Foreseen Risks

Abstract

Financial crises tend to follow rapid credit expansions. Causality, however, is far from obvious. We show how this pattern arises naturally when financial intermediaries optimally exploit economic rents that drive their franchise value. As this franchise value fluctuates over the business cycle, so too do the incentives to engage in risky lending. The model leads to novel insights on the effects of recent unconventional monetary policies in developed economies. We argue that bank lending might have responded less than expected to these interventions because they enhanced franchise value, inadvertently encouraging banks to pursue safer investments in low-risk government securities.
1 Introduction

In the wake of the financial crisis of 2007–2008 and subsequent Great Recession several empirical studies have found that these collapses in economic activity tend to occur in the aftermath of large credit expansions.\textsuperscript{1} This evidence has led some to argue that large credit expansions are a key cause of severe downturns. Competitive pressures to lend often combined with perverse incentives or behavioral biases are often argued to lie behind the expansions and subsequent downturns.\textsuperscript{2}

In this paper, we argue that the apparent propensity of banks to engage in “riskier lending” over the cycle is instead a result of cyclical variations in economic conditions and their impact on the bank’s own franchise value. As a result, in our work, there is no concept of a credit cycle that “causes” the business cycle. Instead, in our model, bank credit co-moves with – and even precedes – macro aggregates such as investment and output, even when these variables are, by design, fully independent of bank lending behavior.

Instead of suffering from irrational exuberance, our bank managers correctly forecast future economic growth and optimally respond to changes in the economic environment. Moreover, they make investment and financing decisions with the aim of maximizing shareholder value. The key assumption in our model is that banks benefit from economic rents, which here arise because of subsidized deposit insurance.\textsuperscript{3,4} The value of these rents fluctuates over time

\textsuperscript{1}See Borio and Lowe (2002); Reinhart and Rogoff (2009); Jordà, Schularick, and Taylor (2011); Schularick and Taylor (2012); Mian and Sufi (2009); Mian, Sufi, and Verner (2017); Krishnamurthy and Muir (2016).

\textsuperscript{2}Work by Minsky (1977) and Kindleberger (1978) already emphasizes the potential for overoptimism to destabilize the economy. Behavioral explanations include neglected risks (Gennaioli, Shleifer, and Vishny, 2012) and extrapolative beliefs (Barberis, Shleifer, and Vishny, 1998; Greenwood and Hanson, 2013).

\textsuperscript{3}An alternative, and equally compelling, source of rents could be imperfect competition in the banking sector.

\textsuperscript{4}Buser, Chen, and Kane (1981) document that deposit insurance premia are subsidized in the US.
as local and aggregate economic conditions change and generally causes banks to take more risks when franchise values are low.

Our model of the banking sector builds on Merton (1978). Specifically, we treat banks as entities with access to an exogenous supply of deposits, paying a fixed deposit rate priced to reflect the presence of a government guarantee. To this simple structure, we then add an investment decision: banks must decide in each period on the composition of their loan portfolio. They invest their assets in a mixture of risky loans to the private sector and safer floating-rate government notes. The government guarantee on deposits provides banks with a source of economic rents. The discounted value of this stream of rents is effectively the bank’s franchise value and its fluctuations over the business cycle drive lending behavior. During expansions, the franchise value is generally large and banks protect it by avoiding excessive risks that may lead to early bankruptcy. Over time however, as aggregate risks eventually build, franchise values begin to fall while risk premia rises and the bank’s equity holders may find it preferable to exploit the additional reward from investing in risky assets.

To quantify the links between bank lending and macroeconomic activity we then complete our model to include a corporate sector making investment and production decisions. Crucially, we assume banks lend only to households, ensuring corporate behavior remains fully independent of bank credit decisions. We then confront our model’s quantitative implications with recent empirical evidence on the relationship between bank lending and financial crises. In particular, we show how our model replicates the empirical patterns in Schularick and Taylor (2012) and Jordà, Schularick, and Taylor (2016) showing that financial crises often follow periods of very fast credit growth, as well as the related findings in Mian, Sufi, and Verner (2017) documenting the strong predictability of future GDP growth by the growth in household debt for 30 countries.

Beyond these core findings, our model provides some important implications
for the evaluation of recent unconventional monetary policy interventions. After the recent recession, central banks in advanced economies responded by providing the banking sector with additional guarantees on funding. Although policymakers intended for banks to increase their lending to the private sector, this did not happen in many cases. Instead, many banks invested heavily in government bonds or excess reserves with the central bank. We show that this behavior need not be puzzling if we plausibly interpret those guarantees as lowering the cost of financing for banks. In this case our model suggests that by effectively increasing the franchise value of banks these policies only worked to reinforce their incentives to hold more safe assets.

Our work is related to several bodies of literature on banking, corporate finance and macroeconomics. Starting from the empirical evidence that the banking industry is both highly regulated and subject to limited entry, a large literature has shown that competition reduces banks’ franchise value and induces banks to assume more risk.\footnote{Although Boyd and De Nicoló (2005) offers an early dissent.}

In his seminal paper, Merton (1978) shows that, in the presence of deposit insurance, the usual relation between asset volatility and equity value does not necessarily hold since franchise values would be lost when banks default. Marcus (1984) and Hellmann, Murdock, and Stiglitz (2000) refine Merton’s idea by explicitly linking a bank’s preference for risky investments to the lower rents in the aftermath of higher competition in the banking industry. The mechanism they propose is one example of the “risk-shifting” idea developed by Jensen and Meckling (1976). Keeley (1990) explains the savings and loan crisis of the 1980s and 1990s combining the intuition that higher competition destroys banks’ oligopolistic rents with the idea that deposit insurance creates moral hazard incentives.

In common with our setting, the sizable franchise value in those models leads banks to self-impose strict discipline on risk taking. The key difference
in our work is that we argue franchise values must fluctuate endogenously with local and aggregate economic conditions. This in turn, induce to important changes in bank lending behavior over the business cycle.

Our work is also related to recent studies analyzing the problem of jointly determining monetary and regulatory policies (Acharya and Yorulmazer, 2007, 2008; Farhi and Tirole, 2012). While those papers focus on ex-ante optimal policy to deal with financial crises, our model focuses specifically, and offers a novel perspective, on the role of uncompetitive pricing of deposit insurance premia on ex-ante incentives.

Finally, and more broadly, this paper is also connected to the recent literature examining the causal links between credit market conditions and economic fluctuations. In particular, our paper is closely related to Santos and Veronesi (2016) and Gomes, Grotteria, and Wachter (2018) which show how endogenous co-movements between leverage and several macroeconomic aggregates are the natural outcome of standard models without requiring financial frictions or behavioral biases. While those two papers focus on the demand side of credit, by either households or firms, our attention here is on the supply side, through the behavior of the banking sector.

The rest of the paper is organized as follows. Section 2 presents a theoretical framework to study the optimal composition of bank lending in the presence of deposit insurance and time variation in economic rents. The model's results are quantitatively assessed in Section 3, while Section 4 studies our key policy implications. Section 5 then discusses novel empirical evidence in support of the role deposit insurance in financial crises. Section 6 concludes.

## 2 Model

Our model economy consists of three elementary units: a banking sector, a representative investor/consumer and a productive sector. They all share a
common exposure to an extreme economic adverse event, or “crisis,” that occurs with a time-varying probability, $p_t$. To avoid clouding on our key underlying mechanism, we do not fully integrate these sectors in a general equilibrium setting.

The representative investor owns both banks and the production sector; all of these entities’ decisions are made in a manner consistent with this agent’s pricing of risk. Banks lend to households which may differ from the representative investor, and may also lend to the firms in the productive sector. However, the productive sector faces no financial frictions and may equivalently be financed with equity alone.

### 2.1 The Stochastic Discount Factor

We assume that all financial claims are owned and priced by an infinitely-lived representative investor with an Epstein and Zin (1989) utility function. The representative agent’s utility is identified by a time preference rate $\beta \in (0, 1)$, a relative risk aversion parameter $\gamma$, and an elasticity of intertemporal substitution $\psi$. It follows that the stochastic discount factor is given by:

$$M_{t+1} = \beta^\theta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{S_{t+1} + 1}{S_t} \right)^{-1+\theta},$$

where $S$ denotes the wealth-consumption ratio and $\theta = \frac{1-\gamma}{1-\psi}$.

### 2.2 Consumption and Uncertainty

We assume the following stochastic process for the representative investor’s consumption:

$$C_{t+1} = C_t e^{\mu_c + \sigma_c \epsilon_{c,t+1} + \xi_{t+1}},$$

where $\epsilon_{c,t}$ is a standard normal random variable that is iid over time. Importantly, this process allows for the possibility of a rare collapse in economic activity...
when consumption drops by a large fraction, \( \xi \), as in (Rietz, 1988; Barro, 2006). If a crisis materializes, an event that occurs with probability \( p_t \), we set \( x_{t+1} = 1 \). Otherwise \( x_{t+1} = 0 \). The realization of \( x_{t+1} \), conditional on \( p_t \), is independent of \( \varepsilon_{c,t+1} \).

