To Pool or Not to Pool?
Security Design in OTC Markets

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Abstract

This paper studies the optimality of pooling and tranching for a privately informed originator who offers securities in an over-the-counter (OTC) market in which buyers have market power. Such a market scenario is particularly relevant when many of the natural counterparties face binding regulatory constraints. Contrary to the standard result that pooling and tranching are optimal practices, we find that selling assets separately may be optimal for originators, as doing so weakens buyers' incentives to inefficiently screen them. Our results shed light on recently observed time-variation in the prevalence of pooling and tranching in financial markets.

1 Introduction

Structured products are typically originated in over-the-counter (OTC) markets, where asymmetric information and market power have been shown to be prevalent frictions.[1] Market power is particularly likely to emerge on the demand side of the origination market when few of the typical buyers are well-positioned to acquire new securities; for example, when the financial institutions that would usually purchase structured products face binding regulatory constraints and thus, increased holding costs. To account for these frictions, we study the optimality of pooling and tranching for a privately informed originator who offers securities in an OTC market where buyers have market power. In doing so, our environment deviates from the standard settings considered in the existing literature on pooling and tranching, where buyers are competitive and deep-pocketed.

In this setting, we find that, counter to conventional wisdom, the separate sale of assets may be optimal for a privately informed originator. This result emerges because the issuer anticipates that pooling assets causes diversification, thereby reducing information asymmetry and associated information rents. Moreover, diversification can cause strategic buyers with market power to choose pricing strategies that lead to greater inefficient rationing. That is, selling assets separately may not only be beneficial for the originator but also sustain greater trading volume and improve the social efficiency of trade.

Our paper emphasizes how in recent years liquidity shortages among major institutions participating in OTC markets might have been an important driver of the dramatic declines in asset-backed security (ABS) issuances, which occurred concurrently with an increase in the volume of assets sold separately.[2]

The existing literature on security design has already identified circumstances under which, in "centralized" markets, an issuer may prefer not to pool assets. Compared with this literature, our focus on decentralized markets and market power on the demand side allows us to derive

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[1]For evidence that OTC trading often involves heterogeneously informed traders, see Green, Hollifield, and Schürhoff (2007), Jiang and Sun (2015), and Hollifield, Neklyudov, and Spatt (2017). For evidence that trade tends to be concentrated among a small set of dealers in OTC markets, see Cetorelli et al. (2007), Atkeson, Eisfeldt, and Weill (2012), Li and Schürhoff (2014), Begenau, Piazzesi, and Schneider (2015), Hendershott et al. (2015), Di Maggio, Kermani, and Song (2017), and Siriwardane (2017).

[2]In 2015, issuance volume of ABS in the U.S. was 60% lower than it was in 2006, while the issuance volume of CDO was 80% lower. In contrast, the total issuance volume in fixed income markets was 3% higher in 2015 than in 2006. For more data, see the Securities Industry and Financial Markets Association: http://www.sifma.org/research/statistics.aspx.

novel insights. DeMarzo (2005) considers the signaling-through-retention model of DeMarzo and Duffie (1999) with price-taking (i.e., competitive) buyers and shows that the simple pooling of assets decreases profits for the issuer since profits are a convex function of quality. Yet DeMarzo (2005) also shows that the originator optimally issues debt on the pool of assets if the number of assets grows large, as this practice reduces residual risks and the information sensitivity of the security being issued.[3] In contrast to DeMarzo (2005), whose setup can be thought of as a centralized market where (price-taking) buyers compete for the asset, we model an issuer who cannot credibly signal the quality of his assets and faces a buyer endowed with market power, capturing the above-described empirical regularities of over-the-counter markets.

The closest paper to ours is that of Biais and Mariotti (2005), who analyze a model where the security design stage is followed by a stage where either the issuer or the prospective buyer chooses a trading mechanism (i.e., a price-quantity menu) for selling the designed security. The paper shows that in both cases issuers with low quality securities participate in the market, whereas high quality issuers might not (despite the presence of gains to trade). In particular, when the buyer chooses the mechanism, he effectively screens the issuer, trading off higher volume with lower issuer participation. In contrast, when the issuer chooses the mechanism, the setup is equivalent to one with multiple competitive buyers. Biais and Mariotti (2005) show that issuing debt on a risky asset is optimal in both cases, since the debt contract's low information sensitivity helps avoid market exclusion. However, unlike our paper, Biais and Mariotti (2005) do not consider the situation where the issuer wishes to sell multiple assets and thus do not address the optimality of pooling and tranching.

Axelson (2007) studies a security design problem with multiple assets where the designed security is (centrally) traded through a uniform price auction among several informed buyers. In contrast to our setting, *buyers* have superior information relative to the issuer and prices are determined by a marginal bidder who is indifferent about buying the asset. The uninformed issuer then aims to minimize underpricing associated with a winner's curse problem. Axelson (2007) finds that pooling several assets and issuing debt on these assets is optimal for the issuer when competition among buyers is low or when the signal distribution is continuous, whereas selling assets

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[3]See also Hartman-Glaser, Piskorski, and Tchistyi (2012) who model a moral hazard problem between a principal and a mortgage issuer and show that the optimal contract features pooling of mortgages with independent defaults, which facilitates effort monitoring.

separately is preferred when competition is high and the signal distribution is discrete, contrasting with our results. The latter result arises in the uniform price auction setting, because for a given number of buyers, the probability that the pivotal bidder has the highest possible signal is higher when a single asset is sold than when a pool of assets is sold. From a social perspective, pooling is not harmful in Axelson (2007) since social surplus is redistributed to the buy side, as high quality pools are simply traded at a discount. In contrast, in our model, high quality pools may not be traded at all in equilibrium (due to buyers' optimal screening strategies), thereby preventing the realization of gains to trade.

Farhi and Tirole (2015) study whether an issuer bargaining with a single buyer prefers to sell the asset as a whole or to separate the information sensitive part of the asset from the riskless part. The paper focuses on how this choice affects information acquisition by both parties. As an extension, Farhi and Tirole (2015) consider splitting an asset viewed as a bundle of an information sensitive part and a riskless part into smaller bundles. Yet Farhi and Tirole (2015) assume that the information sensitive securities issued on these smaller bundles are all perfectly correlated, as they are fractions of the same risky asset. Thus, Farhi and Tirole's analysis does not feature diversification, which is an important channel in the context of the pooling decisions we consider.

In the next section, we describe the general environment of our model. In Section 3, we study a simple example where the originator must decide whether to issue an equity security on a pool of a continuum of assets, or issue a separate equity security for each asset. This example introduces the intuition underlying our main results and illustrates how the presence of a buyer with market power changes the optimal security design relative to the case with competitive buyers. Section 4 formalizes our main results by allowing for a discrete number of assets, more general payoff distributions, and debt securities. Section 5 discusses the robustness of our results to various alternative specifications of the environment. The last section concludes.

2   The Model

Suppose an issuer has $n \geq 2$ assets to sell, each with future payoff X_i for $i \in \{1, 2, ..., n\}$. We consider the security design problem in which the issuer must decide whether to offer the n assets separately or in bundles, and whether to offer equity or debt securities on these underlying assets.

Following the existing literature (see, e.g., Biais and Mariotti 2005), we assume that the issuer does not possess private information at the security design stage. Yet, the issuer learns future realizations of $X_1, \ldots, X_n$ before trading occurs and is thereby informed about the value of the security he is offering for sale. Buyers do not know the realizations of X_i when trade occurs, but they know the composition of the pool, that is, they know which random payoffs X_i are bundled together and which security types are issued.

We study and compare two market scenarios to highlight the importance of market power in the decision whether to pool assets. In the first scenario, *several* deep-pocketed buyers are better equipped to hold claims to future cash flows than the seller is (who needs liquidity today). In this case, these buyers compete for the securities issued by the seller. In the second market scenario, *only one* buyer is better equipped to hold the claims to future cash flows than the seller is. This type of situation may, for example, arise when most potential counterparties in the market face similar regulatory constraints as the seller. In this case, the one buyer with a superior liquidity position has market power; he is the only one making relevant offers to the issuer.[4]

As is common in the security design literature, we capture differential liquidity needs across traders through differences in discount factors. Whereas the issuer has a discount factor $\delta \in (0, 1)$, one buyer or multiple buyers have a discount factor of 1. Thus, the issuer's reservation value for a security with future payoff v is δv. In contrast, a buyer with flush liquidity values this future payoff at v. This difference in valuations implies that gains from trade equal to $(1 - \delta)v$ are realized if trade takes place.

The timing of the game is as follows. First, the issuer chooses the number of assets to bundle in each pool and a security type for each pool. Second, the issuer becomes informed about the realizations of each payoff X_i. Third, the buyer(s) make(s) take-it-or-leave offers to the issuer. Fourth, the issuer decides whether or not to accept any of these offer(s); if multiple buyers offer an identical price that is accepted by the issuer, the asset is randomly allocated among these highest bidders. Finally, the payoffs are realized.

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[4]We show in Appendix B that this second market scenario is similar to the more general case in which multiple buyers face position limits (e.g., due to capital requirements) constraining their total demand to be at least marginally below the total supply of assets for sale.

3 An Illustrative Example

In this section, we present a simple example that illustrates the main intuition for why the seller's pooling decision crucially depends on the presence of market power on the demand side of the origination market. In this example, the issuer only decides between selling a pool of assets and selling assets separately; all securities sold are assumed to be equity securities. In the next section, we will present the general analysis of the model, in which we consider a finite number of assets, more general payoff distributions, and debt securities.

Example. *The issuer owns a continuum of assets of measure one with i.i.d. payoffs $X_i \sim U[0,1]$. The seller decides between selling the assets as a pool and selling them separately.*

In the following, we first consider the market scenario in which multiple potential buyers in the market have flush liquidity (that is, they have a discount factor equal to one). Afterwards, we will consider the scenario in which only one buyer is better equipped to hold claims to the assets' future cash flows than the seller is.[5]

Competitive demand: Multiple buyers with flush liquidity. When multiple buyers have a discount factor of 1, they effectively compete in quotes à la Bertrand and offer a price p that is equal to the expected security payoff conditional on the seller accepting the offer. That is, given a security with future payoffs v, the equilibrium price $\hat{p}$ is the highest price that solves

$$\hat{p} = \mathbb{E}[v|\delta v \le \hat{p}]. \tag{1}$$

Correspondingly, when the issuer sells the assets separately, that is, when he offers a continuum of securities with payoffs $v_i = X_i$, the equilibrium price $\hat{p}_i$ for each security is the highest price that satisfies the relation

$$\hat{p}_i = \mathbb{E}[v_i|\delta v_i \le \hat{p}_i] = \min\left[\frac{\hat{p}_i}{2\delta}, 0.5\right]. \tag{2}$$

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[5]Note that, whether the issuer pools the assets or not, he is still offering all assets to the buyer(s). Hence, even if we allowed for risk-aversion, pooling assets would not by itself lead to better risk sharing among traders. The main impediment to risk sharing would then be the fact that the issuer's private information may result in socially inefficient trade breakdowns, which is already the key channel we study in this paper.

If the issuer accepts the quoted prices $\hat{p}_i$ with probability one, then the prices must satisfy $\hat{p}_i = \mathbb{E}[v_i] = 0.5$. In contrast, when the issuer rejects prices with positive probability—and adversely selects the buyers' quotes—the equilibrium prices have to be below 0.5. It is straightforward to see that the solution to Equation (2) depends on the issuer's discount factor δ. For $\delta < 0.5$, the prices quoted on the separately sold assets are $\hat{p}_i = 0.5$. Since the issuer's outside option is poor, he is effectively committed to always accept these quotes. As a result, the seller's ex ante surplus from trade is $0.5 \cdot (1 - \delta)$, which is also the total surplus from trade (the competitive buyers receive zero ex ante surplus). In contrast, for $\delta > 0.5$, the prices $\hat{p}_i = 0$ obtain, which imply zero trade volume and zero trade surplus for all parties involved. In this case, the adverse selection problem impedes trade, because the seller has a greater incentive to reject quotes and strategically retain the asset when it pays a high cash flow.

In contrast, when the issuer sells the assets as one pool with payoff $v = \int_0^1 X_i di$, the law of large numbers applies, that is, $v = 0.5$ almost surely. Thus, buyers quote a price $\hat{p} = 0.5$ for this pool independent of the issuer's discount factor δ. The seller's ex ante surplus is equal to the total surplus from trade, $0.5 \cdot (1 - \delta)$.

In sum, when the gains to trade are sufficiently large and adverse selection is not a concern ($\delta < 0.5$), selling separately and selling as a pool is equally beneficial from the perspective of the issuer. In contrast, when adverse selection is a concern (for $\delta > 0.5$), pooling is strictly beneficial for the seller, because it avoids the adverse selection problem. Eliminating information asymmetry has only upside for the issuer in this market scenario with *competitive* buyers, as he does not need to retain private information to extract all gains from trade.

Monopolistic demand: One buyer with flush liquidity. In the second scenario, only one potential buyer has a discount factor of 1. When the issuer sells each asset separately to this buyer, that is, when he offers securities with payoffs $v_i = X_i$, the buyer optimally chooses for each security i a price quote $p_i^* \in [0, \delta]$ that maximize his ex ante profit:

$$\Pr(\delta v_i \leq p_i^*)(\mathbb{E}[v_i | \delta v_i \leq p_i^*] - p_i^*) = \frac{p_i^*}{\delta}\left(\frac{1}{2\delta} - 1\right). \tag{3}$$

Similar to before, the solution to this problem depends on the magnitude of the gains from trade,

which are governed by the issuer's discount factor δ. When $\delta < 0.5$, the buyer optimally quotes the prices $p_i^* = \delta$, which ensures that trade takes place with probability one. Because the issuer has strong liquidity needs (i.e., a low discount factor) and the gains to trade are large, the buyer optimally quotes prices that are high enough to ensure the issuer always accepts those offers. The seller's ex ante surplus is 0.5δ. In contrast, when $\delta > 0.5$ and the gains to trade are smaller, the buyer quotes the prices $p_i^* = 0$, implying zero trade volume and zero trade surplus for all parties involved. Again, it is the adverse selection problem that impedes trade in this case.

