Response of the Macroeconomy to Uncertainty Shocks: the Risk Premium Channel

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Abstract

Uncertainty shocks are also risk premium shocks. With countercyclical risk aversion (RA), a positive shock to uncertainty not only increases risk, but it also elevates RA as consumption growth falls. The combination of high RA and high uncertainty produces significant risk premia in bad times, which in turn exacerbate the decline of macroeconomic aggregates and equity prices. Empirically, we document that local projection coefficients capturing the data response to the interaction of risk aversion and uncertainty are statistically significant and economically large. Indeed, heightened levels of RA during the 2008 crisis amplified the drop at the recession trough in output and investment by 41% and 28%, respectively. Theoretically, we show that a New-Keynesian model with endogenously time-varying risk aversion a la Campbell and Cochrane (1999) produces large falls in investment and equity prices and closely matches state-dependent data responses following positive uncertainty shocks.

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1 Introduction

Risk premium matters to macroeconomic outcomes. For a long while, asset pricing is often treated as the byproduct of the equilibrium economy. Given the marginal rate of intertemporal substitution, the pricing kernel can be constructed, which in turn drives expected returns across financial assets. This view has evolved steadily since the 2008 financial crisis, and a number of empirical and theoretical studies have zoomed in on the impact of the financial market on aggregate economic dynamics. These studies generally rely on frictions such as financial intermediation, collateral constraints, or incomplete information. In this paper, we add to this line of research by exploring the causal relationship between asset pricing and economic performance. We show that, in the presence of time-varying risk aversion, the risk premium of financial assets is a crucial determinant of the macroeconomic response to uncertainty shocks. To the best of our knowledge, this is the first paper to examine the risk feedback loop directly connecting asset pricing and the macroeconomy.

By inspecting the joint mechanism of risk aversion and risk, our paper is closely related to Menzly, Santos and Veronesi (2004), Bekaert, Engstrom and Xing (2009), and Wachter (2013) in which the authors combine habit preferences with sophisticated aggregate consumption processes to understand expected returns. Differently from them, we analyze the effects of uncertainty and risk aversion on macroeconomic and financial variables in a production economy: the model setup herein is key to highlight the feedback loop connecting asset pricing and the macroeconomy. Furthermore,

\footnote{For example, Schularick and Taylor (2012), Brunnermeier and Sannikov (2014), Mian et al. (2017), and Mian and Sun (2018) to name a few.}

\footnote{The interaction between risk and risk aversion is well understood in the literature. For example, risk premium under the Consumption CAPM (Breeden, 1979) can be expressed as the product of risk aversion, risk in the form of consumption growth uncertainty, and the correlation between returns and consumption growth.}

\footnote{In particular, Menzly et al. (2004) study the effects of the interaction of time-varying expected dividend growth and time-varying aggregate risk preferences on the relation between price/dividend ratios and expected excess returns. Bekaert et al. (2009) quantify the relative importance of changes in the conditional variance of fundamentals (“uncertainty”) and changes in risk aversion in the determination of the term structure, equity prices, and risk premiums. Wachter (2013) uses habit with time-varying probability of consumption disasters to interpret asset pricing facts for both stocks and bonds.}
Croce (2014) extends the Bansal and Yaron (2004) results to a production economy and shows that the introduction of time-varying volatility reduces the equity premium. We demonstrate that by nesting habit utility inside Epstein and Zin (1989)-Weil (1990) recursive preferences to induce time-varying risk aversion, uncertainty simultaneously elevates asset risk premium and generates large variability in, e.g., output and investment. With countercyclical risk aversion, uncertainty shocks not only increase risk but also the level of risk aversion as consumption growth declines. The combined effect leads to large risk premia in bad times, which causes firms to optimally lower investment and produce less goods. As such, output and consumption further deteriorate. We designate this endogenous feedback loop connecting asset pricing and the macroeconomy the risk premium channel. In this paper, we document that this risk premium channel is statistically and economically important to aggregate dynamics.

Empirically, we estimate impulse responses to uncertainty shocks in the presence of time-varying risk aversion employing the Smoothed Local Projection (SLP) method of Barnichon and Brownlees (2016). Essentially, we run predictive regressions over a range of horizons for output, consumption, investment, or equity prices on uncertainty and the interaction of uncertainty with risk aversion, while controlling for standard predictors. The coefficient loading on the interaction term is called state multiplier and can be interpreted as the amplification of the response of the dependent variable to uncertainty shocks due to the risk premium channel. Empirical estimates of state multipliers are negative across all forecast horizons. That is, higher risk aversion amplifies the decline of macroeconomic and financial variables upon a positive uncertainty shock.

Relying on the estimated coefficients, we perform a counter-factual analysis by comparing the fitted value of the forecast with the fitted value when the state multiplier is set to zero. In particular, our estimates imply that elevated risk aversion during the 2008 crisis, amplified the fall in output and investment in response to uncertainty by 41% and 28%, respectively. Hence, the interaction...
between risk aversion and uncertainty is a potentially important channel through which financial market conditions contributed to the deterioration of the macroeconomy during the crisis.

Theoretically, we discuss the interaction between risk aversion and uncertainty within a DSGE model with stochastic volatility (SV) in productivity. Specifically, we build a New-Keynesian model with Epstein and Zin (1989) – Weil (1990) preferences to allow for the separation of risk aversion from the elasticity of intertemporal substitution. Moreover, we nest Campbell and Cochrane (1999) external habit inside the Epstein-Zin-Weil recursive utility to generate endogenous time variation in risk aversion, which is a function of the inverse of the surplus consumption ratio. We establish the following two main findings. First, risk aversion amplifies the magnitude of macroeconomic responses to uncertainty shocks through the risk premium channel. Higher uncertainty precipitates an economic decline that lowers surplus consumption and increases risk aversion. The interaction between risk aversion and uncertainty produces countercyclical risk premia that aggravates the negative impact of uncertainty further. Second, habit-induced time-varying risk aversion is a powerful mechanism that allows the model to reproduce the observed decline of output and investment in the data following a positive shock to uncertainty. In particular, the presence of time-varying risk aversion causes the drop in output to double, the fall in investment to quadruple, and the decline in the stock market to be amplified by about three times.

We validate our model by reproducing the SLP analysis on simulated data. Our model successfully reproduces significantly negative state multipliers. Importantly, estimated state multipliers from a model without habits are essentially zero across horizons as risk aversion is no longer time-varying. Further, the conditional impulse responses of simulated output, consumption, investment, and stock valuation are in line with their empirical counterparts. In general, the dependent macroeconomic and financial variables fall following a positive uncertainty shock, and the decline

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5State dependent impulse response functions have recently received significant attention in the quantitative macroeconomics literature. E.g., recent work by Elenev (2018), Wong (2018), and Greenwald (2018) emphasize the relevance of household-level credit frictions for the transmission of monetary policy and other aggregate shocks. Diamond and Landvoigt (2018) study how the economy responds to a housing risk shock, conditional on past elevated demand for safe assets.
is amplified in the model with time-varying risk aversion.

We employ higher-order perturbation techniques to solve our DSGE model. It is well known that risk premia is unaffected by first-order terms and completely determined by second- and higher-order terms. A widespread macro-finance separation paradigm, first proposed by Tallarini (2000), suggests that the moments of macroeconomic quantities are not very sensitive to the addition of second- and higher-order terms. This implies that varying risk aversion while holding other model parameters fixed, allows to match asset pricing facts without compromising the model’s fit of the macroeconomy. We document that in a model with uncertainty, risk aversion matters not only for the level of asset returns but also for the dynamics of macroeconomic variables. Put differently, the separation result no longer holds for non-linear DSGE models with stochastic volatility. The interaction of risk aversion and uncertainty poses an additional challenge in our understanding of time-varying expected returns, as risk cannot be filtered solely by observing macroeconomic volatilities.

Our paper is related to various streams of literature in economics. First, in the postmortem of the 2008 Great Recession, numerous papers have highlighted the significance of the financial market, consisting of institutions, firms, and households, in generating and prolonging the crisis. Examples include Schularick and Taylor (2012), Mian et al. (2017), and Mian and Sufi (2018). Of course, the idea of a debt or credit cycle is not new. Bernanke et al. (1999) establish the importance of external financing premium in affecting business activity. Geanakoplos (2010) discusses the leverage cycle as a primary driver of macroeconomic fluctuations through collateralized borrowing. Brunnermeier and Sannikov (2014) explicitly incorporate debt financing in a general equilibrium model to show that endogenous risk faced by market participants produces large declines in macro aggregates when adverse shocks are realized.

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6See Aruoba et al. (2006) for a discussion about perturbation and alternative solution methods.

7In models with Epstein-Zin-Weil preferences, the parameter of relative risk aversion only appears in the perturbation solution in higher than first-order terms, see van Binsbergen, Fernández-Villaverde, Koijen and Rubio-Ramírez (2012).

8Also see Kiyotaki and Moore (1997).
Second, the notion of time-varying risk aversion has gained traction in the macroeconomics and finance literature in recent decades. Grounded in theoretical models with habit (Abel 1990; Constantinides 1990; Campbell and Cochrane 1999), aggregate risk aversion in the economy exhibits counter-cyclical variation as evidenced by the countercyclical equity risk premium. At the same time, the use of time-varying uncertainty has a long tradition in the financial economics literature. Kandel and Stambaugh (1991) study the implications for asset returns of time-varying first and second moments of consumption growth in a model with a representative investor having Epstein and Zin (1989) utilities. In a similar spirit, Bansal and Yaron (2004) incorporate time-varying first and second moments of consumption growth and recursive preferences in an endowment economy, and show that stochastic volatility not only generates time-variation in risk premium but also significantly increases the average equity risk premium. We amalgamate these strands of research to establish the asset risk premium channel.

Third, an increasing body of research studies how uncertainty fluctuations influence business cycle dynamics. Within the framework of irreversible investment (see Bernanke 1983; Dixit and Pindyck 1994; Abel and Eberly 1996; Hassler 1996), Bloom (2009) studies the propagation of firm-level uncertainty shocks. Related, another growing literature stresses the interaction of risk and economic activity propagated through financial, rather than physical frictions (see, e.g., Gilchrist et al. 2014; Arellano et al. 2016; Christiano et al. 2014; Alfaro et al. 2018). The analysis presented here shows that when risk aversion is elevated, uncertainty shocks have larger and prolonged impact. Our risk aversion channel is intimately tied to the financial market frictions channel such that volatility fluctuations drive macroeconomic outcomes: risk aversion can rise with tightening financial constraints or vice versa, and the macroeconomic reaction to uncertainty shocks may intensify as risk premium increase.