The natural log of the crisis probability \( p_t \) follows a first-order autoregressive process with persistence \( \rho_{p} \) and mean log \( \bar{p} \):

\[
\log p_{t+1} = (1 - \rho_{p}) \log \bar{p} + \rho_{p} \log p_t + \sigma_{p} \varepsilon_{p,t+1},
\]

(3)

where \( \varepsilon_{p,t} \) is standard normal, iid over time, and independent of \( \varepsilon_{c,t} \) and \( x_t \). Under these assumptions, the wealth-consumption ratio satisfies

\[
E_t \left[ \beta^\theta \left( \frac{C_{t+1}}{C_t} \right)^{1-\gamma} \left( S(p_{t+1}) + 1 \right)^\theta \right] = S(p_t)^\theta,
\]

(4)

so that the stochastic discount factor (SDF) can be written as

\[
M_{t+1} = \beta^\theta e^{-\gamma (\mu_c + \sigma_c \varepsilon_{c,t+1} + \xi_{t+1})} \left( \frac{S(p_{t+1}) + 1}{S(p_t)} \right)^{-1+\theta}.
\]

(5)

Following Barro (2006), we consider a government bill that is subject to default in times of crisis. We let \( q \) denote the loss in case of default. The price of the government bill is thus given by

\[
P_{G_t} = E_t[M_{t+1}(1 - qx_{t+1})].
\]

(6)

The ex-post realized return on government debt is given by

\[
r_{t+1}^G = \frac{1 - qx_{t+1}}{P_{G_t}} - 1.
\]

(7)

\(^6\)In our simulations, we discretize the process (3) so that \( p_t < 1 \).
2.3 Banks

Key to our analysis is the definition of a bank:

**Definition 1.** A bank is a licensed investment management company whose risky investments or loans are financed by equity and guaranteed deposits.

In our model, a bank is able to extract rents from subsidized deposits and takes advantage of stochastic investment/lending opportunities by responding optimally to unexpected changes in the economic environment.⁷

Every period, bank managers maximize the value of the equity holders by making optimal investment and payout decisions. More specifically, managers decide how much capital to allocate to a portfolio of risky loans and to holdings of government securities as well as on the amount of equity to fund these investments. A bank’s risky loan portfolio consists of a diversified pool of collateralized loans which is subject to bank specific and aggregate shocks.

2.3.1 The Bank’s Balance Sheet

Bank \( i \) enters time \( t \) with book equity \( BE_{it} \) and deposits \( D_{it} \). Following Merton (1978), we assume \( D_{i,t+1} = D_{it}e^g \), namely that deposits grow at a constant rate.⁸,⁹

When a bank is not in default (discussed below), it decides on the overall size of its current loan portfolio (its assets) denoted by \( A_{it} \) and on how much to repay its equity-holders, \( \text{Div}_{it} \). A bank must also pay operational, or non-interest expenses, \( \Phi_{it} \) in every period, so that its resource constraint at time \( t \)

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⁷Deposit guarantees are funded with taxes on the aggregate economy and their impact is not internalized by the bank managers.

⁸It is easy to allow the demand for deposits to be stochastic but this feature is not essential to our results.

⁹We calibrate \( g \) to equal expected consumption growth:

\[
g = \log((1 - E\phi)e^{\mu+c+\sigma^2/2} + E\phi e^{\mu+c+\sigma^2/2+\xi}).
\]  

(8)
is:

\[ A_{it} = BE_{it} + D_{it} - Div_{it} - \Phi_{it}. \]  

(9)

The evolution of book equity over time depends on the ex-post rates of return between \( t \) and \( t + 1 \) on the bank’s assets, \( r^A_{i,t+1} \), and liabilities, \( r^D_{t+1} \). Given these ex post returns, book equity in the next period equals

\[ BE_{i,t+1} = (1 + r^A_{i,t+1})A_{it} - (1 + r^D_{t+1})D_{it}. \]  

(10)

2.3.2 Loans and the Return on Assets

Asset returns depend on the banks’ loan portfolio and overall economic conditions. If \( \varphi_{it} \in [0, 1] \) is the share of bank \( i \)’s total assets that is allocated to a pool of private sector loans, and \( r^L_{i,t+1} \) is the ex-post rate of return on this portfolio, the return on the bank’s assets equals:

\[ r^A_{i,t+1} = \varphi_{it}r^L_{i,t+1} + (1 - \varphi_{it})r^G_{t+1}. \]  

(11)

Each bank’s portfolio of private sector loans is made of a large number of individual loans within a local economy. We think of these as collateralized loans (e.g. mortgages) to households that are not the marginal investor and thus price no assets.\(^{10}\) We let the time-\( t \) collateral value for each individual loan \( j = 1, \ldots, n \) of bank \( i \) equal

\[ W_{ijt} = e^{\alpha \varepsilon_{ct} + \xi x_{t} + \omega_{it} + \sigma_j \epsilon_{jt}}. \]  

(12)

Note that this value depends on the state of the aggregate economy \((\varepsilon_{ct}, x_t)\), a borrower-specific shock, \( \epsilon_{jt} \), and a measure of the health of local market conditions, \( \omega_{it} \).

\(^{10}\)Yeager (2004) shows the vast majority of the U.S. banks remain small and geographically concentrated and 61% have operated within a single county. Mortgages (and other household loans) account for the majority of most bank’s assets.
As an example, the bank-specific variable \( \omega_{it} \) could represent a local determinant of house prices. A persistent bank-specific determinant of loan performance ensures the cross-section of banks will remain non-trivial. We assume \( \omega_{it} \) evolves according to the Markov process:

\[
\omega_{i,t+1} = \rho \omega_{it} + \sigma \varepsilon_{\omega_{i,t+1}}.
\]  

(13)

We assume both \( \varepsilon_{jt} \) and \( \varepsilon_{\omega_{it}} \) to be iid over time, independent of each other and also of \( \varepsilon_{ct}, x_t, \) and \( \varepsilon_{pt} \). Shocks to all these variables will change both the collateral value of an individual loan and the probability it will default.

We assume a common face value of each individual loan of \( \kappa \) so that borrower \( j \) is said to default at time \( t \) if \( W_{ijt} < \kappa \). In this case the bank recovers a fraction \( 1 - L \) of the collateral value. In Appendix A we use the central limit theorem to integrate out borrower risk and derive the distribution of the ex-post return on the bank’s pool of private sector loans, \( r_{i,t+1}^L \). As a result, the ex-ante distribution of \( r_{i,t+1}^L \) depends only on \( p_t \) and \( \omega_{it} \).

Figure 1 shows how the spread between the rates of return on these two investments changes with macroeconomic conditions. Like other risky spreads this is increasing in the probability of a crisis, \( p_t \). In addition, risk premia on the bank loan portfolio decline when local market conditions improve, as measured by collateral values, \( \omega_{it} \). An improvement in local market conditions decreases the chance/severity of default in the loan portfolio, given a crisis, and hence lowers the exposure to \( p_t \).

2.3.3 The Deposit Rate

Following Merton (1978), we assume that the interest rate on deposits is constant over time and below the unconditional average of government bill rate, so that \( r^D < E[r_{i,t+1}^G] \).

As is well known, this wedge can readily arise when deposits provide liquidity
services as in Sidrauski (1967) or Van den Heuvel (2008). Here we prefer instead to invoke the existence of deposit insurance guaranteeing that bank depositors receive at least partial compensation in the event of a bank default. More generally, however, this wedge also arises in any imperfectly competitive model where banks have the ability to earn excess rents on their operations (Drechsler, Savov, and Schnabl, 2017).

Regardless of the precise reason, the notion that deposit rates are both sticky and below the rates on money market accounts and government bills is well-grounded in data. Figure 2 shows the rate on the three-month Treasury bill and the average deposit rate earned on large-denomination interest checking accounts over the last 20 years. Although not constant, deposit rates are very slow moving and, on average, well below those on Treasuries.

2.3.4 Regulation and Termination

Bank regulation takes two forms. First, banks face regulatory requirements on their use of leverage: whenever the bank’s chosen debt-to-asset ratio at time $t$, $D_t/A_t$, exceeds the regulatory threshold, $\chi$, the bank must incur an additional cost $f$ per unit of deposits. Generally, even a small cost will be enough to ensure that banks comply with the regulatory constraint.

Second, as in Merton (1978), we assume that regulators monitoring the bank intervene and seize the bank’s operating license whenever the value of its book equity at the beginning of the period, $BE_{it}$, drops below 0. Formally, this means that whenever $BE_{it} < 0$ a bank cannot raise equity ($Div_{it} < 0$) to avoid being shut down. If the bank is terminated, its assets are seized, the deposits are paid and its equity holders receive nothing. As a result, from the perspective of its equity holders, excessive risk taking by the bank may result in sub-optimal termination.

11Note that this cost is fixed except for the scale factor.
2.3.5 The problem of the bank

It follows from the description above that the market value of bank $i$’s equity at time $t$ is given by:

$$V_{it} = \begin{cases} 
    E_t \left[ \sum_{s=t}^{T_i^* - 1} M_{t+s} \text{Div}_{is} \right], & t < T_i^* \\
    0, & t \geq T_i^* 
\end{cases}$$

(14)

where

$$T_i^* = \inf \{ t : BE_{it} < 0 \}$$

(15)

denotes bank $i$’s (stochastic) termination time, and $M_{t+s}$ is the SDF of the bank’s shareholder between times $t$ and $t + s$ which can be directly derived from (5).