Next, consider the possibility that the issuer pools the assets and sells a security with payoff $v = \int_0^1 X_i di = 0.5$. Now, the buyer will quote an offer price $p^* = 0.5\delta$. As in the scenario with multiple buyers, pooling yields perfect diversification and eliminates adverse selection concerns. Yet, now that the demand side has market power, fully eliminating information asymmetries has *no upside* for the issuer. Facing no informational disadvantage, the buyer can extract all trade surplus, leaving the seller just as well off as without trade.

This last result strikingly highlights the relevance of market power for the optimality of pooling from the perspective of the originator. In the presence of such market power, the issuer's only source of surplus are information rents, which require retaining private information. Thus, perfect diversification is never strictly optimal for the issuer under this scenario.

In sum, if the gains to trade are large enough to contain trade inefficiencies due to adverse selection ($\delta < 0.5$), the issuer prefers to sell assets separately. In contrast, when the gains to trade are small ($\delta > 0.5$) the seller is indifferent between selling assets separately and selling assets as a pool. Selling separately yields zero surplus for the seller, because adverse selection impedes trade. Selling as a pool also yields zero surplus for the seller, as the buyer can extract all rents absent information asymmetries (due to perfect diversification).

4   Main Analysis

In this section, we show that the key insights from the previous illustrative example hold more generally, when the issuer has a finite number of assets with more general payoff distributions, and debt securities are allowed. Moreover, this more general analysis will reveal how diversification's impact on the shape of distributions affects buyers' screening behavior in the presence of market

power. The issuer can, in turn, preempt inefficient screening through optimal security design.

To streamline the exposition, our baseline analysis will feature an issuer who has $n = 2$ assets for sale, although Section 5 shows that it is straightforward to extend our results to $n > 2$. The assets' payoffs X_i are assumed to be identically and independently distributed according to the cumulative distribution function $F(\cdot)$ with a positive density $f(\cdot)$ everywhere on its domain $[0, \bar{x}]$.[6] We also assume that the distribution is "well-behaved," in line with the existing literature (see, e.g., Myerson 1981).

Assumption 1. *The function*

$$h(x) \equiv x\frac{f(x)}{F(x)} \tag{4}$$

is monotonically decreasing on the support of the distribution $[0, \bar{x}]$.

As in the previous section, we first consider a market scenario in which multiple buyers have more liquidity that the seller (that is, a discount factor of 1 instead of δ), and then move to the scenario in which only one buyer satisfies this property. Before proceeding with these analyses, we establish a set of useful preliminary results.

4.1 Preliminary Results

For our analysis of separate asset sales it is useful to establish the following result.

Lemma 1. *The sale of two separate securities, each issued on an asset producing a random payoff X_i distributed according to a c.d.f. $F(x)$ that satisfies Assumption 1, is equivalent from all traders' perspective to the sale of the same type of security issued on an asset producing a random payoff $2X_i$ with c.d.f. $F_2(x) = F(\frac{x}{2})$.*

Proof. See Appendix A. □

In the following, the subscripts p and 2 are used to identify variables pertaining to the case where the underlying asset is a pool (e.g., $F_p(x)$) and where assets are kept separate (e.g., $F_2(x)$),

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[6]The necessary condition for our results is that assets' payoffs are not perfectly correlated as the analysis can be generalized to the case where assets' payoffs exhibit some correlation.

respectively. Lemma 1 implies that the issuer's choice of whether to pool the two assets is equivalent to choosing whether to sell securities associated with the underlying payoff $(X_1 + X_2)$ or with the payoff $2X_i$. This comparison is convenient since both $2X_i$ and $(X_1 + X_2)$ are distributed on the same interval $x \in [0, 2\bar{x}]$, and the difference in the optimal trading decisions of the issuer and buyer(s) will be solely due to differences in the shapes of the underlying distributions. Next, we establish an important feature of these distributional differences.

Lemma 2. *The distribution of the payoff* $(X_1 + X_2)$*, where* X_1 *and* X_2 *are i.i.d. random variables with c.d.f.* $F(x)$*, second-order stochastically dominates the distribution of the payoff* $2X_i$*, that is,*

$$\int_0^x [F_2(y) - F_p(y)]dy \geq 0 \tag{5}$$

for any $x \in [0, 2\bar{x}]$*.*

Proof. See Appendix A. □

As highlighted in the analysis below, the impact of pooling on the shape of the distribution will crucially affect the monopolistic buyer's incentives to screen the issuer. In particular, it may be the case that separate securities can be sold efficiently, whereas a pool of the same assets will lead to inefficient rationing.

To illustrate our formal results, we will repeatedly revisit a parameterized example. In this example, the distribution function associated with each asset's payoff X_i is the sum of two uniformly distributed random variables on the interval $[0, 1]$. Figure 1 compares the probability density function for the pooled payoff $(X_1 + X_2)$ with the one for the normalized payoff $2X_i$. As formalized in Lemma 2, pooling assets results in a second-order stochastically dominant payoff distribution.

Equipped with these preliminary results, we now proceed to the main analysis of the optimality of pooling, first in a market with competitive buyers and then in a market with a monopolistic buyer.

4.2   Competitive Demand

We now solve for the equilibrium pricing decisions by several competing buyers who are offered an equity or a debt security with an arbitrary underlying asset, and then compare the issuer's profits
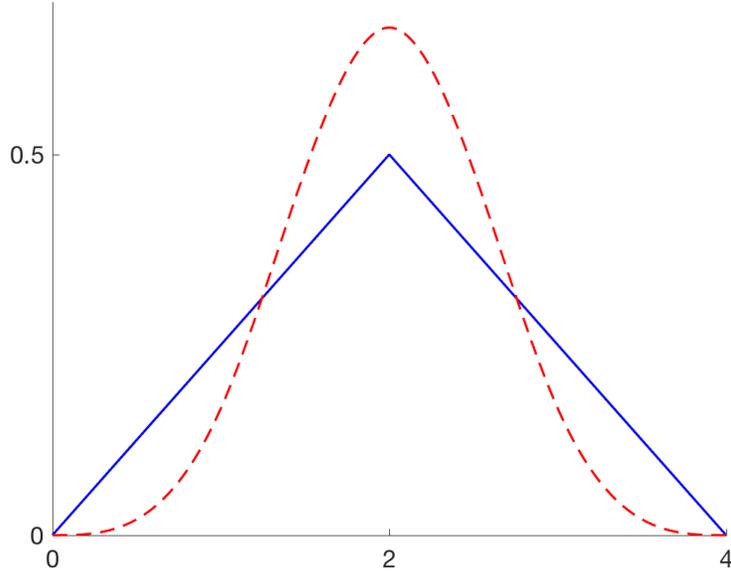
Figure 1: Density functions with and without pooling. The graph illustrates the probability density functions for the pooled payoff $(X_1 + X_2)$ (red, dashed) and for the payoff $2X_i$ (blue, solid). The parameterization assumes that each X_i, for $i = 1, 2$, is the sum of two uniformly distributed random variables on the interval $[0, 1]$.

from designing equity and debt securities on two separate assets and on a pool of two assets.

4.2.1 Buyers' Equilibrium Pricing of Equity and Debt Securities

In general, if the issuer offers a security with payoff v and receives a price quote p, his ex-ante expected profit is:

$$S(p) = \Pr(\delta v \leq p)(p - \delta \mathbb{E}[v | \delta v \leq p]). \tag{6}$$

Similarly, a buyer's ex-ante profit from purchasing an equity security on an asset with payoff v at a price quote p is:

$$B(p) = \Pr(\delta v \leq p)(\mathbb{E}[v | \delta v \leq p] - p). \tag{7}$$

As buyers are effectively competing in quotes à la Bertrand, the issuer is able to extract all the surplus from trade. Accordingly, each buyer offers a price $\hat{p}$ that is equal to his ex ante valuation of the security. This valuation is equal to the expected security payoff conditional on trade occurring (which happens for $\delta v \leq \hat{p}$):

$$\hat{p} = \mathbb{E}[v | \delta v \leq \hat{p}]. \tag{8}$$

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Hence, each buyer's ex-ante equilibrium profit at this price is equal to zero, $B(\hat{p}) = 0$. In contrast, the issuer's ex-ante profit simplifies to:

$$S(\hat{p}) = (1 - \delta) \Pr(\delta v \le \hat{p}) \, \mathbb{E}[v | \delta v \le \hat{p}], \tag{9}$$

which is also equal to the expected total surplus from trade.

If the information asymmetry between the issuer and the buyers is severe, some high issuer types are excluded from the market. In this case, the equilibrium price is lower than the unconditional expected value of the security, $\hat{p} < \mathbb{E}[v]$. If an equity security leads to this type of rationing, the issuer is better off designing a debt security, as shown in Biais and Mariotti (2005).[7]

By definition, a debt security with a face value D issued on an asset with a payoff v has a payoff function $v^D = \min\{v, D\}$. Analytically, the only difference with respect to an equity security is that there is now a positive mass, $\Pr(v \ge D)$, of high issuer types offering a constant payoff D rather than v. If the issuer is able to sell this debt security for a price quote $p = \delta D$, which guarantees participation of all issuer types, his profit is given by:

$$S^D(p) = p - \delta \, \mathbb{E}[v^D | \delta v^D \le p] = \delta(D - \mathbb{E}[v^D | \delta v^D \le p]), \tag{10}$$

and the buyer's profit is given by:

$$B^D(p) = \Pr(\delta v^D \le p)(\mathbb{E}[v^D | \delta v^D \le p] - p) = B(\delta D) + (1 - \delta)(1 - F(D))D. \tag{11}$$

The issuer chooses the face value D that maximizes his profits. Biais and Mariotti (2005) establish that it is optimal to choose the highest face value $\hat{D}$ such that there is no exclusion of high issuer types,[8] i.e., the equilibrium quoted price is $\hat{p} = \delta \hat{D}$, and the buyers' profit is equal to zero, $B^{\hat{D}}(\delta \hat{D}) = 0$. Hence, for this choice of a face value, the equilibrium price is $\hat{p} = \mathbb{E}[v^{\hat{D}} | \delta v^{\hat{D}} \le \hat{p}]$ and the issuer's profit simplifies to:

$$S^{\hat{D}}(\hat{p}) = \delta(1 - \delta)\hat{D}. \tag{12}$$

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[7]See Proposition 5 on p.631 and Proposition 7 on p.634.
[8]See Proposition 4 on p.629.

We now proceed to the comparison of the issuer's profits from designing equity and debt securities either on two separate assets or on a pool of two assets.

4.2.2 Comparing Payoffs from Equity Securities

The next two propositions show that the issuer prefers to pool assets when he faces competitive buyers.

Proposition 1. *In a market with competitive buyers, the issuer finds it strictly optimal to issue debt rather than equity for the same range of values for δ, independent of whether he chooses to pool assets or not.*

Proof. From (8) there is no exclusion of high issuer types when the unconditional expected value of an asset v is higher than the reservation value of the highest issuer type, that is, $\mathbb{E}[v] \geq \delta \max\{v\}$. Since the unconditional expected value as well as the maximum payoff of a pool of two assets is the same as those for separate assets, i.e., $\mathbb{E}[X_p] = \mathbb{E}[X_2]$ and $\max\{X_p\} = \max\{X_2\} = 2\bar{x}$, exclusion happens in the two cases whenever:

$$\delta > \frac{\mathbb{E}[X_2]}{2\bar{x}}. \tag{13}$$

Therefore, if $\delta \leq \mathbb{E}[X_2]/[2\bar{x}]$, i.e., if the gains from trade are high enough, there is no exclusion and the issuer obtains the equilibrium price $\hat{p}_p = \mathbb{E}[X_p]$ for the pool of two assets. Similarly, the issuer obtains the equilibrium price $\hat{p}_2 = \mathbb{E}[X_2]$ for separate assets. Hence, the issuer's profits are the same in the two cases.

On the other hand, if $\delta > \mathbb{E}[X_2]/[2\bar{x}]$, issuing equity would lead to some exclusion. As a result, the issuer finds it optimal to offer debt securities in both cases. This situation is analyzed below. □

Figure 2 compares the issuer's payoff from issuing equity securities on two separate assets or on a pool of these assets in our earlier parameterization, assuming competitive buyers. As formalized in Proposition 1, for any level of δ the issuer is weakly better off selling equity on the pool than on the separate assets when facing competitive buyers.
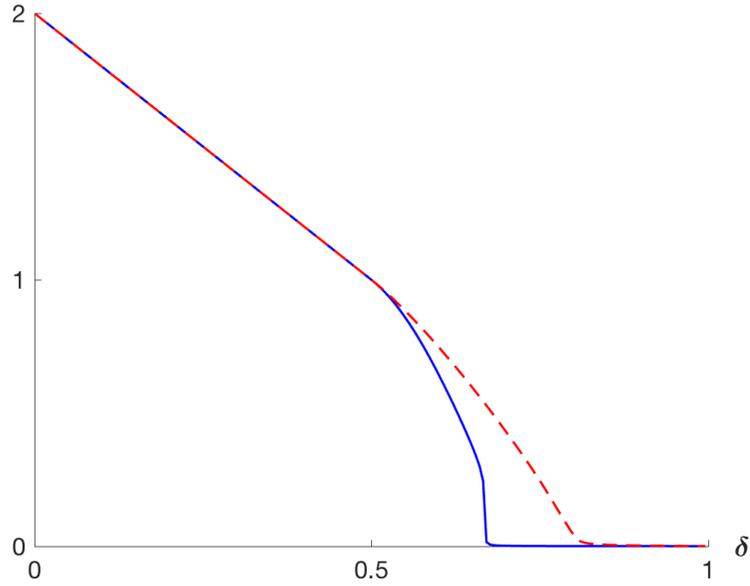
Figure 2: Issuer profits from pooling vs. separate sales. The figure illustrates the seller's ex ante profit when issuing equity securities separately (blue) or on a pool of assets (red) in a market with competitive buyers. The parameterization assumes that each X_i, for $i = 1, 2$, is the sum of two uniformly distributed random variables on $[0, 1]$.

