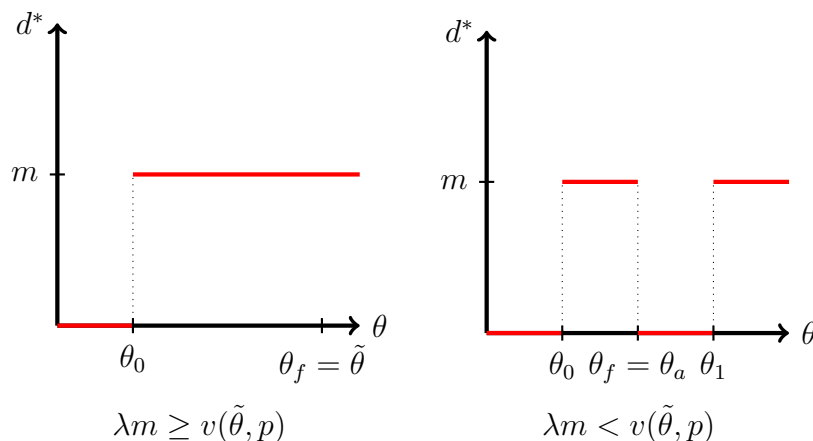
Proposition 8 in the Appendix shows that there is a unique equilibrium in monotone strategies, and that this is the unique equilibrium of the rollover game. Given the discrete choice set for dividends, there are two cases to consider when characterizing the equilibrium. Importantly, the structure of the equilibrium depends on how high the utility of paying dividends is. Formally, let $\tilde{\theta} \equiv \bar{\ell}^{-1}(m + p)$ denote the bank type which can just afford to pay the dividend and sustain a run of size p . The structure of the equilibrium then depends on whether $\lambda m \geq v(\tilde{\theta}, p)$ or $\lambda m < v(\tilde{\theta}, p)$. Figure 1 shows the equilibrium dividend payout

knowledge of agents' information sets in environments with coordination. In addition to its technical role, we can interpret the private noise in dividend observations as a reduced-form for limited attention by some lenders (Sims (2003), Myatt and Wallace (2012)).

¹³Without A1, banks with θ for which it is not feasible to pay the dividend ($\bar{\ell}(\theta) < m$) survive exogenously, as long as $A(\theta^f, \hat{\theta}) \leq \bar{\ell}(\theta)$ and, so, there may be two disjoint failure regions.

and failure cutoff for the two cases.

Figure 1: Equilibrium dividend policy and failure threshold.



If $\lambda m \geq v(\tilde{\theta}, p)$, then given a run of size p , the $t = 0$ payoff from paying a dividend for a type $\tilde{\theta}$ bank is higher than the continuation value of the bank conditional on not paying a dividend and surviving. This is also true for banks with $\theta < \tilde{\theta}$, since these banks have a strictly lower continuation value.¹⁴ Banks with $\theta < \tilde{\theta}$ would then pay dividends, knowing that they will fail the rollover episode and, so, $\theta_f \geq \tilde{\theta}$.

However, as in other global games models (Morris and Shin, 2003), the strategic uncertainty resulting from dispersed private information determines a run size of $A(\theta_f, \hat{\theta}) = p$ for a bank at the equilibrium failure threshold, θ_f . Therefore, it cannot be the case that $\theta_f > \tilde{\theta}$, since given the definition of $\tilde{\theta}$, there are bank types $\theta \in (\tilde{\theta}, \theta_f)$ that can both survive the run and pay out the dividend m , which contradicts the definition of θ_f . Consequently, $\theta_f = \tilde{\theta}$.

If $\lambda m < v(\tilde{\theta}, p)$, then $\theta_f < \tilde{\theta}$, since in that case a $\tilde{\theta}$ bank is better off not paying a dividend and surviving a run of size p . Furthermore, $\theta_f \leq \theta_a$, where $\theta_a \leq \tilde{\theta}$ is implicitly defined by $\lambda m = v(\theta_a, p)$.¹⁵ Hence, θ_a denotes the type of bank that is indifferent between paying a dividend m and failing or not paying a dividend and surviving when the run is of size p . On the other hand, banks with $\theta < \theta_a$ always fail either because they prefer to pay a dividend over surviving or because they do not have sufficient resources to survive (even if they do not pay a dividend). Thus, $\theta_f = \theta_a$.

Equilibrium dividend payouts also differ across the two cases as shown in Figure 1. In the Figure, $\theta_0 \equiv \bar{\ell}^{-1}(m)$ denotes a bank that is just able to pay a dividend m , so paying a dividend is not feasible for banks with $\theta < \theta_0$. On the other hand, for banks with $\theta \in [\theta_0, \theta_f]$,

¹⁴Recall that $v_\theta > 0$.

¹⁵To see this, note that if $\theta_f > \theta_a$, then by the properties of v , $\lambda m < v(\theta_f, p)$, which contradicts the definition of θ_f .

paying a dividend is feasible but by doing so, these bank types lower the liquidity available at $t = 1$ to survive the rollover episode. Some of these bank types cannot survive even if they do not pay a dividend (e.g. bank types sufficiently close to θ_0). However, for bank types sufficiently close to θ_f not paying a dividend ensures survival of the rollover episode. Nevertheless, these banks *choose* to fail by paying a dividend.¹⁶

Therefore, in the case where lenders observe exogenous signals about θ , a bank that pays out a dividend ends up unambiguously increasing the total liquidity outflow it will experience. Since a higher liquidity outflow can be met only by a bank with higher θ , having the option to pay a dividend increases the bank failure threshold in equilibrium. This is the resilience effect associated with dividend payments.

3.1.1 Negative dividend externality and amplification

The resilience effect implies that by paying a dividend a bank weakens its ability to survive. Therefore stronger incentives to pay a dividend, given, for example, by a higher value of λ , increase the bank failure threshold keeping the behavior of lenders unchanged. However, by paying a dividend and choosing to fail, a bank exerts a negative externality on other banks through the equilibrium response of lenders. To illustrate this, suppose that $\lambda m < v(\tilde{\theta}, p)$, so that the failure and strategic cutoffs are jointly determined by

$$\lambda m = v\left(\theta_f, A\left(\theta_f, \hat{\theta}\right)\right). \quad (13)$$

and

$$\Pr\left\{\theta > \theta_f | \hat{\theta}\right\} = A\left(\theta_f, \hat{\theta}\right) = p. \quad (14)$$

By equation (14), $\frac{\partial \hat{\theta}}{\partial \theta_f} > 0$, so that if lenders anticipate that more banks (choose to) fail, they become more aggressive. Intuitively, since lenders have imperfect and dispersed information about the type of bank they are facing, a larger set of failing banks implies that the lender that is indifferent between running and rolling over must be more optimistic and thus observe a higher signal $\hat{\theta}$.

However, $A_{\hat{\theta}} > 0$, and more aggressive lenders lead to larger runs for all banks. Formally,

¹⁶Whenever $\lambda m < v(\tilde{\theta}, p)$, some surviving banks choose not to pay dividends in order to ensure their survival. Only when the utility of paying dividends is sufficiently high relative to the continuation value of the bank without dividend payments will they start paying dividends. This happens at θ_1 , implicitly defined by $\lambda m + v\left(\theta_1, A\left(\theta_1, \hat{\theta}\right) + m\right) = v\left(\theta_1, A\left(\theta_1, \hat{\theta}\right)\right)$.

from equation (13), $\frac{\partial \theta_f}{\partial \hat{\theta}} > 0$. Combining these two effects, we have that,

$$\begin{aligned} \frac{d\theta_f}{d\lambda} &= \frac{\partial \theta_f}{\partial \lambda} + \frac{\partial \hat{\theta}}{\partial \lambda} \frac{\partial \theta_f}{\partial \hat{\theta}} = \frac{\partial \theta_f}{\partial \lambda} + \left(\frac{\partial \hat{\theta}}{\partial \theta_f} \Big|_{A(\theta_f, \hat{\theta})=p} \frac{d\theta_f}{d\lambda} \right) \frac{\partial \theta_f}{\partial \hat{\theta}} \\ &= \frac{\frac{\partial \theta_f}{\partial \lambda}}{1 - \frac{\partial \hat{\theta}}{\partial \theta_f} \Big|_{A(\theta_f, \hat{\theta})=p} \frac{\partial \theta_f}{\partial \hat{\theta}}} > \frac{\partial \theta_f}{\partial \lambda}, \end{aligned}$$

since $\frac{\partial \hat{\theta}}{\partial \theta_f} \Big|_{A(\theta_f, \hat{\theta})=p} = 1$, and $\frac{\partial \theta_f}{\partial \hat{\theta}} < 1$. Thus, a strengthening of the incentives of banks to pay dividends results in an amplified response to the bank failure threshold via the equilibrium feedback between the failure cutoff and strategic cutoff.¹⁷

Therefore, the resilience effect induces a negative dividend externality between banks which ends up amplifying financial instability. This negative dividend externality is conceptually different from the dividend externality in Acharya, Le, and Shin (2013), which operates through the network of inter-bank claims.

3.2 Endogenous information and the signaling effect

Next, we consider the case where lenders observe the dividend action of the bank and update their beliefs about the bank's type based on (a noisy signal of) that action. In this case, when lender signals are sufficiently precise, paying a dividend may actually *decrease* the total liquidity outflow that the bank experiences because of its indirect effect on the run size.

As before, we consider equilibria in monotone strategies by lenders, i.e. lenders attack iff their signal $d_i \leq \hat{d}$.¹⁸ The bank's problem is still characterized by a failure cutoff θ_f . We characterize equilibria under several assumptions.

Assumption A1': $m < \frac{1}{2}$.

Assumption A2. $\Pr \{ \theta \in (\underline{\theta}, \bar{\theta}) \} < \Pr \{ \theta < \underline{\theta} \} \Pr \{ \theta > \bar{\theta} \}$.

Assumption A3. $p < \frac{\Pr \{ \theta > \bar{\theta} \}}{\Pr \{ \theta > \theta_0 \}}$.

¹⁷This amplification effect is also the reason for the two cases that one has to consider when characterizing the equilibrium. Whenever the change in λ is sufficiently large, the economy may move from the $\lambda m < v(\tilde{\theta}, p)$ to the $\lambda m \geq v(\tilde{\theta}, p)$ case, with the failure threshold changing discretely from θ_a to $\tilde{\theta}$.

¹⁸Proposition 2 below shows that under some conditions, this restriction is without loss of generality.

Assumption A1' is a technical condition similar to Assumption A1 above, which ensures that the bank failure threshold is unique.¹⁹ Similarly, Assumption A3 is a technical condition which ensures that \hat{d} always exists. It is a sufficient condition ensuring the existence of a marginal lender, i.e. a lender which has received a signal that makes her indifferent between running and rolling over.²⁰ Intuitively, the assumption puts an upper bound on how aggressive lenders are when deciding whether to roll over or run. Finally, Assumption A2 is a condition on the size of the dominance regions relative to the multiplicity region. This assumption guarantees that a higher dividend signal d_i is always good news about bank survival. The assumption implies that the dominance regions are sufficiently large relative to the multiplicity region, so that the location of the failure threshold does not change the lenders' inference.²¹

Given the distribution of signals and the cutoff strategies of lenders, the fraction of lenders running is given by

$$A(d, \hat{d}) = \Phi\left(\sqrt{\alpha_d}(\hat{d} - d)\right) \quad (15)$$

Therefore, $A_d < 0$, so a higher dividend is associated with a lower run size. This dependence of the run size on the dividend is what we call the signaling effect.

To characterize equilibria, note, as in the previous section, that the binary dividend choice implies that the structure of the equilibrium may differ depending on parameter values. Two Lemmas given in the Appendix (Lemmas 2 and 3) characterize the bank's and lenders' problems. Here we briefly summarize the main insights.

First, suppose that $m + A(m, \hat{d}) > A(0, \hat{d})$, so the liquidity outflow given a dividend payment is higher than the outflow given no dividend payment. In this case the signaling effect is weak and the resilience effect dominates. The liquidity outflow of paying dividends relative to not paying dividends is as in the exogenous information case – higher dividends unambiguously worsen the bank's liquidity position. Thus, as in the exogenous information case, the bank faces a trade-off between paying a dividend and surviving the rollover episode.

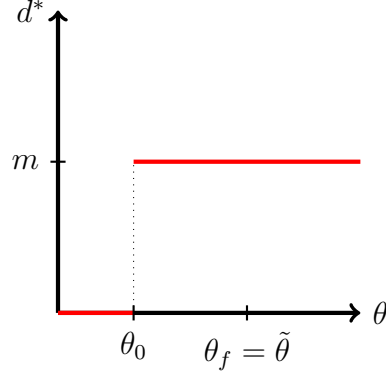
Similar to the exogenous information case, let $\tilde{\theta} \equiv \bar{\ell}^{-1}\left(m + A(m, \hat{d})\right)$. Suppose that

¹⁹Specifically, it ensures that $m \leq \bar{\ell}(\theta)$ whenever $A(0, \hat{d}) \leq \bar{\ell}(\theta)$. If $A(0, \hat{d}) < \bar{\ell}(\theta)$, while $m > \bar{\ell}(\theta)$, some banks for which it is not feasible to pay the dividend survive exogenously.

²⁰In the limit, as $\alpha_d \rightarrow \infty$, one needs a weaker condition to ensure this. In particular, having $p < 1$ is sufficient in that case.

²¹If A2 does not hold, it may be the case that how a higher dividend signal is interpreted by lenders (whether as good or bad news about bank survival) may depend on the position of the bank failure threshold in the multiplicity region. As a result there may be multiple equilibria similar to Angeletos, Hellwig, and Pavan (2006). In that case, the results of this section will apply to the equilibrium in which a higher dividend signal is interpreted by lenders as good news for bank survival. A less restrictive and more easily satisfied condition for monotone inference of lenders is needed in the case where dividends are not restricted to one of two levels (see Section 4).

Figure 2: Dividend policy and failure threshold with strong signaling effect.



$\lambda m \geq v(\tilde{\theta}, A(0, \hat{d}))$, so that a bank of type $\tilde{\theta}$ which can just afford to both pay a dividend and survive a run of size $A(m, \hat{d})$ is better off paying the dividend. In that case, as in the exogenous information case, $\theta_f = \tilde{\theta}$. Furthermore, banks with $\theta \in [\theta_0, \tilde{\theta}]$ pay the dividend and fail. Conversely, if $\lambda m < v(\tilde{\theta}, A(0, \hat{d}))$, then $\theta_f = \theta_a$, which solves $\lambda m = v(\theta_a, A(0, \hat{d}))$, and some banks with $\theta > \theta_f$ choose to not pay a dividend and survive. Therefore, the dividend payout and failure threshold are similar to those in Figure 1.

Next, suppose that $A(m, \hat{d}) + m \leq A(0, \hat{d})$. Then the liquidity outflow from paying a dividend is *lower* than the liquidity outflow from not paying a dividend. In this case the signaling effect is strong and dominates over the resilience effect, so that there is no longer a trade-off for the bank between paying a dividend and surviving the run. Hence, if a bank can pay the dividend it would always do so, and the only failing banks in this case are either banks for which paying the dividend is not feasible (i.e. banks with $\theta < \theta_0 \equiv \bar{\ell}^{-1}(m)$) or banks which can pay the dividend but do not have enough liquidity to survive. Therefore, $\theta_f = \tilde{\theta} = \bar{\ell}^{-1}(m + A(m, \hat{d}))$. The dividend payout and failure threshold in that case are as shown in Figure 2.

Whether the signaling effect is weak or strong depends on how coordinated lenders are, which depends on the idiosyncratic signal precision α_d . When α_d is sufficiently high, the signaling effect dominates the resilience effect and paying a dividend ends up lowering the liquidity outflow from the bank, i.e. $A(m, \hat{d}) + m < A(0, \hat{d})$. Also, if the signaling effect is sufficiently strong for sufficiently large values of α_d , the equilibrium in monotone strategies is the unique equilibrium of this economy, as we show in the next proposition.

Proposition 2. (*Dividend signaling equilibrium*) Consider the endogenous information model, in which lenders follow a monotone strategy with cutoff \hat{d} , and banks fail according to a cutoff θ_f . Suppose that assumptions A1', A2, and A3 hold. Then, there is an $\bar{\alpha} > 0$, such that for

$\alpha_d > \bar{\alpha}$, \hat{d} , and θ_f are uniquely determined by

$$\hat{d} = \frac{m}{2} + \frac{1}{\alpha_d m} \log \left(\frac{p \Pr \{\theta < \theta_0\}}{\Pr \{\theta > \theta_f\} - p \Pr \{\theta > \theta_0\}} \right), \quad (16)$$

and

$$\theta_f = \bar{\ell}^{-1} \left(m + A \left(m, \hat{d} \right) \right), \quad (17)$$

and a bank pays a dividend iff $\theta \geq \theta_0 = \bar{\ell}^{-1}(m)$. Furthermore, the unique monotone strategy equilibrium is also the unique equilibrium of this economy.

Proof. See Appendix. □

To further clarify that a higher α_d strengthens the signaling effect, consider the following comparative static result.

Proposition 3. (*Stronger signaling effect*). For sufficiently high α_d , θ_f is decreasing in α_d .

Proof. See Appendix. □

A higher value of α_d ends up decreasing the liquidity outflow $m + A \left(m, \hat{d} \right)$ associated with paying a dividend. Intuitively, when α_d is larger, lenders are more coordinated and the dividend choice influences the actions of a larger group of lenders. Thus, a bank at the failure threshold that has just enough liquidity to meet the liquidity outflow associated with paying a dividend and survive a run by $A \left(m, \hat{d} \right)$ lenders has strictly more liquidity when α_d is increased, so that even weaker banks can now survive. As $\alpha_d \rightarrow \infty$, lenders become almost perfectly coordinated and the signaling effect becomes extremely powerful as we show next.

Proposition 4. Suppose that assumptions A1' and A2 hold and that $p < 1$. In the limit, as $\alpha_d \rightarrow \infty$, $\theta_f \rightarrow \theta_0 = \bar{\ell}^{-1}(m)$ and $\hat{d} \rightarrow \frac{m}{2}$.

Proof. See Appendix. □

In the limit, when lenders receive arbitrarily precise signals and, so, are almost perfectly coordinated, paying a dividend leads to a liquidity outflow equal only to that dividend payment. Consequently, all banks for which it is feasible to pay the dividend do so and survive. Figure 3 illustrates the equilibrium dividend policy and failure threshold in this limiting case.