Conditional on survival at time $t$, the market value of bank $i$ satisfies the recursion

$$V_i(BE_{it}, A_{i,t-1}, D_{it}, p_t, \omega_{it}) = 
\max_{\varphi_{it}, \text{Div}_{iit}} \text{Div}_{it} + E_t \left[ M_{t+1} V_i(BE_{i,t+1}, A_{it}, e^{\delta} D_{it}, p_{t+1}, \omega_{i,t+1}) \mathbb{1}_{BE_{i,t+1} > 0} \right],$$

(16)

subject to (10),

$$\begin{align*}
    r_{i,t+1}^A &= \varphi_{it} r_{i,t+1}^L + (1 - \varphi_{it}) r_{i,t+1}^G \\
    \log p_{t+1} &= (1 - \rho_p) \log \bar{p} + \rho_p \log p_t + \sigma_p \epsilon_{p,t+1} \\
    \omega_{i,t+1} &= \rho_\omega \omega_{it} + \sigma_\omega \epsilon_{\omega,i,t+1},
\end{align*}$$

(17)

and

$$A_{it} = BE_{it} + D_{it} - \text{Div}_{it} - \Phi(A_{it}, D_{it}, A_{i,t-1}),$$

\[11\]
where

\[
\Phi(A_{it}, D_{it}, A_{i,t-1}) = \eta B A_{i,t-1} \left( \frac{A_{it} - A_{i,t-1}}{A_{i,t-1}} \right)^2 + f D_{it} \mathbb{1}_{D_{it} > \chi A_{it}}.
\]

The cost function \( \Phi \) summarizes the non-interest expenses, inclusive of regulatory charges, incurred by the bank. Operating expenses are assumed to depend on the growth in bank assets over time.

We greatly simplify the computation of the bank’s problem using two economic insights. First, the problem is jointly homogeneous of degree 1 in assets and deposits, because both the current stream of cash flows and the constraints are linear in \( A_{i,t} \) and \( D_{i,t-1} \). Second, we solve for the gap between (scaled) market and book equity:

\[
\bar{v}(a_{i,t-1}, p_t, \omega_t) = \frac{V_i(BE_{it}, A_{i,t-1}, D_{it}, p_t, \omega_t) - BE_{it}}{D_{it}}, \tag{18}
\]

where \( a_{it} = \frac{A_{it}}{D_{it}} \) and \( be_{it} = \frac{BE_{it}}{D_{it}} \). Appendix B shows that (18) is indeed a function of lagged scaled assets \( (a_{i,t-1}) \), crisis probability \( (p_t) \), and local conditions \( (\omega_t) \).\(^{12}\) We refer to (18), the (scaled) difference between market equity and book equity, as the bank’s \textit{franchise value}.

In our model, franchise value is driven by the ability of the bank to earn greater returns, in a risk-adjusted sense, on its asset portfolio, than it is required to pay to its debtholders. In what follows, we show that banks seek to protect this franchise value; this mitigates the moral hazard problem resulting from deposit insurance. When franchise value falls, however, incentives change.

Figure 3 depicts scaled franchise value as a function of the crisis probability, \( p_t \), for alternative values of (scaled) lagged assets. Franchise value is strictly decreasing in the crisis probability and is increasing in lagged assets. The negative relation between the franchise value and the crisis probability arises

\(^{12}\)Technically, (18) is well-defined only when \( BE_{it} \geq 0 \). See Appendix B for details.
endogenously. When $p_t$ rises, safe asset values rise because of precautionary savings. Risky asset values might either rise or fall, depending on whether the negative effect of the crisis probability on expected cash flows and on the risk premium outweighs the precautionary savings effect. For the bank, there is an additional consideration: the bank can choose its portfolio and therefore its level of risk in response to changes in $p_t$.

### 2.4 Bank Risk-Taking

Figures 4 and 5 illustrate the two key choices of a bank. Figure 4 depicts the bank’s decision with respect to the size of its overall loan portfolio, $a_i$. Given an exogenous supply of deposits, this is also the optimal leverage decision of the bank, with a higher $a_i$ corresponding to lower leverage. As Figure 4 shows, current leverage choices will be generally increasing in past leverage. This is because asset growth is costly. The nature of the bank’s operating costs generates a plausibly strong persistence in lending and leverage decisions.

Figure 4 also shows the rich dynamics generated by the model as the previous leverage choice interacts with the crisis probability. For banks beginning the period with low leverage, the optimal level of assets (relative to deposits) decreases as a function of the crisis probability; for low values of the crisis probability the slope is relatively flat, and then steepens as the probability rises. For banks beginning the period with moderate leverage, optimal assets increase, and then decrease. Finally, highly levered banks, optimal assets in the next period are virtually flat in $p_t$.

How does the relatively simple model of Section 2 generate these patterns? All else equal, assets are costly to the bank (this is modeled through the cost function $\phi_i$, which multiplies the level of assets). However, the main business of the bank, taking deposits and investing in assets, is profitable, so the bank would like to avoid being shut down. Thus the bank would like to maintain
positive book equity, not only in the present, but also in the future (provided that the benefits are high, and the costs are sufficiently low). When the probability of a crisis, $p_t$, is low, this effect dominates for all banks but the ones with the highest leverage. If a bank happens to start the period with a high level of assets, it slowly reduces assets to gradually get to the (stochastic) steady state.\(^\text{13}\) This is shown by the dotted line in Figure 4. If a bank happens to start the period at moderate leverage, it increases assets (decreasing leverage). The higher is $p_t$, the more it seeks to increase assets. This effect, illustrated by the dashed line in Figure 4, is due to precautionary motives specific to the bank – mainly the desire to have high book equity in the future. Thus, when $p_t$ is low, the bank’s primary incentive is to stay in business, not just in the present period, but in the future, to protect its franchise value. It is noteworthy that this occurs despite the presence of the moral hazard problem due to deposit insurance.

As the probability of a crisis rises, however, the bank’s incentives change in a dramatic way. The probability of shutdown increases, and avoiding it entirely becomes too costly. The bank shifts from being a “good bank”, making safe investment and seeking to stay in business, to being a “bad bank,” in effect taking advantage of the subsidy offered to depositors. This is illustrated by the kinks in the policy functions shown in Figure 4. The threshold for $p_t$ at which this occurs depends on leverage from the previous period. For the bank with low leverage the shift does not occur until the probability of a crisis is as high as 5%. For the bank with with the middle value, it occurs at 2%. For the bank with the highest leverage, all value of $p_t$ lead it to maintain assets at their lowest value.\(^\text{14}\)

\(^{13}\)For low values of $p_t$, $a_t$ declines as a function of $p_t$. This is because of the usual trade off between the income and substitution effect. At higher $p_t$, investment opportunities are less favorable and the bank returns capital to its equity holders.

\(^{14}\)Recall that this maximum leverage position is defined by need to pay a fine proportional to deposits when leverage exceeds this value.
We can see the same mechanisms at work in the optimal portfolio allocation of the bank, as Figure 5 shows. When the probability of a crisis is low, well-capitalized banks avoid risky loans to households; these are made, however, by poorly capitalized banks (contrast the solid line with the dotted and dashed lines in Figure 4). At a threshold level of \( p_t \), however, the loan portfolio shifts toward the risky household loans. This shift occurs at the same point at which the bank decides to hold less equity in Figure 4.

What explains the shift from “good bank” to “bad bank” at higher levels of the crisis probability? As discussed above, franchise value decreases in the crisis probability. At higher levels of \( p_t \), the bank is not as incentivized to protect this lower value, and so engages in risk shifting. That is, the claim of bank equity holders resembles a call option, which benefits from increased volatility in a way that the overall asset does not. By increasing leverage and investing in risky household loans, the bank “gambles for resurrection.” A good outcome generates high returns for the equity holders. A bad outcome results in being shut down; however, if shutdown is likely regardless, equityholders cannot be further penalized. As for any call option, the sensitivity to volatility increases the more the underlying asset is out of the money. Thus the greater is \( p_t \), the lower is franchise value, and the greater the incentive to gamble for resurrection. Exacerbating this effect is an endogenous decline in the market interest rate as \( p_t \) rises, due to the precautionary motive of the representative agent. It becomes costlier for the bank to protect its franchise value, even as the bank has less of an incentive to do so. This realistic mechanism leads to behavior sometimes referred to as “reaching for yield.”

Figure 6 summarizes our findings by showing the implications of optimal bank behavior to its overall probability of default. For well-capitalized banks the expected failure rate remains essentially at 0 as long as a crisis is somewhat unlikely. As \( p_t \) rises however, risk premia widens, expected returns on govern-

\[15\] The argument in this paragraph shows why this is in fact an equilibrium outcome.
ment debt fall and even well-capitalized banks can no longer be assured of survival. Increased risk taking exposes these banks to more and more systematic risk and raises overall default probabilities until they become indistinguishable from $p_t$ itself.

2.5 Firms, Production and Output

As we will show, the model has realistic implications for the relation between leverage, risky lending, and growth in GDP. These implications arise naturally from a production sector. For simplicity, we assume a representative firm maximizing the present value of cash flows, taking the investors’ stochastic discount factor (5) as given. We assume this sector faces no financial frictions, and is all-equity financed.

2.5.1 Technology

A firm uses capital $K_t$ to produce output $Y_t$ according to the Cobb-Douglas production function

$$Y_t = z_t^{1-\alpha} K_t^\alpha,$$

(19)

where $\alpha$ determines the returns to scale of production and $z_t$ is the productivity level. We assume $z_t$ follows the process

$$\log z_{t+1} = \log z_t + \mu_c + \epsilon_{c,t+1} + \phi \xi x_{t+1}.$$

(20)

During normal-times, productivity grows at rate $\mu$ and is subject to the same shocks as consumption ($\epsilon_{c,t+1}$). Importantly, this process implies that the productive sector is exposed to the same Bernoulli shocks as consumers and banks through the term $\phi \xi x_{t+1}$. $\phi$ is the sensitivity of TFP to an economic crisis.
2.5.2 Investment Opportunities

The law of motion for the firm’s capital stock is

\[
K_{t+1} = \left[ (1 - \delta)K_t + I_t \right] e^{\phi \xi x_{t+1}},
\]

where \( \delta \) is depreciation and \( I_t \) is firm’s investment at time \( t \). Equation (21) captures the depreciation cost necessary to maintain existing capital. Following the formulation of Gabaix (2011) and Gourio (2012), it also captures the impact of a possible destruction of productive capital during a crisis. This can proxy for either literal capital destruction (in the case of war), or simply misallocation due to economic disruption.

