We develop measures of time-varying risk aversion and economic uncertainty that are calculated from financial variables at high frequencies. To do so, we formulate a dynamic no-arbitrage asset pricing model for equities and corporate bonds. The joint dynamics that we specify among asset-specific cash flows, macroeconomic fundamentals and risk aversion accommodate both heteroskedasticity and non-Gaussianity. Variance risk premiums on equity are very informative about risk aversion, whereas credit spreads and corporate bond volatility are highly correlated with economic uncertainty. Our model-implied risk premiums outperform standard instruments for predicting excess returns on equity and corporate bonds. A financial proxy to our economic uncertainty predicts output growth significantly and negatively. [108 words]

**JEL Classification:** C1, G10, G12, G13.

**Keywords:** Risk aversion, Economic uncertainty, Dynamic asset pricing model, Asymmetric state variables, VIX, Variance risk premium.
1 Introduction

Many economic models combine assumptions regarding the preferences of economic agents with assumptions regarding the data generating process for consumption growth or productivity shocks to model endogenous variables such as financial asset prices. A large class of models (see e.g. Bansal, Kiku, Shaliastovich, and Yaron, 2014) relies on time variation in economic uncertainty as the main mechanism to generate variation over time in financial risk premiums. Moreover, uncertainty shocks play an increasingly important role in dynamic macro-models (see Bloom, 2009; Christiano, Motto, and Rostagno, 2014), perhaps contributing to a recent cottage industry for developing indices of economic uncertainty (see e.g. Baker, Bloom, and Davis, 2016; Jurado, Ludvigson, and Ng, 2015).

While many of the models featuring time-varying uncertainty assume the risk aversion of households to be time invariant, another class of models featuring habit-forming utility, starting with Campbell and Cochrane (1999), stresses stochastic risk aversion as the main driver of financial market risk premiums. In addition, changes in risk aversion have become prominent features of models in other areas as well. In monetary economics, recent research suggest a potential link between loose monetary policy and greater risk appetite of market participants, spurring a literature that explores what structural economic factors drive risk aversion changes (see, e.g., Rajan, 2006; Adrian and Shin, 2009; Bekaert, Hoerova, and Lo Duca, 2013). He and Krishnamurthy (2013) and Adrian and Shin (2013) suggest that what appears to be aggregate risk aversion may be related to financial constraints, or time-varying leverage or risk preferences of financial intermediaries. In international finance, Miranda-Agrippino and Rey (2015) and Rey (2015) suggest that global risk aversion is a key transmission vector by which US monetary policy is “exported” to foreign countries and is a major source of asset return comovements across countries (see also Xu, 2017). Finally, reduced-form asset pricing models that aim to simultaneously explain stock return dynamics and option prices, have also concluded that time-varying prices of risk are important drivers (see, e.g., Bakshi and Wu, 2010; Broadie, Chernov, and Johannes, 2007; Pan, 2002).

In this article, we separately identify time-varying uncertainty in fundamentals, using macro data, and time-varying aggregate risk aversion (or its inverse, which we call “risk appetite”), using financial asset prices, through the lens of a dynamic asset pricing model. To
do so, we build on the habit models of Campbell and Cochrane (1999), Menzly, Santos, and Veronesi (2004) and Wachter (2006), but we allow stochastic risk aversion to be imperfectly correlated with fundamentals. We refer to this uncorrelated component of risk aversion as “non-fundamental.” Our approach is agnostic regarding the economic sources of the non-fundamental component of risk aversion. It may reflect sentiment induced by news (see the recent experimental evidence in Cohn, Engelmann, Fehr, and Maréchal, 2015), or even reflect mood swings induced by the weather (Kamstra, Kramer, and Levi, 2003). It could also reflect those unmodeled institutional factors (such as risk constraints faced by financial institutions) that end up affecting what appears to be aggregate risk aversion.

In implementing our approach, we confront several challenges. Most obviously, we must separately identify both the price of risk (risk aversion) and the amount of risk (economic uncertainty). Accounting for uncertainty in fundamentals empirically is challenging; the empirical macro literature suggests that macro shocks feature non-Gaussian distributions with time-varying second and higher order moments (Hamilton, 1990; Fagiolo, Napoletano and Roventini, 2008; Gambetti, Pappa and Canova, 2008; Adrian, Boyarchenko, and Giannone, 2019). To accommodate these features of the data in a tractable fashion, we use the Bad Environment-Good Environment (BEGE, henceforth) framework developed in Bekaert and Engstrom (2017). In this framework, shocks to key variables are modeled as the sum of two components that are distributed as de-meaned gamma distributions, with shape parameters that vary through time. These shape parameters advise changes in “bad” (“good”) volatility, associated with negative (positive) skewness. By realistically modeling economic uncertainty, our model delivers an economic uncertainty index as a by-product.

Second, to develop the risk aversion measure in an internally consistent manner, we must solve for asset prices as a function of preferences and cash flow dynamics. Focusing on two prominent risky asset classes, corporate bonds and equities, we also use the BEGE-class of models to capture their cash flow dynamics (dividends or earnings for equities; default rates for corporate bonds). Despite the fact that the model accommodates state variables with time-
varying non-Gaussian shocks, our formulation admits (quasi) closed-form solutions for asset prices within the affine class. Summing up, the model has three channels for generating time-variation in risk premiums and volatility for asset returns: (1) fundamental macroeconomic uncertainty – of the good and bad varieties – (2) stochastic risk aversion and (3) cash flow uncertainty.

Third, to identify the model parameters and stochastic risk aversion, we go beyond using only information in historical returns, which are known to be noisy. In particular, we use both realized variances and option-implied variances in the estimation of the model parameters. A large empirical literature (see e.g. Andersen, Bollerslev, Diebold and Labys, 2003) shows that realized variances can be measured fairly precisely and provide accurate forecasts of future return variances. Moreover, conditional return variances are an exact function of the relevant state variables (including risk aversion) in our pricing framework (see Joslin, Le, Singleton (2013) for a similar observation in a term structure model). Therefore, realized variances greatly facilitate identifying the risk aversion process. There is also a large literature on inferring risk and risk preferences from option prices, which we discuss in more detail in Section II. Option-implied volatility, such as the famous VIX index in the equity market, reflects both the physical return distribution, including the probability of crashes, and risk aversion. The risk aversion of rational agents creates a demand for insurance against potential losses, making (out-of-money) put options relatively more expensive than call options. Such expensive put options are the source of the consistent presence of a positive variance risk premium (often empirically measured as the difference between the VIX index-squared and the physical conditional return variance) (see Bekaert and Hoerova, 2016; Bakshi and Madan (2006) for formal arguments). Option data should also be informative about conditional risk premiums, which are difficult to observe from the data. Martin (2017) uses option-implied variances to provide bounds on equity premiums, and several articles (see Bollerslev and Todorov, 2011; Liu, Pan, and Wang, 2004; Santa-Clara and Yan, 2010) suggest that compensation for rare events (“jumps”) accounts for a large fraction of equity risk premiums.

The main output of our model is a measure of time-varying aggregate risk aversion that consistently helps price assets through a structural asset pricing model but is easily tracked over

---

between fundamentals and asset prices (see e.g. Bekaert and Hoerova, 2016).
time, even at high frequencies. It is because we solve for its dependence on observable financial variables, such as credit spreads and equity risk-neutral variances. The measure should be a useful model-based complement to “sentiment indices” developed in the behavioral finance literature (see, e.g., Lemmon and Portnaiquina, 2006; and Baker and Wurgler, 2006), or practitioner indices developed by financial institutions (see Coudert and Gex, 2008 for a survey).

Our main results are as follows: First, we find significant time variation in the volatilities and higher-order moments of the fundamentals, especially in real activity. The time variation in uncertainty is dominated by strongly countercyclical “bad” volatility. Moreover, we find that macroeconomic uncertainty is informative about uncertainty regarding risky asset cash flows, both for the equity and corporate bond markets. Nonetheless, we do find evidence of independent time variation in the volatility of corporate bond loss rates.

Second, the extracted risk aversion process loads most significantly on equity risk-neutral variances (with a positive sign) and realized variances (with a negative sign), supporting the variance premium as a good proxy for aggregate risk aversion. This finding is consistent with recent work in the consumption-based asset pricing literature, showing the variance premium to be very informative for identifying equilibrium models featuring complex data generating processes for the fundamentals (see Drechsler and Yaron, 2010; Bollerslev, Tauchen, and Zhou, 2009; Bekaert and Engstrom, 2017). Nevertheless, corporate bond market variables – the credit spread and realized corporate bond variance – still account for 15% of the variation in risk aversion. The risk aversion process is much more rapidly mean reverting than would be implied by habit models, which is consistent with the results in Martin (2017).

Third, economic uncertainty is highly correlated with corporate bond volatility and, especially, with credit spreads. Our model-implied risk premiums significantly predict equity returns, whereas our economic uncertainty index predicts output negatively and significantly. Because equity risk premium variation is dominated by changes in risk aversion, but the conditional variance of equity returns also loads strongly on economic uncertainty, our results help explain the failure of a large literature in finance (starting with French, Schwert and Stambaugh, 1987) to find a robust link between future equity returns and the conditional variance of equity returns, while assuming a constant price of variance risk.

The remainder of the paper is organized as follows. Sections 2 and 3 present the model
and estimation strategy in detail. Section 4 briefly outlines the data we use. Section 5 presents the estimation results and extracts risk aversion and uncertainty from asset prices. In Section 6, we investigate how our measures of risk aversion and macroeconomic uncertainty correlate with (and predict) macroeconomic activity as asset price changes. We also illustrate the use of the indices at high frequencies by examining their behavior around the Bear Stearns and Lehman Brothers bankruptcies. Concluding remarks are in Section 7.

2 Modeling Risk Appetite and Uncertainty

In this section, we first define our concept of risk aversion. We then build a dynamic model with stochastic risk aversion and macro-economic factors affecting the cash flows processes of two main risky asset classes, corporate bonds and equity. The state variables are described in Section 2.2 and the pricing kernel in Section 2.3.

2.1 General Strategy

An ideal measure of risk aversion would be model-free and would not confound time variation in economic uncertainty with time variation in risk aversion. There are many attempts in the literature to approximate this ideal, but invariably various modeling and statistical assumptions are necessary to identify risk aversion. For example, in the options literature, a number of articles (Aıt-Sahalia and Lo, 2000; Engle and Rosenberg, 2002; Jackwerth, 2000; Bakshi, Kapadia and Madan, 2003; Britten-Jones and Neuberger, 2000; Bliss and Panigirtzoglou, 2004) appear at first glance to infer risk aversion from equity options prices in a model-free fashion, but it is generally the case that the utility function is assumed to be of a particular form and/or to depend only on stock prices.

Another strand of the literature relies on general properties of pricing kernels. A strictly positive pricing kernel or stochastic discount factor, \( M_{t+1} \), under no-arbitrage conditions, implies that for all gross returns, \( R_t \),

\[
E_t [M_{t+1} R_{t+1}] = 1
\]  

This is also true in the recent debate about the recoverability of physical probabilities from option prices, which claims to identify risk aversion as well (Ross, 2015; Carr and Wu, 2016).
It is then straightforward to derive that any asset’s expected excess return can be written as an asset specific risk exposure ("beta", or $\beta_t$) times a price of risk (or $\lambda_t$), which applies to all assets (see also Coudert and Gex, 2008):

$$E_t [R_{t+1}] - R^f_t = \beta_t \lambda_t$$

(2)

where $R^f_t$ is the risk free rate, $\beta_t = -\frac{Cov_p (R_{t+1} , M_{t+1})}{Var_t (M_{t+1})}$, and $\lambda_t = \frac{Var_t (M_{t+1})}{E_t (M_{t+1})}$.

Importantly, this price of risk is not equal to time-varying risk aversion, and in particular may confound economic uncertainty with risk aversion. In a simple power utility framework, it is easy to show that the price of risk is linked to both the coefficient of relative risk aversion and the volatility of consumption growth, the latter being a reasonable measure of economic uncertainty. We use the terms “risk aversion” and “risk appetite” as each other’s inverse.4

Our approach is to start from a utility function defined over both consumption (“fundamentals”) and a “non-fundamental” factor. Our measure of risk aversion is then the coefficient of relative risk aversion implied by the utility function. We specify a consumption process accommodating time variation in economic uncertainty and use the utility framework to price assets, given general processes for their cash flows. Therefore, while certainly not model free, our risk aversion process is consistent with a wide set of economic models that respect no-arbitrage conditions.

Consider a period utility function in the HARA class:

$$U \left( \frac{C}{Q} \right) = \left( \frac{C}{Q} \right)^{1-\gamma}$$

(3)

where $C$ is consumption and $Q$ is a process that will be shown to drive time-variation in risk aversion. Essentially, when $Q$ is high, consumption delivers less utility and marginal utility increases. Critically, $Q$ is a function of $C$, that is:

$$Q = \left( \frac{a}{\gamma} - \frac{b}{C} \right)^{-1} = f(C)$$

(4)

Gai and Vause (2006) and Pfleeger, Siriwardane, and Sunderam (2018) however use the term “risk appetite” to indicate the price of risk, that is the product of “risk aversion” and “the amount of risk.” In an older empirical literature testing the conditional CAPM, a number of authors let the price of risk (which constitutes aggregate risk aversion under the null of the model) vary through time, see e.g. Chou, Engle and Kane (1992) and Bekaert and Harvey (1995).
where $a$ and $\gamma$ are positive parameters, and $b$ is an exogenous benchmark parameter or process, which is everywhere positive. Note that $\gamma$ (the curvature parameter) is not equal to risk aversion in this framework. If $Q$ were simply an exogenous process, risk aversion is equal to $\gamma$ and does not vary over time (see Abel, 1990, for such “multiplicative” habit models). Internal habit models are excluded by assuming that the $b$ process can vary through time but is “exogenous” to the agent’s optimization problem, as in the well-known “catching up with the Jones’s” preferences (see also Abel, 1990).

With $\frac{\partial C}{\partial C}$ simplifying to $\frac{a}{\gamma}$, the coefficient of relative risk aversion for this class of models is given by

$$RRA = -\frac{CU''(C)}{U'(C)} = aQ$$

and is thus proportional to $Q$. Note that $\frac{dQ}{dC} = -b \left( \frac{Q}{C} \right)^2 < 0$; in good times when consumption increases, risk aversion decreases.

For pricing assets, it is helpful to derive the log pricing kernel which is the intertemporal marginal rate of substitution in a dynamic economy. We assume an infinitely lived agent, facing a constant discount factor of $\beta$, and the HARA period utility function given above. The pricing kernel is then given by

$$m_{t+1} = \ln(\beta) + \ln \left[ \frac{U'(C_{t+1})}{U'(C_t)} \right] = \ln(\beta) - \gamma \Delta c_{t+1} + \gamma \Delta q_{t+1}$$

where we use $t$ to indicate time, lower case letters to indicate logs of uppercase variables, and $\Delta$ to represent the difference operator.

To get more intuition for this framework, consider the Campbell and Cochrane (1999) (CC henceforth) utility function. CC use an external habit model, with utility being a power function over $C_t - H_t$, where $H_t$ is the habit stock. Of course, we can also write

$$C_t - H_t = C_t Q_t$$

with $Q_t = \frac{C_t}{C_t - H_t}$. Therefore, the CC utility function is a special case of our framework with $a = \gamma$ and $b = H$. As $C_t$ gets closer to the habit stock, risk aversion increases. $Q_t$ is thus the inverse of the surplus ratio in the CC article. CC also model $q_t$ exogenously as a slow-moving,
persistent process, but restrict the correlation between shocks to $q_t$ and $\Delta c_t$ to be perfect. The “moody investor” economy in Bekärt, Engstrom, and Grenadier (2010) is also a special case. In that model, $q_t$ is also exogenously modeled, but has its own shock; that is, there are preference shocks not correlated with fundamentals. Another special case is the model in Brandt and Wang (2003), but their risk aversion process specifically depends on inflation in addition to consumption growth. DSGE models in macroeconomics often attempt to match variation in financial risk premiums by resorting to exogenously specified asset-specific risk premium shocks (see e.g. the seminal New Keynesian model of Smets and Wouters, 2007). Instead, in our model, the pricing kernel, with one stochastic risk aversion process, prices all assets consistently.

Non-fundamental risk aversion may rise in a variety of ways. The experimental literature (see e.g. Cohn, Engelmann, Fehr, and Maréchal, 2015) shows that the subjective willingness to take risk is indeed lower during a recession, which is simulated by “priming” people with a stock market crash (versus boom), and that this risk aversion is rooted in emotions of fear. Even if this is the dominant source of preference variation, it is unlikely that the aggregate component of this type of counter-cyclical risk aversion is perfectly correlated with aggregate consumption growth. Da, Engelberg, and Gao (2014) create a household sentiment measure based on the internet searches for words such as “recession” and “bankruptcies” and find it to temporarily affect stock prices.

Thinking of the risk aversion of the representative agent as the wealth-weighted average of individual risk aversion in the economy suggests additional sources of slow-moving changes in aggregate risk aversion. First, changes in the wealth distribution across individuals may cause changes in aggregate risk aversion and even induce counter-cyclical risk aversion. In bad times, the wealth of the richer people goes conceivably down proportionally more than that of poorer people (because more of their wealth is tied up in risky asset classes). This in turn would then increase aggregate risk aversion. Alternatively, people’s experiences with macro-economic shocks may affect their risk aversion (see e.g. Malmendier and Nagel, 2011), so that changes in the age distribution over time may induce changes in aggregate risk aversion.

Therefore, our approach specifies a stochastic process for $q$ (risk aversion), which is partly but not fully driven by persistent macroeconomic fundamentals (consumption growth) and fea-

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5 There is an active literature on the neural basis of risk taking behavior in a financial context, see e.g. Knutson and Kuhnen (2005) for an early paper.
tures an independent preference shock.

2.2 Economic Environment: State Variables

2.2.1 Macroeconomic Factors

In typical asset pricing models agents have utility over consumption, but it is well known that consumption growth and asset returns show very little correlation. Moreover, consumption data are only available at the quarterly frequency. Because the use of options data is key to our identification strategy and these data are only available since 1986, it is important to use macroeconomic data that are available at the monthly frequency. We therefore chose to use industrial production, which is available at the monthly frequency, as our main macroeconomic factor. In the macroeconomic literature, much attention has been devoted recently to the measurement of “real” uncertainty (see e.g. Jurado, Ludvigson and Ng, 2015) and its effects on the real economy (see e.g. Bloom, 2009). We add to this literature by using a novel econometric framework to extract two macro risk factors from industrial production: “good” uncertainty, denoted by $p_t$, and “bad” uncertainty, denoted by $n_t$.

Specifically, under our model the change in log industrial production index, $\theta_t$, has time-varying conditional moments governed by two state variables: $p_t$ and $n_t$. The conditional mean is modeled as a persistent process to accommodate a time-varying long-run mean of output growth:

$$\theta_{t+1} = \bar{\theta} + \rho_\theta (\theta_t - \bar{\theta}) + m_p (p_t - \bar{p}) + m_n (n_t - \bar{n}) + u^\theta_{t+1},$$

(8)

where the growth shock is decomposed into two independent centered gamma shocks,

$$u^\theta_{t+1} = \sigma_{\theta_p} \omega_{p,t+1} - \sigma_{\theta_n} \omega_{n,t+1}.$$  

(9)

The shocks follow centered gamma distributions with time-varying shape parameters,

$$\omega_{p,t+1} \sim \tilde{\Gamma}(p_t, 1)$$ \hspace{1cm} (10)

$$\omega_{n,t+1} \sim \tilde{\Gamma}(n_t, 1),$$ \hspace{1cm} (11)
where $\tilde{\Gamma}(x, 1)$ denotes a centered gamma distribution with shape parameter $x$ and a unit scale parameter. The shape factors, $p_t$ and $n_t$, follow autoregressive processes,

$$p_{t+1} = \bar{p} + \rho_p(p_t - \bar{p}) + \sigma_{pp}\omega_{p,t+1}$$

(12)

$$n_{t+1} = \bar{n} + \rho_n(n_t - \bar{n}) + \sigma_{nn}\omega_{n,t+1},$$

(13)

where $\rho_x$ denotes the autoregressive term of process $x_{t+1}$, $\sigma_{xx}$ the sensitivity to shock $\omega_{x,t+1}$, and $\bar{x}$ the long-run mean. We denote the macroeconomic state variables as, $Y_{t}^{mac} = [\theta_t \ p_t \ n_t]'$, and the set of unknown parameters are $\theta, \rho_{\theta}, m_p, m_n, \bar{p}, \sigma_{\theta p}, \sigma_{\theta n}, \rho_p, \sigma_{pp}, \rho_n, \sigma_{nn}$.

In this model, the conditional mean has an autoregressive component, but macro risks also affect expected growth. These features can both accommodate cyclical effects (e.g., lower conditional means in bad times), or the uncertainty effect described in Bloom (2009). The shocks reflect the BEGE framework of Bekaert and Engstrom (2017), implying that the conditional higher moments of output growth are linear functions of the bad and good uncertainties. For example, the conditional variance and the conditional unscaled skewness are as follows,

$$E_t \left[ (u_{t+1}^\theta)^2 \right] = \sigma_{\theta p}^2 p_t + \sigma_{\theta n}^2 n_t,$$

Conditional Variance:

$$E_t \left[ (u_{t+1}^\theta)^3 \right] = 2\sigma_{\theta p}^3 p_t - 2\sigma_{\theta n}^3 n_t.$$

Conditional Unscaled Skewness:

This reveals the sense in which $p_t$ represents “good” and $n_t$ “bad” volatility: $p_t$ ($n_t$) increases (decreases) the skewness of industrial production growth.

The state variables and shocks derived from industrial production growth serve as key macro determinants for consumption growth and cash flows.

2.2.2 Cash Flows and Cash Flow Uncertainty

To model the cash flows for equities and corporate bonds, we focus attention on two variables that exhibit strong cyclical movements, namely earnings (see e.g. Longstaff and Piazzesi, 2004) and corporate defaults (see e.g. Gilchrist and Zakrjsek, 2012).

**Corporate Bond Loss Rate** To model corporate bonds, we must model the possibility of defaults. Suppose a portfolio of one-period nominal bonds has a promised payoff of $C \equiv e^{xR}$
at \((t+1)\), but will in fact only pay an unknown fraction \(F_{t+1} \leq 1\) of that amount. Therefore, the nominal payoff for a one-period zero-coupon defaultable corporate bond at period \(t+1\) is \(C \times F_{t+1} = \exp(c + \ln(F_{t+1})) = \exp(c - l_{t+1})\). Thus, \(l_{t+1}\) is defined as \(-\ln(F_{t+1}) = -\ln(1 - L_{t+1})\) where \(L_{t+1}\) (i.e., \(1 - F_{t+1}\)) is the aggregate corporate loss rate, which can be computed as the default rate times one minus the recovery rate. We provide more detail on the pricing of defaultable bonds in the asset pricing section (Section 2.3).

The dynamic system of the aggregate corporate bond log loss rate, \(l_t\), is modeled as follows:

\[
l_{t+1} = l_0 + \rho l_t + m_l p_t + m_{ln} n_t + \sigma_{lp} \omega_{p,t+1} + \sigma_{ln} \omega_{n,t+1} + u_{l,t+1} \tag{14}
\]

\[
u_{l,t+1} = \sigma_{lp} \omega_{p,t+1} - \sigma_{ln} \omega_{ln,t+1} \tag{15}
\]

\[
\omega_{lp,t+1} \sim \hat{\Gamma}(l_p, 1), \tag{16}
\]

\[
\omega_{ln,t+1} \sim \hat{\Gamma}(\ln, 1), \tag{17}
\]

where the variance equation is,

\[
l_{p,t+1} = \bar{l}_p + \rho_{lp}(l_p - \bar{l}_p) + \sigma_{lp} \omega_{p,t+1}, \tag{18}
\]

\[
\ln > 0. \tag{19}
\]

The conditional mean depends on an autoregressive term and the good and bad macro uncertainty state variables \(p_t\) and \(n_t\). The loss rate total disturbance is governed by three independent heteroskedastic shocks: the good and bad environment macro shocks \(\{\omega_{p,t+1}, \omega_{n,t+1}\}\) and the (orthogonal) loss rate shock \(u_{l,t+1}\). The loss rate shock follows a typical BEGE process, but we only allow \(\omega_{lp}\)'s shape parameter to be time-varying.

This dynamic system allows macro-economic uncertainty to affect both the conditional mean and conditional variance of the loss rate process. However, it also allows the loss rate to have an autonomous autoregressive component in its conditional mean (making \(l_t\) a state variable) and accommodates heteroskedasticity not spanned by macro-economic uncertainty.

\[\text{This final model is not chosen at random. We experimented with 5 other models including letting only } l_n \text{ follow a BEGE process, letting both } l_n \text{ and } l_p \text{ follow a BEGE process (with or without restricting the parameters to be the same across } l_n \text{ and } l_p \text{) and, finally, a version of the last two models where } l_n \text{ and } l_p \text{ also enter the loss rate mean. Our final loss rate model outperforms other models based on standard model selection criteria. Details on alternative models are available upon request.}\]
This “financial” cash flow uncertainty has a time-varying component, denoted by $lp_t$, and a constant component denoted by $ln$. If $\sigma_{lp}$ and $\sigma_{lplp}$ are positive, as we would expect, the loss rate and its volatility are positively correlated; that is, in bad times with a high incidence of defaults, there is also more uncertainty about the loss rate, and because the gamma distribution is positively skewed, the (unscaled) skewness of the process increases. We would also expect the sensitivities to the good (bad) economic environment shocks, $\sigma_{lp}$ ($\sigma_{ln}$) to be negative (positive): intuitively, defaults should decrease (increase) in relatively good (bad) times.