Fourth and more recently, the literature has also started investigating the impact of shocks

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to aggregate uncertainty. Justiniano and Primiceri (2008) and Fernández-Villaverde and Rubio-Ramírez (2007) estimate dynamic equilibrium models with heteroskedastic shocks and show that time-varying volatility helps to explain the Great Moderation between 1984 and 2007. Fernández-Villaverde et al. (2011) find that risk shocks are an important factor in explaining business cycles in emerging market economies. Fernández-Villaverde et al. (2015) document the important role of fiscal volatility for output fluctuations. Croce, Nguyen and Schmid (2012) study the effect of fiscal policies on long-run risk and model uncertainty, and show that, in an endogenous-growth economy with agents that are sensitive to model uncertainty, the reduction of model uncertainty can come at the cost of depressing growth for the long-run. Pastor and Veronesi (2013) analyze how the equity risk premium as well as other properties of stock prices such as their level, volatility, and correlations respond to government-induced (political) uncertainty. Basu and Bundick (2017) study the interaction of aggregate risk shocks with precautionary saving in an environment with nominal rigidities. We contribute to this literature by theoretically and empirically investigating the interaction of aggregate uncertainty with risk aversion. Our paper complements the empirical analysis of Pflueger et al. (2018). Pflueger et al. (2018) develop a measure of risk appetite that reflects both expectations of risk and risk aversion. They show that high risk appetite is associated with a high real rate on average and is followed by a boom in investment and output.

Finally, Gourio (2012) examines the joint implication of risk aversion and time-varying risk on macroeconomic dynamics in the context of time-varying probability of disaster risk. In contrast, our paper explores the interaction between stochastic volatility and risk aversion. In a related paper, Chen, Cooper, Ehling and Xiouros (2018) study the impact of time-varying risk aversion on the real business cycle as risk aversion affects the optimal production decision of firms.

In Proposition 3, Gourio (2012) shows that uncertainty in the probability of disaster translates to a level shock to the time discount factor with constant volatility. Thus, our analysis of second moment shocks to productivity is different.
2 Risk Aversion and Uncertainty: Empirical Evidence

In this section we study the amplifying effect of risk aversion on uncertainty shocks by examining the dynamic impulse responses of macroeconomic aggregates in the data. Furthermore, we illustrate how elevated level of risk aversion during the 2008 financial crisis directly contributed to the sharpened decline in output and investment in the recession. Later, we repeat the empirical exercise on simulated data generated by the NK-EZ-Habit model to validate our finding.

2.1 Macroeconomic Responses to Uncertainty Shocks and Time-Varying Risk Aversion

In this section we estimate the dynamic responses of macroeconomic quantities to uncertainty shocks when risk aversion is time-varying. The estimation of state-dependent impulse response functions has recently been the subject of expressed interest in macroeconomics, see e.g. Auerbach and Gorodnichenko (2012a), Auerbach and Gorodnichenko (2012b), and Ramey and Zubairy (2018) for investigations of the size of fiscal multipliers when the economy is in recession, or more broadly, during periods of economic slack. Tenreyro and Thwaites (2016) examine the response of the U.S. economy to monetary policy shocks predicated on the state of the business cycle. To the best of our knowledge, the role of risk aversion underlying the macroeconomic response to uncertainty shocks is unexplored so far.

To estimate the state-dependent IRFs, we rely on the smoothed version of local projections developed by Barnichon and Brownles (2016). The Smooth Local Projections (SLP) strikes a balance between the efficiency of Vector Autoregressions (VAR) and the robustness (to model misspecification) of the Local Projections (LP) approach. In practice, SLP consists in estimating LP under the assumption that the impulse response is a smooth function of the forecast

\footnote{We thank C. Brownless for clarifying various aspects about the SLP technique.}
horizon. Specifically, we estimate an $h$-step ahead predictive regressions,

$$
y_{t+h} = \alpha_h + (\beta_{0,h} + \beta_{1,h}R_{A_{t-1}})\text{UNC}_t + \sum_{i=1}^{p} \gamma_{i,h}w_{t-i} + u_{t+h}
$$

where $h$ ranges from 0 to $H$ and $p$ is the number of lags used for the control variables, $w_t$. $y_{t+h}$ is the $h$ period ahead realization of the outcome variable of interest. $R_{A_t}$ denotes the state variable of interest, in our case risk aversion. $\text{UNC}_t$ is a proxy for uncertainty. To capture state dependence, the response of $y_{t+h}$ to uncertainty at time $t$ is a linear function, $\beta_{0,h} + \beta_{1,h}R_{A_{t-1}}$, of risk aversion. In what follows, the $\beta_{1,h}$ coefficient capturing the amplification/contraction effect due to risk aversion is called the state multiplier. We are interested in knowing whether uncertainty shock has a larger effect on, e.g., output during high risk aversion states.

For our empirical application, we include gross domestic product (GDP), consumption, investment, hours worked, the GDP deflator, the Standard & Poor’s 500 Stock Price Index, and a measure of the stance of monetary policy as control variables. Appendix A provides details on the data construction. We employ two alternative measures of uncertainty and of risk aversion, which gives a total of four possible specifications. Specifically, to proxy for uncertainty we use either the index of economic policy uncertainty (EPU) created by Baker, Bloom and Davis (2016), or the financial uncertainty series constructed in Ludvigson, Ma and Ng (2017) using the framework of Jurado, Ludvigson and Ng (2015). To proxy for risk aversion we use either the (inverse of) surplus consumption computed as the (negative) moving average of past consumption growth, $-\sum_{j=0}^{\infty} \phi^j \Delta c_{t+j}$, or the consumption-price ratio. With regard to the former, we follow Wachter (2006) and truncate the moving average to 40 quarters and employ a decay rate $\phi = 0.97$. Finally, both measures of risk aversion are standardized to have mean zero and unit variance, and we plot the response of the outcome variable $y_{t+h}$ when $R_{A_t}$ is at its average value (i.e. $R_{A_t} = 0$, dubbed average state), and when it is one standard deviation above ($R_{A_t} = 1$) or below ($R_{A_t} = -1$) its average value.
Our choice of risk aversion proxies is motivated by the Campbell and Cochrane (1999) habit model. In this model, risk aversion is time-varying and inversely related to the surplus consumption ratio. In turn, the surplus consumption is essentially consumption relative to a slow-moving average of past consumption (see also discussion in Wachter, 2006; Muir, 2017). Under the original calibration of the Campbell and Cochrane (1999) model, the surplus consumption ratio is nearly linear in the price-dividend ratio. However, we prefer to work with the consumption-price ratio since Menzly, Santos and Veronesi (2004) show that - in an economy with multiple securities and predictable dividend growth - the consumption-price ratio is less sensitive than the dividend-price ratio to changes in expected dividend growth, and more sensitive to changes in the surplus ratio. As a consequence, the consumption-price ratio captures better variation in the aggregate discount rate induced by time-varying risk aversion. Analogously, Lettau and Ludvigson (2001) argue that $\hat{c}_t y_t$, the consumption-wealth ratio, is countercyclical in nature, and it is tied to time varying risk aversion in the Campbell and Cochrane (1999) framework, and Haddad and Muir (2018) employ $\hat{c}_t y_t$ to capture aggregate or household risk aversion. Finally, in Section 3, we develop a DSGE model with habit and show theoretically that the relation between the surplus consumption and risk aversion extends from the Campbell and Cochrane (1999) economy to our model economy. Furthermore, using model-implied data, we confirm the validity of the consumption-price ratio as a proxy for risk aversion.

In our empirical exercise, we choose $p = 4$, and let all variables enter in log levels with the exception of the monetary policy measure. Also, to estimate the state dependent IRFs, we follow Barnichon and Brownless (2016) and include the set of controls $w_t$ and their interaction with the state variable, $R_A$. We use a recursive identification scheme with the uncertainty measure ordered first and estimate our baseline empirical model using quarterly data over the 1967–2018

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12 See Figure 3 in Campbell and Cochrane (1999) and Figure 3 in Wachter (2006).
13 Specifically, see Proposition 1 and 2 in Menzly, Santos and Veronesi (2004).
14 Haddad and Muir (2018) try to disentangle the effect of intermediary risk aversion from household risk aversion on asset prices.
15 Adding a quadratic function of $R_A$, or $UNC$, to the control variables, or replacing the interaction terms with risk aversion, does not alter our conclusions.
sample period.\footnote{A similar identification strategy has been adopted in previous works (e.g. Bloom 2009) and is standard in the literature.}

Figure 1 shows the results when we employ EPU as our measure of uncertainty. Figure 2 reports the same analysis when we replace EPU with the financial uncertainty index of Ludvigson, Ma and Ng (2017). In both figures, Panels (a) – (d) shows the responses when the surplus consumption ratio is used as a proxy for risk aversion; Panels (e) – (h) instead use the consumption-price ratio as a proxy. In each figure, the left column plot the responses of GDP, consumption, investment, and stock prices to an uncertainty shock that is realized (i) in a high risk aversion state ($RA_t = 1$), (ii) in an average state ($RA_t = 0$), and (iii) in a low risk aversion state ($RA_t = -1$).

We start with EPU. Looking at the response when the risk premium channel is shut off ($RA_t = 0$), we obtain the standard result in the literature (see, e.g., Baker et al. 2016; Basu and Bundick, 2017; Ludvigson et al. 2017) that higher uncertainty causes declines in output, consumption, and investment.\footnote{Empirically, we find that the response obtained by estimating Eq. (1) and then setting $RA_t = 0$ is close to the response obtained in absence of the interaction term between risk aversion and uncertainty (i.e. the standard unconditional impulse response). Therefore, we do not report the unconditional IRFs.} When the risk premium channel is active, Figure 1 presents a novel and clear message about the state-dependent nature of uncertainty shocks: the response of the macroeconomy to a EPU shock is substantially larger when risk aversion is heightened. The peak decline in output, consumption, investment, and stock prices when $RA_t = 1$ is roughly twice as large as the decline obtained in a low risk aversion environment when $RA_t = -1$. The gap between the responses in the high and low risk aversion state generally closes after two years.

The second and fourth rows in Figure 1 plot the state multipliers obtained from SLP when we proxy for risk aversion with the surplus consumption ratio or consumption-price ratio, respectively. Recall that the state multiplier, $\beta_{1,h}$, captures the extent to which time-varying risk aversion...
affects the IRF at each horizon. A negative value of the state multiplier implies that the IRF response to a positive uncertainty shock is more negative when the risk aversion is high in the economy ($RA_t = 1$). Independently from the risk aversion proxy, we find that the state multiplier is economically large and statistically significant for about eight quarters, and it generally becomes insignificant afterwards. Also, with regard to the macroeconomic variables, the state multiplier attains its minimum value after about one year, its value being slightly below $-0.1\%$ for output and consumption, and between $-0.2\%$ and $-0.3\%$ for investment. This implies that four quarters after the occurrence of an EPU shock, investment is $-0.3\%$ below its steady state level when risk aversion is at its average value, and $-0.6\%(= -0.3 - 0.3)$ below steady state level when risk aversion is high (c.f. Panel (c) in Figure 1). With regard to the stock market, we observe a state multiplier that is negative on impact and economically large at about $-1\%$; afterward, the multiplier returns steadily to zero.