4 Unrestricted dividend choice

In this section we relax the assumption of binary dividend choice and instead assume that a type θ bank can choose any dividend $d \in [0, \bar{l}(\theta)]$. We assume that lenders have a uniform

Figure 3: θ_f and dividend policies as $\alpha_d \rightarrow \infty$

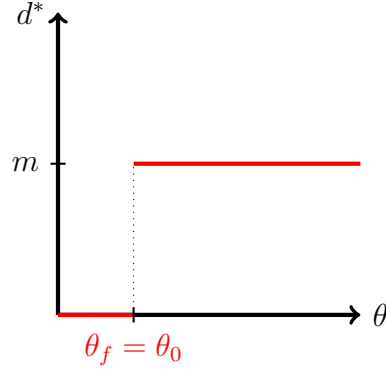
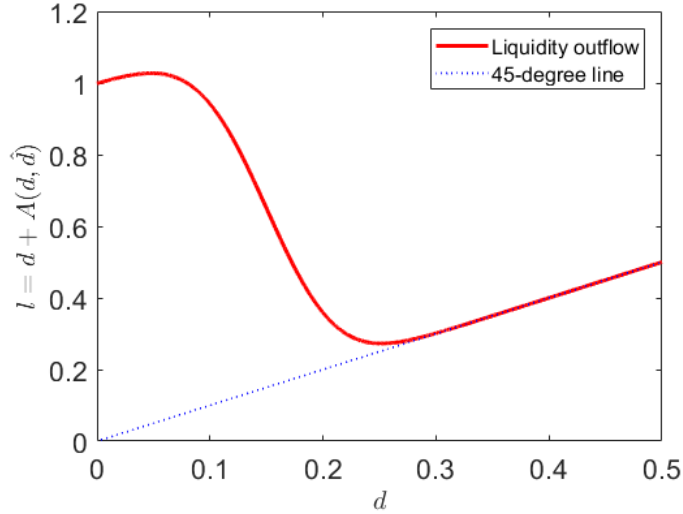


Figure 4: Liquidity outflow $(d + A(d, \hat{d}))$ as a function of dividends paid.



prior about θ over $[-K, K]$ for $K > 0$.²² As in Section 3.2, lenders observe the bank's dividend d with normally distributed idiosyncratic noise with precision α_d . Below we present only the main insights from relaxing the two-dividend assumption. The details of the analysis are contained in the appendix (Appendix C).

We again restrict attention to equilibria in monotone strategies, in which a lender refuses to roll over if $d_i < \hat{d}$, for some $\hat{d} \in \mathbb{R}$.²³ With normally distributed dividend signals and monotone strategies, the fraction of agents attacking given \hat{d} , is

$$A(d(\theta), \hat{d}) = \Phi(\sqrt{\alpha_d}(\hat{d} - d(\theta))). \quad (18)$$

²²We choose K to be sufficiently large, so that the lenders' inference is monotone in their signals, i.e. lenders will essentially have an improper prior over the real line. See Lemma 8 in the appendix for details.

²³Proposition 9 in Appendix C shows conditions under which that restriction is without loss of generality.

It is useful to define d_{\min} as the solutions to

$$1 + A_d(d, \hat{d}) = 0, \quad (19)$$

such that $\hat{d} < d_{\min}$. In addition, we consider economies with $\hat{d} \in (0, 1)$ and α_d sufficiently large, so that $d_{\min} > 0$ is the minimizer of $d + A(d, \hat{d})$.²⁴ In that case, as Figure 4 shows, the liquidity outflow that a bank experiences is decreasing for some values of d , so that the signaling effect dominates the resilience effect. A decreasing liquidity outflow plays an important role in the optimal dividend payout decision of banks as we show next.

Lemma 1. *Suppose that lenders follow a monotone strategy with cutoff at \hat{d} . Then there is a unique critical threshold over bank fundamentals given by θ_f , such that banks with $\theta < \theta_f$ fail and banks with $\theta > \theta_f$ survive. Furthermore, θ_f satisfies*

$$\lambda \bar{\ell}(\theta_f) = \lambda d^*(\theta_f) + v\left(\theta_f, d^*(\theta_f) + A(d^*(\theta_f), \hat{d})\right), \quad (20)$$

where $d^*(\theta) > d_{\min}$ satisfies the condition

$$\frac{\lambda}{1 + A_d(d^*(\theta), \hat{d})} = -v_l\left(\theta, d^*(\theta) + A(d^*(\theta), \hat{d})\right). \quad (21)$$

The bank's optimal dividend policy is given by

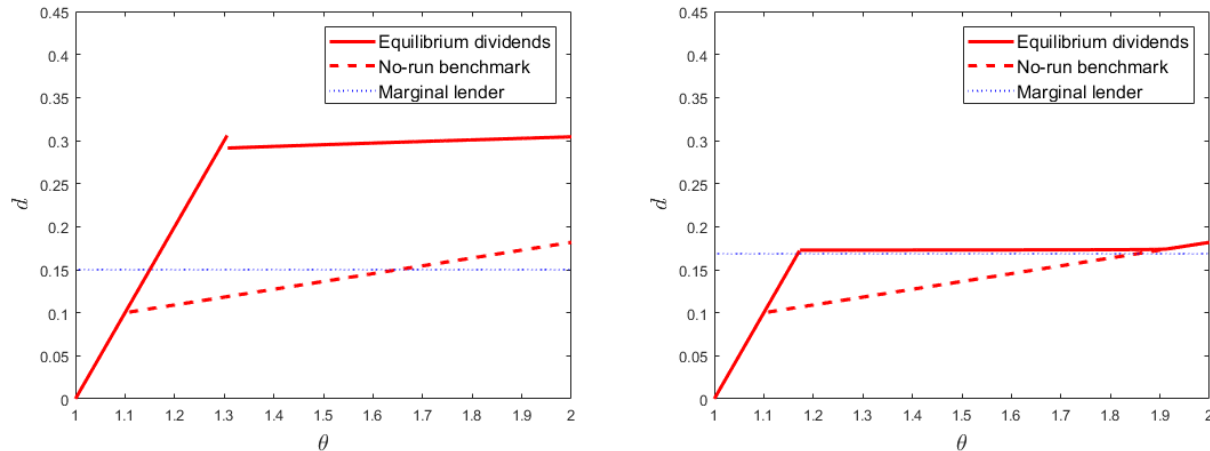
$$d(\theta) = \begin{cases} \bar{\ell}(\theta) & , \theta < \theta_f \\ \{\bar{\ell}(\theta), d^*(\theta)\} & , \theta = \theta_f \\ d^*(\theta) & , \theta > \theta_f \end{cases} \quad (22)$$

Proof. See Appendix. □

It is instructive to compare the optimal dividend choice of the bank in this case to the case when there is no run ($d_{nr}(\theta)$ given by equation (11)). First, if the bank chooses to survive, Lemma 1 implies that it might have to distort its dividend payment above the no-run dividend level, $d_{nr}(\theta)$. This is specifically the case for banks with θ for which $d_{nr}(\theta) \leq d_{\min}$. These banks do not want to be on the downward sloping segment of the liquidity outflow schedule, $l(d) = d + A(d, \hat{d})$. Since a higher dividend is associated with a lower liquidity

²⁴Conditions B3 and B4 in Appendix C ensure that in any monotone strategy equilibrium of this economy, $\hat{d} \in (0, 1)$, while the observation that $d_{\min} \rightarrow \hat{d}$ and $A(0, \hat{d}) \rightarrow 1$ as $\alpha_d \rightarrow \infty$ ensure that $d_{\min} + A(d_{\min}, \hat{d}) \leq A(0, \hat{d})$ for sufficiently large α_d . Focusing on economies in which $\hat{d} \in (0, 1)$ is the most interesting given the assumption on feasible dividend payouts for banks at the lower and upper dominance thresholds.

Figure 5: Equilibrium dividend policies for low (left panel) and high (right panel) dividend signal precisions.



Notes: Parametric assumptions: $v(\theta, l) = \theta \ln(\theta - l)$, for $1 \leq \theta < 2$ (multiplicity region); $p = 0.5$, $\lambda = 1.1$. $\sigma = 0.05$ for the left panel and $\sigma = 0.001$ for the right panel.

outflow in that segment, they optimally choose a dividend of at least d_{\min} . Intuitively, such banks have strong incentives to distort their dividends upward and pay a dividend similar to higher-quality banks to help more lenders coordinate on rolling over.

In addition, the marginal impact on the size of the run given by $A_d(d, \hat{d})$, decreases strongly in d around d_{\min} . Intuitively, since the lenders care about whether the bank fails or survives the rollover episode (rather than the specific bank type), the dividend payouts of banks with higher fundamentals are already interpreted by most lenders as strong evidence that the bank will survive the rollover episode, so any upward distortion in dividends has only a small effect on lenders' inference.

By equation (21), the combination of these two effects implies that banks which distort their dividends relative to the no-run level end up choosing payouts around d_{\min} . Hence, as in the two-dividend case, the strong signaling effect lowers the sensitivity of dividends to the bank fundamentals relative to their dividend payout in the absence of rollover and, so, banks with different fundamentals choose similar dividends.

Figure 5 illustrates this feature by showing the (equilibrium) dividend policy of banks for one particular example.²⁵ The figure plots the equilibrium dividend policies (solid line), as well as the dividend policies in the no-run case (dashed line) for banks in the multiplicity region. Banks below the failure threshold liquidate all assets and pay them as dividends. Otherwise, banks pay a dividend that is higher than their no-run dividend. Furthermore, the dividends of surviving banks are more compressed and vary less with θ relative to the no-run

²⁵See the Appendix for details on the example.

case. As the signal precision is increased and the signaling effect is strengthened (Figure 5(right panel)), the dividend policy becomes even less sensitive to θ for banks close to the failure threshold.

Turning to equilibrium characterization, Proposition 9 in the Appendix characterized equilibria in monotone strategies for this economy. It shows that if the equilibrium in monotone strategies is unique, it is the unique equilibrium of this economy. Similar to the two-dividend case, equilibrium uniqueness in this case results from the combination of sufficiently large dominance regions together with the monotonicity of bank actions when there is no coordination problem among lenders (by the single-crossing assumption $v_{l\theta} > 0$). Specifically, banks with very low values of the fundamental always pay lower dividends compared to banks with very high fundamentals, *irrespective* of the actions of lenders and the inferences they make about θ from the dividend level they observe. Thus, very low (high) private signals are always interpreted as bad (good) news about θ , regardless of the actions of lenders with intermediate signals or the dividend policies of banks with intermediate values of θ . Therefore, despite being endogenous, the information structure of lenders always has this property.

In addition to equilibrium characterization, we can show the following stark result for the limiting case when lender signals become arbitrarily precise.

Proposition 5. *In the limit, as $\alpha_d \rightarrow \infty$, there is a unique equilibrium with $\theta_f \rightarrow \theta^*$, where θ^* solves $\lambda = -v_l(\theta^*, \bar{l}(\theta^*))$, $\hat{d} \rightarrow \bar{l}(\theta^*)$. Furthermore, the bank's dividend policy, $d(\theta) \rightarrow d_{nr}(\theta)$, $\forall \theta$.*

Proof. See Appendix. □

As in Proposition 4, the reason for this stark result is the extreme strengthening of the signaling effect. As $\alpha_d \rightarrow \infty$, lenders become perfectly coordinated and, so, $A(d, \hat{d}) \rightarrow 0$ for $d > \hat{d}$. However, since a bank that chooses to survive always selects a dividend $d^* > d_{\min} > \hat{d}$, it follows that $A \rightarrow 0$ for any surviving bank, including for a bank with type θ_f . This, however, can only be consistent with indifference between survival and failure if $\theta_f = \theta^*$.

Figure 5 provides an illustration of how a strengthening of the signaling effect ends up influencing the equilibrium away from this limit. More precise dividend signals reduce the failure threshold and bring it closer to the no-run failure threshold. Intuitively, more precise dividend signals imply that lenders are more coordinated. As a result, the marginal bank that is indifferent between failing and surviving for a lower signal precision now faces a smaller run in case of survival, making it strictly better off from surviving. Additionally, d_{\min} decreases, which further reduces the cost associated with having to distort dividends away from the no-run level, in order to survive the run.²⁶

²⁶These direct effects ends up dominating any indirect effects arising from changes in the marginal lender.

One interesting implication of the limiting case, which is suggested by the behavior of the equilibrium dividend schedules in Figure 5, is that in the limiting case of arbitrarily precise lender signals, $d^{*'}(\theta_f) \rightarrow 0$. Therefore, around the failure threshold, surviving banks pool on dividend issuance.²⁷ Hence, while in the two-dividend case in Section 3, surviving banks are forced to pool on a positive dividend payout, when dividend payouts are unrestricted, banks close to the failure threshold endogenously choose to (approximately) pool in equilibrium.

5 Policy implications

The possibility that banks can influence the coordination-based run they face during a period of financial market stress via their dividend choices has important implications for dividend regulation aimed at improving financial stability. In particular, suppose that a policy maker cares about minimizing the set of banks experiencing a coordination-based run (i.e. minimizing the failure cutoff θ_f). Consider the economy from Section 4 and suppose that lenders' idiosyncratic signals are arbitrarily precise ($\alpha_d \rightarrow \infty$). We analyze the effects of two dividend regulation policies in this environment – a full restriction on dividend payments and a tax on dividends.

Fully restricting dividend payments has been proposed as a macroprudential tool for stabilizing the financial system (Goodhart, Peiris, Tsomocos, and Vardoulakis, 2010). In our framework, if banks are restricted from paying dividends, lenders have to only rely on their prior beliefs, which leads to multiple equilibria, in which the bank failure threshold can lie anywhere in the multiplicity region $(\underline{\theta}, \bar{\theta})$. Given Proposition 5, this means that the bank failure threshold could be either higher or lower in that case compared to the case without restricting dividends. For a more precise comparison, suppose that restricting bank dividends and the information available in them causes the direct acquisition of private information about the bank's fundamental by market participants (e.g. as in He and Manela (2014), Szkup and Trevino (2015), or Ahnert and Kakhbod (2016)).

Proposition 6. *Consider the case with a continuum of dividend levels and arbitrarily precise dividend signals ($\alpha_d \rightarrow \infty$). Suppose that a policy that restricts dividend payments by banks to $d = 0$, $\forall \theta$, induces private information acquisition about θ by market participants. Then*

In fact, as Proposition 5 suggests, for sufficiently high signal precision, changes in the marginal lender, \hat{d} , should reinforce these effects on θ_f .

²⁷The intuition for this pooling result is the following. When lenders observe very precise signals, to have a marginal lender who observes a signal \hat{d} lower than the dividend payout of all surviving banks be indifferent between running and rolling over, it must be the case that the set of surviving banks issuing a dividend close to \hat{d} is relatively large in equilibrium. Thus, a large set of banks must be paying dividends close to \hat{d} . As signals become arbitrarily precise, having an equilibrium marginal lender with these properties entails that surviving banks close to the failure threshold pool on dividend issuance, which is ensured by $d^{*'}(\theta_f) \rightarrow 0$.

the bank failure threshold is lower with a restriction on dividends than without a restriction, iff $\theta^* > \tilde{\theta}_f$, where θ^* solves $\lambda = -v_l(\theta^*, \bar{\ell}(\theta^*))$ and $\tilde{\theta}_f = \bar{\ell}^{-1}(p)$.

Proof. See Appendix. □

There are two forces that drive a bank's dividend decisions in our model. On the one hand, in the absence of runs, the risk-shifting incentives associated with a high private payoff from dividends (a higher λ) lead to a failure threshold of θ^* . The higher is λ – the marginal value that the bank (owner) derives from paying one more unit of dividends – the higher is θ^* . On the other hand, without the ability to utilize the signaling effect of dividends, a bank is subject to a coordination-based run, which leads to a failure threshold of $\tilde{\theta}_f$. The higher is p – the parameter governing the strength of lenders' incentives to run – the higher is $\tilde{\theta}_f$.

Thus, if corporate governance issues are of less concern for immediate financial stability compared to the financial stress due to a coordination-based rollover crisis (i.e. λ is low, relative to p , so that $\theta^* < \tilde{\theta}_f$), restricting dividends leads to a higher bank failure threshold. On the other hand, if coordination-based runs are not of concern in a crisis episode, and corporate governance issues are particularly salient in that case (i.e. λ is high relative to p , so that $\theta^* > \tilde{\theta}_f$), then restricting dividends leads to a lower bank failure threshold.

Therefore, the effect of dividend restrictions is *ambiguous*. In contrast, a (proportional) tax on dividends *unambiguously* decreases the bank failure threshold, as we show in the next Proposition.

Proposition 7. *Consider the case with a continuum of dividend levels and arbitrarily precise dividend signals (i.e. $\alpha_d \rightarrow \infty$). In that setting, a higher proportional tax on dividend payments, $\tau \in (0, 1)$, decreases the bank failure threshold θ_f .*

Proof. See Appendix. □

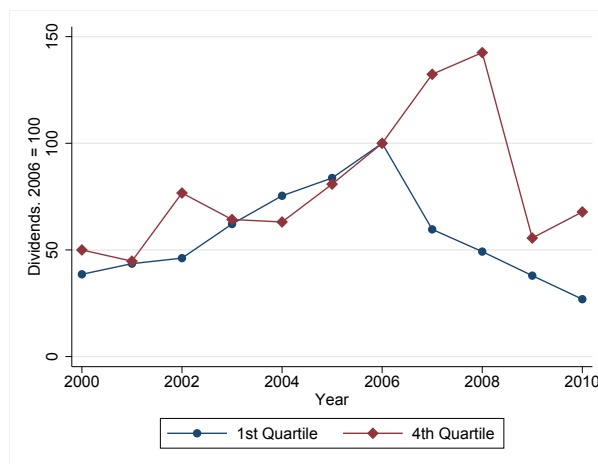
It may be *a priori* unclear to a policy maker whether corporate governance or coordination-based runs are more important in a period of financial stress, so that the sign of comparison between θ^* and $\tilde{\theta}_f$ is not clear. In that case, Proposition 7 suggests that a policy maker can instead use a tax on dividend payments. Such a tax induces banks to adjust their risk-shifting behavior, so that corporate governance concerns are alleviated. In addition, the tax does not distort the signaling effect of dividends, which has a powerful effect on the coordination-based run.²⁸

²⁸Notice that a tax on dividends also works to correct for any negative externalities arising from a bank's dividend payments beyond correcting for spillovers between bank equity and debt holders. For example, suppose that the marginal social benefit from paying a dividend is $\lambda^S < \lambda$, for example, because banks do not internalize the effects of their actions for financial stability through other channels than the ones emphasized in our paper. In that case, setting $\tau = \frac{\lambda}{\lambda^S} - 1$ aligns the bank's incentives with those of the policy maker.