The conditional variance of the loss rate is $\sigma_{lp}^2 p_t + \sigma_{ln}^2 n_t + \sigma_{lplp}^2 p_t + \sigma_{lln}^2 n_t$, and its conditional unscaled skewness is $2 \left( \sigma_{lp}^3 p_t + \sigma_{ln}^3 n_t + \sigma_{lplp}^3 p_t - \sigma_{lln}^3 n_t \right)$. We denote the financial state variables as, $Y_{t}^{fin} = \begin{bmatrix} l_t & lp_t \end{bmatrix}'$. The set of unknown parameters are $l_0, \rho_{ll}, m_{lp}, m_{ln}, \sigma_{lp}, \sigma_{ln}, \sigma_{lplp}, \sigma_{lln}, \ln, lp, \rho_{lp}, \sigma_{llplp}$, and $ln$.

**Log Earnings Growth** Log earnings growth, $g_t$, is defined as the change in log real earnings of the aggregate stock market. It is modeled as follows:

$$
g_{t+1} = g_0 + \rho_{gg} g_t + \rho_{g,mac}' Y_{t}^{mac} + \rho_{g,fin}' Y_{t}^{fin} + \sigma_{g}^2 \omega_{g,t+1}
$$

$$
\sigma_{g}^2 \omega_{g,t+1} \sim N(0, 1).
$$

The conditional mean is governed by an autoregressive component and the three macro factors; the time variation in the conditional variance comes from the good and bad uncertainty factors, and the loss rate uncertainty factor. The earnings shock is assumed to be Gaussian and homoskedastic, because we fail to reject the null that the residuals series, after controlling for the heteroskedastic fundamental shocks, is Gaussian and homoskedastic. Thus, the time variation in the conditional variance of earnings growth is spanned by macro-economic uncertainty and the financial uncertainty present in default rates. The set of unknown parameters is \{ $g_0, \rho_{gg}, \rho_{g,mac}', \rho_{g,fin}', \sigma_{g}, \sigma_{g}, \sigma_{glp}, \sigma_{gln}, \sigma_{gg}$ \}.

**Log Consumption-Earnings Ratio** We model consumption as stochastically cointegrated with earnings so that the consumption-earnings ratio becomes a relevant state variable. Define
\( \kappa_t \equiv \ln \left( \frac{C_t}{E_t} \right) \) which is assumed to follow:

\[
\kappa_{t+1} = \kappa_0 + \rho_{\kappa_0} \kappa_t + \rho_{\kappa,mac}' Y_{t+1}^{mac} + \rho_{\kappa,fin}' Y_{t+1}^{fin}
+ \sigma_{\kappa p} \omega_{p, t+1} + \sigma_{\kappa n} \omega_{n, t+1} + \sigma_{\kappa lp} \omega_{lp, t+1} + \sigma_{\kappa ln} \omega_{ln, t+1} + u_{t+1}^\kappa
\]

(23)

\[
u_{t+1}^\kappa = \sigma_{\kappa n} \omega_{n, t+1}
\]

(24)

\[
\omega_{\kappa, t+1} \sim N(0, 1).
\]

(25)

Similarly to earnings growth, there is an autonomous conditional mean component but the heteroskedasticity of \( \kappa_t \) is spanned by other state variables. As with log earnings growth, we fail to reject Gaussianity and homoskedasticity of \( u_{t+1}^\kappa \). The set of unknown parameters is \( \{ \kappa_0, \rho_{\kappa_0}, \rho_{\kappa,mac}', \rho_{\kappa,fin}', \sigma_{\kappa p}, \sigma_{\kappa n}, \sigma_{\kappa lp}, \sigma_{\kappa ln}, \sigma_{\kappa \kappa} \} \).

**Log Dividend Payout Ratio** The log dividend payout ratio, \( \eta_t \), is expressed as the log ratio of dividends to earnings. Recent evidence in Kostakis, Magdalinos, and Stamatogiannis (2015) shows that the monthly dividend payout ratio is stationary. We model \( \eta_t \) analogously to \( \kappa_t \) and \( g_t \), again assuming a Gaussian and homoskedastic residual shock (as justified by the data):

\[
\eta_{t+1} = \eta_0 + \rho_{\eta_0} \eta_t + \rho_{\eta,mac}' Y_{t+1}^{mac} + \rho_{\eta,fin}' Y_{t+1}^{fin}
+ \sigma_{\eta p} \omega_{p, t+1} + \sigma_{\eta n} \omega_{n, t+1} + \sigma_{\eta lp} \omega_{lp, t+1} + \sigma_{\eta ln} \omega_{ln, t+1} + u_{t+1}^\eta
\]

(26)

\[
u_{t+1}^\eta = \sigma_{\eta n} \omega_{n, t+1}
\]

(27)

\[
\omega_{\eta, t+1} \sim N(0, 1).
\]

(28)

The set of unknown parameters is \( \{ \eta_0, \rho_{\eta_0}, \rho_{\eta,mac}', \rho_{\eta,fin}', \sigma_{\eta p}, \sigma_{\eta n}, \sigma_{\eta lp}, \sigma_{\eta ln}, \sigma_{\eta \eta} \} \). Using \( \eta_{t+1} \) and \( g_{t+1} \), dividend growth \( \Delta d_{t+1} \), is given by \( \eta_{t+1} - \eta_t + g_{t+1} \).

**2.2.3 Pricing Kernel State Variables**

In the model we introduced above, the real pricing kernel depends on consumption growth and changes in risk aversion. To price nominal cash flows (or to price default free nominal bonds), we also need an inflation process. We discuss the modeling of these variables here.
Consumption Growth  By definition, log real consumption growth, \( \Delta c_{t+1} = \ln \left( \frac{C_{t+1}}{C_t} \right) = \Delta c_t + \Delta \kappa_{t+1} \). Therefore, consumption growth is spanned by the previously defined state variables and shocks.

Risk Aversion  The state variable capturing risk aversion, \( q_t \equiv \ln \left( \frac{C_t}{C_t - H_t} \right) \) is, by definition, nonnegative. We impose the following structure,

\[
q_{t+1} = q_0 + \rho_{qq} q_t + \rho_{qp} p_t + \rho_{qn} n_t + \sigma_{qp} \omega_{p,t+1} + \sigma_{qn} \omega_{n,t+1} + u_{q,t+1}^q
\]

(29)

\[
u_{q,t+1}^q = \sigma_{qq} \omega_{q,t+1}
\]

(30)

\[\omega_{q,t+1} \sim \tilde{\Gamma}(q_t, 1).\]

(31)

The risk aversion disturbance is comprised of three parts, exposure to the good uncertainty shock, exposure to the bad uncertainty shock, and an orthogonal preference shock. Thus, given the distributional assumptions on these shocks, the model-implied conditional variance is \( \sigma_{qp}^2 p_t + \sigma_{qn}^2 n_t + \sigma_{qq}^2 q_t \), and the conditional unscaled skewness \( 2 \left( \sigma_{qp}^3 p_t + \sigma_{qn}^3 n_t + \sigma_{qq}^3 q_t \right) \). With \( \sigma_{qq} = 0 \) and certain restrictions on \( \sigma_{qp} \) and \( \sigma_{qn} \), the model would imply a perfect correlation between the conditional correlation between risk aversion and real activity, as in the Campbell-Cochrane (1999) model.

We model the pure preference shock also with a demeaned gamma distributed shock, so that its variance and (unscaled) skewness are proportional to its own level. Controlling for current business conditions, when risk aversion is high, so is its conditional variability and unscaled skewness. This seems like a plausible assumption. For example, option-implied volatilities, which are partially driven by risk aversion, are much more volatile in stress times. The higher moments of risk aversion are perfectly spanned by macroeconomic uncertainty on the one hand and pure risk aversion itself \( (q_t) \) on the other hand. Note that our identifying assumption is that \( q_t \) does not affect the macro variables and \( u_{q,t+1} \) represents a pure preference shock. The conditional mean is modeled as before: an autonomous autoregressive component and dependence on \( p_t \) and \( n_t \). The set of unknown parameters describing the risk aversion process is \( \{q_0, \rho_{qq}, \rho_{qp}, \rho_{qn}, \sigma_{qp}, \sigma_{qn}, \sigma_{qq}\} \).
Inflation  To price nominal cash flows, we must specify an inflation process. The conditional mean of inflation depends on an autoregressive term and the three macro factors $Y_{mac}^t$. The conditional variance and higher moments of inflation are proportional to the good and bad uncertainty factors $\{p_t, n_t\}$. The inflation innovation $\nu_{t+1}^{\pi}$ is assumed to be Gaussian and homoskedastic. There is no feedback from inflation to the macro variables:

$$
\pi_{t+1} = \pi_0 + \rho_{\pi \pi} \pi_t + \rho'_{\pi, mac} Y_{mac}^t + \sigma_{\pi p} \omega_{p,t+1} + \sigma_{\pi n} \omega_{n,t+1} + u_{t+1}^{\pi}, \quad (32)
$$

$$
u_{t+1}^{\pi} = \sigma_{\pi \pi} \omega_{\pi,t+1}, \quad (33)
$$

$$
\omega_{\pi,t+1} \sim N(0,1). \quad (34)
$$

The set of unknown parameters is $\{\pi_0, \rho_{\pi \pi}, \rho'_{\pi, mac}, \sigma_{\pi p}, \sigma_{\pi n}, \sigma_{\pi \pi}\}$.

2.2.4 Matrix Representation

The dynamics of all state variables introduced above can be written compactly in matrix notation. We define the macro factors $Y_{mac}^t = \left[ \theta_t \ p_t \ n_t \right]'$ and other state variables $Y_{other}^t = \left[ \pi_t \ l_t \ g_t \ \kappa_t \ \eta_t \ l_p_t \ q_t \right]'$. Among the ten state variables, the industrial production growth $\theta_t$, the inflation rate $\pi_t$, the loss rate $l_t$, earnings growth $g_t$, the log consumption-earnings ratio $\kappa_t$ and the log divided payout ratio $\eta_t$ are observable, while the other four state variables, $\{p_t, n_t, l_p_t, q_t\}$ are latent. There are eight independent centered gamma and Gaussian shocks in this economy. The system can be formally described as follows (technical details are relegated to the Online Appendix):

$$
Y_{t+1} = \mu + A Y_t + \Sigma \omega_{t+1}, \quad (35)
$$

where constant matrices, $\mu$ (10 x 1), $A$ (10 x 10) and $\Sigma$ (10 x 9), are implicitly defined, $Y_t = \left[ Y_{mac}^t \ Y_{other}^t \right]'$ (10 x 1) is a vector comprised of the state variable levels, and $\omega_{t+1} = \left[ \omega_{p,t+1} \ \omega_{n,t+1} \ \omega_{\pi,t+1} \ \omega_{l_p,t+1} \ \omega_{l_n,t+1} \ \omega_{g,t+1} \ \omega_{\kappa,t+1} \ \omega_{\eta,t+1} \ \omega_{q,t+1} \right]'$ (9 x 1) is a vector comprised of all the independent shocks in the economy.

Note that, among the nine shocks, five shocks are gamma distributed—the good uncertainty shock ($\omega_{p,t+1}$), the bad uncertainty shock ($\omega_{n,t+1}$), the right-tail loss rate shock ($\omega_{l_p,t+1}$), the left-tail loss rate shock ($\omega_{l_n,t+1}$), and the risk aversion shock ($\omega_{q,t+1}$). The remaining four
shocks are standard homoskedastic Gaussian shocks (i.e., \(N(0,1)\)). Importantly, given our preference structure, the state variables driving the time variation in the higher order moments of these shocks are the only ones driving the time variation in asset risk premiums and their higher order moments. Economically, we therefore rely on time variation in risk aversion—as in the classic Campbell-Cochrane model and its variants (see e.g. Bekaert, Engstrom, and Grenadier, 2010; Wachter, 2006)—and time variation in economic uncertainty—as in the Bansal-Yaron (2004) model—to explain risk premiums. The model’s implications for conditional asset return variances are critical in identifying the dynamics of risk aversion (see also Joslin, Le, and Singleton, 2013).

Our specific structure admits conditional non-Gaussianity yet generates affine pricing solutions. The model is tractable because the moment generating functions of gamma and Gaussian distributed variables can be derived in closed form, delivering exponentiated affine functions of the state variables. In particular,

\[
E_t \left[ \exp(\nu' Y_{t+1}) \right] = \exp \left[ \nu' S_0 + \frac{1}{2} \nu' S_1 \Sigma^{other} S'_1 \nu + f_S(\nu) Y_t + S_2(\nu) \ln \right],
\]

where \(S_0 (10 \times 1)\) is a vector of drifts; \(S_1 (10 \times 4)\) is a selection matrix of 0s and 1s which picks out the Jensen’s inequality terms of the four Gaussian shocks; \(\Sigma^{other} (4 \times 4)\) represents the covariance of the Gaussian shocks. The matrix \(f_S(\nu)\) (the scalar \(S_2(\nu)\)) is a non-linear function of \(\nu\), involving the feedback matrix and the scale parameters of the gamma-distributed variables, see the Online Appendix for more details.

### 2.3 Asset Pricing

In this section, we present the model solutions. First, we formally define the real and nominal pricing kernel as a function of the previously defined state variables. Assuming complete markets, this kernel prices any cash flow pattern spanned by our state variable dynamics. Second, asset prices for two risky assets—defaultable corporate bonds and equities—are derived. The solution of the model shows that asset prices are (quasi) affine functions of the state variables, which is crucial in developing the estimation procedure in this paper. In particular, we derive

---

\(\text{Previous research by Bekaert, Engstrom, and Xing (2009) and Bekaert and Engstrom (2017) also combines time variation in economic uncertainty with changes in risk aversion.}\)
approximate expressions for endogenous returns to use in estimating the model parameters, and deriving a risk appetite index.

2.3.1 The pricing kernel

Taking the ratio of marginal utilities at time $t + 1$ and $t$, we obtain the intertemporal marginal rate of substitution which constitutes the real pricing kernel denoted by $M_{t+1}$. As Equation (6) indicates, it has the same form as the pricing kernel in the Campbell and Cochrane model, however, the kernel state variables and kernel shocks are quite different. Unlike the CC model, changes in the log surplus consumption ratio (the inverse of risk aversion) are not perfectly correlated with the consumption growth shock, and consumption growth is heteroskedastic. The real pricing kernel in our model follows an affine process as well:

$$m_{t+1} = m_0 + m_2' Y_t + m_1' \Sigma \omega_{t+1}, \quad (37)$$

where $m_0$, $m_1$ (10×1), $m_2$ (10×1) are constant scalar or matrices that are implicitly defined using Equations (18)-(23) and (27)-(29). To price nominal assets, we define the nominal pricing kernel, $\tilde{m}_{t+1}$, which is a simple transformation of the log real pricing kernel, $m_{t+1}$,

$$\tilde{m}_{t+1} = m_{t+1} - \pi_{t+1}, \quad (38)$$

$$\tilde{m}_{t+1} = \tilde{m}_0 + \tilde{m}_2' Y_t + \tilde{m}_1' \Sigma \omega_{t+1}, \quad (39)$$

where $\tilde{m}_0$, $\tilde{m}_1$ (10×1) and $\tilde{m}_2$ (10×1) are implicitly defined. The nominal risk free rate, $\tilde{r}_f$, is defined as $- \ln \{ E_t [\exp (\tilde{m}_{t+1})] \}$ which can be expressed as an affine function of the state vector.

2.3.2 Asset prices

In this section, we further discuss the pricing of the two risky assets—corporate bonds and equities. The Online Appendix contains detailed proofs and derivations.

Defaultable Nominal Bonds Above, we assume that a one period nominal bond faces a fractional (logarithmic) loss of $l_t$. Given the structure assumed for $l_t$ and Equation (34), the log
price-coupon ratio of the one-period defaultable bond portfolio is

\[ pc_t^1 = \ln \{ E_t [\exp (\tilde{m}_{t+1} - l_{t+1})]\} \]  
\[ = b_{10} + b_{11}' Y_t, \]  

where \( b_{10} \) and \( b_{11}' \) are implicitly defined. It is straightforward to show that a portfolio of zero-coupon nominally defaultable corporate bonds, maturing in \( N \) periods, has a price that is affine in the state variables. The assumed zero-coupon structure of the payments before maturity implies that the unexpected returns to this portfolio are exactly linearly spanned by the shocks to \( Y_t \).

**Equities** Equity is a claim to the dividend stream. The Online Appendix shows that the price-dividend ratio is the sum of an infinite number of exponential affine functions of the state vector, with the coefficients following simple difference equations.

### 2.3.3 Asset Returns

Given that the log price-coupon ratio of a defaultable corporate bond can be expressed as an exact affine function of the state variables, it immediately implies that the log nominal return (before maturity), \( \tilde{r}_{t+1}^{cb} = pc_{t+1} - pc_t \), can be represented in closed-form. For equities, the log nominal equity return is derived as follows, \( \tilde{r}_{t+1}^{eq} = \ln \left( \frac{PD_{t+1} + D_{t+1} \exp(\pi_{t+1})}{PD_{t} + D_{t}} \right) \). It is therefore a non-linear but known function of the state variables. We approximate this function by a linear function (See the Online Appendix for details). Note that this procedure is very different from the very popular Campbell-Shiller (1988) model which approximates returns with a linear expression. Because they approximate the return expression and then price future cash flows with approximate expected returns, their procedure accumulates pricing errors. We approximate a known quasi-affine pricing function in deriving a return expression.

To account for the approximation error, we allow for two asset-specific homoskedastic shocks that are orthogonal to the state variable innovations. As a result, log nominal asset returns approximately satisfy the following factor model,

\[ \tilde{r}_{t+1}^i = \tilde{\xi}_0^i + \tilde{\xi}_1^i Y_t + \tilde{\rho}^i \tilde{\omega}_{t+1} + \tilde{\epsilon}_{t+1}^i, \]
where $\tilde{r}_{t+1}^i$ is the log nominal asset return $i$ from $t$ to $t+1$, $\forall i = \{eq, cb\}$; $\tilde{\xi}_i^t$ ($10 \times 1$) is the loading vector on the state vector; $\tilde{\mathbf{R}}^i$ ($10 \times 1$) is the loading vector on the state variable shocks, and $\tilde{\epsilon}_{t+1}^i$ is a homoskedastic error term with unconditional volatility $\sigma_i$.

Rather than exploiting the model restrictions on prices, we exploit the restrictions the economy imposes on asset returns, physical variances and risk-neutral variances. Given Equation (42) and the pricing kernel, the model implies that one period expected log excess returns are given by:

$$RP_t^i \equiv E_t(\tilde{r}_{t+1}^i) - rf_t = \left\{ \begin{array}{ll} \sigma_p(\tilde{r}^i) + \ln \left[ \frac{1 - \sigma_p(m_1 + \tilde{r}^i)}{1 - \sigma_p(m_1)} \right] & \text{pt} \\ \sigma_n(\tilde{r}^i) + \ln \left[ \frac{1 - \sigma_n(m_1 + \tilde{r}^i)}{1 - \sigma_n(m_1)} \right] & \text{nt} \\ \sigma_{lp}(\tilde{r}^i) + \ln \left[ \frac{1 - \sigma_{lp}(m_1 + \tilde{r}^i)}{1 - \sigma_{lp}(m_1)} \right] & \text{lp} \\ \sigma_{ln}(\tilde{r}^i) + \ln \left[ \frac{1 - \sigma_{ln}(m_1 + \tilde{r}^i)}{1 - \sigma_{ln}(m_1)} \right] & \text{ln} \\ \sigma_q(\tilde{r}^i) + \ln \left[ \frac{1 - \sigma_q(m_1 + \tilde{r}^i)}{1 - \sigma_q(m_1)} \right] & \text{qt} \\ - m_1' S_1 \Sigma_{\text{other}} S_1' \tilde{r}^i - \frac{1}{2} \left[ \tilde{\mathbf{r}}^i \Sigma_{\text{other}} \tilde{\mathbf{r}}^i + \sigma_i^2 \right] & \end{array} \right.$$  

(43)

Here (as before), $\tilde{\mathbf{m}}_1$ and $\tilde{\mathbf{r}}^i$ are vectors containing the sensitivities of the log nominal pricing kernel and the log nominal asset returns to the state variable shocks, respectively. The symbols $\sigma_p(\mathbf{x})$, $\sigma_n(\mathbf{x})$, $\sigma_{lp}(\mathbf{x})$, $\sigma_{ln}(\mathbf{x})$ and $\sigma_q(\mathbf{x})$ represent linear functions of state variables’ sensitivities to the good uncertainty shock ($\omega_{p,t+1}$), the bad uncertainty shock ($\omega_{n,t+1}$), the right-tail loss rate shock ($\omega_{l,t+1}$), the left-tail loss rate shock ($\omega_{l,t+1}$) and the risk aversion shock ($\omega_{r,t+1}$).

The signs of state variable coefficients are also intuitive. For instance, because $\tilde{\mathbf{m}}_1 = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & -\gamma & -\gamma & 0 & 0 \end{bmatrix}'$ and $\Sigma_{\text{eq}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{qq} \end{bmatrix}'$,

$\tilde{\mathbf{m}}_1 ' \Sigma_{\text{eq}} = \gamma \sigma_{qq} > 0$, where $\gamma > 0$ follows from the concavity of the utility function and $\sigma_{qq} > 0$ implies positive skewness of risk aversion in Equation (29). It immediately implies that an asset with a negative return sensitivity to the risk aversion shock exhibits a higher risk premium when risk aversion is high. That is, for such an asset, $\sigma_q(\tilde{r}^i) < 0$; then, it can be easily shown that

$$\sigma_q(\tilde{r}^i) + \ln \left[ \frac{1 - \sigma_q(m_1 + \tilde{r}^i)}{1 - \sigma_q(m_1)} \right] \approx \sigma_q(\tilde{r}^i) - \frac{\sigma_q(\tilde{r}^i)}{1 - \sigma_q(m_1)} > 0.$$  

Under realistic parameter values, expected

---

8Matrix $\Sigma_{eq}$ is the j-th column of the shock coefficient matrix in the state variable process, or $\Sigma$ in Equation (35).
excess returns thus vary through time and are affine in $p_t, n_t, l_{pt}$ (macroeconomic and cash flow uncertainties) and $q_t$ (aggregate risk aversion).

The physical conditional return variance is obtained given the return loadings of Equation (42):

$$\text{VAR}_t \equiv \text{VAR}_t (\tilde{r}_{t+1}) = \left( \sigma_p(\tilde{r}_t) \right)^2 p_t + \left( \sigma_n(\tilde{r}_t) \right)^2 n_t + \left( \sigma_{lp}(\tilde{r}_t) \right)^2 l_{pt} + \left( \sigma_q(\tilde{r}_t) \right)^2 q_t \tag{44}$$

where $S_1$ is defined in the Online Appendix. The expected variance under the physical measure is time-varying and affine in $p_t, n_t, l_{pt}$ and $q_t$.

The one-period risk-neutral conditional return variance is:

$$\text{VAR}_t^{Q} \equiv \text{VAR}_t^{Q} (\tilde{r}_{t+1}) = \left( \sigma_p(\tilde{r}_t) \right)^2 p_t + \left( \sigma_n(\tilde{r}_t) \right)^2 n_t + \left( \sigma_{lp}(\tilde{r}_t) \right)^2 l_{pt} \tag{45}$$

Note that the functions in Equation (45) are affine transformations from the ones in Equation (44), using the “$\sigma(m)$” functions. Under normal circumstances, we would expect that the relative importance of “bad” uncertainty, the loss rate’s uncertainty and risk aversion increases under the risk neutral measure relative to the importance of “good” uncertainty. In Equation (45), this intuition can be formally established as $\sigma_n(m), \sigma_{lp}(m), \sigma_q(m)$ are positive and $\sigma_p(m)$ is negative. For example, as derived above, $\sigma_q(\tilde{m}_1) = \gamma \sigma_{qq}$ is strictly positive. Therefore, because $\sigma_{qq}$ is likely a small number, as long as $\gamma$ is not very large, the risk neutral variance should load more heavily on $q_t$ than does the physical variance.
3 The Identification and Estimation of Risk Aversion and Uncertainty

In this section, we first describe our general estimation strategy which is focused on retrieving a risk aversion process that can be measured at high frequencies, and then outline the estimation methodology in detail, which consists of two steps. The first step is the identification of macro-economic and cash flow uncertainties; the second step is the actual estimation of the remainder of the model parameters and the identification of risk aversion.

3.1 General Estimation Philosophy

While there are 10 state variables in the model, there are only four latent state variables that drive risk premiums and conditional physical and risk neutral variances in the model as described in Equations (43)–(45). Three of these state variables, good uncertainty, \( p_t \), bad uncertainty, \( n_t \) and cash flow uncertainty, \( l p_t \), describe economic uncertainty. We want to ensure that these variables are identified from macro-economic and cash flow information alone and are not contaminated by asset prices. We therefore pre-estimate these variables. This constitutes the first step in the estimation methodology.