Finally, to allow us to match the relative volatility of investment and output in the data the firm is assumed to face convex costs when adjusting its stock of capital (Hayashi, 1982). To be precise, we assume that each dollar of added productive capacity requires \( 1 + \lambda(I_t, K_t) \) dollars of expenditures, where

\[
\lambda(I_t, K_t) = \eta_F \left( \frac{I_t}{K_t} \right)^2 K_t,
\]

and the parameter \( \eta_F > 0 \) determines the severity of the adjustment cost.

Optimal production and investment decisions, can then be constructed by computing the total value of the firm, \( V^F \), which obeys the recursion

\[
V^F(K_t, z_t, p_t) = \max_{I_t, K_{t+1}} \left[ z_t^{1-\alpha} K_t^{\alpha} - I_t - \lambda(I_t, K_t) + E_t[M_{t+1} V^F(K_{t+1}, z_{t+1}, p_{t+1})] \right],
\]

subject to (21) and (22).
3 Crisis, Bank Lending and the Predictability of Macro Aggregates

The joint exposure of consumers, firms and banks to common aggregate shocks generates interesting co-movements between the various macroeconomic aggregates and bank lending over the business cycle. In this section we investigate the implications of a quantitative version of our model for these movements with a special focus on the role of bank risk-taking decisions.

3.1 Parameter Values

We begin by picking a set of values for our model’s parameters. We calibrate the model at an annual frequency. Tables 1-3 summarize our choices for the parameters used to solve the problems of investors, banks and firms, respectively.

The representative investor prices all risky claims in our economy. Thus, we pick the parameters of preferences (1) and the consumption process (2) to match key asset pricing moments or well-established macro patterns. Specifically, we take the values for the parameters $\gamma$ and $\psi$ from the recent literature on asset pricing with rare events (e.g. Gourio (2012) and Gomes, Grotteria, and Wachter (2018)), while the values chosen for the parameters $\mu_c, \sigma_c$ and $\beta$ follow from a long tradition in extant macro literature (e.g. Cooley and Prescott (1995))

Due to their rare nature, precise calculations of the probabilities and distributions implied by (3) are difficult. We generally follow Barro and Ursua (2008) and set the average probability of an economic collapse $\bar{p}$ to be 2% per annum and an associated drop in consumption of $\xi = 30\%$.\footnote{Our estimate of $p_t$ is slightly below Barro and Ursua (2008) estimates of an average probability of disaster of 2.9% on OECD countries and 3.7% for all countries.} Next, we set the autoregressive coefficient to be 0.8 (annually) with an unconditional standard deviation of 0.42, values that are consistent with those used in Gourio (2013).
Finally, we assume the government bills experience a loss of $q = 12\%$ during a crisis.

To solve the problem of the bank, we set the loss given default on private loans to 60\% so that it matches the observed average recovery rate on secured senior debt (Ou, Chlu, and Metz, 2011). The face value of an individual private loan $\kappa$ is set so that the average loan-to-value ratio equals 80\%, the typical value for newly originated or refinanced residential mortgages (Korteweg and Sorensen, 2016). The parameters governing the evolution of local conditions, $\sigma_\omega$ and $\rho_\omega$, are determined from volatility and persistence U.S. house prices, at the individual state level. The value for the idiosyncratic component of volatility, $\sigma_j$, is borrowed from Landvoigt, Piazzesi, and Schneider (2015) who estimated an annual volatility of individual house prices between 8\% and 11\%.

The regulatory capital requirement parameter $\chi$ is set to be 0.92, corresponding to an 8\% equity to asset ratio, in accordance to Basel rules. Finally, the value of the operating cost parameter $\eta_B$ is chosen to generate a plausible cross sectional dispersion in the asset-to-debt ratio in the model that approximates that for US bank holding companies.

Parameter values used to solve the problem of the firm are either in line with standard choices in the macroeconomics literature ($\alpha$ and $\delta$) or chosen to match specific facts, like the relative drop in GDP during crises ($\phi$) or the volatility of investment growth relative to the volatility of output growth in the data ($\eta_F$).

### 3.2 Quantitative Results

We now quantify the links between bank lending and macroeconomic activity. We focus on a well-known set of empirical results that have been interpreted to indicate a causal relation between credit and poor subsequent economic performance (e.g. Gennaioli and Shleifer (2018)). We show that our model
can quantitatively account for these findings, though the interpretation is quite different. We simulate 10,000 years of artificial data from the economy described above (see Appendix C for further detail). We consider a crisis to have occurred in simulated data if realized GDP growth is in the bottom 4% of its unconditional distribution, matching the empirical frequency of crisis in the Schularick and Taylor (2012) data. This definition captures not only all the observations in which \( x_t = 1 \) but also a number of periods during which the probability of a crisis, \( p_t \), rose sharply, leading firms in the model to reduce investment and thus output to fall. It addresses the concern that the econometrician, in identifying crises, does not observe the variable \( x_t \); indeed there may be no clear line between \( x_t = 1 \) events and events in which there is a large positive shock to \( p_t \) in terms of observables. Our results are not sensitive to this definition.\(^{17}\)

Each simulation contains a cross-section of 1,000 ex-ante identical banks. To quantitatively match the model’s behavior to the data, we focus on variables that are stationary. Recall that, normalized by deposits, the dollar value of the loans of bank \( i \) at time \( t \) equals \( \varphi_{it} a_{it} \). We therefore define aggregate lending as this quantity, summed over banks. That is, aggregate lending equals

\[
L_t = \sum_i \varphi_{it} a_{it}.
\]

Table 4 replicates the results of Schularick and Taylor (2012) that an increase in lending raises the probability of a crisis. That is, crisis occurrence is regressed on lagged values of bank loans in an international sample, spanning the years 1870 to 2008. Schularick and Taylor (2012) find that an increase in lending is a statistically significant predictor of a crisis. The standard interpretation is that increased bank lending causes a crisis. We find a similar effect in the model, which holds with similar magnitude. In our model, however, time-varying exogenous risk drives both, and the relation between lagged bank lending and

\(^{17}\)We use this definition of crisis only for comparison with existing empirical results. In later sections of the paper, we will continue to use the terminology “crisis” to refer to the exogenous event that \( x_t = 1 \).
crises is merely a correlation.

Figure 7 compares our model’s findings with the evidence in Jordà, Schularick, and Taylor (2016) showing that financial crises often follow periods of very fast credit growth. It breaks down the frequency of a crisis for each quintile of lagged credit growth. The top figure replicates the evidence in the original paper showing that crises frequencies increase significantly after periods of fast credit-to-GDP growth. However, Panel B, taken from our artificial dataset, shows how we can substantively replicate these same facts, even when our model assumes that a crisis is never caused by changes in bank lending. High credit growth is correlated with more frequent realizations of crises.\footnote{In the model, we look directly at growth in credit \( L_t \), rather than growth in credit scaled by GDP. This is because, as we have defined defined it, credit growth is stationary. However, the ratio of loans to GDP may not be stationary.}

We also examine our model’s implications for the related findings in Mian, Sufi, and Verner (2017), documenting the strong predictability of future GDP growth by the lagged growth in household debt for 30 countries. Figure 8 replicates and updates their work to show negative relation between growth in lagged household debt (scaled by GDP) and GDP growth over a three-year window. In our model, the growth rate in bank loans also negatively predicts the growth rate in GDP. Note that this empirical exercise has no explicit link to financial crises. In the model, however, it is the increased probability of a crisis that leads to lower growth.

Why is the model able to produce these findings? The key mechanism is the fluctuating value of the bank’s franchise. During periods of high probability of financial crises, banks’ franchise value falls. Some banks, and in particular those with poor balance sheets, find it optimal to gamble for resurrection, taking on risky household loans.\footnote{A direct empirical implication of our findings is that firms with the highest ex post profit are those that have taken on the most risk, and are thus most prone to fail. Recent work by Meiselman, Nagel, and Purnanandam (2018) shows that this is indeed the case in the data.} Thus growth in risky loans predicts crises. It also predicts lower GDP growth because non-financial firms, perceiving the
same economic instability, reduce their investment, leading to lower output.

To conclude then, our quantitative model is broadly consistent with the observed empirical patterns in bank credit that generally precedes economic collapses. In the model, however, these patterns merely reflect optimal decisions taken in response to *exogenous* fluctuations in the probability of a financial and economic collapse and thus, by construction, have no effect on the odds that this event will occur.\(^{20}\)

### 4 Policy Evaluation

In responding to the recent financial crisis fiscal and monetary authorities unleashed an array of polices aimed at influencing the behavior of the banking sector. These included the first round of quantitative easing measures in the United States (QE1) and the long-term refinancing operations (LTRO) in Europe. In this section we show how our model offers an important new perspective on the impact of these policies.