6 Empirical relevance

We now discuss the empirical relevance of our theory. One implication of our model is that the combination of rollover risk and a strong signaling effect may lead to a lower sensitivity of dividends to a change in fundamentals compared to an environment in which rollover risk is absent (Section 4). Second, signaling should be associated with a negative relation between dividend payments and short-term funding outflows. Finally, the impact of a change in dividends on short-term funding outflows increases in the strength of the signaling effect. Below we document three novel empirical facts and discuss related empirical work that is broadly consistent with these predictions.²⁹

Figure 6: Yearly nominal dividend payments for large U.S. banks with different reliance on short-term debt.



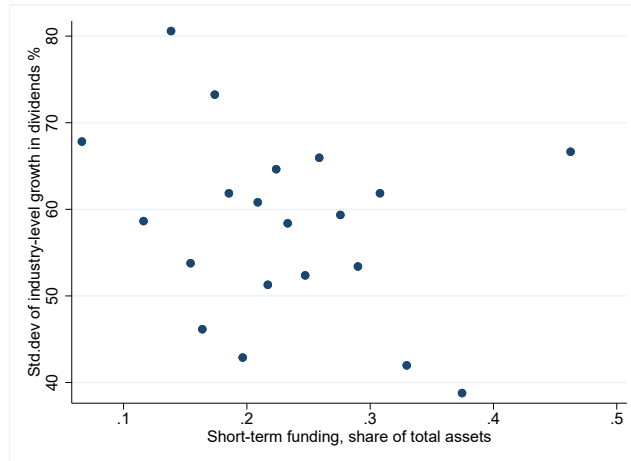
Source: Bank holding companies Y-9C reports. Large bank are defined as bank holding companies with more than \$500 million in assets as of Q1-2006. Banks holding companies are grouped into four quartiles based on their short-term debt relative to total liabilities in 2006. The figure compares the dividend payments of the 1st and the 4th quartile. Short-term debt is defined as the sum of repurchase agreements and federal funds.

Consistent with the first implication, we show that banks that relied more heavily on short-term funding prior to the crisis were more reluctant to cut dividends during the crisis. Specifically, we rank U.S. banks according to the share of liabilities in short-term debt (defined as the sum of federal funds and repurchase agreements) in 2006 and examine the behavior of banks in the first and fourth quartiles. Figure 6 plots the dividend payments of these two groups of banks. While both groups have similar trends in dividend growth before 2007, their dividend payments diverge sharply thereafter. Banks that relied relatively less on short-term debt decreased their dividend payments starting in 2007. In contrast, banks that relied relatively more on short-term debt stayed on their pre-2007 dividend growth trend during

²⁹Additional information on data and measurement are included in Appendix D.

2007 and 2008.³⁰

Figure 7: Industry level standard deviation of dividend growth and average share of short-term debt relative to total assets, 2005-2016 (binned scatter plot).



Source: CRSP and Compustat. Short-term debt defined as current liabilities. Industry correspond to 4-digit SIC code, excluding financials (SIC codes 6000-6999) and utilities (SIC codes 4900-4949).

Second, and again consistent with the first implication, we show that industries in which firms rely relatively more on short-term funding also have more stable dividend payments.³¹ Figure 7 plots the standard deviation of dividend growth against the average share of short-term debt relative to assets for different U.S. industries. As the Figure shows, there is a negative relation between these two quantities.

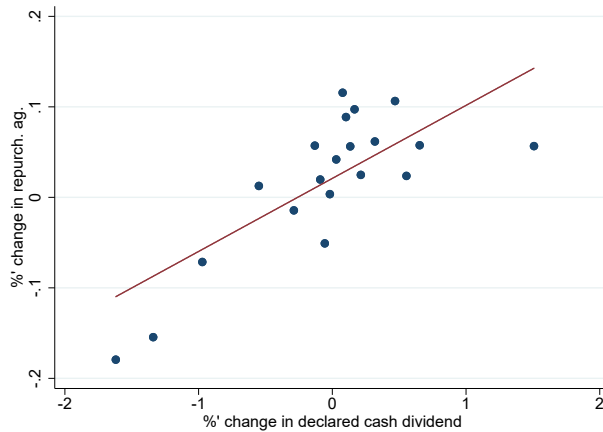
Third, and consistent with the second implication of our theory, we show that banks that cut dividends less during 2007-2009 also faced a lower outflow of short-term funding during the crisis. Figure 8 plots the percentage change in dividend payments for individual banks (relative to the mean change across all banks) against the percentage change of repurchase agreement liabilities during that period. There is a strong positive relationship between dividend changes and the change in repurchase agreements during that period.

Finally, the strength of the signaling effect in our model depends on the precision of lender signals. The higher this precision, the more coordinated are lenders and the larger their aggregate response to changes in dividends. One interpretation of the noise in lender signals is that it is a reduced-form representation for limited attention by some lenders (Sims (2003), Myatt and Wallace (2012)). According to this interpretation, more precise signals reflect lenders who are more attentive to changes in the bank’s dividends. Given this interpretation, the signaling effect should be stronger for banks that face more attentive lenders.

³⁰Including the 2nd and the 3rd quartile would show a similar pattern, with quartile three being slower to cut dividend payments relative to quartiles one and two.

³¹Our theoretical analysis naturally carries over to industries beyond the banking sector where firms also face coordination based runs from short-term lenders.

Figure 8: Symmetric changes in dividends and changes in repurchase agreement liabilities. 2007-2009. Relative to year mean (binned scatter plot).



Source: Bank holding companies Y-9C reports.

Recent empirical work by Forti and Schiozer (2015) provide suggestive evidence for such a link. Specifically, the authors show for a sample of Brazilian banks that banks with more informationally-sensitive depositors such as institutional investors were more likely to increase their dividends during the 2007-2008 financial crisis.

7 Concluding comments

U.S banks paid large amounts in dividends during the financial crisis. In our paper we study a framework that incorporates two distinct views of the underlying reasons for this behavior – risk shifting and signaling. When signaling incentives are weak, paying dividends lowers a bank’s resilience to rollover crises. However, the bank also exerts a negative externality on other banks through the rollover actions of lenders. In contrast, when changes in dividends affect the behavior of short-term lenders, lowering dividends may worsen a bank’s ability to survive the rollover crisis.

The signaling effect and related reluctance of banks to cut dividends due to an increased risk of a rollover crisis may also explain why banks were reluctant to issue new equity (essentially a negative dividend) during the financial crisis (Bigio, 2012). Examining the implications of new equity issuance in an environment with rollover crises is a potentially important extension of our framework that we leave for future research.

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Appendix A: Additional Results

A.1. Equilibrium definitions

We will use the following related equilibrium concepts that reflect the different information structures that we work with.

Definition 1. (Exogenous information) A symmetric perfect Bayesian equilibrium in the exogenous information model consists of individual beliefs $\theta|\theta_i$, a rollover decision $a(\theta_i)$, an aggregate attack fraction, $A(\theta)$, and a bank policy pair $(d(\theta), g(\theta))$, such that 1) lenders’ beliefs about θ given their signal θ_i satisfy Bayes’ rule; 2) $a(\theta_i)$ maximizes individual expected utility (8), given individual beliefs; 3) the aggregate attack fraction is consistent with individual decisions, i.e. $A(\theta) = \int_i a(\theta_i) di$; and (4) $(d(\theta), g(\theta))$ maximizes the bank owner’s utility (3), given the aggregate attack fraction $A(\theta)$.

For the endogenous information case, we have:

Definition 2. (Endogenous information) A symmetric perfect Bayesian equilibrium in the endogenous information model consists of individual beliefs $\theta|d_i$, a rollover-decision $a(d_i)$, an aggregate attack fraction, $A(d)$, and a bank policy pair $(d(\theta), g(\theta))$, such that 1) lenders' beliefs about θ given their signal d_i satisfy Bayes' rule; 2) $a(d_i)$ maximizes individual expected utility (8), given individual beliefs; 3) the aggregate attack fraction is consistent with individual decisions, i.e. $A(d) = \int_i a(d_i) di$; and (4) $(d(\theta), g(\theta))$ maximizes the bank owner's utility (3), given the aggregate attack fraction.

A.3. Microfounded examples

One can treat the two examples jointly by assuming that $X_\theta \geq 0$ and $\rho_\theta \leq 0$ with one of the two inequalities strict for any θ . First of all, note that

$$\tilde{a}_l = \frac{1}{(1-\rho)X} > 0.$$

Thus, if the bank wants to hold an additional dollar in cash, it has to liquidate more assets. Also,

$$\tilde{a}_\theta = - \int_0^{\tilde{a}} \frac{\partial}{\partial \theta} [(1-\rho)X] da \cdot \tilde{a}_l < 0.$$

Thus, a bank with higher portfolio quality has to liquidate a smaller fraction of its portfolio. The remaining part of the bank's portfolio has a value of

$$v(\theta, l) = \int_{\tilde{a}}^1 X(a, \theta) da. \quad (23)$$

In this case,

$$v_l = - \frac{1}{1-\rho(\tilde{a}, \theta)} < 0, \quad (24)$$

which is the opportunity cost to the bank of marginally increasing its cash holdings. Intuitively, it is inversely related to the liquidation discount of the asset. An asset with no liquidation discount is a perfectly substitutable to cash, while a fully illiquid asset is never liquidated. Also, note that $v_l \propto -\rho_a \tilde{a}_l < 0$. Intuitively, to obtain an additional dollar of liquidity, the bank has to liquidate assets with larger discounts. Furthermore,

$$v_\theta = \int_{\tilde{a}}^1 X_\theta da - X(\tilde{a}, \theta) \tilde{a}_\theta > 0.$$

The asset value at $t = 2$ is increasing in θ for two reasons. First, a bank with higher θ may have assets with higher expected payoffs. Second, a bank with higher θ may have to liquidate fewer assets to obtain the same amount of cash. Finally,

$$v_{l\theta} \propto -(\rho_\theta + \rho_a \tilde{a}_\theta) > 0. \quad (25)$$

Therefore, a bank with higher θ has a lower opportunity cost of increasing cash holdings, since its assets have lower discounts, and also, since it has to liquidate fewer assets.

A.2. Exogenous information and two dividend levels

The following proposition characterizes the unique equilibrium with exogenous information.

Proposition 8. (*Exogenous information*) *Suppose that lenders have a diffuse prior over $\theta \in \mathbb{R}$ and that assumption A1 holds. There exists a unique equilibrium, which is in monotone strategies and is given by a failure cutoff θ_f , a strategic cutoff $\hat{\theta}$, such that a lender with signal x attacks iff $x \leq \hat{\theta}$ and the bank fails iff $\theta \leq \theta_f$. Additionally, let $\theta_0 \equiv \bar{\ell}^{-1}(m)$, $\tilde{\theta} \equiv \bar{\ell}^{-1}(m+p)$, and let θ_a solve*

$$\lambda m = v(\theta_a, p). \quad (26)$$

If $\lambda m \geq v(\tilde{\theta}, p)$, then $\theta_f = \tilde{\theta}$, and the optimal bank dividend policy satisfies

$$d(\theta) = \begin{cases} m & , \theta \in [\theta_0, \infty) \\ 0 & , o.w. \end{cases}.$$

Otherwise, $\theta_f = \theta_a < \tilde{\theta}$, and the optimal bank dividend policy satisfies

$$d(\theta) = \begin{cases} m & , \theta \in [\theta_0, \theta_f) \cup [\theta_1, \infty) \\ 0 & , o.w. \end{cases},$$

where $\theta_1 > \theta_f$ solves

$$\lambda m + v(\theta_1, A(\theta_1, \hat{\theta}) + m) = v(\theta_1, A(\theta_1, \hat{\theta})). \quad (27)$$

In either case, $\hat{\theta}$ is the solution to

$$A(\theta_f, \hat{\theta}) = p. \quad (28)$$

Proof. We show this result in several steps. First we show that for every strategic cutoff $\hat{\theta}$, there is a unique $\theta_f \in (\underline{\theta}, \bar{\theta})$, such that the bank fails for $\theta \leq \theta_f$ and survives otherwise.

Second, we characterize the bank's optimal policies. Next we show some properties of $\hat{\pi}(x, \hat{\theta})$ and conclude that there is a unique strategic cutoff $\hat{\theta}$. Finally, we argue using an interim rationalizability argument that the monotone strategy equilibrium is the unique equilibrium of the game.

Bank's problem. Consider the bank's optimization problem, given by

$$\max_{d,g} W(d, g, \theta, \hat{\theta}) = \lambda d + \mathbf{1}_{\{g \geq A(\theta, \hat{\theta})\}} v(\theta, g + d) \quad (29)$$

$$s.t. \ g + d \leq \bar{\ell}(\theta),$$

$$d \in \{0, m\},$$

where

$$A(\theta, \hat{\theta}) = \Phi\left(\sqrt{\alpha_\theta}(\hat{\theta} - \theta)\right).$$

There are several cases to consider.

1. Suppose that $\bar{\ell}(\theta) < A(\theta, \hat{\theta})$. Therefore, the bank cannot survive even if it uses the maximum available liquid assets $\bar{\ell}(\theta)$. In that case it is optimal for the bank to choose the highest feasible dividend payment, i.e. $d(\theta) = \begin{cases} m & , m \leq \bar{\ell}(\theta) \\ 0 & , o.w. \end{cases}$.

2. Suppose that $\bar{\ell}(\theta) \geq A(\theta, \hat{\theta})$ and $\bar{\ell}(\theta) - m < A(\theta, \hat{\theta})$. Therefore, survival is feasible provided that the bank does not pay out m in dividends. Suppose that $m > \bar{\ell}(\theta)$. Since choosing $d = 0$ is always feasible, and the bank survives if it pays 0, it always prefers survival. Suppose instead that $m \leq \bar{\ell}(\theta)$. In that case, the bank chooses between paying out m and failing or paying out 0 and surviving. It chooses the former if

$$\lambda m > v\left(\theta, A(\theta, \hat{\theta})\right).$$

As we show below, at θ_f , $A(\theta_f, \hat{\theta}) = p$, so Assumption A1 implies that at θ_f , $m < A(\theta_f, \hat{\theta}) \leq \bar{\ell}(\theta_f)$. This in turn implies that $m < \bar{\ell}(\theta)$ whenever $\bar{\ell}(\theta) \geq A(\theta, \hat{\theta})$. Therefore, only the case $m \leq \bar{\ell}(\theta)$ is relevant.

3. Suppose that $\bar{\ell}(\theta) - m \geq A(\theta, \hat{\theta})$. Therefore, survival is feasible even if the bank pays out m . In that case, the bank chooses to survive and pays m if

$$\lambda m + v\left(\theta, A(\theta, \hat{\theta}) + m\right) > v\left(\theta, A(\theta, \hat{\theta})\right).$$

Next, define $\tilde{\theta}(\hat{\theta})$ as the solution to

$$\bar{l}(\tilde{\theta}) = m + A(\tilde{\theta}, \hat{\theta}) \quad (30)$$

Therefore, $\tilde{\theta}$ gives the value of fundamentals at which a bank can both pay out m and survive. Also, define $\theta_a(\hat{\theta})$ as the solution to

$$\lambda m = v(\theta_a, A(\theta_a, \hat{\theta})). \quad (31)$$

Therefore, θ_a gives the value of fundamentals at which a bank is indifferent between paying out m and failing or paying out 0 and surviving.

By the definition of $\tilde{\theta}$, $\theta_f(\hat{\theta}) \leq \tilde{\theta}$, since for $\theta > \tilde{\theta}$ the bank always chooses to survive. Since $v(\theta, A(\theta, \hat{\theta}))$ is increasing in θ , it follows that if at $\tilde{\theta}$, $\lambda m \geq v(\tilde{\theta}, A(\tilde{\theta}, \hat{\theta}))$, then for $\theta < \tilde{\theta}$, the bank is better off failing even when it is feasible to survive, and so $\theta_f(\hat{\theta}) = \tilde{\theta}$. Also, it is better off paying m whenever feasible. Also, notice that

$$\begin{aligned} \frac{\partial}{\partial \theta} [v(\theta, A(\theta, \hat{\theta}) + m) - v(\theta, A(\theta, \hat{\theta}))] &= [v_\theta(\theta, A(\theta, \hat{\theta}) + m) - v_\theta(\theta, A(\theta, \hat{\theta}))] + \\ &+ [v_l(\theta, A(\theta, \hat{\theta}) + m) - v_l(\theta, A(\theta, \hat{\theta}))] \frac{\partial}{\partial \theta} A(\theta, \hat{\theta}) > 0 \end{aligned}$$

by the properties of $v(\theta, l)$, and since $A(\theta, \hat{\theta})$ is monotone decreasing in θ . Therefore, for $\theta > \tilde{\theta}$, $\lambda m + v(\theta, A(\theta, \hat{\theta}) + m) > v(\tilde{\theta}, A(\tilde{\theta}, \hat{\theta}))$ and the bank is better off paying m .

On the other hand, if at $\tilde{\theta}$, $\lambda m < v(\tilde{\theta}, A(\tilde{\theta}, \hat{\theta}))$, then $\theta_f(\hat{\theta}) = \theta_a \leq \tilde{\theta}$. In that case the bank pays m whenever feasible for $\theta \leq \theta_f$, and it pays 0 for $\theta \in (\theta_f, \theta_1)$, and m for $\theta > \theta_1$. Therefore,

$$\theta_f(\hat{\theta}) = \begin{cases} \tilde{\theta}(\hat{\theta}) & , \tilde{\theta} \leq \theta_a \\ \theta_a(\hat{\theta}) & , \tilde{\theta} > \theta_a \end{cases}. \quad (32)$$

Finally, notice that in either case, $0 \leq \frac{\partial \theta_f}{\partial \hat{\theta}} < 1$ whenever θ_f is differentiable. To see this, note first that by the implicit function theorem, $\frac{\partial \tilde{\theta}}{\partial \hat{\theta}} = \frac{A_{\hat{\theta}}}{\bar{l} - A_\theta} = \frac{A_{\hat{\theta}}}{\bar{l} + A_{\hat{\theta}}}$, where the second equality comes from using $A_{\hat{\theta}} = -A_\theta$. Therefore, $0 \leq \frac{\partial \tilde{\theta}}{\partial \hat{\theta}} < 1$ by the properties of A and \bar{l} . Similarly, $\frac{\partial \theta_a}{\partial \hat{\theta}} = -\frac{v_l A_{\hat{\theta}}}{v_\theta + v_l A_\theta} = \frac{v_l A_\theta}{v_\theta + v_l A_\theta}$, so again $0 \leq \frac{\partial \theta_a}{\partial \hat{\theta}} < 1$ by the properties of v and A .