After identifying the dynamics of these variables, there are a variety of ways that we can retrieve risk aversion from the model and data on corporate bonds and equities. However, an important goal of the paper is to make risk aversion observable, even at high frequencies. Under the null of the model, asset prices, risk premiums and variances are an exact function of the state variables, including risk aversion. It thus follows that risk aversion should be spanned by a set of asset prices and risk variables. Given our desire to generate a high frequency risk aversion index, we select these instruments to be observable at high frequencies and to reflect risk and return information for our two asset classes. In particular, we assume

\[ q_t = \chi' z_t, \]  

(46)

where \( z_t \) is a vector of 6 observed asset prices (and a vector of ones), including (1) the credit spread, (2) a “detrended” dividend yield or earnings yield, (3) the realized equity return variance, (4) the risk-neutral equity return variance, (5) the realized corporate bond return variance, and
(6) term spread.

The term spread may reflect information about the macro-economy (see e.g. Harvey, 1988) and was also included in the risk appetite index of Bekaert and Hoerova (2016). The credit spread and price yields contain direct price information from the corporate bond and equity market respectively and thus reflect partially information about risk premiums. Ideally, we would include information on both risk-neutral and physical variances for both equities and corporate bonds, but we do not have data on the risk neutral corporate bond return variance. We use the realized variance for both markets, rather than an estimate of the physical conditional variance, because realized variances are effectively observed, whereas conditional variances must be estimated. Given a loading vector $\chi$, the risk aversion process can be computed daily from observable data.

So far, the methodology is reminiscent of the FAVAR literature (see Bernanke, Boivin, and Eliaasz, 2005) and Stock and Watson (2002), where unobserved macro-factors are identified using large date sets of observable macro-data using a spanning assumption. However, in contrast to the above literature and all “principal component” type analysis, we exploit the restrictions the economy imposes on risk premiums, and physical and risk neutral variances to estimate the loadings of the time-varying risk aversion process. That is, our risk aversion estimate is forced to satisfy the properties of risk aversion implied by the above model: it is an element of the pricing kernel, which must, in turn, correctly price asset returns and be consistent with observed measures of return volatility under both the physical and risk-neutral measures. To do so, we adopt a GMM procedure detailed in Section 3.3.

3.2 Identifying Economic and Cash Flow Uncertainties

Given that there is no feedback from risk aversion to the three uncertainty state variables, we can pre-estimate the uncertainty factors without using financial asset prices.

First, we use the monthly log real growth rate of industrial production to measure $\theta_t$. In the system for $\theta_t$, described in Equations (8)–(13), there are three state variables, which we

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Footnote: Imposing the model restrictions and no arbitrage through a positive pricing kernel also differentiates the estimation from the approach taken in Bekaert and Hoerova (2016).
collect in $Y_t^{mac}$:

$$Y_t^{mac} = \left[ \theta_t, \hat{p}_t, \hat{n}_t \right]'$$

We denote the filtered shocks,

$$\omega_t^{mac} = \left[ \hat{\omega}_{p,t}, \hat{\omega}_{n,t} \right]'$$

The system is estimated using Bates (2006)’s approximate MLE procedure (see the Online Appendix for details).

Second, we must determine the latent cash flow uncertainty factor $l_{p,t}$, which determines the time variation in the conditional variance of the log corporate bond default rate. The dynamics of the variables are described in Equations (14)–(18). We use Bates’ approximate MLE to estimate the model parameters. Unlike the BEGE structure for real shocks, for the idiosyncratic loss rate shock, only the right-tail shock (i.e., the adverse tail) is heteroskedastic. We denote the estimated right-tail loss rate shape parameter as $\hat{l}_{p,t}$, and the loss rate shocks as $\hat{\omega}_{l_{p,t+1}}$ and $\hat{\omega}_{l_{n,t+1}}$.

3.3 Identifying Risk Aversion

To identify the risk aversion process and the parameters in the spanning condition (Equation (46) above), we exploit the restrictions the model imposes on return risk premiums (equities and corporate bonds), physical conditional variances (equities and corporate bonds) and risk neutral variances (for equities only). The estimation is a GMM system in which we use instruments the same variables that are used to span risk aversion ($z_t$). Apart from the $\chi$ parameters, we must also identify the parameters in the kernel ($\beta$, the discount factor, and $\gamma$, the curvature parameter), and the scale parameter of the preference shock, $\sigma_{qq}$. Note that the level of risk aversion is also driven by the $q_t$ process, so that $\gamma$ and $\beta$ are not well identified. We impose $\gamma = 2$ and $\beta = 0.999$. The GMM system thus has 8 unknown parameters,

$$\Theta = \left[ \chi_0, \chi_{tsprd}, \chi_{esprd}, \chi_{CF5yr}, \chi_{rvareq}, \chi_{qvareq}, \chi_{rvareb}, \sigma_{qq} \right]$$

where the notation is obvious, and $CF5yr$ refers to either a detrended dividend or earnings yield (“DY5yr” or “EY5yr”). Before the moment conditions can be evaluated, we must identify

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10 In the remainder of the paper, a hat superscript is used to indicate estimated variables or matrices.
the state variables and their shocks, the pricing kernel, and the return shocks. The estimation process consists of six steps: for each candidate $\hat{\Theta} = [\hat{\Theta}', \hat{\sigma}_{gq}]$ vector,

1. Identify the implied risk aversion series given the loading choices, $\hat{q}_t = \hat{\chi}'z_t$. We impose a lower boundary of $10^{-8}$ on $q_t$ during the estimation. This is consistent with theory, as $q_t$ is motivated from a habit model ($q_t = \ln(Q_t) = \ln \left( \frac{C_t}{c_t - \bar{H}} \right) > 0$). It is also consistent with the distributional assumption for $q_t$ as the positive shape parameter of the $\omega_q$ shock.\[11\]

2. Identify the state variable levels ($Y_t$) and shocks ($\Sigma_{\omega_{t+1}}$).

The parameters of the state variable processes, $\{\theta_t, g_t, \kappa_t, \eta_t\}$, are pre-determined according to Section 3.2. For the remaining cash flow state variables $\{\pi_t, g_t, \kappa_t, \eta_t\}$, we estimate the parameters in each iteration using simple projections. To identify the risk aversion-specific shock in the risk aversion process, we first project $\hat{q}_{t+1}$ on $\hat{q}_t$, $\hat{p}_t$, $\hat{n}_t$, $\hat{\omega}_{p,t+1}$ and $\hat{\omega}_{n,t+1}$ to obtain the residual term $\hat{u}_t q_{t+1}$, and then divide it by $\hat{\sigma}_{gq}$ to obtain the preference shock $\hat{w}_{q,t+1}$ (see Equations (29)-(31)). Now, given the choice of $\hat{\chi}_t$, a full set of state variables levels, $\hat{Y}_t = \left[ \hat{Y}_t^{mac} \pi_t g_t \kappa_t \eta_t \hat{p}_t \hat{q}_t \right]'$, and the eight independent shocks, $\hat{\omega}_{t+1}$ including $\hat{w}_{q,t+1}$, can be identified.\[12\]

3. Identify the nominal pricing kernel.

Consumption growth in this model is (endogenously) implied by two state variables, real log earnings growth and (changes in) the log consumption-earnings ratio. Given consumption growth (i.e., $g_t + \Delta \kappa_t$), the risk aversion process $\hat{q}_t$, $\gamma$, and $\beta$, the monthly nominal kernel is obtained:

$$\hat{m}_{t+1} = \ln(\beta) - \gamma \Delta \kappa_t + \gamma (\hat{q}_{t+1} - \hat{q}_t) - \pi_{t+1}.$$ 

Constant matrices related to the log nominal kernel—$\tilde{m}_0$, $\tilde{m}_1$, $\tilde{m}_2$ (as in the affine representation of the kernel; see Equation (30))—are implicitly identified.

4. Estimate the return loadings.

In this step, we obtain the loadings of nominal asset returns on the state variable shocks, controlling for time-varying conditional means. Note that there are 8 state variables.

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\[11\] However, for the best model, the minimum $q$ is 0.1436 and the boundary is non-binding.

\[12\] The parameters obtained from this substep are $\pi_0$, $\rho_\pi$, $\rho_\theta$, $\rho_{\theta_\theta}$, $\rho_\eta$, $\sigma_\pi$, $\sigma_\theta$, $\sigma_\eta$, $\sigma_{\pi_\pi}$, $\sigma_{\pi_\theta}$, $\sigma_{\pi_\eta}$, $\sigma_{\theta_\theta}$, $\sigma_{\theta_\eta}$, $\sigma_{\eta_\eta}$, $\sigma_{\pi_\pi}$, $\sigma_{\pi_\theta}$, $\sigma_{\pi_\eta}$, $\sigma_{\theta_\theta}$, $\sigma_{\theta_\eta}$, $\sigma_{\eta_\eta}$, $\sigma_{\pi_\pi}$, $\sigma_{\pi_\theta}$, $\sigma_{\pi_\eta}$, $\sigma_{\theta_\theta}$, $\sigma_{\theta_\eta}$, $\sigma_{\eta_\eta}$, $\sigma_{\pi_\pi}$, $\sigma_{\pi_\theta}$, $\sigma_{\pi_\eta}$, $\sigma_{\theta_\theta}$, $\sigma_{\theta_\eta}$, $\sigma_{\eta_\eta}$, $\sigma_{\pi_\pi}$, $\sigma_{\pi_\theta}$, $\sigma_{\pi_\eta}$, $\sigma_{\theta_\theta}$, $\sigma_{\theta_\eta}$, $\sigma_{\eta_\eta}$, $\sigma_{\pi_\pi}$, $\sigma_{\pi_\theta}$, $\sigma_{\pi_\eta}$, $\sigma_{\theta_\theta}$, $\sigma_{\theta_\eta}$, $\sigma_{\eta_\eta}$, $\sigma_{\pi_\pi}$, $\sigma_{\pi_\theta}$, $\sigma_{\pi_\eta}$, $\sigma_{\theta_\theta}$, $\sigma_{\theta_\eta}$, $\sigma_{\eta_\eta}$, $\sigma_{\pi_\pi}$, $\sigma_{\pi_\theta}$, $\sigma_{\pi_\eta}$, $\sigma_{\theta_\theta}$, $\sigma_{\theta_\eta}$, $\sigma_{\eta_\eta}$, $\sigma_{\pi_\pi}$, $\sigma_{\pi_\theta}$, $\sigma_{\pi_\eta}$, $\sigma_{\theta_\theta}$, $\sigma_{\theta_\eta}$, $\sigma_{\eta_\eta}$, $\sigma_{\pi_\pi}$, $\sigma_{\pi_\theta}$, $\sigma_{\pi_\eta}$, $\sigma_{\theta_\theta}$, $\sigma_{\theta_\eta}$, $\sigma_{\eta_\eta}$, $\sigma_{\pi_\pi}$, $\sigma_{\pi_\theta}$, $\sigma_{\pi_\eta}$, $\sigma_{\theta_\theta}$, $\sigma_{\theta_\eta}$, $\sigma_{\eta_\eta}$, $\sigma_{\pi_\pi}$, $\sigma_{\pi_\theta}$, $\sigma_{\pi_\eta}$, $\sigma_{\theta_\theta}$, $\sigma_{\theta_\eta}$, $\sigma_{\eta_\eta}$, $\sigma_{\pi_\pi}$, $\sigma_{\pi_\theta}$, $\sigma_{\pi_\eta}$, $\sigma_{\theta_\theta}$, $\sigma_{\theta_\eta}$, $\sigma_{\eta_\eta}$, $\sigma_{\pi_\pi}$, $\sigma_{\pi_\theta}$, $\sigma_{\pi_\eta}$, $\sigma_{\theta_\theta}$, $\sigma_{\theta_\eta}$, $\sigma_{\eta_\eta}$, $\sigma_{\pi_\pi}$, $\sigma_{\pi_\theta}$, $\sigma_{\pi_\eta}$, $\sigma_{\theta_\theta}$, $\sigma_{\theta_\eta}$, $\sigma_{\eta_\eta}$, $\sigma_{\pi_\pi}$, $\sigma_{\pi_\theta}$, $\sigma_{\pi_\eta}$, $\sigma_{\theta_\theta}$, $\sigma_{\theta_\eta}$, $\sigma_{\eta_\eta}$, $\sigma_{\pi_\pi}$, $\sigma_{\pi_\theta}$, $\sigma_{\pi_\eta}$, $\sigma_{\theta_\theta}$, $\sigma_{\theta_\eta}$, $\sigma_{\eta_\eta}$, $\sigma_{\pi_\pi}$, $\sigma_{\pi_\theta}$, $\sigma_{\pi_\eta}$, $\sigma_{\theta_\theta}$, $\sigma_{\theta_\eta}$, $\sigma_{\eta_\eta}$, $\sigma_{\pi_\pi}$, $\sigma_{\pi_\theta}$, $\sigma_{\pi_\eta}$, $\sigma_{\theta_\theta}$, $\sigma_{\theta_\eta}$, $\sigma_{\eta_\eta}$,
\{\theta_t, p_t, n_t, \pi_t, g_t, \kappa_t, l_p, q_t\} \text{ affecting the pricing kernel. The remaining state variables,} \\
\{l_t, \eta_t\}, \text{ correspond to cash flow state variables in the corporate bond and equity markets.} \\
\text{We estimate the loadings by simple projections, assuming the asset-specific approximation} \\
\text{shock is homoskedastic:} \\
\tilde{r}_{t+1}^i = \xi_0^i + \xi_1^i \tilde{Y}_t + \tilde{\rho}^i \tilde{\omega}_{t+1} + \varepsilon_{t+1}^i, \quad (47) \\
\text{where } \tilde{r}_{t+1}^i \text{ is the log nominal return for asset } i, \tilde{\Sigma} \text{ and } \tilde{\omega}_{t+1} \text{ are identified previously, and} \\
\varepsilon_{t+1}^i \text{ has mean 0 and variance } \sigma_i^2. \text{ To obtain asset moments, } \tilde{\rho}^i \text{ is the crucial shock loading vector, but we also need } \tilde{\sigma}_t. \\
\text{5. Obtain the model-implied endogenous moments.} \\
\text{We derive three moments for the asset returns: 1) the expected excess return implied by} \\
\text{the model (using the pricing kernel), } R_P^i; \text{ 2) the physical (conditional expected) return} \\
\text{variance, } \text{VAR}^i, \text{ which only depends on the return definition in Equation } (47) \text{ and 3) } \\
\text{the risk neutral conditional variance, } \text{VAR}^i_Q, \text{ which also uses the pricing kernel. The} \\
\text{expressions for these variables are derived in Equations } (43)-(45) \text{ where } p_t, n_t, l_p, q_t, \tilde{r}^i, \\
\tilde{\Sigma}^{\text{other}} \text{ and } \sigma_i \text{ have been estimated in previous steps.} \\
\text{6. Obtain the moment conditions } \varepsilon(\Theta; \Psi_t). \text{ Given data on asset returns and options, we use} \\
\text{the derived moments to define 7 error terms that can be used to create GMM orthogonality} \\
\text{conditions. There are three types of errors we use in the system. First, neither risk} \\
\text{ premiums nor physical conditional variances are observed in the data, but we use the} \\
\text{restriction that the observed returns/realized variances minus their expectations under} \\
\text{the null of the model ought to have a conditional mean of zero:} \\
\varepsilon_1(\Theta; \Psi_t) = \begin{bmatrix} \left( \tilde{r}_{t+1}^{eq} - \tilde{r}_{ft} \right) - \tilde{R}_{P_t}^{eq} \\
\text{\text{RVAR}_{t+1}^{eq} - \text{VAR}_{t}^{eq}} \\
\text{\text{\left( \tilde{r}_{t+1}^{cb} - \tilde{r}_{ft} \right) - \tilde{R}_{P_t}^{cb}} \\
\text{\text{\text{RVAR}_{t+1}^{cb} - \text{VAR}_{t}^{cb}} \\
\end{bmatrix}, \quad (48) \\
\text{where } \tilde{r}_{t+1}^i \text{ is the realized nominal return from } t \text{ to } t + 1, r_{ft} \text{ is the risk free rate, and}
RVAR_{t+1}^r is the realized nominal variance from t to t + 1 defined as the sum of the squares of the log high-frequency returns from t to t + 1 (see the Data section for details). Here \(\Psi_t\) denotes the information set at time t. The risk neutral variance can be measured from options data (see e.g. Bakshi and Madan, 2000; Bakshi, Kapadia, and Madan, 2003), and so we use the error:

\[
\varepsilon_2(\Theta; \Psi_t) = \left[ QVAR^eq_t - \widehat{VAR}^{eq,Q}_t \right], \tag{49}
\]

where \(QVAR^eq_t\) is the ex-ante risk-neutral variance of \(r^eq_{t+1}\) calculated from the data. We assume that \(\varepsilon_2(\Theta; \Psi_t)\) reflects model and measurement error, orthogonal to \(\Psi_t\). Finally, we also construct two moment conditions to identify \(\sigma_{qq}\), exploiting the model dynamics for \(u^q_{t+1}\) (i.e., the shock to the risk aversion process as in Equation (29)):

\[
\varepsilon_3(\Theta; \Psi_t) = \begin{bmatrix}
(\hat{u}^q_{t+1})^2 - (\hat{\sigma}_{qq})^2 \hat{q}_t \\
(\hat{u}^q_{t+1})^3 - 2(\hat{\sigma}_{qq})^3 \hat{q}_t
\end{bmatrix} \tag{50}
\]

Let \(\varepsilon_{1,2}(\Theta; \Psi_t) = \begin{bmatrix} \varepsilon_1(\Theta; \Psi_t)' & \varepsilon_2(\Theta; \Psi_t) \end{bmatrix} \). Under our assumptions these errors are mean zero given the information set, \(\Psi_t\). We can therefore use them to create the usual GMM moment conditions. Given our previously defined set of instruments, \(z_t\) (7 \times 1, including a vector of 1’s), we define the moment conditions as:

\[
E [g_t(\Theta; \Psi_t, z_t)] = E \left[ \begin{bmatrix} \varepsilon_{1,2}(\Theta; \Psi_t) \otimes z_t \\ \varepsilon_3(\Theta; \Psi_t) \end{bmatrix} \right]_{5 \times 1} = 0_{7 \times 1}. \tag{51}
\]

Note that to keep the set of moment conditions manageable, we only use two moment conditions for the identification of \(\sigma_{qq}\). Denote \(g_t(\Theta; \Psi_t, z_t)\) (37 \times 1) as the vector of errors at time t, and \(g_T(\Theta; \Psi, z)\) (37 \times 1) the sample mean of \(g_t(\Theta; \Psi_t, z_t)\) from \(t = 1\) to \(t = T\). Then, the GMM objective function is,

\[
J(\Theta; \Psi, z) \equiv T g_T'(\Theta; \Psi, z) W g_T(\Theta; \Psi, z),
\]

26
where $W$ is the weighting matrix. We use the standard GMM procedure, first using an identity weighting matrix, yielding a first stage set of parameters $\hat{\Theta}_1$. We then compute the usual optimal weighting matrix as the inverse of the spectral density at frequency zero of the orthogonality conditions, $\hat{S}_1$, using 5 Newey-West (1987) lags:

$$
\hat{S}_1 = \sum_{j=-5}^{j=5} \frac{5-|j|}{5} E[g_t(\hat{\Theta}_1; \Psi_t, z_t)g_{t-j}(\hat{\Theta}_1; \Psi_{t-1}, z_{t-1})'].
$$

(52)

Then, the inverse of $\hat{S}$ is shrunk towards the identity matrix with a shrinkage parameter of 0.1 in obtaining the second-step weight matrix $W_2$:

$$
W_2 = 0.9\hat{S}^{-1} + 0.1I_{37 \times 37},
$$

(53)

where $I_{37 \times 37}$ is a identity matrix of dimension $37 \times 37$. This gives rise to a second-round $\hat{\Theta}_2$ estimator. To ensure that poor first round estimates do not affect the estimation, we conduct one more iteration with shrinkage, compute $\hat{S}_2(\hat{\Theta}_2)$, and produce a third-round GMM estimator, $\hat{\Theta}_3$. Lastly, the asymptotic distribution for the third-step GMM estimation parameter is, $\sqrt{T}(\hat{\Theta}_3 - \Theta_0) \rightarrow_d N(0, \text{Avar}(\hat{\Theta}_3))$, where $\text{Avar}(\hat{\Theta}_3) = (G_T(\hat{\Theta}_3)\hat{S}_2^{-1}G_T(\hat{\Theta}_3))^{-1}$ and where $G_T$ denotes the gradient of $g_T$.

Because the estimation involves several steps and is quite non-linear in the parameters, we increase the chance of finding the true global optimum by starting from 24,000 different starting values for $\hat{\chi}$ drawn randomly from a large set of possible starting values for each parameter. The global optimum is defined as the parameter estimates generating the lowest minimum objective function value.

4 Data

Because we combine macro and cash flow data to estimate the dynamics of the state variables, with financial data in the GMM estimation, we use the longest data available for the various estimations of the state variable dynamics. The estimation of the macroeconomic uncertainty state variables uses the period from January 1947 to February 2015, and the estimation of the loss rate uncertainty state variable uses data from January 1982 to February 2015. For
the GMM estimation, the sample spans the period from June 1986 to February 2015 (T=345 months). All estimations are conducted at the monthly frequency.

4.1 State variables

Our output variable—delivering three state variables ($\theta_t$, $p_t$ and $n_t$)—is the change in log real industrial production where the monthly real industrial production index is obtained from the Federal Reserve Bank at St. Louis. Inflation ($\pi$), is defined as the change in the log of the consumer price index (CPI) obtained from the Bureau of Labor Statistics (BLS).

The fifth state variable, the log corporate bond loss rate ($l$), as indicated before requires data on default rates and recovery rates for the US corporate bond market. We obtain data on 3-month trailing all-corporate bond default rates and monthly recovery rates spanning November 1980 to February 2015 from the Federal Reserve Board. We use 6 month moving averages of these raw data to compute the log loss rate representative for each month.

The sixth state variable, real earnings growth ($g$), is defined as the change in log real earnings per capita. Real earnings is the product of real earnings per share and the number of shares outstanding during the same month. The seventh state variable, the log consumption-earnings ratio ($\kappa$), uses real per capita consumption and real per capita earnings. Real monthly consumption is defined as the sum of seasonally-adjusted real personal consumption expenditures on nondurable goods and services; as widely recognized in the literature, the consumption deflator is different from the CPI and is computed using monthly data in this paper. The eighth state variable, the log dividend payout ratio ($\eta$), uses the log ratio of real dividends and real earnings. Therefore, consumption growth (dividend growth) is implicitly defined given $g$ and $\kappa$ ($g$ and $\eta$).

The source for the consumption data is the U.S. Bureau of Economic Analysis (BEA). The source for the dividends and earnings data is Robert Shiller’s website. We use the 12-month trailing dividends and earnings, i.e., $E_t^{12} = E_{t-12} + ...E_{t-1}$ where $E_t$ denotes the monthly earnings. There are no true monthly earnings data because almost all firms report earnings results only quarterly. According to Shiller’s website, the monthly dividend and earnings data provided are inferred from the S&P four-quarter totals, which are available since 1926. Calculating 12-month trailing values of earnings and dividends is common practice to control for the strong seasonality.
in the data. Total market shares are obtained from CRSP. To obtain per capita units, we divide real consumption and real earnings by the population numbers provided by BEA.

4.2 Financial Variables

Daily equity returns are the continuously compounded value-weighted nominal market returns with dividends from CRSP. The monthly return ($r^e$) is the sum of daily returns within the same month. To create excess returns, we subtract the one-month Treasury bill rate from the end of the prior month, also from CRSP. We use the square of the month-end VIX index (divided by 120000) as the one-period risk-neutral conditional variance of equity returns ($QVAR^e$) which is obtained from the Chicago Board Options Exchange (CBOE) and is only available from the end of January 1990. We use the VXO index prior to 1990, also from CBOE, going back to June 1986. We construct the monthly one-period physical conditional variance of equity returns ($PVAR^e$) as in Bekaert, Hoerova, and Lo Duca (2013). We calculate the monthly realized variance as the sum of the squared daily equity returns within the same month; then, we project the monthly realized variance onto the lagged risk-neutral variance and the lagged realized variance to obtain the monthly $PVAR^e$.

The daily corporate bond market return is the continuously compounded log change in the daily Dow Jones corporate bond total return index (source: Global Financial Data). The monthly return ($r^{cb}$) is the sum of daily returns within the same month. The conditional variance under the physical measure ($PVAR^{cb}$) is the projection of monthly realized variance onto the lagged realized variance and the lagged credit spread (defined as the difference between the month-end Baa yield and the 10-year zero-coupon Treasury yield).