Quantitative easing in the United States is usually understood as an unconventional monetary policy that involved large scale asset purchases of both long term government bonds and securities with some private risks (Gertler and Karadi, 2015; Curdia and Woodford, 2010; Del Negro, Eggertsson, Ferrero, and Kiyotaki, 2017; Williamson, 2012). Less emphasized is the fact that this policy also provided banks with funding at very favorable terms, in effect subsidizing the banking sector. By comparison, LTROs in the Eurosystem were more transparently designed as bank subsidies; the aim was to give banks more liquidity by allowing them to borrow at sub-market rates.

Our model has precise implications regarding the impact of government interventions aimed at lowering the marginal cost of debt financing for banks.

\(^{20}\)In our simulated time series the raw correlation between the growth in aggregate bank lending and the crises probability (in log) is about 40\%.\)
Within our model, this is a switch to a new deposit rate $\bar{r}^D < r^D$. Unsurprisingly, in our model, as Figure 9 shows, this policy directly leads to an increase in the franchise value of banks, since they can now secure better terms to fund themselves. Figure 10 then shows that a consequence of this intervention is that banks will now also actually rely on (relatively) more equity. This is because with increased franchise values, default will now trigger larger losses for equity holders. As a result this policy intervention will produce a decline in expected bank failure rates.

However, this increased conservatism by equity holders will also manifest itself in the optimal portfolio composition of banks. We can see in Figure 11 that the optimal asset composition now generally tilts more towards government bonds and away from risky private loans. Only poorly-capitalized banks eschew this behavior to remain fully invested in private sector loans.

Thus policies that effectively subsidize bank equity holders by allowing them tap debt markets at below-market rates lead many banks to reduce overall risk taking. Moreover, Figures 10 and 11 show that this effect is particularly strong when the likelihood of a crisis is high.

We believe these findings add a fresh perspective to the ongoing debate about the effects of unconventional monetary policies on bank lending. In particular they suggest an explanation for the perceived limited success of unconventional monetary policies in stimulating bank credit to the private sector during the economic recovery after the recent financial crisis. As Bocola (2016) shows, European banks mainly used LTROs to cheaply substitute liabilities, while in the US Di Maggio, Kermani, and Palmer (2016) describe a “flypaper effect” in which banks chose to hold excess reserves with the central bank rather than expand credit to the private sector.

Our results are also consistent with the evidence of Rodnyansky and Darmouni (2017), who find that U.S. banks with mortgage-backed securities on their books increased lending relative to their peers after QE1. In our model
this is unsurprising since those are the banks who optimally chose $\varphi = 1$. These banks will remain the most eager to replace safe assets with risky ones. However, we argue that this evidence should not necessarily be interpreted as a measure of the impact of QE on bank lending. Instead, the effectiveness of these policies should be measured through their impact on marginal banks who, we predict, will now take less risk.

5 The Role of Deposit Insurance: Empirical Evidence

Rent-seeking behavior from banks is a crucial ingredient in delivering many of our results. Although, in practice, this behavior can also arise from a lack of competition in the sector, our model focuses on rents derived from explicit government guarantees on bank deposits. As we have shown above, access to subsidized financing can meaningfully alter a bank’s incentives to hold risky securities in its loan portfolio over time.

In this section we provide independent supporting evidence on the link between the availability of deposit insurance and economic crises. We combine several databases to create a country-level unbalanced panel dataset that contains observations on aggregate household and non-financial firm debt to GDP, macro quantities and the availability of deposit insurance in both advanced and emerging economies. Effectively, this extends the sample used by Mian, Sufi, and Verner (2017) to include more countries, a longer time period and data on the use of deposit insurance.\textsuperscript{21}

Our basic procedure is adapted from Mian et al. (2017). Let $\Delta_3y_{t+h}$ be the three year change in log real GDP per capita in local currency between

\textsuperscript{21}Our data adds together the Bank of International Settlements (BIS) “Long series on total credit to the non-financial sectors”, the World Bank’s World Development Indicators (WDI) database and the Global Financial Database.
year $t + h - 3$ and $t + h$. Similarly, define $\Delta_3 d_{i,t-1}^{HH}$ and $\Delta_3 d_{i,t-1}^{F}$ as the three year rates of growth in the household and firm debt to GDP ratios. Our baseline regression, reported in Panel A in Table 5, reports the estimates for the following equation:

$$\Delta_3 y_{i,t+h} = \alpha_i + \beta_H \Delta_3 d_{i,t-1}^{HH} + \beta_F \Delta_3 d_{i,t-1}^{F} + u_{it}, \quad (23)$$

when $h = -1, \ldots, 5$. Consistent with prior evidence (Mian et al., 2017) we find that a 1 percentage point increase in household debt to GDP ratio is correlated with a 0.4 percentage point drop in GDP per capita after 3 years.

We next combine our data with the country-level database on deposit insurance schemes, constructed by Demirgüç-Kunt, Karacaoglu, and Laeven (2005). For countries where no explicit scheme was reported before 2005, we hand collected the dates of enactment, if any. Overall, we find that in about 25% of our country-year observations there is no deposit insurance scheme in place.

We then interact a zero-one dummy variable for the presence of explicit deposit insurance in the past three years to (23) and estimate the following equation:

$$\Delta_3 y_{i,t+h} = \alpha_i + (\beta_H^{DI} + \beta_H^{HH} \mathbb{1}_{DI}) \Delta_3 d_{i,t-1}^{HH} + (\beta_F^{DI} + \beta_F^{HH} \mathbb{1}_{DI}) \Delta_3 d_{i,t-1}^{F} + u_{it}, \quad (24)$$

for $h = -1, \ldots, 5$.

Panel B in Table 5 shows that the coefficients on the interaction between growth in household credit and the presence of deposit insurance are generally statistically significant, suggesting the variation captured by our regressors is mostly concentrated in periods and countries where deposit insurance is in place. Notably, the relation between credit and GDP is essentially flat and not

\footnote{While US introduced deposit insurance as early as 1934, it became common in most countries only in the late 80s.}
significant in countries without explicit government insurance. By contrast, we find that when deposit insurance schemes are present, a 1 percentage point increase in household debt is correlated with a 0.51 percentage drop in GDP after 3 years.

While a detailed empirical assessment of the role of deposit insurance in financial crises is outside the scope of this paper, Table 5 strongly suggests that the relation between credit growth and financial crises is mediated through deposit insurance.

6 Conclusions

A large literature, motivated by empirical linkages between leverage and crises, argues that excessive household leverage is a cause of subsequent crises, and specifically the financial crisis of 2008. However, leverage is itself an outcome of endogenous decision-making. While it may be plausible that households, perhaps based on lack of experience, overoptimism, or simply rule-of-thumb behavior, took more risk than, ex post, proved optimal, it is harder to believe that banks, en masse, decided to lend to such households purely based on overoptimism, as economic conditions worsened.

This paper offers a quantitative resolution of this conundrum based on a dynamic model of risk-shifting by banks. In our model, banks endogenously provide more leverage to households in times of worsening economic conditions. The subsequent economic decline is in no way caused by household’s overleveraging. Rather, leverage and the subsequent crises are caused by the same economic phenomenon: in this model, a time-varying likelihood of an economic crisis.

Our study suggests that recent policy toward banks might have effects

---

23It is also noteworthy that there is no significant relation between firm credit and subsequent economic growth. The relation is confined to growth in the riskiest form of credit, that is, household credit. This is consistent with our model.
counter to what is intended. Banks’ decisions over time are driven by fluctuations in their franchise value. Methods to strengthen banks, while conferring long-run benefits, might actually result in less lending because they increase the franchise value. On the flip side, any policy with the side effect that weakens banks might actually result in more undesirable lending, and further bank instability, as banks gamble for resurrection. In both cases, ignoring the incentive effects of policy on banks, which operate through fluctuating franchise values, could itself exacerbate underlying risks.
References


Appendix A  Bank Lending

Following Vasicek (2002), Gornall and Strebulaev (2015), and Nagel and Purnanandam (2017), we assume an exogenous process for bank loans. Define a payoff on an individual loan based on the random variable

$$W_{ijt} = e^{\sigma_t \epsilon_{it} + \xi_{jt} + \omega_{jt} + \sigma_j \epsilon_{jt}},$$

(A.1)

where $j$ indexes the borrower and $i$ indexes the bank. Define a constant default threshold $\kappa$. If we assume (A.1) is a two-period process that has the value 1 at time $t - 1$, then $\kappa$ has the interpretation of the loan-to-value ratio. The lender receives repayment

$$\text{Rep}_j(\epsilon_{c,t}, x_t, \omega_t, \epsilon_{jt}) = \kappa 1_{W_{jt, t} \geq \kappa} + (1 - \mathcal{L})W_{jt, t} 1_{W_{jt, t} < \kappa},$$

for a constant $\mathcal{L}$, interpreted as the loss given default. In what follows, we suppress the bank-specific $i$ subscript.