Lender's problem. We have

$$\begin{aligned}\hat{\pi}(x, \theta_f) &= p - \Pr\{g > A|x\} \\ &= p - \Pr\{\theta > \theta_f|x\}.\end{aligned}$$

With a diffuse prior over θ , Bayes' rule implies that $\theta|x \sim N(x, \alpha_\theta^{-1})$. Therefore,

$$\Pr\{\theta > \theta_f|x\} = 1 - \Phi(\sqrt{\alpha_\theta}(\theta_f - x)) = \Phi(\sqrt{\alpha_\theta}(x - \theta_f)).$$

Therefore, $\hat{\theta}(\theta_f)$ satisfies

$$\Phi\left(\sqrt{\alpha_\theta}\left(\hat{\theta}(\theta_f) - \theta_f\right)\right) = A\left(\theta_f, \hat{\theta}(\theta_f)\right) = p. \quad (33)$$

By the implicit function theorem this condition defines an implicit relation between $\hat{\theta}$ and θ_f , such that $\frac{\partial \hat{\theta}}{\partial \theta_f} = -\frac{A_\theta}{A_{\hat{\theta}}} = 1$.

Thus, $\hat{\pi}$ is continuously differentiable in both arguments, strictly increasing in θ_f and strictly decreasing in x . Furthermore, $\lim_{x \rightarrow -\infty} \hat{\pi}(x, \theta_f) = p > 0$ and $\lim_{x \rightarrow \infty} \hat{\pi}(x, \theta_f) = p - 1 < 0$.

Unique monotone equilibrium. Therefore, a monotone equilibrium of this economy is given by the intersection of condition (32) and the relation $\hat{\theta}(\theta_f)$ implicitly defined by condition (33) in $(\hat{\theta}, \theta_f)$ -space. Both are continuous and increasing but the first relation has a slope strictly less than one, while the second relation has a slope equal to one. Therefore, the two relations can cross at most once.³²

Rationalizability. We now show that the unique monotone equilibrium is also the unique equilibrium of this economy by using iterated elimination of strictly dominated strategies. Define $h(\hat{\theta})$ implicitly via $\hat{\pi}(h(\hat{\theta}), \theta_f(\hat{\theta})) = 0$. This function is continuous, since the functions $\hat{\pi}$ and θ_f are continuous. It is also increasing in $\hat{\theta}$, since θ_f is increasing in $\hat{\theta}$ and $\hat{\pi}$ is decreasing in its first argument and increasing in the second argument. Also, notice that a fixed point of $h(\hat{\theta})$ gives a monotone equilibrium for this economy. Since that monotone equilibrium is unique, it follows that $h(\cdot)$ has a unique fixed point.

The most optimistic scenario for any lender is that only banks with $\theta < \underline{\theta}$ fail. Let $\Pr(\theta < \underline{\theta}|x_i)$ denote the probability of bank failure in this case for a lender that observes signal x_i . Since $\Pr(\theta < \underline{\theta}|x_i)$ is continuous and strictly decreasing in x_i with $\lim_{x \rightarrow -\infty} \Pr(\theta < \underline{\theta}|x_i) = 1$ and $\lim_{x \rightarrow \infty} \Pr(\theta < \underline{\theta}|x_i) = 0$, there exists a signal \underline{x} such that $\Pr(\theta < \underline{\theta}|\underline{x}) = p$. For $x_i \leq \underline{x}$ it is strictly dominant to choose $a_i = 1$. Similarly, one can establish the existence of a \bar{x} , which gives the signal of an indifferent agent in the most pessimistic scenarios when only banks

³²Also standard fixed point results ensure equilibrium existence.

with $\theta > \bar{\theta}$ survive.

Next, set $x_0 = \underline{x}$. Therefore, from the definition of $h(\cdot)$, running for signal $x_i \leq x_1 \equiv h(x_0)$ is strictly dominant. By the properties of the function $h(\cdot)$, we can therefore construct a sequence $\{x_n\}_{n=0}^{\infty}$ defined recursively by $x_n = h(x_{n-1})$, which is increasing. Furthermore, it's bounded above by \bar{x} . Therefore, this sequence converges and by continuity of h it converges to the unique fixed point of $h(\cdot)$, $\hat{\theta}$. Similarly, once can construct a sequence starting from $x_0 = \bar{x}$, which is decreasing and bounded below by \underline{x} . That sequence then also converges to the unique fixed point of $h(\cdot)$. \square

We also have the following immediate corollary.

Corollary 1. *In the limit, as $\alpha_\theta \rightarrow \infty$, $\hat{\theta} \rightarrow \theta_f$. Furthermore, θ_f still satisfies $\theta_f = \tilde{\theta}$ or $\theta_f = \theta_a$ depending on whether $\lambda m \geq v(\tilde{\theta}, p)$.*

Proof. We use (28) to write $\hat{\theta} - \theta_f = -\frac{1}{\sqrt{\alpha_\theta}} \Phi^{-1}(p-1)$. Thus $\alpha_\theta \rightarrow \infty$ implies that $\hat{\theta} \rightarrow \theta_f$. Finally, notice also that either of the two equations that pin down θ_f do not depend on α_θ . \square

Thus the precision of private information that lenders receive has no effect on the failure threshold. What it does affect is only the cutoff θ_1 at which surviving banks pay dividends in the case when $\lambda m < v(\tilde{\theta}, p)$.

A.3. Endogenous information and two dividend levels

For a given cutoff \hat{d} , the solution to the bank's problem is summarized in Lemma 2.

Lemma 2. *(Bank problem). Suppose assumption A1' holds and that lenders follow a monotone strategy with cutoff \hat{d} . Then there exists a unique θ_f such that the bank fails iff $\theta \leq \theta_f$. If $A(m, \hat{d}) + m \leq A(0, \hat{d})$ or $A(m, \hat{d}) + m > A(0, \hat{d})$ and $\lambda m \geq v(\tilde{\theta}, A(0, \hat{d}))$ (Case 1'), $\theta_f = \tilde{\theta} \equiv \bar{\ell}^{-1}(m + A(m, \hat{d}))$, and the optimal bank dividend policy satisfies*

$$d(\theta) = \begin{cases} m & , \theta \in [\theta_0, \infty) \\ 0 & , o.w. \end{cases}, \quad (34)$$

where $\theta_0 \equiv \bar{\ell}^{-1}(m)$. Otherwise, (Case 2') $\theta_f = \theta_a$, where θ_a solves

$$\lambda m = v(\theta_a, A(0, \hat{d})), \quad (35)$$

and banks with $\theta \in \Theta_1 = [\theta_0, \theta_f] \cup [\theta_1, \infty)$ pay dividends, where θ_1 satisfies $\lambda m + v \left(\theta_1, A(m, \hat{d}) + m \right) = v \left(\theta_1, A(0, \hat{d}) \right)$. In both cases $\frac{\partial \theta_f}{\partial \hat{d}} \geq 0$.

Proof. Consider the bank's optimization problem, given by

$$\max_{d, g} W(d, g, \theta, \hat{d}) = \lambda d + \mathbf{1}_{\{g \geq A(d, \hat{d})\}} v(\theta, g + d) \quad (36)$$

$$s.t. \ g + d \leq \bar{\ell}(\theta),$$

$$d \in \{0, m\},$$

where

$$A(d, \hat{d}) = \Phi \left(\sqrt{\alpha_d} (\hat{d} - d) \right).$$

First, define $\tilde{\theta}$ as the solution to

$$m + A(m, \hat{d}) = \bar{\ell}(\tilde{\theta}), \quad (37)$$

θ_0 as the solution

$$m = \bar{\ell}(\theta_0), \quad (38)$$

θ_1 as the solution to

$$\lambda m + v \left(\theta_1, A(m, \hat{d}) + m \right) = v \left(\theta_1, A(0, \hat{d}) \right), \quad (39)$$

and θ_2 as the solution to

$$A(0, \hat{d}) = \bar{\ell}(\theta_2). \quad (40)$$

First, notice that if $\hat{d} > 0$, assumption A1' ensures that $A(0, \hat{d}) > \frac{1}{2}$, so $m < \frac{1}{2}$ ensures that $A(0, \hat{d}) > m$. We, therefore, have two main cases to consider, the case where $A(0, \hat{d}) \geq A(m, \hat{d}) + m$ and $A(0, \hat{d}) < A(m, \hat{d}) + m$.

Consider first the case where $A(0, \hat{d}) \geq A(m, \hat{d}) + m$. We then have that $\theta_2 \geq \tilde{\theta} \geq \theta_0$. For $\theta \geq \theta_2$, banks in this range always survive since $A(0, \hat{d}) \leq \bar{\ell}(\theta)$. If $\lambda m + v \left(\theta, A(m, \hat{d}) + m \right) \geq v \left(\theta, A(0, \hat{d}) \right)$ the bank sets $d = m$ and zero otherwise. Since $A(0, \hat{d}) \geq A(m, \hat{d}) + m$, the bank always pays dividends in this region. For $\theta \in [\tilde{\theta}, \theta_2)$ the bank survives if it pays dividends and fails if not. Since $v \left(\theta, A(m, \hat{d}) + m \right) > v \left(\theta, \bar{\ell}(\theta) \right) = 0$ the bank pays dividends $d = m$ and survive. For $\theta \in [\theta_0, \tilde{\theta})$ the bank always fail. Since $\bar{\ell}(\theta) \geq m$ for $\theta \in [\theta_0, \tilde{\theta})$, the bank sets

$d = m$. For $\theta < \theta_0$ both paying dividends and surviving is not feasible and the bank fails with $d = 0$. Thus, if $A(0, \hat{d}) \geq A(m, \hat{d}) + m$, $\theta_f = \tilde{\theta}$ with $d = m$ if $\theta \in [\theta, \infty)$ and $d = 0$ otherwise.

Next, consider the case where $A(0, \hat{d}) < A(m, \hat{d}) + m$. We then have that $\tilde{\theta} \geq \theta_2 \geq \theta_0$. For $\theta \geq \tilde{\theta}$, the bank survives regardless. For $\theta \in [\theta_2, \tilde{\theta})$ the bank can pay dividends and fail or don't pay dividends and survive. We then have two subcases. If $\lambda m \geq v(\tilde{\theta}, A(0, \hat{d}))$ (Case 1') the bank pay dividends and fail. If $\lambda m < v(\tilde{\theta}, A(0, \hat{d}))$ (Case 2') the bank don't pay dividends and survive. If $\theta \in [\theta_0, \theta_2)$ dividends is feasible but survival without dividends is not, so the bank set $d = m$ and fail. If $\theta \in (-\infty, \theta_0)$ both dividends and survival is not feasible and the bank thus fails with $d = 0$. Thus, if we are in case 1', $\theta_f = \tilde{\theta}$. If we are in case 2', $\theta_f = \theta_a$, where θ_a solves $\lambda m = v(\theta_a, A(0, \hat{d}))$. Furthermore, in Case 1' $d = m$ for $\theta \in (\theta_0, \infty)$ and $d = 0$ otherwise. In Case 2', $d = m$ for $\theta \in [\theta_0, \theta^f] \cup [\theta_1, \infty)$ and $d = 0$ otherwise. To see the latter, notice that from the properties of $v(\theta, l)$, $\theta_1 \geq \tilde{\theta}$ in this case since $v_\theta(\theta, A(m, \hat{d}) + m) - v_\theta(\theta, A(0, \hat{d})) > 0$ by the properties of $v(\theta, l)$ and from the fact that $A(m, \hat{d}) + m > A(0, \hat{d})$.

In either case,

$$g = \begin{cases} [0, \bar{l}(\theta) - d(\theta) & \theta \leq \theta_f \\ A(d, \hat{d}) & \theta > \theta_f \end{cases}. \quad (41)$$

Finally, notice that if $\theta_f = \tilde{\theta}$, applying the implicit function theorem on the failure condition $m + A(m, \hat{d}) - \bar{l}(\theta_f) = 0$ yields $\frac{\partial \theta_f}{\partial \hat{d}} = \frac{A_{\hat{d}}}{\bar{l}'(\theta_f)} > 0$. If $\theta_f = \theta_a$, applying the implicit function theorem on the failure condition $\lambda m - v(\theta_f, A(0, \hat{d})) = 0$ yields $\frac{\partial \theta_f}{\partial \hat{d}} = -\frac{v_l}{v_\theta} A_{\hat{d}} > 0$. \square

Next, we characterize the lenders' problem.

Lemma 3. (Strategic cutoff). *Suppose that assumptions A2 and A3 hold, and let θ_f denote the bank failure threshold. If $\Theta_1 = [\theta_0, \infty)$, then there exists a unique strategic cutoff $\hat{d}(\theta_f)$, such that a lender runs iff $d_i \leq \hat{d}(\theta_f)$. Furthermore, $\hat{d}(\theta_f)$ satisfies*

$$\hat{d} = \frac{m}{2} + \frac{1}{\alpha_d m} \log \left(\frac{p \Pr \{\theta < \theta_0\}}{\Pr \{\theta > \theta_f\} - p \Pr \{\theta > \theta_0\}} \right). \quad (42)$$

If $\Theta_1 = [\theta_0, \theta_f] \cup [\theta_1, \infty)$ and $\Pr \{\theta < \underline{\theta}\} > 1 - p$, then there exists a unique strateigc cutoff $\hat{d}(\theta_f)$, such that a lender runs iff $d_i \leq h(\hat{d})$. Furthermore, $\hat{d}(\theta_f)$ satisfies

$$\hat{d} = \frac{m}{2} + \frac{1}{\alpha_d m} \log \left(\frac{p \Pr \{\theta < \theta_0\} - (1 - p) \Pr \{\theta \in (\theta_f, \theta_1)\}}{\Pr \{\theta > \theta_f\} - p \Pr \{\theta > \theta_0\} - (1 - p) \Pr \{\theta \in (\theta_f, \theta_1)\}} \right). \quad (43)$$

Proof. We first show that given Assumption A2, a higher dividend signal constitutes good news about bank survival. Let $\Theta_0 \subset \Theta$ denote the set of banks that choose $d(\theta) = 0$, and $\Theta_1 \subset \Theta$ denote the set of banks that choose $d(\theta) = m$. Then

$$\Pr \{\theta < y | d_i\}$$

is strictly decreasing in d_i , iff

$$\Pr \{\Theta_1 | \theta < y\} < \Pr \{\Theta_1\}. \quad (44)$$

To show this, from Bayes' rule, we have

$$\begin{aligned} \Pr \{\theta < y | d_i\} &= \frac{f(d_i | \theta < y) \Pr \{\theta < y\}}{f(d_i | \theta < y) \Pr \{\theta < y\} + f(d_i | \theta > y) \Pr \{\theta > y\}} \\ &= \frac{\int_{-\infty}^y f_d(d_i | \theta) f_p(\theta) d\theta}{\int_{-\infty}^y f_d(d_i | \theta) f_p(\theta) d\theta + \int_y^{\infty} f_d(d_i | \theta) f_p(\theta) d\theta}, \end{aligned}$$

where $f_p(\theta)$ denotes the pdf for the prior belief over θ and $f_d(d_i | \theta)$ is the conditional pdf of d_i given θ . Notice that $d_i | \theta \sim N(d(\theta), \alpha_d^{-1})$. Therefore,

$$\begin{aligned} \int_{-\infty}^y f_d(d_i | \theta) f_p(\theta) d\theta &= \int_{(-\infty, y) \cap \Theta_0} \sqrt{\alpha_d} \phi(\sqrt{\alpha_d}(d_i - d^0)) f_p(\theta) d\theta \\ &\quad + \int_{(-\infty, y) \cap \Theta_1} \sqrt{\alpha_d} \phi(\sqrt{\alpha_d}(d_i - d^1)) f_p(\theta) d\theta \\ &= \sqrt{\alpha_d} \phi(\sqrt{\alpha_d}(d_i - d^0)) \Pr \{\{\theta < y\} \cap \Theta_0\} \\ &\quad + \sqrt{\alpha_d} \phi(\sqrt{\alpha_d}(d_i - d^1)) \Pr \{\{\theta < y\} \cap \Theta_1\}, \end{aligned}$$

and analogously for $\int_y^{\infty} f_d(d_i | \theta) f_p(\theta) d\theta$. Simplifying, we get the following expression for $\Pr \{\theta < y | d_i\}$,

$$\Pr \{\theta < y | d_i\} = \Pr \{\theta < y\} \frac{1 + (LR(d_i) - 1) \Pr \{\Theta_1 | \theta < y\}}{1 + (LR(d_i) - 1) \Pr \{\Theta_1\}},$$

where

$$LR(x) \equiv \frac{f_d(x | d^1, \alpha_d)}{f_d(x | d^0, \alpha_d)} = \exp \left\{ \alpha_d (d^1 - d^0) \left(x - \frac{d^1 + d^0}{2} \right) \right\}.$$

Notice that $LR' > 0$, so higher realizations of d_i imply that it is more likely that it is drawn

from a distribution with a mean of d^1 . Thus,

$$\begin{aligned} \frac{\partial}{\partial d_i} (\Pr \{\theta < y | d_i\}) &= \Pr \{\theta < y\} LR' \frac{\Pr \{\Theta_1 | \theta < y\} - \Pr \{\Theta_1\}}{(1 + [LR(d_i) - 1] \Pr \{\Theta_1\})^2} < 0 \\ &\iff \Pr \{\Theta_1 | \theta < y\} < \Pr \{\Theta_1\}. \end{aligned}$$

Therefore, for $y = \theta_f$, we have that

$$\Pr \{\Theta_1 | \theta < \theta_f\} = \frac{\Pr \{\theta \in (\underline{\theta}, \theta_f)\}}{\Pr \{\theta < \underline{\theta}\} + \Pr \{\theta \in (\underline{\theta}, \theta_f)\}} < \frac{\Pr \{\theta \in (\underline{\theta}, \bar{\theta})\}}{\Pr \{\theta < \underline{\theta}\}},$$

since banks in the lower-dominance region never pay a dividend. Furthermore, since banks in the upper dominance region always pay a dividend, it follows that

$$\Pr \{\Theta_1\} > \Pr \{\theta > \bar{\theta}\}.$$

Therefore,

$$\Pr \{\Theta_1 | \theta < \theta_f\} < \frac{\Pr \{\theta \in (\underline{\theta}, \bar{\theta})\}}{\Pr \{\theta < \underline{\theta}\}} < \Pr \{\theta > \bar{\theta}\} < \Pr \{\Theta_1\},$$

where the second inequality follows by Assumption A2.