In some of our work below, we also use data on speculative corporate bonds, which may be more informative about the economic environment than the universe of corporate bonds dominated by investment grade securities (source: FRED, “ICE BofAML US High Yield Total Return Index”). We obtain monthly realized speculative corporate bond return variances using the methodology described above for overall corporate returns. Because the daily index only starts in February 1990, we use an empirical model to fill in the missing data from June 1986 to January 1990.\textsuperscript{13}

\textsuperscript{13}Th exact model is described in the Online Appendix.
In terms of the financial variables that span risk aversion, the credit spread is the difference between Moody’s Baa yield and the 10-year Treasury bond yield. The detrended dividend yield is calculated as the difference between the raw dividend yield and an moving average term that takes the 5 year average of monthly dividend yields, starting one year before, or
\[
DY_{5yr_t} = DY_t - \sum_{i=1}^{60} DY_{t-12-i}
\]
where \(DY_t\) denotes the dividend yield level at time \(t\) (the ratio of 12-month trailing dividends and the equity market price). We create an analogous detrended earnings yield variable using earnings data. The term spread is the difference between the 10-year Treasury yield and the 3-month Treasury yield, where the yield data is obtained from the Federal Reserve Bank of St. Louis.

5 Estimation Results

In this section, we describe the estimation of the state variable processes, and the actual risk aversion process.

5.1 State Variable Dynamics

5.1.1 Macro-economic factors

While we entertained a number of alternative model specifications for industrial production growth, the model described in Section 2.2 was best in terms of the standard BIC criterion. The parameter estimates are reported in Table I. Industrial production growth features slight positive auto-correlation and high realizations of “bad” volatility decrease its conditional mean significantly. The \(p_t\) process is extremely persistent (almost a unit root) and nearly Gaussian, forcing us to fix its unconditional mean at 500 (for such values, the skewness and kurtosis of shocks to \(p_t\) are effectively zero). The shape parameter \(n_t\) has a much lower mean, featuring gamma-distributed shocks \(\omega_{n,t+1}\) with a higher unconditional skewness coefficient of 0.50 (\(\frac{2}{\sqrt{16.14}}\)) and a higher excess kurtosis coefficient of 0.37 (\(\frac{6}{16.14}\)). It is also less persistent than the \(p_t\) process.

We graph the conditional mean of \(\theta_t\) and the \(p_t\) and \(n_t\) process in Figure I together with
NBER recessions. The strong countercyclicality of the $n_t$ process and the procyclicality of the conditional mean of the growth rate of industrial production are apparent from the graph. We also confirmed the cyclicality by running a regression of the three processes (conditional mean, $p_t$, and $n_t$) on a constant and a NBER dummy. The NBER dummy obtains a highly statistically significant positive (negative) coefficient for the $n_t$ (conditional mean) equation. The coefficient is in fact positive in the $p_t$ equation as well, but not statistically significant. In fact, the $n_t$ regression features an adjusted $R^2$ of almost 45%.

The conditional variance of industrial production and its conditional unscaled skewness (see the Online Appendix for a plot) are dominated by $n_t$ and therefore highly countercyclical. Thus, exposure to such macroeconomic uncertainty may render asset prices countercyclical as well.

5.1.2 Cash flow dynamics

As shown in Table II, the loss rate process is persistent with the autocorrelation coefficient close to 0.83. The $p_t$-process does not significantly affect the loss rate process, either through the conditional mean or through shock exposures. However, the $\omega_{n,t}$ shock has a statistically significant effect on the loss rate process; moreover $n_t$ affects the loss rate’s conditional mean with a statistically significant positive coefficient. The time-varying part of the conditional variance, $lp_t$, is persistent with an autoregressive coefficient of 0.86. The idiosyncratic shocks to the loss rate process also exhibit substantial excess kurtosis (unconditional kurtosis = 1.15) and positive skewness (unconditional skewness = 0.90). The gamma shock generating negative skewness, which has a time-invariant shape parameter, is nearly Gaussian, with the shape parameter exceeding 100, so that while it contributes to the variance of the loss rate, there is no meaningful negative skewness associated with this shock.
In Figure II we first plot the loss rate process $l$. The loss rate clearly spikes around recessions, from an overall average of 0.6% to 2.1% on average in recessions (the maximum value is 5.6% during February 2009). The conditional mean of the loss rate in fact inherits the countercyclicality of the loss rate itself, given the loss rate’s high persistence and it positive dependence on $n_t$. Our model fits the positive skewness of the loss rate process through the positively skewed $u^l$ shocks and the positive dependence on $\omega_t$.

Next, in the second and third plots of Figure II we show the conditional higher-order moments of the loss rate process, including the $lp_t$ process and the total conditional variance. While $lp_t$ is overall countercyclical, it appears to peak a few months after recessions. The conditional variance in the second panel ($Var_t(l_{t+1})$) also appears countercyclical, which is the combined result of a countercyclical $lp_t$ process and a strongly countercyclical $n_t$ process ($\sigma_{ln}$ being positive). In fact, a regression of $lp_t$ on a constant and a NBER dummy, yields a NBER coefficient of 6.78 with a t statistic of 3.03, but the t statistic increases to 8.87 when regressing total variance on the NBER dummy.

We decompose the total conditional variance of the loss rate in its contributions coming from shocks associated with $lp_t$, $p_t$ and $n_t$ in the fourth plot of Figure II. The dominant sources of variation are $lp_t$ (accounting for 29% of the total variance on average) and $n_t$ (accounting for 40%). The relative importance of $lp_t$ drops slightly in recessions while that of $n_t$ increases. It tends to peak when the economy starts recovering, reaching as high as 93%. The $p_t$ process has a negligible effect on the loss rate variance. Clearly, the loss rate variance has substantial independent variation not spanned by macro-economic uncertainty.

With the loss rate process estimated, the dynamics of the other cash flow state variables (earnings growth, the consumption-earnings ratio and the payout ratio) follow straightforwardly. We simply use linear projections of the variables onto the previously identified state variables and shocks. The results are reported in Table III. Earnings growth is less persistent than the two equity yield variables, but loads positively and significantly on industrial production growth. The $n_t$ state variable has a positive effect on the conditional mean of the consumption-earnings.
and dividend-earnings ratio, indicating that in recessions these ratios are expected to be larger than in normal times. This makes economic sense as consumption and dividends are likely smoothed over the cycle whereas earnings are particularly cycle sensitive (see also Longstaff and Piazzesi, 2004). Yet, the cyclicality of earnings growth does not show through a significant effect of $n_t$ but rather appears through its positive dependence on industrial production growth directly and its negative dependence on the loss rate. Again, the ratio variables load significantly, but positively on the loss rate. The same intuition explains why the ratio variables load positively on $\omega_n$ shocks and earnings growth loads negatively on this shock. The $\omega_p$ and $\omega_{lp}$ shocks do not have a significant effect on these state variables. The $\omega_{ln}$ shock, which has a negative effect on loss rates, somewhat surprisingly affects the ratio variables positively and significantly.

The projections implicitly define the residuals shock for the cash flow variables, which we found to be homoskedastic (in unreported results). Table III indicates that these shocks still feature substantial and significant variability. We do not impose any correlation structure on these shocks, and Table IV shows that they are quite correlated. Essentially, because earnings growth is quite variable, the ratio variables are positively correlated with one another and negatively correlated with earnings growth. When pricing assets with the model, this correlation structure must be accounted for (see below). The correlations with the other state variable shocks and between these state variable shocks ($\omega_p$, $\omega_n$, $\omega_{lp}$, $\omega_{ln}$) ought to be zero in theory and the table shows that they are economically indeed close to zero.

5.2 Risk Aversion

Recall that we assume risk aversion to be spanned by 6 financial instruments. We report some properties of these financial instruments in the Online Appendix and offer a summary here. First, all of the instruments are highly persistent. This high correlation is the main reason we use a stochastically detrended dividend (AR(1)=0.982) or earnings yield (AR(1)=0.984) series rather than the actual dividend or earnings yield series. Second, the various instruments are positively

\[14\text{The dividend yield shows a secular decline over part of the sample that induces much autocorrelation. This decline is likely due to the introduction of a tax policy favoring repurchases rather than dividends as a means of} \]
correlated but the correlations never exceed 85%. Perhaps surprisingly, the term spread is also positively correlated with the other instruments, even though it is generally believed that high term spreads indicate good times, whereas the yield and variance instruments would tend to be high in bad times. Third, 4 of the instruments show significant positive skewness. This is consistent with our assumptions that the risk aversion dynamics are positively skewed through its gamma distributed shock (see Equation (31)), and we need the linear spanning model to be consistent with the assumed dynamics for risk aversion. The term spread, and earnings yields are significantly negatively skewed so that a negative weight on one of them could also induce positive skewness in risk aversion, but their skewness coefficients are much smaller in magnitude.

Insert Table V

Table V reports the reduced form estimates in the spanning relation. The system estimates 8 parameters with 37 moment conditions. The test of the over-identifying restrictions fails to reject at the 5% level but we investigate the fit of the model along various dimensions in more detail later. The significant determinants of the risk aversion process are the credit spread, realized equity return and corporate bond return variances and the equity return risk neutral variance. The positive coefficient on the risk neutral and the negative coefficient on the physical realized equity return variances is consistent with the idea that the variance risk premium may be quite informative about risk aversion in financial markets (see also Bekaert and Hoerova, 2016). To translate the coefficients into statistics of economic importance, we also report a variance decomposition, reporting the ratio of the covariance of the estimated beta times the instrument with risk aversion over the variance of risk aversion (these statistics sum up to 100%). Jointly the realized and risk neutral variance account for 85% of the total risk aversion variation; the credit spread and the realized corporate bond variance for 15%. The implied risk aversion process shows a 0.40 correlation with the NBER indicator and is thus highly counter-cyclical.\footnote{We also estimated the system using the detrended dividend yield instead of the earnings yield, which produces very similar results. The two resulting $q_t$ processes are 98% correlated.}
In Table VI, we estimate the dynamic properties of the risk aversion process according to Equation (29). All the parameters are estimated by OLS, except for the $\sigma_{qq}$ parameter, which is delivered by the GMM estimation (see Section 3.3). The process shows moderate persistence (an autocorrelation coefficient of 0.67), and the conditional mean shows significant positive loadings on $q_t$ and $n_t$ which account for 80% and 19% of the variation in the conditional mean, respectively. Risk aversion shocks do not load significantly on the macro-economic uncertainty shocks and therefore most of their variation is driven by the risk aversion specific shock. It appears that economic models that impose a very tight link between aggregate fundamentals and risk aversion, such as pure habit models (Campbell and Cochrane, 1999) are missing important variation in actual risk aversion. In addition, risk aversion is much less persistent than the risk aversion implied by these models; the autocorrelation coefficient of the surplus ratio process in the CC model is 0.99 at the monthly level; the first-order autocorrelation coefficient of $q_t$ derived in this paper is 0.67.

While the test of the over-identifying restrictions fails to reject, Table VII examines in more detail how well the estimated dynamic system fits critical asset price moments in the data. The moments are reported in monthly units; for example, the monthly equity premium produced by the model is 78.5 basis points. All model moments are within two standard errors of the data moments and most are within one standard error of the data moment. The model over-estimates the equity premium but is still close to within one standard error of the data moment. The corporate bond risk premium is only 4 basis points higher than the data moment. The model implied variance moments are all quite close to their empirical counterparts. Finally,

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16 Bootstrapped standard errors for the five asset price moments (equity risk premium, equity physical variance, equity risk-neutral variance, corporate bond risk premium, and corporate bond physical variance) use different block sizes to accommodate different serial auto-correlations, to ensure that the sampled blocks are approximately i.i.d.. In particular, Politis and Romano (1995) (and later discussed in Politis and White, 2004) suggest looking for the smallest integer after which the correlogram appears negligible, where the significance of the autocorrelation estimates is tested using the Ljung-Box Q Test (Ljung and Box, 1978).
the table also reports the model-implied variance and unscaled skewness of the risk aversion innovation, \( \sigma_{qq}^2 q_t \) and \( 2\sigma_{qq}^3 q_t \) (respectively).

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**Figure III** graphs the empirical and model implied risk neutral variance. While the model fails to match the distinct spikes of the VIX in several crisis periods, the fit is remarkably good, with the correlation between the two series being 90.06%.

### 6 Risk Aversion, Uncertainty and Asset Prices

In this section, we first characterize the link between risk aversion and macroeconomic uncertainty, and asset prices. We compare the time variation in risk aversion and macroeconomic uncertainty and document how our measures correlate with extant measures of uncertainty and risk aversion.

#### 6.1 Risk Aversion, Macro-Economic Uncertainty and the First and Second Moments of Asset Returns

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**Figure IV** graphs the risk aversion process (black line), which in our model is:

\[
ra_t^{BEX} = \gamma \exp(q_t).
\]

The weak countercyclicality of the process is immediately apparent with risk aversion spiking in all three recessions, but also showing distinct peaks in other periods. The highest risk aversion of 6.31 is reached at the end of February in 2009, at the height of the Great Recession. However, the risk aversion process also peaks in the October 1987 crash, the August 1998 crisis (Russia default and LTCM collapse), after the TMT bull market ended in August 2002 and in August 2011 (Euro area debt crisis).
How important is risk aversion for asset prices? In this article’s model, the priced state variables for risk premiums and variances are those entering the conditional covariance between asset returns and the pricing kernel and therefore are limited to risk aversion \( q_t \), the macro-economic uncertainty state variables, \( p_t \) and \( n_t \) and the loss rate variability \( lp_t \). In Table VIII, we report the loadings of risk premiums and variances on the 4 state variables. To help interpret these coefficients, we scaled the projection coefficients by the standard deviation of the state variables so that they can be interpreted as the response to a one standard deviation move in the state variable. For the equity premium, by far the most important state variable is \( q_t \), which has an effect more than 10 times larger than that of \( n_t \). The effects of \( p_t \) and \( lp_t \) are trivially small. The economic effect of a one standard deviation change in \( q_t \) is large representing 52 basis points at the monthly level (this is a bit lower than the average monthly equity premium).

For the corporate bond premium, \( n_t \) and \( q_t \) are again the most important state variables. A one standard deviation increase in \( n_t \) increases the monthly corporate bond risk premium by 5 basis points, representing more than 10% of the average monthly premium. The effect of \( q_t \) is about twice as large as the effect of \( n_t \).

The coefficients for variances are somewhat harder to interpret, but \( n_t \) and \( q_t \) remain the most important state variables with the former (latter) more important for corporate bond (equity) variances. The one variable for which \( q_t \) is only the third most important variable is the corporate bond physical variance, which reacts more strongly to \( n_t \), and \( lp_t \). Recall that \( lp_t \) measures the idiosyncratic component in corporate loss rates but that loss rates load very significantly on our business cycle variable.

Because the relationship between asset prices and state variables is affine, we also compute a variance decomposition. That is, we compute, for \( x \in \{p, n, v, q\} \), coefficient on \( x_t \times \frac{\text{Cov}(x_t, \text{Mom}_t)}{\text{Var}(\text{Mom}_t)} \) where \( \text{Mom} \) represents an asset price moment like the equity risk premium, or corporate bond physical variance. These variance proportions add up to one. In the model, 96% of the equity risk premium’s variance is driven by risk aversion; only 72.5% of the corporate bond risk premium is driven by risk aversion, while more than 27% is accounted for by
“bad” macro-economic uncertainty. The physical equity variance is predominantly driven by risk aversion (69%) while 89% of the corporate bond return’s physical variance is driven by bad macroeconomic uncertainty. Nevertheless, macro-economic uncertainty also accounts for 31% of the variance of the physical equity variance. It would be logical that the risk neutral variance would load more on risk aversion and less on macroeconomic uncertainty than the physical variance and this is indeed the case, with risk aversion accounting for 85% of the variance of the risk neutral variance.

Bekaert, Hoerova and Lo Duca (2013) argue that the variance risk premium houses much information about risk aversion. Is this true in our model? To answer this question, we compute the model-implied variance risk premium as the difference between the risk neutral variance and the physical variance. A projection on the 4 state variables reveals that 96.8% of the variance of the variance risk premium is accounted for by risk aversion. Conversely, regressing risk aversion on the variance premium, the coefficient is 160.40 with a t-stat of 93.37, and the $R^2$ is 96.2%. Through the lens of our model, the variance premium is clearly a good proxy for risk aversion. However, they are not identical. As Cheng (2019) discusses in detail, the estimates of the variance risk premium occasionally go negative but our risk aversion process never does.

Finally, because the state variables perfectly explain conditional first and second moments of asset returns in the model, they should help predict realized returns and variances in the data. We test this in Table IX. We regress realized monthly excess returns and variances in both the equity and corporate bond markets on 1) the 4 state variables, or 2) the model-implied conditional moment (either the conditional risk premium or the conditional variance). Not imposing the model restrictions on how the state variables combine to the model implied conditional moment, slightly decreases the adjusted $R^2$, but for equity returns the $R^2$ decreases strongly from 5.5% to 0.1%. In this case, the coefficient on the model implied moment is about 0.61 and not significantly different from 1, but it is significantly different from zero. For corporate bond risk premiums, the coefficient is only borderline significantly different from zero. Not surprisingly, the $R^2$’s are higher for the realized equity and corporate bond variances hovering
around 20–25%. The coefficients on the model-implied conditional moments are higher than 1 in this case.

When investigating the coefficients of individual state variables, risk aversion significantly predicts equity returns and both realized variances. Bad economic uncertainty predicts the realized variances and equity returns, the earlier with a negative sign. The latter result is reminiscent of a result in Bekaert and Hoerova (2014). They show that the variance risk premium predicts stock returns with a positive sign whereas uncertainty, as measured by an estimate of the conditional variance in the stock market, predicts stock returns with a negative sign. As \( n_t \) is highly correlated with the stock market physical variance and \( q_t \) with the variance risk premium, this result is therefore no surprise. The loss rate variance has an intuitive positive coefficient in the variance and corporate bond return regressions but is not statistically significantly different from zero. However, it predicts equity returns with a negative sign, with the coefficient significant at the 10% level. The last line reports correlations of the model-implied moments with the NBER recession indicator, showing all of them to be significantly countercyclical.

Given the vast literature on return predictability, it is informative to contrast the predictive power of our model implied premiums with the predictive power of the usual instruments used in the literature. We do this exercise out of sample as the literature has shown huge biases due to in sample over-fitting (Goyal and Welch, 2008). Our model premium candidates are either derived from a projection of excess returns on the 4 state variables (Model 2) or the actual model-implied risk premium (Model 1). We consider three empirical models, depending on the instruments used: 1) earning yield, 2) earnings yield, term spread and credit spread, 3) physical uncertainty and variance risk premium estimate. For equity (corporate bond) returns, we use the physical uncertainty derived from equity (corporate bond) returns. We then generate out-of-sample predictions for the risk premiums according to the various empirical models by starting the sample after five years of data and then running rolling samples to generate predictions from the five-year point to one month before the end of the sample. For the model implied risk premiums, Model 1 uses whatever the model predicts the premium to be. For Model 2, the projections are also conducted in a rolling fashion, but note that the construction of the state variables uses information from the full sample. With those competing risk premium estimates
in hand, we then run simple horse races over the full sample by estimating:

\[ \tilde{r}_{t+1} - r_f = a \text{ Mod}(t, i) + (1 - a) \text{ Emp Mod}(t, j) + \epsilon_{t+1}, \text{ for } i = 1, 2, j = 1, 2, 3. \]  

(55)

The results are reported in Table X. The table then reports the optimal weight on Mod (1) or Mod (2), with the complement of the weight being on Emp Mod (1), (2), or (3). Using the implied risk premiums form the model (Model 1) clearly outperforms the empirical model for equity returns with the “a”-coefficients being well over 0.50, varying between 0.73 and 0.85. For corporate bond returns, the “a”-coefficient varies between 0.45 and 0.56, with all “a”-coefficients highly statistically different from zero. Thus, while the data suggest that the model implied risk premiums are very useful in forecasting excess corporate bond returns out of sample, the empirical instruments yield information about risk premiums that is equally important.

Model 2 fares less well, with the coefficients only being above 0.5 for the equity return regressions when the model is pitted against empirical models 2 and 3. Yet, the “a”-coefficients remain statistically significant different from zero in all cases except for corporate bond returns when pitted against the first instrument set. The “a”-coefficients vary between 0.36 and 0.56 in the other 5 cases. We conclude that our model seems to capture the predictable variation better, or at least as well, than the fitted values extracted from standard instruments used in the literature. While the model premium is not strictly out of sample, the model imposes numerous restrictions relative to the empirical models.

6.2 Interpreting Economic Uncertainty

An advantage of the risk aversion process we estimated is that because of its dependence on financial instruments it can be computed at even a daily level. In contrast, our filtered macro-economic uncertainty variables were extracted from industrial production which is only available at the monthly level. Here we consider whether we can use the financial instruments to approximate macro uncertainty.
First, total macro-economic uncertainty, the conditional variance of industrial production growth, is a function of both \( p_t \) and \( n_t \), \( \sigma_{\theta p}^2 p_t + \sigma_{\theta n}^2 n_t \), where the coefficient estimates of \( \sigma_{\theta p} \) and \( \sigma_{\theta n} \) are provided in Table I. In Table XI, we show the coefficients from a regression of uncertainty on the financial instruments used to span risk aversion and two additional instruments, the detrended dividend yield and realized variances of speculative bond returns. The \( R^2 \) is almost 50\% and uncertainty loads significantly on all instruments except for the realized equity and speculative bond return variances. Unlike the risk aversion process, uncertainty loads very strongly on both credit spreads and the physical corporate bond variance. The term spread also has a significant negative effect on uncertainty (and no effect on risk aversion). This makes sense as flattening yield curves are associated with future economic downturns. The table also reports regressions from the two components in macro-economic uncertainty, bad and good uncertainty, onto the instruments. Clearly, the variation in macro-economic uncertainty is dominated by the bad component and the coefficients for the bad component projection coefficients are very similar to those of total uncertainty. We also report the results from a variance decomposition applied to the fitted values of the regression. The credit spread explains almost 63\% of the explained uncertainty variation. The dividend and earnings yield variables likely offset one another partially with one contributing a positive, the other a negative amount to the total variation but jointly the equity yield variables still explain close to 20\%. Finally, the risk neutral equity variance and the physical corporate bond return variance each contribute about 13-14\% of the explained variation of uncertainty.

From this analysis, we create an uncertainty index representing the part of total uncertainty that is explained by the financial instruments:

\[
\text{unc}_t^{BEX} = \chi^{\text{unc}} z_t. 
\]  

(56)

In Figure IV, we graph the financial instrument proxies to uncertainty and risk aversion for comparison. The correlation between actual uncertainty and risk aversion is 51\%; when we use the proxy the correlation increases to 70\%. Obviously, most of the time crisis periods feature
both high uncertainty and high risk aversion. There are exceptions however. For example, the October 1987 crash happened during a time of relatively low economic uncertainty. It also appears that at the end of the 90s, macro-uncertainty seems to secularly increase, consistent with the Great Moderation ending around that time (see also Baele et al., 2015).

Bloom (2009) has argued that uncertainty precedes bad economic outcomes. We regress future real industrial production growth at various horizons on our uncertainty index—its financial proxy and the actual one—and the risk aversion process. In addition, we use the VIX as suggested in Bloom (2009). The results are in Table XII. We use Hodrick (1992) standard errors to accommodate the overlap in the data. Panel A shows univariate results. All indices predict growth with a negative sign at the one month, one quarter and one year horizons. Our financial instrument uncertainty index generates the highest $R^2$ by far. This suggests that it is indeed macro uncertainty predicting output growth, with the VIX having much lower predictive power in univariate regressions. The actual macroeconomic uncertainty (Column “unc$^{true}$”) exhibits substantially more predictive power than the VIX (Column “QVAR”), but still substantially less than the combination of financial instruments most correlated with it (Column “unc$^{BEX}$”). This is likely due to the important role played by the credit spread in unc$^{BEX}$; with the credit spread known to predict future economic activity (see De Santis, 2018, and the references therein).

This result is confirmed in multivariate regressions. In Panel B, we use our risk aversion index ($ra^{BEX}$), “financial instruments” uncertainty index ($unc^{BEX}$), and the VIX simultaneously. The “financial instruments” uncertainty index is statistically significant for all horizons, whereas risk aversion (with a positive sign) and the VIX squared (with a negative sign) are significant at the quarterly horizon and beyond. Thus, our uncertainty index dominates the VIX index in terms of its predictive power for real activity. In Panel C, the “financial instruments” uncertainty index is replaced by the actual economic uncertainty. The latter variable is statistically significant at all three horizons. However, the predictive power of the multivariate model drops substantially at all horizons being only 1/4 to 2/3’s of that present in Panel B (by 24% ~
64%). Not surprisingly, Panel D shows that the “financial instruments” uncertainty proxy also dominates actual economic uncertainty ($unc^{true}$), when both are present. The actual economic uncertainty is insignificant at all horizons.

Finally, we also examine the correlation between $lp_t$, the idiosyncratic variance component of corporate bond loss rates, with financial instruments, but the R2 in such a regression is only 9% (see the Online Appendix for detailed results).

### 6.3 Correlations with Extant Measures

In this section, we examine the correlation of our risk aversion and uncertainty indices with existing measures. For risk aversion, we consider three categories: risk aversion indices based on “fundamental” habit models, sentiment indices and commercially available risk aversions indices. For uncertainty, we focus on the recent uncertainty index developed by Jurado, Ludvigson and Ng (2015) and the Baker, Bloom and Davis (2016) political uncertainty measure.

Recall that in an external habit model such as Campbell and Cochrane (1999), the curvature of the utility function is a negative affine function of the log “consumption surplus ratio,” which in turns follows a heteroskedastic autoregressive process with shocks perfectly correlated with consumption growth. We follow Wachter (2006) and create a “fundamental” risk aversion process from consumption data and CC’s parameter estimates, which we denote by $RA_{CC}$.

