Illiquidity, Closure Policies & the Role of LOLR

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Abstract

We develop a dynamic model of a bank which chooses its optimal liquidity buffers, equity issuance policies, and asset portfolio size when facing frictions in equity issuance, asset liquidations, and closure policies. The bank has access to central bank liquidity facilities. The central bank policies generally lead to a greater investment in new loans, lower asset liquidations and a lower expected cost of closure. On the other hand, the central bank’s provision of liquidity causes the bank to hold lower cash buffers than would be desirable. It also reduces the incentives of the bank to issue equity or cut dividends. We show that the collateral framework inherently produces these tradeoffs. Using novel data sets we estimate that the average borrowing capacity of banks at the ECB is about 50% of the assets held by the banks. In addition, we show that there are significant cross-sectional variations in the borrowing capacity, with small banks having relatively more borrowing capacity than the big banks. The data also shows that the banks with higher borrowing capacity tend to originate more new loans. These stylized facts are broadly consistent with the model’s implications. We run some policy experiments within the modeling framework, wherein we explore the effects of changes in ex-ante margins, penalty rates, and bank closure boundaries on the lending channel, equity issuance and liquidity buffers.
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1 Introduction

Central Banks provide liquidity to banks in their domain through their standing facilities - such as the Main Refinancing Operations (MRO) of the European Central Bank (ECB) and the Discount window of the Federal Reserve Bank of New York) - against an eligible menu of collateral or asset pools. They decide what asset pools are eligible and set margins or haircuts on eligible asset pools submitted by the depository institutions. Such decisions are a part of the collateral framework of central banks. The famous dictum of Walter Bagehot \cite{Bagehot1873} that Central Banks should “lend freely at a high rate, on good collateral” motivates us to think about the effects of collateralized provision of liquidity by Central Banks on banks’ decisions about their asset portfolio size, cash (liquidity) buffers, equity issuances choices, dividends and expected dead-weight losses from closure policies.

Central to assessing the effects of collateral policies is estimating the borrowing capacity of the bank at the Central Bank’s liquidity facilities. Once this is done, we can examine the economic costs and benefits of collateral policies pursued by the Central banks, in the context of a dynamic model of a bank.

One of the costs is the possible subsidy provided by the Central Banks in expanding the menu of eligible collateral and enforcing “sticky margins.” For example, the knowledge that the borrowing banks possess about their ability to access central bank liquidity may incentivize them, ex ante, to hold lower liquidity buffers than what they would have held in the absence of central bank liquidity provision. Likewise, they may choose to issue equity less frequently in the presence of central banks’ liquidity facilities. Additional potential costs may arise from the fact that the Central Bank debt is senior to existing private debt of bank: this will be internalized by creditors, potentially resulting in higher credit spreads, ex-ante.

The benefits of Central Bank liquidity arise from the fact that illiquid banks, with a positive probability, are able to return to liquid status by availing of Central Bank liquidity.

\footnote{To the extent that the level of liquidity drawn from the central bank is higher than initially anticipated, the credit spreads of private debt of banks will spike, squeezing them out of private credit markets, in “bad states.”}
in “illiquid states” and potentially escape costly closure, and avoid liquidating assets in states with correlated macro-shocks. This in turn may allow them to continue to lend, keeping the credit channel open during periods of illiquidity. Another key consideration is the extent to which banks care about how their risk-taking may correlate with the overall macro risks. Central banks will also care about the costs arising from asset liquidations, especially when they are correlated with macroeconomic shocks, of banks to address their liquidity needs. Hence, they will set their collateral framework in such a way to attenuate the social costs arising from interconnected positions. This is clearly another potential benefit provided by the central bank.

These potential costs and benefits lead us to ask the following questions. From an empirical perspective, what fraction of banks’ assets are eligible to be pledged at the liquidity facilities of the central banks? This leads us to construct an empirical estimate of the banks’ borrowing capacity at ECB. Next, what would be the effect of Central Bank margining policies on the endogenous demand for Central Bank liquidity? How do they influence the optimal asset portfolio size, equity issuance and cash (liquidity) buffer decisions of banks? Are the expected closure costs lower when banks have access to Central Bank liquidity?

Through a specific dynamic model, we address these questions. A novel feature of our modeling framework is that we model the bank closure policies based on a capital framework explicitly, which enables us to study the interactions between the closure rules and the LOLR functions of the central bank through its collateral policies. Since closure policies may subtly interact with LOLR and closure policies (see Stein (2013) and Cœuré (2013) for example), our unified framework enables us to formally explore interactions of different policy tools in affecting the bank’s decisions on asset portfolio size and liquidity buffers.

\footnote{Collateral framework is not the only policy tool that may be used for this purpose. We abstract from other macro-prudential policy tools for simplicity.}
1.1 Summary of Results

We document some important stylized facts as they pertain to central bank provision of liquidity and their possible effects. They are based on ECB proprietary data from multiple sources. We use a novel proprietary dataset that contains monthly bank-level records of all banks’ ECB borrowings for the period August 2007 to April 2016, as well as, security level information on the collateral each bank pledges. We match these data to ECB proprietary data on banks’ characteristics known as IBSI (Individual Balance Sheet Data), which covers around 250 monetary financial institutions (MFIs) from 17 euro-area countries. The data covers about 70% of euro area banking sector. Our objective is to measure the bank’s borrowing capacity with the ECB. The pledged collateral of a bank may, however, not be representative of its assets’ holdings: it is possible that a significant fraction of the bank’s assets may not be eligible for ECB liquidity. To overcome this limitation and to get at an empirical measure of a bank’s borrowing capacity at the ECB’s liquidity facilities, we implement a novel approach using data on security-level portfolio holdings of euro-area banks from the Securities Holding Statistics (SHS). Here, we find two important patterns: the mean percentage of eligible collateral that banks possess for borrowing from the central bank is only about 50% of their assets.\footnote{The central bank thus restricts the asset pool that is eligible for extending its liquidity. To put this in context, it should be noted that private creditors are likely to restrict the collateral pool even more strictly.} By examining the attributes of ineligible collateral pool (interbank loans (to take cognizance of interconnectedness), FX-denominated loans, secondary market liquidity, etc.), we gain some insights about how Central bank takes into account systematic risk of asset sales in bad states. In addition, we show that there are significant cross sectional variations in the borrowing capacity of the banks. Big banks, on average, tend to have a lower borrowing capacity over total assets as many of their assets are ineligible form the perspective of ECB’s lending facilities: inter-bank loans, non-Euro denominated assets, etc. are not eligible asset pools.

Finally, we show that there is a positive association between the banks’ borrowing ca-
capacity at the ECB and their origination of new business, primarily new loans.

We use these stylized facts in two ways: first, we use the empirically observed mean borrowing capacity as a guide in setting the level of haircuts in the model for generating the results from the model. With this in place, we next use our model to develop endogenous links between the leverage of the banks and the amount of liquidity that they draw from the central bank. We also show that the model implies that the banks with greater borrowing capacity, on average, invest more in new loans.

The paper develops a dynamic, theoretical framework, and then uses the theory as a lens to analyze formally how specific provisions of Central Bank liquidity such as the lending rate, haircuts, penalty rates, etc., affect the decisions of banks (loan portfolio decisions, asset liquidations, equity issuance, and cash buffers) and the expected costs of closures. We establish the following theoretical results:

Banks, faced with costly equity issuance, pay dividends only in “good states” when their cash balances exceed a threshold high level. Below this level, they do not pay dividends and hold cash buffers. The level of cash buffers held by the banks depend on whether or not they have access to central bank liquidity, the collateral policies, and the penalty rates. In general, banks hold lower cash buffers and issue equity less often when they have access to central bank liquidity facilities, and the levels are sensitive to the collateral policies. The presence of private debt (such as deposits and subordinated debt), the extent of maturity transformation, and the closure policies of resolution authorities all have qualitatively important implications. In addition, we find that the banks tend to issue equity less often in the presence of central bank liquidity facilities, relative to the counterfactual.

The amount of new loans extended by the bank depends crucially on the banks’ access to central bank liquidity facilities, which in turn depends on the amount of eligible collateral (net of haircuts) held by the banks: banks with greater access to central banks (i.e., with more eligible collateral) tend to invest more in new loan portfolios. This result is sensitive to the closure policies of the resolution authorities.
Banks dynamically change the level of central bank debt in their liability structure, relative to their private market debt level to manage their “illiquidity”: this has implications for the credit spreads of the banks’ debt in so far as the central bank debt is senior to the private debt claims issued by the banks.

Finally, we highlight a potential conflict between incumbent equity holders and new equity holders that may induce an asset liquidation bias: incumbent equity holders find that the costly issuance of equity dilutes their claims, and hence may prefer to sell assets in some states. This “asset liquidation bias” hurts the creditors.

The roadmap of the paper is as follows. In section 2, we provide an overview of the relevant literature and place our paper in a proper perspective. Section 3 describes the dynamic model of a bank and the key economic ingredients of the modeling approach. Section 4 describes the data and contains our empirical results and stylized facts. In Section 5 we provide the model results with and without access to Central Bank liquidity. Section 6 develops several policy experiments covering LOLR policies, and closure policies. Section 7 concludes.

2 Literature Review

There is a significant amount of literature, which address the Lender of Last Resort (LOLR) role of the Central Bank and its impact on banks, which utilize the liquidity provided at the LOLR. In addition, this literature also examines the consequences of the banks’ behavior at the LOLR for the fragility of the markets, and risk-taking propensity. In this section, we briefly review the most relevant work as they pertain to the main goals of our research.