4.2.3 Comparing Payoffs from Debt Securities

We now show that even once the issuer can sell debt securities he weakly prefers pooling assets.

Proposition 2. *In a market with competitive buyers, the issuer obtains weakly higher profits by issuing an optimal debt security on the pool of two assets rather than by issuing optimal debt securities on each asset.*

Proof. As argued above, if $\delta > \mathbb{E}[X_2]/[2\bar{x}]$ and the sale of the equity security involves exclusion of some issuer types, it is optimal for the issuer to design a debt security with a face value $D \in (0, 2\bar{x})$. In this case, the buyers' profit (11) can be written as:

$$B^D(\delta D) = (1 - \delta)D - \int_0^D F(x)dx. \tag{14}$$

Therefore, it follows from Lemma 2 and equation (5) that this profit function for the pooled sale is

higher that the same profit function for the separate sale:

$$B_p^D(\delta D) \geq B_2^D(\delta D) \tag{15}$$

for any $D \in (0, 2\bar{x})$.

The two profit functions have the same value at $D = 2\bar{x}$,

$$B_p^{2\bar{x}}(\delta 2\bar{x}) = B_2^{2\bar{x}}(\delta 2\bar{x}) = B(\delta 2\bar{x}), \tag{16}$$

and this value is negative as long as there is some exclusion in the equity case, $B(\delta 2\bar{x}) < 0$. Moreover, both functions are decreasing at the right end.

Hence, since the optimal face value of debt $\hat{D}$ is determined implicitly by $B^D(\delta \hat{D}) = 0$, we have $\hat{D}_p \geq \hat{D}_2$. The optimal face value of debt issued on the pool of two assets is higher than the optimal face value of debt issued on separate assets.

Finally, since evaluating (12) at the equilibrium price $\hat{p} = \delta \hat{D}$ yields that the issuer's profit is equal to $S^{\hat{D}}(\hat{p}) = \delta(1 - \delta)\hat{D}$, we know that $S_p^{\hat{D}_p}(\delta \hat{D}_p) \geq S_2^{\hat{D}_2}(\delta \hat{D}_2)$. The issuer obtains weakly higher profits by issuing optimal debt on the pool of two assets rather than by issuing optimal debt on separate assets. □

In Figure 3, we revisit the earlier parameterization where each asset produces a payoff X_i that is the sum of two uniformly distributed random variables and compare the issuer's payoff from issuing equity or debt securities on two separate assets or on a pool of these assets. As formalized in Proposition 2, for any level of δ the issuer is weakly better off pooling the assets and selling debt on the pool. In fact, for large enough δ issuing a debt security on the pool of assets strictly dominates the three alternative strategies.

4.3 Monopolistic Demand

We now examine the optimal pricing decisions of a monopolistic buyer who is offered an equity or a debt security with arbitrary payoffs and then compare the issuer's profits from designing equity and debt securities on two separate assets and on a pool of two assets.

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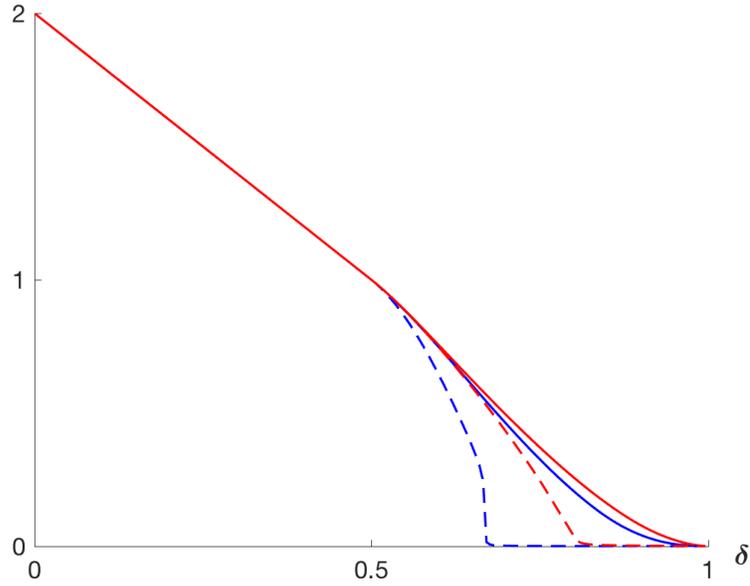
Figure 3: Issuer profits for all available security designs. The figure illustrates the seller's ex ante profit when issuing equity (dashed) or debt (solid) securities separately (blue) or on a pool of assets (red) in a market with competitive buyers. The parameterization assumes that each X_i, for $i = 1, 2$, is the sum of two uniformly distributed random variables on $[0, 1]$.

4.3.1 Buyer's Optimal Pricing of Equity and Debt Securities

In the second market scenario, only one buyer has a discount factor of one. This buyer thus acts as a monopolist. He optimally uses a take-it-or-leave-it offer to screen the privately informed issuer.[9]

Recall that a buyer's ex-ante profit from purchasing an equity security on an asset with payoff v for a price quote p is given by:

$$B(p) = \Pr(\delta v \leq p)(\mathbb{E}[v|\delta v \leq p] - p). \tag{7}$$

It follows that the buyer with market power chooses to quote a price p^* that maximizes his profits:

$$p^* = \arg \max_{p \in [0, \delta \bar{v}]} B(p). \tag{17}$$

The optimal price p^* uniquely corresponds to the threshold for the marginal participating issuer

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[9]As in Biais and Mariotti (2005), a take-it-or-leave-it offer is the optimal trading mechanism for a buyer with market power.

type $v^* = p^*/\delta$. Issuer types with payoffs below v^* participate in the trade while issuer types with payoffs above v^* get excluded (screened out) from the trade.

Using a density function $f(v)$ for an asset's random payoff v, we can rewrite the buyer's profit as:

$$B(p) = \int_0^{p/\delta} (v - p) f(v) dv. \tag{18}$$

Under the regularity condition we imposed (Assumption 1), we can solve the buyer's profit maximization problem (17) using its first-order condition. The optimal price p^* or, equivalently, the marginal participating issuer type v^*, is then characterized by:

$$(1 - \delta) v^* f(v^*) - \delta F(v^*) = 0. \tag{19}$$

When choosing p^*, the buyer trades off benefits of convincing more issuer types to participate in the trade with the losses associated with a higher price paid to participating issuer types.

As with competitive buyers, Biais and Mariotti (2005) show[10] that if some high issuer types get excluded from the market, i.e., $v^* < \bar{v}$, when the equity security on an asset v is traded, the issuer is better off designing and offering a debt security on the asset v instead of equity.

As described above, if the issuer offers a debt security with a face value D, its payoff is given by $v^D = \min\{v, D\}$. Consequently, the buyer's pricing decision is equivalent to what he would face if the issuer types with $v \in [D, \bar{v}]$ were replaced by a positive mass $\Pr(v \geq D)$ of issuers whose asset's payoff is D. Since the reservation value of this issuer type is δD, if the buyer were to find it optimal to offer a price below δD, the positive mass of this type would reject the offer and we would be back to a situation consistent with the issuance of equity. In other words, trading debt is equivalent to trading equity whenever the buyer quotes a price $p < \delta D$.

As in the competitive market scenario, if the buyer quotes a price $p = \delta D$ for a debt security with a face value D his profit is given by:

$$B^D(p) = \Pr(\delta v^D \leq p)(\mathbb{E}[v^D | \delta v^D \leq p] - p) = B(\delta D) + (1 - \delta)(1 - F(D)) D, \tag{11}$$

while the issuer's profit is given by (10). Therefore, when trying to buy a debt security with a face

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[10]See Proposition 5 on p.631 and Proposition 8 on p.636.

value D, the buyer is now comparing two strategies: (i) offer the high price $p = \delta D$, which yields the profit $B^D(\delta D)$, and (ii) offer the optimal equity price $p^* = \delta v^*$, which yields the profit $B(\delta v^*)$.

Choosing between different face values D the issuer maximizes his profits taking into account the buyer's optimal decision. Biais and Mariotti (2005) establish that it is optimal to choose the highest face value $D^* > v^*$ such that there is no high issuer type exclusion,[11] i.e., the buyer quotes the price $p^* = \delta D^*$, and the buyer is indifferent between his two options:

$$B^{D^*}(\delta D^*) = B(\delta v^*). \tag{20}$$

Using the derivations above for arbitrary payoff distributions, we can now compare the issuer's profits from designing equity and debt securities when he pools the two assets and when he does not.

4.3.2 Comparing Payoffs from Equity Securities

Here we analyze the screening incentives of a buyer with market power. We show that pooling assets increases the buyer's incentives to screen an issuer for low values of the discount factor δ and can thereby be detrimental to an issuer facing a buyer with market power.

Consider again the first-order condition for the buyer's profit maximization problem (19). At the upper boundary, the buyer finds it optimal not to screen the issuer and quote the efficient price $p = \delta \bar{v}$ whenever $(1 - \delta)\bar{v}f(\bar{v}) - \delta F(\bar{v}) \geq 0$. Note that when $f(\bar{v}) = 0$ efficient trade is impossible since the buyer can obtain a positive gain by virtually costlessly decreasing the marginal price. If, however, $f(\bar{v}) > 0$ the threshold for efficient trade on δ is:

$$\delta \leq \bar{\delta} \equiv \frac{\bar{v}f(\bar{v})}{1 + \bar{v}f(\bar{v})}. \tag{21}$$

Therefore, if $f(\bar{v}) = 0$ or if $f(\bar{v}) \neq 0$ but $\delta > \bar{\delta}$, an equity security is screened and, as argued above, the issuer is better off issuing a debt security.

The discussion above can be used to show that there might be situations where a sale of equity on separate assets is not screened while a sale of equity on a pool of two assets is screened. In-

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[11]See Proposition 4 on p.629.

deed, as argued in the proof of Lemma 2, there might be circumstances when $f_2(2\bar{x}) \neq 0$ while $f_p(2\bar{x}) = 0$. For example, this holds when each asset is uniformly distributed on $[0, \bar{x}]$.[12] Therefore, under these conditions, when $\delta \in (0, \bar{\delta}_2]$, the issuer obtains strictly higher profits from selling assets separately than from selling assets in a pool. This is in contrast to the competitive market scenario where efficient trade happens in both cases for the same region of discount factor $\delta \in (0, \mathbb{E}[X_2]/[2\bar{x}])$.

In the rest of this section we consider situations where both $f_2(2\bar{x}) = 0$ and $f_p(2\bar{x}) = 0$, i.e., the first-order condition for the buyer's profit maximization problem (19) has an interior solution in both cases. Although we argued above that in this case the issuer is strictly better off designing a debt security than an equity security, we start by comparing the issuer's profits from issuing separate equity securities for each asset and from issuing one equity security on a pool of assets. This exercise further highlights the intuition that pooling assets increases the buyer's incentives to screen the issuer and can thereby be detrimental to an issuer facing a buyer with market power. It also serves as a building block for the analysis of optimal debt securities.

Using equation (4), we can write the buyer's first-order condition as:

$$\frac{1}{h(v^*)} = \frac{(1-\delta)}{\delta}, \tag{22}$$

and define a function $k(v) \equiv \frac{1}{h(v)}$. The regularity condition in Assumption 1 states that this function monotonically increases on the support $[0, \bar{v}]$. For a given δ, the function $k(v)$ measures the buyer's incentives to screen the issuer by offering a lower price. The incentives are highest at the right tail as $k(\bar{v}) \to \infty$. Analogously, if the value of this function at the left tail, i.e., $k(0)$, is greater than the right side of (22) the equity security is completely screened out for a given δ. This implies that for all $\delta > \frac{1}{1+k(0)}$ the issuer is completely excluded from trade.

The following lemma studies properties of the function $k(v)$ for the distributions of $2X_i$ and $X_1 + X_2$ and allows us to compare the quoted prices in the two cases for different values of δ.

Lemma 3. *In a market with a monopolistic buyer, there exist thresholds δ' and δ'' such that: (i) for $\delta \in (0, \delta')$ the equilibrium quoted prices are $p_p^* < p_2^*$ while (ii) for $\delta \in (\delta', \delta'')$ the equilibrium quoted prices are $p_p^* > p_2^*$.*

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[12]In this case the threshold for the efficient sale of separate assets is $\bar{\delta}_2 = \frac{1}{2}$.