Table XIII shows that it is only weakly correlated with our risk aversion measure but the correlation is still significantly different from zero. Work by Bekaert, Engstrom and Grenadier (2010) and Martin (2017) also suggests the existence of more variable risk aversion in financial markets, imperfectly correlated with fundamentals.\(^\text{17}\)

The behavioral finance literature suggests that the sentiment of retail investors may drive asset prices and cause non-fundamental price swings. As a well-known representative of this work, we use the sentiment index from Baker and Wurgler (2006). The index is based on the

\(^{17}\text{When computing the same correlations, with our alternative risk aversion index (using the dividend rather than the earnings yield as the equity yield instrument), all the correlations in Table XIII are quite similar (but mostly slightly higher in absolute magnitude).}
first principal component of six (standardized) sentiment proxies including: the closed-end fund discount, the NYSE share turnover, the number and the average first-day returns of IPOs, the share of equity issues in total equity and debt issues, and the dividend premium (the log-difference of the average market-to-book ratios of payers and nonpayers). We denote their index by $Sent_{BW}$. High values mean positive sentiment so we expect a negative correlation with our risk aversion indicator, and indeed the correlation is significantly negative but still relatively small at -0.17.

Because the Baker-Wurgler index relies on financial data, it may not directly reflect the sentiment of retail investors. Lemmon and Portnaiguina (2006) and Qiu and Welch (2006) therefore suggest using a consumer sentiment index such as the Michigan Consumer Sentiment Index (MCSI). The correlation with this index is also negative, as expected, and larger in absolute magnitude at -0.28. Hence, our risk aversion index correlates more with a pure consumer sentiment index than with $Sent_{BW}$, derived from financial variables.

Finally, many financial services companies create their own risk appetite indices. As a well-known example, we obtain data on the Credit Suisse First Boston Risk Appetite Index (RAI). The indicator draws on the correlation between risk appetite and the relative performance of safe assets (proxied by seven to ten-year government bonds) and risky assets (equities and emerging market bonds). The underlying assumption is that an increasing risk preference shifts the demand from less risky investments to assets associated with higher risks, thus pushing their prices up relative to low-risk assets (and vice versa). The indicator is based on a cross-sectional linear regression of excess returns of 64 international stock and bond indices on their risk, approximated by historic volatility. The slope of the regression line represents the risk appetite index. The index shows a -0.48 correlation with our index and is thus highly correlated with our concept of risk aversion.

Our uncertainty measure only uses industrial production data. Jurado, Ludvigson and Ng (2015) use the weighted sum of the conditional volatilities of 132 financial and macroeconomic series, with the bulk of them being macroeconomic. They have three versions of the measure depending on the forecasting horizon, but we focus on the one month horizon, which is most consistent with our model. The correlation with our uncertainty index is highly significant and substantial at 81%.
Macroeconomic uncertainty may be correlated with political uncertainty, which has recently been proposed as a source of asset market risk premiums (Pastor and Veronesi, 2013). Baker, Bloom and Davis (2016) create a policy uncertainty measure, based on newspaper coverage frequency, which we denote by $UC_{BBD}$. The index shows a highly significant correlation of 0.34 with our uncertainty index. One advantage of $UC_{BBD}$ relative to the Jurado, Ludvigson, and Ng (2015) measure is that it can also be computed at the daily frequency. However, our financial proxy to uncertainty can also be computed at the daily frequency.

6.4 Daily Risk Aversion and Uncertainty Indices

Monthly indices may hide important variation within the month in uncertainty and risk aversion. To demonstrate this, Figure V shows how the indices behaved around two critical events in the recent global financial crisis plotting 2-month intervals of our daily risk aversion index (top plots) and our daily financial instrument proxy to macroeconomic uncertainty (bottom plots): the Bear Stearns collapse and bail out and the Lehman Brothers bankruptcy. While there are some local peaks in risk aversion and uncertainty around the Bear Stearns’ collapse on March 14, 2008, the overall effects are relatively small. During August and September 2008, both risk aversion and especially uncertainty gradually increase, with risk aversion rising to around the same level that it reached during the Bear Stearns’ collapse. However, as the systemic nature of the Lehman Brothers bankruptcy became clear to financial market participants, both risk aversion and especially uncertainty continued to rise, and remained relatively elevated. Risk aversion shows two major peaks towards the end of October and the end of November, corresponding to the aftermath of central banks around the world acknowledging the ramifications of the crisis by cutting their interest rates at the end of October and the fall out for car companies at the end of November.
7 Conclusion

We formulate a no-arbitrage model where fundamentals such as industrial production, consumption earnings ratios, corporate loss rates, etc. follow dynamic processes that admit time-variation in both conditional variances and the shape of the shock distribution. The agent in the economy takes this time-varying uncertainty into account when pricing equity and corporate bonds, but also faces preference shocks imperfectly correlated with fundamentals. The state variables in the economy that drive risk premiums and higher order moments of asset prices involve risk aversion, good and bad economic uncertainty and the conditional variance of loss rates on corporate bonds. We use equity and corporate bond returns, physical equity and corporate bond return variances and the risk neutral equity variance to estimate the model parameters and simultaneously derive a risk aversion spanning process. Risk aversion is a function of 6 financial instruments, namely the term spread, credit spread, a detrended earnings yield, realized and risk-neutral equity return variance, and realized corporate bond return variance.

We find that risk aversion loads significantly and positively on the risk neutral equity variance and the realized corporate bond variance, and negatively on the realized equity return variance. Risk aversion is much less persistent than the risk aversion process implied by standard habit models. It is the main driver of the equity premium and the equity return risk neutral variance. It also accounts for 68% of the conditional variance of equity returns with the remainder accounted for by bad macro uncertainty. For corporate bonds, bad economic uncertainty plays a relatively more important role. It accounts for 27% of the risk premium variation and 89% of the corporate bond physical variance. Hence, different asset markets reflect differential information about risk appetite versus economic uncertainty. Our model-implied risk premiums often beat standard predictors of equity and corporate bond returns in an out-of-sample horse race, but their performance for corporate bonds is weaker than for equities.

While our risk aversion measure is correlated with some existing risk appetite and sentiment indices, the simplest approximation may be the variance risk premium in equity markets which is 96% correlated with our risk appetite index.

Because the spanning instruments are financial data, we can track the risk aversion index at higher frequencies. Similarly, we obtain a financial proxy to economic uncertainty (the conditional variance of industrial production growth) which can be obtained at the daily frequency.
as well. This measure is 81% correlated with the well-known Jurado, Ludvigson and Ng (2015) measure, extracted from macro data. The financial proxy to economic uncertainty predicts output growth negatively and significantly and is a much stronger predictor of output growth than is the VIX. We plan to make both our risk aversion and uncertainty indices available on our websites and update them regularly, which could potentially be useful for both academic researchers and practitioners.

Our work has important implications for the dynamic asset pricing literature. Clearly, to provide a unified framework explaining asset return dynamics in different asset classes, both changes in risk aversion and economic uncertainty must be accommodated. In addition, aggregate risk aversion must contain a relatively non-persistent variable component.

Finally, we only used risky asset classes to create the risk appetite index, omitting Treasury bonds, arguably an additional important asset class. In principle, given a process for inflation our model should also price Treasury bonds. In fact, Cremers, Fleckenstein and Gandhi (2017) claim that an implied volatility measure computed from Treasury bonds predicts the level and volatility of macro-economic indicators better than stock market implied indicators do. However, for our purposes, the problem with considering Treasuries as determining general risk aversion is that they are often viewed as the benchmark “safe” assets and are subject to occasional flights-to safety (see Baele et al, forthcoming). This makes it ex-ante unlikely that a simple pricing model such as ours can jointly price the three assets classes. We therefore defer incorporating Treasury bonds to future work.
References


Rey, H. Dilemma not trilemma: the global financial cycle and monetary policy independence.


Table I: The Dynamics of the Macro Factors

This table reports parameter estimates from the model below using the monthly log growth rate of U.S. industrial production from January 1947 to February 2015. The model involves latent processes ("good" gamma shock shape parameter governing positive skewness, \( p_t \), and "bad" gamma shock shape parameter governing negative skewness, \( n_t \)) and is estimated using the MLE-filtration methodology described in Bates (2006).

\[
\begin{align*}
\theta_{t+1} &= \bar{\theta} + \rho_\theta (\theta_t - \bar{\theta}) + m_p (p_t - 500) + m_n (n_t - \bar{n}) + u^\theta_{t+1} \\
p_{t+1} &= 500 + \rho_p (p_t - 500) + \sigma_{pp} \omega_{p,t+1} \\
n_{t+1} &= \bar{n} + \rho_n (n_t - \bar{n}) + \sigma_{nn} \omega_{n,t+1}
\end{align*}
\]

, where

\[
\begin{align*}
  u^\theta_{t+1} &= \sigma_\theta \omega_{p,t+1} - \sigma_\theta \omega_{n,t+1} \\
  \omega_{p,t+1} &\sim \tilde{\Gamma}(p_t, 1) \\
  \omega_{n,t+1} &\sim \tilde{\Gamma}(n_t, 1) \\
  \sigma_{pp} &> 0 \\
  \sigma_{nn} &> 0.
\end{align*}
\]

Other details of the model are discussed in Section 2. Standard errors are displayed in parentheses. Note that the effective loading of \( \theta_{t+1} \) on \( \omega_{n,t+1} \) is \(-\sigma_{\theta n}\). Bold (italic) coefficients have <5% (10%) p-values.

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<th>( n_{t+1} )</th>
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52
Table II: The Dynamics of the Corporate Loss Rate

This table reports parameter estimates for the corporate loss rate model using monthly data from June 1984 to February 2015. The actual nominal payoff of a one-period zero-coupon defaultable (risky) corporate bond at period \( t + 1 \) is \( C \times F_{t+1} \) where \( F_{t+1} < 1 \) is a stochastic fraction of the total promised payment \( C \). The nominal payment can be rewritten as, \( CF_{t+1} = \exp(c + \ln(F_{t+1})) = \exp(c - l_{t+1}) \); equivalently, \( l_{t+1} = -\ln(F_{t+1}) = -\ln(1 - L_{t+1}) \) where \( L_{t+1} \) is corporate loss rate. The empirical proxy for \( L \) is obtained using the following equality, \( \text{DEF}_t \times (1 - \text{RECOV}_t) \). Both the default rate \( \text{DEF}_t \) and recovery rate \( \text{RECOV}_t \) pertain to the overall corporate bond market and are empirically constructed as 6-month trailing measures from 3-month trailing all-corporate bond default rate and monthly recovery rate, respectively; both raw series are obtained from the Federal Reserve Board. The dynamic process of \( l_{t+1} \) is as follows:

\[
\begin{align*}
l_{t+1} &= l_0 + \rho_l l_t + m_{lp} p_t + m_{ln} n_t + \sigma_{lp} \omega_{p,t+1} + \sigma_{ln} \omega_{n,t+1} + u_{t+1} \\
\omega_{p,t+1} &\sim \tilde{\Gamma}(lp_{t}, 1), \\
\omega_{n,t+1} &\sim \tilde{\Gamma}(ln_{t}, 1),
\end{align*}
\]

where the variance equation is,

\[
\begin{align*}
lp_{t+1} &= \bar{lp} + \rho_{lp} (lp_{t} - \bar{lp}) + \sigma_{lp} \omega_{lp,t+1}, \\
\overline{n} &> 0.
\end{align*}
\]

The mean equation is estimated by projection, the variance equation by Bates’ approximate MLE-filtration. Standard errors are displayed in parentheses. Other details of the model are discussed in Section 2. Bold (italic) coefficients have <5% (10%) p-values.

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53
Table III: Cash Flow Dynamics

This table shows the projection results of cash flow dynamics. The dynamic processes of cash flow variables are expressed in Equation (20) for the log earnings growth, $g_{t+1}$, Equation (23) for the log consumption-earnings ratio, $\kappa_{t+1}$, Equation (26) for the log dividend-earnings ratio, $\eta_{t+1}$, and Equation (32) for the inflation rate, $\pi_{t+1}$. These coefficients are estimated using simple linear projections. Robust standard errors are shown in parentheses. The adjusted $R^2$ of the conditional mean part (with information set $t$) is reported in the last row. Bold (italic) coefficients have <5% (10%) p-values. The sample period is 1986/06 to 2015/02 (345 months).

<table>
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<td>0.0037</td>
<td>-0.0001</td>
</tr>
<tr>
<td>(0.0007)</td>
<td>(0.0008)</td>
<td>(0.0008)</td>
<td>(0.0000)</td>
<td></td>
</tr>
<tr>
<td>$l_t$</td>
<td>-1.2518</td>
<td>2.3791</td>
<td>2.150</td>
<td></td>
</tr>
<tr>
<td>(0.7376)</td>
<td>(0.9352)</td>
<td>(0.9596)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$lp_t$</td>
<td>0.0007</td>
<td>-0.0008</td>
<td>-0.0008</td>
<td></td>
</tr>
<tr>
<td>(0.0004)</td>
<td>(0.0004)</td>
<td>(0.0004)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega_{p,t+1}$</td>
<td>-0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0000</td>
</tr>
<tr>
<td>(0.0001)</td>
<td>(0.0001)</td>
<td>(0.0001)</td>
<td>(0.0000)</td>
<td></td>
</tr>
<tr>
<td>$\omega_{n,t+1}$</td>
<td>-0.0033</td>
<td>0.0066</td>
<td>0.0068</td>
<td>0.0001</td>
</tr>
<tr>
<td>(0.0011)</td>
<td>(0.0013)</td>
<td>(0.0014)</td>
<td>(0.0001)</td>
<td></td>
</tr>
<tr>
<td>$\omega_{lp,t+1}$</td>
<td>-0.0005</td>
<td>0.0008</td>
<td>0.0008</td>
<td></td>
</tr>
<tr>
<td>(0.0010)</td>
<td>(0.0013)</td>
<td>(0.0013)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega_{ln,t+1}$</td>
<td>-0.0002</td>
<td>0.0004</td>
<td>0.0004</td>
<td></td>
</tr>
<tr>
<td>(0.0001)</td>
<td>(0.0001)</td>
<td>(0.0002)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gaussian shock volatility</td>
<td>0.0462</td>
<td>0.0558</td>
<td>0.0574</td>
<td>0.0023</td>
</tr>
<tr>
<td>(0.0018)</td>
<td>(0.0021)</td>
<td>(0.0022)</td>
<td>(0.0001)</td>
<td></td>
</tr>
<tr>
<td>Adjusted $R^2$ (conditional mean)</td>
<td>56.76%</td>
<td>98.34%</td>
<td>98.06%</td>
<td>20.85%</td>
</tr>
</tbody>
</table>
Table IV: Filtered Shock Correlation Matrix

The table provides a correlation matrix of the shock structure of the economy. The shocks are summarized as follows:

<table>
<thead>
<tr>
<th>Shock Type</th>
<th>Description</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_{p,t+1}$</td>
<td>good uncertainty shock</td>
<td>$\Gamma(p_t, 1)$;</td>
</tr>
<tr>
<td>$\omega_{n,t+1}$</td>
<td>bad uncertainty shock</td>
<td>$\Gamma(n_t, 1)$;</td>
</tr>
<tr>
<td>$\omega_{lp,t+1}$</td>
<td>cash flow good uncertainty shock</td>
<td>$\Gamma(lp_t, 1)$;</td>
</tr>
<tr>
<td>$\omega_{ln,t+1}$</td>
<td>cash flow bad uncertainty shock</td>
<td>$\Gamma(ln_t, 1)$;</td>
</tr>
<tr>
<td>$\omega_{g,t+1}$</td>
<td>log earnings growth-specific shock</td>
<td>$N(0,1)$;</td>
</tr>
<tr>
<td>$\omega_{κ,t+1}$</td>
<td>log C/E-specific shock</td>
<td>$N(0,1)$;</td>
</tr>
<tr>
<td>$\omega_{η,t+1}$</td>
<td>log D/E-specific shock</td>
<td>$N(0,1)$;</td>
</tr>
<tr>
<td>$\omega_{q,t+1}$</td>
<td>risk aversion-specific shock</td>
<td>$\Gamma(q_t, 1)$;</td>
</tr>
</tbody>
</table>

Bold (italic) coefficients have <5% (10%) p-values. The sample period is 1986/06 to 2015/02 (345 months).

<table>
<thead>
<tr>
<th></th>
<th>$\omega_p$</th>
<th>$\omega_n$</th>
<th>$\omega_p$</th>
<th>$\omega_n$</th>
<th>$\omega_p$</th>
<th>$\omega_n$</th>
<th>$\omega_p$</th>
<th>$\omega_n$</th>
<th>$\omega_p$</th>
<th>$\omega_n$</th>
<th>$\omega_p$</th>
<th>$\omega_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_p$</td>
<td>1</td>
<td>-0.1129</td>
<td>0.0000</td>
<td>-0.0011</td>
<td>-0.0007</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>1</td>
<td>0.0000</td>
<td>-0.0912</td>
<td>-0.0828</td>
<td>-0.0442</td>
<td>0.1060</td>
<td>-0.0120</td>
<td>-0.0536</td>
<td>0.0360</td>
<td>0.0000</td>
<td>0.0000</td>
<td>-0.1789</td>
</tr>
<tr>
<td>$\omega_p$</td>
<td>1</td>
<td>0.0943</td>
<td>-0.0442</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>1</td>
<td>-0.1877</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\omega_p$</td>
<td>1</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>1</td>
<td>0.6765</td>
<td>-0.6765</td>
<td>-0.6589</td>
<td>0.0613</td>
<td>0.0413</td>
<td>0.0351</td>
<td>0.0351</td>
<td>0.0351</td>
<td>0.0351</td>
<td>0.0351</td>
<td>0.0351</td>
</tr>
<tr>
<td>$\omega_p$</td>
<td>1</td>
<td>0.9863</td>
<td>-0.9863</td>
<td>-0.9863</td>
<td>0.0613</td>
<td>0.0413</td>
<td>0.0351</td>
<td>0.0351</td>
<td>0.0351</td>
<td>0.0351</td>
<td>0.0351</td>
<td>0.0351</td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>1</td>
<td>-0.0351</td>
<td>0.0351</td>
<td>0.0351</td>
<td>0.0351</td>
<td>0.0351</td>
<td>0.0351</td>
<td>0.0351</td>
<td>0.0351</td>
<td>0.0351</td>
<td>0.0351</td>
<td>0.0351</td>
</tr>
</tbody>
</table>
Table V: Reduced-Form Risk Aversion Parameters

This table presents the GMM estimation results for risk aversion, \( q_t = \chi'z_t \), using equity market and corporate bond market asset prices. The utility curvature parameter, \( \gamma \), is fixed at 2 in the estimation. The first-step GMM weighting matrix is an identity matrix; the second-step weighting matrix builds on the Newey-West spectral density function with 5-month lags, and then is shrunk towards an identity matrix where the shrinkage parameter is 0.1. The GMM system also consistently estimates \( \sigma_{qq} \). Therefore, the system has 8 unknown parameters. The p-value of Hansen’s over-identification test (\( J \) test) is calculated from the asymptotic \( \chi^2 \) distribution with the degree of freedom being 29 (37-8).

Variance decomposition, or “VARC”, of a linear variable \( z_t \) is defined as \( \beta_z \frac{\text{corr}(q_t,z_t)}{\text{var}(q_t)} \) (the sum=100%). Efficient standard errors are shown in parentheses. Bold (italic) coefficients have <5% (10%) p-values. The sample period is 1986/06 to 2015/02 (345 months).

<table>
<thead>
<tr>
<th>( q_t )</th>
<th>Efficient GMM Estimators</th>
<th>VARC</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.059</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td></td>
</tr>
<tr>
<td>( \chi_{t,spred} )</td>
<td>0.222</td>
<td>0.28%</td>
</tr>
<tr>
<td></td>
<td>(0.511)</td>
<td></td>
</tr>
<tr>
<td>( \chi_{c,spred} )</td>
<td>1.948</td>
<td>5.97%</td>
</tr>
<tr>
<td></td>
<td>(0.564)</td>
<td></td>
</tr>
<tr>
<td>( \chi_{EY5yr} )</td>
<td>0.126</td>
<td>0.26%</td>
</tr>
<tr>
<td></td>
<td>(0.265)</td>
<td></td>
</tr>
<tr>
<td>( \chi_{revarc} )</td>
<td>-20.195</td>
<td>-42.69%</td>
</tr>
<tr>
<td></td>
<td>(0.460)</td>
<td></td>
</tr>
<tr>
<td>( \chi_{rqarreq} )</td>
<td>61.196</td>
<td>126.65%</td>
</tr>
<tr>
<td></td>
<td>(1.551)</td>
<td></td>
</tr>
<tr>
<td>( \chi_{rqarch} )</td>
<td>105.721</td>
<td>9.52%</td>
</tr>
<tr>
<td></td>
<td>(11.500)</td>
<td></td>
</tr>
</tbody>
</table>

Correlation with the NBER Indicator

| \( \rho(q_t,NBER_t) \) | 0.401 |
|                        | (0.045) |

Model Specifications

<table>
<thead>
<tr>
<th></th>
<th>Hansen’s J</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>41.2904</td>
<td>0.0649</td>
</tr>
</tbody>
</table>
Table VI: Structural Risk Aversion Parameters

This table presents the model-implied risk aversion process parameters. In the first panel, parameter estimates are obtained either from simple projection or from the GMM procedure introduced earlier (see Table V). The second and third panels report the variance decomposition results of the conditional mean and shock structure of $\hat{q}_{t+1}$, respectively. In the second panel, VARC of a linear variable $x$ is as follows, $VARC = \beta_x \frac{\text{cov}(\hat{y},x)}{\text{var}(\hat{y})}$, where $\hat{y} = \hat{E}_t(\hat{q}_{t+1})$. VARC in the third panel is calculated using the residual, $\hat{\omega}_{t+1} = \hat{q}_{t+1} - \hat{E}_t(\hat{q}_{t+1})$. Therefore, the sum of VARC in explaining the conditional mean and the shock structure is 100% each. Robust and efficient standard errors are shown in parentheses. Bold (italic) coefficients have <5% (10%) p-values. The sample period is 1986/06 to 2015/02 (345 months).

\begin{align*}
\hat{q}_{t+1} &= q_0 + \rho_{qq}\hat{q}_t + \rho_{qp}\hat{p}_t + \rho_{qn}\hat{n}_t + \sigma_{qp}\hat{\omega}_{p,t+1} + \sigma_{qn}\hat{\omega}_{n,t+1} + u^{q}_{t+1}, \\
u^{q}_{t+1} &= \sigma_{qq}\hat{\omega}_q,t+1, \\
\omega_{q,t+1} &= \tilde{\Gamma}(q_t, 1),
\end{align*}

<table>
<thead>
<tr>
<th>Structural Risk Aversion Parameters, $q_{t+1}$</th>
<th>Projection</th>
<th>GMM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Est</td>
<td>$p_t$</td>
<td>$n_t$</td>
</tr>
<tr>
<td>(SE)</td>
<td>(0.0566)</td>
<td>(0.0001)</td>
</tr>
</tbody>
</table>

\begin{align*}
\text{Conditional Mean Variance Decomposition (65% of Total Variance)} \\
& p_t \\
& n_t \\
& q_t \\
& 1.06\% \\
& 18.74\% \\
& 80.20\% \\
\end{align*}

\begin{align*}
\text{Shock Structure Variance Decomposition (35% of Total Variance)} \\
& \omega_{p,t+1} \\
& \omega_{n,t+1} \\
& \omega_{q,t+1} \\
& 0.77\% \\
& 0.02\% \\
& 99.22\%
\end{align*}
Table VII: Fit of Moments

This table evaluates the fit of conditional moments of equity and corporate bond returns. Column “Model” reports the averages of the relevant model-implied conditional moments. The “Empirical Averages” represent the sample averages of the excess returns (for “Mom 1” and “Mom 4”), the sample averages of empirical conditional variances (for “Mom 2”, “Mom 3”, and “Mom 5”). For “Mom 6” and “Mom 7”, “Risk Aversion Innovation” refers to $u^q_{t+1}$ in Equation (29). The variance and unscaled skewness rows compare the average model-implied conditional moments with the unconditional moments. Block bootstrapped standard errors are shown in parentheses; we allow the block size to vary for different moments: block sizes=[0 6 15 1 10] for Mom 1 to Mom 5, respectively. Asymptotic standard errors (standard deviation divided by square root of the number of observations) are reported for Mom 6 and Mom 7. Bold numbers denote a distance of less than 1.645 standard errors from the corresponding empirical point estimate. The sample period is 1986/06 to 2015/02 (345 months).