A number of papers have addressed the implications of the Central Bank’s margin policies, penalty rates and interest rates. In this context, the work of Garleanu and Pedersen (2011), Ashcraft, Garleanu, and Pedersen (2011), Repullo (2005), and Woodford (2010) are close in spirit to our paper. Garleanu and Pedersen (2011) examines the key distinctions between the
margin policy and the interest rate policy on their pricing implications for high margin and low margin assets. Woodford (2010) argues that the margin as a monetary policy tool may be more important in unusual states of the world, when the economy is in stress. Repullo (2005) shows that the penalty rates and margins may have very different implications on risk-taking by banks. Ashcraft, Garleanu, and Pedersen (2011) provide evidence from TALF and survey evidence that haircuts affect the required rates of returns. Corradin and Rodriguez-Moreno (2016) compare a matched sample of euro area sovereign bonds issued in both euro and US dollar and study the effect of ECB collateral and liquidity factors on the pricing variations.

Our paper takes a more detailed investigation of a bank’s response to central bank policies, but we abstract from equilibrium pricing implications considered in Garleanu and Pedersen (2011). We, however, are able to characterize, the response of the bank in terms of its optimal liquidity buffers, equity issuance, and dividend payments to central bank policies, and relate it to stylized facts.

In the context of our paper, Drechsler et al. (2016) show that weakly-capitalized banks borrowed more from the central bank. Their dataset is drawn from the ECB and covers all the Eurozone banks. Fecht, Nyborg, and Rocholl (2011) show that weak banks tend to demand more liquidity from LOLR. The dataset used in this study is confined to German banks. Nyborg (2016) provides a detailed analysis of the collateral framework of ECB. He argues that the collateral framework of the Central Banks may have a distortionary effect: they may bias the private provision of liquidity.

Bindseil et al. (2017) argue that the collateral framework has helped to “prevent large-scale liquidity-driven defaults of financial institutions in major advanced economies.” It stresses that the broad collateral policy has helped to create and maintain an elastic supply of credit to the economy, and protected the economy from financial losses. In addition, the paper emphasizes an ex-ante effect: banks anticipating a supportive collateral framework, which accepts a broad range of collateral may engage more in maturity transformation and liquidity transformation. The paper notes that potential adverse effects of a broad collateral
policy such as zombie banks, excessive risk-taking, etc. can be alleviated through monitoring by equity holders, private creditors and prudential bank supervision.

Our framework allows us to examine these issues in a precise way. First to properly judge the effects of collateral framework, it is necessary to set up a counterfactual, which can be used as a benchmark to evaluate the implications of collateral policies of the central bank. Our framework provides such a counterfactual. We also conduct policy experiments in which we vary the haircuts and penalty rates to examine the implications of conservative and generous collateral policies and bank-specific outcomes such as optimal cash buffers, optimal new loans investments, expected dead-weight losses associated with asset sales and closure.

We show that the collateral policy can lead to lower asset liquidations in bad states of the world, a higher level of solvency through improved equity values and lower credit spreads. More importantly, banks invest more by making additional loans relative to the counterfactual. The flip side is, however, that bank tends to hold lower cash buffers. In our framework, lower cash balances is a channel to increase the overall riskiness of the portfolio of assets held by the bank. In addition, we show that the access to LOLR can diminish the incentives to issue equity, relative to the counterfactual. It may also promote greater dividend payments by the banks.

De Fiore, Hoerova, and Uhlig (2017) examine the role of central bank collateral policy when the bank faces frictions in money markets. Their general equilibrium framework shows that when banks face decreasing access to unsecured funding, it can lead to moderate output contractions. By turning to secured funding markets, banks can alleviate this but may face higher haircuts. This is where central banks by expanding their balance sheets can help restore the output losses.

Alves, Bonfim, and Soares (2016) use the Portuguese Central Credit Register (CRC), which has monthly data on virtually all bank loans granted by Portuguese Financial institutions. They combine this with the monthly information on banks’ liquidity, capital
and balance sheet items, as well as on their holdings of Portuguese government bonds. In addition, they gather bank-level data on the recourse to monetary policy operations and standing facilities, the collateral pool and reserve requirements compliance. They show that the access to central bank liquidity allowed the banks to maintain their loan portfolio at a normal level despite the collapse of private credit markets. Crosignani and Carpinelli (2016) investigate how the extension of a pool of eligible collateral by the Italian government for the ECB 3–year Long Term Refinancing Operations (LTROs) restored bank credit supply after the previous unsecured wholesale funding dry-up. Van Bekkum, Gabarro, and Irani (2017) study the impact of the lower rating requirement for residential mortgage-backed securities announced in the context of the 3–year LTROs on bank lending in the Netherlands. They document that most affected banks by the policy increase loan supply and lower interest rates on new mortgage originations. Jasova et al. (2018) provide evidence that 3–year LTROs providing long-term funding reduced debt rollover risk of Portuguese banks has a positive effect and economically sizable impact on bank lending to the real economy.

From a modeling perspective, our approach draws from the insights of three papers: Asvanunt, Broadie, and Sundaresan (2011), Bolton, Chen, and Wang (2011), and Hugonnier and Morellec (2017). These models share a common view about an important friction in the markets: they all recognize that there are costs associated with issuing new equity. There are both direct costs of floatation as well as costs associated with dilution of the stakes of “inside equity holders.” This incentivizes the borrower to accumulate cash buffers internally. Asvanunt, Broadie, and Sundaresan (2011) show that illiquidity in the presence of realistic capital market frictions can be managed by (a) equity dilution, (b) carrying positive cash balances, or (c) entering into loan commitments with a syndicate of lenders. They also show that carrying positive cash balances for managing illiquidity is generally less efficient than entering into loan commitments, since cash balances (a) may have agency costs, (b) reduce the riskiness of the firm thereby lowering the option value to default, (c) postpone or reduce dividends in good states, and (d) tend to inject liquidity in both good and bad states.
Loan commitments, on the other hand, (a) reduce agency costs, and (b) permit injection of liquidity in bad states as and when needed. They however assume that the borrower takes the asset side as given: we relax this assumption. They also do not model a Central Bank, which is the focus of our work.

Hugonnier and Morellec (2017) examine the liquidity accumulation strategy of a bank when the bank faces costly equity dilution. They model a fixed cost and focus on equity-maximizing strategy for cash accumulation and optimal default. Their work showcases the importance of tail risk, by modeling jumps, but they abstract from Central Bank liquidity provision nor do they explicitly allow the possibility of changing the size of the asset portfolio, which we model in this paper.

Bolton, Chen, and Wang (2011) provide a framework wherein a firm can alter optimally its investment policy and will also have the ability to manage its liquidity through both cash balances and lines of credit. By modeling both the asset side and the liability side, they come close to the ideas developed in our paper. In their framework, the key variable is the ratio of cash to capital, which effectively serves as the single, key state variable. They get a “double barrier” characterization of liquidity management in which their key state variable plays an important role. In our formulation, both the loan portfolio size and the amount of cash buffer both enter as state variables, and the problem cannot be collapsed into a single state variable as in Bolton, Chen, and Wang (2011), due to the richer structure of the economy studied in the paper. Our model allows for private debt in addition to adjustments on the asset side of the balance sheet. In addition, in our model the Central Bank is explicitly considered. With the presence of private debt in our model, accessing Central Bank liquidity, which sits senior to existing private debt, is a nuanced decision for the bank - we model this explicitly. First, we allow the banks to be closed by resolution authorities when the bank capital falls below a certain threshold. Thus, banks in our model can be shut down by the closure authorities. In this sense, the closure policies of the regulators inform on the decisions of banks to draw on central bank liquidity. Second, banks perform maturity transformation: a fraction of their
debt matures each instant, whereas the assets have a larger average life. The assets of the banks are also illiquid, and depending on their correlation with macroeconomic shocks, the liquidation costs can be very high. These are important dimensions on which our model differs from the work of Bolton, Chen, and Wang (2011).

The work by De Nicolo, Gamba, and Lucchetta (2014) is also closely related to our work. They provide a dynamic model of banking in which the banks transform short-dated insured deposits into long-dated loans, and are subject to regulation. The banks in their model are also subject to default risk due to costly external equity issuance and inability to issue un-collateralized debt, leaving a clear role for regulation of banks. They use their framework to link capital requirements and liquidity requirements on banks’ optimal decisions.

3 A Dynamic Model of Banking

We model a representative bank in the economy. The bank in our model starts with a risky asset portfolio, risk-free deposits and a level of risky private market debt that can be redeemed prematurely with a positive probability. The bank faces the following key frictions: a) it can issue equity, but the process is costly$^4$; b) it can increase or decrease the size of its assets, but faces portfolio adjustment costs that are asymmetric. We assume that it may be costlier to liquidate assets, especially when the bank’s asset returns are highly correlated with macroeconomic shocks; c) the risky private debt can be prematurely redeemed leading to premature costly liquidation of the bank; and, d) the bank can be shut down by the closure/resolution authorities when its book value of equity drops to a low threshold. When this happens, there are additional dead-weight losses.

Faced with these frictions, the bank must decide at each instant, the size of their asset portfolio, how much cash buffer to hold, how much central bank liquidity to draw, and decide on their de-leveraging and costly equity issuance decisions in illiquid states. The bank can liquidate a part of its assets portfolio or issue equity to meet its liquidity needs. The

$^4$The costs arise from the informational asymmetry, discussed in Myers and Majluf (1984).
bank maximizes its equity value in making the above choices taking into account the fact that the closure/resolution authorities can shut down the bank when the book value of bank equity falls below an exogenously specified level.