Therefore, given a failure threshold $\theta_f \in [\underline{\theta}, \bar{\theta}]$, the expected net payoff of an agent i with signal d_i is given by

$$\begin{aligned} \hat{\pi}_i(d_i, \theta_f) &= p - \Pr \{g > A | d_i\} \\ &= p - \Pr \{\theta > \theta_f | d_i\} \\ &= p - 1 + \Pr \{\theta < \theta_f | d_i\}. \end{aligned}$$

So,

$$\hat{\pi}(d_i, \theta_f) = p - 1 + \Pr \{\theta \leq \theta_f\} \frac{1 + (LR(d_i) - 1) \Pr \{\Theta_1 | \theta \leq \theta_f\}}{1 + (LR(d_i) - 1) \Pr \{\Theta_1\}}. \quad (45)$$

By the above discussion, we have that $\frac{\partial \hat{\pi}(d_i, \theta_f)}{\partial d_i} < 0$.

Next, we proceed to characterize the signal \hat{d} which makes a lender indifferent between rolling over the debt and running satisfies

$$\hat{\pi}(\hat{d}, \theta_f) = 0. \quad (46)$$

Since $\frac{\partial \hat{\pi}(d_i, \theta_f)}{\partial d_i} < 0$ and $\hat{\pi}(d_i, \theta_f)$ is continuous, a threshold signal \hat{d} such that an agent runs iff $d_i \leq \hat{d}$ exists and is unique if $\lim_{d_i \rightarrow -\infty} \hat{\pi}(d_i, \theta_f) > 0$ and $\lim_{d_i \rightarrow \infty} \hat{\pi}(d_i, \theta_f) < 0$. To see that this is indeed the case, observe that $\lim_{d_i \rightarrow \infty} LR(d_i) = \infty$ and $\lim_{d_i \rightarrow -\infty} LR(d_i) = 0$. We then have that

$$\lim_{d_i \rightarrow -\infty} \hat{\pi}(d_i, \theta_f) = p - 1 + \Pr\{\theta \leq \theta_f\} \frac{1 - \Pr\{\Theta_1 | \theta \leq \theta_f\}}{1 - \Pr\{\Theta_1\}}. \quad (47)$$

From Lemma 2, we can either have that $\Theta_1 = [\theta_0, \infty)$ with $\theta_f = \tilde{\theta}$ or $\Theta_1 = [\theta_0, \theta_f) \cup [\theta_1, \infty)$ with $\theta_f = \theta_a$.

If $\Theta_1 = [\theta_0, \infty)$, then

$$\lim_{d_i \rightarrow -\infty} \hat{\pi}(d_i, \theta_f) = p - 1 + 1 = p > 0. \quad (48)$$

If $\Theta_1 = [\theta_0, \theta_f) \cup [\theta_1, \infty)$, then

$$\lim_{d_i \rightarrow -\infty} \hat{\pi}(d_i, \theta_f) = p - 1 + \frac{\Pr\{\theta < \theta_0\}}{\Pr\{\theta < \theta_0\} + \Pr\{\theta \in (\theta_f, \theta_1)\}}. \quad (49)$$

Notice that $\Pr\{\theta < \theta_0\} + \Pr\{\theta \in (\theta_f, \theta_1)\} < 1$. Also, if $\Pr\{\theta < \theta_0\} > 1 - p$ then $\Pr\{\theta < \theta_0\} > 1 - p$. Therefore, $\lim_{d_i \rightarrow -\infty} \hat{\pi}(d_i, \theta_f) > 0$. Thus, in both cases $\lim_{d_i \rightarrow -\infty} \hat{\pi}(d_i, \theta_f) > 0$.

Similarly, we have that when $\Theta_1 = [\theta_0, \infty)$

$$\lim_{d_i \rightarrow \infty} \hat{\pi}(d_i, \theta_f) = p - 1 + \frac{\Pr\{\theta < \theta_f\} - \Pr\{\theta < \theta_0\}}{1 - \Pr\{\theta < \theta_0\}}, \quad (50)$$

and when $\Theta_1 = [\theta_0, \theta_f) \cup [\theta_1, \infty)$

$$\lim_{d_i \rightarrow \infty} \hat{\pi}(d_i, \theta_f) = p - 1 + \frac{\Pr\{\theta < \theta_f\} - \Pr\{\theta < \theta_0\}}{\Pr\{\theta < \theta_f\} + \Pr\{\theta > \theta_1\} - \Pr\{\theta < \theta_0\}}. \quad (51)$$

Again, $\Pr\{\theta < \theta_f\} + \Pr\{\theta > \theta_1\} < 1$. Thus, if we can show that $\lim_{d_i \rightarrow \infty} \hat{\pi}(d_i, \theta_f) < 0$ for $\Theta_1 = [\theta_0, \theta_f) \cup [\theta_1, \infty)$, it must also be so for $\Theta_1 = [\theta_0, \infty)$.

We can write

$$\begin{aligned} \lim_{d_i \rightarrow \infty} \hat{\pi}(d_i, \theta_f) &= p - 1 + \frac{\Pr\{\theta < \theta_f\} - \Pr\{\theta < \theta_0\}}{\Pr\{\theta < \theta_f\} + \Pr\{\theta > \theta_1\} - \Pr\{\theta < \theta_0\}} \\ &= p - 1 + \frac{\Pr\{\theta < \theta_f\} - \Pr\{\theta < \theta_0\}}{1 - \Pr\{\theta < \theta_0\} - \Pr\{\theta \in (\theta_f, \theta_1)\}} \\ &= p - \frac{1 - \Pr\{\theta < \theta_f\} - \Pr\{\theta \in (\theta_f, \theta_1)\}}{1 - \Pr\{\theta < \theta_0\} - \Pr\{\theta \in (\theta_f, \theta_1)\}}, \end{aligned}$$

so

$$\lim_{d_i \rightarrow \infty} \hat{\pi}(d_i, \theta_f) < 0 \Leftrightarrow p \leq \frac{\Pr\{\theta > \theta_f\} - \Pr\{\theta \in (\theta_f, \theta_1)\}}{\Pr\{\theta > \theta_0\} - \Pr\{\theta \in (\theta_f, \theta_1)\}}. \quad (52)$$

By assumption A3, $p < \frac{\Pr\{\theta > \bar{\theta}\}}{\Pr\{\theta > \theta_0\}}$. Notice also that $\Pr\{\theta > \theta_f\} \geq \Pr\{\theta > \bar{\theta}\}$, so $p < \frac{\Pr\{\theta > \theta_f\} - \Pr\{\theta \in (\theta_f, \theta_1)\}}{\Pr\{\theta > \theta_0\} - \Pr\{\theta \in (\theta_f, \theta_1)\}}$. Thus, $\lim_{d_i \rightarrow \infty} \hat{\pi}(d_i, \theta_f) < 0$ in both cases and that there exists a unique $\hat{d}(\theta_f)$ such that $\hat{\pi}(\hat{d}, \theta_f) = 0$.

Finally, to characterize the strategic cutoff, notice that $\hat{\pi}(d_i, \theta_f) = 0$ can be written as

$$p - 1 + \Pr\{\theta \leq \theta_f\} \frac{1 + \left(LR(\hat{d}) - 1\right) \Pr\{\Theta_1 | \theta \leq \theta_f\}}{1 + \left(LR(\hat{d}) - 1\right) \Pr\{\Theta_1\}} = 0. \quad (53)$$

When $\Theta_1 = [\theta_0, \infty)$, we have that $\Pr\{\Theta_1 | \theta \leq \theta_f\} = \Pr\{\theta < \theta_f\} - \Pr\{\theta < \theta_0\}$, $\Pr\{\Theta_1\} = \Pr\{\theta > \theta_0\}$ and thus solving for \hat{d} yields

$$\hat{d} = \frac{m}{2} + \frac{1}{\alpha_d m} \log \left(\frac{p \Pr\{\theta < \theta_0\}}{\Pr\{\theta > \theta_f\} - p \Pr\{\theta > \theta_0\}} \right). \quad (54)$$

When $\Theta_1 = [\theta_0, \theta_f) \cup [\theta_1, \infty)$ we have that $\Pr\{\Theta_1 | \theta \leq \theta_f\} = \Pr\{\theta < \theta_f\} - \Pr\{\theta < \theta_0\}$, $\Pr\{\Theta_1\} = \Pr\{\theta < \theta_f\} - \Pr\{\theta < \theta_0\} + \Pr\{\theta > \theta_1\}$ and thus solving for \hat{d} yields

$$\hat{d} = \frac{m}{2} + \frac{1}{\alpha_d m} \log \left(\frac{p \Pr\{\theta < \theta_0\} - (1-p) \Pr\{\theta \in (\theta_f, \theta_1)\}}{\Pr\{\theta > \theta_f\} - p \Pr\{\theta > \theta_0\} - (1-p) \Pr\{\theta \in (\theta_f, \theta_1)\}} \right). \quad (55)$$

□

Lemma 3 shows that for a given θ_f and with dividend policies as either one of the two different policy profiles in Lemma 2, there exists a unique dividend signal at which a lender is indifferent between running and rolling over.³³ When α_d is large, an agent knows with certainty that the bank's dividend policy is 0 if she observes a signal close to 0. If $\Theta_1 = [\theta_0, \infty)$, she would refuse to roll over, since banks paying $d = 0$ always fail. Alternatively, if she observes a signal close to m , she knows for sure that the bank dividend policy is m . In that case, the dividend policy can come from both a failing and a surviving bank. Given that $p < \frac{\Pr\{\theta > \bar{\theta}\}}{\Pr\{\theta > \theta_0\}}$, she believes that the mass of surviving banks relative to all banks paying m is relatively large. Thus, she is better off rolling over in this case.

³³The condition $\Pr\{\theta < \underline{\theta}\} > 1 - p$ in Lemma 3 puts a lower bound on how pessimistic agents can be in terms of the probability of the run succeeding.

Appendix B: Omitted Proofs

Proofs of results in Section 2

Proof of Proposition 1.

The bank-owner solves

$$\tilde{W}(\theta) = \max_{d \in [0, \bar{\ell}(\theta)]} \{\lambda d + v(\theta, d)\}.$$

Taking the f.o.c. with respect to d , the optimal d^* solves

$$\lambda \leq -v_l(\theta, d^*) + \kappa_{\bar{\ell}},$$

where $\kappa_{\bar{\ell}}$ and d^* satisfy the complementary slackness condition $\kappa_{\bar{\ell}}(\bar{\ell}(\theta) - d^*) = 0$. Therefore, d^* satisfies

$$[\lambda + v_l(\theta, d^*)](\bar{\ell}(\theta) - d^*)d^* = 0, \quad (56)$$

Let

$$\varphi(\theta) \equiv \lambda \bar{\ell}(\theta) - \tilde{W}(\theta).$$

Given Assumption B1, $\varphi(\theta)$ is (weakly) decreasing in θ . To show this, note first that $\tilde{W}(\theta) \geq \lambda \bar{\ell}(\theta)$ with equality, whenever $d^* = \bar{\ell}(\theta)$, so $\varphi(\theta) \leq 0$. By the theorem of the maximum, $\tilde{W}(\theta)$ is continuous in θ , so $\varphi(\theta)$ is also continuous in θ . Also, d^* is continuous in θ . Therefore, there are intervals of $\{\theta \geq \underline{\theta}\}$, where d^* may be on the lower boundary, interior or upper boundary of the feasible set. Denote those by Θ_L , Θ_I , Θ_U , respectively. For $\theta \in \Theta_U$, $\varphi(\theta) = 0$. For $\theta \in \Theta_L$, $\varphi(\theta) = \lambda \bar{\ell}(\theta) - v(\theta, 0)$, so $\varphi'(\theta) = \lambda \bar{\ell}'(\theta) - v_\theta(\theta, 0)$. Similarly, for $\theta \in \Theta_I$, applying the envelope theorem, we get

$$\varphi'(\theta) = \lambda \bar{\ell}'(\theta) - v_\theta(\theta, d^*). \quad (57)$$

Suppose, toward a contradiction, that $\varphi(\theta)$ is strictly increasing in θ , for some θ_0 . Since, $\varphi(\theta) = 0$ for $\theta \in \Theta_U$, it follows that $\theta_0 \in \Theta_L \cup \Theta_I$, and so

$$\varphi'(\theta_0) = \lambda \bar{\ell}'(\theta_0) - v_\theta(\theta_0, d^*(\theta_0)). \quad (58)$$

Noting that $\vec{\ell}' = -\frac{v_\theta(\theta_0, \bar{\ell}(\theta_0))}{v_l(\theta_0, \bar{\ell}(\theta_0))}$ and using the first-order condition for d^* at θ_0 , $\lambda \leq -v_l(\theta_0, d^*(\theta_0))$, we get that

$$\varphi'(\theta_0) \leq v_l(\theta_0, d^*(\theta_0)) \frac{v_\theta(\theta_0, \bar{\ell}(\theta_0))}{v_l(\theta_0, \bar{\ell}(\theta_0))} - v_\theta(\theta_0, d^*(\theta_0)) \quad (59)$$

$$= v_l(\theta_0, d^*(\theta_0)) \left(\frac{v_\theta(\theta_0, \bar{\ell}(\theta_0))}{v_l(\theta_0, \bar{\ell}(\theta_0))} - \frac{v_\theta(\theta_0, d^*(\theta_0))}{v_l(\theta_0, d^*(\theta_0))} \right) < 0, \quad (60)$$

where the last inequality comes from Assumption B1 and from $v_l < 0$. Thus, we reach a contradiction.

Therefore, by this property of $\varphi(\theta)$, if $\lambda < -v_l(\underline{\theta}, 0)$, then $\varphi(\theta) < 0$, for $\theta > \underline{\theta}$, and so, $d^*(\theta) < \bar{\ell}(\theta)$, for $\theta > \underline{\theta}$, with $d^*(\theta)$ determined by (9). Furthermore, $v_{l\theta} > 0$, implies that d^* is increasing in θ . To show this, note that for $\theta \in \Theta_I$, $\frac{\partial d^*}{\partial \theta} = -\frac{v_{l\theta}}{v_{ll}} > 0$, and since d^* is continuous in θ , it follows that for any $\theta_U \in \Theta_U$, for which $d^* = 0$, $\theta_U \leq \theta$, $\forall \theta \in \Theta_I$.

Similarly, if $\lambda \geq -v_l(\underline{\theta}, 0)$, then a non-increasing $\varphi(\theta)$ implies that there is a unique critical value of θ , θ^* , such that for a bank with $\theta \leq \theta^*$, $d^* = \bar{\ell}(\theta)$ and for a bank with $\theta > \theta^*$, $d^* < \bar{\ell}(\theta)$. Furthermore, $v_{l\theta} > 0$, and continuity of d^* imply that $d^* > 0$ for $\theta > \theta^*$. To show this, note that for $\theta \in \Theta_I$, $\frac{\partial d^*}{\partial \theta} = -\frac{v_{l\theta}}{v_{ll}} > 0$, and since $d^* > 0$, for $\theta \leq \theta^*$ and is continuous, there can be no value of $\theta > \theta^*$, for which $d^* = 0$. Finally, d^* is increasing in θ , since $\frac{\partial d^*}{\partial \theta} = \vec{\ell}'(\theta)$, for $\theta < \theta^*$. Finally, notice that continuity of d^* implies that θ^* must solve

$$\lambda = -v_l(\theta^*, \bar{\ell}(\theta^*)). \quad (61)$$

□

Proofs of results in Section 3

Proof of Proposition 2.

We show this proposition in three Lemmas. First, we show that for α_d sufficiently large, it is always the case that $\lambda m \geq v\left(\bar{\ell}^{-1}\left(m + A\left(m, \hat{d}\right)\right), A(0, \hat{d})\right)$. Next, we show that there is a unique equilibrium in monotone strategies. Finally, we show that that equilibrium is the unique equilibrium of this economy.

Lemma 4. *There is an $\bar{\alpha}_1 > 0$, such that for $\alpha_d > \bar{\alpha}_1$, $\lambda m \geq v\left(\bar{\ell}^{-1}\left(m + A\left(m, \hat{d}\right)\right), A(0, \hat{d})\right)$.*

Proof. Notice that an equilibrium value of \hat{d} always have to satisfy either

$$\hat{d} = \frac{m}{2} + \frac{1}{\alpha_d m} \log \left(\frac{p \Pr\{\theta < \theta_0\}}{\Pr\{\theta > \theta_f\} - p \Pr\{\theta > \theta_0\}} \right)$$

or

$$\hat{d} = \frac{m}{2} + \frac{1}{\alpha_d m} \log \left(\frac{p \Pr \{\theta < \theta_0\} - (1-p) \Pr \{\theta \in (\theta_f, \theta_1)\}}{\Pr \{\theta > \theta_f\} - p \Pr \{\theta > \theta_0\} - (1-p) \Pr \{\theta \in (\theta_f, \theta_1)\}} \right)$$

depending on whether we are in Case 1' with $\Theta_1 = [\theta_0, \infty)$ and $\theta_f = \tilde{\theta}$ or in Case 2' with $\Theta_1 = [\theta_0, \theta_f) \cup [\theta_1, \infty)$ and $\theta_f = \theta_a$. Thus, in both cases \hat{d} is a continuous function of α_d . Furthermore, $\lim_{\alpha_d \rightarrow \infty} \hat{d} = \frac{m}{2}$ in both cases. Since \hat{d} is continuous in α_d , this means that \hat{d} can be made arbitrarily close to $\frac{m}{2}$ for sufficiently high values of α_d and for any $\theta_f \in (\underline{\theta}, \bar{\theta})$. However, as $\hat{d} \rightarrow \frac{m}{2}$, $v \left(\bar{\ell}^{-1} \left(m + A \left(m, \hat{d} \right) \right), A \left(0, \hat{d} \right) \right) \rightarrow v \left(\bar{\ell}^{-1} (m), 1 \right) = 0$ for $\theta \in (\underline{\theta}, \bar{\theta})$ and, thus, $\lambda m \geq v \left(\bar{\ell}^{-1} \left(m + A \left(m, \hat{d} \right) \right), A \left(0, \hat{d} \right) \right)$. Therefore, there is a $\bar{\alpha}_1 > 0$, such that for $\alpha_d > \bar{\alpha}_1$. \square

Lemma 5. *There is an $\bar{\alpha}$, such that for $\alpha_d > \bar{\alpha}$, there is a unique monotone strategy equilibrium, in which \hat{d} , and θ_f are uniquely determined by conditions (16) and (17) and a bank pays a dividend iff $\theta \geq \theta_0$.*

Proof. From Lemma 2, we have that $\frac{\partial \theta_f}{\partial \hat{d}} > 0$. Hence, the expected net payoff from attacking $\hat{\pi}(d_i, \theta_f)$ is an increasing function of the strategic cutoff. Given that all other lenders follow a strategy \hat{d} denote $h(\hat{d})$ as the solution to

$$\hat{\pi} \left(h(\hat{d}), \theta_f \left(\hat{d} \right) \right) = 0. \quad (62)$$

By the properties of $\hat{\pi}$ and the implicit function theorem, $h(\hat{d})$ is increasing in \hat{d} . To show that there exists a unique equilibrium in monotone strategies, we show that $h(\hat{d})$ has a unique fixed point $\hat{d} = h(\hat{d})$. To show this, it is sufficient to show that $h' \left(\hat{d} \right) \in [0, 1)$, so that that h is a contraction mapping. Let f_p denote the pdf of the lenders' prior belief. Furthermore, suppose that $\alpha_d > \bar{\alpha}_1$. In that case, we are in Case 1'. Then, applying the implicit function theorem on equation 46 yields

$$h' \left(\hat{d} \right) = \frac{1}{\alpha_d m} \frac{1}{p \Pr \{\theta < \theta_0\} + 1 - \Pr \{\theta \leq \theta_f\} - p} f_p \left(\theta_f \right) \frac{\partial \theta_f}{\partial \hat{d}}. \quad (63)$$

We then have that $h' \left(\hat{d} \right) \geq 0 \Leftrightarrow p \Pr \{\theta < \theta_0\} + 1 - \Pr \{\theta \leq \theta_f\} - p > 0$. Assumption A2 ensures that this is the case.