Turning to financial uncertainty we see results that are largely consistent with those discussed so far. In particular, the second and fourth rows in Figure 2 show that replacing EPU with the uncertainty index leaves the behavior of the state multiplier largely unaffected: the state multiplier for the macroeconomic outcomes still bottoms at about one quarter with values that are about $-0.05\%$ and $-0.1\%$ for output and consumption, and about $-0.15\%$ for investment. This implies that four quarters after the occurrence of a financial uncertainty shock, investment is $-0.8\%$ below its steady state level when risk aversion is at its average value, and $-0.95\%(= -0.8 - 0.15)$ below steady state level when risk aversion is high (c.f. Panel (c) in Figures 2). The state multiplier for the stock market behaves similarly to that estimated in Figure 1 using EPU. In particular, the state multiplier takes values on impact that are economically large and negative at $-0.4\%$ and $-1.5\%$ depending on the proxy for risk aversion.

Interestingly, in both Figure 1 and 2 with few exceptions, using the consumption-price ratio (panels (e)-(h)) instead of the surplus consumption (panels (a)-(d)) lead to smaller and less precisely estimated state multipliers. This evidence will be further confirmed in Section 4 using model-
simulated data. Intuitively, in our model both the surplus consumption and the consumption-price ratio proxy for risk aversion, but the latter is noisier.

The Appendix shows additional robustness checks. First, motivated by the work of Santos and Veronesi (2016), we employ the financial intermediary leverage as measured by He et al. (2017) as a proxy for risk aversion. Panels (a) – (d) in Figure C.1 show the results when EPU is employed, whereas panels (e) – (h) display the results when financial uncertainty is used. In general, using leverage leads to more persistent differences between the response in the high and low risk aversion states. E.g., looking at panels (e) and (f), we observe that it takes about four years for the response of output and consumption in the high risk aversion state to converge to the respective response in the low risk aversion state. Also, the response of the stock market in panel (h) is very close in shape and magnitude to that reported in Figure 2: the state multiplier is large and negative on impact at −1.5%, and it returns steadily to zero afterward.

Figure C.2 shows the results when we use two alternative proxies for uncertainty and risk aversion recently proposed by Bekaert et al. (2017). Similar to Figures 1 - 2, the state multiplier significant decreases during the first six quarters. Thereafter, the state multiplier gradually converges to zero and become insignificant. The maximum value of the state multiplier is about −0.1 for output (at five quarters), −0.15 for consumption (at seven quarters), −0.4 for investment (at six quarters), and −1.5 for the stock market (at five quarters).

2.2 The Financial Crisis: An Application

To quantify the amplifying effect of time-varying risk aversion on the economic impact of uncertainty shocks, we use the estimates from Eq. (1) to generate the fitted values of output and investment when the risk premium channel is turned on and off. In other words, we construct

\[^{18}\text{Our measure of leverage is based on market prices (market leverage). In the model of Santos and Veronesi (2016), the debt-to-wealth ratio is monotonically decreasing in the surplus consumption ratio (see their Corollary 13), which can be seen as the inverse of risk aversion.}\]
fitted values with the state multiplier, $\beta_{1,h}$, set to either the estimated value (high RA) or to zero (average RA). Specifically, we examine output and investment declines during the financial crisis using the post 2007Q4 sample and choose the forecast horizon, $h$, to be one quarter.

Panels (a) and (b) in Figure 3 present the time series plots of realized and fitted values of output, while panels (c) and (d) present the same plots for investment. RA is proxied by either the (inverse of) surplus consumption in subplot (a) and (c), or by the consumption-price ratio in subplot (b) and (d).\footnote{In the interest of space, we report only the case where uncertainty is proxied by EPU. Please refer to Appendix C.2 for the corresponding figures using financial uncertainty as the uncertainty proxy.} Focusing on panels (a) and (b), we see the one-quarter ahead forecast of output (dashed line) from the SLP matches the realized path of output (solid line) in both subplots well. In particular, the forecasted maximal drop in output appears within four quarters of the actual minimal output during the financial crisis. Next, we set the state multiplier to zero and repeat the forecast while keeping all other coefficient estimates from the SLP. The resulting fitted output path (square-dashed line) is plotted along the original forecast for counter-factual analysis. Subplot (a) in Figure 3 shows that relative to the level of output at the onset of the crisis, the maximal decline in output due to uncertainty is exacerbated by more than 300\% when the risk premium channel is active (dashed vs. square-dashed lines), where risk aversion is proxied by surplus consumption. In subplot (b), the amplifying effect is roughly 41\% when the risk premium channel is turned on, and risk aversion is approximated by the consumption-price ratio.

Quantitatively, heightened risk aversion during the financial crisis generates significantly larger decline in investment due to uncertainty. Panels (c) and (d) in Figure 3 present the realized (solid line), the actual one-quarter ahead forecast (dashed line), and the counter-factual forecast (square-dashed line) of investment when risk aversion is zeroed. Similar to Panels (a) and (b), the SLP forecast of investment matches reasonably well with the realized path, especially in subplot (c)...
when surplus consumption is used as the risk aversion proxy. Relative to the level of investment at the end of 2007, Panels (c) and (d) in Figure 3 show that the maximal forecasted decline in investment is between 28% and 38% greater than when we allow for risk-aversion-dependence of uncertainty in the SLP.

Overall, the economic significance of time-varying risk aversion on macroeconomic dynamics cannot be overlooked. Our results from applying the SLP methodology to examine the financial crisis can perhaps be viewed in one of two ways. First, in the absence of the risk premium channel, the econometrician cannot decipher the true impact of uncertainty shocks on economic aggregates. Second, conversely, intensified risk aversion aggravated the depth of the recession by causing uncertainty shocks to be more effective through general equilibrium mechanism.

In the next section we present our DSGE model which we then use in Section 4 to revisit the interaction between risk aversion and uncertainty in relation to the macroeconomy using model simulated data.

3 A Theoretical Model of Uncertainty Shocks in the Presence of Time-Varying Risk Aversion

In this section, we build a small-scale New-Keynesian model that accommodates endogenous risk aversion. In particular, we nest the Campbell and Cochrane (1999) external habit process in Epstein-Zin-Weil recursive preferences such that the surplus consumption ratio becomes a key state variable that delivers time variation in risk aversion. In what follows, we refer to our baseline model as the “NK-EZ-Habit” model. Since the model economy is rather standard, we highlight, in particular, the two key ingredients for our analysis: the source of risk and the preference specification of the agent.
3.1 Uncertainty

We consider intermediate goods-producing firms $i$ with the same constant returns to scale Cobb-Douglas production function, subject to a fixed cost of production $\Psi$ and their level of productivity $Z_t$:

$$Y_t(i) = [U_t(i)K_{t-1}(i)]^\alpha [A_tZ_tN_t(i)]^{1-\alpha} - \Psi,$$

where $\alpha$ is the capital share, $U_t(i)$ is the rate of utilization of their installed physical capital, $K_{t-1}(i)$ is capital subject to the usual law of motion with adjustment cost, $N_t(i)$ is labor input, and $Y_t(i)$ is the intermediate good. $A_t$ is the growth trend of labor productivity. The growth rate of $A_t$, or $\Delta A_t$, follows the process $\Delta A_t = \Delta A_{ss} + \sigma_a \varepsilon_{a,t}$ such that $\Delta A_{ss}$ is the steady state growth rate and $\varepsilon_{a,t} \sim i.i.d. N(0,1)$. The transitory technological process $Z_t$ evolves according to a first-order autoregressive process with stochastic volatility:

$$Z_t = (1 - \rho_z) \bar{Z} + \rho_z Z_{t-1} + \sqrt{\sigma^2_{z,t}} \varepsilon_{z,t}$$

$$\sigma^2_{z,t} = (1 - \rho_{\sigma}) \sigma_z^2 + \rho_{\sigma} \sigma^2_{z,t-1} + \sigma_{\sigma} \varepsilon_{\sigma,t}$$

with $\varepsilon_{z,t} \sim i.i.d. N(0,1)$ and $\varepsilon_{\sigma,t} \sim i.i.d. N(0,1)$. The innovations $\varepsilon_{z,t}$ and $\varepsilon_{\sigma,t}$ are assumed to be mutually independent at all leads and lags. In words, two independent innovations affect the level of productivity. The first innovation, $\varepsilon_{z,t}$, changes the level of productivity itself, while the second innovation, $\varepsilon_{\sigma,t}$, determines the spread of values for the productivity level.

3.2 Time-varying risk aversion

To generate endogenous time-varying risk aversion, we augment the Epstein and Zin (1989) recursive preference specification with Campbell and Cochrane (1999) external habit. Specifically, the Epstein-Zin utility function is defined over habit-adjusted consumption rather than consumption.
\[
V_t = \left[ \left( C_t^{h\omega} (1 - N_t)^{1-\omega} A_t^{1-\omega} \right)^{1-\psi} + \beta \mathbb{E}_t \left[ V_{t+1}^{1-\gamma} \right]^{1-\psi} \right]^{1-\psi},
\]

(4)

where \( \gamma \) is the coefficient of risk aversion, \( \psi \) is the inverse of elasticity of intertemporal substitution (EIS), \( \beta \) is the subjective discount factor, \( \omega \) determines the Frisch elasticity of labor supply, and \( A_t^{1-\omega} \) is the labor productivity needed in order for the value function to achieve the balanced growth path. \( C_t^{h} \) is consumption with external habit such that \( C_t^{h} = C_t \times S_t \times X_t \). \( X_t \) is a preference shock with mean equal to 1 and variance \( \sigma_x \). Consistent with Campbell and Cochrane (1999), we define \( S_t \) as the surplus consumption ratio, which evolves according to the following process:

\[
\log(S_t) = (1 - \rho_s) \bar{s} + \rho_s \log(S_{t-1}) + \lambda_h \log \left( \frac{C_t}{C_{t-1}} \right).
\]

(5)

This is a generalized AR(1) process with mean \( \bar{s} \) and autoregressive coefficient \( \rho_s \). \( \lambda_h \) is the sensitivity function of the surplus consumption ratio to consumption growth. It introduces non-linearity in the original Campbell and Cochrane (1999) model and it is key for generating time-varying equity risk premium within their endowment framework. Denoting \( \log(S_t) \) as \( s_t \), we define \( \lambda_h \) to be a constant following Chen (2017). Campbell and Cochrane (1999) further show that the time-varying local risk aversion coefficient is equal to the inverse of the surplus consumption ratio. In other words, when \( S_t \) is high, risk aversion of the representative agent is low and vice versa.