The bank has a finite horizon of $T$, at which time it liquidates its asset portfolio and cash buffers, pays off its obligations to its creditors and keeps any remaining amount as dividends to equity holders. We assume that the bank can raise external funds once at time $t = 0$ by issuing equity, debt and/or collateralized lending with the central bank. After $t = 0$ the bank can roll over its debt and issue equity, but equity issuance is assumed to be costly.

At $t = 0$ the bank raises some risky private debt on which it pays a coupon $C$ per unit time, with a par value of $P$. In addition, the bank issues risk-free deposits with a par value of $P_D$ and a contractual coupon rate of $C_D = Pr$ per unit time. The risky debt is thus subordinated to the deposits. A fraction of the risky debt matures at each instant, and the bank rolls over to maintain the par value of the risky debt at $P$. When the market value of the risky debt is below par, the roll over will be an expensive proposition for the bank, reflecting the tight credit market conditions for the bank to borrow (Leland and Toft (1996)). In the Appendix we provide an extension of our framework where in these states of the world, we model, with a positive probability, that the bank will simply be unable to refinance its debt, and must liquidate with the proceeds used to pay off the depositors. Any remaining amount will be allocated to creditors and equity holders with strict absolute priority rights. In this way we capture in our model, the costs of premature liquidations stemming from freezes in capital markets.

Given the constraints faced by the bank, it would have an incentive to have cash buffers, and seek Central Bank liquidity when needed, if it is optimal for the bank to do so.

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5We enforce the necessary mathematical conditions to ensure that the deposits are risk-free when the bank is shut down by the resolution authorities.
The bank has an initial cash balance of $W_0$. Its asset portfolio $A$ follows the dynamics

$$dA_t = \left( -\frac{1}{\delta} A_t + I_t \right) dt,$$

where $\delta > 0$ is the rate at which the asset portfolio matures. Thus, the average maturity of the existing stock of loans is $\delta$. The bank can increase or decrease the stock of its loans investing or disinvesting in the asset portfolio itself, $I_t$. This is a control variable at the bank’s disposal: the bank can sell of its loan portfolio in “bad states” to meet the liquidity needs, and expand its loan portfolio in “good states”.

The bank’s operating revenue at time $t$ is proportional to its asset stock $A_t$, and is given by $A_t dX_t$, where $dX_t$ is the bank’s revenue shock over time increment $dt$. The bank’s revenue shock evolves according to

$$dX_t = \mu_X dt + \sigma_{X,1} \rho dZ_{1,t} + \sigma_{X,2} \sqrt{1 - \rho^2} dZ_{2,t},$$

where $Z_{1,t}$ and $Z_{2,t}$ are standard Brownian motions under the risk-neutral measure and $\rho$ is the correlation coefficient. The first shock captures the macroeconomic shocks to the economy, while the second represents the idiosyncratic shocks to the value of the asset portfolio of the bank. Thus, the bank’s revenue shocks are assumed to be i.i.d., and the parameters $\mu_X$, $\sigma_{X,1}$ and $\sigma_{X,2}$ are the mean and volatility of the risk-adjusted revenue shocks $dX_t$. We assume that $\sigma_{X,1}$ is higher than $\sigma_{X,2}$ to reflect the fact that the shocks to the economy can have a much greater effect on the bank’s revenue than idiosyncratic shocks. This assumption reflects the intuition that the bank’s asset portfolio is diversified so that the idiosyncratic risk is low, but the bank is still exposed to economy wide shocks over which it has limited control.

As seen from equation (2), the uncertainty in the economy in our model is characterized by both idiosyncratic shocks to the value of the asset portfolio of the bank, as well as by the
Macroeconomic shocks to the economy. In “bad macro-economic states”, both the cash flows of the asset portfolio as well as their resale values are adversely impacted. The assets of the bank can be thought of as a portfolio of loans and non-loan assets such as debt securities. We will explicitly account for the fact that only a fraction of the assets held by the bank may be eligible as collateral in drawing liquidity from the Central Bank.

The bank’s incremental operating profit cash flows \( dY_t \) over time increment \( dt \) is given by

\[
dY_t = \frac{1}{\delta} A_t dX_t + \frac{1}{\delta} A_t dt - I^*_t dt - g(I^*_t, \rho) dt - C_D dt - C_D dt - \frac{1}{m} (D(W_t, A_t; s) - P) dt,
\]

where \( (1/\delta)A_t \) is the repayment of maturing loans, \( I^*_t \) is the optimal investment in new loans if it is positive, or the amount of cash obtained by liquidating loans if it is negative. \( g(I^*_t, \rho) \) is the additional adjustment cost that the bank incurs in the investment process. We introduce a convex asymmetric loan adjustment cost as in the \( Q \)-theory of investment to capture banks information production costs about credit quality. The adjustment-costs function for loans is quadratic and increasing in the correlation between systematic and idiosyncratic shocks:

\[
g(I^*_t, \rho) = \frac{(\psi^+ \mathbb{1}_{I^*_t \geq 0} + \psi^- \mathbb{1}_{I^*_t < 0})(1 + \rho)I^{*2}}{2},
\]

where \( \mathbb{1} \) is an indicator function. A bank incurs screening and monitoring per-unit costs \( \psi^+ \) when it increases lending, and per-unit liquidation costs \( \psi^- \) when loans are reduced, where \( \psi^- \geq \psi^+ \geq 0 \). If \( \psi^- > \psi^+ \), then there is costly reversibility, since a bank would typically face higher costs to liquidate its investments rather than expanding them. This assumption is consistent with the view that the costs of breaking bank relationships may be higher than
those associated with an expansion of lending to old as well as new customers. Another important feature of the cost function is that it explicitly recognizes the possibility the bank may be liquidating its assets when there are not enough bidders. We capture this idea by letting the liquidation costs to increase with the correlation coefficient $\rho$: clearly, when the bank sells its assets when there is an aggregate macroeconomic shock, it has to do so by experiencing a discount. When $\rho = 0$, this discount disappears as the assets of the selling banks have greater value to the banks acquiring them. This distinction is emphasized in models of bank capital structure, as in Gale and Gottardi (2015) and Gale and Gottardi (2017). When we set $\psi^+ = \psi^-$, we let the costs of adjusting the loan portfolio to just depend on the correlation of the bank’s cash flows to macroeconomic shocks. The asset market is otherwise liquid. Thus, our specification can shed some light on the extent of liquidity transformation undertaken by the bank, and how that influences its choice of loan portfolio and its decision to access LOLR.

The last terms are related to the debt service. Deposits are the most important source of funds for banks to finance their assets. Let $P_D$ denote the amount of deposits that a bank takes. Deposits are risk-free, the fair interest rate on deposits is the risk-free rate $r$. Thus $C_D = rP$. We model the external debt as follows. A constant fraction $m$ of the outstanding debt matures at any instant of time. $C$ is the coupon of the external debt with face value $P$. Thus, the bank retires the debt at the rate $mP$ but it continuously replaces it by the issuance of new debt with identical principal value, coupon rate, and seniority. New debt is issued at market value which may diverge from par value as in Leland and Toft (1996). Thus the net refunding cost occurs at the rate $m(P - D(W_t, A_t; s))$ where $D(W_t, A_t; s)$ is the market value of the total debt given the current value of cash $W_t$ and asset portfolio $A_t$ and $s$ is the credit spread.

Ignoring closure and debt repurchase, the average maturity of a debt contract is then $1/m$ years. This way of modeling the debt allows us to examine two important issues. First, we can compare the maturity of the debt contract with the maturity of the loan book, $1/\delta$,
to quantify the extent of maturity transformation performed by the bank. Second, we can explore the relationship between the share of short-term liabilities of the bank and the bank’s reliance on cash buffers and central bank liquidity.

We next turn to the bank’s cash buffers. Let $W_t$ denote the bank’s cash buffer at time $t$. When $W_t > 0$, the bank is in the cash region. The rate of return that the bank earns on its cash buffer is the risk-free rate $r$ minus a carry cost $\lambda > 0$ that captures in a simple way the agency costs that may be associated with free cash in the bank. Intuitively, when the cash buffer is very high, the bank is better off paying out the excess cash to shareholders to avoid the cash-carrying cost. The benefit of a payout is that shareholders can invest at the risk-free rate $r$, which is higher than $r - \lambda$, the net rate of return on cash within the bank. However, paying out cash also reduces the bank’s cash balance, which potentially exposes the bank to current and future underinvestment and future reliance on Central Bank liquidity. The tradeoff between these two factors determines the optimal payout policy. Let $W(A)$ denote the endogenous payout boundary. If the bank starts with a large amount of cash ($W > W(A)$), then it is optimal to distribute the excess cash as a lump sum and bring the cash holdings down to $W(A)$. Because the bank’s equity value must be continuous before and after cash distribution, we have

$$E(W, A) = E(W(A), A) + (W - W(A))$$

$$W > W(A) > 0.$$  

Let $U_t$ be the bank’s cumulative payout to shareholders up to time $t$, and by $dU_t$ the incremental payout over time interval $dt$. Thus, the dynamics of the state variable $W$ in the cash region ($W_t > 0$) is

$$dW_t = dY_t + (r - \lambda) W_t - dU_t.$$  \hspace{1cm} (5)$$

If the bank runs out of cash ($W_t = 0$), it has to either raise Central Bank liquidity to
continue operating, or raise equity. The latter case is our counterfactual. Bank’s equity value is continuous before and after equity issuance which implies the following condition at \( W(A) = 0 \)

\[
E(0, A) = E(\tilde{W}(A), A) - \phi A - (1 + \gamma) \tilde{W}(A)
\]

(6)

where \( \tilde{W}(A) \) is the equity issuance amount, \( \gamma \) is the marginal cost of equity issuance and \( \phi \) is a monetary cost that depends on the asset value \( A \).\(^6\)

Alternatively, the bank can also borrow from the liquidity facilities of the Central Bank, \( W_t \leq 0 \). The bank optimally choose the amount of liquidity to be drawn before issuing equity. The boundary condition for the amount of equity issuance is similar to Equation (6):

\[
E(W(A), A) = E(W(A) + \tilde{W}(A), A) - \phi A - (1 + \gamma) \tilde{W}(A),
\]

(7)

where \( W(A) < 0 \) is the endogenous boundary.