Proof. From Lemma 2, we know that the payoff distribution has thinner tails for the pooled security $X_1 + X_2$ than for separate sales $2X_i$, which means that $F_p(x) > F_2(x)$ and $f_p(x) < f_2(x)$ as $x \to 2\bar{x}$. This in turn implies that $k_p(x) > k_2(x) \to \infty$ as $x \to 2\bar{x}$. Thus, there exists δ' such that for any $\delta \in (0, \delta')$ the marginal issuers in the two cases satisfy $x_p^* < x_2^*$, which directly implies that $p_p^* < p_2^*$. In this region, the buyer has stronger incentives to screen an issuer who sells a pool of assets than an issuer who sells these assets separately.

In contrast, using arguments in the proof of Lemma 2, it can be shown that the order of the two functions is reversed at the left tail, $k_p(0) < k_2(0)$. Define $\delta'' = \frac{1}{1+k_2(0)}$ and, if the monotonically increasing functions $k_p(x)$ and $k_2(x)$ intersect only once at some $x = x'$, denote by δ' the unique value of δ such that $k_p(x') = k_2(x') = \frac{1-\delta'}{\delta'}$.

For low values of the discount factor, i.e., $\delta < \delta'$, we have $k_p(x) > k_2(x)$ and the optimal marginal issuer targeted by the buyer when two assets are sold in a pool, x_p^*, has a lower valuation than the optimal marginal issuer targeted by the buyer when two assets are sold separately, x_2^*. By unique correspondence between screening thresholds and quoted prices, this means that $p_p^* < p_2^*$.

On the other hand, for high values of the discount factor, i.e., $\delta \in (\delta', \delta'')$, the order of the two functions is reversed, i.e., $k_p(x) < k_2(x)$, which also means that the orders of the screening thresholds and the quoted prices are reversed: $x_p^* > x_2^*$ and $p_p^* > p_2^*$.

Lastly, if $\delta \in [\delta'', \frac{1}{1+k_p(0)})$ the assets cannot be sold separately, as the issuer is completely screened out by the buyer, but a pooled security is only partially screened and some gains from trade are realized. □

Figure 4 illustrates Lemma 3 by plotting the functions $k_p(x)$ and $k_2(x)$, which respectively capture the buyer's incentives to lower the price below $p = \delta x$ when an equity security is issued on a pool of two assets or when separate equity securities are issued for each asset, using our earlier parameterization.

Finally, having established the relative order for the quoted prices p^*, for an equity security with a payoff $2X_i$ and for an equity security with a payoff $X_1 + X_2$, we can now compare the issuer's expected profits in the two cases.

Proposition 3. *In a market with a monopolistic buyer, when $\delta \in (0, \delta')$ the issuer obtains higher profits by selling assets separately rather than by selling them in a pool.*
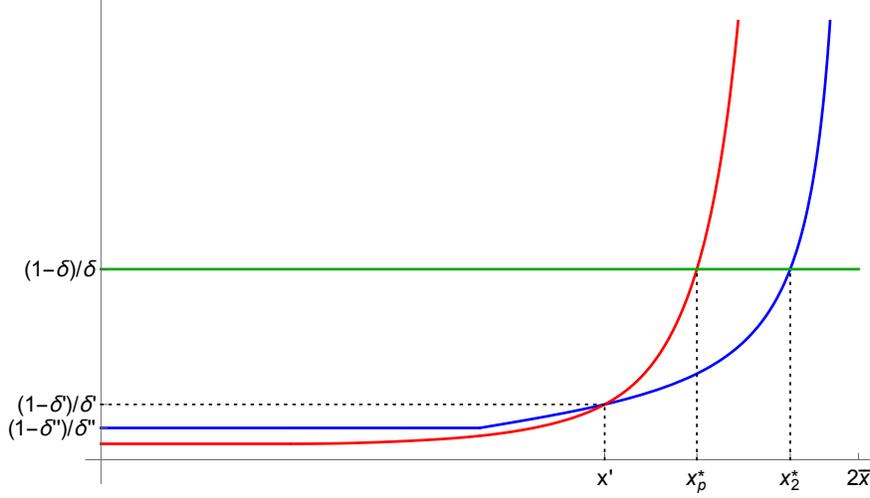
Figure 4: Screening incentives under pooling and separate sales. The figure illustrates the function $k_2(x)$ for separate sales (blue) and function $k_p(x)$ for a pool of assets (red). The parameterization assumes that each X_i, for $i = 1, 2$, is the sum of two uniformly distributed random variables on $[0, 1]$. Thus, $\delta' = 0.53$, $\delta'' = 0.5$ and $\bar{x} = 2$.

Proof. The issuer's profit (6) being offered a price p can be written as:

$$S(p) = pF(p/\delta) - \delta \int_0^{p/\delta} xf(x)dx. \tag{23}$$

Integrating by parts the equation above, the issuer's profit is reduced to:

$$S(p) = \delta \int_0^{p/\delta} F(x)dx. \tag{24}$$

By Lemma 2, X_p second-order stochastically dominates $2X$ and we can use equation (5) to show that:

$$S_2(p) \geq S_p(p) \tag{25}$$

for any $p \in [0, \delta 2\bar{x}]$. If the buyer were to offer the same price $p = \delta x$ regardless of the security, the issuer would have strictly higher profits by selling separately the two assets than by selling them as a pool.

However, as we know, the buyer optimally offers different prices in the two cases. As shown in Lemma 3, whenever the discount factor is small enough, the buyer tries to screen the issuer more aggressively if he is selling a pool of two assets than if he is selling two separate assets:

$p_2^* > p_p^*$. Given that the issuer's profit is an increasing function of the offered price, we also have that $S_2(p_2^*) > S_2(p_p^*)$.

Thus, combining the last inequality and inequality (25) evaluated at p_p^* implies that for $\delta \in (0, \delta')$ the issuer is better off selling equity securities on separate assets than selling one equity security on the pool, i.e., $S_2(p_2^*) > S_p(p_p^*)$. □
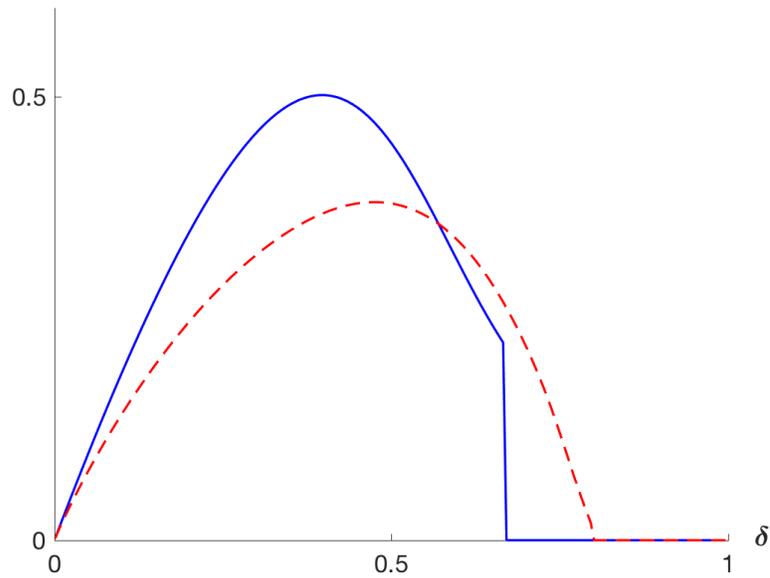
Figure 5: Issuer profits with a monopolistic buyer. The figure illustrates the seller's ex ante profit when issuing equity securities separately (blue, solid) or on a pool of assets (red, dashed) in a market with a monopolistic buyer. The parameterization assumes that each X_i, for $i = 1, 2$, is the sum of two uniformly distributed random variables on $[0, 1]$.

As we did for the competitive market, we now compare in Figure 5 the issuer's payoff from issuing equity securities on two separate assets or on a pool of these assets in a market with a monopolistic buyer. As formalized in Proposition 3, for small enough δ (< 0.57) the issuer is strictly better off selling separate equity securities than pooling the assets.

4.3.3 Comparing Payoffs from Debt Securities

As argued in subsection 4.3.1, the issuer is always able to design a debt security that generates a higher expected profit than a comparable equity security if there is some screening in the later

case. In this subsection, we will show that when δ is sufficiently low, the issuer is better off selling separate debt securities on each asset than selling a debt security on the pool of assets. In other words, when the buyer is endowed with market power, pooling assets may be suboptimal, even if debt securities are allowed. We focus on the region $\delta \in (0, \delta')$ where the buyer's screening behavior was more aggressive for pooled securities than non-pooled securities in the case with equity securities only.

The issuer's profit for debt securities is given by the same function as for equity securities (6) when the buyer quotes a price $p = \delta D$, since the marginal issuer, who has a reservation value δD, earns zero rents in this case. Furthermore, it is established that the issuer finds it optimal to choose a face value D^* such that the price $p^* = \delta D^*$ is quoted. Therefore, if we can show that the optimal face value of debt when the payoff is $2X_i$ is higher than the optimal face value of debt when the payoff is $X_1 + X_2$, i.e., $D_2^* > D_p^*$, then the same argument as in the proof of Proposition 3 for the equity case can be carried out for debt securities.

Proposition 4. *In a market with a monopolistic buyer, there exists a threshold δ^* such that for any $\delta \in (0, \delta^*)$ the optimal face value of debt D_2^* for assets sold separately is higher than the optimal face value of debt D_p^* for a pool of assets.*

Proof. Recall that the issuer chooses the optimal D^* such that the buyer's profit from buying the debt security without screening, at a price δD^*, is the same as his optimal profit from buying an equity security at the price $p^* = \delta x^*$. Using our earlier notation, the optimal face values D_2^* and D_p^* are determined by $B_2^{D_2^*}(\delta D_2^*) = B_2(\delta x_2^*)$ and $B_p^{D_p^*}(\delta D_p^*) = B_p(\delta x_p^*)$. While our main results are derived analytically below, we use Figure 6 to illustrate how the relevant payoff functions behave for a simple numerical example.

To compare the relative positions of D_2^* and D_p^*, we must first study the relative positions of functions $B_2^D(\delta D)$ and $B_p^D(\delta D)$ and levels $B_2(\delta x_2^*)$ and $B_p(\delta x_p^*)$. We can establish that $B_2(\delta x_2^*) < B_p(\delta x_p^*)$ when x_2^* and x_p^* are both above $\bar{x}$. Integrating by parts, the buyer's profit (18) from buying an equity security at a price δx simplifies to:

$$
\begin{aligned}
B(\delta x) &= \int_0^x (v - \delta x) f(v) dv \\
&= (1 - \delta) x F(x) - \int_0^x F(v) dv.
\end{aligned}
\tag{26}
$$
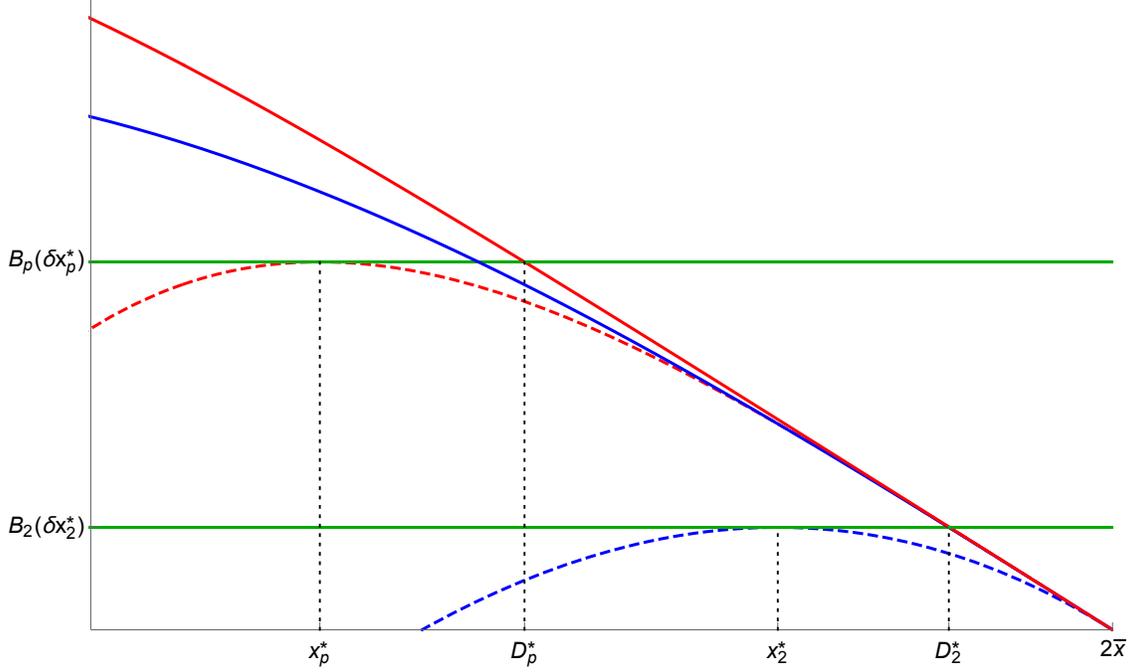
Figure 6: Profits functions for the monopolistic buyer. The figure illustrates the buyer's profit functions for an equity security $B(\delta x)$ (dashed) and for a debt security $B^D(\delta D)$ (solid), each drawn for the two cases: selling two assets separately (blue) and selling a pool of two assets (red). The figure also plots the buyer's maximum profit levels $B_2(\delta x_2^*)$ and $B_p(\delta x_p^*)$ for equity (green), and the optimal values for D_p^* and D_2^* (black). The parameterization assumes that each X_i, for $i = 1, 2$, is the sum of two uniformly distributed random variables on $[0, 1]$ and $\delta = 0.25$. Thus, $\bar{x} = 2$.