Define

$$\text{Rep}(\epsilon_{c,t}, x_t, \omega_t) = \kappa \text{Prob}(W_{jt, t} \geq \kappa | \epsilon_{c,t}, \omega_t, x_t)$$

$$+ (1 - \mathcal{L})E[W_{jt, t} 1_{W_{jt, t} < \kappa} | \epsilon_{c,t}, \omega_t, x_t].$$

(A.2)

It follows from the law of large numbers that

$$\lim_{n \to \infty} \frac{1}{n} \sum_{j=1}^{n} \text{Rep}_j(\epsilon_{c,t}, x_t, \omega_t, \epsilon_{jt}) = \text{Rep}(\epsilon_{c,t}, x_t, \omega_t).$$

(A.3)

We assume, for simplicity, that the bank holds an equal-weighted portfolio of an arbitrarily large number of loans. Equation A.3 justifies the use of (A.2) as the repayment on the loan portfolio.
We now discuss the computation of (A.2). Define

\[
\begin{align*}
    f(\bar{\epsilon}, \bar{\omega}, 0) &= \log(\kappa) - \sigma_\epsilon \bar{\epsilon} - \bar{\omega} \\
    f(\bar{\epsilon}, \bar{\omega}, 1) &= \log(\kappa) - \sigma_\epsilon \bar{\epsilon} - \xi - \bar{\omega}.
\end{align*}
\]

Note that the function \(f\) is the inverse of the normal cumulative density function (cdf), applied at the default probability. The probability of default conditional on no crisis at time \(t\) equals

\[
\begin{align*}
    p(\bar{\epsilon}, \bar{\omega}, 0) &= \text{Prob} \left( \log W_{jt} < \log \kappa \mid \epsilon_{ct} = \bar{\epsilon}, \omega_t = \bar{\omega}, x_t = 0 \right) \\
    &= \mathcal{N} \left( \frac{1}{\sigma_\epsilon} \left( \log(\kappa) - \sigma_\epsilon \bar{\epsilon} - \bar{\omega} \right) \right) \\
    &= \mathcal{N} \left( f(\bar{\epsilon}, \bar{\omega}, 0) \right).
\end{align*}
\]

where \(\mathcal{N}(\cdot)\) denotes the normal cdf. Similarly, the probability of default conditional on a crisis at time \(t\) equals

\[
\begin{align*}
    p(\bar{\epsilon}, \bar{\omega}, 1) &= \text{Prob} \left( \log W_{jt} < \log \kappa \mid \epsilon_{ct} = \bar{\epsilon}, \omega_t = \bar{\omega}, x_t = 1 \right) \\
    &= \mathcal{N} \left( \frac{1}{\sigma_\epsilon} \left( \log(\kappa) - \sigma_\epsilon \bar{\epsilon} - \xi - \bar{\omega} \right) \right) \\
    &= \mathcal{N} \left( f(\bar{\epsilon}, \bar{\omega}, 1) \right).
\end{align*}
\]

Note that \(p(\bar{\epsilon}, \bar{\omega}, 1) > p(\bar{\epsilon}, \bar{\omega}, 0)\). Default is more likely if a crisis occurs. It is also the case that \(f(\bar{\epsilon}, \bar{\omega}, 1) > f(\bar{\epsilon}, \bar{\omega}, 0)\); there is a higher effective threshold for avoiding default if a crisis occurs.
To compute repayment (A.2), note that

\[ E \left[ W_{j,t} 1_{W_{j,t} < \epsilon} | \epsilon_{c,t}, \omega_t, x_t \right] = \\
\begin{cases} 
    e^{\sigma_{\epsilon_{c,t}} + \omega_t + \frac{\sigma^2}{2}} \int_{-\infty}^{f(\epsilon_{c,t}, \omega_t, 0)} (2\pi)^{-1/2} e^{-\frac{(z-\sigma)^2}{2}} dz & x_t = 0 \\
    e^{\sigma_{\epsilon_{c,t}} + \xi + \omega_t + \frac{\sigma^2}{2}} \int_{-\infty}^{f(\epsilon_{c,t}, \omega_t, 1)} (2\pi)^{-1/2} e^{-\frac{(z-\sigma)^2}{2}} dz & x_t = 1,
\end{cases} \]

where we use the result that, for any \( a \),

\[ \int_{-\infty}^{a} e^{-\sigma_j z^2} dz = e^{\frac{\sigma_j a^2}{2}} \int_{-\infty}^{a} e^{-\frac{(z-\sigma)^2}{2}} dz. \]  \( \text{(A.4)} \)

A loan portfolio is thus an asset whose time-\( t \) payoff is defined by the random variable (A.2). Consider a time-\( t \) investment in the time-(\( t + 1 \)) loan portfolio. The price of the loan portfolio equals

\[ P^L(p_t, \omega_t) = E_t \left[ M_{t+1} \text{Rep}(\epsilon_{c,t+1}, x_{t+1}, \omega_{t+1}) \right]. \]  \( \text{(A.5)} \)

It follows that the ex-post return on the portfolio of loans equals

\[ r^L_{t+1} = \frac{\text{Rep}(\epsilon_{c,t+1}, x_{t+1}, \omega_{t+1})}{P^L(p_t, \omega_t)} - 1. \]  \( \text{(A.6)} \)

Note that \( p_t \) and \( \omega_t \) are sufficient statistics for the distribution of the return on the loan portfolio.

**Appendix B  Franchise value**

Define scaled franchise value:

\[ \tilde{v}(a_{i,t-1}, p_t, \omega_{it}) = \frac{V(BE_{it}, A_{i,t-1}, D_{it}, p_t, \omega_{it}) - BE_{it}}{D_{it}}, \]  \( \text{(B.1)} \)

where we conjecture that the left-hand side is a function of \( a_{i,t-1}, p_t \) and \( \omega_{it} \).

The definition (B.1) holds as long as \( BE_{it} \geq 0 \). In this Appendix, we derive a
recursion for (B.1), thereby verifying the conjecture.

First, substituting (9) into (16) implies that, conditional on \( BE_{it} \geq 0 \),

\[
V_i(BE_{it}, A_{i,t-1}, D_{it}, p_t, \omega_{it}) = \\
\max_{\phi_{it}, A_{it}} BE_{it} + D_{it} - A_{it} - \Phi(A_{it}, D_{it}, A_{i,t-1}) + \\
E_t \left[ M_{t+1} V(BE_{i,t+1}, A_{it}, D_{it}e^g, p_{t+1}, \omega_{i,t+1} \mathbb{1}_{BE_{i,t+1}>0}) \right],
\]

(B.2)

subject to (10) and (17). Otherwise \( V_{it} = 0 \).

Define scaled market value and conjecture that this is a function of \( be_{it}, a_{it}, p_t, \) and \( \omega_{it} \):

\[
v_i(be_{it}, a_{i,t-1}, p_t, \omega_{it}) = V(BE_{it}, A_{i,t-1}, D_{it}, p_t, \omega_{it}) / D_{it}.
\]

(B.3)

We further define

\[
\phi(a_{it}, a_{i,t-1}) \equiv \eta_B a_{i,t-1} e^{-g} \left( \frac{a_{it} - a_{i,t-1} e^{-g}}{a_{i,t-1} e^{-g}} \right)^2 + f \mathbb{1}_{a_{it}^{\top} < \chi}.
\]

Note that \( \phi(a_{it}, a_{i,t-1}) = \frac{\Phi(A_{it}, D_{it}, A_{i,t-1})}{D_{it}} \).

Recursively define \( v_i(be_{it}, a_{i,t-1}, p_t, \omega_{it}) \) as

\[
v_i(be_{it}, a_{i,t-1}, p_t, \omega_{it}) = \max_{\phi_{it}, a_{it}} be_{it} + 1 - a_{it} - \phi(a_{i,t-1}, a_{it}) + \\
E_t \left[ M_{t+1} e^g v(be_{i,t+1}, a_{it}, p_t, \omega_{t+1} \mathbb{1}_{be_{i,t+1}>0}) \right],
\]

(B.4)

subject to

\[
be_{i,t+1} = e^{-g} \left( (1 + r_{i,t+1}^A) a_{it} - (1 + r_{t+1}^D) \right),
\]

(B.5)

and (17), for \( be_{it} \geq 0 \); otherwise \( v_{it} = 0 \). Dividing both sides of (16) by \( D_{it} \) and applying the law of motion for deposits shows that the definitions (B.4) and (B.3) are consistent, verifying the conjecture.
Finally, define $\tilde{v}(a_{i,t-1}, p_t, \omega_{it})$ as the solution to the recursion

$$
\tilde{v}(a_{i,t-1}, p_t, \omega_{it}) = \max_{\phi_{it}, a_{it}} \left[ 1 - a_{i,t-1} - \phi(a_{i,t-1}, a_{it}) + \mathbb{E}_t \left[ M_{t+1} e^g (be_{i,t+1} + \tilde{v}(a_{i,t}, p_{t+1}, \omega_{i,t+1})) \mathbb{1}_{be_{i,t+1} > 0} \right] \right], \quad (B.6)
$$

subject to (B.5) and (17). Then

$$
\nu(b_{e_{it}}, a_{i,t-1}, p_t, \omega_t) = \begin{cases} 
\tilde{v}(a_{i,t-1}, p_t, \omega_{it}) + be_{it} & \text{if } be_{it} \geq 0 \\
0 & \text{otherwise}
\end{cases}
$$

It follows that, provided that $be_{it} \geq 0$, we can define scaled franchise value as

$$
\tilde{v}(a_{i,t-1}, p_t, \omega_t) = \nu(b_{e_{it}}, a_{i,t-1}, p_t, \omega_t) - be_{it}.
$$
Appendix C  Solution Algorithm

We discretize the stochastic processes for the probability of crisis $p$, the collateral value $\omega$, and the i.i.d. $\epsilon_c$ shocks following the method developed by Rouwenhorst (1995). For $p$ we use a 10-node Markov chain, while for $\omega$, and $\epsilon_c$ we use 5 nodes.