The central bank liquidity account is fully collateralized by the bank’s asset portfolio, net of the central bank haircut \( \theta \). This implies that the borrowing capacity of the bank from the Central Bank is:

\[
0 \geq W_t > -(1 - \theta) A_t.
\]

(8)

We assume that the liquidity is offered by the central bank at penalty rate captured by the spread \( s^{CB} \). However, the bank doesn’t have to pay any commitment fee on the unused amount of the liquidity account or upfront fee for having access to the central bank liquidity. These assumptions are consistent with what one observes in real life. Moreover, the bank has to be solvent to draw the Central Bank liquidity. We assume that the bank’s closure is

\(^6\)This gives the following smooth pasting boundary condition:

\[
E_W(\tilde{W}(A), A) = 1 + \gamma.
\]
triggered when

\[
\frac{\text{Equity Book Value}}{\text{Assets}} = \frac{\max(A + W - P_D - P, 0)}{A + W} \leq \alpha \quad \text{no CB}
\]

\[
\frac{\text{Equity Book Value}}{\text{Assets}} = \frac{\max(A + W - P_D - P, 0)}{A} \leq \alpha \quad \text{with CB}
\]

where \(\alpha\) is a constant (e.g. 5\%) and \(W_b\) and \(A_b\) denote the cash and asset portfolio levels at which the bank resolution authorities will shut down the “problem bank”. The resolution authority shuts down the bank when the Tier-1 capital (equity) falls below a certain fraction of the sum of cash and loan portfolio held by the bank.\(^7\)

Combining cash flow from operations \(dY_t\) with the bank’s by the cumulative payout process \(U\) and the debt payments the bank’s cash inventory \(W\) evolves according to the following cash accumulation equation:

\[
dW_t = \frac{dY_t}{\text{Bank’s profits}} + (r + s^{CB} W_t + dH_t) \text{ Equity funding}
\]

We denote by \(H_t\) the cumulative external financing up to time \(t\) and by \(dH_t\) the incremental external financing over time interval \(dt\). \(E(W, A)\) denotes the bank’s equity value that depends on the two state variables. Our model, in fact, is a two-dimensional (2-D) framework with both loans stock and cash as state variables that will provide sharp quantitative predictions that we will link to the empirical stylized facts previously discussed. We will show that bank decision-making and bank value depend on which of the following regions it finds itself in: (i) closure region, (ii) equity issuance or access to central bank liquidity region, (iii) an internal financing region, and (iv) a payout region. The bank is in the equity funding region when its cash stock \(W\) is less than or equal to an endogenous lower barrier \(W(A)\). It is in the payout region when its cash stock \(W\) is greater than or equal to an endogenous upper barrier \(\bar{W}(A)\). Then, it is in the internal financing region when \(W\) is in

\(^7\)We can attach a risk weight to the loan portfolio by linking the risk weight to the volatility parameters, and the secondary market liquidity parameter. We abstract from risk weights for simplicity.
between $W(A)$ and $W(A)$. The Appendix provides a characterization of the model solution for the equity and debt value and a discussion of the numerical approach adopted.

4 Data

4.1 Borrowing Capacity with the Central Bank

One key empirical challenge is to measure the bank’s borrowing capacity with the ECB. In fact, the pledged collateral by a bank may not necessarily be representative of its assets’ holding. Public information about banks asset holdings is extremely limited since these data are considered proprietary or are only available to bank regulators.\footnote{Drechsler et al. (2016) for example use information on bank holdings of distressed-sovereign debt published for the European bank stress tests. However, European banks conducted only three separate rounds of bank stress tests (March 2010, December 2010, September 2011), the information is limited to bank holdings of distressed-country sovereign debt and the bank stress tests were only designed to include the largest banks in euro area.}

We overcome this limitation implementing a novel approach to gauge the bank’s borrowing capacity with the ECB. For marketable assets, we use data on security-level portfolio holdings of euro-area investors from the Securities Holding Statistics (SHS). The data are collected on a quarterly basis from custodian banks in the euro area since 2009 – Q1. Securities in our sample are identified by a unique International Securities Identification Number (ISIN). Investors in the SHS are defined by sector and by country of domicile. There are six aggregate sectors: households, monetary and financial institutions (MFI), insurance companies and pension funds (ICPF), other financial institutions (OFI), general government, and non-financial corporations. We refer to MFI as banks. The assets covered include both government and corporate debt, equities, mutual fund shares, asset-backed securities (ABS), and covered bonds providing a unique overview of the portfolios of banks in the euro area. For each country, we compute how much the banking sector holds of a specific ISIN over its outstanding amount. Thus, we merge the SHS with data on the eligible securities published by the ECB to verify whether an ISIN in SHS is eligible for ECB liquidity operations and the
haircut applied by the ECB if the security is eligible. If the security is ineligible we set the haircut at 100%. Finally, we link the holdings data to IBSI to compute the aggregate ECB haircut on the main IBSI asset balance sheet items at MFI level on a monthly frequency. The borrowing capacity with the ECB is defined as the value of the asset balance sheet item (e.g. domestic sovereign bonds) at net of the ECB haircuts. For non-marketable assets, we rely on the ECB eligibility criteria. Three types of non-marketable assets are eligible as collateral: fixed-term deposits from eligible counterparties, credit claims and non-marketable retail mortgage-backed debt instruments (RMBDs).

This in turn, allows us to measure the overall borrowing capacity of each bank over time for the 250 banks during 2008 – 2016 and investigate the time-series and cross-sectional patterns of such measure.

Table 1 provides summary statistics for our main sample. Average bank size is euro 97,039 million. About 61% of assets are loans and 16% of assets are fixed income securities. 59% of liabilities are financed with deposits. On average, about 47% of banks borrows from the ECB in a given month. The average total borrowing per bank is euro 2,097 million\textsuperscript{9}, which represents about 21% of the collateral pledged. Therefore, banks on average are over-collateralized when they draw liquidity from the ECB. About 17% of collateral is sovereign bonds and about 12% of collateral is domestic sovereign bonds indicating that banks tend to pledge sovereign bonds issued by its own country. The category ”Other securities” includes mortgage backed securities, covered bonds and other debt instruments and approximately represents 67% of collateral. The average ECB haircut on the collateral pledged is about 9%. The same statistics are separately provided for banks with low and high ECB borrowing capacity. The first group pools the banks with a borrowing capacity below the 25\textsuperscript{th} percentile of the distribution that is 39%, while the second one pools the banks with a borrowing capacity above the 75\textsuperscript{th} percentile of the distribution that is 65%. We note that large banks have on average a lower borrowing capacity over total assets. These banks are relatively

\textsuperscript{9}This includes observations with zero borrowing.
more diversified in terms of asset types but most of these assets are not eligible for ECB credit operations (e.g. loans to banks).

All central bank borrowings are collateralized by ECB-eligible assets, and are subject to margins (haircuts). Thus, the available pool of ECB-eligible assets net of haircuts provides an upper bound on the borrowing capacity of banks with respect to Central Bank liquidity. We examine in Figure 1 the cross-sectional variation in the borrowing capacity of banks. As can be seen, there is a significant cross sectional variation in the borrowing capacity of the banks. The mean borrowing capacity is, roughly, 50% of the eligible assets. Further analysis shows that smaller banks tend to have larger borrowing capacity. Since we do not know whether ECB-eligible collateral has been pledged in the private markets, our estimates should be thought of as an upper bound on the banks’ ability to draw liquidity at the ECB.
4.2 Other Parameters

Table 2 summarizes the parameters in the model. We set the theta parameter, which is the haircuts used by the central bank at 50% consistent with our estimates. For the marginal costs of equity issuance, we rely on the estimates of Hennessy and Whited (2007). They estimate the costs between 5% to about 11% for non-financial firms. We set the costs at 6%. The risk-free rate is set at 1% reflecting the low interest rates over the past decade. The idiosyncratic loan shock are set at the median levels of about 5% reported in Sundaresan and Wang (2015). We set the volatility of systematic loan shock at a higher level. We assume that the asset liquidation costs are higher at 5% compared to loan acquisition costs, which is set at 3%. We set the level of deposits at 70 and the level of risky debt at 30. These are broadly consistent with the statistics that we have reported in Table 1.