<table>
<thead>
<tr>
<th>Moment</th>
<th>Model</th>
<th>Empirical Average</th>
<th>Boot.SE/SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mom 1  Equity Risk Premium</td>
<td><strong>0.00785</strong></td>
<td>0.00530</td>
<td>(0.00246)</td>
</tr>
<tr>
<td>Mom 2  Equity Physical Variance</td>
<td><strong>0.00346</strong></td>
<td>0.00286</td>
<td>(0.00051)</td>
</tr>
<tr>
<td>Mom 3  Equity Risk-neutral Variance</td>
<td><strong>0.00412</strong></td>
<td>0.00397</td>
<td>(0.00049)</td>
</tr>
<tr>
<td>Mom 4  Corporate Bond Risk Premium</td>
<td><strong>0.00429</strong></td>
<td>0.00388</td>
<td>(0.00050)</td>
</tr>
<tr>
<td>Mom 5  Corporate Bond Physical Variance</td>
<td><strong>0.00023</strong></td>
<td>0.00024</td>
<td>(0.00003)</td>
</tr>
<tr>
<td>Mom 6  Risk Aversion Innovation Variance</td>
<td><strong>0.00767</strong></td>
<td>0.000929</td>
<td>(0.00195)</td>
</tr>
<tr>
<td>Mom 7  Risk Aversion Innovation Unscaled Skewness</td>
<td><strong>0.00239</strong></td>
<td>0.00216</td>
<td>(0.00111)</td>
</tr>
</tbody>
</table>
Table VIII: Asset Prices and the State Variables

This table reports the decomposition of model-implied conditional moments by the underlying dynamic drivers. The closed-form solution of each conditional moment is shown in Section 2 (see a detailed derivation in the Online Appendix). The asset conditional moments are explained by four premium state variables, \( \{p_t, n_t, l_p, q_t\} \). This table presents 1. scaled coefficients for interpretation purpose and 2. variance decomposition. 1. The coefficients are multiplied by standard deviations of the corresponding state variables of the same column and then multiplied by 10000. 2. The variance decomposition (VARC) is reported in a bold italic font and is calculated from coefficient \( \times \frac{\text{Cov}(x_t, \text{Mom}_t)}{\text{Var}(\text{Mom}_t)} \) where \( x \in \{p, n, v, q\} \) and \( \text{Mom} \) is from Mom 1 to Mom 5. The sum of the four VARCs in the same row/moment add up to 100% by design.

<table>
<thead>
<tr>
<th>Moment</th>
<th>( p_t )</th>
<th>( n_t )</th>
<th>( l_p )</th>
<th>( q_t )</th>
<th>( \text{VARC} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mom 1 Equity Risk Premium</td>
<td>0.1473</td>
<td>4.0169</td>
<td>-0.0852</td>
<td>52.6093</td>
<td><strong>0.014%</strong></td>
</tr>
<tr>
<td><strong>VARC</strong></td>
<td><strong>0.014%</strong></td>
<td><strong>4.014%</strong></td>
<td><strong>-0.033%</strong></td>
<td><strong>96.004%</strong></td>
<td></td>
</tr>
<tr>
<td>Mom 2 Equity Physical Variance</td>
<td>0.0492</td>
<td>3.1165</td>
<td>0.0555</td>
<td>5.6090</td>
<td><strong>-0.042%</strong></td>
</tr>
<tr>
<td><strong>VARC</strong></td>
<td><strong>-0.042%</strong></td>
<td><strong>31.249%</strong></td>
<td><strong>0.148%</strong></td>
<td><strong>68.645%</strong></td>
<td></td>
</tr>
<tr>
<td>Mom 3 Equity Risk-neutral Variance</td>
<td>0.0493</td>
<td>3.0708</td>
<td>0.0554</td>
<td>11.8273</td>
<td><strong>0.000%</strong></td>
</tr>
<tr>
<td><strong>VARC</strong></td>
<td><strong>0.000%</strong></td>
<td><strong>14.717%</strong></td>
<td><strong>0.086%</strong></td>
<td><strong>85.197%</strong></td>
<td></td>
</tr>
<tr>
<td>Mom 4 Corporate Bond Risk Premium</td>
<td>0.0025</td>
<td>4.7338</td>
<td>0.1157</td>
<td>9.8541</td>
<td><strong>-0.025%</strong></td>
</tr>
<tr>
<td><strong>VARC</strong></td>
<td><strong>-0.025%</strong></td>
<td><strong>27.310%</strong></td>
<td><strong>0.187%</strong></td>
<td><strong>72.528%</strong></td>
<td></td>
</tr>
<tr>
<td>Mom 5 Corporate Bond Physical Variance</td>
<td>0.0004</td>
<td>0.1610</td>
<td>0.0408</td>
<td>0.0101</td>
<td><strong>-0.076%</strong></td>
</tr>
<tr>
<td><strong>VARC</strong></td>
<td><strong>-0.076%</strong></td>
<td><strong>88.928%</strong></td>
<td><strong>7.921%</strong></td>
<td><strong>3.227%</strong></td>
<td></td>
</tr>
</tbody>
</table>

59
Table IX: Predicting Realized Excess Returns and Realized Variances

This table reports the regression coefficients of realized future excess returns and realized variances of equity and corporate bond returns on model-implied state variables \( \{p_t, n_t, lp_t, q_t\} \) and moments (i.e., risk premiums, physical variances). For interpretation purpose, the coefficients on \( \{p_t, n_t, lp_t, q_t\} \) are multiplied by standard deviations of the corresponding state variables of the same column and then multiplied by 100. Bold (italic) coefficients have <5% (10%) p-values. Adjusted \( R^2 \)'s are reported. Standard errors are shown in parentheses. The last line reports the correlation of the fitted value with a NBER dummy. The sample period is 1986/06 to 2015/02 (345 months).

\[
\begin{align*}
\hat{r}_{t+1} - r_f & \quad RVAR_{t+1}^e \\
\hat{r}_{t+1} - r_f & \quad RVAR_{t+1}^g
\end{align*}
\]

| \( p_t \) & -0.2263 & 0.0146 & -0.0821 & -0.0067 \\
| \( n_t \) & -1.1843 & 0.1461 & 0.0530 & 0.0081 \\
| \( lp_t \) & -0.4346 & 0.0014 & 0.0674 & 0.0010 \\
| \( q_t \) & 0.8623 & 0.1976 & 0.0911 & 0.0071 \\
| \text{Model-Implied Moments} & 0.6108 & 2.1525 & 0.1553 & 3.9400 \\
| \text{Adjusted } R^2 & 5.5\% & 0.1\% & 24.8\% & 0.9\% \\
| \rho(NBER) & 0.4361 & 0.5798 & 0.5651 & 0.7041 \\

\text{Adjusted } R^2 \text{ and } \rho \text{ are reported.}
Table X: Out-Of-Sample Exercise

This table evaluates the relative importance of in-sample/out-of-sample model-implied risk premium estimates and empirical risk premium estimates by econometricians in predicting future excess returns.

1. **Model-implied risk premium estimates, “Mod”:** “Mod (1)” denotes the in-sample estimates of model-implied risk premiums whose dynamics are fully spanned by \( \{ p_t, n_t, l_t, q_t \} \) in this paper. “Mod (2)” denotes the out-of-sample estimates of model-implied risk premiums: Define a 60-month rolling window from \( t - 60 \) to \( t - 1 \), then project one-period ahead excess returns onto the 4 premium state variables and use the coefficient estimates to obtain \( E_t(\tilde{r}_{t+1} - r_f) \), repeatedly.

2. **Empirical risk premium estimates, “Emp Mod”:** We consider three out-of-sample empirical risk premium estimates using three instruments sets (subsets of \( z_t \)): (1) 5-year detrended earnings-price ratio; (2) 5-year detrended earnings-price ratio + term spread + credit spread; (3) market return physical uncertainty plus variance risk premium estimate. The table then reports the optimal combination of Mod and Emp Mod estimates which minimize the sum of squared innovations. Least Square standard errors are shown in parentheses. Bold (italic) coefficients have <5% (10%) \( p \)-values (against zero).

<table>
<thead>
<tr>
<th></th>
<th>Least-Square Estimate of ( a ) in ( \tilde{r}<em>{t+1} - r_f = a \times \text{Mod}(t, i) + (1 - a) \times \text{Emp Mod}(t, j) + e</em>{t+1} ) ( i = 1, 2; j = 1, 2, 3 )</th>
<th>( \circ ) Equity:</th>
<th>( \circ ) Corporate Bond:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>\text{Mod (1)} \text{ Mod (2)} \text{ Mod (1)} \text{ Mod (2)}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Emp Mod (1)</td>
<td>\textbf{0.7322} \text{ 0.3987} \text{ 0.4501} \text{ 0.1051}</td>
<td>(0.0958) (0.0894)</td>
<td>(0.0740) (0.0778)</td>
</tr>
<tr>
<td>Emp Mod (2)</td>
<td>\textbf{0.8483} \text{ 0.5628} \text{ 0.5509} \text{ 0.3614}</td>
<td>(0.0878) (0.0848)</td>
<td>(0.0591) (0.0720)</td>
</tr>
<tr>
<td>Emp Mod (3)</td>
<td>\textbf{0.7609} \text{ 0.5619} \text{ 0.5579} \text{ 0.4507}</td>
<td>(0.0758) (0.0763)</td>
<td>(0.0579) (0.0836)</td>
</tr>
</tbody>
</table>
Table XI: Projecting Macroeconomic Uncertainty on Financial Instruments

This table presents regression results of the monthly macroeconomic uncertainties (estimated from industrial production growth; see Table I) on monthly observable instruments; some are used to span the time-varying risk aversion. “Total” is the total industrial production growth conditional variance, or $\sigma^2_{\theta_i}p_t + \sigma^2_{\theta_n}n_t$; “Upside”, $\sigma^2_{\theta_i}p_t$; “Downside”, $\sigma^2_{\theta_n}n_t$. “×10^3” in the header means that the coefficients and their SEs reported are multiplied by 1000 for reporting convenience. VARC reports the variance decomposition. Robust and efficient standard errors are shown in parentheses. Adjusted $R^2$s are reported. Bold (italic) coefficients have <5% (10%) p-values. The sample period is 1986/06 to 2015/02 (345 months).

<table>
<thead>
<tr>
<th></th>
<th>(×10^3)</th>
<th>(×10^3)</th>
<th>(×10^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
<td>VARC</td>
<td>Upside</td>
</tr>
<tr>
<td>constant</td>
<td>-0.009</td>
<td>0.006</td>
<td>-0.015</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.000)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>$\chi_{tsprd}$</td>
<td>-0.577</td>
<td>-2.33%</td>
<td>-0.004</td>
</tr>
<tr>
<td></td>
<td>(0.112)</td>
<td>(0.002)</td>
<td>(0.112)</td>
</tr>
<tr>
<td>$\chi_{cspd}$</td>
<td>2.024</td>
<td>62.69%</td>
<td>-0.016</td>
</tr>
<tr>
<td></td>
<td>(0.246)</td>
<td>(0.004)</td>
<td>(0.246)</td>
</tr>
<tr>
<td>$\chi_{DY5yr}$</td>
<td>2.343</td>
<td>41.57%</td>
<td>-0.162</td>
</tr>
<tr>
<td></td>
<td>(0.456)</td>
<td>(0.007)</td>
<td>(0.456)</td>
</tr>
<tr>
<td>$\chi_{EY5yr}$</td>
<td>-0.609</td>
<td>-22.57%</td>
<td>0.048</td>
</tr>
<tr>
<td></td>
<td>(0.189)</td>
<td>(0.003)</td>
<td>(0.189)</td>
</tr>
<tr>
<td>$\chi_{rvareq}$</td>
<td>-0.257</td>
<td>-3.76%</td>
<td>-0.002</td>
</tr>
<tr>
<td></td>
<td>(0.620)</td>
<td>(0.010)</td>
<td>(0.621)</td>
</tr>
<tr>
<td>$\chi_{qvareq}$</td>
<td>1.190</td>
<td>13.25%</td>
<td>0.066</td>
</tr>
<tr>
<td></td>
<td>(0.669)</td>
<td>(0.010)</td>
<td>(0.670)</td>
</tr>
<tr>
<td>$\chi_{rvarcb}$</td>
<td>17.792</td>
<td>13.67%</td>
<td>-0.056</td>
</tr>
<tr>
<td></td>
<td>(5.927)</td>
<td>(0.092)</td>
<td>(5.935)</td>
</tr>
<tr>
<td>$\chi_{rvarcbSPEC}$</td>
<td>-2.233</td>
<td>-2.51%</td>
<td>-0.108</td>
</tr>
<tr>
<td></td>
<td>(5.564)</td>
<td>(0.087)</td>
<td>(5.571)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>50.20%</td>
<td>70.80%</td>
<td>50.60%</td>
</tr>
</tbody>
</table>
Table XII: On the Predictive Power of Risk Aversion and Uncertainty for Future Output Growth

This table reports the coefficient estimates of the following predictive regression,

$$\theta_{t+k} = a_k + b_k' x_t + \omega_{t+k},$$

where $\theta_{t+k}$ represents future log industrial production growth during period $t+1$ and $t+k$ ($\sum_{r=1}^{k} \theta_{t+r}$) and $x_t$ represents a vector of current predictors: (1) our GMM-implied risk aversion index, $r^B_{BEX}$, (2) our financial instrument proxy of economic uncertainty, $unc^B_{BEX}$, (3) the risk-neutral conditional variance (the square of the month-end VIX (after 1990) / VXO (prior to 1990) index divided by 120000), $QVAR$, and (4) the true total macroeconomic uncertainty filtered from industrial production growth $unc^{true}$. The coefficients are scaled by the standard deviation of the predictor in the same column for interpretation purposes. Hodrick (1992) standard errors are reported in parentheses, and adjusted $R^2$s are in %. Bold (italic) coefficients have <5% (10%) p-values.

<table>
<thead>
<tr>
<th></th>
<th>$r^B_{BEX}$</th>
<th>$unc^B_{BEX}$</th>
<th>$QVAR$</th>
<th>$unc^{true}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Univariate</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1m</td>
<td>-0.002</td>
<td>-0.003</td>
<td>-0.002</td>
<td>-0.002</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.000)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td></td>
<td>7.2%</td>
<td>20.6%</td>
<td>6.5%</td>
<td>13.1%</td>
</tr>
<tr>
<td>1q</td>
<td>-0.005</td>
<td>-0.008</td>
<td>-0.005</td>
<td>-0.007</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.001)</td>
</tr>
<tr>
<td></td>
<td>14.9%</td>
<td>37.9%</td>
<td>15.3%</td>
<td>26.5%</td>
</tr>
<tr>
<td>4q</td>
<td>-0.006</td>
<td>-0.017</td>
<td>-0.008</td>
<td>-0.010</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.003)</td>
<td>(0.004)</td>
</tr>
<tr>
<td></td>
<td>2.5%</td>
<td>17.7%</td>
<td>3.7%</td>
<td>6.5%</td>
</tr>
</tbody>
</table>

| **B. Multivariate (1)** | $R^2$ | | | |
| 1m    | 0.001       | -0.003        | 0.000  | 21.2%        |
|       | (0.001)     | (0.001)       | (0.001) |             |
| 1q    | 0.004       | -0.009        | -0.003 | 39.1%        |
|       | (0.001)     | (0.002)       | (0.001) |             |
| 4q    | 0.021       | -0.025        | -0.011 | 22.8%        |
|       | (0.004)     | (0.005)       | (0.002) |             |

| **C. Multivariate (2)** | $R^2$ | | | |
| 1m    | 0.000       | 0.000         | -0.002 | 14.1%        |
|       | (0.001)     | (0.001)       | (0.001) |             |
| 1q    | 0.001       | -0.003        | -0.006 | 29.6%        |
|       | (0.001)     | (0.002)       | (0.001) |             |
| 4q    | 0.010       | -0.012        | -0.010 | 8.1%         |
|       | (0.003)     | (0.003)       | (0.003) |             |

| **D. Multivariate (3)** | $R^2$ | | | |
| 1m    | 0.001       | -0.003        | 0.000  | -0.001       |
|       | (0.001)     | (0.001)       | (0.001) |             |
| 1q    | 0.004       | -0.008        | -0.003 | -0.002       |
|       | (0.001)     | (0.001)       | (0.001) |             |
| 4q    | 0.021       | -0.027        | -0.011 | 0.003        |
|       | (0.004)     | (0.006)       | (0.002) |             |
Table XIII: Alternative Risk Aversion and Uncertainty Measures.

This table reports the correlation between our risk aversion and economic uncertainty indices and extant measures. For risk aversion in Panel A: A.1) We follow Wachter (2006) to create a fundamental risk aversion process from inflation-adjusted (real) quarterly consumption growth ($\sum_{j=0}^{4} \Delta c_{t-j}$); A.2) sentiment index by Baker and Wurgler (2006) from the behavior finance literature, and the Michigan Consumer Sentiment Index; A.3) an industry index, the Credit Suisse First Boston Risk Appetite Index. For economic uncertainty in Panel B: B.1) the macroeconomic uncertainty index created by Jurado, Ludvigson, and Ng (2015); B.2) the Economic Policy Uncertainty Index created by Baker, Bloom, and Davis (2016). Correlations are calculated using overlapping samples at the monthly frequency. Side-by-side graphical comparisons are shown in the Online Appendix. Standard errors are shown in parentheses. Bold correlation coefficients have <5% p-values.

<table>
<thead>
<tr>
<th>A. Correlations with Extant Risk Aversion Indices</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A.1) “Fundamental” Habit Model:</strong></td>
</tr>
<tr>
<td>Wachter (2006) / Campbell and Cochrane (1999)</td>
</tr>
<tr>
<td>(0.0534)</td>
</tr>
<tr>
<td><strong>A.2) Sentiment Index:</strong></td>
</tr>
<tr>
<td>Baker and Wurgler (2006)</td>
</tr>
<tr>
<td>(0.0531)</td>
</tr>
<tr>
<td>Michigan Consumer Sentiment Index</td>
</tr>
<tr>
<td>(0.0518)</td>
</tr>
<tr>
<td><strong>A.3) Industry Index</strong></td>
</tr>
<tr>
<td>Credit Suisse First Boston Risk Appetite Index</td>
</tr>
<tr>
<td>(0.0471)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. Correlations with Extant Uncertainty Indices</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>B.1) Macroeconomic Uncertainty:</strong></td>
</tr>
<tr>
<td>Jurado, Ludvigson, and Ng (2015)</td>
</tr>
<tr>
<td>(0.0316)</td>
</tr>
<tr>
<td><strong>B.2) Political Uncertainty:</strong></td>
</tr>
<tr>
<td>Baker, Bloom, and Davis (2016)</td>
</tr>
<tr>
<td>(0.0506)</td>
</tr>
</tbody>
</table>
The estimation results are displayed in Table I. The plot covers the estimation period from January 1947 to February 2015. The shaded regions are NBER recession months from the NBER website.
The dynamics and estimation results are shown in Table II. From top to bottom: loss rate $l_{t+1}$, the cash flow uncertainty state variable $lp_t$, the total conditional variance, and its decomposition. There are four independent gamma shocks in the system that contribute to the magnitude and the dynamics of the total loss rate disturbance: real economic shocks, $\omega_p$ and $\omega_n$, and pure cash flow shocks, $\omega_{lp}$ and $\omega_{ln}$. The fourth plot depicts the shares of total conditional variances that are explained by each of the shocks at each point of time. Among them, only $\omega_{ln}$ is homoskedastic. The loss rate estimation uses the longest sample available from June 1984 to February 2015. The shaded regions are NBER recession months from the NBER website.
Figure III: Model-Implied and Empirical Risk-Neutral Conditional Variances of Equity Market Returns.

The shaded regions are NBER recession months from the NBER website. The two series are 90.06% correlated.
Figure IV: Risk Aversion Index (black/left y-axis) and Uncertainty Index (red/right y-axis).

The risk aversion index denoted as $ra^{BEX} = \gamma \exp(q_t)$ and the uncertainty index denoted as $unc^{BEX}$ are linear functions of a set of financial instruments. The utility curvature parameter $\gamma$ is 2. Correlation between the two series is 70.43%. The shaded regions are NBER recession months from the NBER website.
Figure V: Risk aversion and economic uncertainty at daily frequencies around the Bear Stearns and Lehman Brothers Collapses in 2008.
Online Appendices for
“The Time Variation in Risk Appetite and Uncertainty”

A The state variables

A.1 Matrix representation of the state variables

In this section, we show the matrix representation of the system of ten state variables in this economy. The ten state variables, as introduced in Section 3, are as follows,

\[ Y_t = [\theta_t, p_t, n_t, \pi_t, l_t, g_t, \kappa_t, \eta_t, t_g, q_t]^\prime, \]

where \{p_t, n_t\} denote the upside uncertainty factor and the downside uncertainty factor, as latent variables extracted from the system of corporate loss rate; \(l_t\) represents the log consumption-earnings ratio; \(g_t\) represents the log of corporate loss rate; \(\kappa_t\) represents the log dividend payout ratio; \(\eta_t\) represents the cash flow uncertainty factor, as the latent variable extracted from the system of corporate loss rate \(l_t\); \(q_t\) represents the latent risk aversion of the economy. The state variables have the following matrix representation:

\[ Y_{t+1} = \mu + AY_t + \Sigma \omega_{t+1}, \] (A.1)

where \(\omega_{t+1} = [\omega_{p_t+1}, \omega_{n_t+1}, \omega_{\pi_t+1}, \omega_{l_p+1}, \omega_{l_n+1}, \omega_{g_p+1}, \omega_{g_n+1}, \omega_{\eta_p+1}, \omega_{\eta_n+1}]^\prime \) is a vector comprised of eight independent shocks in the economy. Among the nine shocks, \(\{\omega_{\pi_t+1}, \omega_{l_p+1}, \omega_{l_n+1}, \omega_{g_p+1}, \omega_{g_n+1}\}\) shocks are homoskedastic. The conditional variance, skewness and higher-order moments of the following four centered gamma shocks—\(\omega_{p_t+1}, \omega_{n_t+1}, \omega_{\pi_t+1}, \omega_{g_p+1}\)—are assumed to be proportional to \(p_t, n_t, l_p\), and \(q_t\) respectively. The underlying distributions for the rest four shocks are assumed to be Gaussian with unit standard deviation.

The constant matrices are defined implicitly,

\[
\begin{align*}
\mu &= \begin{bmatrix}
(1 - \rho_0) \tilde{\beta} - m_p \tilde{\beta} - m_n \tilde{\beta} \equiv \theta_0 \\
\pi_0 \\
l_0 \\
g_0 \\
\kappa_0 \\
\eta_0 \\
v_0 \\
q_0
\end{bmatrix}, \\
A &= \begin{bmatrix}
\rho_0 & m_p & m_n & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \rho_p & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \rho_n & 0 & 0 & 0 & 0 & 0 & 0 \\
\rho_{\pi p} & \rho_{g p} & \rho_{g n} & \rho_{g \pi} & \rho_{l p} & \rho_{l n} & 0 & 0 & 0 \\
0 & \rho_{l p} & \rho_{n p} & 0 & 0 & 0 & 0 & 0 & 0 \\
\rho_{\pi \eta} & \rho_{g \eta} & \rho_{g \eta} & \rho_{g \pi} & \rho_{l \eta} & \rho_{l \eta} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \rho_{\pi q} & \rho_{g q} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}, \\
\Sigma &= \begin{bmatrix}
\sigma_{\pi p} & -\sigma_{\pi n} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\sigma_{g p} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \sigma_{\pi n} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\sigma_{\pi \pi} & \sigma_{g \pi} & \sigma_{g \pi} & \sigma_{g \pi} & \sigma_{l \pi} & \sigma_{l \pi} & 0 & 0 & 0 \\
\sigma_{l p} & \sigma_{l p} & \sigma_{l p} & \sigma_{l p} & \sigma_{l \pi} & \sigma_{l \pi} & 0 & 0 & 0 \\
\sigma_{g n} & \sigma_{g n} & \sigma_{g n} & \sigma_{g n} & \sigma_{g n} & \sigma_{g \eta} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \sigma_{\pi \eta} & \sigma_{\pi \eta} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \sigma_{g \eta} & \sigma_{g \eta} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\sigma_{q p} & \sigma_{q n} & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}.
\end{align*}
\] (A.2, A.3, A.4)
Given the moment generating functions (mgf) of gamma and Gaussian distributions, we show that the model is affine, \( \forall \nu \in \mathbb{R}^{10} \),

\[
M_Y(\nu) := E_t \left[ \exp(\nu' Y_{t+1}) \right] = \exp(\nu' \mu + \nu' AY_t) E_t \left[ \exp(\nu' \Sigma \omega_{t+1}) \right] \\
= \exp \left[ \nu' S_0 + \frac{1}{2} \nu' S_1 \Sigma_{\text{other}} S'_1 \nu + f_S(\nu) Y_t + S_2(\nu) \nu \right],
\]

(A.5)

where \( S_0 = \mu \) (10 \times 1),

\[
S_1 = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix},
\]

(A.6)

\[
\Sigma_{\text{other}} = \begin{bmatrix}
\sigma_{\pi \pi} & \sigma_{\pi \eta} & \sigma_{\pi \eta} & \sigma_{\pi \eta} \\
\sigma_{\eta \pi} & \sigma_{\eta \eta} & \sigma_{\eta \eta} & \sigma_{\eta \eta} \\
\sigma_{\pi \pi} & \sigma_{\pi \eta} & \sigma_{\pi \eta} & \sigma_{\pi \eta} \\
\sigma_{\eta \pi} & \sigma_{\eta \eta} & \sigma_{\eta \eta} & \sigma_{\eta \eta}
\end{bmatrix} \quad \text{(cov-var matrix of \( \{\omega_{\pi}, \omega_{\eta}, \omega_{\kappa}, \omega_{\eta}\} \))},
\]

(A.7)

\[
f_S(\nu) = \nu' A + \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
-\sigma_p(\nu) - \ln(1 - \sigma_p(\nu)) \\
-\sigma_n(\nu) - \ln(1 - \sigma_n(\nu)) \\
0 \\
0 \\
0 \\
-\sigma_l(\nu) - \ln(1 - \sigma_l(\nu)) \\
-\sigma_q(\nu) - \ln(1 - \sigma_q(\nu))
\end{bmatrix},
\]

(A.8)

\[
S_2(\nu) = -\sigma_{\nu \nu}(\nu) - \ln(1 - \sigma_{\nu \nu}(\nu)),
\]

(A.9)

\[
\sigma_p(\nu) = \nu' \Sigma_{\pi 1},
\]

(A.10)

\[
\sigma_n(\nu) = \nu' \Sigma_{\eta 2},
\]

(A.11)

\[
\sigma_l(\nu) = \nu' \Sigma_{\kappa 4},
\]

(A.12)

\[
\sigma_{\nu \nu}(\nu) = \nu' \Sigma_{\nu 5},
\]

(A.13)

\[
\sigma_q(\nu) = \nu' \Sigma_{\eta 9},
\]

(A.14)

where \( M_{e_j} \) denotes the \( j \)-th column of the matrix \( M \).