The corresponding real stochastic discount factor (SDF) of the NK-EZ-Habit model is defined as:

\[
M_{t,t+1} = \beta \left( \frac{C_{t+1}^{h\omega} (1 - N_{t+1})^{1-\omega}}{C_t^{h\omega} (1 - N_t)^{1-\omega}} \right)^{1-\psi} \left( \frac{C_{t+1}^{h}}{C_t^{h}} \right)^{-1} \left( \frac{V_{t+1}}{\mathbb{E}_t[V_{t+1}^{1-\gamma}]/(1-\gamma)} \right)^{\psi-\gamma},
\]

(6)

where habit-adjusted consumption is, again, constructed over raw consumption and the surplus consumption ratio. As is common to habit driven SDFs, \( M_{t,t+1} \) can be further decomposed into a consumption growth term, a term containing the growth rate of surplus consumption ratio, and, in

\[\text{In the Campbell and Cochrane (1999) setting, this particular parameterization of } \lambda_h \text{ allows to achieve a constant risk free rate. This is no longer the case in our DSGE model with Epstein-Zin preferences.}\]
this case, the continuation utility term resulting from recursive preferences.

When household labor is not fixed (as in our setting), the measure of risk aversion needs to be suitably modified. In particular, Swanson (2018) derives that the coefficient of absolute wealth-gamble risk aversion when the household has generalized recursive preferences and can vary its labor supply is given by:

\[ R_a^t = \frac{-E_t[V_{t+1}^{\gamma-1}V_{t+1}'' - \gamma V_{t+1}^{\gamma-1}V_{t+1}']^2}{E_t[V_{t+1}^{\gamma}V_{t+1}']}, \]

where \( V_{t+1}' \) and \( V_{t+1}'' \) denote the first and second derivatives of the utility function with respect to total wealth. In Appendix B we show that with our preference specification, similar to Campbell and Cochrane (1999), wealth-gamble risk aversion is a function of the inverse of the surplus consumption ratio.

### 3.3 Intermediate Goods Producers

There is a continuum of intermediate goods producers that rent labor from households. The intermediate goods market is monopolistically competitive and producers face each period quadratic costs \( \phi_P \) when changing their nominal price \( P_t(i) \). The firms own their capital stocks \( K_{t-1}(i) \) and face convex costs \( \phi_K \) of changing the quantity of capital installed. In addition to prices, firms choose the rate of utilization of their installed physical capital \( U(i) \) which affects its depreciation rate. Firm \( i \) chooses labor input \( N_t(i) \), investment \( I_t(i) \), and prices \( P_t(i) \) to maximize cash flows \( D_t(i)/P_t(i) \) given aggregate demand \( Y_t \) and the aggregate goods price index \( P_t \). Further, the intermediate goods firms all have the same constant-returns-to-scale Cobb-Douglas production function, subject to a fixed production cost \( \Psi \).

Each firm maximizes the discounted cash flows:

\[
\max \mathbb{E}_t \sum_{s=0}^{\infty} M_{t,t+s} \frac{D_{t+n}(i)}{P_{t+s}}
\]
subject to the production function,

$$
\left[ \frac{P_t(i)}{P_t} \right]^{-\epsilon \mu} Y_t = \left[ U_t(i) K_{t-1}(i) \right]^{\alpha} [A_t Z_t N_t(i)]^{1-\alpha} - \Psi,
$$

and subject to the capital accumulation equation,

$$
K_t(i) = \frac{1}{\delta} \frac{U_t(i)}{K_{t-1}(i)} - \frac{\Phi}{2} \left( \frac{I_t(i)}{K_{t-1}(i)} - \delta \right)^2 K_{t-1}(i) + I_t(i).
$$

The cash flows are defined as follows:

$$
\frac{D_t(i)}{P_t} = \left[ \frac{P_t(i)}{P_t} \right]^{1-\epsilon \mu} Y_t - \frac{W_t}{P_t} N_t(i) - I_t(i) - \frac{\Phi P_t(i)}{\Pi P_{t-1}(i)} - 1 \right] Y_t,
$$

and capital depreciation is non-linearly determined by utilization:

$$
\delta(U_t(i)) = \delta + \delta_1 (U_t(i) - U) + \left( \frac{\delta_2}{2} \right) (U_t(i) - U)^2.
$$

Finally, each intermediate goods firm finances a fraction $\nu$ of its capital stock each period with one-period risk-less bonds which pay the one-period real risk-free interest rate. Therefore, total firm cash flows are split into payments to bond and equity holders:

$$
\frac{D_t^E(i)}{P_t} = \frac{D_t(i)}{P_t} - \nu \left( K_{t-1}(i) - \frac{1}{R_t^R} K_t(i) \right).
$$

Each intermediate goods firm optimizes the cash flows by choosing price, $P_t(i)$. The first order
condition is

\[-\phi_P \left( \frac{P_t(i)}{\Pi P_{t-1}(i)} - 1 \right) \left[ \frac{1}{\Pi P_{t-1}(i)} \right] Y_t + \phi_P \mathbb{E} \left[ M_{t,t+1} \left\{ \frac{P_{t+1}(i)}{\Pi P_t(i)} - 1 \right\} \left\{ \frac{P_{t+1}(i)}{\Pi P_t(i)^2} \right\} Y_{t+1} \right] = 0\]

\[-\phi_P \left( \frac{P_t(i)}{\Pi P_{t-1}(i)} - 1 \right) \left[ \frac{P_t}{\Pi P_{t-1}(i)} \right] + \phi_P \mathbb{E} \left[ M_{t,t+1} \left\{ \frac{P_{t+1}(i)}{\Pi P_t(i)} - 1 \right\} \left\{ \frac{P_{t+1}(i)}{\Pi P_t(i)^2} \right\} Y_{t+1} \right] = 0,\] (13)

where \(MC_t\) is the marginal cost of producing one extra unit of goods \(i\). This is the Phillips curve that relates current inflation to future expected inflation and output.

### 3.4 Final Goods Producers

The representative final goods uses \(Y_t(i)\) units of each intermediate good, where \(i \in [0, 1]\) to assemble the final good. The market for final goods is perfectly competitive which results in zero profits in equilibrium. The aggregate goods price index \(P_t\) is defined as:

\[P_t = \left[ \int_0^1 P_t(i)^{1-\epsilon\mu} di \right]^{\frac{1}{1-\epsilon\mu}}.\] (14)

#### 3.4.1 Monetary Policy

The monetary authority sets the nominal interest rate, \(r_t\), to stabilize inflation and output growth. Doing so, the monetary policy is in accordance with the Taylor rule:

\[r_t = r + \rho_\pi (\pi_t - \pi) + \rho_y \Delta y_t,\] (15)
where $r_t = \ln(R_t)$, $\pi_t = \ln(\Pi_t)$, and $\Delta y_t = \ln(Y_t/Y_{t-1})$. The gross nominal interest rate $R_t$ is further pinned down by the standard Euler equation:

$$1 = R_t \mathbb{E}_t \left[ M_{t,t+1} \frac{1}{\Pi_{t+1}} \right].$$ (16)

### 3.5 Equilibrium

In the symmetric equilibrium, all intermediate goods producers choose the same price, employ the same amount of labor, the same amount of capital and the same utilization rate. As a result, all intermediate goods firms have the same cash flows that are financed with the exogenously determined mix of bonds and equity. Intuitively, one can interpret the continuum of firms as one representative intermediate goods producer.

### 4 Model Analysis

We calibrate the model and examine the dynamic response of the macroeconomy to uncertainty shocks. We show that our benchmark model with time-varying risk aversion produces amplifying effects on shocks to SV that are quantitatively similar to those observed in the data. Importantly, a calibration featuring high coefficient of risk aversion but no habit has hard time in reproducing our empirical facts.

#### 4.1 Parametrization

We choose the parameters in our baseline model to match the empirical evidence both in terms of moments and impulse response functions. In line with our empirical analysis, we calibrate the model to quarterly frequency. We set the intertemporal elasticity of substitution, $1/\psi$, is to 0.95.
which is in line with papers that find it to be less than one, see e.g. Hall (1988). The curvature parameter $\gamma$ equals 35 which is considerably lower than values typically used in the literature, see e.g. Binsbergen et al. (2012) ($\gamma = 66$), Basu and Bundick (2017) ($\gamma = 80$), and Rudebusch and Swanson (2012) ($\gamma = 110$). Moreover, our calibration of the surplus consumption dynamics follows Campbell and Cochrane (1999). Finally, we choose the steady-state of hours worked and $\omega = 0.38$ such that the model has a Frisch labor supply elasticity of 1.8 in line with the estimate of Smets and Wouters (2007).

The choice of firm related parameters is motivated by moments observed in the data: The capital share $\alpha$ is set to 0.3, the capital discount rate $\delta$ is chosen to match the investment/output ratio and $\delta_1$ follows from the first-order condition for the capacity utilization in steady state. Moreover, we set the price adjustment cost parameter to $\Phi_P = 120$ which implies a Calvo parameter of 0.76 or, put differently, firms can reset prices about once a year. The capital adjustment costs $\Phi_K$ are chosen to match the ratio of investment and output volatility. Finally, the leverage ratio affects only the mean and volatility of the equity risk premium in our model and is set to 0.45 in our calibration.

Monetary policy related parameters such as the response to inflation ($\rho_\pi$) and output growth ($\rho_y$), targeted inflation ($\Pi$), and the persistence of the interest rate rule ($\rho_r$) are set to standard values in the literature.

Finally, parameters of the exogenous productivity process are set to conservative values found in the literature. For example, the persistence of productivity $\rho_z$ and the volatility of the productivity volatility shock $\sigma_\sigma_z$ are in line with the values from Kung (2015). Finally, the steady-state volatility of the productivity level shock $\bar{\sigma}_z$ is consistent with Schmitt-Grohé and Uribe (2012) and a lower bound to Andreasen (2012).
Table 2 compares model-implied moments against the data. Overall, our model matches well moments observed in the data. For example, the model-implied standard deviation of output, consumption, investment, and hours are close to the data. Importantly, in addition to macro moments, the model also successfully reproduces the mean, standard deviation and autocorrelation of the risk-free rate. Finally, the model generates a sizeable equity risk premium which is as volatile as in the data.

[Insert Table 2 about here.]

4.2 Uncertainty Shocks, Risk Aversion and Macroeconomic Dynamics

With habit-adjusted consumption in the utility function, the representative agent’s risk aversion (RA) displays time variation driven by the surplus consumption ratio (see Eq. (5), and discussion in Appendix B). In particular, when consumption growth is low (bad times), $S_t$ is also low, making RA high. We show here that this endogenous counter-cyclicality of RA interacts with uncertainty shocks to generate large movements in economic variables through risk premium channel. The details of the mechanism is laid bare in the next subsection.

Figure 4 presents the impulse responses of output, consumption, investment, and stock market value, in order, following a positive productivity uncertainty shock in our model. We further contrast these model implied responses to the empirical responses in the data. The subplots demonstrate that the NK-EZ-Habit model does a reasonable job in producing impulse responses that are largely consistent with those obtained in the data when EPU is employed as a proxy for uncertainty. In particular, we observe large falls in output, consumption, investment, and stock prices following an uncertainty spike in line with data. Thus, in addition to the unconditional moments reported in Table 2, the calibrated model is performing well also in terms of dynamics.