Before analyzing the impact of having access to Central Bank liquidity, we first consider the benchmark case in which the bank is forced to liquidate when it runs out of cash. Next, we extend our framework to allowing to the bank to rely on a secured credit line with the Central Bank. We model the credit line as a source of funding the bank can draw on at any time it chooses up to a limit that depends on the Central Bank haircuts policy. Thus, we set the credit limit to a maximum fraction of the banks loans stock, so that the bank can borrow up to \((1 - \theta)A\), where \(\theta\) is the haircut.
Table 1: Summary statistics

<table>
<thead>
<tr>
<th></th>
<th>Full Sample</th>
<th>Borrowing capacity &lt; p25</th>
<th>Borrowing capacity &gt; p75</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>St.Dev</td>
<td>Mean</td>
</tr>
<tr>
<td>Assets</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Assets</td>
<td>97,038</td>
<td>170,412</td>
<td>221,759</td>
</tr>
<tr>
<td>Cash/Assets (%)</td>
<td>0.27</td>
<td>0.32</td>
<td>0.09</td>
</tr>
<tr>
<td>Loans/Assets (%)</td>
<td>61.03</td>
<td>14.62</td>
<td>53.95</td>
</tr>
<tr>
<td>Fixed income securities / Assets (%)</td>
<td>16.41</td>
<td>11.25</td>
<td>13.04</td>
</tr>
<tr>
<td>Equity/Assets (%)</td>
<td>3.55</td>
<td>3.73</td>
<td>4.58</td>
</tr>
<tr>
<td>Own sovereign sec / Fixed income securities (%)</td>
<td>26.04</td>
<td>25.34</td>
<td>17.40</td>
</tr>
<tr>
<td>Liabilities</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Liabilities</td>
<td>94,490</td>
<td>166,059</td>
<td>214,452</td>
</tr>
<tr>
<td>Deposits / Liabilities (%)</td>
<td>59.44</td>
<td>20.40</td>
<td>43.09</td>
</tr>
<tr>
<td>Debt / Liabilities (%)</td>
<td>15.13</td>
<td>16.19</td>
<td>14.44</td>
</tr>
<tr>
<td>ECB Collateral and Liquidity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Liquidity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Liquidity</td>
<td>2,097</td>
<td>5,184</td>
<td>3,087</td>
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<tr>
<td>Dummy borrowing (%)</td>
<td>47.96</td>
<td>49.96</td>
<td>43.01</td>
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<tr>
<td>Borrowing / Assets (%)</td>
<td>2.21</td>
<td>3.93</td>
<td>1.61</td>
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<td>Borrowing / Collateral (%)</td>
<td>21.74</td>
<td>30.56</td>
<td>17.14</td>
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<tr>
<td>MRO liquidity / Collateral (%)</td>
<td>3.37</td>
<td>11.64</td>
<td>3.06</td>
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<tr>
<td>LTRO liquidity / Collateral (%)</td>
<td>13.63</td>
<td>25.31</td>
<td>10.35</td>
</tr>
<tr>
<td>Collateral</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Collateral value (after haircuts)</td>
<td>5,952</td>
<td>11,574</td>
<td>11,180</td>
</tr>
<tr>
<td>Sovereign Sec / Collateral (%)</td>
<td>17.64</td>
<td>27.87</td>
<td>16.73</td>
</tr>
<tr>
<td>Own Sovereign Sec / Collateral (%)</td>
<td>12.59</td>
<td>25.56</td>
<td>6.95</td>
</tr>
<tr>
<td>Other Securities / Collateral (%)</td>
<td>67.32</td>
<td>33.69</td>
<td>66.82</td>
</tr>
<tr>
<td>Non-marketable loans / Collateral (%)</td>
<td>15.04</td>
<td>26.14</td>
<td>16.45</td>
</tr>
<tr>
<td>Implied ECB haircut on collateral pledged (%)</td>
<td>9.06</td>
<td>54.35</td>
<td>8.03</td>
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</table>
Table 2: **Parameters**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-free rate</td>
<td>$r$</td>
<td>1%</td>
</tr>
<tr>
<td>Cash carrying cost</td>
<td>$\lambda$</td>
<td>1%</td>
</tr>
<tr>
<td>Mean loan-shock</td>
<td>$\mu_X$</td>
<td>7.2%</td>
</tr>
<tr>
<td>Volatility of systematic loan shock</td>
<td>$\sigma_{X,1}$</td>
<td>5%</td>
</tr>
<tr>
<td>Volatility of idiosyncratic loan shock</td>
<td>$\sigma_{X,2}$</td>
<td>5%</td>
</tr>
<tr>
<td>Correlation between systematic and idiosyncratic loan shock</td>
<td>$\rho$</td>
<td>0%</td>
</tr>
<tr>
<td>Years of reimbursed loan</td>
<td>$\delta$</td>
<td>10</td>
</tr>
<tr>
<td>Unit price for loan investment</td>
<td>$\psi^+$</td>
<td>3%</td>
</tr>
<tr>
<td>Unit price for loan fire sales</td>
<td>$\psi^-$</td>
<td>5%</td>
</tr>
<tr>
<td>Marginal cost of equity issuance</td>
<td>$\gamma$</td>
<td>6%</td>
</tr>
<tr>
<td>Face value deposits</td>
<td>$P_D$</td>
<td>70</td>
</tr>
<tr>
<td>Face value external risky debt</td>
<td>$P$</td>
<td>30</td>
</tr>
<tr>
<td>Years to maturity debt</td>
<td>$m$</td>
<td>1</td>
</tr>
<tr>
<td>Bankruptcy trigger</td>
<td>$\alpha$</td>
<td>5%</td>
</tr>
<tr>
<td>Central bank haircut</td>
<td>$\theta$</td>
<td>50%&amp;80%</td>
</tr>
</tbody>
</table>
5 Model Results

5.1 Counterfactual

Figure 2 characterizes the counterfactual in which the bank has no access to the central bank’s liquidity facilities.

The red dotted line represents the closure boundary \( W_D(A) \): the bank is shut down by the resolution authorities when its ratio of the book value of equity to assets drops to a threshold of 5%. The solid black line represents the upper boundary of cash \( W_L(A) \) when the bank does not have the ability to issue equity and it is forced into liquidation when it runs out of cash. Above this solid black line, the bank pays all excess cash as dividends. The cash buffer, \( W_L(A) \), increases with respect to asset values \( A \) but is concave. When asset values increase, the marginal value of holding cash decreases as liquidation becomes less likely. This implication is a key departure from the work of Bolton, Chen, and Wang (2011) where the ratio, \( W/A \), is a sufficient state variable. In their framework, this boundary would be a straight line. The concavity is pronounced especially near the closure boundary, indicating that the bank prefers cash when it is threatened with closure by resolution authorities, in a more than proportionate manner relative to changes in the asset value. Thus, the region between the solid black line and the closure boundary is the cash buffer strategy of the bank when \( A \leq 105 \). Above asset values of 105, the region is simply the area below the solid black line and the X-axis (\( W = 0 \)).

When the bank has the ability to issue equity, its upper boundary for dividend payments \( W_E(A) \) goes down as denoted by the black dashed curve, below the solid curve \( W_L(A) \). The bank holds less cash balance at every asset value in the solvent region, where the asset values are higher than 105. The amount of equity issued by the bank is endogenous and is denoted by the black circles, \( \tilde{W}_E(A) \). The bank issues equity when the cash buffers reach a level \( W = 0 \). The bank issues more equity at higher asset levels to alleviate the illiquidity problem but the marginal benefit from a larger equity issuance is lower because the
Figure 2: **Counterfactual:** This figure plots, the closure boundary, dividend payout region, and the equity issuance decisions under the counterfactual. Closure by the resolution authorities occurs when $A + W < 105$. The bank issues equity when $W = 0$ to push the cash balance to a positive level. The amount of equity issued, $\tilde{W}_E(A)$, is endogenously determined. When the equity issuance costs are prohibitive, the bank must sell assets to address illiquidity, and hence holds higher cash balances, relative to the case when equity issuance costs are finite. Increase in the cost of issuing equity ($\phi A + \gamma \tilde{W}_E(A)$) outweighs the benefit from otherwise relaxing the bank’s financing constraints.

In Figure 3, we characterize the new loan originations and/or asset liquidations decisions of the bank under different cash buffer levels for two cases: the solid curve at the top of the figure represents the schedule of investments, when the bank is able to issue equity. The lower curve with solid circles denotes the investment schedule when the equity issuance costs are infinite (liquidation case). Consider the case where the bank is unable to issue equity: it starts to deplete the cash buffer from the level $\tilde{W}_L(A)$. At this stage, the bank also starts to disinvest. By contrast, note that the bank generally invests more in new loans when it
Figure 3: **Investment under the Counterfactual**: This figure plots the investment in new assets by the bank under the counterfactual with and without equity issuance. The dotted horizontal line is the zero new investment case. The key takeaway is that bank’s asset liquidations are generally much lower when they are able to issue equity and they invest more in additional loans.

has sufficient liquidity. At low cash levels it liquidates its loan portfolio to alleviate the illiquidity problem. When the bank is unable to issue equity, the bank invests less in new loans and disinvests more when it runs of cash. When the bank is able to issue equity, it invests more in new loans. It also disinvests much later when the cash balance falls to a lower level. In order to understand these implications, we first reproduce below the first order conditions associated with the investment decisions. Note that the investment policy
satisfies the following first-order condition:

\[
I_t^* = \begin{cases} 
\frac{1}{\psi^+(1 + \rho)} \left( \frac{E_A}{E_W} - 1 \right) & \text{if } I_t^* \geq 0, \\
\frac{1}{\psi^-(1 + \rho)} \left( \frac{E_A}{E_W} - 1 \right) & \text{if } I_t^* < 0.
\end{cases}
\]

The key trade off is between the marginal value of assets, $E_A$, and the marginal value of cash, $E_W$. It shows that investment region internalizes the cost parameter $\psi^+$ whereas the dis-investment region incorporates the cost parameter $\psi^-$ which is higher than $\psi^+$. The bank disinvests where $E_W > E_A$ implying $E_A/E_W < 1$. In these states the marginal value of cash increases as the bank becomes more constrained and liquidation or equity issuance becomes more likely. As a result the bank holds cash to reduce the likelihood that it will be liquidated or it has to raise equity. The effect is stronger for the liquidation case implying a more pronounced concavity of the equity value that gives rise to the demand for cash.