### A.2 Consumption growth

Consumption growth in this economy is endogenous defined and can be expressed in an affine function:

\[
\Delta c_{t+1} = \gamma_{t+1} + \Delta \kappa_{t+1} \\
= c_0 + e'_2 Y_t + c'_1 \Sigma \omega_{t+1},
\]

(A.15)

(A.16)

(A.17)
where \( c_0 = g_0 + \kappa_0 \), \( c_1 = [0 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0]' \), and

\[
c_2 = \begin{bmatrix}
\rho_{\theta\theta} + \rho_{\theta\theta} \\
\rho_{\theta\theta} + \rho_{\theta\theta} \\
\rho_{\theta\gamma} + \rho_{\gamma\theta} \\
0 \\
0 \\
\rho_{\gamma\gamma} \\
\rho_{\gamma\gamma} - 1 \\
0 \\
0
\end{bmatrix}.
\] (A.18)

\section*{B Asset Pricing}

In this section, we solve the model analytically. First, given consumption growth and changes in risk aversion, the log of real pricing kernel of the economy is derived as an affine function of the state variables. Next, we show that asset prices of claims on cash flows from three different asset markets can be expressed in (quasi) affine equations. The model is solved using the non-arbitrage condition. The goal of this section is to derive the analytical solutions for the expected excess returns, the physical variance of asset returns and the risk-neutral variance of asset returns in closed forms. The implied moments are crucial for the estimation procedure.

\subsection*{B.1 The real pricing kernel}

The log real pricing kernel for this economy is given by,

\[
m_{t+1} = \ln(\beta) - \gamma \Delta c_{t+1} + \gamma \Delta q_{t+1} \\
= m_0 + m_2 Y_t + m_1 \Sigma \omega_{t+1},
\] (B.1)

where \( m_0 = \ln(\beta) + \gamma(g_0 - g_0 - \kappa_0) \), \( m_1 = [0 \ 0 \ 0 \ 0 \ 0 \ -\gamma \ -\gamma \ 0 \ 0 \ \gamma]' \), and

\[
m_2 = \begin{bmatrix}
\gamma(\rho_{\theta\theta} - \rho_{\theta\theta}) \\
\gamma(\rho_{\theta\theta} - \rho_{\theta\theta} - \rho_{\theta\theta}) \\
\gamma(\rho_{\theta\theta} - \rho_{\theta\theta}) \\
0 \\
0 \\
-\gamma\rho_{\theta\theta} \\
-\gamma(\rho_{\theta\theta} - 1) \\
0 \\
0 \\
\gamma(\rho_{\theta\theta} - 1)
\end{bmatrix}.
\] (B.3)

As a result, the moment generating function of the real pricing kernel is, \( \forall \nu \in \mathbb{R} \)

\[
E_t [\exp(\nu m_{t+1})] = \exp \left[ \nu m_0 + \nu m_2 Y_t \right]
\] · \( \exp \left[ \left\{ -\nu \sigma_p (m_1) - \ln (1 - \nu \sigma_p (m_1)) \right\} p_t + \left\{ -\nu \sigma_n (m_1) - \ln (1 - \nu \sigma_n (m_1)) \right\} n_t \right] \\
\] · \( \exp \left[ \left\{ -\nu \sigma_{p\gamma} (m_1) - \ln (1 - \nu \sigma_{p\gamma} (m_1)) \right\} p_t + \left\{ -\nu \sigma_{n\gamma} (m_1) - \ln (1 - \nu \sigma_{n\gamma} (m_1)) \right\} n_t \right] \\
\) · \( \exp \left[ \left\{ -\nu \sigma_{v\gamma} (m_1) - \ln (1 - \nu \sigma_{v\gamma} (m_1)) \right\} v_t + \frac{1}{2} \nu^2 \left[ m_1' \Sigma_{\text{other}} S_1' m_1 \right] \right], 
\] (B.4)

where \( m_0, m_1, m_2, S_1 \), and \( \Sigma_{\text{other}} \) are constant matrices defined earlier, and

\[
\sigma_p (m_1) = m_1' \Sigma_{\bullet 1}.
\] (B.5)

\[
\sigma_n (m_1) = m_1' \Sigma_{\bullet 2}.
\] (B.6)

\[
\sigma_{p\gamma} (m_1) = m_1' \Sigma_{\bullet 4}.
\] (B.7)

\[
\sigma_{v\gamma} (m_1) = m_1' \Sigma_{\bullet 5}.
\] (B.8)

\[
\sigma_q (m_1) = m_1' \Sigma_{\bullet 9}.
\] (B.9)
Accordingly, the model-implied short rate $r_f$ is,

$$
rf = - \ln \{ E_t [\exp (m_{t+1})] \}
$$

(B.10)

$$
= -m_0 - m_2^Y t
$$

(B.11)

$$
+ [\sigma_p (m_1) + \ln (1 - \sigma_p (m_1))] p_t + [\sigma_n (m_1) + \ln (1 - \sigma_n (m_1))] n_t
$$

(B.12)

$$
+ [\sigma_p (m_1) + \ln (1 - \sigma_p (m_1))] p_t + [\sigma_n (m_1) + \ln (1 - \sigma_n (m_1))] q_t
$$

(B.13)

$$
+ [\sigma_v (m_1) + \ln (1 - \sigma_v (m_1))] v_n - 1 \left\{ m_1^l S_1 \Sigma^{other} S'_1 m_1 \right\}.
$$

(B.14)

$$
= r_{f0} + r_{f2}^Y Y_t.
$$

(B.15)

To price nominal assets, we define the nominal pricing kernel, $\tilde{m}_{t+1}$, which is a simple transformation of the log real pricing kernel, $m_{t+1}$,

$$
\tilde{m}_{t+1} = m_{t+1} - \pi_{t+1},
$$

(B.16)

$$
= \tilde{m}_0 + \tilde{m}_2^Y Y_t + \tilde{m}_1^\omega \Sigma \omega_{t+1},
$$

(B.17)

where $\tilde{m}_0 = m_0 - \pi_0, \tilde{m}_1 = m_1 - [0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]'$, and

$$
\tilde{m}_2 = m_2 - \begin{bmatrix}
\rho_{eg}

\rho_{ep}

\rho_{en}

\rho_{eo}

0

0

0

0

0
\end{bmatrix},
$$

(B.18)

The nominal risk free rate $\tilde{r}_f$ is defined as $- \ln \{ E_t [\exp (\tilde{m}_{t+1})] \}$.

### B.2 Valuation ratio

It is a crucial step in this paper to show that asset prices are (quasi) affine functions of the state variables.

**Defaultable Nominal Bonds** In the paper, we assume that a one period nominal bond faces a fractional (logarithmic) loss of $l_t$. Given the structures assumed for $l_t$ and $\pi_t$ and the model-implied log pricing kernel, the price-coupon ratio of the one-period defaultable bond portfolio is

$$
PC_t^1 = E_t [\exp (\tilde{m}_{t+1} - l_{t+1})]
$$

(B.19)

$$
= \exp (b_0^d + b_1^d Y_t),
$$

(B.20)

where $b_0^d$ and $b_1^d$ are implicitly defined. Consider next a portfolio of multi-period zero-coupon defaultable bonds with a promised terminal payment of $C$ at period $(t + N)$. As for the $N$-period bond, the actual payment will be less than or equal to the promised payment, and the ex-post nominal payoff can be expressed as $\exp (c - l_{t+N})$. We ignore the possibility of early default or prepayment. Then, the price-coupon ratio of this bond at period $(t + N - 1)$, one period before maturity, $PC_t^{t+N-1}$, is $\exp (b_0^d + b_1^d Y_{t+N-1})$. Given the Euler equation and the law of iterated expectations, it then follows by induction that all earlier dated zero-coupon nominally defaultable corporate bond (maturing in $N$ period) prices are similarly affine in the state variables, in particular:

$$
PC_t^N = E_t \left[ \tilde{M}_{t+1}^{N-1} \right],
$$

$$
= \exp \left( b_0^N + b_1^N Y_t \right).
$$

(B.21)

Therefore, the assumed zero-coupon structure of the payments before maturity implies that the unexpected returns to this portfolio are exactly linearly spanned by the shocks to $Y_t$. 

4
Equity  It is especially not obvious for equity price-dividend ratio, of which we provide proofs below. First, we rewrite the real dividend growth in a general matrix expression:

\[ \Delta d_{t+1} = g_{t+1} + \Delta \eta_{t+1} \]
\[ = h_0 + h'_2 Y_t + h'_1 \Sigma \omega_{t+1}, \tag{B.22} \]

where \( h_0 = g_0 + \eta_0, \) \( h_1 = [0 0 0 0 1 0 0 0 0] \), and

\[ h_2 = \begin{bmatrix} \rho_{\theta \theta} + \rho_{\theta \phi} \\ \rho_{\gamma \gamma} + \rho_{\eta \gamma} \\ 0 \\ \rho_{\eta \eta} \\ 0 \\ 0 \end{bmatrix}. \tag{B.23} \]

The price-dividend ratio, \( PD_t = E_t \left[ M_{t+1} \left( \frac{P_{t+1}}{P_t} \right) \right] \), can be rewritten as,

\[ PD_t = \sum_{n=1}^{\infty} E_t \left[ \exp \left( \sum_{j=1}^{n} m_{t+j} + \Delta d_{t+j} \right) \right]. \tag{B.24} \]

Let \( F^n_t \) denote the \( n \)-th term in the summation:

\[ F^n_t = E_t \left[ \exp \left( \sum_{j=1}^{n} m_{t+j} + \Delta d_{t+j} \right) \right], \tag{B.25} \]

and \( F^n_t D_t \) is the price of zero-coupon equity that matures in \( n \) periods.

To show that equity price is an approximate affine function of the state variables, we first prove that \( F^n_t (\forall n \geq 1) \) is exactly affine using induction. First, when \( n = 1 \),

\[ F^1_t = E_t \left[ \exp \left( m_{t+1} + \Delta d_{t+1} \right) \right] \]
\[ = E_t \left[ \exp \left( \left( m_0 + h_0 \right) + \left( m'_2 + h'_2 \right) Y_t + \left( m'_1 + h'_1 \right) \Sigma \omega_{t+1} \right) \right] \]
\[ = \exp \left( \left( m_0 + h_0 \right) + \left( m'_2 + h'_2 \right) Y_t \right) \times \exp \left( \left[ -\sigma_{\theta}(m_1 + h_1) - \ln \left( 1 - \sigma_{\theta}(m_1 + h_1) \right) \right] p_t + \left[ -\sigma_{\gamma}(m_1 + h_1) - \ln \left( 1 - \sigma_{\gamma}(m_1 + h_1) \right) \right] q_t \right) \]
\[ \times \exp \left( \left[ -\sigma_{\eta}(m_1 + h_1) - \ln \left( 1 - \sigma_{\eta}(m_1 + h_1) \right) \right] v_t + \frac{1}{2} \left( m'_1 + h'_1 \right) S_1 \Sigma^{\text{other}} S_1' \left( m_1 + h_1 \right) \right) \]
\[ = \exp \left( \epsilon_0 + \epsilon'_1 Y_t \right), \tag{B.26} \]

where \( m_0, m_1, m_2, h_0, h_1, h_2, S_1, \) and \( \Sigma^{\text{other}} \) are constant matrices defined earlier, and

\[ \sigma_{\theta}(m_1 + h_1) = (m'_1 + h'_1) \Sigma_{\theta 1}, \tag{B.27} \]
\[ \sigma_{\gamma}(m_1 + h_1) = (m'_1 + h'_1) \Sigma_{\gamma 2}, \tag{B.28} \]
\[ \sigma_{\eta}(m_1 + h_1) = (m'_1 + h'_1) \Sigma_{\eta 3}, \tag{B.29} \]
\[ \sigma_{\eta}(m_1 + h_1) = (m'_1 + h'_1) \Sigma_{\theta 4}, \tag{B.30} \]
\[ \sigma_{\eta}(m_1 + h_1) = (m'_1 + h'_1) \Sigma_{\gamma 5}, \tag{B.31} \]
and $e_0^1 = m_0 + h_0 + [-\sigma_v(m_1 + h_1) - \ln (1 - \sigma_v(m_1 + h_1))] v_0 + \frac{1}{2} [m'_1 + h'_1] \Sigma v_{other} S'(m_1 + h_1)$.

\[
e_1^1 = m_2 + h_2 + \begin{bmatrix}
-\sigma_t (m_1 + h_1) - \ln (1 - \sigma_t (m_1 + h_1)) \\
-\sigma_n (m_1 + h_1) - \ln (1 - \sigma_n (m_1 + h_1)) \\
0 \\
0 \\
0 \\
0 \\
-\sigma_{t'} (m_1 + h_1) - \ln (1 - \sigma_{t'} (m_1 + h_1)) \\
-\sigma_{n'} (m_1 + h_1) - \ln (1 - \sigma_{n'} (m_1 + h_1))
\end{bmatrix}.
\]

(B.32)

Now, suppose that the $(n - 1)$-th term $F_{t+1}^{n-1} = \exp \left( e_{0}^{n-1} + e_{1}^{n-1'} Y_t \right)$, then

\[
F_t^n = E_1 \left[ \exp \left( \sum_{i=1}^{n} m_{i} t_{i} + \Delta d_{i} t_{i} \right) \right]
\]

\[
= E_1 \left[ E_{t+1} \left[ \exp(m_{t+1} + \Delta d_{t+1}) \exp \left( \sum_{j=1}^{n-1} m_{j} t_{j+1} + \Delta d_{j+1} t_{j+1} \right) \right] \right]
\]

\[
= E_1 \left[ \exp(m_{t+1} + \Delta d_{t+1}) E_{t+1} \left[ \exp \left( \sum_{j=1}^{n-1} m_{j} t_{j+1} + \Delta d_{j+1} t_{j+1} \right) \right] \right]
\]

\[
= E_1 \left[ \exp(m_{t+1} + \Delta d_{t+1}) \exp \left( e_{0}^{n-1} + e_{1}^{n-1'} Y_{t+1} \right) \right]
\]

\[
= \exp \left( e_{0}^{n} + e_{1}^{n'} Y_t \right).
\]

(B.33)

where $e_{0}^{n}$ and $e_{1}^{n'}$ are defined implicitly.

Hence, the price-dividend ratio is approximately affine:

\[
P_{D_t} = \sum_{n=1}^{\infty} \left[ \exp \left( \sum_{j=1}^{n} m_{j} t_{j} + \Delta d_{j} t_{j} \right) \right]
\]

\[
= \sum_{n=1}^{\infty} F_t^n
\]

\[
= \sum_{n=1}^{\infty} \exp \left( e_{0}^{n} + e_{1}^{n'} Y_t \right).
\]

(B.34)

\section{B.3 Log asset returns}

\textbf{Log return of zero-coupon nominally defautable corporate bonds maturing at $t + N$}

Given the exact exponential affine expression of the valuation ratio of this asset (see derivations above), the log return can be derived an approximate linear closed form:

\[
\pi_{t+1} = \ln \left[ \frac{PC_{t+1}^{N-1} + 1}{PC_t^{N}} \right] \ln \left( \frac{C}{C_t} \right) \ln (\Pi_{t+1})
\]

\[
= \pi_{t+1} + \ln \left[ \frac{1 + \exp \left( b_0^{N-1} + b_1^{N-1'} Y_{t+1} \right)}{\exp \left( b_0^{N} + b_1^{N'} Y_t \right)} \right]
\]

\[
\approx \pi_{t+1} + \text{const.} + \frac{\exp \left( b_0^{N-1} + b_1^{N-1'} Y \right)}{1 + \exp \left( b_0^{N-1} + b_1^{N-1'} Y \right)} Y_{t+1} - b_1^{N'} Y_t
\]
where $\tilde{r}_{t+1}^b$ is the log nominal return of corporate bond from $t$ to $t+1$, $\tilde{c}_0^b$ is constant, $\tilde{c}_1^b$ is a vector of state vector coefficients, and $\tilde{r}^a$ is a vector of shock coefficients. Thus, this step involves linear approximation.

**Log nominal equity return** We apply first-order Taylor approximations to the log nominal equity return, and obtain a linear system,

$$
\tilde{r}_{t+1}^{eq} = \ln \left( \frac{P_{t+1} + D_{t+1} \Pi_{t+1}}{P_t} \right) = \ln \left( \frac{PD_{t+1} + \Pi_{t+1}}{PD_t} \right) = \Delta d_{t+1} + \pi_{t+1} + \ln \left[ \frac{1 + \sum_{m=1}^{\infty} \exp (e_0^n + e_1^n \tilde{Y}_{t+1})}{\sum_{m=1}^{\infty} \exp (e_0^n + e_1^n \tilde{Y}_t)} \right] \\
\approx \Delta d_{t+1} + \pi_{t+1} + \text{const.} + \frac{\sum_{m=1}^{\infty} \exp (e_0^n + e_1^n \tilde{Y})}{\sum_{m=1}^{\infty} \exp (e_0^n + e_1^n \tilde{Y})} \tilde{Y}_{t+1} + \frac{\sum_{m=1}^{\infty} \exp (e_0^n + e_1^n \tilde{Y})}{\sum_{m=1}^{\infty} \exp (e_0^n + e_1^n \tilde{Y})} \tilde{Y}_t \\
= \tilde{c}_0^{eq} + \tilde{c}_1^{eq} \tilde{Y}_t + \tilde{c}_2^{eq} \Sigma \tilde{w}_{t+1}.
$$

(B.36)

where $\tilde{r}_{t+1}^{eq}$ is the log nominal return of equity from $t$ to $t+1$, $\tilde{c}_0^{eq}$ is constant, $\tilde{c}_1^{eq}$ is a vector of state vector coefficients, and $\tilde{r}^a$ is a vector of shock coefficients. Thus, this step involves linear approximation.

**General expression** To acknowledge the errors that are potentially caused by the linear approximations (the Taylor approximation in log price-dividend ratio in the return equation), we write down the return innovations for asset $i$ with an idiosyncratic shock:

$$
\tilde{r}_{t+1}^i - E_t \left( \tilde{r}_{t+1}^i \right) = \tilde{r}^{\prime} \Sigma \omega_{t+1} + \epsilon_{t+1}^i. 
$$

(B.37)

where $E_t (\tilde{r}_{t+1}^i)$ is the expected return, $\tilde{r}^i$ ($10 \times 1$) is the asset $i$ return loadings on selected state variable innovations (the choice of which depends on the asset classes), and $\epsilon_{t+1}^i$ is the Gaussian noise uncorrelated with the state variable shocks but may be cross-correlated (with other asset-specific shocks). The Gaussian shock $\epsilon_{t+1}^i$ has an unconditional variance $\sigma_i^2$.

**B.4 Model-implied moments**

In this section, we derive three model-implied asset conditional moments—expected excess returns, physical and risk-neutral conditional variances of nominal asset returns. The moments are crucial in creating the moment conditions during the third step of model estimation.

**B.4.1 One-period expected excess return**

We impose the no-arbitrage condition, $1 = E_t [\exp (\tilde{m}_{t+1} + \tilde{r}_{t+1}^i)]$ ($\forall i \in \{\text{equity, treasury bond, corporate bond}\}$), and obtain the expected excess returns. Expand the law of one price (LOOP) equation:

$$
1 = E_t [\exp (\tilde{m}_{t+1} + \tilde{r}_{t+1}^i)] \\
= \exp \left[ E_t (\tilde{m}_{t+1}) + E_t (\tilde{r}_{t+1}^i) \right] \\
\times \exp \left[ -\sigma_p (\tilde{m} + \tilde{r}) - \ln (1 - \sigma_p (\tilde{m} + \tilde{r})) \right] p_i + \left[ -\sigma_v (\tilde{m} + \tilde{r}) - \ln (1 - \sigma_v (\tilde{m} + \tilde{r})) \right] v_i \\
\times \exp \left[ -\sigma_v (\tilde{m} + \tilde{r}) - \ln (1 - \sigma_v (\tilde{m} + \tilde{r})) \right] q_i \\
\times \exp \left[ \frac{1}{2} \left( \tilde{m}^t + \tilde{r}^{it} \right) S_1 \Sigma^\text{other} S_1^t (\tilde{m} + \tilde{r}) + \sigma_i^2 \right],
$$

(B.38)

where $\tilde{m}, \tilde{r}, \sigma_i, S_1$, and $\Sigma^\text{other}$ are constant matrices defined earlier, and

$$
\sigma_p (\tilde{m} + \tilde{r}) = (\tilde{m}^t + \tilde{r}^{it}) \Sigma_{1,1},
$$
\[ \sigma_n(m_1 + \bar{r}^i) = (\bar{m}_1 + \bar{r}^i)\Sigma^2_2, \]
\[ \sigma_p(m_1 + \bar{r}^i) = (\bar{m}_1 + \bar{r}^i)\Sigma^4_2. \]
\[ \sigma_v(m_1 + \bar{r}^i) = (\bar{m}_1 + \bar{r}^i)\Sigma_5. \]
\[ \sigma_q(m_1 + \bar{r}^i) = (\bar{m}_1 + \bar{r}^i)\Sigma_9. \] (B.39)

Given the nominal risk free rate derived earlier using real pricing kernel and inflation, the nominal excess return is,
\[
E_t(\bar{r}^i) - rf_t = \left\{ \begin{array}{l}
\sigma_p(\bar{r}^i) + \ln \left[ \frac{1 - \sigma_p(m_1 + \bar{r}^i)}{1 - \sigma_p(m_1)} \right] p_t \\
+ \sigma_v(\bar{r}^i) + \ln \left[ \frac{1 - \sigma_v(m_1 + \bar{r}^i)}{1 - \sigma_v(m_1)} \right] n_t \\
+ \sigma_p(\bar{r}^i) + \ln \left[ \frac{1 - \sigma_p(m_1 + \bar{r}^i)}{1 - \sigma_p(m_1)} \right] l p_t \\
+ \sigma_q(\bar{r}^i) + \ln \left[ \frac{1 - \sigma_q(m_1 + \bar{r}^i)}{1 - \sigma_q(m_1)} \right] q_t \\
+ \sigma_v(\bar{r}^i) + \ln \left[ \frac{1 - \sigma_v(m_1 + \bar{r}^i)}{1 - \sigma_v(m_1)} \right] v_n - \bar{m}_1 S_1 \Sigma^other S_1' \bar{r}^i - \frac{1}{2} \bar{r}^i S_1 \Sigma^other S_1' \bar{r}^i + \sigma^2 \\
\end{array} \right. 
\] (B.40)

where
\[ \sigma_p(\bar{r}^i) = \bar{r}^i \Sigma^1_1, \] (B.41)
\[ \sigma_v(\bar{r}^i) = \bar{r}^i \Sigma^2_2. \] (B.42)
\[ \sigma_p(\bar{r}^i) = \bar{r}^i \Sigma^4_2. \] (B.43)
\[ \sigma_v(\bar{r}^i) = \bar{r}^i \Sigma_5. \] (B.44)
\[ \sigma_q(\bar{r}^i) = \bar{r}^i \Sigma_9. \] (B.45)
\[ \sigma_p(m_1 + \bar{r}^i) = (\bar{m}_1 + \bar{r}^i) \Sigma^1_1. \] (B.46)
\[ \sigma_v(m_1 + \bar{r}^i) = (\bar{m}_1 + \bar{r}^i) \Sigma^2_2. \] (B.47)
\[ \sigma_p(m_1 + \bar{r}^i) = (\bar{m}_1 + \bar{r}^i) \Sigma^4_2. \] (B.48)
\[ \sigma_v(m_1 + \bar{r}^i) = (\bar{m}_1 + \bar{r}^i) \Sigma_5. \] (B.49)
\[ \sigma_q(m_1 + \bar{r}^i) = (\bar{m}_1 + \bar{r}^i) \Sigma_9. \] (B.50)

### B.4.2 One-period physical conditional return variance

The physical variance is easily obtained given the loadings:
\[
VAR_t(\bar{r}^i) = \left\{ \begin{array}{l}
\sigma_p(\bar{r}^i)^2 p_t + \sigma_v(\bar{r}^i)^2 n_t + \sigma_p(\bar{r}^i)^2 l p_t + \sigma_q(\bar{r}^i)^2 q_t \\
+ \sigma_v(\bar{r}^i)^2 v_n - \bar{m}_1 S_1 \Sigma^other S_1' \bar{r}^i + \sigma^2 \\
\end{array} \right. 
\] (B.51)