Therefore, for $x > \bar{x}$ the second-order stochastic dominance of $F_p(x)$ implies that $F_p(x) \geq F_2(x)$ and that:

$$B_p(\delta x) - B_2(\delta x) = (1 - \delta)x(F_p(x) - F_2(x)) + \int_0^x [F_2(v) - F_p(v)]dv \geq 0. \qquad (27)$$

Geometrically, the function $B_p(\delta x)$ lies above the function $B_2(\delta x)$ for any $x > \bar{x}$ and the maximum of the former function must therefore be higher than the maximum of the latter function. The buyer's profit at the optimal equity screening threshold of $X_1 + X_2$ is strictly higher than the buyer's profit at the optimal equity screening threshold of $2X_i$:

$$B_p(\delta x_p^*) > B_2(\delta x_2^*). \qquad (28)$$

As in the proof of Proposition 2, we can similarly establish that the function $B_p^D(\delta D)$ lies above the function $B_2^D(\delta D)$. Indeed, the buyer's profit from buying debt with the face value D at a price

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δD is:

$$B^D(\delta D) = B(\delta D) + (1 - \delta)(1 - F(D))D = (1 - \delta)D - \int_0^D F(x)dx. \qquad (29)$$

Using the second-order stochastic dominance of $F_p(\cdot)$, we know that the function $B_p^D(\delta D)$ lies above the function $B_2^D(\delta D)$ for any $D \in (0, 2\bar{x})$. Additionally, at the right tail of the distribution (at $x = 2\bar{x}$), all the four profit functions have the same value:

$$B_p(\delta 2\bar{x}) = B_2(\delta 2\bar{x}) = B_p^{2\bar{x}}(\delta 2\bar{x}) = B_2^{2\bar{x}}(\delta 2\bar{x}) = 2\,\mathbb{E}[X_i] - \delta 2\bar{x}. \qquad (30)$$

Having determined the relative positions of the functions $B_2^D(\delta D)$ and $B_p^D(\delta D)$ and the levels $B_2(\delta x_2^*)$ and $B_p(\delta x_p^*)$, we now identify the relative positions of D_2^* and D_p^*. Since the function $B_p^D(\delta D)$ lies above the function $B_2^D(\delta D)$ for any D, the former must cross both levels $B_2(\delta x_2^*)$ and $B_p(\delta x_p^*)$ closer to the right boundary $2\bar{x}$ than the latter. However, the main question is whether it crosses the higher level $B_p(\delta x_p^*)$, the point D_p^*, further from $2\bar{x}$ than $B_2^D(\delta D)$ crosses the lower level $B_2(\delta x_2^*)$, the point D_2^*.

Going back to our parameterized example, Figure 7 zooms in on the region of interest for a case where the buyer's incentives to screen are strengthened (i.e., $\delta = 0.53$ instead of 0.25). It can be seen that the profit functions $B_2^D(\delta D)$ and $B_p^D(\delta D)$ as well as their slopes are close near the right tail, $2\bar{x}$. The derivative of the buyer's profit function which determines the slope is:

$$[B^D(\delta D)]' = 1 - \delta - F(D). \qquad (31)$$

Hence, the slopes of functions $B_2^D(\delta D)$ and $B_p^D(\delta D)$ are indeed the same at the right tail $2\bar{x}$ where both $F_2(2\bar{x}) = F_p(2\bar{x}) = 1$.

Denote as D' the face value of debt for which $B_p^{D'}(\delta D') = B_2(\delta x_2^*)$. By the properties of the profit functions established above, $D' > D_2$ and $D' > D_p$. Thus, to match up D_2^* and D_p^* we can compare $D' - D_2^*$ and $D' - D_p^*$.

Up to a linear approximation, these differences are each given by:

$$D' - D_p^* \approx \frac{B_p(\delta x_p^*) - B_2(\delta x_2^*)}{-[B_p^{D'}(\delta D')]'}. \qquad (32)$$
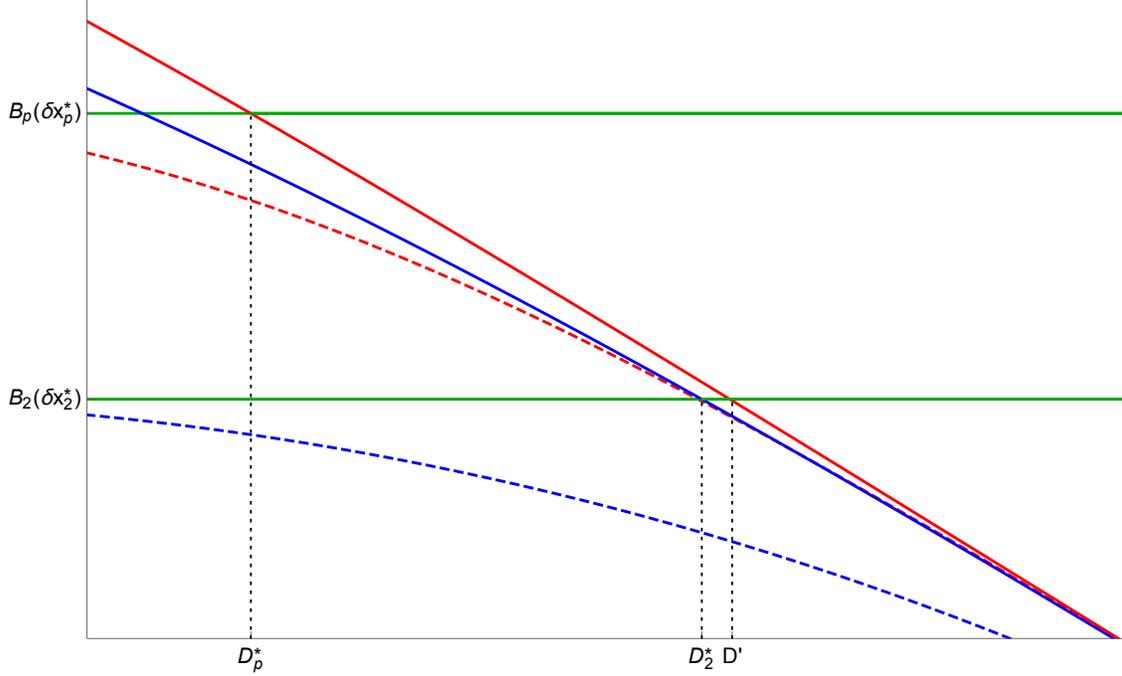
Figure 7: Profits functions for the monopolistic buyer. The figure illustrates the buyer's profit functions for an equity security $B(\delta x)$ (dashed) and for a debt security $B^D(\delta D)$ (solid), each drawn for the two cases: selling two assets separately (blue) and selling a pool of two assets (red). The figure also plots the buyer's maximum profit levels $B_2(\delta x_2^*)$ and $B_p(\delta x_p^*)$ for equity (green), and the optimal values for D_p^* and D_2^* (black). The parameterization assumes that each X_i for $i = 1, 2$ is the sum of two uniformly distributed random variables on $[0, 1]$ and $\delta = 0.53$. Thus, $\bar{x} = 2$.

$$D' - D_2^* \approx (2\bar{x} - D_2^*) - (2\bar{x} - D') = \frac{B_2(\delta x_2^*) - B_2(\delta 2\bar{x})}{-[B_2^{D_2^*}(\delta D_2^*)]'} - \frac{B_2(\delta x_2^*) - B_2(\delta 2\bar{x})}{-[B_p^{D'}(\delta D')]'}. \tag{33}$$

Finally, $D_p^* < D_2^*$ whenever $D' - D_p^* > D' - D_2^*$ which, substituting the formulas from the above, is expanded to:

$$\frac{B_p(\delta x_p^*) - B_2(\delta x_2^*)}{-[B_p^{D'}(\delta D')]'} > \frac{B_2(\delta x_2^*) - B_2(\delta 2\bar{x})}{-[B_p^{D_2^*}(\delta D_2^*)]'} - \frac{B_2(\delta x_2^*) - B_2(\delta 2\bar{x})}{-[B_p^{D'}(\delta D')]'}, \tag{34}$$

which is equivalent to:

$$\frac{B_p(\delta x_p^*) - B_2(\delta x_2^*)}{B_2(\delta x_2^*) - B_2(\delta 2\bar{x})} > \frac{-[B_p^{D'}(\delta D')]'}{-[B_2^{D_2^*}(\delta D_2^*)]'} - 1 = \frac{\delta - 1 + F_p(D_2^*)}{\delta - 1 + F_2(D')} - 1 = \frac{F_p(D') - F_2(D_2^*)}{\delta - 1 + F_2(D_2^*)}. \tag{35}$$

This shows that $D_p^* < D_2^*$ whenever either $B_p(\delta x_p^*) - B_2(\delta x_2^*)$ is high, $B_2(\delta x_2^*) - B_2(\delta 2\bar{x})$ is low, or $F_p(D') - F_2(D_2^*)$ is low.

Lastly, we show that condition (35) is satisfied for δ close to zero. For low values of the

discount factor $\delta \to 0$ the buyer almost does not screen and $x_2^* \to 2\bar{x}$ and $x_p^* \to 2\bar{x}$. Therefore, both differences $B_2(\delta x_2^*) - B_2(\delta 2\bar{x}) \to 0$ and $B_p(\delta x_p^*) - B_2(\delta x_2^*) \to 0$. However, because of the thinner tails of $X_1 + X_2$, x_p^* approaches the right tail $2\bar{x}$ at a slower rate than x_2^*. Hence, $B_p(\delta x_p^*) - B_2(\delta x_2^*) > B_2(\delta x_2^*) - B_2(\delta 2\bar{x}) \approx 0$. At the same time, $D_2 \to 2\bar{x}$ and $D' \to 2\bar{x}$ when $\delta \to 0$ and an application of the L'Hôpital's rule to the right-hand side of condition (35) yields:

$$\left. \frac{F_p(D') - F_2(D_2)}{\delta - 1 + F_2(D_2)} \right|_{\delta \to 0} \approx \frac{f_p(2\bar{x}) - f_2(2\bar{x})}{1 + f_2(2\bar{x})} = 0. \tag{36}$$

Summing up the above analysis, the condition is satisfied for $\delta \to 0$ and, therefore, by monotonicity in some interval around zero, i.e., for $\delta \in (0, \delta^*)$ with $\delta^* > 0$.

Thus, in this interval where δ is small, the optimal face value of debt for an asset with payoff $2X_i$ is higher than the optimal face value of debt for an asset with payoff $X_1 + X_2$. □
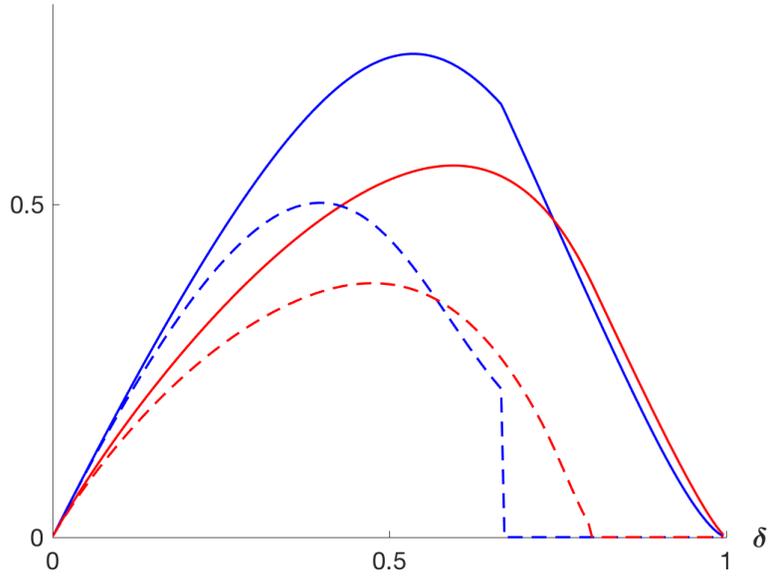
Proposition 5. *In a market with a monopolistic buyer, if $\delta \in (0, \delta^*)$ the issuer obtains higher profits by separately selling optimal debt securities on each asset rather than selling the optimal debt security on the pool of assets.*

Proof. Employing the argument from the beginning of this section, the optimal debt security issued on a pool of two assets yields a lower profit compared to the profit obtained from selling two optimal debt securities issued on each asset. Equivalently, the issuer is strictly better off selling debt on separate assets than on the pool of assets. □

In Figure 8, we compare the issuer's payoff from issuing equity or debt securities on two separate assets or on a pool of these assets in a market with a monopolistic buyer. As formalized in Proposition 5, for small enough δ (< 0.75) the issuer is strictly better off selling debt securities on the separate assets.

5 Discussion

In this section we argue that the main result holds under various alternative scenarios. First, we separately consider scenarios where there are more than 2 assets, where the scarcity of liquidity

Figure 8: Issuer profits with a monopolistic buyer. The graph illustrates the seller's ex ante profit when issuing equity (dashed) or debt (solid) securities separately (blue) or on a pool of assets (red) in a market with a monopolistic buyer. The parameterization assumes that each X_i, for $i = 1, 2$, is the sum of two uniformly distributed random variables on $[0, 1]$.

is modeled differently with multiple constrained buyers instead of one monopolistic buyer, and where the issuer could signal asset quality. Finally, we highlight the possibility of adding a time dimension to the setup.