Figure 4 illustrates this prediction. It provides equity values under equity issuance and asset liquidations alternatives to better understand the investment decisions of the bank. As can be seen, the marginal value of equity and cash are nearly the same in “high liquidity” states. But when the cash level drops, the marginal value of equity goes up relative to the marginal value of cash leading to disinvestments.

<table>
<thead>
<tr>
<th>Table 3: Results: $W_0 = 5, A_0 = 110, P = 70, P_D = 30$ and $E = \max(W_0 + A_0 - P - P_D, 0) = 15$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
</tr>
<tr>
<td>$\Theta$</td>
</tr>
<tr>
<td>Spread (%)</td>
</tr>
<tr>
<td>Liquidation</td>
</tr>
<tr>
<td>Equity Financing</td>
</tr>
</tbody>
</table>

In Table 3, we examine the differences in the economic implications of equity issuance versus asset liquidations. Note that when the costs of issuing equity is infinite (the first
Figure 4: **Equity values under the Counterfactual:** This figure plots the equity values when the bank has no access to equity issuance, and compares that outcome with the case where the bank can issue costly equity.

The first row of Table 3 illustrates this case), the expected investments in asset portfolio is low as are the equity values. In fact, the bank, in expectation, disinvests. The bank also ends up facing very high credit spreads in private markets, as less assets are available to the creditors when the bank is shut down. When the bank can issue equity (the second row of Table 3 illustrates this case), the credit spreads go down dramatically. This illustrates the “debt overhang” effects and the reluctance that equity holders may have in issuing equity. But, the equity values also go up. The expected investment in asset portfolio is much higher.

### 5.2 Central Bank

Having characterized the bank’s optimal decisions under the counterfactual, we focus in this section on the optimal response of banks when they have access to central bank liquidity
facilities under the collateral policy framework. We have chosen the haircut at 50% in line with our estimates presented earlier. Figure 5 characterizes bank’s the optimal dividend payment regions and the lower boundary when they access central bank liquidity. The red line with circles is the closure boundary. The red dashed line plots the borrowing capacity of the bank with the central bank, $-(1-\theta)A$. Note that the closure boundary is well above the borrowing capacity for this level of haircuts. It is clear from Figure 5 that i) the bank holds less cash buffers and ii) pays more dividends when they are solvent. This follows from the fact that $W_{CB}(L) < W(L)$. Also note that the bank is drawing liquidity from the central bank when the cash buffer $W \leq 0$. Therefore, relative to the counterfactual, the bank is able to avoid liquidating assets in more states of the world.

Figure 5 also shows that the banks do not draw liquidity to the full extent of their borrowing capacity: the equity value maximizing strategy is to leave some excess collateral at the central bank in relation to the liquidity drawn from the central bank.

Access to central bank liquidity also substantially mitigates the bank’s underinvestment problem (see Figure 6). With access to central bank liquidity, the bank’s investment in new loans (as a fraction of its assets) when it runs out of cash is $I^*(W = 0, A = 130) \approx 1\%$ which is substantially higher that $I^*(W = 0, A = 130) \approx -12\%$ when the bank is about to run out of cash and has no access to a secured line of credit with the central bank. There are two clear implications. First, the bank invests more in loan portfolios in the presence of the central bank. Second, disinvestment occurs in fewer states of the world - the main implication here is that the expected losses due to asset liquidations should be lower when the central bank liquidity is available. The figure also demonstrates that the bank pays dividends in more states of the world when it has access to central bank.

Next, in Figure 7, we plot the bank equity values with and without access to the central bank. The bank equity values are higher (recall that banks pay dividends in more states of the world when they have access to the central bank).

Taken together, Figures 5, 6 and 7 show that the access to the central bank, while
Figure 5: **Central Bank and Liquidation Case:** This figure plots the optimal decisions of banks when they have access to the central bank, and contrasts their decisions with the ones that would have taken under the counterfactual. The red dashed line at the bottom of the figure is the borrowing capacity of the bank (net of haircuts). The black solid curve at the top is the endogenous upper boundary (under the counterfactual) above which they pay dividends. The blue dashed curve right below the black solid curve is the upper boundary for dividend payments when banks have access to the central bank. The blue solid curve right above the closure boundary is the locus of asset values when the banks use the central bank liquidity.

Improving the equity values of the banks (and hence their solvency) leads the banks to hold lower levels of liquidity buffers than they would have under the counterfactual. In other words, banks improve their solvency, but hold lower liquidity internally since they can draw from the central bank in illiquid states. Another implication of this lowered liquidity buffers is that the bank ends up paying dividends in more states of the world, when it has access to central bank’s liquidity facility.

Figures 8 and 9 plot the relevant boundaries when the bank has access to the central
Figure 6: **Investment under the Central Bank and Liquidation Case**: This figure plots the bank’s optimal investment with and without access to the central bank. The red curve with solid circles represents the optimal investment strategy of the bank when they do not have access to the central bank when the asset value is held fixed at $A = 130$. The black curve with solid circles represents the optimal investment strategy of the bank when they have access to the central bank when the asset value is held fixed at $A = 130$. The three vertical lines represent the endogenous boundaries: the line on the left is the liquidation boundary when the central bank debt reaches a level of $\approx -20$. The dividends payments occur when the boundary represented by the middle vertical line (≈ $W = 18$): note that this happens much earlier compared to the counterfactual when the boundary for dividend payments is $\approx W = 27$ as given by the last vertical line. Under the counterfactual, the bank liquidates when $W = 0$.

Bank and is also able to issue equity. Figure 8 shows that banks pay more dividends, and use their ability to issue equity to address their illiquidity. In figure 8 the bank’s borrowing capacity is very high, but the closure boundary is very tight. This results in the bank relying more on equity issuance.

In figure 9, we plot the optimal investments in the asset portfolio with and without access...
Figure 7: **Equity under the Central Bank and Liquidation Case:** This figure plots the equity values with and without access to the central bank. The two vertical lines denote the upper boundary for the cash above which dividends are paid for the two cases. The black solid curve is the equity value when the bank has access to the central bank. The black curve with solid circles is the equity value when the bank does not have access to the central bank.

To the central bank, when the bank has the ability to issue equity (under both scenarios). Note that the investment levels are higher when the bank has access to the central bank’s liquidity facility. In addition, asset liquidations are avoided before closure. Dividend payments occur to the right of the vertical lines, which represent the upper boundaries.

Table 4 summarizes the implications of central bank access under two options: when banks can issue equity and when banks are closed by the resolution authorities when they run out of cash and their tier-1 capital falls to the closure threshold. It is instructive to compare the results reported in Table 4 with the ones reported, under the counterfactual, in Table 3. They key finding is that the bank’s expected investment in the loan portfolio is
Table 4: Results: $W_0 = 5, A_0 = 110, P = 70, P_D = 30$ and $E = \max(W_0 + A_0 - P - P_D, 0) = 15$

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquidation</td>
<td>50</td>
<td>19.01</td>
<td>6.23</td>
<td>15.29</td>
</tr>
<tr>
<td>Equity financing</td>
<td>50</td>
<td>26.22</td>
<td>0.52</td>
<td>17.39</td>
</tr>
</tbody>
</table>

order of magnitude higher when they have access to LOLR, relative to the counterfactual, when banks do not issue equity. Even in the case where banks can issue equity, expected investments are higher, though only marginal. Credit spreads are lower and equity values are higher with access to LOLR, thereby relaxing the bank’s access to private credit markets.

We now turn to an interesting implication of the model about the extent of dilution that can occur to the stakes of original equity holders when the bank was formed at $t = 0$. Figure 10 is the result of a possible sample path of the evolution of the equity stake of incumbent (the original) equity holders at $t = 0$. We simulate 50,000 paths for the asset portfolio $A$ and cash $W$ state variables for 30 years on a monthly frequency. The details are provided in the Appendix. Note that under the counterfactual, there would have been a significant dilution of the stakes of the incumbent equity holders. In fact, within a period of 10 years, their stake goes down from 100% to a mere 30%, under the counterfactual. In sharp contrast, when the bank has access to the central bank liquidity facilities, the equity stake remains at 100% even after 10 years.

Figure 11 represents the histogram of equity issuances based on simulating the sample paths of the evolution of the equity issuances in the future. Note that under the counterfactual, there would have been significantly more issuance of equity as represented by the blue solid curve. In sharp contrast, when the bank has access to the central bank liquidity facilities, the equity issuance region shifts to the left.

Figures 10 and 11, when viewed together, suggest that banks are reluctant to issue equity
Figure 8: **Central Bank and Equity Financing Case:** This figure plots the optimal decisions of banks when they have access to the central bank, and contrasts their decisions with the ones that would have taken under the counterfactual. The red dashed line at the bottom of the figure is the borrowing capacity of the bank (net of haircuts). The black solid curve at the top is the endogenous upper boundary (under the counterfactual) above which they pay dividends. The blue dashed curve right below the black solid curve is the upper boundary for dividend payments when banks have access to the central bank. The blue dot-dashed curve right below the blue dashed curve is the equity issuance boundary. The blue solid curve right above the closure boundary is the locus of asset values when the banks use the central bank liquidity.

when they have access to central bank liquidity. This reluctance was pointed out by Stein (2013) who noted - “Moreover, during the interval from the start of 2007 through the third quarter of 2008, the largest U.S. financial firms - which, collectively, would go on to charge off $375 billion of loans over the next 12 quarters - paid out almost $125 billion in cash to their shareholders via common dividends and share repurchases, while raising only $41 billion in new common equity.”