### B.4.3 One-period risk-neutral conditional return variance

To obtain the risk-neutral variance of the asset returns, we use the moment generating function under the risk-neutral measure:
\[
m_{gf}^{1/2}(\bar{r}^i; \nu) = \frac{E_t [\exp (\bar{m}_t + \nu \bar{r}^i)]}{E_t [\exp (\bar{m}_t)]} \\
= \exp \left\{ E_t (\bar{m}_t + \nu \bar{r}^i) \right\} \\
\cdot \exp \left\{ -\sigma_p(\bar{m}_1 + \nu \bar{r}^i) - \ln \left( 1 - \sigma_p(\bar{m}_1 + \nu \bar{r}^i) \right) p_t \right\} \\
\cdot \exp \left\{ -\sigma_v(\bar{m}_1 + \nu \bar{r}^i) - \ln \left( 1 - \sigma_v(\bar{m}_1 + \nu \bar{r}^i) \right) n_t \right\} \\
\cdot \exp \left\{ -\sigma_p(\bar{m}_1 + \nu \bar{r}^i) - \ln \left( 1 - \sigma_p(\bar{m}_1 + \nu \bar{r}^i) \right) l p_t \right\} \\
\cdot \exp \left\{ -\sigma_v(\bar{m}_1 + \nu \bar{r}^i) - \ln \left( 1 - \sigma_v(\bar{m}_1 + \nu \bar{r}^i) \right) v_n \right\} \\
\]
The first-order moment is the first-order derivative at the no-arbitrage condition. The second-order moment is derived,

\[
\begin{align*}
\text{VAR}^t_r & = \left( \mu_r^t \right)^2 - \left( E^Q_r \right)^2 \\
& = \frac{\partial^2 m \mathcal{G}^2_r (\tilde{r}^t_{t+1}; \nu) \big|_{\nu=0}}{\partial \nu^2} - \left( \frac{\partial m \mathcal{G}^2_r (\tilde{r}^t_{t+1}; \nu) \big|_{\nu=0}}{\partial \nu} \right)^2 \\
& = \left( \frac{\sigma_p^2}{1 - \sigma_p(m_1)} \right) p_t + \left( \frac{\sigma_n^2}{1 - \sigma_n(m_1)} \right) n_t + \left( \frac{\sigma_{ip}^2}{1 - \sigma_{ip}(m_1)} \right) l_p + \left( \frac{\sigma_q^2}{1 - \sigma_q(m_1)} \right) q_t
\end{align*}
\]

\[B.52\]

where

\[
A(\nu) = \exp \left\{ -\sigma_{vn}(m_1 + \nu \tilde{r}^i) - \ln \left( \frac{1 - \sigma_{vn}(m_1 + \nu \tilde{r}^i)}{1 - \sigma_{vn}(m_1)} \right) \right\} v_n + \frac{1}{2} \left[ (m_1 + \nu \tilde{r}^i) S_1 \Sigma_{other} S_1^t (m_1 + \nu \tilde{r}^i) + \nu^2 \sigma^2_r \right]
\]

\[B.53\]

and

\[
\begin{align*}
\sigma_p(m_1 + \nu \tilde{r}^i) &= (m_1 + \nu \tilde{r}^i) \Sigma_{a1}^t. \\
\sigma_n(m_1 + \nu \tilde{r}^i) &= (m_1 + \nu \tilde{r}^i) \Sigma_{a2}^t. \\
\sigma_{ip}(m_1 + \nu \tilde{r}^i) &= (m_1 + \nu \tilde{r}^i) \Sigma_{a4}^t. \\
\sigma_{vn}(m_1 + \nu \tilde{r}^i) &= (m_1 + \nu \tilde{r}^i) \Sigma_{a5}^t. \\
\sigma_q(m_1 + \nu \tilde{r}^i) &= (m_1 + \nu \tilde{r}^i) \Sigma_{a9}^t.
\end{align*}
\]

\[B.54-B.58\]

The first-order moment is the first-order derivative at \( \nu = 0 \):

\[
E^Q_t \left( \tilde{r}^t_{t+1} \right) = \frac{\partial m \mathcal{G}^Q_t (\tilde{r}^t_{t+1}; \nu)}{\partial \nu} \big|_{\nu=0}
\]

\[
= E_t \left( \tilde{r}^t_{t+1} \right) + \sigma_p(m_1) \sigma_p(m_1) \sigma_n(m_1) \sigma_n(m_1) \sigma_{ip}(m_1) \sigma_{ip}(m_1) l_p + \sigma_q(m_1) \sigma_q(m_1) l_q + \sigma_{ip}(m_1) \sigma_{ip}(m_1) l_p + \sigma_q(m_1) \sigma_q(m_1) l_q
\]

\[B.59\]

Note the similarity between \( E_t \left( \tilde{r}^t_{t+1} \right) - E^Q_t \left( \tilde{r}^t_{t+1} \right) \) from this equation and the equity premium derived before using the no-arbitrage condition. The second-order moment is derived,

\[
VAR^Q_t \left( \tilde{r}^t_{t+1} \right) = E^Q_t \left( \tilde{r}^t_{t+1} \right)^2 - \left( E^Q_t \left( \tilde{r}^t_{t+1} \right) \right)^2
\]

\[
= \frac{\partial^2 m \mathcal{G}^Q_t (\tilde{r}^t_{t+1}; \nu) \big|_{\nu=0}}{\partial \nu^2} - \left( \frac{\partial m \mathcal{G}^Q_t (\tilde{r}^t_{t+1}; \nu) \big|_{\nu=0}}{\partial \nu} \right)^2
\]

\[
= \left( \frac{\sigma_p^2}{1 - \sigma_p(m_1)} \right) p_t + \left( \frac{\sigma_n^2}{1 - \sigma_n(m_1)} \right) n_t + \left( \frac{\sigma_{ip}^2}{1 - \sigma_{ip}(m_1)} \right) l_p + \left( \frac{\sigma_q^2}{1 - \sigma_q(m_1)} \right) q_t
\]
\[
+ \left( \frac{\sigma(v\tilde{r}^i)}{1 - \sigma(v\tilde{m}_1)} \right)^2 v_n + \tilde{r}^i S_1 \Sigma \text{other } S_1 \tilde{r}^i + \sigma_i^2.
\]

(B.60)

### C Variables and parameters

#### Table C.1: Variables. (In order of first appearance)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_t)</td>
<td>consumption level</td>
</tr>
<tr>
<td>(Q_t)</td>
<td>the relative risk aversion state (RRA) variable</td>
</tr>
<tr>
<td>(m_t)</td>
<td>log real pricing kernel</td>
</tr>
<tr>
<td>(c_t)</td>
<td>(\ln(C_t))</td>
</tr>
<tr>
<td>(q_t)</td>
<td>(\ln(Q_t))</td>
</tr>
<tr>
<td>(\Delta c_t)</td>
<td>log change in consumption</td>
</tr>
<tr>
<td>(\Delta q_t)</td>
<td>log change in RRA of per period utility of the representative agent</td>
</tr>
<tr>
<td>(H_t)</td>
<td>external habit level (as in Campbell and Cochrane, 1999)</td>
</tr>
<tr>
<td>(\theta_t)</td>
<td>log change in the real industrial production index, or growth</td>
</tr>
<tr>
<td>(p_t)</td>
<td>upside macroeconomic uncertainty state variable, or “good” uncertainty, or shape parameter of the upside macroeconomic shock</td>
</tr>
<tr>
<td>(n_t)</td>
<td>downside macroeconomic uncertainty state variable, or “bad” uncertainty, or shape parameter of the downside macroeconomic shock</td>
</tr>
<tr>
<td>(\omega_{\theta,t})</td>
<td>growth disturbance</td>
</tr>
<tr>
<td>(\omega_{p,t})</td>
<td>upside macroeconomic shock</td>
</tr>
<tr>
<td>(\omega_{n,t})</td>
<td>downside macroeconomic shock</td>
</tr>
<tr>
<td>(Y_{t,\text{mac}})</td>
<td>macroeconomic state variables consisting of ({\theta_t, p_t, n_t})</td>
</tr>
<tr>
<td>(l_t)</td>
<td>log corporate bond loss rate</td>
</tr>
<tr>
<td>(u_{l,t}^g)</td>
<td>loss-rate-specific shock</td>
</tr>
<tr>
<td>(\omega_{l,p,t})</td>
<td>upside loss rate (cash flow) shock</td>
</tr>
<tr>
<td>(\omega_{l,n,t})</td>
<td>downside loss rate (cash flow) shock</td>
</tr>
<tr>
<td>(lp_t)</td>
<td>upside loss rate (cash flow) uncertainty state variable, or shape parameter of the upside loss rate shock</td>
</tr>
<tr>
<td>(Y_{t,\text{fin}})</td>
<td>financial state variables consisting of ({l_t, lp_t})</td>
</tr>
<tr>
<td>(g_t)</td>
<td>change in log earnings</td>
</tr>
<tr>
<td>(u_{g,t}^g)</td>
<td>earnings growth-specific disturbance</td>
</tr>
<tr>
<td>(\omega_{g,t})</td>
<td>standardized earnings growth-specific shock</td>
</tr>
<tr>
<td>(\kappa_t)</td>
<td>log consumption-earnings ratio</td>
</tr>
<tr>
<td>(u_{\kappa,t}^g)</td>
<td>consumption-earnings ratio-specific disturbance</td>
</tr>
<tr>
<td>(\omega_{\kappa,t})</td>
<td>standardized consumption-earnings ratio-specific shock</td>
</tr>
<tr>
<td>(\eta_t)</td>
<td>log dividend payout ratio</td>
</tr>
<tr>
<td>(u_{\eta,t}^d)</td>
<td>dividend payout ratio-specific disturbance</td>
</tr>
<tr>
<td>(\omega_{\eta,t})</td>
<td>standardized dividend payout ratio-specific shock</td>
</tr>
<tr>
<td>(\Delta d_t)</td>
<td>log change in dividend</td>
</tr>
<tr>
<td>(u_{d,t})</td>
<td>risk aversion-specific disturbance</td>
</tr>
<tr>
<td>(\omega_{d,t})</td>
<td>risk aversion shock</td>
</tr>
<tr>
<td>(\pi_t)</td>
<td>inflation</td>
</tr>
<tr>
<td>(u_{\pi,t})</td>
<td>inflation-specific disturbance</td>
</tr>
<tr>
<td>(\omega_{\pi,t})</td>
<td>standardized inflation-specific shock</td>
</tr>
<tr>
<td>(Y_{t,\text{other}})</td>
<td>a vector of non-macro state variables, ([\pi_t, l_t, g_t, \kappa_t, \eta_t, lp_t, qi])</td>
</tr>
<tr>
<td>(Y_t)</td>
<td>a vector of all 10 state variables, ([Y_{t,\text{mac}}, Y_{t,\text{fin}}])</td>
</tr>
<tr>
<td>(\bar{m}_t)</td>
<td>log nominal pricing kernel</td>
</tr>
<tr>
<td>(\bar{r}_t)</td>
<td>nominal risk free rate</td>
</tr>
<tr>
<td>(pc_t)</td>
<td>log price-coupon ratio of one period defaultable bond portfolio</td>
</tr>
<tr>
<td>(pc_t^N)</td>
<td>log price-coupon ratio of N-period defaultable bond portfolio</td>
</tr>
<tr>
<td>(PD_t)</td>
<td>price-dividend ratio</td>
</tr>
<tr>
<td>(r^i)</td>
<td>log nominal asset return for asset (i, i \in {eq, cb})</td>
</tr>
<tr>
<td>(E_t(r^i_{t+1}))</td>
<td>expected return for asset (i)</td>
</tr>
<tr>
<td>(RP_t^i)</td>
<td>model-implied one-month expected excess returns for asset (i)</td>
</tr>
</tbody>
</table>
$$\text{VAR}_i^t \equiv \text{VAR}_i(t_{i+1}^t)$$  \text{model-implied one-month expected physical variances for asset } i
$$\text{VAR}_i^{Q_t} \equiv \text{VAR}_i^Q(t_{i+1}^t)$$  \text{model-implied one-month expected risk-neutral variances for asset } i
$$\text{RVAR}_i^t$$  \text{empirical benchmark of one-month realized physical variances for asset } i
$$\text{QVAR}_i^{eq}$$  \text{empirical benchmark of one-month expected risk-neutral variances for equity}

$E_t$  monthly earnings

$\text{ra}^\text{BEX}_t$  Bekaelt-Engstrom-Xu’s financial proxy to risk aversion

$\text{unc}^\text{BEX}_t$  Bekaelt-Engstrom-Xu’s financial proxy to macroeconomic uncertainty

### Table C.2: Parameters.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>constant utility curvature parameter</td>
</tr>
<tr>
<td>$a, b$</td>
<td>parameters in $Q_t$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>constant discount factor</td>
</tr>
<tr>
<td>$\hat{\theta}$</td>
<td>unconditional mean of growth</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>AR(1) coefficient of growth</td>
</tr>
<tr>
<td>$m_p$</td>
<td>sensitivity of output growth on lagged upside macroeconomic uncertainty</td>
</tr>
<tr>
<td>$m_n$</td>
<td>sensitivity of output growth on lagged downside macroeconomic uncertainty</td>
</tr>
<tr>
<td>$\hat{p}$</td>
<td>unconditional mean of $p_t$</td>
</tr>
<tr>
<td>$\hat{n}$</td>
<td>unconditional mean of $n_t$</td>
</tr>
<tr>
<td>$\sigma_{\omega_p}$</td>
<td>scale parameter associated with the upside macroeconomic shock, $\omega_{p,t}$, in $\theta_t$</td>
</tr>
<tr>
<td>$\sigma_{\omega_n}$</td>
<td>scale parameter associated with the downside macroeconomic shock, $\omega_{n,t}$, in $\theta_t$</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>AR(1) coefficient of $p_t$</td>
</tr>
<tr>
<td>$\rho_n$</td>
<td>AR(1) coefficient of $n_t$</td>
</tr>
<tr>
<td>$\sigma_{\omega_p}$</td>
<td>sensitivity of $p_t$ on $\omega_{p,t}$</td>
</tr>
<tr>
<td>$\sigma_{\omega_n}$</td>
<td>sensitivity of $n_t$ on $\omega_{n,t}$</td>
</tr>
<tr>
<td>$l_0$</td>
<td>constant in the dynamic process of $l_t$</td>
</tr>
<tr>
<td>$\rho_l$</td>
<td>AR(1) coefficient of $l_t$</td>
</tr>
<tr>
<td>$m_{lp}$</td>
<td>sensitivity of loss rate on lagged upside macroeconomic uncertainty</td>
</tr>
<tr>
<td>$m_{ln}$</td>
<td>sensitivity of loss rate on lagged downside macroeconomic uncertainty</td>
</tr>
<tr>
<td>$\sigma_{lp}$</td>
<td>sensitivity of loss rate on the upside macroeconomic shock $\omega_{p,t}$ in $l_t$</td>
</tr>
<tr>
<td>$\sigma_{ln}$</td>
<td>sensitivity of loss rate on the downside macroeconomic shock $\omega_{n,t}$ in $l_t$</td>
</tr>
<tr>
<td>$\hat{l}$</td>
<td>unconditional mean of $l_p$</td>
</tr>
<tr>
<td>$l_n$</td>
<td>constant shape parameter of $\omega_{ln,t}$</td>
</tr>
<tr>
<td>$\sigma_{lp}$</td>
<td>sensitivity of $l_p$ on $\omega_{lp,t}$</td>
</tr>
<tr>
<td>$\rho_{lp}$</td>
<td>AR(1) coefficient of $lp$</td>
</tr>
<tr>
<td>$j_0$</td>
<td>constant in the dynamic process of variable $j_t$</td>
</tr>
<tr>
<td>$\rho_{j_*}$</td>
<td>AR(1) coefficient of variable $j_*$</td>
</tr>
<tr>
<td>$\rho_{j,\text{mac}}$</td>
<td>sensitivities of variable $j_t$ on lagged macroeconomic state variables $Y_{t}^{\text{mac}}$</td>
</tr>
<tr>
<td>$\rho_{j,\text{fin}}$</td>
<td>sensitivities of variable $j_t$ on lagged financial state variables $Y_{t}^{\text{fin}}$</td>
</tr>
<tr>
<td>$\sigma_{jp}$</td>
<td>sensitivity of variable $j_t$ on the upside macroeconomic shock $\omega_{p,t}$</td>
</tr>
<tr>
<td>$\sigma_{jn}$</td>
<td>sensitivity of variable $j_t$ on the downside macroeconomic shock $\omega_{n,t}$</td>
</tr>
<tr>
<td>$\sigma_{lp}$</td>
<td>sensitivity of variable $j_t$ on the upside loss rate shock $\omega_{lp,t}$</td>
</tr>
<tr>
<td>$\sigma_{ln}$</td>
<td>sensitivity of variable $j_t$ on the downside loss rate shock $\omega_{ln,t}$</td>
</tr>
<tr>
<td>$\sigma_{j*}$</td>
<td>standard deviation of the variable $j_t$ residual</td>
</tr>
<tr>
<td>$q_0$</td>
<td>constant in the dynamic process of risk aversion $q_t$</td>
</tr>
<tr>
<td>$\rho_{qq}$</td>
<td>AR(1) coefficient of $q_t$</td>
</tr>
<tr>
<td>$\sigma_{qq}$</td>
<td>scale parameter associated with the risk aversion shock $\omega_{q,t}$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>constant vector in the state variable system (10 × 1)</td>
</tr>
<tr>
<td>$A$</td>
<td>autocorrelation vector in the state variable system (10 × 10)</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>scale / volatility parameter matrix of the 9 shocks (10 × 9)</td>
</tr>
<tr>
<td>$m_0$</td>
<td>constant in the real pricing kernel process</td>
</tr>
<tr>
<td>$m_1$</td>
<td>sensitivity vector of real pricing kernel to state variable shocks</td>
</tr>
</tbody>
</table>
**m**₂ sensitivity vector of real pricing kernel to state variable levels

*mn* constant in the nominal pricing kernel process

*ml* sensitivity vector of nominal pricing kernel to state variable shocks

*ml₂* sensitivity vector of nominal pricing kernel to state variable levels

*b*₁ constant in the log price-coupon ratio of one period defaultable bond portfolio

*b*₁ sensitivity vector of the log price-coupon ratio on state variable levels

*b*N* constant in the log price-coupon ratio of N-period defaultable bond portfolio

*b*N₁* sensitivity vector of the log price-coupon ratio on state variable levels

*ξ*₀ constant in the log return generating process of asset *i*

*ξ*₁ sensitivity vector of log asset return *i* on lagged state variable levels

*σᵢ* unconditional volatility of idiosyncratic return residuals

*χ* loadings on the instruments

*χ*ₜₜ macroeconomic uncertainty loadings on the instruments

*Θ* a vector of unknown parameters in the GMM system

*ˆX* estimates of *X* where *X* can be a parameter or a variable

* for all *j* ∈ {g, κ, η, π}:

** for all *j* ∈ {g, κ, η}:

### D Supplementary Tables and Figures

#### Table D.1: Summary Statistics of Financial Instruments Spanning Risk Aversion

This table presents summary statistics of the 6 financial instruments that are used to span our risk aversion measure: “tsprd” is the difference between 10-year treasury yield and 3-month Treasury yield; “csprd” is the difference between Moody’s Baa yield and the 10-year zero-coupon Treasury yield; “EY5yr” (“DY5yr”) is the detrended earnings (dividend) yield where the moving average takes the 5 year average of monthly earnings yield, starting one year before; “rvareq” and “rvarcb” are realized variances of log equity returns and log corporate bond returns, calculated from daily returns; “qvareq” is the risk-neutral conditional variance of log equity returns; for the early years (before 1990), we use VXO and authors’ calculations. Bold (italic) coefficients have <5% (10%) p-values. Block bootstrapped errors are shown in parentheses. The sample period is from 1986/06 to 2015/02 (345 months).

<table>
<thead>
<tr>
<th></th>
<th>tsprd</th>
<th>csprd</th>
<th>DY5yr</th>
<th>EY5yr</th>
<th>rvareq</th>
<th>qvareq</th>
<th>rvarcb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation Matrix</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tsprd</td>
<td>1</td>
<td>0.3524</td>
<td>0.2595</td>
<td>0.2526</td>
<td>0.1269</td>
<td>0.1244</td>
<td>0.2952</td>
</tr>
<tr>
<td>csprd</td>
<td>0.4990</td>
<td>1</td>
<td>0.5083</td>
<td>0.4786</td>
<td>0.5988</td>
<td>0.5330</td>
<td></td>
</tr>
<tr>
<td>DY5yr</td>
<td>1.0000</td>
<td>0.8966</td>
<td>1</td>
<td>0.1678</td>
<td>0.1650</td>
<td>0.3101</td>
<td></td>
</tr>
<tr>
<td>EY5yr</td>
<td></td>
<td>0.1399</td>
<td>0.1564</td>
<td>0.3359</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>rvareq</td>
<td></td>
<td></td>
<td>1</td>
<td>0.1399</td>
<td>0.1564</td>
<td>0.3359</td>
<td></td>
</tr>
<tr>
<td>qvareq</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>0.8431</td>
<td>0.5943</td>
<td></td>
</tr>
<tr>
<td>rvarcb</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.5376</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Summary Statistics

<p>| | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0179</td>
<td>0.0231</td>
<td>-0.0030</td>
<td>-0.0074</td>
<td>0.0029</td>
<td>0.0040</td>
<td>0.0002</td>
</tr>
<tr>
<td>Boot.SE</td>
<td>(0.0006)</td>
<td>(0.0004)</td>
<td>(0.0003)</td>
<td>(0.0008)</td>
<td>(0.0003)</td>
<td>(0.0002)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>S.D.</td>
<td>0.0116</td>
<td>0.0075</td>
<td>0.0061</td>
<td>0.0049</td>
<td>0.0059</td>
<td>0.0037</td>
<td>0.0003</td>
</tr>
<tr>
<td>Boot.SE</td>
<td>(0.0003)</td>
<td>(0.0005)</td>
<td>(0.0003)</td>
<td>(0.0007)</td>
<td>(0.0014)</td>
<td>(0.0005)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.2322</td>
<td>1.7891</td>
<td>0.0959</td>
<td>-0.3495</td>
<td>8.1198</td>
<td>3.7225</td>
<td>4.2227</td>
</tr>
<tr>
<td>Boot.SE</td>
<td>(0.0810)</td>
<td>(0.2515)</td>
<td>(0.1882)</td>
<td>(0.1502)</td>
<td>(1.5951)</td>
<td>(0.5123)</td>
<td>(0.6872)</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.9668</td>
<td>0.9640</td>
<td>0.9822</td>
<td>0.9843</td>
<td>0.4312</td>
<td>0.7462</td>
<td>0.5775</td>
</tr>
<tr>
<td>SE</td>
<td>(0.0137)</td>
<td>(0.0143)</td>
<td>(0.0083)</td>
<td>(0.0068)</td>
<td>(0.0488)</td>
<td>(0.0360)</td>
<td>(0.0441)</td>
</tr>
</tbody>
</table>
Table D.2: Projecting Pure Cash Flow Uncertainty using OLS

This table presents regression results of the estimated monthly pure cash flow uncertainty (from loss rate) on a set of monthly asset prices; some are used to span the time-varying risk aversion. The dependent variable is \( lp_t \), the time-varying shape parameter of the pure right-tail loss rate residual (after controlling for macroeconomic shocks) as demonstrated in Table 2. \( \times 10^{-3} \) in the header means that the coefficients and their SEs reported are divided by 1000 for reporting convenience. “VARC” reports the variance decomposition. Bold (italic) coefficients have <5% (10%) p-values. Robust and efficient standard errors are shown in parentheses. Adjusted \( R^2 \)s are reported. The sample period is 1986/06 to 2015/02 (345 months).

<table>
<thead>
<tr>
<th>( lp_t ) (( \times 10^{-3} ))</th>
<th>VARC</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.001</td>
</tr>
<tr>
<td>( \chi_{\text{tsprd}} )</td>
<td>-0.058</td>
</tr>
<tr>
<td>( \chi_{\text{csprd}} )</td>
<td>0.202</td>
</tr>
<tr>
<td>( \chi_{\text{DY5yr}} )</td>
<td>0.234</td>
</tr>
<tr>
<td>( \chi_{\text{EY5yr}} )</td>
<td>-0.061</td>
</tr>
<tr>
<td>( \chi_{\text{rvareq}} )</td>
<td>-0.026</td>
</tr>
<tr>
<td>( \chi_{\text{qvareq}} )</td>
<td>0.119</td>
</tr>
<tr>
<td>( \chi_{\text{rvarcb}} )</td>
<td>1.779</td>
</tr>
<tr>
<td>( \chi_{\text{rvarcbSPEC}} )</td>
<td>-0.223</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>9.11%</td>
</tr>
</tbody>
</table>
Figure D.1: Model-implied conditional moments of industrial production growth. The shaded regions are NBER recession months from the NBER website.
Figure D.2: Risk aversion/sentiment measures: A comparison.
Figure D.3: Uncertainty measures: A comparison.