More than two assets. Throughout the paper, we have analyzed the decision of an issuer to pool two assets and issue a security on this pool. Our main result that pooling assets can be suboptimal when facing a buyer with market power, however, extends to cases with more than two assets. The limiting case with $n \to +\infty$ assets provides a clear intuition for why that is. Suppose the issuer has access to an infinite number of assets indexed by i. Each asset i produces a payoff that is independently distributed. If the issuer faces a group of competitive buyers, pooling all these assets into one security guaranteed, by the law of large numbers, to always deliver its expected payoff, say μ, is optimal, as all buyers offer a price μ and the issuer extracts the full surplus $(1 - \delta)\mu$. Now if the issuer instead faces a monopolistic buyer and were to pool all the assets into one security guaranteed to deliver a payoff μ, the monopolistic buyer would find it optimal to offer a price $\delta\mu$, which would leave the issuer with no surplus. Pooling an infinite number of assets leads to non-

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existent tails in the distribution of payoffs (i.e., payoff realizations always equal their expectation) and leaves the issuer with no information rents. The issuer can therefore do better by separately selling a subset of these assets such that he is not fully screened out and is able to extract some rents, consistent with the above analysis for $n = 2$ assets. (See Appendix B for a formal extension of our model to the case with $n > 2$ assets producing identically distributed payoffs that follow a binomial distribution.)

Constrained buyers. The main result of the paper, that is, pooling might be suboptimal when buyers have market power, is derived in an environment with only one deep-pocketed buyer endowed with full market power. We, however, show in Appendix B that this result holds in an extended model containing several buyers who have position limits which might be due to scarce market liquidity. Buyers with position limits cannot compete aggressively and thereby quote prices similar to the monopolist buyer prices.[13] Because of this the issuer prefers the same securities as in the baseline model and might find pooling suboptimal.

In particular, we consider two extensions of the model with different types of constraints. In the first extension, deep-pocketed buyers are limited by the number of risk units they can take in their inventory. We show that when the total number of risk units demanded is lower than the number of supplied units the constrained buyers quote the same prices as a monopolist buyer would. In the second extension, buyers have limited wealth to post quotes and acquire assets. We show that under this scenario, if the buyers' wealth is sufficiently constrained, the sale of a security on a pool might be inefficient and separate sales are preferred by the issuer.

Signaling through retention. In the scenario with competing buyers, allowing the issuer to signal the quality of the assets through partial retention would yield results consistent with DeMarzo (2005) — issuers with assets of higher quality would retain a higher fraction of the issue. Signaling would allow the high type issuers to separate themselves from the low types and would resolve the lemons problem for high values of δ. In contrast, when facing a buyer with market power, the issuer can be made worse off by signaling the quality of his assets. Since the buyer has all bargaining power once he knows the issuer type he is able to extract all the surplus from trade and leave the issuer with zero profit. In this case the issuer's profit with fully revealing retention policies is

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[13]The result that capacity constraints can hamper competition is long standing in the literature, see, for example, Green (2007).

weakly lower than his profit without any signaling through retention. (See Glode, Opp, and Zhang (2018) for related arguments.)

Alternative interpretation. The model assumes that the cash flows of different assets occur at the same time and we study whether pooling such assets is optimal for the issuer. However, the model allows an alternative interpretation where a time dimension is added to the assets' payoffs. Suppose the issuer has an asset that pays cash flows in different time periods. To map this situation to our model each particular cash flow can be viewed as an asset from our setup, while the asset itself can be considered as a pool of such cash flows. For such a mapping, we would also need to assume that the issuer is better informed than the buyers about all future cash flows. The question would be then whether it is optimal for the issuer to sell the asset as it is, pooling all cash flows across time, or to separate them and sell, for example, cash flows occurring earlier separately from those occurring at later time periods. The prediction of the model is that when the demand side has market power we should see more separation across the time dimension of cash flows.

6 Conclusion

This paper studies the optimality of pooling and tranching under asymmetric information when security originators face a market where liquidity is scarce and buyers endowed with such liquidity may have market power. Contrary to the standard result that pooling and tranching are optimal practices, we find that selling assets separately may be preferred by originators to avoid being inefficiently screened by buyers. While our results suggest that the dramatic decline of the ABS market post crisis may represent an efficient response by originators to changes in liquidity and market power in OTC markets, it also highlights the potential welfare implications of liquidity constraints imposed on financial institutions in the new market environment.

Appendix A   Proofs of Lemmas

Proof of Lemma 1: Recall that if $F(x)$ and $f(x)$ are the c.d.f. and the p.d.f. of a random variable X then a random variable $2X$ has a c.d.f. $F_2(x) = F(\frac{x}{2})$ and a p.d.f. $f_2(x) = \frac{f(\frac{x}{2})}{2}$. Next, we establish the equivalence for each of the two market scenarios.

Market with competitive buyers: Since the price quoted by competitive buyers is given by

$$\hat{p} = \mathbb{E}[X|\delta X \le \hat{p}] \tag{A1}$$

it can be checked that the price obtained for the equity security issued on $2X$ is twice as large as the price quoted for the equity security issued on X, $\hat{p}_2 = 2\hat{p}$. Similarly, the optimal debt level $\hat{D}_2$ issued on an asset $2X$ is twice as large as the optimal debt level issued on an asset X, $\hat{D}_2 = 2\hat{D}$.

Moreover, the issuer's profit from selling a security issued on an asset $2X$ is twice as large as that from a sale of the same security issued on an asset X:

$$S(\hat{p}_2) = \Pr(\delta 2X \le \hat{p}_2)(\hat{p}_2 - \delta\mathbb{E}[2X|\delta 2X \le \hat{p}_2]) = \Pr(\delta 2X \le 2\hat{p})(2\hat{p} - \delta\mathbb{E}[2X|\delta 2X \le 2\hat{p}])$$
$$= 2\Pr(\delta X \le \hat{p})(\hat{p} - \delta\mathbb{E}[X|\delta X \le \hat{p}]) = 2S(\hat{p}). \tag{A2}$$

Analogously, the buyers' profit is twice as large as that from buying a security issued on an asset with payoff X.

Thus, from the issuer's and the buyers' perspectives two separate sales of an asset with random payoff X is the same as a sale of one asset with random payoff $2X$.

Market with monopolistic buyer: First, we analyze a sale of equity issued on an asset paying $2X$. Since the regularity condition in Assumption 1 is also satisfied by the distribution $F_2(x)$, using (19), the optimal screening threshold x_2^* is given by the FOC of the buyer's profit maximization problem:

$$(1-\delta)x_2^* f_2(x_2^*) = \delta F_2(x_2^*). \tag{A3}$$

Substituting in the above the p.d.f. and the c.d.f. of the random variable X we obtain:

$$(1-\delta)x_2^* f(x_2^*/2)/2 = \delta F(x_2^*/2). \tag{A4}$$

This is the FOC of the buyer's optimization problem if the underlying asset is X, (19), with $x^* = x_2^*/2$. Therefore, the optimal equity screening threshold for an asset $2X$ is twice as large as the optimal equity screening threshold for an asset X, $x_2^* = 2x^*$.

Analogous steps can be taken in the case of a debt issued on an asset $2X$ to find that the optimal debt level D_2^* is twice as large as the optimal debt level for a debt issued on an asset X, $D_2^* = 2D^*$.

Furthermore, given the established properties, the issuer's profit from selling a security issued on an asset $2X$ is twice as large as that from a sale of the same security issued on an asset X:

$$S(\delta 2y) = \delta \int_0^{2y} (2y - x)f_2(x)dx = \int_0^y (2y - 2x)f_2(2x)d(2x) =$$
$$= 2\delta \int_0^y (y - x)f_1(x)dx = 2S(\delta y). \tag{A5}$$

Similarly, the buyer's profit is twice as large as that from buying a security issued on an asset with payoff X.

Therefore, from the issuer's and the buyer's perspectives two separate sales of an asset with random payoff X is the same as a sale of one asset with random payoff $2X$. □

Proof of Lemma 2: Follows from the fact that $2X_i$ is a mean preserving spread of $X_1 + X_2$. Indeed, the former can be written as the sum of the latter and $X_1 - X_2$:

$$2X_1 = X_1 + X_2 + (X_1 - X_2), \tag{A6}$$

and $X_1 - X_2$ has conditional expected value of zero:

$$\mathbb{E}[X_1 - X_2 | X_1 + X_2] = \mathbb{E}[X_1 | X_1 + X_2] - \mathbb{E}[X_2 | X_1 + X_2] \overset{a.s.}{=} 0. \tag{A7}$$

Therefore, the distribution of $X_1 + X_2$ second-order stochastically dominates the distribution of $2X_i$ and, thus, has thinner tails.

Another way to see that $X_1 + X_2$ has thinner tails is to analyze its density function directly. Denote as $F_p(x)$ and $f_p(x)$ the c.d.f. and a p.d.f. of a random variable $X_1 + X_2$. Since X_1 and X_2 are independent and both have a density function $f(x)$ the function $f_p(x)$ is, by definition, the

convolution of the two functions, $f(x)$ and $f(x)$, and is given by:

$$f_p(x) = \int_{-\infty}^{+\infty} f(x-y)f(y)dy = \int_0^{\bar{x}} f(x-y)f(y)dy. \tag{A8}$$

This can be further reduced, since $f(x) > 0$ only on $x \in [0, \bar{x}]$, to obtain

$$f_p(x) = \begin{cases} \int_0^x f(x-y)f(y)dy & \text{if } 0 \le x \le \bar{x} \\ \int_{x-\bar{x}}^{\bar{x}} f(x-y)f(y)dy & \text{if } \bar{x} < x \le 2\bar{x}. \end{cases} \tag{A9}$$

Now, consider the shape of the density function $f_p(x)$ at its left tail, close to zero. The first two derivatives of the density $f_p(x)$ are

$$f_p'(x) = f(0)f(x) + \int_0^x f'(x-y)f(y)dy, \tag{A10}$$

$$f_p''(x) = f(0)f'(x) + f'(0)f(x) + \int_0^x f''(x-y)f(y)dy. \tag{A11}$$

Hence, from the equation (A9), $f_p(0) = 0$ even if $f_2(0) = \frac{f(0)}{2}$ might not be equal to zero. Furthermore, from (A10) and (A11), the values of the derivatives at the left boundary are $f_p'(0) = f^2(0)$ and $f_p''(0) = 2f(0)f'(0)$. Thus, it is possible that $f_p'(0) = f_p''(0) = 0$ when $f_2(0) = \frac{f(0)}{2} = 0$ even if $f_2'(0) = \frac{f'(0)}{4}$ and/or $f_2''(0) = \frac{f''(0)}{8}$ might not be equal to zero. Since the same results can be obtained for the right tail it follows that the distribution of $X_1 + X_2$ has smoother, thinner tails than the distribution of $2X_i$. $\qquad\square$

Appendix B Extensions to the Baseline Model

B.1 More Assets and Binomially Distributed Payoffs

This section generalizes our results to pools of $k \le n$ assets when the distribution of an asset's payoff is binomial. In particular, each asset i produces a payoff $X_i = \varphi_i \sigma$ where φ_i is an independent random variable that takes the value 1 with probability $(1 - q_i)$ and the value 0 otherwise. If the issuer bundles the first k assets together the total payoff from the pool is given by

$v_k \equiv \varphi_1\sigma + \cdots + \varphi_k\sigma$. If the issuer decides to sell equity on this pool of k assets, the payoff is simply v_k. In contrast, if he decides to issue debt with a face value of $D\sigma$, the payoff from this security becomes $v_k^D \equiv \min\{v_k, D\sigma\}$.

B.1.1 Competitive (Deep-pocketed) Buyers

In this subsection, we consider a market with competitive buyers. Suppose first that the issuer offers an equity claim on a pool of k assets and there are several competitive unconstrained buyers. Again, any buyer quotes the price $p_k = \mathbb{E}[v_k|v_k \leq m\sigma]$ when it is higher than the reservation value of the highest participating issuer: $m\delta\sigma$. Writing out the conditional expectation, this is equivalent to requiring that:

$$\frac{\sum_{i=0}^m \Pr(v_k = i\sigma)(i\sigma)}{\sum_{i=0}^m \Pr(v_k = i\sigma)} \geq m\delta\sigma. \tag{B1}$$

As before, these inequalities allow us to characterize the thresholds for the discount factor δ at which trade at prices $E(v_k|v_k \leq m\sigma)$ is possible:

$$\delta_{km} \equiv \frac{\sum_{i=0}^m \Pr(v_k = i\sigma)i}{m\sum_{i=0}^m \Pr(v_k = i\sigma)}. \tag{B2}$$

Whenever δ is higher than the threshold δ_{km}, it means that the gains to trade are too low to sustain trade at a price $\mathbb{E}[v_k|v_k \leq m\sigma]$. Specifically, the upper bound on δ, which corresponds to the lowest gains from trade where trade is efficient can be written as:

$$\delta_{kk} \equiv \delta_k \equiv \frac{\mathbb{E}[v_k]}{k}. \tag{B3}$$

Later, we will obtain analogous thresholds for a market with a monopolistic buyer and show how they differ in the two cases and how they can be used to find the solution to the issuer's problem.