[Insert Figure 4 about here.]
Figure 5 contrasts the impulse responses of the NK-EZ-Habit model to those in the baseline NK-EZ model, which has no habit formation. The NK-EZ model has a fixed value of relative risk aversion of 75. We see that when RA exhibits habit-induced time variation, the quantitative impact of the uncertainty shock is significantly larger. Specifically, when external habit is active, the declines in consumption doubles, the fall in output, investment, and the stock market more than quadruples. The drop in investment is especially noticeable. Because a positive uncertainty shock increases the expected productivity in the future, marginal cost of renting an extra unit of capital is expected to be high. As a result, firms optimally choose to not disinvest now due to the fact that capital acts as a hedging asset in production models. In the absence of time-varying risk aversion, this tension forces investment to stay relatively stable after productivity uncertainty is elevated, which leads to the minuscule response under NK-EZ seen in subplot (c) in Figure 5.

When external habit is active, however, the negative impact of uncertainty shock is so severe that the risk premium on investment becomes a significant cost that overwhelms the benefit of keeping investment stable. Furthermore, the risk premium on the dividend claim also rises, lowering the present value of future profits such that high investment is suboptimal. Thus, a large disinvestment is observed in the NK-EZ-Habit model due to a positive uncertainty shock. The risk premium channel plays a leading role here.

We conclude that counter-cyclical RA is a powerful mechanism that intensifies the effectiveness of productivity uncertainty shocks. The interaction of risk aversion with risk generates quantitatively large movements in the New-Keynesian model that is consistent with those in the data without resorting to extreme calibrations of volatility shock size.
4.3 Dissecting the Risk Premium Channel

Following the risky log-linearization used by Bianchi et al. (2018), there are four primary sources of risk premium in the model:

\[ r_t^r = -\log(\beta) + \psi \mathbb{E}_t[c^h_{t+1} - c^h_t] + \frac{1}{2}(1 - \psi)(1 - \gamma) \text{var}_t(v_{t+1}) - \frac{1}{2} \text{var}_t(m_{t,t+1}); \]  

Precautionary Savings Motive

\[ r_t - r_t^r = \mathbb{E}_t[\pi_{t+1}] + \text{cov}_t(m_{t,t+1}; \pi_{t+1}) - \frac{1}{2} \text{var}_t(\pi_{t+1}); \]  

Inflation Risk Premium

\[ \mathbb{E}_t[r_t^{Inv} - r_t^r] = -\text{cov}_t(m_{t,t+1}; r_t^{Inv}) - \frac{1}{2} \text{var}_t(r_t^{Inv}); \]  

Investment Risk Premium

\[ \mathbb{E}_t[r_t^{E} - r_t^r] = -\text{cov}_t(m_{t,t+1}; r_t^{E}) - \frac{1}{2} \text{var}_t(r_t^{E}); \]  

Equity Risk Premium

where \( c^h_t \) is log habit adjusted consumption, \( v_t \) is log of the value function under Epstein-Zin utility, \( m_{t,t+1} \) is log of the SDF, and \( \pi_t \) is log inflation. \( r_t^r \) is the log real short rate, \( r_t \) is the log nominal short rate, \( r_t^{Inv} \) is log return on investment, and \( r_t^{E} \) is log return on equity.

Equation (17) says that the log real short rate is driven by the precautionary savings motive. The more volatile the SDF is, the stronger the household’s incentive to save. This leads to higher bond prices and lower interest rates in equilibrium. Equation (18) shows that the difference between nominal and real rates is driven by expected inflation and inflation risk premium. If the SDF covaries strongly with inflation, \( \text{cov}_t(m_{t,t+1}; \pi_{t+1}) > 0 \), then payoffs on nominal bonds is low in real terms due to high inflation precisely when marginal utility is high. As a result, nominal bond prices are lower because bond yields contain compensation for inflation risk.
Equation (19) decomposes the expected excess return of investment over the real risk free rate. In our model economy, the return on investment is given by:

\[
R_{\text{Inv}}^t = U_t \frac{R^K_t}{P_t} + q_t \left[ 1 - \delta (U_t) - \frac{\Phi K}{2} \left( \frac{J_t}{J_{t-1}} - \delta \right) \right]^2 + \Phi K \left( \frac{J_t}{J_{t-1}} - \delta \right) \left( \frac{J_t}{J_{t-1}} \right),
\]

where \( R^K_t \) is the marginal productivity of capital and \( q_t \) is Tobin’s q. The investment risk premium component, \( \text{cov}_t(m_{t,t+1}; r_{\text{Inv}}^{t+1}) \), shows that if return on investment is low when marginal utility is high, then investment is risky and generates positive investment risk premium. The concept is analogous for return on equity in Equation (20). Return on equity is defined as:

\[
R^E_t = \frac{D^E_t}{P_t^E} + \frac{P^E_t}{P_{t-1}^E},
\]

where \( P^E_t \) denotes the firm’s market capitalization. Similarly, if return on equity is low when marginal utility is high, then \( \text{cov}_t(m_{t,t+1}; r^E_{t+1}) \) is negative, which leads to higher expected excess return.

In the model, uncertainty shocks simultaneously produce variability in the SDF as well as interest rates and returns. To understand the channel through which Campbell-Cochrane habit assists uncertainty in generating amplified responses of macroeconomic aggregates as seen in Figure 5, we plot in Columns (a)-(d) of Figure 6 the impulse responses of, respectively, the SDF, inflation, return on investment, and return on equity following a positive uncertainty shock. Each panel contrasts the responses of the NK-EZ model with those of the NK-EZ-Habit model to decipher the source of risk premium that is responsible for the amplification effect.

Top row of Figure 6 shows the IRFs under the baseline calibration of the two models. Two facts are important for our discussion. First, consistent with the previous finding, time-varying risk
aversion due to habit makes the model more volatile in general. This is particular so for the SDF in Column (a). When habit is turned on, uncertainty shock causes marginal utility to intensify by around 10 times relative to the model with no habit. This suggests that the precautionary saving motive outlined in Equation (17) plays a significant role in the mechanism. Declines in inflation and returns are also more dramatic following the positive uncertainty shock under the model with habit. Second, the large decrease in equity return in Column (d) in the presence of habit stands out compared to the decreases in inflation and return on investment, in Columns (b) and (c), respectively. In examining the covariances, this evidence points to the fact that time-varying risk aversion is particularly impactful on the covariance between the SDF and the return on equity. In other words, equity risk premium summarized in Equation (20) dominates inflation and investment risk premia in allowing risk aversion to amplify the economic reaction to uncertainty shocks.

Taken together, the top row of Figure 6 shows that time-varying risk aversion coming from habit is effective in our general equilibrium model with uncertainty because of two sources of risk: precautionary savings and equity risk premium. The increased variability of the SDF due to habit is straightforward to see. Substituting $C^h_t = C_t \times S_t \times X_t$ in the pricing kernel expression (6) yields:

$$M_{t,t+1} = \beta \left( \frac{C^h_{t+1}}{C_t} \omega \left(1 - N_{t+1}\right)^{1-\omega} \right)^{-1} \left( \frac{C^h_{t+1}}{C_t} \omega \left(1 - N_t\right)^{1-\omega} \right)^{-1} \left( \frac{V_{t+1}}{E_t [V_{t+1}^{1-\gamma}/(1-\gamma)]} \right)^{\psi-\gamma}$$

$$= \beta \left( \frac{C_{t+1} \omega S_{t+1}^{1-\omega} \left(1 - N_{t+1}\right)^{1-\omega}}{C_t \omega S_t^{1-\omega} \left(1 - N_t\right)^{1-\omega}} \right)^{-1} \left( C_{t+1} S_{t+1} \right)^{-1} \left( \frac{V_{t+1}}{E_t [V_{t+1}^{1-\gamma}/(1-\gamma)]} \right)^{\psi-\gamma}$$

$$= \beta \left( \frac{(1 - N_{t+1})^{1-\omega}}{(1 - N_t)^{1-\omega}} \right)^{-1} \left( C_{t+1} S_{t+1} \right)^{-1} \omega \left(1 - \psi\right)^{-1} \left( \frac{V_{t+1}}{E_t [V_{t+1}^{1-\gamma}/(1-\gamma)]} \right)^{\psi-\gamma},$$

where $X_t$ is assumed to be 1. Given that the surplus consumption ratio ($S_t$) is a function of past consumption growth, see Eq. (5), the growth rate of $S_t$ is even more volatile than consumption growth itself. Moreover, as the covariance between marginal utility and the return on equity intensifies under the habit specification, a positive uncertainty shock generates large equity risk premium such that households demand high compensation for the dividend claim. This is consistent
with the significant drop in stock market valuation observed in subplot (d) in Figure 5. So in bad times, following a realized positive shock to productivity uncertainty, households want to save more through safe bonds rather than stocks. This negative demand shock causes bonds to be expensive and dividend claims to be cheap. The resulting optimization problem, shown in Eq. (7), on the production side leads to amplified declines in investment and output.

To further dissect the mechanism in the NK-EZ-Habit model, we shut off a number of key frictions in the calibrated model to see how the impulse responses differ following the realization of a positive uncertainty shock.

To start, in the second row of Figure 6, we shut down the feedback mechanism in habit by setting \( \lambda^h = 0 \). This makes the surplus consumption ratio essentially fixed at \( \bar{s} \) in Eq. (5). Without this feedback between the surplus consumption ratio and consumption growth, the habit model reverts back to the NK-EZ model. With the surplus consumption ratio fixed, risk aversion is no longer time-varying. Thus, the amplification effect of the uncertainty shock on the SDF and return on equity disappears.

In the third row of Figure 6, we set the models to their respective flexible price versions by setting the price rigidity parameter, \( \Phi^P \), to 0. This transforms Equation (13) to the standard relationship between price and marginal cost. As seen in Column (a) for the SDF and Column (d) for equity return, the key mechanism driving the risk premium channel with Campbell-Cochrane habit are not affected by price rigidities: the SDF is still volatile and its covariance with the return on equity is still significantly negative. Counter intuitively, under flexible prices, the return on investment is now positive after a positive uncertainty shock.

Finally, the last row of Figure 6 displays the IRFs when recursive preferences is shutdown \((\psi = \gamma)\). Immediately, we see the differential responses between the NK-EZ model and the NK-EZ-Habit model ceased to exist. The implication of this outcome is that news to the continuation
utility term in the pricing kernel,

\[
\left( \frac{V_{t+1}}{E_t[V_{t+1}^{1-\gamma}/(1-\gamma)]} \right)^{\psi-\gamma},
\]

is crucial to the model success. Habit alone is not sufficient to produce the intensifying effect of uncertainty shocks on macroeconomic aggregates. This is consistent with our understanding of uncertainty. Since uncertainty works through expectations about future realizations of shocks, news to continuation utility reacts strongly to uncertainty shocks. Subsequently, the variability of the SDF due to uncertainty shocks is primarily driven by the continuation term. Accordingly, when Epstein-Zin preferences is turned off, the responses of the model to uncertainty shocks become very mild as seen in the last row of Figure 6. In subplot (a), a positive uncertainty shock actually produces a drop in marginal utility because habit-adjusted consumption growth increases. Furthermore, in the absence of recursive preferences, inflation rises following a positive uncertainty shock in subplot (b) due to the fact that marginal cost increases as expected productivity gains. However, the magnitude of the inflation response is trivial when Epstein-Zin is relegated.