To show that $h' \left(\hat{d} \right) < 1$, first note that

$$\begin{aligned}
\frac{\partial \theta_f}{\partial \hat{d}} &= -A_{\hat{d}} \frac{v_l}{v_\theta} \Big|_{\bar{\theta}} \\
&= -\sqrt{\alpha_d} \phi_d \left(\sqrt{\alpha_d} (\hat{d} - m) \right) \left(\bar{\ell}^{-1} \left(m + A \left(m, \hat{d} \right) \right) \right)',
\end{aligned}$$

so

$$h'(\hat{d}) < 1 \Leftrightarrow -\sqrt{\frac{1}{\alpha_d}} \frac{1}{m p \Pr\{\theta < \theta_0\} + 1 - \Pr\{\theta \leq \theta_f\} - p} \frac{f_p(\theta_f) \phi_d \left(\sqrt{\alpha_d} (\hat{d} - m) \right)}{\left(\bar{\ell}^{-1} \left(m + A \left(m, \hat{d} \right) \right) \right)'} < 1. \quad (64)$$

Noting that $\hat{d} \rightarrow \frac{m}{2}$ as $\alpha_d \rightarrow \infty$, it follows that $\phi_d \left(\sqrt{\alpha_d} (\hat{d} - m) \right) \rightarrow 0$ as $\alpha_d \rightarrow \infty$. Hence, there exists a $\bar{\alpha}_2$, such that for $\alpha_d > \bar{\alpha}_2$, the inequality holds. Then defining $\bar{\alpha} = \min \{ \bar{\alpha}_1, \bar{\alpha}_2 \}$, we can conclude that for $\alpha_d > \bar{\alpha}$, $h(\cdot)$ has a unique fixed point $\hat{d} = h(\hat{d})$. This is the unique equilibrium in monotone strategies. \square

Lemma 6. *There is an $\bar{\alpha}$, such that for $\alpha_d > \bar{\alpha}$, the unique monotone strategy equilibrium of this economy is also the unique equilibrium.*

Proof. We take $\bar{\alpha}$ to be the value from Lemma 5. If $\alpha_d > \bar{\alpha}$, we are always in Case 1' for all equilibrium values of \hat{d} and θ_f . Furthermore, (\hat{d}, θ_f) is the unique equilibrium in monotone strategies.

Let us first establish that there exists a pair $\underline{\hat{d}} < \bar{\hat{d}}$ such that attacking is strictly dominant for all $d_i \leq \underline{\hat{d}}$ and not attacking is strictly dominant for $d_i \geq \bar{\hat{d}}$. Consider $\underline{\hat{d}}$ first. The most pessimistic scenario an agent can consider is that $\theta_f = \underline{\theta}$. With this belief, the strategic threshold for an agent is pinned down by the condition

$$\Pr \left\{ \theta < \underline{\theta} | \underline{\hat{d}} \right\} = 1 - p. \quad (65)$$

Notice that in this case

$$\Pr \left\{ \theta < \underline{\theta} | \underline{\hat{d}} \right\} = \Pr \left\{ \theta < \underline{\theta} \right\} \frac{1}{1 + \left(LR(\underline{\hat{d}}) - 1 \right) \Pr \left\{ \Theta_1 \right\}}, \quad (66)$$

since $\Pr \left\{ \Theta_1 | \theta < \underline{\theta} \right\} = 0$. $\Pr \left\{ \theta < \underline{\theta} | \underline{\hat{d}} \right\}$ is a continuous and strictly decreasing function of $\underline{\hat{d}}$ with $\lim_{\underline{\hat{d}} \rightarrow -\infty} \Pr \left\{ \theta < \underline{\theta} | \underline{\hat{d}} \right\} = \frac{\Pr \left\{ \theta < \underline{\theta} \right\}}{\Pr \left\{ \theta \leq \theta_0 \right\}} > 1 - p$ and $\lim_{\underline{\hat{d}} \rightarrow \infty} \Pr \left\{ \theta < \underline{\theta} | \underline{\hat{d}} \right\} = 0$. Thus, by Bolzano's theorem there is a unique $\underline{\hat{d}}$ such that $\Pr \left\{ \theta < \underline{\theta} | \underline{\hat{d}} \right\} = 1 - p$. A similar argument, where the most optimistic belief is that $\theta_f = \bar{\theta}$, establishes $\bar{\hat{d}}$.

Next, let's set $\hat{d}_0 = \hat{d}$. It is strictly dominant to refuse to rollover if $d_i < \hat{d}_0$. But if all agents follow a strategy \hat{d}_0 , it is strictly dominant to attack for $d_i < h(\hat{d}_0)$. Similarly, set $\bar{d}_0 = \bar{d}$. If it is strictly dominant to not attack for $d_i < \bar{d}_0$. But if all agents follow a strategy \bar{d}_0 , it is strictly dominant to not attack for $d_i < h(\bar{d}_0)$.

Therefore, we can construct two monotone sequences, with $\{\hat{d}_n\}_{n=0}^{\infty}$ and $\{\bar{d}_n\}_{n=0}^{\infty}$ where $\hat{d}_{n+1} = h(\hat{d}_n)$ and $\bar{d}_{n+1} = h(\bar{d}_n)$. $\{\hat{d}_n\}_{n=0}^{\infty}$ is bounded above by a \hat{d}^* and $\{\bar{d}_n\}_{n=0}^{\infty}$ is bounded below by the same \hat{d}^* . Thus, by the discussion above, both these sequences converge to the same fixed point \hat{d}^* as $n \rightarrow \infty$. \square

Proof of Proposition 3.

Suppose that $\alpha_d > \bar{\alpha}$. In that case, θ_f is implicitly defined by $\bar{\ell}(\theta_f) = m + A(m, \hat{d})$ and with $\hat{d}(\theta_f; \alpha_d) = \frac{m}{2} + \frac{1}{\alpha_d m} \log \left(\frac{p \Pr\{\theta < \theta_0\}}{p \Pr\{\theta < \theta_0\} + \Pr\{\theta > \theta_f\} - p} \right)$. Since $A(m, \hat{d}) = \Phi \left(\sqrt{\alpha_d} (\hat{d} - m) \right)$, we can write $\bar{\ell}(\theta_f) = m + A(m, \hat{d}(\theta_f, \alpha_d); \alpha_d)$.

Then, by the implicit function theorem

$$\frac{\partial \theta_f}{\partial \alpha_d} = - \frac{-A_{\hat{d}} \frac{\partial \hat{d}}{\partial \alpha_d} - A_{\alpha}}{\bar{\ell}'(\theta_f) - A_{\hat{d}} \frac{\partial \hat{d}}{\partial \theta_f}} \quad (67)$$

We have that $A_{\hat{d}} = \phi \left(\sqrt{\alpha_d} (\hat{d} - m) \right) \sqrt{\alpha_d}$ and $A_{\alpha} = \phi \left(\sqrt{\alpha_d} (\hat{d} - m) \right) \frac{1}{\sqrt{\alpha_d}} (\hat{d} - m)$. Furthermore

$$\frac{\partial \hat{d}}{\partial \alpha_d} = - \frac{1}{\alpha_d^2 m} \log \left(\frac{p \Pr\{\theta < \theta_0\}}{\Pr\{\theta > \theta_f\} - p \Pr\{\theta > \theta_0\}} \right) \quad (68)$$

and

$$\frac{\partial \hat{d}}{\partial \theta_f} = - \frac{1}{\alpha_d m} \frac{1}{\Pr\{\theta > \theta_f\} - p \Pr\{\theta > \theta_0\}} \frac{\partial \Pr\{\theta > \theta_f\}}{\partial \theta_f} \quad (69)$$

Inserting the partial derivatives into equation 67 and rearranging gives

$$\frac{\partial \theta_f}{\partial \alpha_d} = \frac{-\phi(\cdot) \frac{1}{\alpha_d^{3/2}} \log \left(\frac{p \Pr\{\theta < \theta_0\}}{\Pr\{\theta > \theta_f\} - p \Pr\{\theta > \theta_0\}} \right) + \phi(\cdot) \frac{1}{2} \frac{1}{\sqrt{\alpha_d}} (\hat{d} - m)}{\ell'(\theta_f) + \phi(\cdot) \frac{1}{\sqrt{\alpha_d m} \Pr\{\theta > \theta_f\} - p \Pr\{\theta > \theta_0\}} \frac{\partial \Pr\{\theta > \theta_f\}}{\partial \theta_f}} \quad (70)$$

where $\phi(\cdot) = \phi\left(\sqrt{\alpha_d}(\hat{d} - m)\right)$. Notice that, for an α_d sufficiently large, the denominator is positive. In that case,

$$\frac{\partial \theta_f}{\partial \alpha_d} \leq 0 \Leftrightarrow -\frac{1}{\alpha_d} \log\left(\frac{p \Pr\{\theta < \theta_0\}}{\Pr\{\theta > \theta_f\} - p \Pr\{\theta > \theta_0\}}\right) + \frac{1}{2}(\hat{d} - m) \leq 0$$

Since $\hat{d} \rightarrow \frac{m}{2}$, this condition holds for an α_d sufficiently large. \square

Proof of Proposition 4.

First, notice that if $p < 1$, \hat{d} given by

$$\hat{d} = \frac{m}{2} + \frac{1}{\alpha_d m} \log\left(\frac{p \Pr\{\theta < \theta_0\}}{p \Pr\{\theta < \theta_0\} + \Pr\{\theta > \theta_f\} - p}\right). \quad (71)$$

is well-defined, if $\theta_f = \theta_0$. Furthermore as $\alpha_d \rightarrow \infty$, $\hat{d} \rightarrow \frac{m}{2}$ and we are in Case 1' by Lemma 4. Turning to the share of lenders that run in the limit, using $\hat{d} \rightarrow \frac{m}{2}$, we have that $A(0, \hat{d}) \rightarrow 1$ and $A(m, \hat{d}) \rightarrow 1$. Therefore, in the limit, θ_f satisfies

$$\bar{\ell}\left(\lim_{\alpha_d \rightarrow \infty} \theta_f\right) = m + \lim_{\alpha_d \rightarrow \infty} A(m, \hat{d}) = m, \quad (72)$$

which is precisely the condition that defines θ_0 . Finally, to verify the conjecture that the economy is in Case 1', notice that given $m < \frac{1}{2}$, at $\theta = \theta_0$, a bank cannot survive to a run of size 1, which will be the case if it were to set $d = 0$. Therefore,

$$\lambda m > v\left(\theta_0, A\left(0, \hat{d}\right)\right), \quad (73)$$

verifying the condition for the economy to be in Case 1'. \square

Proofs of results in Section 4

Proof of Lemma 1.

The bank-owner solves

$$W(\theta) = \max_{d \in [0, \bar{\ell}(\theta)]} \left\{ \lambda d + \mathbf{1}_{\{d + A(d, \hat{d}) \leq \bar{\ell}(\theta)\}} v\left(\theta, d + A(d, \hat{d})\right) \right\}.$$

Let $d_{nr}^*(\theta)$ and $\tilde{W}_{nr}(\theta)$ denote that optimal dividend policy and value function for a bank that faces no run. Also, let θ_{nr}^* denote that value of θ for which $d_{nr}^*(\theta) = \bar{\ell}(\theta)$ (see Lemma

1). Additionally, let us define $\mathcal{D}(\theta, \hat{d}) = \{d : d \geq 0, d + A(d, \hat{d}) \leq \bar{\ell}(\theta)\} \subset [0, \bar{\ell}(\theta)]$. Given the properties of $\bar{\ell}(\theta)$, $\mathcal{D}(\theta_1, \hat{d}) \subset \mathcal{D}(\theta_2, \hat{d})$, $\forall \theta_1 < \theta_2$. Since d_{\min} is the global minimizer of $d + A(d, \hat{d})$, it follows that

$$\mathcal{D}(\theta, \hat{d}) = \begin{cases} \emptyset & , \theta < \theta_0 \\ [d_{\min}, \bar{d}(\theta)] & , \theta \geq \theta_0 \end{cases}, \quad (74)$$

where $\bar{d}(\theta) > d_{\min}$ solves $\bar{d}(\theta) + A(\bar{d}(\theta), \hat{d}) = \bar{\ell}(\theta)$, and θ_0 solves

$$\bar{\ell}(\theta_0) = d_{\min} + A(d_{\min}, \hat{d}). \quad (75)$$

If $\mathcal{D}(\theta, \hat{d}) = \emptyset$, then the bank cannot meet the withdrawals of lenders. In that case it is optimal for the bank to set $d = \bar{\ell}(\theta)$ and $g = 0$, the bank-owner obtains $\tilde{W}(\theta) = \lambda \bar{\ell}(\theta)$, and the bank fails. If $\mathcal{D}(\theta, \hat{d}) \neq \emptyset$, then the bank can choose between:

- Setting $d = \bar{\ell}(\theta)$ and obtaining $\tilde{W}(\theta) = \lambda \bar{\ell}(\theta)$
- Solving

$$\max_{d \in [d_{\min}, \bar{d}(\theta)]} \lambda d + v(\theta, d + A(d, \hat{d})). \quad (76)$$

Taking the f.o.c. with respect to d , the optimal d^* satisfies

$$\lambda \leq -v_l(\theta, d^* + A(d^*, \hat{d})) (1 + A_d(d^*, \hat{d})) + \kappa_{\bar{l}}$$

where $\kappa_{\bar{l}}$ and d^* satisfy the complementary slackness condition $\kappa_{\bar{l}} (\bar{\ell}(\theta) - d^* - A(d^*, \hat{d})) = 0$.³⁴ Notice, however, that whenever, $d^* + A(d^*, \hat{d}) = \bar{\ell}(\theta)$, the bank is better off setting $d = \bar{\ell}(\theta)$ and $g = 0$, so one can disregard this case. Similarly, $d^* = d_{\min}$ only if $\bar{\ell}(\theta) = d_{\min}$ but in that case the bank is better off setting $d = \bar{\ell}(\theta)$ and $g = 0$ as well.

For a value of $d^* > d_{\min}$ that satisfies

$$\lambda + v_l(\theta, d^* + A(d^*, \hat{d})) (1 + A_d(d^*, \hat{d})) = 0, \quad (78)$$

³⁴It is straightforward to show that $d^* \leq d_{\min}$ also satisfies the second-order condition

$$v_{ll}(\theta, d^* + A) (1 + A_d) + v_l(\theta, d^* + A) A_{dd} < 0. \quad (77)$$

the bank owner compares

$$\lambda \bar{\ell}(\theta) \geq \lambda d^* + v\left(\theta, d^* + A(d^*, \hat{d})\right). \quad (79)$$

Whenever the left-hand side is higher than the right-hand side, the bank sets $d = \bar{\ell}(\theta)$ and $g = 0$. Otherwise, it sets $d = d^*$ and $g = A(d^*, \hat{d})$.

Next, define

$$\varphi(\theta) \equiv \lambda \bar{\ell}(\theta) - \lambda d^* - v\left(\theta, d^* + A\left(\theta, \hat{\theta}\right)\right), \quad (80)$$

for $\theta \geq \theta_0$, where d^* solves (78), and let $\theta_f \geq \theta_0$ solve

$$\varphi(\theta_f) = 0. \quad (81)$$

The lower and upper dominance region assumptions ensure that θ_f exists. To show this, note that at $\theta = \theta_0$,

$$\begin{aligned} \varphi(\theta_0) &= \lambda \bar{\ell}(\theta_0) - \lambda d_{\min} - v\left(\theta_0, \bar{\ell}(\theta_0)\right) \\ &= \lambda \bar{\ell}(\theta_0) - \lambda d_{\min} > 0. \end{aligned} \quad (82)$$

Also, for $\theta > \bar{\theta}$, the upper dominance region assumption implies that

$$\begin{aligned} \lambda \bar{\ell}(\theta) &< v(\theta, 1) \leq \lambda \tilde{d}_1 + v(\theta, 1) \\ &\leq \lambda d^* + v\left(\theta, d^* + A\left(d^*, \hat{d}\right)\right), \end{aligned} \quad (83)$$

where $\tilde{d}_1 \geq d_{\min}$ solves $\tilde{d}_1 + A(\tilde{d}_1, \hat{d}) = 1$. The last inequality in (83) comes from individual optimality and feasibility of choosing $d = \tilde{d}_1$ (revealed preference). Thus, $\varphi(\theta) < 0$ for $\theta > \underline{\theta}$.

Next, Assumption B1 ensures that θ_f is unique. To show this, first note that by the theorem of the maximum $\varphi(\theta)$ is continuous in θ . Next, notice that (78) also implies that d^* is continuous in θ . Differentiating $\varphi(\theta)$ and applying the envelope theorem, we get

$$\varphi'(\theta) = \lambda \bar{\ell}'(\theta) - v_\theta\left(\theta, d^* + A\left(d^*, \hat{d}\right)\right). \quad (84)$$

Using $\bar{\ell}' = -\frac{v_\theta}{v_l}$, we get

$$\begin{aligned}
\varphi'(\theta) &= -\lambda \frac{v_\theta(\theta_0, \bar{\ell}(\theta_0))}{v_l(\theta_0, \bar{\ell}(\theta_0))} - v_\theta\left(\theta, d^* + A(d^*, \hat{d})\right) \\
&= v_l\left(\theta, d^* + A(d^*, \hat{d})\right) \frac{v_\theta(\theta_0, \bar{\ell}(\theta_0))}{v_l(\theta_0, \bar{\ell}(\theta_0))} - v_\theta\left(\theta, d^* + A(d^*, \hat{d})\right) \\
&= v_l\left(\theta, d^* + A(d^*, \hat{d})\right) \left(\frac{v_\theta(\theta_0, \bar{\ell}(\theta_0))}{v_l(\theta_0, \bar{\ell}(\theta_0))} - \frac{v_\theta\left(\theta, d^* + A(d^*, \hat{d})\right)}{v_l\left(\theta, d^* + A(d^*, \hat{d})\right)} \right) < 0,
\end{aligned} \tag{85}$$

where the second line follows from the first order condition $\lambda = -v_l\left(\theta, d^* + A(d^*, \hat{d})\right)$, and the last inequality comes from Assumption B1.