We then calculate asset prices. The equilibrium wealth-consumption ratio is found solving the fixed-point problem in (4). Under the assumptions described in the main text, the wealth-consumption ratio is function of $p$ only. The investor’s stochastic discount factor is computed from (5). Prices and returns for the Treasury bill and the loans to households are derived from the Euler equations presented in (6) and (A.5), respectively.

With this information at hand, we solve the problem of the bank. We solve for scaled franchise value on the discretized state space, by iterating on (B.4). The bank takes prices as given, and jointly decides on its capital and portfolio allocation to maximize the sum of current cash-flows and continuation value.

The solution to the firm’s problem is given in Appendix C of Gomes, Grotteria, and Wachter (2018).

We obtain model-implied moments by simulating 10,000 banks for 10,000 periods. The burn-out sample consists of the first 1,000 periods. Simulations yield a series for the exogenous state variables $\omega_{j,t}$, $p_t$, the endogenous state variables, $a_{j,t}$ and firm capital, as well as a series of shocks that determine the ex-post return on the bank investments and the ex post output of the firm. Using these series, we can calculate all quantities of interest based on the functions for the value of the bank and the value of the firm.

\[\text{We assume, for simplicity, that when a bank defaults, an identical bank is created with the same state variables. This implies we do not need to keep track of past defaults (the bank’s optimal decisions depend only on the current value of the state variables). This assumption allows us to maintain a stationary distribution of banks.}\]
<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Elasticity of intertemporal substitution</td>
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<td>Relative risk aversion</td>
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<tr>
<td>Volatility of log consumption growth (normal times)</td>
<td>$\sigma_c$</td>
<td>0.015</td>
</tr>
<tr>
<td>Average probability of crisis</td>
<td>$\bar{p}$</td>
<td>0.02</td>
</tr>
<tr>
<td>Impact of crisis on consumption size</td>
<td>$\xi$</td>
<td>$\log(1 - 0.30)$</td>
</tr>
<tr>
<td>Persistence in crisis probability</td>
<td>$\rho_p$</td>
<td>0.8</td>
</tr>
<tr>
<td>Volatility of crisis probability</td>
<td>$\sigma_p$</td>
<td>0.42</td>
</tr>
<tr>
<td>Government bill loss given crisis</td>
<td>$q$</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Notes: The table shows the parameter values used to solve the representative investor’s problem. The investor has Epstein and Zin (1989) utility with risk aversion $\gamma$, elasticity of intertemporal substitution $\psi$, and time discount factor $\beta$. Her consumption process is given by

$$C_{t+1} = C_t e^{\mu_c + \epsilon_{c,t+1} + \xi x_{t+1}},$$

where $x_{t+1}$ is a crisis indicator that takes a value of 1 with probability $p_t$. We assume that the logarithm of $p_t$ follows a Markov process with persistence $\rho_p$ and volatility $\sigma_p$. Conditional on a crisis realization, government bills experience a loss of $q$ per unit invested. The model is calibrated at annual frequency.
Table 2. Parameter Values – Bank

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return on deposits</td>
<td>$r_D$</td>
<td>0.48%</td>
</tr>
<tr>
<td>Loss given default on loans to households</td>
<td>$\mathcal{L}$</td>
<td>0.40</td>
</tr>
<tr>
<td>Loan-to-value ratio</td>
<td>$\kappa$</td>
<td>0.80</td>
</tr>
<tr>
<td>Volatility of local market component of collateral</td>
<td>$\sigma_\omega$</td>
<td>0.02</td>
</tr>
<tr>
<td>Persistence of local market component of collateral</td>
<td>$\rho_\omega$</td>
<td>0.90</td>
</tr>
<tr>
<td>Volatility of household component of collateral</td>
<td>$\sigma_j$</td>
<td>0.10</td>
</tr>
<tr>
<td>Capital regulation requirement</td>
<td>$\chi$</td>
<td>0.92</td>
</tr>
<tr>
<td>Adjustment cost on capital</td>
<td>$\eta_B$</td>
<td>3</td>
</tr>
</tbody>
</table>

Notes: The table shows the parameter values used to solve the problem of an individual bank. Each bank $i$ has access to a portfolio of $n$ infinitesimal one-period loans with the same loan-to-value ratio ($\kappa$) at issuance. Let $j = 1, \ldots, n$ index borrowers for bank $i$. Let

$$W_{ijt} = e^{\sigma_c e_{ct} + \xi x_t + \omega_{it} + \sigma_j \epsilon_{jt}}$$

denote the time-$t$ collateral value for borrower $j$ of bank $i$ (assuming a time-$t-1$ value of 1). If a loan defaults, the bank recovers $1 - \mathcal{L}$ of its collateral value $W_{ijt}$. The dividends for bank $i$ are:

$$\text{Div}_{it} = BE_{it} + D_{it} - A_{it} - \Phi(A_{it}, D_{it}, A_{i,t-1}).$$

where $\Phi(\cdot)$ are non-interest costs, inclusive of regulatory charges. They are given by:

$$\Phi(A_{it}, D_{it}, A_{i,t-1}) = \eta_B A_{i,t-1} \left( \frac{A_{it} - A_{i,t-1}}{A_{i,t-1}} \right)^2 + f D_{it} I_{D_{it} > \chi A_{it}}.$$ 

The bank $i$ deposits grow exogenously according to:

$$D_{i,t+1} = D_{i,t} e^g.$$ 

The model is calibrated at annual frequency.
Table 3. Parameter Values – Representative Firm

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Returns to scale</td>
<td>$\alpha$</td>
<td>0.40</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>$\delta$</td>
<td>0.08</td>
</tr>
<tr>
<td>Sensitivity to crises</td>
<td>$\phi$</td>
<td>2</td>
</tr>
<tr>
<td>Adjustment cost on capital</td>
<td>$\eta_F$</td>
<td>5</td>
</tr>
</tbody>
</table>

Notes: The table shows the parameter values used to solve the firm’s problem. The firm has a Cobb-Douglas production function of the form

$$Y_t = z_t^{1-\alpha}K_t^\alpha,$$

where the logarithm of the firm productivity level, $z_t$, follows a random walk process given by:

$$\log z_{t+1} = \log z_t + \mu_c + \epsilon_{c,t+1} + \phi \xi x_{t+1}.$$

The law of motion for each firm’s capital stock is: $K_{t+1} = (1-\delta)K_t + I_t \exp{\phi \xi x_{t+1}}$. The model is calibrated at annual frequency.
**Table 4.** Predicting crises in data and model

<table>
<thead>
<tr>
<th></th>
<th>LPM – Data</th>
<th>LPM – Model</th>
<th>Logit – Data</th>
<th>Logit – Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta L_{t-1}$</td>
<td>-0.0182</td>
<td>0.1579</td>
<td>-0.0917</td>
<td>3.4280</td>
</tr>
<tr>
<td>$\Delta L_{t-2}$</td>
<td>0.260</td>
<td>0.1580</td>
<td>6.641</td>
<td>3.4335</td>
</tr>
<tr>
<td>$\Delta L_{t-3}$</td>
<td>0.0638</td>
<td>0.0200</td>
<td>1.675</td>
<td>0.5877</td>
</tr>
<tr>
<td>$\Delta L_{t-4}$</td>
<td>-0.00423</td>
<td>0.0807</td>
<td>0.0881</td>
<td>1.9856</td>
</tr>
<tr>
<td>$\Delta L_{t-5}$</td>
<td>0.0443</td>
<td>0.0347</td>
<td>0.998</td>
<td>0.9774</td>
</tr>
</tbody>
</table>

| Sum of lag coefficients | 0.345 | 0.4513 | 9.311 | 10.4122 |
| $R^2$                  | 0.0126 | 0.0048 | 0.0379 | 0.0047 |

**Notes:** The table reports the coefficients and $R^2$ for the crises prediction equation as estimated by Schularick and Taylor (2012). Let the crisis event be identified by a binary variable equal to 1 if a crisis occurs and 0 otherwise. The first two columns report estimates from the following linear probability model (LPM)

$$\text{crisis}_{it} = \beta_0 + \sum_{j=1}^{5} \beta_j \Delta L_{t-j} + \epsilon_{it}. $$

The third and fourth columns report estimates from the following logit model:

$$P(\text{crisis} = 1) = \frac{e^{\beta_0 + \sum_{j=1}^{5} \beta_j \Delta L_{t-j}}}{1 + e^{\beta_0 + \sum_{j=1}^{5} \beta_j \Delta L_{t-j}}},$$

where financial crises in the data are as identified by Schularick and Taylor (2012) and $L$ stands for the total dollar value of bank loans in real terms. The data cover 14 developed countries between 1870 and 2008. In the model, a crisis is defined based on contemporaneous GDP growth so that the frequency equals that in the data (4%) and $L_t$ is defined as the sum of the dollar value of bank loans for each bank, scaled by that bank’s deposits.
Table 5. Dependent Variable: \( \Delta_{3y_{t+h}} \)

### Panel A: Benchmark Estimates

<table>
<thead>
<tr>
<th>( -1)</th>
<th>(0)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta_{3d_{i,t-1}}^{HH} )</td>
<td>0.15**</td>
<td>0.06</td>
<td>-0.07</td>
<td>-0.25***</td>
<td>-0.41***</td>
<td>-0.45***</td>
</tr>
<tr>
<td>(0.06)</td>
<td>(0.07)</td>
<td>(0.06)</td>
<td>(0.07)</td>
<td>(0.07)</td>
<td>(0.08)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>( \Delta_{3d_{i,t-1}}^{F} )</td>
<td>-0.04</td>
<td>-0.10**</td>
<td>-0.11***</td>
<td>-0.06**</td>
<td>-0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>(0.04)</td>
<td>(0.05)</td>
<td>(0.04)</td>
<td>(0.03)</td>
<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.02</td>
<td>0.06</td>
<td>0.10</td>
<td>0.10</td>
<td>0.14</td>
<td>0.15</td>
</tr>
</tbody>
</table>