Taken together, the results highlighted in Figure 6 tell a comprehensive story of how uncertainty shocks, in combination with time-varying risk aversion due to habit, can generate large declines in output, consumption, investment, and stock valuation through the risk premium channel. Habit makes marginal utility more volatile, which raises the precautionary savings motive of the households. At the same time, habit causes equity return to be rather negative precisely when marginal utility spikes, which increases the equity risk premium the households demand for holding the dividend claim. As the present value of future dividends fall, firms optimally choose to invest less in capital and decrease output. Finally, recursive preferences is an essential ingredient of the model such that uncertainty matters to the discount factor.
4.4 Model Implied Conditional Responses to Uncertainty Shocks

The NK-EZ-Habit setup allows us to study the endogenous response of the economy to productivity uncertainty shocks conditional on the level of risk aversion displayed by the representative agent. To do so, we use the model to simulate economies that span 208 periods (to match the data used in our empirical exercise in Section 2.1).

As we did in the data, within the model we proxy for risk aversion with either the inverse of the surplus consumption ratio or the consumption-price ratio. Indeed, Appendix B shows that with our preference specification, wealth-gamble risk aversion is a function of the inverse of the surplus consumption ratio. Moreover, Appendix B.1 shows that the behavior of the surplus consumption ratio in the model is well captured by the price-consumption ratio. We then use the simulated series of output, consumption, investment, stock prices, consumption-price ratio, and the surplus consumption ratio to perform the SLPs as outlined in Section 2.

Figure 7 plots the SLP results from simulated data where we employ the (inverse of the) surplus consumption ratio as a proxy for risk aversion. The first row shows the conditional IRFs of output, consumption, investment, and the stock market following a positive one standard deviation shock to productivity volatility. The third row shows analogous responses when productivity uncertainty is replaced with financial uncertainty. The second and fourth rows show the state multipliers from estimating equation (1), $\beta_1$, over horizons $h \in \{1, \ldots, 24\}$ for the same four variables. There are two main takeaways in Figure 7. First, conditional on high risk aversion (surplus consumption ratio is low and $RA_t$ is high), output, consumption, investment, and the stock market react more negatively to a positive uncertainty shock relative to the scenario where risk aversion is neutral ($RA_t = 0$). This is consistent with the corresponding empirical IRFs shown in Figures 1 and 2.

---

21 These two results extend the Campbell and Cochrane (1999) observation that the price-dividend ratio is nearly linear in the surplus consumption ratio (see their Figure 3) to a production economy.

22 In the model, we proxy for financial uncertainty with the expected conditional volatility of the return on firm equity. This aligns the SLP estimations on simulated data to the empirical analysis in Figure 4 and, in particular, Figure 2 as closely as possible.
Second, the estimated state multipliers are significantly negative for both macro and financial series in the simulated data causing high risk aversion ($RA_t = 1$ and $\beta_{1,h}RA_t < 0$) to generate stronger declines in these variables following an increase in uncertainty.

[Insert Figures 7 and 8 about here.]

Figure 8 shows the results when we use the consumption-price ratio as a proxy for risk aversion. In line with Figure 7, the IRFs in Figure 8 suggest that conditional on a high consumption-price ratio relative to its average (risk aversion is high), output, consumption, investment, and the stock price drop more after a positive uncertainty shock. Conversely, when consumption-price ratio is relatively low, the economic responses to the same shock are significantly milder. Comparing Figure 7 to Figure 8 we observe that the state multiplier becomes smaller and less precisely estimated (this is particularly the case for output and investment). This is to be expected since, in our model, the consumption-price ratio is a noisier proxy for the risk aversion than the surplus consumption ratio. Nonetheless, the similarities between Figures 7 and 8 provide some assurance that the consumption-price ratio is a credible instrument to proxy for the level of risk aversion in the data.

Finally, we can use our model to address an important identification issue in our SLP design. In particular, should risk aversion be constant, would the state multipliers continue to be negative due to movements in our proxies (moving average of past consumption and the consumption-price ratio) unrelated to risk aversion? Or would our specification be able to detect absence of risk aversion movements and estimate a state multiplier indistinguishable from zero? To answer this question, we re-run our SLP regressions on data simulated from the NK-EZ model, which by construction does not feature time varying risk aversion. The results are reported in Figure 9. In order, Columns (a) to (d) present the state multiplier for the SDF, consumption, investment and the stock market. The first two rows report the analysis for a response to a shock in productivity volatility. The third and fourth rows show analogous responses when productivity uncertainty is replaced with financial
uncertainty. The first and third rows are obtained using data simulated from the NK-EZ-Habit economy (these state multipliers coincide with those in Figure 8 and are reproduced here only for reader’s convenience); the second and fourth rows use data simulated from the NK-EZ model. Importantly, when habit-induced time-varying risk aversion is absent, the state multipliers become statistically insignificant. Hence, even with a sample of length equal to our data, our procedure would be able to detect absence of amplification due to risk aversion.

[Insert Figure 9 about here.]

Overall, the SLP results using simulated data presented here serve as validation bridging the NK-EZ-Habit model in Section 3 and the empirical findings in Section 2.1. First, we verify the theoretical implication from the model that risk aversion is a significant state variable in determining the impact of uncertainty shocks to macroeconomic aggregates. Second, our model is doing a reasonable job in capturing the amplifying effect of risk aversion on the impact of uncertainty shocks observed in the data. Indeed, our conclusion continues to hold when we replace productivity uncertainty with the model-implied market variance. Finally, our regressions on model-implied data validate (1) the empirical usage of the surplus consumption and the price-consumption ratios as risk aversion proxies; (2) the ability of our procedure to pick up genuine amplification of uncertainty by time-varying risk aversion.

5 Conclusion

Our study shows that not only risk matters to equilibrium outcomes, but more importantly, the degree of risk aversion determines the magnitude of these outcomes.

The effects we document here are quantitatively important: after a positive shock to uncertainty, the higher the risk aversion, the larger and more prolonged the decline in economic activity.
Empirically, we show that conditional on the fact that risk aversion was elevated during the 2008 crisis, the fall in output and investment driven by uncertainty was deepened by 41% and 28%, respectively. From a theoretical perspective, a New-Keynesian model with endogenous time variation in risk aversion caused by habit is able to reproduce our empirical evidence using simulated data.

Our results could be relevant for policymakers to consider stochastic volatility, and its interplay with financial quantities via risk aversion, when implementing fiscal and monetary policy.
References


### Tables

**Table 1: Model Parameters:** This table reports the calibrated parameters for the baseline model. The parameters are organized in 4 subgroups that relate them to preferences, firms, shocks, and monetary policy.

<table>
<thead>
<tr>
<th>Preferences:</th>
<th>Firms:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$  time discount parameter</td>
<td>$\alpha$ capital share</td>
</tr>
<tr>
<td>$\gamma$ risk aversion parameter</td>
<td>$\delta$ capital discount rate</td>
</tr>
<tr>
<td>$1/\psi$ intertemporal elasticity of substitution</td>
<td>$\delta_1$ capital discount rate (%)</td>
</tr>
<tr>
<td>$\rho_s$ AR(1) surplus consumption ratio</td>
<td>$\Phi_K$ capital adjustment cost parameter</td>
</tr>
<tr>
<td>$\bar{s}$ steady-state surplus consumption</td>
<td>$\Phi_P$ price rigidity parameter</td>
</tr>
<tr>
<td>$\mu$ productivity growth</td>
<td>$\epsilon$ leverage</td>
</tr>
<tr>
<td>$\rho_z$ AR(1) productivity</td>
<td>$\omega$ consumption share</td>
</tr>
<tr>
<td>$\sigma_z$ volatility productivity</td>
<td>$\Pi$ targeted inflation</td>
</tr>
<tr>
<td>$\rho_{\sigma_z}$ AR(1) volatility productivity</td>
<td>$\rho_r$ monetary policy inertia</td>
</tr>
<tr>
<td>$\sigma_{\sigma_z}$ volatility of volatility productivity (%)</td>
<td>$\rho_z$ response inflation gap</td>
</tr>
<tr>
<td>$\sigma_z$ volatility iid shock to s</td>
<td>$\rho_y$ response output growth</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Shocks:</th>
<th>Monetary Policy:</th>
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<tr>
<td>$\mu$ productivity growth</td>
<td>$\Pi$ targeted inflation</td>
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<tr>
<td>$\rho_z$ AR(1) productivity</td>
<td>$\rho_r$ monetary policy inertia</td>
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<tr>
<td>$\rho_{\sigma_z}$ AR(1) volatility productivity</td>
<td>$\rho_y$ response output growth</td>
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<table>
<thead>
<tr>
<th>Monetary Policy:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_z$ volatility iid shock to s</td>
</tr>
</tbody>
</table>
Panel A reports empirical and model-implied unconditional moments of output ($\Delta y$), consumption ($\Delta c$), investment ($\Delta i$), hours ($\Delta h$), and wage growth ($\Delta w$), and the real excess return on equity ($\text{ret}^{ex}$), the risk-free rate ($r^f$), and inflation ($\pi$). Empirical moments are obtained for a sample period from 1960:Q2 to 2017:Q4. Model-implied moments are obtained from simulating the model 50 times for 1000 periods. The table reports the median model-implied moments along with the 95% confidence bands (in brackets).