As before, the issuer can improve his profits by issuing debt instead of equity on a pool of k assets. If the face value is $D\sigma$ with $D \in (m-1, m]$ then the offered price which is equal to the expected security payoff, assuming that all issuer types participate in the trade, is:

$$p_k = \mathbb{E}[v_k^D|v_k \leq D\sigma] = \sum_{i=0}^{m-1} \Pr(v_k = i\sigma)(i\sigma) + \sum_{i=m}^{k} \Pr(v_k = i\sigma)D\sigma. \tag{B4}$$

Setting $D \in (0, 1)$ is however never optimal. An optimal level of $D = 1$ can be sustained when δ reaches its highest bound where any trade is possible in equilibrium:

$$\delta_{k0} \equiv \sum_{i=1}^{k} \Pr(v_k = i\sigma) = 1 - \Pr(v_k = 0). \tag{B5}$$

As δ decreases within the interval $[\delta_k, \delta_{k0})$ the optimal level of face value $D\sigma$ rises and the issuer's profit increases. In that region, the issuer's profit is higher than the proceeds from selling an equity stake on the pool. Unlike with an equity stake, there is no exclusion with the optimal debt security and all issuer types participate in the trade, although the higher types have to retain some exposure to the payoff in the form of a call option.

The analysis of the optimal decision to pool is analytically involved if we consider it for general levels of δ. Instead we focus below on the region of δ where efficient trade is possible and pooling is optimal (i.e., below the highest bound δ_k). Any asset or pool of assets can be characterized by this bound. The higher δ_k the larger the region where there is efficient trade. Note also that with competitive buyers δ_k depends only on the mean of distribution of v_k but not on the shape of its density. To see what implications this property has on the decision to pool we consider adding one asset to an already existing pool.

Adding an asset to the pool. Suppose the issuer hesitates between selling a pool of $k - 1$ assets and selling a pool of k assets. The bounds allowing for efficient trade with the two candidate securities are related as follows:

$$\delta_k = \frac{\mathbb{E}[v_k]}{k} = \frac{k-1}{k} \delta_{k-1} + \frac{1}{k} \delta_1^k. \tag{B6}$$

The bound on the larger pool is the weighted average of the bounds for efficient trade of the existing pool δ_{k-1} and the additional asset δ_1^k. If the issuer adds an asset with the same mean payoff as the existing pool, the bound for efficient trade of the pool does not change, as $\delta_k = \delta_{k-1}$. In particular, if the issuer pools assets with the same mean payoff, which might be less than perfectly correlated, δ_k is constant for all k. This means that pooling such assets when buyers are competitive does not harm trade efficiency, in a sense that the region of δ where trade is efficient does not change as k increases. If $\delta \leq \delta_1$ the decision to pool is optimal and remains so as long as the issuer adds

assets to the pool without changing the mean payoff. This is in sharp contrast to the case with a monopolistic buyer, as we will see later.

If the additional asset added to the existing pool of assets has higher mean payoff than the average payoff of the assets already in the pool the bound increases. This means that the region of efficient trade and optimal pooling expands. To be more precise, if $\delta_1 > \delta_{k-1}$ then $\delta_k \in (\delta_{k-1}, \delta_1)$. If $\delta \in (\delta_{k-1}, \delta_k)$ the decision to add this asset to the existing pool is optimal since otherwise the trade would involve the sale of the optimal debt security on the existing pool, as $\delta > \delta_{k-1}$, and the efficient sale of the additional asset, as $\delta < \delta_1$. Pooling the additional asset increases the issuer's profit because he is able to sell all assets and does not require to use a debt security (which would result in the issuer retaining a call option).

The opposite is, however, true if the asset being added has a lower mean than the average asset payoff of the existing pool. If $\delta_1 < \delta_{k-1}$ then $\delta_k \in (\delta_1, \delta_{k-1})$. The benefit of pooling in this case is that it allows to sell the additional asset, which could not be sold separately. If $\delta \in (\delta_1, \delta_k)$ it is the only change in the issuer's payoff and it leads to an increase in the issuer's profit. However, if $\delta \in (\delta_k, \delta_{k-1})$, the impact of pooling on the issuer's profit is ambiguous, since in this region the issuer has to use the optimal debt security and retains some cash flows in the form of a call option.

B.1.2   Buyer with Market Power

Turning to the market where one buyer has market power, we derive analogous thresholds for the discount factor δ and determine how they change if some assets are added to the existing pool. Suppose first that the issuer offers an equity claim on the pool of k assets for sale and the buyer quotes $p_k = m\delta\sigma$, for $0 \leq m \leq k$. The buyer's ex-ante profit can be written as:

$$
\begin{aligned}
B(m\delta\sigma) &= \Pr(v_k \leq m\sigma)(\mathbb{E}[v_k|v_k \leq m\sigma] - m\delta\sigma) \\
&= \sum_{i=0}^{m} \Pr(v_k = i\sigma)(i\sigma - m\delta\sigma). \tag{B7}
\end{aligned}
$$

Similarly, the ex-ante profit to the issuer is:

$$
\begin{aligned}
S(m\delta\sigma) &= \Pr(v_k \leq m\sigma)(m\delta\sigma - \delta\,\mathbb{E}[v_k|v_k \leq m\sigma]) \\
&= \sum_{i=0}^{m} \Pr(v_k = i\sigma)(m\delta\sigma - i\delta\sigma).
\end{aligned}
\tag{B8}
$$

Again, the issuer's profit increases with m, meaning that the issuer prefers to avoid being screened and to receive the highest possible offer: $p_k = k\delta\sigma$.

The buyer prefers a quote of $m\delta\sigma$ to a quote of $(m-1)\delta\sigma$ whenever $B(m\delta\sigma) - B((m-1)\delta\sigma) \geq 0$. From equation (B7), this condition is equivalent to:

$$
\begin{aligned}
0 &\leq \sum_{i=0}^{m} \Pr(v_k = i\sigma)(i\sigma - m\delta\sigma) - \sum_{i=0}^{m-1} \Pr(v_k = i\sigma)(i\sigma - (m-1)\delta\sigma) \\
&= \Pr(v_k = m\sigma)(m\sigma - m\delta\sigma) - \sum_{i=0}^{m-1} \Pr(v_k = i\sigma)\delta\sigma \\
&= \Pr(v_k = m\sigma)m(1-\delta)\sigma - \Pr(v_k \leq (m-1)\sigma)\delta\sigma.
\end{aligned}
\tag{B9}
$$

Again, the above inequality can be written as a threshold on the discount factor:

$$
\delta \leq \bar{\delta}_{km} \equiv \frac{m\Pr(v_k = m\sigma)}{\Pr(v_k \leq (m-1)\sigma) + m\Pr(v_k = m\sigma)}.
\tag{B10}
$$

The buyer prefers to quote the highest quote $p_k = k\delta\sigma$ whenever $B(m\delta\sigma) - B((m-1)\delta\sigma) \geq 0$ for $\forall m \in \{1, \ldots, k\}$. For tractability, we restrict our attention to distributions for which $\frac{m\Pr(v_k = m\sigma)}{\Pr(v_k \leq (m-1)\sigma)}$ monotonically declines with m, resulting into decreasing $\bar{\delta}_{km}$.[14]

Therefore, the m inequalities reduce to one condition $B(k\sigma) - B((k-1)\sigma) \geq 0$ which can be written as:

$$
\delta \leq \bar{\delta}_{kk} \equiv \bar{\delta}_k \equiv \frac{k\Pr(v_k = k\sigma)}{1 + (k-1)\Pr(v_k = k\sigma)}.
\tag{B11}
$$

This is the highest value of δ for which screening is not profitable and trade can thus be efficient. Before conducting an analysis on how the thresholds change with the size of the pool, we consider issuing debt on the pool. Debt can improve the issuer's ex-ante profits whenever an equity claim

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[14]This restriction is satisfied, for example, if $q_i = 1/2$ for all assets but can be violated for some different q_i.

on the same pool would end up being screened by the issuer. For a face value of debt $D\sigma$, the whole issue is sold when the buyer offers $p_k^D = D\delta\sigma$ as every issuer type values the issue at most at $D\delta\sigma$. If the buyer instead makes a lower offer, the outcome becomes equivalent to the screened sale of equity with higher issuer types refusing to trade. If $D \in (m-1, m]$ for $0 \leq m \leq k$ and the buyer offers $D\delta\sigma$ then his ex-ante profit is:

$$
\begin{aligned}
B(D\delta\sigma) &= \Pr(v_k^D \leq D\sigma)(\mathbb{E}[v_k^D | v_k^D \leq D\sigma] - D\delta\sigma) \\
&= \sum_{i=0}^{m-1} \Pr(v_k = i\sigma)(i\sigma) + \sum_{i=m}^{k} \Pr(v_k = i\sigma)(D\sigma) - D\delta\sigma, \quad \text{(B12)}
\end{aligned}
$$

while the issuer's profit is:

$$
\begin{aligned}
S(D\delta\sigma) &= \Pr(v_k^D \leq D\sigma)(D\delta\sigma - \delta\,\mathbb{E}[v_k^D | v_k^D \leq D\sigma]) \\
&= \delta \sum_{i=0}^{m-1} \Pr(v_k = i\sigma)(D\sigma - i\sigma). \quad \text{(B13)}
\end{aligned}
$$

Note that the issuer prefers to sell debt with higher face value d. As before, a debt security with $D \in (0,1)$ is suboptimal. Moreover, the boundary above which no trade occurs (i.e., $\bar{\delta}_{k0}$) is the same as with competitive buyers.

For any other $\delta \in [\bar{\delta}_k, \bar{\delta}_{k0})$ the optimal level of the face value of debt $D \in (1, k]$ increases as δ decreases. In particular, if $\delta \in (\bar{\delta}_{k(m+1)}, \bar{\delta}_{km}]$ the optimal value of d makes the buyer indifferent between the two options: (i) not screening, paying $\delta D\sigma$ for the debt security, and receiving a profit of $B(D\delta\sigma)$, and (ii) optimally screening, offering a price of $\delta m\sigma$, and receiving a profit of $B(m\delta\sigma)$. It can be shown that the optimal face value $d > m$ and the condition $B(D\delta\sigma) = B(m\delta\sigma)$ pins down its level. Moreover if $m = k - 1$, i.e., $\delta \in (\bar{\delta}_{kk}, \bar{\delta}_{k(k-1)}]$, the optimal $D \in (k-1, k) = (m, m+1)$. However, if $m < k - 1$ it might be the case that $d > m + 1$.

As with competitive buyers, we will now focus on the region of δ where efficient trade is possible and pooling is optimal (i.e., below the highest bound $\bar{\delta}_k$). Any asset or pool of assets can be characterized by this bound. The higher $\bar{\delta}_k$ the more likely it is that there is trade without screening. Note also that unlike with competitive buyers, $\bar{\delta}_k$ depends on the shape of the distribution of v_k, in particular, on its density in the tail. To see how this property changes the optimal decision to pool relative to a market with competitive buyers we consider the same application as in the previous

section, namely the addition of one asset to an existing pool.

Adding an asset to the pool. Suppose the issuer hesitates between selling a pool of $k-1$ assets and selling a pool of k assets. The first pool is characterized by a threshold $\bar{\delta}_{k-1}$ and the larger pool is characterized by $\bar{\delta}_k$. Using definition in (B11), we can identify conditions when $\bar{\delta}_{k-1} < \bar{\delta}_k$ in the following way:

$$\frac{(k-1)\Pr(v_{k-1}=(k-1)\sigma)}{1+(k-2)Pr(v_{k-1}=(k-1)\sigma)} < \frac{kPr(v_k=k\sigma)}{1+(k-1)\Pr(v_k=k\sigma)} \tag{B14}$$

$$\Leftrightarrow (k-1)\Pr(v_{k-1}=(k-1)\sigma)+\Pr(v_k=k\sigma)\Pr(v_{k-1}=(k-1)\sigma) < k\Pr(v_k=k\sigma).$$

Since $\Pr(v_{k-1}=(k-1)\sigma)=\Pr(v_k=k\sigma)+\Pr(v_{k-1}=(k-1)\sigma,\varphi_k=0)$, the above becomes:

$$(k-1)\Pr(v_{k-1}=(k-1)\sigma,\varphi_k=0) < \Pr(v_k=k\sigma)(1-\Pr(v_{k-1}=(k-1)\sigma)) \tag{B15}$$

$$\Leftrightarrow (k-1)\frac{\Pr(v_{k-1}=(k-1)\sigma,\varphi_k=0)}{\Pr(v_k=k\sigma)(1-\Pr(v_{k-1}=(k-1)\sigma))} < 1. \tag{B16}$$

If this condition is satisfied, we know that the threshold for the pool with the additional asset to be traded efficiently (i.e., $\bar{\delta}_k$) is higher than the threshold for the existing pool to be traded efficiently (i.e., $\bar{\delta}_{k-1}$). Intuitively, adding one more asset to the existing pool of assets is beneficial, in terms of reducing the region with screening, whenever assets in the current pool are relatively bad, $1-\Pr(v_{k-1}=(k-1)\sigma)$ is high, or the additional asset $\sigma\varphi_k$ is relatively good, $\frac{\Pr(v_{k-1}=(k-1)\sigma,\varphi_k=0)}{\Pr(v_k=k\sigma)}$ is low.

B.1.3 Simple Parametric Example

We illustrate our general result with the following example where assets have the same quality ex ante: $\varphi_i \sim Ber(1-q)$. Then condition (B16) simplifies to:

$$(k-1)\frac{q(1-q)^{k-1}}{(1-q)^k(1-(1-q)^{k-1})} < 1. \tag{B17}$$

Using a Taylor series expansion this expression reduces to:

$$(k-1)\frac{q}{(1-q)} < 1 - (1-(k-1)q) \tag{B18}$$

$$\Leftrightarrow \frac{1}{1-q} < 1. \tag{B19}$$

This inequality never holds and adding one more asset to the pool reduces the region of efficient trade, as the corresponding threshold decreases, $\bar{\delta}_k < \bar{\delta}_{k-1}$. This is in sharp contrast with the case of competitive buyers where even pooling assets with the same mean is harmless in terms of efficiency. Since with a monopolistic buyer the threshold $\bar{\delta}_k$ decreases with k the issuer might prefer to avoid creating one large pool of assets and might instead want to create several smaller pools to sell them separately. For example, if $\delta \in (\bar{\delta}_k, \bar{\delta}_{k/2}]$ the issuer can increase his ex-ante profits by selling two same-size parts of one large pool separately. Since the large pool is screened by the buyer, the issuer has to use the optimal debt security and has to retain some exposure. In contrast, the separate sales of each part are not screened and using equity securities on the smaller pools allows the issuer to sell all assets. Figure 9 illustrates this situation.