Therefore, any bank with $\theta \leq \theta_f$ optimally sets $d = \bar{\ell}(\theta)$ and $g = 0$, while a bank with $\theta > \theta_f$ sets $d = d^*$ that solves (78) and $g = A(d^*, \hat{d})$.

Finally, d is discontinuous at $\theta = \theta_f$. with $d(\theta_f^-) > d(\theta_f^+)$. \square

Proof of Proposition 5.

Consider the condition for the marginal lender (90). First, let us multiply both the numerator and denominator on the left-hand side by $\sqrt{\alpha}$. Next, notice that both $\bar{\ell}(\theta)$ and $d^*(\theta)$ are differentiable, by the Implicit function theorem. Also, $\bar{\ell}(\theta)$ is strictly monotone on $[\underline{\theta}, \theta_f]$ and also $d^*(\theta)$ is strictly monotone for $\theta > \theta_f$ for sufficiently large α . To see the latter, note that by the Implicit function theorem, from (21) we have that

$$\begin{aligned}
d^{*'}(\theta) &= \frac{v_{l\theta}(\theta, d^* + A)}{-v_{ll}(\theta, d^* + A)(1 + A_d) - v_l \frac{A_{dd}}{1 + A_d}} \\
&= - \frac{v_{l\theta}}{\frac{v_l^2}{\lambda} \alpha^{3/2} (d^* - \hat{d}) \phi\left(\sqrt{\alpha} (d^* - \hat{d})\right) - \lambda \frac{v_{ll}}{v_l}},
\end{aligned}$$

where the second line uses condition (21) and the observation that $A_{dd} = \alpha (d^* - \hat{d}) A_d = -\alpha^{3/2} (d^* - \hat{d}) \phi\left(\sqrt{\alpha} (d^* - \hat{d})\right)$, and note that $\lim_{\alpha \rightarrow \infty} \alpha A_d = \lim_{\alpha \rightarrow \infty} \alpha^{3/2} \phi\left(\sqrt{\alpha} (d^* - \hat{d})\right) = 0$ for any $d^* > \hat{d}$. Therefore, we can use a change of variable to re-write the left-hand side of (90) as

$$\frac{\sqrt{\alpha} \phi(\hat{d}) (\underline{\theta} + K) + \int_0^{\bar{\ell}(\theta_f)} \frac{1}{\bar{\ell}'(\bar{\ell}^{-1}(x))} \sqrt{\alpha} \phi\left(\sqrt{\alpha} (x - \hat{d})\right) dx}{\int_{d^*(\theta_f)}^{d^*(K)} \frac{1}{d^{*'}(d^{*-1}(x))} \sqrt{\alpha} \phi\left(\sqrt{\alpha} (x - \hat{d})\right) dx}.$$

Using integration by parts, we further get³⁵

$$\begin{aligned}
& \frac{\sqrt{\alpha}\phi\left(\sqrt{\alpha}\hat{d}\right)\left(\underline{\theta}+K\right)+\frac{\Phi\left(\sqrt{\alpha}\left(\bar{\ell}\left(\theta_f\right)-\hat{d}\right)\right)}{\bar{\ell}'\left(\theta_f\right)}-\frac{\Phi\left(\sqrt{\alpha}\left(-\hat{d}\right)\right)}{\bar{\ell}'\left(\underline{\theta}\right)}-\int_0^{\bar{\ell}\left(\theta_f\right)}\Phi\left(\sqrt{\alpha}\left(x-\hat{d}\right)\right)\frac{\partial}{\partial x}\left(\frac{1}{\bar{\ell}'\left(\bar{\ell}^{-1}\left(x\right)\right)}\right)dx}{\frac{\Phi\left(\sqrt{\alpha}\left(d^*\left(K\right)-\hat{d}\right)\right)}{d^{*'}\left(K\right)}-\frac{\Phi\left(\sqrt{\alpha}\left(d^*\left(\theta_f\right)-\hat{d}\right)\right)}{d^{*'}\left(\theta_f\right)}-\int_{d^*\left(\theta_f\right)}^{d^*\left(K\right)}\Phi\left(\sqrt{\alpha}\left(x-\hat{d}\right)\right)\frac{\partial}{\partial x}\left(\frac{1}{d^{*'}\left(d^{*-1}\left(x\right)\right)}\right)dx} \\
&= \frac{\sqrt{\alpha}\phi\left(\sqrt{\alpha}\hat{d}\right)\left(\underline{\theta}+K\right)+\frac{1-A\left(\bar{\ell}\left(\theta_f\right),\hat{d}\right)}{\bar{\ell}'\left(\theta_f\right)}-\frac{1-A\left(0,\hat{d}\right)}{\bar{\ell}'\left(\underline{\theta}\right)}-\int_0^{\bar{\ell}\left(\theta_f\right)}\left(1-A\left(x,\hat{d}\right)\right)\frac{\partial}{\partial x}\left(\frac{1}{\bar{\ell}'\left(\bar{\ell}^{-1}\left(x\right)\right)}\right)dx}{\frac{1-A\left(d^*\left(K\right),\hat{d}\right)}{d^{*'}\left(K\right)}-\frac{1-A\left(d^*\left(\theta_f\right),\hat{d}\right)}{d^{*'}\left(\theta_f\right)}-\int_{d^*\left(\theta_f\right)}^{d^*\left(K\right)}\left(1-A\left(x,\hat{d}\right)\right)\frac{\partial}{\partial x}\left(\frac{1}{d^{*'}\left(d^{*-1}\left(x\right)\right)}\right)dx} \\
&= \frac{\sqrt{\alpha}\phi\left(\sqrt{\alpha}\hat{d}\right)\left(\underline{\theta}+K\right)+\frac{1-A\left(\bar{\ell}\left(\theta_f\right),\hat{d}\right)}{\bar{\ell}'\left(\theta_f\right)}-\frac{1-A\left(0,\hat{d}\right)}{\bar{\ell}'\left(\underline{\theta}\right)}-\int_0^{\bar{\ell}\left(\theta_f\right)}\left(1-A\left(x,\hat{d}\right)\right)\frac{\partial}{\partial x}\left(\frac{1}{\bar{\ell}'\left(\bar{\ell}^{-1}\left(x\right)\right)}\right)dx}{\frac{1-A\left(d^*\left(K\right),\hat{d}\right)}{d^{*'}\left(K\right)}-\frac{1-A\left(d^*\left(\theta_f\right),\hat{d}\right)}{d^{*'}\left(\theta_f\right)}-\left(\frac{1}{d^{*'}\left(K\right)}-\frac{1}{d^{*'}\left(\theta_f\right)}\right)+\int_{d^*\left(\theta_f\right)}^{d^*\left(K\right)}A\left(x,\hat{d}\right)\frac{\partial}{\partial x}\left(\frac{1}{d^{*'}\left(d^{*-1}\left(x\right)\right)}\right)dx} \\
&= \frac{\sqrt{\alpha}\phi\left(\sqrt{\alpha}\hat{d}\right)\left(\underline{\theta}+K\right)+\frac{1-A\left(\bar{\ell}\left(\theta_f\right),\hat{d}\right)}{\bar{\ell}'\left(\theta_f\right)}-\frac{1-A\left(0,\hat{d}\right)}{\bar{\ell}'\left(\underline{\theta}\right)}-\int_0^{\bar{\ell}\left(\theta_f\right)}\left(1-A\left(x,\hat{d}\right)\right)\frac{\partial}{\partial x}\left(\frac{1}{\bar{\ell}'\left(\bar{\ell}^{-1}\left(x\right)\right)}\right)dx}{\frac{A\left(d^*\left(K\right),\hat{d}\right)}{d^{*'}\left(K\right)}+\frac{A\left(d^*\left(\theta_f\right),\hat{d}\right)}{d^{*'}\left(\theta_f\right)}+\int_{d^*\left(\theta_f\right)}^{d^*\left(K\right)}A\left(x,\hat{d}\right)\frac{\partial}{\partial x}\left(\frac{1}{d^{*'}\left(d^{*-1}\left(x\right)\right)}\right)dx},
\end{aligned}$$

where we use $\Phi\left(\sqrt{\alpha}\left(x-\hat{d}\right)\right)=1-\Phi\left(\sqrt{\alpha}\left(\hat{d}-x\right)\right)=1-A\left(x,\hat{d}\right)$ to substitute for the fraction of lenders attacking for a given dividend level x . Notice that from the bank's problem, $\bar{\ell}\left(\theta_f\right)\geq d^*\left(\theta_f\right)>d_{\min}>\hat{d}$, for sufficiently large α . Also, the integrals in the numerator and denominator exist for any α , and in the limit, as $\alpha\rightarrow\infty$, and lenders are

perfectly coordinated, $A\left(x,\hat{d}\right)=\begin{cases} 0 & , x>\hat{d} \\ [0,1] & , x=\hat{d}. \text{ Thus,} \\ 1 & , x<\hat{d} \end{cases}$

$$\lim_{\alpha\rightarrow\infty}\frac{\int_{-K}^{\theta_f}\phi\left(\sqrt{\alpha}\left(\bar{\ell}\left(\theta\right)-\hat{d}\right)\right)d\theta}{\int_{\theta_f}^K\phi\left(\sqrt{\alpha}\left(d^*\left(\theta\right)-\hat{d}\right)\right)d\theta}=\frac{\lim_{\alpha\rightarrow\infty}\frac{1}{\bar{\ell}'\left(\bar{\ell}^{-1}\left(\hat{d}\right)\right)}}{\lim_{\alpha\rightarrow\infty}\frac{A\left(d^*\left(\theta_f\right),\hat{d}\right)}{d^{*'}\left(\theta_f\right)}}=\frac{1-p}{p}$$

or

$$\lim_{\alpha\rightarrow\infty}\frac{A\left(d^*\left(\theta_f\right),\hat{d}\right)}{d^{*'}\left(\theta_f\right)}=\frac{p}{1-p}\lim_{\alpha\rightarrow\infty}\frac{1}{\bar{\ell}'\left(\bar{\ell}^{-1}\left(\hat{d}\right)\right)}.$$

Since $\lim_{\alpha\rightarrow\infty}A\left(d^*\left(\theta_f\right),\hat{d}\right)=0$, it follows that $d^{*'}\left(\theta_f\right)\rightarrow 0$, as well.

Next, consider the bank's problem. For $\theta>\theta_f$, we can combine conditions (21) and (11)

³⁵We implicitly assume that v is differentiable of sufficient order for the expression below.

and write condition (21) as

$$\frac{v_l(\theta, d_{nr}(\theta))}{v_l(\theta, d^*(\theta) + A(d^*(\theta), \hat{d}))} = 1 + A_d(d^*(\theta), \hat{d}). \quad (86)$$

In the limit, as $\alpha \rightarrow \infty$ and lenders become perfectly coordinated, so $A_d(d^*, \hat{d}) \rightarrow 0$ for $d^* > \hat{d}$. Therefore, from (86) we get that

$$\frac{v_l(\theta, d_{nr}(\theta))}{v_l(\theta, d^*(\theta))} \rightarrow 1,$$

and so $d^*(\theta) \rightarrow d_{nr}(\theta)$. In addition, in the limit, the marginal bank that is indifferent between failing and surviving experiences no run in equilibrium, since $\lim_{\alpha \rightarrow \infty} A(d^*(\theta_f), \hat{d}) = 0$. Therefore, condition (20) implies that

$$\lambda \bar{\ell}(\theta_f) = \lambda d^*(\theta_f) + v(\theta_f, d^*(\theta_f)).$$

However, by the definition of θ^* in (10), this in turn implies that $\theta_f \rightarrow \theta^*$ and $d^*(\theta_f) \rightarrow \bar{\ell}(\theta^*)$. Finally, note that

$$d^{*'}(\theta_f) = -\frac{v_{l\theta}}{\frac{v_l^2}{\lambda} \alpha^{3/2} (d^*(\theta_f) - \hat{d}) \phi(\sqrt{\alpha} (d^*(\theta_f) - \hat{d})) - \lambda \frac{v_{ll}}{v_l}} \rightarrow 0$$

implies that $\alpha^{3/2} (d^*(\theta_f) - \hat{d}) \phi(\sqrt{\alpha} (d^*(\theta_f) - \hat{d})) \rightarrow \infty$, which can only be the case if $\hat{d} \rightarrow d^*(\theta_f)$. \square

Proofs of results in Section 5

Proof of Proposition 6.

Suppose that $d = 0$, $\forall \theta$ and instead lenders obtain dispersed private signals about θ as in Section 3.1. Let us denote the failure threshold in that case by $\tilde{\theta}_f$. In that case, Proposition 8 implies that $\tilde{\theta}_f$ solves

$$\bar{\ell}(\tilde{\theta}_f) = p.$$

In contrast with arbitrarily precise dividend signals, Proposition 5 implies that the failure threshold $\theta_f \rightarrow \theta^*$. Thus whenever $\bar{\ell}(\theta^*) < p$, it follows that $\theta^* < \tilde{\theta}_f$. \square

Proof of Proposition 7.

A proportional tax on dividends $\tau > 0$ decreases the effective value of λ to $\tilde{\lambda} = (1 - \tau) \lambda$. With arbitrarily precise dividend signals, Proposition 5 implies that the failure threshold $\theta_f \rightarrow \theta^*$, where θ^* solves $\lambda = -v_l(\theta^*, \bar{\ell}(\theta^*))$. With a proportional tax, τ , $\theta^*(\tau)$ solves

$$\tilde{\lambda} = (1 - \tau) \lambda = -v_l(\theta^*(\tau), \bar{\ell}(\theta^*(\tau))).$$

By the implicit function theorem, we have

$$\frac{\partial \theta^*}{\partial \tau} = -\frac{\partial \theta^*}{\partial \lambda} = \frac{1}{v_{l\theta} + v_{ll} \frac{\partial \bar{\ell}}{\partial \theta}} = \frac{1}{v_l (v_{l\theta} v_l - v_{ll} v_\theta)}.$$

Note that by Assumption B1,

$$\frac{\partial}{\partial \ell} \left(\frac{v_\theta}{v_l} \right) = \frac{v_{l\theta} v_l - v_{ll} v_\theta}{v_l^2} > 0,$$

it follows that $\frac{\partial \theta^*}{\partial \tau} < 0$. □

Appendix C: A continuum of dividend levels – details

This part of the appendix contains the details of the analysis of the equilibrium when banks can issue any feasible dividend level given their type. First, we derive conditions under which a lender i 's posterior belief about the bank failing, $\Pr(\theta < \theta_f | d_i)$, is decreasing in d_i . Let us define two probability densities,

$$\psi_{N,i}(x) = \frac{\phi(\sqrt{\alpha}(\bar{\ell}(x) - d_i))}{\int_{-K}^{\theta_f} \phi(\sqrt{\alpha}(\bar{\ell}(z) - d_i)) dz}, \text{ for } x \in [-K, \theta_f], \quad (87)$$

and

$$\psi_{D,i}(x) = \frac{\phi(\sqrt{\alpha}(d^*(x) - d_i))}{\int_{\theta_f}^K \phi(\sqrt{\alpha}(d^*(z) - d_i)) dz}, \text{ for } x \in [\theta_f, K]. \quad (88)$$

with corresponding cdf given by $\Psi_{N,i}(x)$ and $\Psi_{D,i}(x)$, respectively. Analogously, we define the expectation operators with respect to the two densities by $E_{N,i}[\cdot]$ (resp., $E_{D,i}[\cdot]$). The following lemma characterizes the lenders' inference based on their signals about d .

Lemma 7. *The posterior belief of a lender observing signal d_i , $\Pr_i\{\theta < \theta_f | d_i\}$, is strictly decreasing in d_i iff*

$$E_{N,i}[\bar{\ell}(\theta)] < E_{D,i}[d^*(\theta)]. \quad (89)$$

Proof. It will be useful to work with the posterior odds that the bank fails, i.e.

$$\begin{aligned} h(d_i, \theta_f) &\equiv \frac{\Pr\{\theta < \theta_f | d_i\}}{\Pr\{\theta > \theta_f | d_i\}} = \frac{\int_{-K}^{\theta_f} f(d_i | \theta) f(\theta | \theta < \theta_f) d\theta \Pr\{\theta < \theta_f\}}{\int_{-K}^{\theta_f} f(d_i | \theta) f(\theta | \theta > \theta_f) d\theta \Pr\{\theta > \theta_f\}} \\ &= \frac{\int_{-K}^{\theta_f} \phi(\sqrt{\alpha}(\bar{\ell}(\theta) - d_i)) d\theta}{\int_{\theta_f}^K \phi(\sqrt{\alpha}(d^*(\theta) - d_i)) d\theta}, \end{aligned}$$

where the last line uses the symmetry of the normal pdf around the mean and an improper prior over θ . Let

$$N(d_i, \theta_f) \equiv \int_{-K}^{\theta_f} \sqrt{\alpha} \phi(\sqrt{\alpha}(\bar{\ell}(\theta) - d_i)) d\theta,$$

and

$$D(d_i, \theta_f) \equiv \int_{\theta_f}^K \sqrt{\alpha} \phi(\sqrt{\alpha}(d^*(\theta) - d_i)) d\theta.$$

Then

$$h(d_i, \theta_f) = \frac{N(d_i, \theta_f)}{D(d_i, \theta_f)},$$

and so

$$\log(h(d_i, \theta_f)) = \log(N(d_i, \theta_f)) - \log(D(d_i, \theta_f)).$$

Therefore, $h(d_i, \theta_f)$ is decreasing in d_i , iff $\log(h(d_i, \theta_f))$ is decreasing in d_i , which will be the case iff

$$\frac{N_{d_i}}{N} < \frac{D_{d_i}}{D}.$$

Note that

$$\begin{aligned} \frac{N_{d_i}}{N} &= \frac{\int_{-K}^{\theta_f} \phi'(\sqrt{\alpha}(\bar{\ell}(\theta) - d_i)) d\theta}{\int_{-K}^{\theta_f} \phi(\sqrt{\alpha}(\bar{\ell}(\theta) - d_i)) d\theta} \\ &= \int_{-K}^{\theta_f} \alpha(\bar{\ell}(\theta) - d_i) \frac{\phi(\sqrt{\alpha}(\bar{\ell}(\theta) - d_i))}{\int_{-K}^{\theta_f} \phi(\sqrt{\alpha}(\bar{\ell}(\theta) - d_i)) dz} d\theta \\ &= \alpha \{E_{N,i}[\bar{\ell}(\theta)] - d_i\}. \end{aligned}$$

Similarly,

$$\frac{D_x}{D} = \alpha \{E_{D,i}[d^*(\theta)] - d_i\}$$

Therefore, the comparison simplifies to

$$\alpha \{E_{N,i}[\bar{\ell}(\theta)] - d_i\} < \alpha \{E_{D,i}[d^*(\theta)] - d_i\}$$

or

$$E_{N,i} [\bar{\ell}(\theta)] < E_{D,i} [d^*(\theta)].$$

□

Intuitively, Lemma 7 states that if a lender with signal d_i expects a lower dividend payment from a bank that fails compared to a bank that survives (where expectations are taken with respect to the densities ψ_N and ψ_D , respectively), then a lender with a marginally higher signal is more optimistic about the bank surviving. Put differently, whenever condition (89) is satisfied for some d_i , observing a marginally higher d_i constitutes good news about the bank surviving.