<table>
<thead>
<tr>
<th>Data</th>
<th>Model</th>
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<th>$\gamma = 70$</th>
<th>Leverage = 0</th>
<th>No Adj Cost</th>
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<td>3.23</td>
<td>3.90</td>
<td>3.23</td>
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<td>[2.79,3.75]</td>
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<td>[2.79,3.75]</td>
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<td>0.53</td>
<td>0.46</td>
<td>0.53</td>
<td>0.65</td>
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<td>0.69</td>
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<td>[0.64,0.75]</td>
<td>[0.61,0.85]</td>
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</tr>
<tr>
<td>$E[I/Y]$</td>
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<td>0.19</td>
<td>0.16</td>
<td>0.19</td>
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<tr>
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<td>0.44</td>
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<tr>
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<tr>
<td>$E[r^f]$</td>
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<td>$AC1 r^f$</td>
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Figures

(Figure 1: State-dependent IR to an uncertainty shock (EPU): Rows 1 and 3 plot the empirical state-dependent impulse responses (estimated using SLP) to an uncertainty shock for different levels of risk aversion for output, consumption, investment, and the stock market. Rows 2 and 4 plot the associated state multipliers with 90% confidence intervals (shaded areas). We measure uncertainty using the economic policy uncertainty (EPU) by [Baker, Bloom and Davis (2016)]. In the top panels, (a) - (d), our proxy for risk aversion is the surplus consumption proxy, \( \sum_{j=1}^{\text{out}} \theta / \Delta c_{t-j} \). In the following panels, (e) - (h), our proxy for risk aversion is the consumption-price ratio.)
Figure 2: State-dependent IR to an uncertainty shock (JLN): Rows 1 and 3 plot the empirical state-dependent impulse responses (estimated using SLP) to an uncertainty shock for different levels of risk aversion for output, consumption, investment, and the stock market. Rows 2 and 4 plot the associated state multipliers with 90% confidence intervals (shaded areas). We measure uncertainty using the financial uncertainty series by Jurado, Ludvigson and Ng (2015). In the eight top panels, (a) - (d), our proxy for risk aversion is the surplus consumption proxy, $\sum_{j=1}^{\infty} \phi^j \Delta c_{t-j}$. In the following panels, (e) - (h), our proxy for risk aversion is the consumption-price ratio.
Figure 3: The interaction between uncertainty and risk aversion in output and investment: The top (bottom) two panels report results for per capita, real GDP (investment). The solid line displays realized GDP and investment, respectively. The dashed line is the 1-quarter ahead forecast from direct regressions that allow for an interaction between risk aversion and uncertainty. The yellow line with squares is the 1-quarter ahead forecast from direct regression with no interaction between risk aversion and uncertainty. Shaded areas indicate NBER recession dates. We measure uncertainty using the economic policy uncertainty (EPU) by Baker, Bloom and Davis (2016). To proxy for risk aversion we use either the surplus consumption proxy \( \sum_{j=1}^{40} \phi_j \Delta c_{t-j} \), Panels a) and c), or the consumption-price ratio, Panels b) and d).
Figure 4: **Data vs Model**: The figure compares empirical impulse responses to a one standard deviation increase in uncertainty with their model counterparts for output, consumption, investment, and the stock market. The empirical responses are obtained from a VAR where uncertainty is measured using the economic policy uncertainty index by [Baker, Bloom and Davis (2016)](https://doi.org/10.1162/rest.2016.95.3.235). The red dashed line (circles) describes the impulse responses in the data, the shaded areas indicate 90% confidence intervals. The blue dashed line (squares) reports the corresponding impulse responses in the NK model with Epstein-Zin-Habit preferences.

Figure 5: **The role of habit**: The figure plots model impulse responses to a one standard deviation increase in uncertainty for output, consumption, investment, and the stock market. The red dashed line (circles) describes the impulse responses in the NK model with Epstein-Zin preferences. The blue dashed line (squares) reports the corresponding impulse responses in an identical NK model with Epstein-Zin-Habit preferences.
Figure 6: The risk premium channel in the model: The figure plots model impulse responses to a one standard deviation increase in uncertainty (i.e. productivity volatility) for the SDF, inflation, return on investment, and the return on equity across columns a) to d). The first row plots the responses in the baseline model. Rows two to four plot the responses in a model without habit ($\lambda_h = 0$), with flexible prices ($\Phi_P = 0$), and constant relative risk aversion ($\gamma = 1/\psi$), respectively.
Figure 7: Model implied state-dependent impulse responses to a uncertainty shock: Rows 1 and 3 plot the baseline model-implied state-dependent impulse responses to an uncertainty shock for different levels of risk aversion for output, consumption, investment, and the stock market. Rows 2 and 4 plot the associated state multipliers with 90% confidence intervals (shaded areas). In the eight top panels, (a) - (d), we measure uncertainty using realized productivity volatility time series. In the following panels, (e) - (h), we approximate uncertainty with the model-implied expected conditional volatility of the stock market. Risk aversion is equal to the inverse of the surplus consumption ratio.
Figure 8: Model implied state-dependent impulse responses to a uncertainty shock: Rows 1 and 3 plot the baseline model-implied state-dependent impulse responses to an uncertainty shock for different levels of risk aversion for output, consumption, investment, and the stock market. Rows 2 and 4 plot the associated state multipliers with 90% confidence intervals (shaded areas). In the eight top panels, (a) - (d), we measure uncertainty using realized productivity volatility time series. In the following panels, (e) - (h), we approximate uncertainty with the model-implied expected conditional volatility of the stock market. Risk aversion is approximated with the consumption price ratio.
Figure 9: Model implied state multipliers: The figure plots model-implied state multipliers of risk aversion together with 90% confidence intervals (shaded areas) for output, consumption, investment, and the stock market. The first (second) eight panels uncertainty using realized productivity volatility (model-implied expected conditional volatility of the stock market). Moreover, rows 1 and 3 plot the state multipliers for the NK model with Epstein-Zin-Habit preferences whereas rows 2 and 4 report the state-multiplier for the NK model with Epstein-Zin preferences. Throughout all panels, risk aversion is approximated with the consumption price ratio.
Online Appendix

A Data Construction

This section provides additional details on the data construction for the empirical evidence from Section 2 of the main text. We estimate our baseline Smooth Local Projection (SLP) using data on the uncertainty, real GDP, real consumption, real investment, hours worked, the GDP deflator, the Standard & Poors 500 Stock Price Index, and the Wu and Xia (2016) shadow rate. We use the Wu and Xia (2016) shadow rate series as our indicator of monetary policy since the Federal Reserve hit the zero lower bound on nominal interest rates at the end of 2008. Away from the zero lower bound, this series equals the federal funds rate. But at the zero lower bound, the shadow rate uses information from the entire yield curve to summarize the stance of monetary policy. To match the concept in the model, we measure consumption in the data as the sum of non-durable and services consumption. Then, we use the sum of consumer durables and private fixed investment as a measure of investment in our baseline empirical model. We convert output, consumption, investment, and hours worked to per capita terms by dividing by population (Civilian Noninstitutional Population: 16 Years and Over). Except for the shadow rate, all other variables enter the SLP in log levels. The series of output, consumption, investment are from NIPA.

To compute the price-consumption ratio, we measure stock market wealth by the quarter-end market capitalization of the CRSP value-weighted index, expressed in real per capita terms for comparability to the consumption data.

The measure of economic policy uncertainty (EPU) is based on Baker et al. (2016). We use the News-Based Policy Uncertainty Index available on EPU’s web site for the US. The series is monthly and spans the period 1985:1−2017:12. We convert it to quarterly values by taking the end of the quarter value. To go further back in time, we merge the News-Based Policy Uncertainty Index series with the US Index, a longer series available from the Historical EPU’s web site.

The measure of financial uncertainty is an updated version of data used in Jurado et al. (2015) and Ludvigson et al. (2017). It is available on Sydney Ludvigson’s home page. We use their 3-month ahead uncertainty series. The series spans the period 1960:07−2017:12. We take the end of the quarter value to construct quarterly series.

As additional robustness checks, we re-estimated our IRFs excluding the Standard & Poor’s 500 Stock Price Index and our conclusions remain unaltered. We have also re-estimated the IRFs when we replace the Standard & Poors 500 Stock Price Index with the M2 money stock. Once again results are not affected by these changes.

B Consumption Surplus and Risk Aversion

From Swanson (2018), wealth-gamble risk aversion with recursive preferences can be written as:

\[ R^\phi(a_t; \theta_t) = -\mathbb{E}_t[U(a^*_{t+1}; \theta_{t+1})^{-\gamma}U_{11}(a^*_{t+1}; \theta_{t+1}) - \gamma U(a^*_{t+1}; \theta_{t+1})^{-\gamma-1}U_1(a^*_{t+1}; \theta_{t+1})^2] \]

where \( a^*_{t+1} \) is optimal wealth through the budget constraint \( a^*_{t+1} \equiv (1 + R_t)a_t + W_t N^*_t + D_t - C^*_t \). \( \theta_t \) represents the set of exogenous shocks driving the dynamics of the economy. Subscripts 1 and 11 denote the first and second derivatives, respectively, with respect to future wealth.

Assuming within-period power utility over habit adjusted consumption only, the Epstein-Zin value function has the following form:

\[ U(a^*_{t+1}; \theta_{t+1}) = \left[ \frac{C_{t+1}^{1-\psi}}{1-\psi} + \beta E_{t+1} \left[ U(a^*_{t+2}; \theta_{t+2})^{1-\gamma} \right]^{\frac{1}{1-\psi}} \right]^{\frac{1}{1-\psi}}. \]
Recall that $C_{t+1}^h = C_{t+1}^* S_{t+1}$.

We can apply the chain rule to calculate the derivative of $U(a_{t+1}^*; \theta_{t+1})$ with respect to $a_{t+1}^*$ such that

$$\frac{\partial U(a_{t+1}^*; \theta_{t+1})}{\partial a_{t+1}^*} = \frac{\partial U(a_{t+1}^*; \theta_{t+1})}{\partial C_{t+1}^h} \frac{\partial C_{t+1}^h}{\partial a_{t+1}^*} = \frac{\partial U(a_{t+1}^*; \theta_{t+1})}{\partial C_{t+1}^h} S_{t+1} (1 + R_{t+1}),$$

and

$$\frac{\partial U(a_{t+1}^*; \theta_{t+1})}{\partial C_{t+1}^h} = \frac{1}{1 - \psi} \left[ U(a_{t+1}^*; \theta_{t+1})^{1 - \psi} \right]^{\frac{1}{1 - \psi}} C_{t+1}^h \psi.$$

The second derivative of $U(a_{t+1}^*; \theta_{t+1})$ with respect to $a_{t+1}^*$ can be found by repeated application of the product rule to the expression $\frac{\partial U(a_{t+1}^*; \theta_{t+1})}{\partial C_{t+1}^h} S_{t+1} (1 + R_{t+1})$ and notice that $\frac{\partial S_{t+1}}{\partial a_{t+1}^*} = 0$ and $\frac{\partial (1 + R_{t+1})}{\partial a_{t+1}^*} = 0$. Thus we have:

$$\frac{\partial^2 U(a_{t+1}^*; \theta_{t+1})}{\partial a_{t+1}^* \partial a_{t+1}^*} = \frac{\partial^2 U(a_{t+1}^*; \theta_{t+1})}{\partial C_{t+1}^* \partial C_{t+1}^*} S_{t+1} (1 + R_{t+1})^2,$$

and

$$\frac{\partial^2 U(a_{t+1}^*; \theta_{t+1})}{\partial C_{t+1}^* \partial C_{t+1}^*} = \frac{1}{1 - \psi} \left[ -\psi U(a_{t+1}^*; \theta_{t+1})^\psi C_{t+1}^h \psi \right]^{\frac{1}{1 - \psi}} C_{t+1}^h \psi U(a_{t+1}^*; \theta_{t+1})^{\psi - 1} \frac{\partial U(a_{t+1}^*; \theta_{t+1})}{\partial C_{t+1}^h} C_{t+1}^h \psi$$

$$= \frac{1}{1 - \psi} \left[ -\psi U(a_{t+1}^*; \theta_{t+1})^\psi C_{t+1}^h \psi \right]^{\frac{1}{1 - \psi}} C_{t+1}^h \psi U(a_{t+1}^*; \theta_{t+1})^{\psi - 1} \frac{1}{1 - \psi} U(a_{t+1}^*; \theta_{t+1})^\psi C_{t+1}^h \psi C_{t+1}^h \psi$$

$$= \frac{\psi}{1 - \psi} - U(a_{t+1}^*; \theta_{t+1})^\psi C_{t+1}^h \psi \left[ C_{t+1}^h \psi \right]^{\frac{1}{1 - \psi}} C_{t+1}^h \psi$$