### Panel B: Control for Deposit Insurance

<table>
<thead>
<tr>
<th>( -1)</th>
<th>(0)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta_{3d_{i,t-1}}^{HH} )</td>
<td>0.30</td>
<td>0.29</td>
<td>0.23</td>
<td>0.11</td>
<td>-0.04</td>
<td>-0.09</td>
</tr>
<tr>
<td>(0.23)</td>
<td>(0.22)</td>
<td>(0.18)</td>
<td>(0.14)</td>
<td>(0.12)</td>
<td>(0.12)</td>
<td>(0.12)</td>
</tr>
<tr>
<td>( \Delta_{3d_{i,t-1}}^{F} )</td>
<td>-0.03</td>
<td>-0.13</td>
<td>-0.15**</td>
<td>-0.12**</td>
<td>-0.07</td>
<td>-0.03</td>
</tr>
<tr>
<td>(0.09)</td>
<td>(0.08)</td>
<td>(0.07)</td>
<td>(0.05)</td>
<td>(0.05)</td>
<td>(0.05)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>( \Delta_{3d_{i,t-1}}^{HH}1_{DI} )</td>
<td>-0.182</td>
<td>-0.28</td>
<td>-0.37*</td>
<td>-0.45***</td>
<td>-0.47***</td>
<td>-0.47***</td>
</tr>
<tr>
<td>(0.23)</td>
<td>(0.23)</td>
<td>(0.19)</td>
<td>(0.16)</td>
<td>(0.15)</td>
<td>(0.15)</td>
<td>(0.15)</td>
</tr>
<tr>
<td>( \Delta_{3d_{i,t-1}}^{F}1_{DI} )</td>
<td>-0.01</td>
<td>0.04</td>
<td>0.05</td>
<td>0.07</td>
<td>0.06</td>
<td>0.04</td>
</tr>
<tr>
<td>(0.10)</td>
<td>(0.09)</td>
<td>(0.07)</td>
<td>(0.05)</td>
<td>(0.06)</td>
<td>(0.06)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.03</td>
<td>0.06</td>
<td>0.11</td>
<td>0.13</td>
<td>0.17</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Notes: Let \( y_{it} \) be the log real GDP per capita in local currency and \( d_{it}^{HH} \) and \( d_{it}^{F} \) be the household and firm debt to GDP ratios, respectively. \( 1_{DI} \) is an indicator function equal to 1 if the country had explicit deposit insurance enacted in time \( t - 3 \). For deposit insurance, dates before 2005 are from Demirgüç-Kunt, Kane, and Laeven (2005). For countries without a deposit insurance by 2005, scheme dates have been hand collected. Panel A presents the estimated coefficients and \( R^2 \) of the following equation

\[
\Delta_{3y_{i,t+h}} = \alpha_i + \beta_H \Delta_{3d_{i,t-1}}^{HH} + \beta_F \Delta_{3d_{i,t-1}}^{F} + u_{it},
\]

for \( h = -1, \ldots, 5 \). Each column gradually leads the left-hand-side variable by one year. Panel B presents the estimated coefficients and \( R^2 \) of the following equation

\[
\Delta_{3y_{i,t+h}} = \alpha_i + (\beta_{HH} + \beta_{HH}^{DI}1_{DI}) \Delta_{3d_{i,t-1}}^{HH} + (\beta_F + \beta_{F}^{DI}1_{DI}) \Delta_{3d_{i,t-1}}^{F} + u_{it},
\]

for \( h = -1, \ldots, 5 \). Each column gradually leads the left-hand-side variable by one year. Reported \( R^2 \) values are from within-country variation. We control for country fixed effects. Standard errors in parentheses are clustered by country. *, ** and *** indicate significance at the 0.1, 0.05 and 0.01 level, respectively. The panel is unbalanced and data are from 1960 to 2015.
Fig. 1. Excess Return on Private Loans. The figure shows the ex-ante expected rate of return on bank loans, $r_{t+1}^L$, relative to the rate of return earned on a one-year government bill, $r_{t+1}^G$, for each level of the probability of crisis, $p_t$, and alternative values of the current-period collateral, $\omega_t$. The expected return and the probability are in annual terms.
Fig. 2. Rates on deposits and Treasury bills The figure shows the deposit rate on checking accounts (US average) and the yield on the 3-month Treasury bill from March 1999 to May 2018. Treasury bill rates are from FRED. Data on checking deposits before 2009 are from Drechsler et al. (2017) while after 2009 are from FDIC.
Fig. 3. Bank Franchise Value. The figure shows the bank’s franchise value, scaled by deposits, $\tilde{v}_t = \left( \frac{V_t}{D_t} \right)$. Alternative levels of crises probability $p_t$ are plotted on the x-axis. Different lines represent different lagged asset-to-debt ratio $a_{t-1} = \left( \frac{A_{t-1}}{D_{t-1}} \right)$. $\omega_t$ is fixed to 0.
Fig. 4. Optimal Bank Lending. The figure shows the optimal amount of bank assets (lending), scaled by deposits. Alternative levels of crises probability $p_t$ are plotted on the x-axis. Different lines represent different lagged asset-to-debt ratio $a_{t-1} = \left( \frac{A_{t-1}}{D_{t-1}} \right)$. $\omega_t$ is fixed to 0.
Fig. 5. Portfolio Allocation. The figure shows the policy for portfolio allocation of an individual bank ($\varphi_t$). Alternative levels of crises probability $p_t$ are plotted on the x-axis. Different lines represent different lagged asset-to-debt ratio $a_{t-1} = \left( \frac{A_{t-1}}{D_{t-1}} \right)$. $\omega_t$ is fixed to 0. $\varphi$ equal to 1 represents investment in the portfolio of household loans, while $\varphi$ equal to 0 stands for investment in the government T-bill.
**Fig. 6. Optimal default probability.** The figure shows the endogenous default probability of an individual bank after optimally deciding on the amount of capital and its portfolio allocation. Alternative levels of crises probability $p_t$ are plotted on the x-axis. Different lines represent different lagged asset-to-debt ratio $a_{t-1} = \left( \frac{A_{t-1}}{D_{t-1}} \right)$. $\omega_t$ is fixed to 0.
**Fig. 7. Frequency of crises by credit growth.** The top figure shows the empirical average frequency of a crisis in year $t$ conditioning on a given quintile of credit-to-GDP growth rates from year $t - 5$ to $t$. Data are from Jordà, Schularick, and Taylor (2016). For each country, we compute the growth rate in the ratio of total loans to GDP between year $t - 5$ and $t$. Empirically, a crisis is a systemic financial crisis, as identified by Jordà et al. (2016). The bottom figure reproduces the relation in data simulated from the model using quintiles of credit growth rates from year $t - 5$ to $t$. Results are from simulating the model with 10,000 banks for 10,000 periods. A crisis occurs when the 1-year GDP growth rate is in the bottom 4% of its distribution.
Fig. 8. GDP and Household Debt growth. The top figure shows the empirical relationship between the (demeaned) GDP growth rate from year $t$ to $t + 3$ and the growth rate of the household debt to GDP ratio from year $t - 4$ to $t - 1$. Data are from the Bank of International Settlements and cover 39 countries between 1961 and 2012. The bottom figure reproduces the same relationship in the model using however the growth rate of aggregate bank’s loans (to household) from year $t - 5$ to $t$. Results are from simulating the model with 10,000 banks for 10,000 periods. The solid line is the estimated regression line from

$$
\Delta_3 y_{t, t+3} - \Delta_3 y_t = \alpha + \beta_H \Delta_3 d_{t, t-1}^{HH} + u_{it},
$$

where $y$ is GDP and $d^{HH}$ is the measure of credit to households.
Fig. 9. Impact of subsidies on bank franchise value. The figure shows bank franchise value scaled by deposits, \( \tilde{\nu}_t = \left( \tilde{V}_t/D_t \right) \), as a function of crisis probability \( p_t \) for two different levels of bank subsidies. We set \( a_{t-1} = 1.12 \) and \( \omega_t = 0 \). The case of low subsidies is our benchmark model. For the high subsidies scenario we lower the benchmark deposit rate by 6 basis points.
Fig. 10. Impact of subsidies on bank leverage. The figure shows the optimal ratio of assets to deposits $a_t = (A_t/D_t)$ as a function of crisis probability $p_t$ for two different levels of bank subsidies. We set $a_{t-1} = 1.12$ and $\omega_t = 0$. The case of low subsidies is our benchmark model. For the high subsidies scenario we lower the benchmark deposit rate by 6 basis points.
Fig. 11. Impact of subsidies on bank’s optimal portfolio composition. The figure shows the portfolio allocation of an individual bank ($\varphi_t$) for high and low subsidies (solid and dashed line respectively) and different levels of the probability of crisis $p_t$ keeping fixed the last period asset-to-debt ratio $a_{t-1} = (A_{t-1}/D_{t-1})$ to 1.123, and $\omega_t = 0$. $\varphi$ equal to 1 represents investment in the portfolio of household loans, while $\varphi$ equal to 0 stands for investment in the government T-bill. The case of low subsidies is our benchmark model. For the high subsidies scenario we lower the benchmark deposit rate by 0.06%.