In our model, we have assumed that the equity holders maximize the discounted value of
their collective dividend payments, irrespective of whether they are incumbent equity holders or not. As figure 10 illustrates, the incumbent equity holders will be even more reluctant to issue equity and have their stakes diluted. This potential conflict between “inside equity” and “outside equity” is a topic worthy of further research.

In figure 12 we show the future dividend distribution to the incumbent (time $t = 0$)

Figure 9: **Investment under the Central Bank and Equity Financing Case**: This figure plots the bank’s optimal investment with and without access to the central bank. The red curve with solid circles represents the optimal investment strategy of the bank when they do not have access to the central bank when the asset value is held fixed at $A = 130$. The black curve with solid circles represents the optimal investment strategy of the bank when they have access to the central bank when the asset value is held fixed at $A = 130$. The three vertical lines represent the endogenous boundaries: the line on the left is the liquidation boundary when the central bank debt reaches a level of $\approx -22$. The dividends payments occur when the boundary represented by the middle vertical line ($\approx W = 8$); note that this happens much earlier compared to the counterfactual when the boundary for dividend payments is $\approx W = 16$ as given by the last vertical line. Under the counterfactual, the bank liquidates when $W = 0$. 
Figure 10: **Equity share evolution (possible path):** This figure plots the equity share of the incumbent equity holders who held 100% of the bank’s equity at $t = 0$. This is represented by the blue step function under the counterfactual and red dashed step function when the bank has access to the central bank.

Equity holders with and without access to LOLR. Consistent with the earlier figures, we find that access to LOLR leads to higher dividend payouts to current equity holders.
Figure 11: **Equity Issuance under the Central Bank and Equity Financing Case:**
This figure plots the histogram of total equity issuance after 30 years, based on Monte Carlo simulation of 50,000 sample paths. The blue curve represents the counterfactual, and the red dashed curve represents the case where the bank has access to the central bank liquidity.
Figure 12: Dividends Payout to Incumbent Equity Holders under the Central Bank and Equity Financing Case: This figure plots the distribution of dividend payouts to the incumbent (time $t = 0$) equity holders with and without access to LOLR.
6 Policy Experiments

We conduct in this section the following policy experiments: first, we change the haircut policy from 50% to 80%. Second, we change the penalty rates from 0.50% to 1.5%. Finally, we change the tier-1 capital requirements from 5% to 6%.

Table 5 shows the key conclusions, which emerge. The base line case, the parameters and results corresponding to the base line case are highlighted so that it is easy to understand the consequences of changing any of the above policy tools. The top panel of Table 5 reports the results when the bank is unable to issue equity. The lower panel reports the results when the bank is able to issue equity, which is costly.

From the top panel we see that increasing the penalty rate does not lead to a significant drop in expected investments by the banks. But it increases the credit spreads by more than 1.5%. It also leads to a fall in equity values. When the central bank increases the haircuts from 50% to 80% we find that the most significant effect is on the expected investment by the bank in its loans and securities. To be sure there is a drop in equity values and an increase in credit spreads as before, but they are much less significant compared to the response when the penalty rates were increased. Finally, we examine the bank’s response to an increase in the tier-1 capital requirements from 5% to 6%. There is a pronounced drop in investments when the bank is subject to a higher capital requirement. The credit spreads marginally increase, and the equity values drop.

The lower panel of Table 5 summarizes the results of policy experiments when the bank is able to issue equity. The key differences from the top panel are: a) the credit spreads are not very sensitive to policy changes; b) expected investment into new loans and assets do not change by much when the penalty rates and haircuts change; and c) the equity values drop in response to all policy changes. But as in the top panel, the closure policy change has a major effect on loan/asset investments by the bank. It drops from the base case value of 17.39% to 14.95%.

In Figure 13 we explore the effects of increasing the tier-1 capital requirements and the
channels through which it influences the decisions of the banks. Note first that the bank increase the upper boundary above which it will start to pay dividends. This is one of the drivers of the fall in the equity values that we saw in Table 5. Second, the bank draws liquidity earlier when it is subject to a higher capital requirement: this underscores the link between the closure policies and the bank’s decision to use LOLR.
Figure 13: **Central Bank and Liquidation**: This figure plots the optimal decisions of banks when they have access to the central bank, and the closure policy tools are activated. We consider an increase in tier-1 capital requirement from 5% to 6%. The increased capital requirements are denoted by the line with small red circles. The base line capital requirements are denoted by the line with red triangles. The black solid line plots the liquidity access by the bank with tier-1 requirement of 5%, and the blue solid line plots the liquidity access by the bank with tier-1 requirement of 6%.

### 7 Conclusion

We develop a model of central bank liquidity provision to banks, which face costly equity issuance, and must manage their liquidity by building cash buffers, and optimally sizing their loan portfolio. We show that the ability to access central bank liquidity has both positive and negative implications. On the positive side, the size of the loan portfolio is higher when the banks have access to central bank liquidity than when they do not. The incremental effect depends crucially on the level of leverage of the bank and the extent of maturity transformation undertaken by the bank. Another positive outcome is that the expected
dead-weight losses are much lower when the banks have access to central bank liquidity. This is a direct consequence of the sticky margining policy of central banks whereby they do not change their haircuts and tend to hold the collateral for a long horizon thereby avoiding a fire sale of collateral. On the negative side, the existence of central bank liquidity facilities causes the banks to hold lower optimal cash buffers. By substituting risky assets for cash the banks increase their overall riskiness. In addition, we have shown that the access to LOLR decreases the incentives of banks to issue equity, or to cut dividends.

We have not modeled a general equilibrium framework: rather, we have worked out in detail how banks will respond when they have access to LOLR and have compared their response with the counterfactual when they do not have access to LOLR. We believe that this is a useful exercise. However, our policy experiments do not shed light on equilibrium feedback effects, which are important for welfare comparisons.
References


A-I Appendix

A-I.1 Model Set Up for the Counterfactual

Equity value depends on two state variables, its stock of cash $W$ and its assets’ portfolio $A$. Let $E(A, W)$ denote equity value. When $W_t > 0$ the bank is in the “cash region”, i.e., the bank is carrying positive cash buffer. When $W_t \leq 0$, the bank is in the “credit region”, i.e., it is drawing liquidity from the central bank liquidity facility. It is suboptimal for the bank to draw down the central bank liquidity account if the bank’s cash holding is positive.

In the counterfactual model, we consider four regions: i) external funding region when the bank raises equity; ii) internal financing region where the bank holds cash; iii) a payout region where the bank distributes dividends to shareholders; and iv) default region. The bank is in the external funding region when its cash stock $W$ is equal to zero, $W(A) = 0$. It is in the payout region when its cash stock $W$ is greater than endogenous upper barrier $W(A) > 0$. Thus, the bank is in the internal financing region when $W$ is in between 0 and $W(A)$. The bank is shut down by the resolution authorities when its tier 1 capital falls below a threshold level. Thus, the default region is one in which the tier 1 capital is below the exogenously specified level $\alpha$.

The bank chooses its investment $I$, payout policy $U$ and external financing policy $H$ to maximize shareholder value:

$$E \left[ \int_0^\tau dU_t - dH_t + e^{-r\tau} \max(W_\tau + lA_\tau - P_D - P, 0) 1_{\tau = \min(\tau_r, \tau_d)} \right]. \quad (A-1)$$

The first term is the discounted value of net payouts to shareholders and the second term is the discounted value from liquidation. The liquidation occurs in two circumstances: i) default triggered by regulatory authorities $\tau = \tau_d$; or ii) premature liquidation due to debt run $\tau = \tau_r$.

We fist characterize the solution in the internal financing region.
**Internal Financing Region** \( W(A) > W > 0 \)

The PDE for the equity value \( E(W, A) \) is

\[
\begin{align*}
    rE &= \left( \mu_X A + \frac{1}{\delta} A - I^* - g(I^*, \rho) - C_D - C + \frac{1}{m} (D(W, A; s) - P) + (r - \lambda) W \right) E_W \\
    &\quad + \frac{(\sigma^2_X + \sigma^2_A) \sigma^2}{2} E_{WW} + \left( I^* - \frac{1}{\delta} A \right) E_A + \epsilon \mathbb{1}_{D(W, A; s) < P} (\tilde{E} - E),
\end{align*}
\]

(A-2)

where the last term \( \epsilon \mathbb{1}_{D(W, A; s) < P} (\tilde{E} - E) \) represents the expected effect of premature liquidation due to debt run which occurs with probability \( \epsilon \mathbb{1}_{D(W, A; s) < P} \) and generates the impact \( (\tilde{E} - E) \) on equity value. \( \tilde{E} \) is the liquidation value:

\[
\tilde{E} = \max(W + lA - P_D - P, 0),
\]

(A-3)

where the liquidation value is proportional to the asset portfolio \( lA_t \) where \( l > 0 \) is a constant.

The boundary conditions are defined below.