Then, we can calculate the terms in $R^a$:

$$U(a_{t+1}^*; \theta_{t+1})^{\gamma - 1} U_1(a_{t+1}^*; \theta_{t+1}) = \frac{1}{1 - \psi} U(a_{t+1}^*; \theta_{t+1})^\psi C_{t+1}^h \psi S_{t+1} (1 + R_{t+1}),$$

$$U(a_{t+1}^*; \theta_{t+1})^{\gamma - 1} U_{11}(a_{t+1}^*; \theta_{t+1}) = \frac{\psi}{1 - \psi} U(a_{t+1}^*; \theta_{t+1})^\psi C_{t+1}^h \psi \left[ C_{t+1}^h \psi \right]^{\frac{1}{1 - \psi}} C_{t+1}^h \psi$$

$$= \frac{1}{1 - \psi} U(a_{t+1}^*; \theta_{t+1})^{\psi - 1} C_{t+1}^h \psi S_{t+1} (1 + R_{t+1})^2,$$

$$U(a_{t+1}^*; \theta_{t+1})^{\gamma - 1} U_2(a_{t+1}^*; \theta_{t+1}) = \left( \frac{1}{1 - \psi} \right)^2 U(a_{t+1}^*; \theta_{t+1})^2 \psi S_{t+1} (1 + R_{t+1})^2.$$
such that

\[ R^\gamma(a_t; \theta_t) = -\mathbb{E}_t \left[ \frac{\psi}{\psi - 1} U(a^*_t; \theta_t) \right] + \gamma \mathbb{E}_t \left[ \frac{1}{\psi - 1} U(a^*_t; \theta_t) \right] (1 + R_{t+1}) \]

where the approximation relies on the fact that the risk involved in the wealth gamble is minuscule.

Recall that \( \psi \) is the inverse of the elasticity of intertemporal substitution. Under standard calibration, \( \psi > 1 \), and \( R^\gamma(a_t; \theta_t) \) can be written as:

\[ R^\gamma(a_t; \theta_t) \approx \mathbb{E}_t \left[ \psi \left\{ \frac{1}{C^*_{t+1}} + \frac{1}{\psi - 1} U(a^*_t; \theta_t) \psi^{-1} \frac{1}{C^*_{t+1}} \psi^{\psi - 1} \right\} (1 + R_{t+1}) \right. \]

\[ \left. + \gamma \left( \frac{1}{\psi - 1} U(a^*_t; \theta_t) \psi^{-1} \frac{1}{C^*_{t+1}} \psi^{\psi - 1} \right) (1 + R_{t+1}) \right] \]

\[ = \mathbb{E}_t \left[ \psi \left\{ \frac{1}{C^*_{t+1}} + \frac{1}{\psi - 1} U(a^*_t; \theta_t) \psi^{-1} \frac{1}{C^*_{t+1}} \psi^{\psi - 1} \right\} (1 + R_{t+1}) \right], \]

which is an inverse function of \( S_{t+1} \). In other words, when the consumption surplus ratio is high, wealth-gamble risk aversion is low and vice versa. The derivation with leisure preference is similar but slightly more involved.

### B.1 Proxies for Risk Aversion: Simulations

Figure B.1 shows one simulation path for model variables that proxy for risk aversion: surplus consumption ratio, and wealth-consumption.
Figure B.1: Different Proxies for Risk Aversion within the Model: This figure plots one simulation paths (1000 periods with a burn-in period of 5000) for the two proxies of risk aversion within our model environment, i.e. the inverse of surplus consumption ratio and consumption-price ratio. The correlation between the two is 0.799.

C  Local Projection: Additional Results

C.1 Alternative Measure of Uncertainty and Risk Aversion

This section shows that our results are robust to alternative measures of uncertainty and risk aversion.

First, Figure C.1 shows the results when we employ the financial intermediary leverage as measured by He, Kelly and Manela (2017) as our proxy for risk aversion. This is motivated by the work of Santos and Veronesi (2016): in this model the debt-to-wealth ratio is monotonically decreasing in the surplus consumption ratio (see their Corollary 13), which can be seen as the inverse of risk aversion.

In Figure C.1 Panels 1(a)–1(d) we proxy uncertainty with the economic policy uncertainty by Baker, Bloom and Davis (2016). In Panels 1(e)–1(h) we proxy uncertainty with the aggregate uncertainty index proposed by Jurado, Ludvigson and Ng (2015).

Second, we replace $\text{UNC}_t$ and $\text{RA}_t$ in Eq. (1) with the proxies to risk aversion and economic uncertainty proposed by Bekaert, Engstrom and Xu (2017). Since this series is available only starting from 1986/06 to 2015/02 we include two instead of four lags in the estimation of the local projections. Also, we find results to be noisy when we use the raw measure of risk aversion. We therefore let $\text{RA}_t$ take value equal to 1 when the risk aversion proxy is above the 75th percentile, $\text{RA}_t$ is equal to −1 when it is below the 25th percentile, and $\text{RA}_t$ is set to zero otherwise. We then standardize the variable $\text{RA}_t$.

The authors kindly thank Nancy Xu for sharing with us the risk aversion and uncertainty indices.
Figure C.2 shows the results, and it confirms the evidence obtained when we proxied risk aversion with the surplus consumption (see Panels (a) to (d) in Figures 1 and 2), and the consumption-price ratio (see Panels (e) to (h) in Figures 1 and 2).
Figure C.1: State-dependent (leverage) IR to an uncertainty shock: Rows 1 and 3 plot the empirical state-dependent impulse responses (estimated using SLP) to an uncertainty shock for different levels of risk aversion for output, consumption, investment, and the stock market. Rows 2 and 4 plot the associated state multipliers with 90% confidence intervals (shaded areas). Our proxy for risk aversion is intermediary leverage by [He, Kelly and Manela (2017)]. In the eight top panels, (a) - (d), we measure uncertainty using the economic policy uncertainty (EPU) by [Baker, Bloom and Davis (2016)]. In the following panels, (e) - (h), measure uncertainty using the financial uncertainty series by [Jurado, Ludvigson and Ng (2015)].
Figure C.2: **State-dependent (leverage) IR to an uncertainty shock**: The first row plots the empirical state-dependent impulse responses (estimated using SLP) to an uncertainty shock for different levels of risk aversion for output, consumption, investment, and the stock market. The second row plots the associated state multipliers with 90% confidence intervals (shaded areas). We measure risk aversion and uncertainty using the financial proxies provided by Bekaert, Engstrom and Xu (2017).

### C.2 Local Projection: Financial Crisis Results Using Alternative Measures of Uncertainty

Figure C.3 presents the time series plots of realized and fitted values of output and investment when RA is proxied by the surplus consumption (Panel (a) and (c)) or the consumption-price ratio (Panel (b) and (d)). Throughout, to proxy for uncertainty we use the financial uncertainty index of Jurado, Ludvigson and Ng (2015).
Figure C.3: The interaction between uncertainty and risk aversion in output and investment: The top (bottom) two panels report results for per capita, real GDP (investment). The solid line displays realized GDP and investment, respectively. The dashed line is the 1-quarter ahead forecast from direct regressions that allow for an interaction between risk aversion and uncertainty. The yellow line with squares is the 1-quarter ahead forecast from direct regression with no interaction between risk aversion and uncertainty. Shaded areas indicate NBER recession dates. We measure uncertainty using the financial uncertainty series by Jurado, Ludvigson and Ng (2015). To proxy for risk aversion we use either the surplus consumption proxy $\sum_{j=1}^{40} \phi_j \Delta c_{t-j}$, Panels a) and c), or the consumption-price ratio, Panels b) and d).
D Perturbation Methods and Generalized Impulse Response Function

This appendix includes a more detailed discussion of the solution of the model and the explanation of how we compute the IRFs and the variance decomposition of the model. We refer the interested reader to the appendix of Born and Pfeifer (2014) for an exhaustive discussion of the use of perturbation and pruning techniques, and their implications for simulation and IRFs.

To judge the importance of volatility shocks for business cycle moments, and their interaction with risk aversion, we rely on perturbation methods in our model solution (see Judd (1998)).

Our investigation faces a number of computational challenges. First, we are interested in the implications of a volatility increase while keeping the level of the variable constant. We thus have to consider a third-order Taylor expansion of the solution of the model, see e.g. Schmitt-Grohe and Uribe (2004), Fernández-Villaverde et al. (2011) and Fernández-Villaverde et al. (2015). Indeed, in a first-order approximation, stochastic volatility does not enter the policy function of the representative agent. In the second-order approximation, only the product of the two innovations appears in the policy function. Only a third- or higher order approximation to the model allows to study the role of innovations to volatility.\(^{24}\)

Second, higher order perturbation solutions tend to explode due to the accumulation of terms of increasing order. For example, in a second order approximated solution, the quadratic term at time \(t\) will be raised to the power of two in the quadratic term at \(t+1\), thus resulting in a quartic term, which will become a term of order 8 at \(t+2\) and so on. As a solution, we adopt the pruning scheme described in Andreasen et al. (2016). This pruning scheme augments the state space to keep track of first- to third-order terms and uses the Kronecker product of the first- and second-order terms to compute the third-order term. In contrast, Fernández-Villaverde et al. (2011) and Born and Pfeifer (2014) use a IRF-pruning scheme were all higher order terms are based on the first-order terms. Also, whereas in Fernández-Villaverde et al. (2011) and Born and Pfeifer (2014) the IRF-pruning scheme differs from the scheme used for simulations, we use the same pruning for both IRFs and simulations.

Third, computing IRFs in a nonlinear environment is somewhat involved, since the IRFs are not invariant to rescaling and to the previous history of shocks. To circumvent this problem, we consider the generalized impulse response function (GIRF) proposed by Koop et al. (1996). In particular, we follow Fernández-Villaverde et al. (2011) and Born and Pfeifer (2014), and start the IRFs at the ergodic mean in the absence of shocks (EMAS).

\(^{24}\) Recently, de Groot (2016) shows that to risk-correct the constant term for the standard deviation of stochastic volatility innovations (a.k.a. vol of vol) a fourth (or sixth, depending on the functional form of the volatility process) order expansion is further needed. de Groot (2016) shows that this risk-correction has important consequences for the bond and equity risk premia as well as for understanding the welfare cost of business cycle fluctuations.