Figure 9: In the parameter region highlighted in red the separate sale of the two halves of the pool is strictly more profitable than the sale of the debt on the pool.

Moreover, the threshold for the resulting pool may be lower not only in the special case with independent identical assets. Suppose the issuer has two assets which can be sold separately. If sold separately, these assets are characterized then by $\bar{\delta}_1^1$ and $\bar{\delta}_1^2$ but as a pool they are characterized by $\bar{\delta}_2$. If $\bar{\delta}_1^1 < \bar{\delta}_2$ there is a region of gains from trade where a sale of individual asset is screened while a sale of a pool is not. According to (B11), $\bar{\delta}_1^1 < \bar{\delta}_2$ implies:

$$\frac{\Pr(v_1 = \sigma)}{1 + (1-1)\Pr(v_1 = \sigma)} < \frac{2\Pr(v_2 = 2\sigma)}{1 + (2-1)\Pr(v_2 = 2\sigma)}, \tag{B20}$$

or equivalently,

$$\Pr(v_1 = \sigma) - 2\Pr(v_2 = 2\sigma) + \Pr(v_2 = 2\sigma)\Pr(v_1 = \sigma) < 0. \tag{B21}$$

The above condition holds for the general case and allows for arbitrary correlations between the two assets. However, to illustrate the point, we can assume that assets φ_i are independent. Then (B21) reduces to $1 - 2(1 - q_2) + (1 - q_1)(1 - q_2) < 0$ or

$$\frac{q_2}{1 - q_2} < q_1. \tag{B22}$$

The threshold for the resulting pool $\bar{\delta}_2$ is higher than the threshold for the first asset $\bar{\delta}_1^1 = 1 - q_1$ when q_1 is sufficiently high, while q_2 is relatively low which means that the first asset should be "bad" while the second added asset should be "good." Note that in a market with a monopolistic buyer the required quality of a "good" asset should be higher than in a market with competitive buyers where the condition was that the first asset's mean is lower than that of the second asset $q_2 < q_1$. With the monopolistic buyer, it is not enough to add an asset that is simply better, this asset needs to be sufficiently better. As a result, if the two added assets are similar in quality, that is, they do not differ much in the means, the threshold for the pool is lower than both thresholds for the individual assets and consequently there is a region $\delta \in [\bar{\delta}_2, \min\{\bar{\delta}_1^1, \bar{\delta}_1^2\}]$ where pooling is not optimal and the separate sale of the assets strictly dominates the sale of equity or debt on the pool.

B.2 Buyers with Position Limits

We consider an extension of the model with a finite number of buyers who are constrained by their position limits. In this section, the position limit is a constraint on the number of risk units a buyer can take where a risk unit is defined as any security with a maximum cash flow of 1. We suppose that the total position limit across all buyers is less than or equal to the total supply offered by the issuer. We show that in this case the issuer's optimal decision to pool assets is similar to the case of one buyer.

Instead of one monopolistic buyer, we now assume that there are two deep-pocketed buyers,

each with a discount factor of 1, who have position limits of one risk unit each. Therefore, the two buyers cannot buy more than two risk units in total. The seller, as before, has two assets with binomial cash flows. This implies that two risk units can be sold by either selling two assets separately or by selling two identical halves — shares of a pool of the two assets. We also assume that buyers submit quotes for both risk units and the seller picks up the best quotes. If the two buyers submit the same quote for a unit the seller allocates it to one of them randomly with equal probabilities.

If the assets are ex-ante identical the seller must choose between offering each asset separately or two halves of a pool containing both assets. In this case each unit is sold separately as if to a monopolistic buyer since it is not optimal for one buyer to undercut the other buyer. Offering slightly higher price than monopolistic for any unit would only result in a trade of this unit, one of the two identical units that could have been obtained by quoting monopolistic price. Formally, there are several equilibria with the buyers quoting monopolistic price for one or both units which differ only in outcomes of buyer-unit matches.

If the seller has more than two assets there are more than two risk units. Thus, some of them are retained while the pricing of the two sold units is the same as above. Therefore, when the total number of risk units demanded is lower than the supplied one the constrained buyers quote the same prices as the monopolist buyer would. Thus, the optimal pooling decision of the issuer does not change compared to the case of one buyer with full market power.

Assets of different quality. If the assets are ex-ante different a new situation arises when the issuer decides to sell assets separately. Now, the risk units are different and the asset with a better quality (i.e., higher expected cash flow) is not priced as in the monopolistic buyer case. Assume the contrary, then by offering slightly higher price than the monopolistic price, a buyer can get the better asset which results into higher profits than the purchase of the lower quality asset for the monopolist price. The last result is due to the fact that the monopolist buyer profits are higher for the the higher quality asset. The buyers keep increasing bids for the higher quality asset until the profit obtained from its purchase is equal to the monopolist buyer's profit from the purchase of the lower quality asset. Therefore, the separate sale of assets of different quality to constrained buyers increases the issuer's profits compared to the monopolist buyer case. Consequently, if the separate

sale is preferred by the issuer facing a monopolist buyer it is also preferred when facing buyers with position limits.

Formally, suppose that asset payoffs are $v_1^i = \varphi_i \sigma$ where $\varphi_i \sim Ber(1 - q_i)$ for $i = 1, 2$. Then in the monopolist buyer case an asset i is traded if $\delta \leq 1 - q_i$ and the profits of the seller and the buyer are

$$S_i(p_i) = q_i p_i = q_i \delta \sigma \tag{B23}$$

$$B_i(p_i) = (1 - q_i)\sigma - p_i = (1 - q_i)\sigma - \delta \sigma \tag{B24}$$

where $p_i = \delta \sigma$ is the efficient price. Assume that $q_1 < q_2$ and both assets can be traded efficiently, i.e., $\delta < 1 - q_2$, then $B_1(p_1) > B_2(p_2)$. If the seller allocates the better quality asset first without considering the quotes for another asset any equilibrium must satisfy the following two properties. First, the profit obtained from the lower quality asset is equal to the monopolist case profit as it can be guaranteed by quoting monopolist price for this asset and zero for the first asset. Second, the buyers' profits obtained from the two assets are equal since otherwise a buyer would either withdraw from competition for the first asset or slightly undercut the competitor. Therefore, the competition for the high quality asset in the case of the two constrained buyers increases the price $\tilde{p}_1$ offered for this asset until $B_1(\tilde{p}_1) = B_2(p_2)$ or

$$(1 - q_1)\sigma - \tilde{p}_1 = (1 - q_2)\sigma - p_2. \tag{B25}$$

Consequently, the price offered for this asset is $\tilde{p}_1 = \delta \sigma + (q_2 - q_1)\sigma$ while the total profit of the seller is

$$S = \delta(q_1 + q_2)\sigma + (q_2 - q_1)\sigma. \tag{B26}$$

The last term is the increase in the issuer's profits compared to the monopolistic buyer case.

If only the high quality asset can be traded, i.e., $1 - q_2 < \delta < 1 - q_1$, the situation becomes equivalent to the case of competitive buyers and one asset. Therefore, the quoted price is $p_1 = (1 - q_1)\sigma$, the buyers' profits are $B = 0$ while the seller's profit is $S = (1 - \delta)(1 - q_1)\sigma$. Finally, if $1 - q_1 < \delta$ no asset is traded.

The analysis illustrates another reason why the separate sale of assets might be beneficial to the issuer in the presence of several buyers with position limits on risk units. If assets are of

different quality buyers compete for the assets of higher quality to fill their limits and that increases the issuer's profits compared to the monopolistic buyer case. In contrast, when the issuer sells homogeneous shares on the pool of the assets buyers with position limits do not compete and quote the monopolist buyer prices. Therefore, the region where the separate sale is preferred by the issuer is larger in the case of several constrained buyers compared to the case of a single buyer with full market power. In the former case this region is $\delta \in [0, \min\{\bar{\delta}_1^1, \bar{\delta}_1^2\}]$ while in the later case it is $\delta \in [\bar{\delta}_{22}, \min\{\bar{\delta}_1^1, \bar{\delta}_1^2\}]$.

B.3   Buyers with Wealth Constraints

In this section, we consider the same extension of the model with two constrained buyers except that now the constraint is in terms of a buyer's wealth w. This constraint limits a buyer's quotes as we assume that their sum cannot be greater than his wealth w.[15] Suppose the seller has two identical assets characterized by the parameter q and creates two risk units as before, by either selling them separately or selling two identical shares issued on the pool of the two assets. We focus on the interval for the discount factor δ where in the monopolistic buyer case the assets can be traded efficiently separately but efficient trade breaks down when they are sold in a pool, $\Delta = [\bar{\delta}_{22}, \bar{\delta}_1^1]$.

Selling separately. First, we analyze the case where the assets are sold separately. In region Δ, the monopolist buyer quotes the efficient price $\delta\sigma$ for each asset. Offering any price below this price results in a negative profit since such offer is rejected by the seller with positive cash flows. Therefore, if a buyer's wealth is constrained as: $\delta\sigma \leq w < 2\delta\sigma$, each buyer quotes $\delta\sigma$ for one of the assets and the outcome is the same as in the case of the risk unit constrained buyers. If a buyer's wealth is higher, $2\delta\sigma \leq w \leq 2(1-q)\sigma$, they compete for the assets and bid half of their wealth, $\frac{w}{2}$, for each asset while the profits are non-negative. There is no profitable deviation since undercutting on one asset results in a surrender of the other asset. The expected profit from each asset is then given by $B_i = \frac{1}{2}((1-q)\sigma - \frac{w}{2})$. Thus, when assets are sold separately and a buyer's wealth is in the region $\delta\sigma \leq w < 2\delta\sigma$ the quoted price is the same as in the monopolist buyer case and assets are sold efficiently.

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[15]The constraint can be motivated by a requirement to post a collateral for a quote. Such constraint might also result from a model with a large punishment imposed on a buyer who cannot fulfill his quotes.

Pooling. Now we consider the case when assets are pooled. From the baseline model we know that in the monopolist buyer case the profit from quoting a price $p \geq 2\delta\sigma$ is $B_2(p) = 2(1-q)\sigma - p$, the profit from quoting a price $\delta\sigma \leq p < 2\delta\sigma$ is $B_1(p) = 2(q - q^2)\sigma - (2q - q^2)p$ and the profit from quoting any price below $\delta\sigma$ is negative since it is rejected by any seller with a positive cashflow. Consequently, in Δ the monopolist buyer quotes the inefficient, low price $\delta\sigma$ for the pool since the profit from quoting the efficient price $2\delta\sigma$ is lower, $B_1(\delta\sigma) > B_2(2\delta\sigma)$. It can also be noted that if the monopolist buyer is offered one of the two identical halves of the pool he simply quotes half of his optimal price for the whole pool.

Therefore in Δ, if $\frac{1}{2}\delta\sigma \leq w < \delta\sigma$ the two buyers quote $\frac{1}{2}\delta\sigma$ for different units and the outcome is the same as in the case of the monopolistic buyer. If $w \geq \delta\sigma$ instead the buyers must compete for the units. If the buyers' wealth w is slightly higher than $\delta\sigma$ they bid half of their wealth, $\frac{w}{2}$, for each unit. This is the equilibrium for $\delta\sigma \leq w \leq p_0$ where p_0 is given by $B_1(p_0) = B_2(2\delta\sigma)$, the inefficient low price that yields to the monopolist buyer the same profit as the efficient price. The expected profit from each unit is then equal to $B_i = \frac{1}{2}((q - q^2)\sigma - (2q - q^2)\frac{w}{2})$. As above, there is no profitable deviation since undercutting on one asset results in a surrender of the other asset.

Due to the discrete nature of the assets' cash flows there is no equilibrium for the values of wealth in the region $p_0 < w < \frac{p_0}{2} + \delta\sigma$ as a buyer has always an option to quote the price $\delta\sigma$. If $\frac{p_0}{2} + \delta\sigma \leq w < 2\delta\sigma$ the buyers quote a high price $\delta\sigma$ for one unit, each buyer for different unit, and a low price $w - \delta\sigma$ for another. Compared to the previous region, there is enough wealth to both win one unit by quoting the price $\delta\sigma$ and to deter deviations by the other buyer. In the equilibrium both units are sold efficiently. Finally, if $2\delta\sigma \leq w \leq 2(1-q)\sigma$ the buyers quote half of their wealth, $\frac{w}{2}$, for each unit while the profits are non-negative. The expected profit from each unit is then given by $B_i = \frac{1}{2}((1-q)\sigma - \frac{w}{2})$.

Overall, comparing the outcomes in the cases of the separate sales and the pooling, when $\delta \in \Delta$ and $\delta\sigma \leq w < 2\delta\sigma$, we can see that separate sales yield efficient outcomes while the sale in the pooling case is inefficient when $\delta\sigma \leq w < \frac{p_0}{2} + \delta\sigma$. Therefore, the main conclusion that the pooling might be inefficient compared to the separate sales holds if the buyers' wealth is sufficiently constrained.

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