**Payout Region** \( W(A) > W \)

When the cash-asset portfolio \( (W/A) \) ratio is very high, the bank is better off paying out the excess cash to shareholders to avoid the cash-carrying cost. The natural question is how high the cash-loan ratio needs to be before the bank pays out. Let \( W(A) \) denote this endogenous payout boundary. Intuitively, if the bank starts with a large amount of cash, then it is optimal for the bank to distribute the excess cash as a lump sum and bring the cash-loan ratio \( W(A) \) down to \( W(A) \). Moreover, equity bank value must be continuous before and after cash distribution. Therefore, for \( W(A) > W(A) \), we have the following equation for \( E(W, A) \):

\[
E(W, A) = \begin{cases} 
    (W(A) - W(A)) + E(W(A), A) & \text{if } W(A) > W(A). \\
    \text{Cash distribution} & 
\end{cases}
\]

(A-4)

Since the above equation also holds for \( W(A) \) close to \( W(A) \), we may take the limit and
obtain the following condition for the endogenous upper boundary:

\[ E_W(\bar{W}(A), A) = 1. \] (A-5)

**External Funding Region** \( W = 0 \)

If the bank runs out of cash \( (W_t = 0) \), it raises equity in the counterfactual case. The alternative option is to draw liquidity form the Central Bank liquidity facility to continue operating. Bank’s equity value is continuous before and after equity issuance which implies the following condition at \( \bar{W}(A) = 0 \)

\[
E(0, A) = E(\tilde{W}(A), A) - \phi A - (1 + \gamma \tilde{W}(A)),
\]

where \( \tilde{W}(A) \) is the equity issuance amount, \( \gamma \) is the marginal cost of equity issuance and \( \phi \) is a fixed monetary cost. We refer to \( \tilde{W}(A) \) as the cash return point after equity issuance.

This gives the following smooth pasting boundary condition:

\[ E_W(\tilde{W}(A), A) = 1 + \gamma. \] (A-6)

**Closure Region** We assume that the bank’s bankruptcy is triggered when

\[
\frac{\text{Equity Book Value}}{\text{Assets}} = \frac{\max(W + A - P_D - P, 0)}{W + A} < \alpha,
\] (A-7)

where \( \alpha \) is a constant (e.g. 5%) and \( W_b \) and \( A_b \) denote the cash and asset portfolio levels at which the bank resolution authorities will shut down the “problem bank”. When the bank is shut down, the equity value is

\[ \tilde{E} = \max(W + lA - P_D - P, 0). \] (A-8)
A-I.2 Numerical Approach

The problem cannot be solved analytically. The numerical method adopted in solving the problem is as described in Kushner and Dupuis (2013), namely a Markov chain approximation. It is an explicit solution approach involving a finite difference approximation of the HJB equation which derives a Markov chain that is locally consistent with the underlying continuous time stochastic process.

A-I.2.1 Hamilton-Jacobi-Bellman (HJB)

First consider the coefficient of \( E_W \) in Equation (A-2). Partition the terms that make up this coefficient into a positive and negative group if \( W_t \geq 0 \) and \( I^*_t \geq 0 \)

\[
\begin{align*}
a^+_W &= \mu X A + \frac{1}{\delta} A + \frac{1}{m} D(W, A; s) + rW \\
a^-_W &= I^* + g(I^*, \rho) + C_D + C + \frac{1}{m} P + \lambda W
\end{align*}
\]

where \( a_W = a^+_W - a^-_W \) and \( a_W \) being the coefficient of \( E_W \). A similar approach can be applied to the coefficient of \( E_A \) if \( I^*_t \geq 0 \)

\[
\begin{align*}
a^+_A &= I^* \\
a^-_A &= \frac{A}{\delta}.
\end{align*}
\]

Thus, let us approximate the partial derivatives in Equation (A-2) as follows:

\[
\begin{align*}
E_t &= \frac{E_{t+\Delta t}^{h,i} - E_{t}^{h,i}}{\Delta t} \\
E^+_W &= \frac{E_{t+\Delta t}^{h+1,i} - E_{t}^{h,i}}{\Delta W} \quad \text{for } a^+_W \\
E^-_W &= \frac{E_{t}^{h,i} - E_{t}^{h-1,i}}{\Delta W} \quad \text{for } a^-_W
\end{align*}
\]
\[
E_{WW} = \frac{E_{h,i+1} - 2E_{h,i} + E_{h,i-1}}{\Delta W \Delta W} \\
E_{A}^+ = \frac{E_{h,i+1} - E_{h,i}}{\Delta A} \quad \text{for} \quad a_A^+ \\
E_{A}^- = \frac{E_{h,i} - E_{h,i-1}}{\Delta A} \quad \text{for} \quad a_A^-.
\]

Substituting these terms into the HJB, we derive a discretize version of it. The like terms are collected together and all terms involving \(E_{h,i}\) are moved to the left-hand side. Simplifying we observe that the coefficient of \(E_{h,i}\) is

\[
Q = \frac{1}{\Delta t} + \frac{a_A^+}{\Delta W} + \frac{a_A^-}{\Delta W} + \frac{a_A^+}{\Delta A} + \frac{a_A^-}{\Delta A} + \frac{\sigma^2 A^2}{2 \Delta W \Delta W}.
\]

A recursive Markov chain optimization problem can be derived as:

\[
E_{h,i}^t = \max_I \sum_{\psi=-1}^{1} \sum_{\phi=-1}^{1} p_{h+\psi,i+\phi}^t E_{h+\psi,i+\phi}^t + p_{h,i}^{t+\Delta t} E_{h,i}^{t+\Delta t} 
\]

where each \(p\) may be interpreted as a transition probability of a Markov chain. For example, the notation \(p_{h+1,i}^t\) refers to the probability of moving from the current state \((W, A)\) \(((h, i)\) at time \(t\) to the state \((W + \Delta W, A)\) \(((h + 1, i)\) at time \(t\):

\[
p_{h+1,i}^t = \frac{1}{Q} \left( \frac{a_W^+}{\Delta W} + \frac{\sigma^2 A^2}{2 \Delta A \Delta A} \right).
\]

Thus the scheme allows only nine possible local movements from the initial position \((W, A)\). Of the nine possible movements, only five are permitted. This is a consequence of the zero off-diagonal values of the variance-covariance matrix in Equation (A-2).

### A-I.2.2 General Method

We adopt an iterative procedure for a sequence of fixed boundary problems:

1. At time \(T\)
• Equity and debt value correspond to book values;
• The bank does not hold or borrow cash.

2. At time $T - 1$ the inaction region $(\overline{W}(A), \underline{W}(A))$ is relaxed

   (a) Jointly solve the PDEs associated with the equity $E(W, A)$ and debt $D(W, A)$ subject to the boundary conditions at $(\overline{W}(A) \text{ and } \underline{W}(A))$.

   (b) Compute the optimal investment policy $I^*$.

   (c) Boundary update procedure - Verify whether smooth pasting conditions at $\overline{W}(A)$ and $\underline{W}(A)$ are satisfied:

   i. No: update the inaction region and solve 2.a with 2.b;
   ii. Yes: convergence.

3. Move backward and repeat the procedure 2 for any time $t$ to get a sequence of regions of inaction.

A-I.3 Equity Dilution and Monte Carlo Simulation

When the bank incurs an external financing cost $\phi A + \gamma \tilde{W}(A)$ as it raises equity $\tilde{W}(A)$ at any time $t$, the banks post-issuance equity value is $E(W_{t+}, A_{t+})$. We can denote by $\eta_{t+}$ the fraction of the newly issued equity held by outside investors:

$$\eta_{t+} = \frac{\tilde{W}(A) + \phi A + \gamma \tilde{W}(A)}{E(W_{t+}, A_{t+})} = 1 - \frac{E(W_t, A_t)}{E(W_{t+}, A_{t+})} \quad (A-9)$$

given the boundary condition

$$E(W_t, A_t) = E(W_{t+}, A_{t+}) - \phi A - (1 + \gamma)\tilde{W}(A) \quad (A-10)$$

and $W_t = 0$ given that equity is issued when the bank runs out of cash.
Via Montecarlo simulation, we can highlight the dynamics of equity dilution by keeping track of the equity ownership of the original investors who have established the bank at $t = 0$. We denote by $\zeta_t$ the ownership share of the original equity holders at time $t$ (where $\zeta_0 = 1$ or (100%) at $t = 0$). As the bank issues equity to finance investment and/or replenish liquidity over time, the original equity investors ownership evolves as follows:

$$\zeta_{t+} = \zeta_t \times (1 - \eta_{t+}).$$  (A-11)

With no issuance $\eta_{t+} = 0$ and $\zeta_{t+} = \zeta_t$. But when new equity is issued at time $t$, with a strictly positive ownership stake for new investors of $\zeta_{t+} > 0$, the original equity investors equity is diluted to $\eta_{t+}$ from $\eta_t$ according to Equation (A-11).

We simulate 50,000 paths for the asset portfolio $A$ and cash $W$ state variables for 30 years on a monthly frequency. The dynamics are

$$dA_t = \left(-\frac{1}{\delta}A_t + I^*_t\right)dt$$
$$dW_t = (r - \lambda)W_t + A_t dX_t + \frac{1}{\delta}A_t dt - I^*_t dt - g(I^*_t, \rho) dt$$
$$- C_d dt - C dt - \frac{1}{m}(D(W_t, A_t; s) - P) dt$$

where

$$dX_t = \mu_X dt + \sigma_{X,1} \rho dZ_{1,t} + \sigma_{X,2} \sqrt{1-\rho^2} dZ_{2,t},$$

and $Z_{1,t}$ and $Z_{2,t}$ are simulated and used for the counterfactual and central bank case.

For each simulation path, we use the optimal investment policy $I^*_t = I^*(W_t, A_t)$ to trace the optimal evolution of $A$ and $W$. We approximate continuous time by evaluating investment rules at discrete time intervals $\Delta t$ (i.e. monthly) given realizations of $W$ and $A$.

The only approximation in moving from continuous to discrete time is that the bank is allowed to adjust the cash variable only at discrete time. This occurs when i) $W(A)$ hits the cash boundary $W(A)$ ($W(A) > 0$) and the bank distributes dividends; ii) $W(A)$ hits the
cash boundary $W(A)$ ($W(A) < 0$). In the equity financing case, the bank issues equity and $W(A)$ readjusts to the optimal point $\tilde{W}(A)$; and iii) $W(A)$ hits the closure boundary and the bank stops to operate.