Notice, however, that by the lower dominance assumption, $\bar{\ell}(\theta) = 0$ for $\theta \leq \underline{\theta}$. Therefore, we can re-write $E_{N,i} [\bar{\ell}(\theta)]$ as

$$\begin{aligned} E_{N,i} [\bar{\ell}(\theta)] &= (1 - \Psi_{N,i}(\underline{\theta})) E_{N,i} [\bar{\ell}(\theta) | \theta \in [\underline{\theta}, \theta_f]], \\ &\leq (1 - \Psi_{N,i}(\underline{\theta})) \bar{\ell}(\theta_f) \leq 1 - \Psi_{N,i}(\underline{\theta}) \end{aligned}$$

where the first inequality follows from the strict monotonicity of $\bar{\ell}(\theta)$ and the second inequality follows from the observation that $\theta_f \leq \bar{\theta}$ and the upper dominance region assumption. On the other hand, by the properties of $d^*(\theta)$, $E_{D,i} [d^*(\theta)] > d_{\min}$. However, as we show in the proof of Lemma 8 below, for any d_i , $\Psi_{N,i}(\underline{\theta})$ can be made arbitrarily close to 1 for sufficiently large K . Intuitively, whenever K is large, the conditional probability that a failing bank has a fundamental below the lower dominance threshold and issues a dividend of 0 given that it fails is large. Therefore, observing a higher value of d_i increases the likelihood that the lender is facing a failing bank with fundamental above the lower dominance threshold only slightly. In contrast, it increases substantially the likelihood that it is a surviving bank. Overall, this implies that a higher value of d_i is good news about the bank being above the failure threshold.

We use this observation to characterize the lenders' actions below.

Lemma 8. *Suppose that banks with $\theta < \theta_f$ fail, where $\theta_f \leq \bar{\theta}$ is given in (20) and that banks follow the dividend policy given in (22). There exists a \bar{K} such that for $K > \bar{K}$, there is a lender with signal $d_i = \hat{d}$, where \hat{d} is uniquely determined by*

$$\frac{\int_{-K}^{\theta_f} \phi \left(\sqrt{\alpha} \left(\bar{\ell}(\theta) - \hat{d} \right) \right) d\theta}{\int_{\theta_f}^K \phi \left(\sqrt{\alpha} \left(d^*(\theta) - \hat{d} \right) \right) d\theta} = \frac{1-p}{p} \quad (90)$$

who is indifferent between running and rolling over, where $\phi(\cdot)$ denotes the standard normal

p.d.f. Furthermore, a lender observing $d_i < \hat{d}$ is strictly better off from running, while a lender observing $d_i > \hat{d}$ is strictly worse off from running.

Proof. A lender with signal \hat{d} is indifferent between running and rolling over whenever

$$\Pr \left\{ \theta < \theta_f | \hat{d} \right\} = 1 - p,$$

or equivalently,

$$\frac{\Pr \left\{ \theta < \theta_f | \hat{d} \right\}}{\Pr \left\{ \theta > \theta_f | \hat{d} \right\}} = \frac{1 - p}{p}.$$

Next, by the discussion immediately after Lemma 7, we have that

$$E_{N,i} [\bar{\ell}(\theta)] \leq 1 - \Psi_{N,i}(\theta).$$

However, observe that

$$\begin{aligned} \Psi_{N,i}(\theta) &= \frac{\int_{-K}^{\theta} \phi(\sqrt{\alpha}(\bar{\ell}(z) - d_i)) dz}{\int_{-K}^{\theta_f} \phi(\sqrt{\alpha}(\bar{\ell}(z) - d_i)) dz} \\ &= \frac{\phi(\sqrt{\alpha}(d_i))(\theta + K)}{\phi(\sqrt{\alpha}(d_i))(\theta + K) + \int_{\theta}^{\theta_f} \phi(\sqrt{\alpha}(\bar{\ell}(z) - d_i)) dz} \\ &= \frac{1}{1 + \frac{1}{\theta + K} \int_{\theta}^{\theta_f} \frac{\phi(\sqrt{\alpha}(\bar{\ell}(z) - d_i))}{\phi(\sqrt{\alpha}d_i)} dz} \\ &\geq \frac{1}{1 + \frac{\theta_f - \theta}{\theta + K} \max_{z \in [\theta, \theta_f]} \frac{\phi(\sqrt{\alpha}(\bar{\ell}(z) - d_i))}{\phi(\sqrt{\alpha}d_i)}} \end{aligned}$$

Therefore, for sufficiently larger values of K one can ensure that $E_{N,i}[\bar{\ell}(\theta)] < d_{\min} < E_{D,i}[d^*(\theta)]$ for any d_i . Therefore, by Lemma 7, $\Pr\{\theta < \theta_f | d_i\}$ is monotone decreasing in d_i , and so is $\frac{\Pr\{\theta < \theta_f | d_i\}}{\Pr\{\theta > \theta_f | d_i\}}$. Also, clearly, $\frac{\Pr\{\theta < \theta_f | d_i\}}{\Pr\{\theta > \theta_f | d_i\}}$ can be made arbitrarily large (arbitrarily close to 0) for sufficiently small (large) d_i . Therefore, by the intermediate value theorem, there exists a unique marginal lender with signal \hat{d} that satisfies (90). By the strict monotonicity of $\frac{\Pr\{\theta < \theta_f | d_i\}}{\Pr\{\theta > \theta_f | d_i\}}$, for any lender with $d_i < \hat{d}$, $\frac{\Pr\{\theta < \theta_f | d_i\}}{\Pr\{\theta > \theta_f | d_i\}} < \frac{1-p}{p}$, so that lender is strictly better off attacking. Similarly, any lender with $d_i > \hat{d}$ is strictly better off not attacking. \square

Next, we derive a condition under which in any monotone strategy equilibrium of this economy, the cutoff $\hat{d} \in (0, 1)$. To this end, we first characterize the optimal dividend policy of a bank that faces a run of $A = 1$ irrespective of its fundamentals θ .

Lemma 9. Consider a bank that does face a run by all lenders, i.e. $A = 1$, suppose that $\lambda \geq -v_l(\underline{\theta}, 0)$, and let $d_r(\theta)$ denote the bank's optimal dividend policy. Then banks with $\theta < \bar{\theta}$ choose $d_r(\theta) = \bar{l}(\theta)$, while banks with $\theta \geq \bar{\theta}$ choose $d_r(\theta) = d^*$, where d^* solves the first-order condition

$$(\lambda + v_l(\theta, d^* + 1)) d^* = 0. \quad (91)$$

Proof. First, notice that by the multiplicity region assumption, if $A = 1$, then any bank with $\theta < \bar{\theta}$ knows that it will fail for sure, so it is optimal for it to set $d_r(\theta) = \bar{l}(\theta)$. Next, for $\theta \geq \bar{\theta}$, the bank is better off surviving, given the upper dominance region assumption. In that case, it solves

$$\tilde{W}(\theta) = \max_{d \in [0, \bar{l}(\theta)]} \{\lambda d + v(\theta, d + 1)\}.$$

Taking the f.o.c. with respect to d , the optimal d^* solves

$$\lambda \leq -v_l(\theta, d^*).$$

Therefore, d^* satisfies condition (91). □

Next, let us define \hat{d}_{\max} as the unique solution to

$$\frac{\int_{-K}^{\bar{\theta}} \phi\left(\sqrt{\alpha}\left(\bar{l}(\theta) - \hat{d}_{\max}\right)\right) d\theta}{\int_{\bar{\theta}}^K \phi\left(\sqrt{\alpha}\left(d_r(\theta) - \hat{d}_{\max}\right)\right) d\theta} = \frac{1-p}{p}. \quad (92)$$

By Lemma 7 and the discussion after it, it follows that for any α , there is a sufficiently large K , such that the left-hand side of (92) is decreasing in \hat{d}_{\max} and so (83) can have at most one solution. Furthermore, the left-hand side of that expression can be made arbitrarily large (arbitrarily close to 0) for sufficiently small (large) values of \hat{d}_{\max} and so there exists a solution. We can now state condition B3 which ensures that in any monotone equilibrium, $\hat{d} < 1$.

Assumption B3. $\hat{d}_{\max} < 1$.

Notice that \hat{d}_{\max} is the signal of a marginal lender who is indifferent between running and rolling over if all other lenders run irrespective of their signal, and hence banks suffer a run of $A = 1$ irrespective of θ . Therefore, this is the lender cutoff in the most pessimistic possible case, when other lenders run irrespective of their signals and only banks in the upper dominance region survive. As we show in the proof of Proposition 9 below, this condition then ensures that in any monotone strategy equilibrium, $\hat{d} < \hat{d}_{\max} < 1$.

Similarly, let \hat{d}_{\min} be the unique solution to

$$\frac{\int_{-K}^{\theta^*} \phi \left(\sqrt{\alpha} \left(\bar{\ell}(\theta) - \hat{d}_{\min} \right) \right) d\theta}{\int_{\theta^*}^K \phi \left(\sqrt{\alpha} \left(d_{nr}(\theta) - \hat{d}_{\min} \right) \right) d\theta} = \frac{1-p}{p}, \quad (93)$$

where θ^* and $d_{nr}(\theta)$ were defined in Lemma 1 that examines optimal bank behavior in the case of no run. As with the case of \hat{d}_{\max} it is straightforward to show that for sufficiently large K , \hat{d}_{\min} is unique. Condition B4 then is analogous to condition B3:

Assumption B4. $\hat{d}_{\min} > 0$.

Therefore, \hat{d}_{\min} is the signal of a marginal lender who is indifferent between running and rolling over if all other lenders roll over irrespective of their signal and hence the bank experiences no run for any θ . Therefore, this is the lender cutoff in the most optimistic possible case, when other lenders do not run irrespective of their signal and only banks with $\theta < \theta^*$ fail. As we show in the proof of Proposition 9 below, this condition then ensures that in any monotone strategy equilibrium, $\hat{d} > \hat{d}_{\min} > 0$.

We can combine the results from Lemmas 1 and 8 to characterize equilibria in monotone strategies for this economy. An equilibrium in monotone strategies consists of a lender cutoff \hat{d} , bank cutoff θ_f and a bank dividend policy $d(\theta)$.

Proposition 9. *Consider equilibria of this economy, in which lenders follow a monotone strategy with cutoff at \hat{d} . In those equilibria banks fail according to a cutoff θ_f , and \hat{d} and θ_f jointly satisfy conditions (90) and (20), where the banks follow a dividend policy $d(\theta)$ given by equation (22). Furthermore, if θ_f and \hat{d} are unique, and assumptions B3 and B4 hold, then the unique monotone strategy equilibrium is also the unique equilibrium of this economy.*

The first part of the proposition follows directly from the partial characterization results in Lemmas 1 and 8. To show the second part, let \hat{d}^* and θ_f^* denote the unique solutions to (90) and (20). We will use the notation $\theta_f(x)$, $x \in (0, 1)$, for the bank failure threshold given that lenders use a monotone strategy with cutoff at x and similarly, $\hat{d}(x) = \hat{d}(\theta_f(x))$, $x \in (0, 1)$, for the signal of a marginal agent given that lenders are using a monotone strategy with cutoff at x , and so, the bank failure cutoff is $\theta_f(x)$. First of all, applying the implicit function theorem and the envelope theorem on equation (20), we get

$$\frac{\partial \theta_f}{\partial \hat{d}} = \frac{v_l A_d}{v_\theta \left(\frac{v_l + \lambda}{v_l} \right)}.$$

Furthermore, substituting for

$$\frac{v_l + \lambda}{v_l} = -A_d$$

from equation (21), we get

$$\frac{\partial \theta_f}{\partial \hat{d}} = -\frac{v_l}{v_\theta} > 0.$$

Similarly, from equation (90), and given that $\Pr \{\theta < \theta_f | d_i\}$ is strictly decreasing in d_i by the discussion preceding Lemma 8, we have that $\frac{\partial \hat{d}}{\partial \theta_f} > 0$, and so,

$$\frac{\partial \hat{d}}{\partial x} = \frac{\partial \hat{d}}{\partial \theta_f} \frac{\partial \theta_f}{\partial x} > 0. \quad (94)$$

Next, let us define recursively two sequences, $\{\underline{\delta}_n\}_{n=0}^\infty$ and $\{\bar{\delta}_n\}_{n=0}^\infty$, in the following way. Let $\bar{\delta}_0 = \infty$, $\bar{\delta}_1 = \hat{d}_{\max}$, where \hat{d}_{\max} was defined in (92) above, and $\bar{\delta}_n = \hat{d}(\bar{\delta}_{n-1})$, for $n \geq 2$. Given Assumption B3, one can define $\bar{\delta}_2 = \hat{d}(\hat{d}_{\max})$ in this way. Similarly, let $\underline{\delta}_0 = -\infty$, $\underline{\delta}_1 = \hat{d}_{\min}$, where \hat{d}_{\min} was defined in (93), and $\underline{\delta}_n = \hat{d}(\underline{\delta}_{n-1})$, for $n \geq 2$. Given Assumption B4, one can define $\underline{\delta}_2 = \hat{d}(\hat{d}_{\min})$ in this way. By condition (94), it follows that $\{\underline{\delta}_n\}_{n=0}^\infty$ is a strictly increasing sequence. Furthermore, it is bounded above by \hat{d}^* . Therefore, $\{\underline{\delta}_n\}_{n=0}^\infty$ converges. Let us denote the limiting value of that sequence by $\underline{\delta}_\infty$. Since $\hat{d}^* = \hat{d}(\hat{d}^*)$ and $\hat{d}(x)$ is continuous in x , it follows that $\underline{\delta}_\infty = \hat{d}^*$. Similarly, $\{\bar{\delta}_n\}_{n=0}^\infty$ is a strictly decreasing sequence bounded below by \hat{d}^* and it converges to $\bar{\delta}_\infty = \hat{d}^*$. In the case when \hat{d} and θ^f are not uniquely determined, let \hat{d}_L and \hat{d}_H denote the smallest and largest values of \hat{d} . In that case, $\underline{\delta}_\infty$ converges to \hat{d}_L and $\bar{\delta}_\infty$ converges to \hat{d}_H . In the latter case, $\bar{\delta}_n \leq \hat{d}_{\max} < 1$, $n \geq 1$, and so $\hat{d}_H < 1$. Thus Assumption B3 implies that $\hat{d} < 1$ in any monotone strategy equilibrium of this economy. Similarly, Assumption B4 implies that $\hat{d} > 0$. \square

Details for the numerical example in Section 4

In Section 4 we show the results from a numerical example to illustrate the equilibrium dividend payments for banks when the precision of lender signals is large but finite. For this numerical example we assume that $v(\theta, l) = \theta \ln(\theta - l)$ for $1 \leq \theta < 2$, $v(\theta, l) = 0$, for $\theta < 1$, and $v(\theta, l) = \theta \ln(\theta - l) + Z$, $Z > 0$, for $\theta \geq 2$.³⁶ Therefore, the lower-dominance region is $\theta \leq \underline{\theta} = 1$, while the upper dominance region (provided Z is sufficiently large) is $\theta \geq \bar{\theta} = 2$.

³⁶This payoff assumption violates the condition that v is continuously differentiable everywhere. However, it is a convenient way to ensure the existence of an upper dominance region. Also, all of the analytical results hold in this case as well.

Note also that $d_{nr}(\theta) = \begin{cases} \theta - 1 & , \theta \leq \lambda \\ (1 - \frac{1}{\lambda})\theta & , \theta > \lambda \end{cases}$, so the bank's dividend policy is piece-wise linear with a kink at $\theta = \lambda$ and the bank liquidates all assets at $\theta^* = \lambda$. We solve for the equilibrium in two cases, a low precision case in which $\sigma = 0.05$ and a high-precision case with $\sigma = 0.001$. The other parameters for this example are $p = 0.5$, and $\lambda = 1.1$

Appendix D: Data

The following table contains an overview over the data we use. All bank data are from the Federal Reserves Y-9C reports and are at the bank-holding company level. All firm data used when comparing across industries are from Compustat. For Compustat firms, we compute dividends paid from CRSP.

Variable	Source	Definition	Variable name in s
Repo	Federal FR Y-9C Reports	Securities sold under agreements to repurchase	BHCKB995
Dividends (bank)	Federal FR Y-9C Reports	Cash dividends declared on common stock	BHCK4460
Liquid assets	Federal FR Y-9C Reports	Cash and cash-like instruments + treasuries	Sum of several varia
Cash	Federal FR Y-9C Reports	Noninterest-bearing balances and currency and coin	BHCK0081
Cash-like instrument	Federal FR Y-9C Reports	Interest-bearing balances in U.S offices	BHCK0395
Treasuries	Federal FR Y-9C Reports	US Treasurers, both market and amortized value.	BHCK0211,BHCK0213,BHCK1
Total Assets	Federal FR Y-9C Reports	Total Assets	BHCP2170
Total Assets	Compustat	Assets - total	at
Current liabilities	Compustat	Current liabilities - total	lct
Liabilities	Compustat	Liabilities - Total	lt
Dividends	CRSP	Dividend per share	divamt
Total number of shares	CRSP	Total number of outstanding shares	shrout

Table 1: Variables and definitions - bank and firm data.